Supersymmetry and its breaking

IDPASC, 10^{th} February 2015

Mark D. Goodsell

LPTHE

Introduction

- Introduction to supersymmetry
- Supersymmetry breaking
- Mediation
- The MSSM
- SUSY dark matter
- The case for SUSY
- Beyond minimal models

References

SUSY has been studied for $\sim$40 years now; there exists a huge literature and many excellent reviews/lectures/books, e.g.:

- Martin, *A supersymmetry primer*, hep-ph/9709356 (the classic reference)
- Murayama, *Supersymmetry phenomenology*, hep-ph/0002232
- Bailin and Love, *Supersymmetric gauge field theory and string theory* (the first part)

More formal:

- The book of Wess and Bagger
- Rather complete, also contains some phenomenological aspects: lectures by Bertolini, `http://people.sissa.it/ bertmat/teaching.htm`
- *Introduction to Supersymmetry* by Bilal, hep-th/0101055
- *2001 Busstepp lectures* by Figueroa-O'farrill, hep-th/0109072
- *1001 lessons in superspace*, hep-th/0108200

Symmetries

Symmetries play a central role in our understanding of nature.
The quest to understand the laws of nature has often been a game of proposing laws and testing them, e.g.:

- "Obvious" ones such as translations, rotations $\to$ relativity added boosts, to have Poincaré algebra
- Parity $\to$ violated by neutrinos
- CP $\to$ violated in weak interactions
- Lepton and baryon number $\to$ only $B-L$ is preserved nonperturbatively, and then only if neutrinos are Dirac $\to$ approximate symmetry
- Gauge symmetries $\to$ although these are "fake" because they admit dual descriptions (c.f. QCD) $\to$ more a redundancy or convenient description.

Discrete symmetries may be preserved (e.g. CPT) but continuous ones must only be approximate, yet they are still important.

Approximate symmetries

- If a theory is invariant under some symmetry, then quantum corrections cannot generate terms in the effective action which violate that symmetry.
- NB even if the symmetry is anomalous it will only be violated *nonperturbatively*.
- E.g. if we have a theory with a massless fermion, it cannot obtain a mass due to the chiral symmetry:

$$\mathcal{L} \supset \overline{\Psi} i \not{D} \Psi \underbrace{\longrightarrow}_{\Psi \to e^{i\alpha\gamma_5}\Psi} \overline{\Psi} e^{i\alpha\gamma_5} i \not{D} e^{i\alpha\gamma_5} \Psi$$
$$= \overline{\Psi} i \not{D} \Psi$$

- If we add a term $\delta\mathcal{L}$ to the Lagrangian that violates this symmetry, then it will induce other terms in the effective action that violate the symmetry
- But they must all be proportional to $\delta\mathcal{L}$, since when we set it to zero the symmetry is restored.
- Hence e.g. chiral symmetry protects fermion masses from large renormalisation, e.g. the electron mass

$$m_e = m_e^{\mathrm{bare}} \left[1 + \frac{3\alpha}{4\pi} \log \frac{m_e}{\Lambda} + \ldots \right]$$

- Approximate symmetries are therefore very important.

What is SUSY?

- The initial study of SUSY can be regarded as an attempt to find a new fundamental symmetry of spacetime.
- Sometimes this is presented as a search for a symmetry between fermions and bosons, to unify their description.
- Otherwise, the Coleman-Mandula theorem told us that, for theories with a mass gap (i.e. some discrete set of masses) and non-trivial interactions, we could only extend the Poincaré group with other Lie groups in a trivial way.
- The Haag, Sohnius, Łopuszanski extension showed an exception is allowed for anticommuting generators, and that the only exception was supersymmetry.

The obligatory supersymmetry algebra:

$$\begin{aligned}
\{Q_\alpha, \overline{Q}_{\dot\alpha}\} =& 2\sigma^\mu_{\alpha\dot\alpha} P_\mu \\
\{Q_\alpha, Q_\beta\} =& \{\overline{Q}_{\dot\alpha}, \overline{Q}_{\dot\beta}\} = 0 \\
[Q_\alpha, P^\mu] =& [\overline{Q}_{\dot\alpha}, P^\mu] = 0. \\
[M_{\mu\nu}, Q_\alpha] =& i(\sigma_{\mu\nu})_\alpha^{\ \beta} Q_\beta, \quad [M_{\mu\nu}, \overline{Q}_{\dot\alpha}] = i(\overline{\sigma}_{\mu\nu})^{\dot\alpha}_{\ \dot\beta} \overline{Q}^{\dot\beta}
\end{aligned}$$

I.e. the charges $Q_\alpha, \overline{Q}_{\dot\alpha}$ are fermionic. N.B. this is for $N = 1$ supersymmetry; we could add more supercharges and central charges ...

n.b. I will talk only about *global* SUSY, i.e. 4d field theory models decoupled from gravity. If we make the SUSY transformations local, then we automatically include diffeomorphisms and are led to *supergravity* $\rightarrow$ see Karim Benakli's lectures.

Two-component spinors

In SUSY theories in four dimensions, two-component spinor notation is most convenient because we find Weyl spinors in one-to-one correspondence with complex scalars or real vectors:

$$\begin{aligned} \mathrm{LH}: \ \psi_\alpha, \qquad & \alpha = 1,2 \\ \mathrm{RH}: \ \overline{\psi}_{\dot\alpha}, \qquad & \dot\alpha = 1,2 \\ 4 \text{ component spinor}: \ \Psi = & \begin{pmatrix} \psi_\alpha \\ \overline{\chi}^{\dot\alpha} \end{pmatrix} \end{aligned}$$

and

$$\begin{aligned} \gamma_\mu = & \begin{pmatrix} 0 & \sigma_\mu \\ \overline{\sigma}_\mu & 0 \end{pmatrix} \\ \psi\chi \equiv & \psi^\alpha \chi_\alpha = \chi\psi \\ \psi^\alpha = & \epsilon^{\alpha\beta}\psi_\beta \end{aligned}$$

So we write the Lagrangian for a Dirac fermion as

$$\mathcal{L} = i\overline{\psi}\overline{\sigma}^\mu \partial_\mu \psi + i\overline{\chi\sigma}^\mu \partial_\mu \chi - m(\chi\psi + \overline{\chi}\overline{\psi})$$

Supersymmetric theories

- Having determined the algebra, we can then search for theories that obey it.
- Clearly they must consist of bosons and fermions. In fact, there must be equal numbers of equal masses: consider $\mathrm{tr}(-1)^{2S}$, which counts the difference:

$$\begin{aligned}
2\sigma^{\mu}_{\alpha\dot{\alpha}} p_\mu \mathrm{tr}(-1)^{2S} =& 2\sigma^{\mu}_{\alpha\dot{\alpha}} \sum_i \langle i|(-1)^{2S} P_\mu |i\rangle \\
=& \sum_i \langle i|(-1)^{2S} (Q_\alpha \overline{Q}_{\dot{\alpha}} + \overline{Q}_{\dot{\alpha}} Q_\alpha)|i\rangle \\
=& \sum_i \langle i|(-1)^{2S} Q_\alpha \overline{Q}_{\dot{\alpha}}|i\rangle + \sum_{i,j} \langle i|(-1)^{2S} \overline{Q}_{\dot{\alpha}}|j\rangle \langle j|Q_\alpha|i\rangle \\
=& \sum_i \langle i|(-1)^{2S} Q_\alpha \overline{Q}_{\dot{\alpha}}|i\rangle + \sum_{j} \langle j|Q_\alpha (-1)^{2S} \overline{Q}_{\dot{\alpha}}|j\rangle \\
=& 0
\end{aligned}$$

- We must then search for theories with fermions and bosons of equal mass where we can find representations of the SUSY algebra relating them; the first such model was the Wess-Zumino model, the free version being

$$\mathcal{L} = |\partial_\mu \phi|^2 - m^2 |\phi|^2 + i\overline{\psi}\overline{\sigma}^\mu \partial_\mu \psi - \frac{1}{2} m(\psi\psi + \overline{\psi\psi})$$

Superfields

- A particularly convenient way to organise the collections of bosons and fermions is to put them in a *superfield*.
- This clearly requires including fermionic coordinates $\theta_\alpha, \overline{\theta}_{\dot{\alpha}}$ so that we have overall a bosonic or fermionic object.
- These then become the partners of the spacetime coordinates x_μ; we can derive the representations of the supercharges as therefore

$$
\begin{aligned}
P_\mu =& i\partial_\mu \\
Q_\alpha =& \frac{\partial}{\partial\theta^\alpha} - i\sigma^\mu_{\alpha\dot{\alpha}}\overline{\theta}^{\dot{\alpha}}\partial_\mu, \qquad \overline{Q}_{\dot{\alpha}} = -\frac{\partial}{\partial\overline{\theta}_{\dot{\alpha}}} + i\theta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu
\end{aligned}
$$

- By demanding that the superfields are invariant under variations generated by $Q_\alpha, \overline{Q}_{\dot{\alpha}}$ we can therefore construct supersymmetric theories by writing Lagrangians with them.
- One approach is to expand a general bosonic function as a series in $\theta, \overline{\theta}$ since $\theta^3 = \overline{\theta}^3 = 0$:

$$
\begin{aligned}
F(x,\theta,\overline{\theta}) =& f + \theta\psi + \overline{\theta\psi} + \theta^2 m + \overline{\theta}^2 n \\
&+ \theta\sigma^\mu\overline{\theta}V_\mu + \theta\theta\overline{\theta\lambda} + \overline{\theta\theta}\theta\chi + (\theta\theta)(\overline{\theta\theta})D
\end{aligned} \tag{1}
$$

- However this contains too many degrees of freedom to be interesting.

Chiral and vector superfields

- Another approach is to define superspace derivatives

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\sigma^\mu_{\alpha\dot\alpha}\overline{\theta}^{\dot\alpha}\partial_\mu, \qquad \overline{D}_{\dot\alpha} = -\frac{\partial}{\partial\overline{\theta}_{\dot\alpha}} - i\theta^\alpha\sigma^\mu_{\alpha\dot\alpha}\partial_\mu$$

- These commute with the supercharges, so we can use them to define constrained superfields with fewer degrees of freedom.
- E.g. *chiral superfields*, satisfying $\overline{D}\Phi = 0$ (and their antichiral version $D\overline{\Phi} = 0$). These are expanded as

$$\Phi = \phi + \theta\psi + \theta^2 F + i\sqrt{2}\theta\sigma^\mu\overline{\theta}\partial_\mu\phi - \frac{i}{\sqrt{2}}\overline{\theta\theta}\theta\psi - \frac{1}{4}(\theta\theta)(\overline{\theta\theta})\partial_\mu\partial^\mu\phi \qquad (2)$$

- They contain a complex boson, a Weyl fermion and an auxiliary field F.
- Otherwise we have *real* superfields

$$V = \theta\sigma^\mu\overline{\theta}A_\mu + \theta\theta\overline{\theta\lambda} + \overline{\theta\theta}\theta\lambda + \frac{1}{2}(\theta\theta)(\overline{\theta\theta})D$$

- These contain a vector, a Weyl fermion and an auxiliary D, so describes a gauge boson and its gaugino, after eliminating spurious degrees of freedom via the supergauge transformations

$$\begin{aligned} e^{2gV} &\to e^{-2ig\overline{\Lambda}}e^{2gV}e^{2ig\overline{\Lambda}} \\ \to V &\to V + i\Lambda - i\overline{\Lambda} \qquad \text{(abelian)} \end{aligned} \qquad (3)$$

Interactions

With these two types of fields, we can now write down interactions

- In order to write actions from a Lagrangian density, we must integrate over the super-coordinates and not just spacetime.
- If we integrate sets of fields $\Phi, \overline{\Phi}$ over $\int d^2\theta d^2\overline{\theta}$ we will only have derivative interactions $\rightarrow$ ok for kinetic terms and gauge interactions, we write

$$\mathcal{L} \supset \int d^2\theta d^2\overline{\theta}\mathrm{tr}(\overline{\Phi}e^{2gV}\Phi)$$
$$\rightarrow |D_\mu\phi|^2 + i\overline{\psi}\overline{\sigma}^\mu D_\mu\psi + \overline{F}F + gD^a\overline{\Phi}T^a\Phi - \sqrt{2}g(\overline{\phi}T^a\lambda^a\psi + h.c.) + \ldots \quad (4)$$

- However, the chiral (matter) superfields contain ϕ and ψ but not their complex conjugates; they are holomorphic fields.
- We can therefore describe their interactions via a superpotential W integrated over only half of the superspace:

$$\mathcal{L} \supset \int d^2\theta W\Big|_{\overline{\theta}=0} + \int d^2\overline{\theta}\overline{W}\Big|_{\theta=0}$$

- To be invariant under changes via $Q, \overline{Q}$, we find W must be a holomorphic function of fields Φ.
- Note that since θ has a mass dimension of $1/2$, W has mass dimension 3 and is at most cubic in Φ to be renormalisable.

$$W = t\Phi + \frac{1}{2}M\Phi^2 + \frac{y}{3}\Phi^3$$

- What does this lead to for interactions?

F-term scalar potential

- If we integrate W and $\overline{W}$ over superspace we find

$$\mathcal{L} \supset \overline{F}F + \frac{\partial W(\phi)}{\partial \phi} F + \frac{\partial \overline{W}(\overline{\phi})}{\partial \overline{\phi}} \overline{F} - \frac{1}{2} \frac{\partial^2 W}{\partial \phi^2} \psi\psi - \frac{1}{2} \frac{\partial^2 \overline{W}}{\partial \overline{\phi}^2} \overline{\psi\psi} \tag{5}$$

- The auxiliary field F has no kinetic term so it can be integrated out

$$V_F = \left| \frac{\partial W}{\partial \phi} \right|^2$$

Hence for the superpotential $W = \frac{1}{2} M \Phi^2 + \frac{y}{3} \Phi^3$ we will have:

- Mass terms $\mathcal{L} \supset -\frac{1}{2} M \psi\psi - |M|^2 |\phi|^2$
- Yukawa couplings $\mathcal{L} \supset -y\phi\psi\psi$
- Cubic couplings $\mathcal{L} \supset -M\overline{y}\phi\overline{\phi}^2 + h.c.$
- Quartic couplings $\mathcal{L} \supset -|y|^2 |\phi|^4 \to$ supersymmetry automatically relates them!

D-term scalar potential

- The kinetic term for the gauge field requires adding some extra derivatives; we could write it in the form $\sim \int d^4\theta V \partial^2 V$ but it is possible to write it in terms of a fermionic chiral superfield strength W_α:

$$
\begin{aligned}
W_\alpha \equiv & -\frac{1}{8}\overline{D}^2 e^{-2gV} D_\alpha e^{2gV} \\
\underset{\text{abelian}}{\rightarrow} & -\frac{1}{4}\overline{D}^2 D_\alpha V = \lambda_\alpha + \theta_\alpha D + \frac{i}{2}(\sigma^\mu \overline{\sigma}^\nu)_\alpha^\beta \theta_\beta F_{\mu\nu} + \ldots
\end{aligned}
$$

- This may look cumbersome, but it can then be written as a holomorphic integral

$$
\begin{aligned}
\mathcal{L} \supset & \int d^2\theta \frac{1}{4} W^\alpha W_\alpha + h.c. \\
\supset & -\frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} + i\overline{\psi}\overline{\sigma}^\mu D_\mu \psi + D^2 + \ldots
\end{aligned}
$$

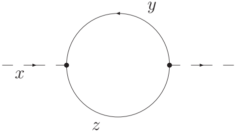
- Again the auxiliary field D has no kinetic term, so integrating out and including the matter terms from $\int d^4\theta \overline{\Phi} e^{2gV} \Phi$ we have

$$
V_D = \frac{1}{2} g^2 (\overline{\Phi} T^a \Phi)^2
$$

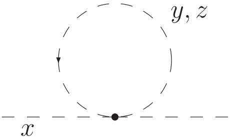
- Hence in supersymmetric models there are quartic couplings given by the gauge couplings!

Quadratic divergences

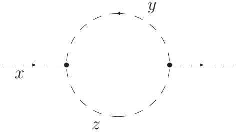
One of the famous properties of SUSY is the lack of quadratic divergences in scalar loops:



$$= -2y^2 \int \frac{d^4q}{(2\pi)^4} \frac{q^2}{(q^2-m_y^2)(q^2-m_z^2)} \sim -\frac{2y^2}{16\pi^2}\Lambda_{UV}^2 + \dots$$



$$= y^2 \int \frac{d^4q}{(2\pi)^4} \frac{1}{(q^2-m_y^2)} + \frac{1}{(q^2-m_z^2)} \sim \frac{2y^2}{16\pi^2}\Lambda_{UV}^2 + \dots$$



$$= y^2 \int \frac{d^4q}{(2\pi)^4} \frac{(m_y^2+m_z^2)}{(q^2-m_y^2)(q^2-m_z^2)} \sim \frac{y^2}{16\pi^2}(m_y^2+m_z^2)\log\Lambda_{UV}^2 + ..$$

This means that if we couple a low-energy supersymmetric theory to a heavier theory, then the low energy paramters depend at most *logarithmically* on the cutoff scale. This behaviour persists to all loops: quadratic divergences cancel between bosons and fermions. This is the origin of the interest in SUSY as a solution to the hierarchy problem.

Other special properties

SUSY field theories exhibit many other beautiful simplifications.
Of most practical application are the non-renormalisation theorems:

- The superpotential only exhibits wavefunction renormalisation (no vertex corrections) due to its holomorphy
- The gauge couplings have a holomorphic correction only at one loop, and can be given *exactly* in terms of matter field anomalous dimensions to all loops (NSVZ formula).

Some more formal properties:

- If we impose additional symmetries, we can restrict the form of quantum corrections to non-perturbative processes $\rightarrow$ can determine exact formulae for instanton contributions to the superpotential.
- Seiberg duality relates supersymmetric gauge theories <u>with different gauge groups</u> in the infra red (other related dualities exist).
- Superconformal fixed points have a very rich structure (a-theorem etc)
- There is currently much work on applying localisation techniques to understand RG flows between such theories.

R-symmetry

An important point:

- The SUSY algebra itself possesses a symmetry: we can rotate the supercharges by $Q_\alpha \to e^{i\alpha} Q_\alpha, \overline{Q}_{\dot{\alpha}} \to e^{-i\alpha} \overline{Q}_{\dot{\alpha}}$.
- This is a global $U(1)$ symmetry known as R-symmetry (we add it to the algebra with generators R and $[R, Q_\alpha] = -Q_\alpha, [R, \overline{Q}_{\dot{\alpha}}] = \overline{Q}_{\dot{\alpha}}$).
- Since $Q \sim \frac{\partial}{\partial \theta^\alpha}$ so $\theta^\alpha \to e^{-i\alpha} \theta^\alpha$
- Also, since the gauge kinetic field strength is $\int d^2\theta \frac{1}{4} W^\alpha W_\alpha$, we need $W_\alpha \to e^{-i\alpha} W_\alpha$ and $W_\alpha = \lambda_\alpha + \ldots$ the gauginos transform (in fact, this is the only global symmetry under which they transform, and we could have used this to define R-symmetry).
- Since the superpotential is the θ^2 component, we need $W \to e^{-2i\alpha} W$ too.
- Therefore not all theories respect an R-symmetry, e.g. $W = \frac{1}{2} M \Phi^2 + \frac{1}{3} y \Phi^3$.

R-symmetry is intimately related to SUSY-breaking, e.g.

- A Majorana mass for the gauginos $\mathcal{L} \supset -\frac{1}{2} M \lambda\lambda$ breaks R-symmetry, while a Dirac mass $\mathcal{L} \supset -m_D \chi\lambda$ does not.
- If we spontaneously break R-symmetry then we expect an R-axion.
- On the other hand, the gravitino mass is actually a measure of R-symmetry breaking $N = 1$ SUGRA with broken SUSY must break R.
- The Nelson-Seiberg “theorem”.

Supersymmetry breaking

- We know that supersymmetry is broken in nature!
- If we believe that it is connected to the hierarchy problem, then the superpartners of SM fields should not be too heavy: the effective SM below M_{SUSY} would lead to

$$\Delta m_H^2 \sim \frac{y_t^2}{16\pi^2} M_{SUSY}{}^2$$

- i.e. naively a "natural" scale for M_{SUSY} is less than $\sim 4\pi/y_t$ times electroweak scale $\rightarrow$ i.e. of $\mathcal{O}(\mathrm{TeV})$.
- So how do we give the superpartners a mass?

First, must look at how to break SUSY:

- We must suppose that some physics spontaneously breaks SUSY (so that at high energies it is restored, cancelling the ultraviolet divergences).
- The vacuum is then not invariant:

$$Q_\alpha|0\rangle \neq 0$$

- (provided we can globally define Q_α). Then

$$\begin{aligned}
\langle 0|P^0|0\rangle =& \frac{1}{4}\mathrm{tr}\langle 0|(Q_\alpha \overline{Q}_{\dot{\alpha}} + \overline{Q}_{\dot{\alpha}} Q_\alpha)|0\rangle \\
=& \frac{1}{4}\sum_\alpha ||Q_\alpha|0\rangle||^2 + \frac{1}{4}\sum_{\dot{\alpha}} ||\overline{Q}_{\dot{\alpha}}|0\rangle||^2 \\
>& 0
\end{aligned}$$

- So then the vacuum energy is greater than zero (n.b. global SUSY)
- $\rightarrow \langle F\rangle \neq$ and/or $\langle D\rangle \neq 0$.

LPTHE PARIS

Goldstino

Since we want to break global SUSY spontaneously, and the generators are fermionic, then there should be a *Goldstino* associated.
If we assume that we have a renormalisable theory then we can identify it by analysing the fermion mass matrix. Write $W_i = \frac{\partial W}{\phi_i}, W_{ij} = \frac{\partial^2 W}{\partial \phi_i \partial \phi_j}$ etc

$$m_{(F)\,ij} = \begin{pmatrix} W_{ij} & \sqrt{2} D^a_i \\ \sqrt{2} D^a_i & 0 \end{pmatrix}$$

N.b. condition for stability of potential is that

$$W_{ij} W^j + D^a_i D^a = 0$$

Therefore $(W^j, \frac{1}{\sqrt{2}} D^a)$ is a zero eigenvector $\rightarrow$ it is proportional to the Goldstino!

When we couple SUSY to gravity, the Goldstino is eaten by the gravitino and so has mass $m_{3/2}$. Whether this is the lightest state in the theory is of crucial importance to collider phenomenology!

LPTHE PARIS

Supertrace formula

If we assume that we have a renormalisable theory then by taking the second derivative of the potential we can obtain simple formulae for the masses:

- Write $D^a_i = -g\overline{\phi}_i T^a$, then the mass of the vectors is

$$|D_\mu \phi|^2 \rightarrow g^2 \overline{\phi}_i T^a T^b \phi_i D^b_j A^a_\mu A^{b\,\mu} \rightarrow m^2_{(V)\,ab} = 2 D^a_i D^{a\,i}$$

- For the scalars we have

$$m^2_{(S)\,ij} = \begin{pmatrix} W_{ik}W^{kj} + D^{a\,j}_i D^a + D^a_i D^{a\,j} & W_{ijk}W^k + D^a_i D^a_j \\ W^{ijk}W_k + D^{a\,i} D^{a\,j} & W^{ik}W_{jk} + D^{a\,i}_j D^a + D^a_j D^{a\,i} \end{pmatrix}$$

So if we write the supertrace we have

$$\begin{aligned} \mathrm{STr}(\mathcal{M}^2) \equiv & \sum_s (-1)^{2s}(2s+1)\mathrm{tr}(m^2_s) \\ =& \mathrm{Tr}(m^2_{(S)} - 2m_{(F)}m^\dagger_{(F)} + 3m^2_{(V)}) \\ =& 2W_{ik}W^{ki} - 2 \times W_{ik}W^{ki} + 2(D^{a\,i}_i D^a + D^a_i D^{a\,i}) - 8D^a_i D^{a\,i} + 6D^a_i D^{a\,i} \\ =& 2D^a D^{a\,i}_i \\ =& -2g\langle D^a\rangle \mathrm{Tr}(T^a) \\ =& 0 \qquad \text{unless we have an anomalous } U(1) \end{aligned}$$

Soft SUSY breaking

- If we break SUSY spontaneously, then at high energies SUSY should be restored and ultraviolet divergences still cancel.
- As an illustration, recall at one loop the Coleman-Weinberg potential with a cutoff can be written

$$
\begin{aligned}
-32\pi^2 V &= \int_{1/\Lambda^2}^{\infty} \frac{dt}{t^3} \mathrm{STr}(e^{-t\mathcal{M}^2}) \\
&= -\frac{1}{2}\lambda^4 \mathrm{STr}(1) - \Lambda^2 \mathrm{STr}(\mathcal{M}^2) - \frac{1}{2}\mathrm{STr}\mathcal{M}^4 \left(\log \frac{\mathcal{M}^2}{\Lambda^2} - \frac{3}{2} + \gamma_E \right) \\
&\quad + \mathcal{O}(1/\Lambda^2)
\end{aligned}
$$

- Hence the supertrace formula at tree level can be seen to guarantee the vanishing of quadratic divergences at loop level.

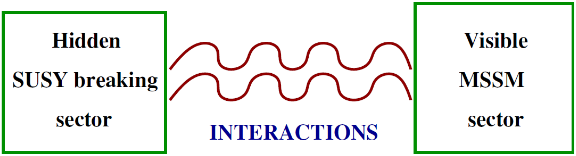
Hidden sectors and SUSY breaking

One of the famous consequences of the supertrace formula is that we cannot break SUSY with only the standard model:

- First, we need to add a goldstino (super)field which must be a Standard Model gauge singlet, but we could suppose we add just the one field.
- Second, we see that the gauginos only obtain *supersymmetric* masses at tree level, and via Higgsing: recal $m_{(F)\,ij} = \begin{pmatrix} W_{ij} & \sqrt{2}D_i^a \\ \sqrt{2}D_i^a & 0 \end{pmatrix}$ where $D_i^a = -g\overline{\phi}_i T^a$.
- Since QCD and QED are unbroken, this means we should have a massless gluino and photino at tree level!
- Then at least some of the the scalar partners of standard model fields would have to be lighter than the standard model fermions $\rightarrow$ which we have not observed!
- Supposing that we could induce masses for these at loop level, they would still be much lighter than the scalars and not invalidate this.
- Hence we conclude that we must add some additional “hidden sector” in which SUSY is broken.

SUSY breaking mediation

We then have the following picture



The main types of mediation mechanism that people consider are usually divided into:

- Gravity mediation $\rightarrow$ see Karim Benakli's talk.
- Anomaly mediation, which is really a part of gravity mediation.
- Gauge mediation:

Exploring the consequences of these was a large undertaking of the community in the build up to the LHC start. A large "inverse problem" was anticipated that was perhaps now premature.

Models of SUSY breaking

There have been many ideas for what to put in the hidden sector, e.g.:

- Strong gauge dynamic effects, e.g. the Intriligator-Seiberg-Shih model.
- The Polonyi model and O'Raiferteagh models are simple renormalisable models that spontaneously break SUSY:

$$W = fX + \frac{1}{2}(\lambda_{ij}X + m_{ij})\Phi_i\Phi_j + \frac{1}{6}\lambda_{ijk}\Phi_i\Phi_j\Phi_k$$

- Polonyi has only $f \neq 0$. Original O'Raiferteagh model has

$$\begin{aligned} W =& fX + m\Phi_1\Phi_2 + \frac{y}{2}X\Phi_2^2 \\ \rightarrow & F_X = -f - \frac{y}{2}\overline{\phi}_2^2, \quad F_1 = -m\overline{\phi}_2, \quad F_2 = -m\phi_1 - y\overline{X\phi}_2 \end{aligned}$$

- Cannot satisfy all of these; minimum for $F_X = -f$
- X is a pseudomodulus gauge singlet.
- Theory possesses an R-symmetry (c.f. Nelson-Seiberg "theorem").
- Subtleties to obtain sufficient gaugino masses in gauge mediation (see Komargodski & Shih 0902.0030).

Soft terms

On the other hand, we can take a more phenomenological approach. All possible terms that can be added to a Lagrangian that do not introduce new quadratic divergences have been categorised.

They must all be dimensionful, to be proportional to the breaking parameter.

- We have the "standard" soft terms:

$$-\mathcal{L}^{\text{Standard}}_{Breaking} = (m^2)^j_i \phi^i \phi_j + (\frac{1}{6} a^{ijk} \phi_i \phi_j \phi_k + \frac{1}{2} b^{ij} \phi_i \phi_j + \frac{1}{2} M_a \lambda_a \lambda_a + h.c.)$$

- And the "non-standard" terms which may be soft:

$$-\mathcal{L}^{\text{Non-standard}}_{Breaking} = t^i \phi_i + \frac{1}{2} r^{jk}_i \phi^i \phi_j \phi_k + m^{ia}_D \chi_i \lambda_a + h.c.$$

- These latter terms are less widely considered. They are guaranteed to be soft only if there are no singlets in the spectrum.
- However, they can come from supersoft terms via the operator

$$\mathcal{L} \supset \sqrt{2} m_D \int d^2\theta \theta^\alpha W^a_\alpha \mathbf{\Sigma}^a \supset -m_D \chi^a \lambda^a - \sqrt{2} g m_D \Sigma^a \phi^i T^a \phi_j$$

- These do not even induce logarithmic divergences, and lead to Dirac masses for the gauginos.

The MSSM

Now we can turn to low-energy phenomenology.

- The SUSY model that has attracted almost all the phenomenological attention is the Minimal Supersymmetric Standard Model (MSSM).
- The idea is to take the fermions of the standard model and promote them to chiral superfields, and promote the gauge bosons into vector superfields.
- The Higgs, being a scalar, must live in a chiral superfield.
- However, in the Standard Model we have Yukawa couplings written in two-component spinor notation:

$$\begin{aligned}\mathcal{L} \supset & -Y_D \overline{q}_L \cdot H \overline{d}_R - Y_U \overline{q}_L \cdot \overline{H} \overline{u}_R - Y_E \overline{l}_L \cdot H \overline{e}_R \\ & - Y_D^\dagger q_L \cdot \overline{H} d_R - Y_U^\dagger q_L \cdot H u_R - Y_E^\dagger l_L \cdot H e_R\end{aligned}$$

- Whether we identify H or $\overline{H}$ as a chiral superfield, some of the Yukawa couplings would violate supersymmetry as they are not (anti)holomorphic!
- Therefore we introduce two Higgs fields H_u, H_d with opposite hypercharge, and the physical Higgs will be a mixture of the neutral components of the two.
- Note that the higgsinos are therefore a vector-like pair under all gauge groups and therefore do not give a net contribution to anomalies!

Fields of the MSSM

Names		Spin 0	Spin 1/2	Spin 1	$SU(3), SU(2), U(1)_Y$
Quarks	$\mathbf{Q}$	$\tilde{Q} = (\tilde{u}_L, \tilde{d}_L)$	(u_L, d_L)		(**3**, **2**, 1/6)
	$\mathbf{u^c}$	$\tilde{u}^c_L$	u^c_L		($\overline{\mathbf{3}}$, **1**, -2/3)
($\times 3$ families)	$\mathbf{d^c}$	$\tilde{d}^c_L$	u^c_L		($\overline{\mathbf{3}}$, **1**, 1/3)
Leptons	$\mathbf{L}$	$(\tilde{\nu}_{eL}, \tilde{e}_L)$	(ν_{eL}, e_L)		(**1**, **2**, -1/2)
($\times 3$ families)	$\mathbf{e^c}$	$\tilde{e}^c_L$	e^c_L		(**1**, **1**, 1)
Higgs	$\mathbf{H_u}$	(H_u^+, H_u^0)	$(\tilde{h}_u^+, \tilde{h}_u^0)$		(**1**, **2**, 1/2)
	$\mathbf{H_d}$	(H_d^0, H_d^-)	$(\tilde{h}_d^0, \tilde{h}_d^-)$		(**1**, **2**, -1/2)
Gluons	$\mathbf{W}_{3\alpha}$		$\lambda_{3\alpha}$ $[\equiv \tilde{g}_\alpha]$	g	(**8**, **1**, 0)
W	$\mathbf{W}_{2\alpha}$		$\lambda_{2\alpha}$ $[\equiv \tilde{W}^\pm, \tilde{w}^0]$	$W^\pm, W^0$	(**1**, **3**, 0)
B	$\mathbf{W}_{1\alpha}$		$\lambda_{1\alpha}$ $[\equiv \tilde{B}]$	B	(**1**, **1**, 0)

Couplings of the MSSM

We can then write the superpotential for the MSSM:

$$W = \mu \mathbf{H_u} \cdot \mathbf{H_d} + Y_U^{ij} \mathbf{Q_i} \cdot \mathbf{H_u} \mathbf{u_j^c} - Y_D^{ij} \mathbf{Q_i} \cdot \mathbf{H_d} \mathbf{d_j^c} - Y_E^{ij} \mathbf{L_i} \cdot \mathbf{H_d} \mathbf{e_j^c}$$

The MSSM is <u>defined</u> to obey an additional discrete symmetry called <u>R-parity</u> (not to be confused with R-symmetry!):

$$R_P = (-1)^{3(B-L)+2S} \tag{6}$$

This performs two jobs:

- It is an economical way of eliminating $B - L$-violating couplings such as

$$W_{\not{R}_P} \supset \mu_i H_u L_i + \lambda_{ijk} L_i L_j E_k + \lambda'_{ijk} L_i Q_j D_k + \lambda'' U_i^c D_j D_k$$

- Since the fermions in the multiplets have a different R-parity, the gauginos and higgsinos have odd R-parity while the Standard Model fields (including the Higgs bosons) have even parity. This means that the lightest SUSY particle (LSP) is stable $\rightarrow$ and can be a dark matter candidate!

The soft-breaking terms

Adding the standard soft-breaking terms to the Lagrangian we have:

$$
\begin{aligned}
\mathcal{L}_{\text{soft}}^{\text{MSSM}} \quad = \quad & -\frac{1}{2}\left(M_3 \widetilde{g}\widetilde{g} + M_2 \widetilde{W}\widetilde{W} + M_1 \widetilde{B}\widetilde{B} + \text{c.c.}\right) \\
& -\left(\widetilde{\overline{u}}\, \mathbf{a_u}\, \widetilde{Q} H_u - \widetilde{\overline{d}}\, \mathbf{a_d}\, \widetilde{Q} H_d - \widetilde{\overline{e}}\, \mathbf{a_e}\, \widetilde{L} H_d + \text{c.c.}\right) \\
& -\widetilde{Q}^\dagger\, \mathbf{m_Q^2}\, \widetilde{Q} - \widetilde{L}^\dagger\, \mathbf{m_L^2}\, \widetilde{L} - \widetilde{\overline{u}}\, \mathbf{m_{\overline{u}}^2}\, \widetilde{\overline{u}}^\dagger - \widetilde{\overline{d}}\, \mathbf{m_{\overline{d}}^2}\, \widetilde{\overline{d}}^\dagger - \widetilde{\overline{e}}\, \mathbf{m_{\overline{e}}^2}\, \widetilde{\overline{e}}^\dagger \\
& - m_{H_u}^2 H_u^* H_u - m_{H_d}^2 H_d^* H_d - (B_\mu H_u H_d + \text{c.c.})\,.
\end{aligned}
$$

Note that the trilinear $\mathbf{a_i}$ terms are 3×3 complex matrices.
The quark/lepton mass-squareds ($\mathbf{m_Q^2}$ etc) are 3×3 Hermitian matrices.

The Higgs sector

At tree level, the Higgs scalar potential is

$$
\begin{aligned}
V =& (|\mu|^2 + m_{H_u}^2)(|H_u^0|^2 + |H_u^+|^2) + (|\mu|^2 + m_{H_d}^2)(|H_d^0|^2 + |H_d^-|^2) \\
&+ \left[B_\mu \left(H_u^+ H_d^- - H_u^0 H_d^0\right) + \text{c.c.}\right] \\
&+ \frac{1}{8}(g^2 + g'^2)(|H_u^0|^2 + |H_u^+|^2 - |H_d^0|^2 - |H_d^-|^2)^2 + \frac{1}{2} g^2 |H_u^+ H_d^{0*} + H_u^0 H_d^{-*}|^2
\end{aligned}
$$

In terms of just the neutral components this gives

$$
\begin{aligned}
V =& (|\mu|^2 + m_{H_u}^2)|H_u^0|^2 + (|\mu|^2 + m_{H_d}^2)|H_d^0|^2 - (B_\mu H_u^0 H_d^0 + \text{c.c.}) \\
&+ \frac{1}{8}(g^2 + g'^2)(|H_u^0|^2 - |H_d^0|^2)^2
\end{aligned}
$$

- The quartic coupling is given by the gauge couplings!
- We need the potential to have a minimum and not a runaway at infinity; at large H_u, H_d this is true except perhaps when $H_u = H_d = H$. Along that (D-flat) line, we have

$$
V \to (m_{H_u}^2 + m_{H_d}^2 + 2|\mu|^2)|H|^2 \to m_{H_u}^2 + m_{H_d}^2 + 2|\mu|^2 > 0.
$$

- Similarly, at the origin of field space, taking the second derivatives wrt H_u^0, H_d^0 we find the mass matrix

$$
\begin{pmatrix} m_{H_u}^2 + \mu^2 & -B_\mu \\ -B_\mu & m_{H_d}^2 + \mu^2 \end{pmatrix}.
$$

- It is only a saddle point if $(m_{H_u}^2 + \mu^2)(m_{H_d}^2 + \mu^2) < B_\mu^2$.

Goldstones and mixing

Let us write

$$H_u = \begin{pmatrix} c_\beta H^+ - s_\beta G^+ \\ \frac{1}{\sqrt{2}}[s_\beta(v+h) + c_\alpha H + i(c_\beta A - s_\beta G^0)] \end{pmatrix}$$
$$H_d = \begin{pmatrix} \frac{1}{\sqrt{2}}[c_\beta(v+h) - s_\alpha H + i(s_\beta A + c_\beta G^0)] \\ c_\beta G^- + s_\beta H^- \end{pmatrix}$$

where s_β, c_β are shorthand for $\sin(\beta), \cos(\beta)$ etc, and $G^- = \overline{G^+}, H^- = \overline{H^+}$. Then look at the kinetic term:

$$\begin{aligned}\mathcal{L} \supset & |(\partial_\mu - i\frac{1}{2}g_Y B_\mu - ig_2 T^a W_\mu^a)H_u|^2 + |(\partial_\mu + i\frac{1}{2}g_Y B_\mu - ig_2 T^a W_\mu^a)H_d|^2 \\ \to & \frac{|v|^2}{8}\left[\left(g_Y B_\mu - g_2 W_\mu^3\right)^2 \sin^2\beta + \left(g_Y B_\mu - g_2 W_\mu^3\right)^2 \cos^2\beta\right] + \ldots \\ = & \frac{e^2}{\sin^2 2\theta_W} v^2 \frac{1}{2} Z_\mu Z^\mu + \ldots\end{aligned}$$

Expanding we see that the Goldstone bosons $G^0, G^\pm$ are eaten by the corresponding gauge fields and they become the longitudinal components of $Z_\mu, W^\pm$.

Minima

- Taking the first derivatives of the potential w.r.t. H_u^0, H_d^0 we find

$$0 = v\sin\beta\left[m_{H_u}^2 + \mu^2 - B_\mu \cot\beta - \frac{1}{2}M_Z^2\cos 2\beta\right]$$
$$0 = v\cos\beta\left[m_{H_d}^2 + \mu^2 - B_\mu \tan\beta + \frac{1}{2}M_Z^2\cos 2\beta\right].$$

- We can take the second derivative w.r.t. the pseudoscalar A to find

$$m_A^2 = \frac{2B_\mu}{\sin 2\beta}$$

- So then we usually write the minimisation conditions as

$$\mu^2 = -\frac{M_Z^2}{2} + \frac{1}{\tan^2\beta - 1}(m_{H_d}^2 - \tan^2\beta m_{H_u}^2)$$
$$m_A^2 = m_{H_u}^2 + m_{H_u}^2 + 2\mu^2 > 0.$$

Masses

The second derivatives give the Higgs mass matrix as

$$\mathcal{M}_h^2 = \begin{pmatrix} M_Z^2 \cos^2 2\beta & -M_Z^2 \sin 2\beta \cos 2\beta \\ -M_Z^2 \sin 2\beta \cos 2\beta & M_A^2 + M_Z^2 \sin^2 2\beta \end{pmatrix}.$$

There is thus mixing between h, H so the true Higgs state needs a further rotation; we usually replace

$$H_u^0 \to \frac{1}{\sqrt{2}}(v \sin\beta + h\cos\alpha + ...), \qquad H_d^0 \to \frac{1}{\sqrt{2}}(v\sin\beta - h\sin\alpha + ...)$$

- However, for large M_A the heavy Higgs H decouples and the two are equivalent.
- More importantly, we see from the above that, at tree level, $m_h^2 \le M_Z^2 \cos^2 2\beta$!
- Therefore in the MSSM loop corrections are at least equal to the tree contribution:
$$\delta m_h^2(\text{loops}) \ge (86\text{GeV})^2 \gtrsim m_h^2(\text{tree})$$
- The loop corrections will be dominated by the stops (partners of the top).
- One loop contributions can easily be sufficient, but the two-loop contributions can give a mass shift of up to ~ 10 GeV – so there is a lot of work in understanding these.

Sparticle masses

- Since SUSY leads us to put fermions into separate left- and right-handed superfields, we have left- and right-handed squarks and sleptons.
- However, just as quarks mix, these mix via the mu and $\mathbf{a}$ terms.
- E.g. for the up-type squarks, we must group them into a vector of

$$\tilde{t} \equiv \begin{pmatrix} \tilde{t}_{L\,i} \\ \tilde{t}^*_{R\,i} \end{pmatrix}$$

- E.g. if we ignore mixing between the generations, the stop mass matrix reads

$$\mathcal{L}_{\text{stop masses}} = -\begin{pmatrix} \widetilde{t}^*_L & \widetilde{t}_R \end{pmatrix} \mathbf{m}^2_{\tilde{\mathbf{t}}} \begin{pmatrix} \widetilde{t}_L \\ \widetilde{t}^*_R \end{pmatrix}$$

$$\rightarrow \mathbf{m}^2_{\tilde{\mathbf{t}}} = \begin{pmatrix} m^2_Q + m^2_t + M^2_Z(\frac{1}{2} - \frac{2}{3}s^2_W)c_{2\beta} & m^*_t(A^*_t - \mu^* \cot\beta) \\ m_t(A_t - \mu \cot\beta) & m^2_{t_R} + m^2_t + \frac{2}{3}M^2_Z s^2_W c_{2\beta} \end{pmatrix}.$$

- We then diagonalise according to

$$\begin{pmatrix} \tilde{t}_1 \\ \tilde{t}_2 \end{pmatrix} \equiv \begin{pmatrix} c_{\theta_t} & s_{\theta_t} e^{-i\phi_t} \\ -s_{\theta_t} e^{i\phi_t} & c_{\theta_t} \end{pmatrix} \begin{pmatrix} \tilde{t}_L \\ \tilde{t}^*_R \end{pmatrix}$$

.

Gauginos

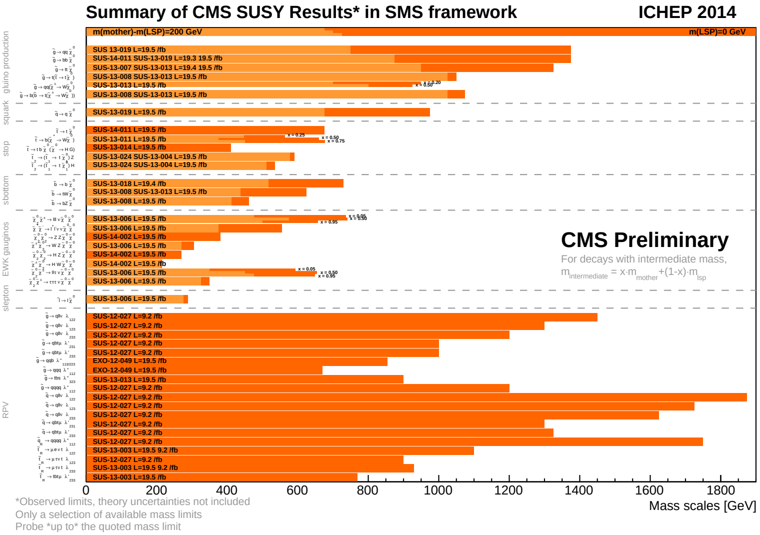
- After electroweak symmetry breaking, the gluino does not mix with anything, although it does obtain large loop corrections from the stops.
- The electroweak gauge bosons, however, mix with themselves and the higginos; they divide into the charginos – which form Dirac fermions with mass matrix

$$\mathcal{L}_{\text{charginos}} = -\begin{pmatrix} \tilde{W}^- & \tilde{h}^- \end{pmatrix} \begin{pmatrix} M_2 & gv\sin\beta \\ gv\cos\beta & \mu \end{pmatrix} \begin{pmatrix} \tilde{W}^+ \\ \tilde{h}^+ \end{pmatrix} + h.c.$$

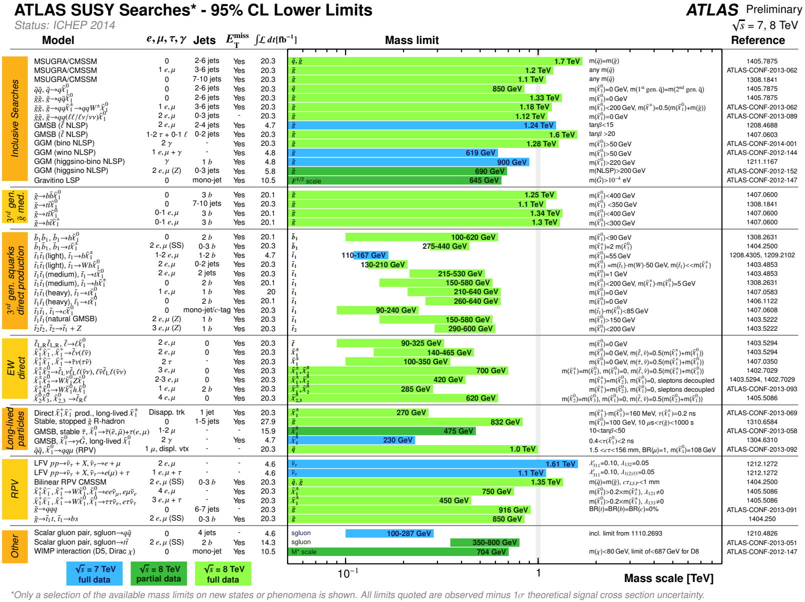
- ... and the neutralinos, which become Majorana fermions. In the basis $(\tilde{B}, \tilde{W}^0, \tilde{h}^0_d, \tilde{h}^0_u)$ we obtain

$$\mathcal{M}_{\text{neutralinos}} = \begin{pmatrix} M_1 & 0 & -c_\beta\, s_W\, M_Z & s_\beta\, s_W\, M_Z \\ 0 & M_2 & c_\beta\, c_W\, M_Z & -s_\beta\, c_W\, M_Z \\ -c_\beta\, s_W\, M_Z & c_\beta\, c_W\, M_Z & 0 & -\mu \\ s_\beta\, s_W\, M_Z & -s_\beta\, c_W\, M_Z & -\mu & 0 \end{pmatrix}$$

CMS constraints



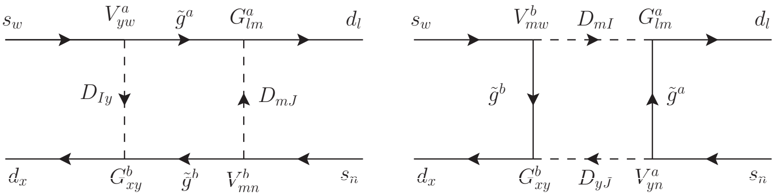
ATLAS constraints



LPTHE PARIS

The SUSY flavour problem

- As I described yesterday in my first lecture, rare decays and meson oscillations can be a powerful probe of new physics.
- For example, if the soft terms $(\mathbf{m_d}^2)_{ij}$ have mixing between the first two generations, then this can lead to Kaon mixing via the diagrams:



- If we define the down-type squark mass matrix as

$$\mathcal{L}_{\mathrm{down-squark}} = -\left(\widetilde{d}_L^* \quad \widetilde{d}_R \right) \begin{pmatrix} \mathcal{M}_{\tilde{d}\,LL} & \mathcal{M}^\dagger_{\tilde{d}\,LR} \\ \mathcal{M}_{\tilde{d}\,LR} & \mathcal{M}_{\tilde{d}\,RR} \end{pmatrix} \begin{pmatrix} \widetilde{d}_L \\ \widetilde{d}_R^* \end{pmatrix}$$

- We see that generic entries will lead to generation mixing amongst the squarks, independent of the quark mixing, which will lead to a large Δm_K and ϵ_K
- Recall the definition

$$\Delta m_K = 2\mathrm{Re}\langle K^0|\mathcal{H}_K|\overline{K}^0\rangle\,, \quad |\epsilon_K| = \left|\frac{\mathrm{Im}\langle K^0|\mathcal{H}_K|\overline{K}^0\rangle}{\sqrt{2}\Delta m_K}\right|$$

Kaon bounds

- Then in the approximation that the diagonal elements are all equal to $M_{\tilde{q}}$, we can define

$$\delta_{LL} = \frac{\mathcal{M}^{12}_{\tilde{d}\,LL}}{M^2_{\tilde{q}}}, \quad \delta_{LL} = \frac{\mathcal{M}^{12}_{\tilde{d}\,LR}}{M_{\tilde{q}}}, \quad \delta_{RR} = \frac{\mathcal{M}^{12}_{\tilde{d}\,LL}}{M_{\tilde{q}}}$$

- Then the bounds on these mixing parameters are, for a gluino mass $M_{\tilde{g}} = 2$ TeV:

$m_{\tilde{q}}$ [GeV]	$\delta^{LL} \neq 0$	$\delta^{LL} = \delta^{RR} \neq 0$	$\delta^{LR} = \delta^{RL} \neq 0$
1500	0.180	0.002	0.014
2000	0.157	0.003	0.008

- These are just for Δm_K!! If we allow CP violation then the bounds are 25 times smaller!
- Therefore SUSY must be mediated in a special way to the soft terms – presumably flavour blind.
- N.B. As I mentioned yesterday, there are also flavour mixing constraints from $b \to s\gamma, \mu \to e\gamma$ etc.

Dark matter

see e.g. Jungman, Kamnionkowski and Griest, “Supersymmetric Dark Matter”, hep-ph/9506380

Due to R-parity, the lightest supersymmetric particle (LSP) is stable – so can be a dark matter candidate!

SUSY models present a couple of possible candidates for a dark matter particle:

- The neutralino
- The sneutrino

Of these, the sneutrino is challenging because it interacts only through the Z-portal and only through the left-handed sneutrino. It may have a reasonable relic abundance if its mass is greater than about 500 GeV, but is challenged by direct detection unless very heavy.

Hence we almost always prefer a neutralino!

Neutralino dark matter

see e.g. Arkani-Hamed, Delgado, Giudice, “The well-tempered neutralino,” hep-ph/0601041

- The neutralino is composed of a mixture of the bino, wino, and higgsinos.
- In general, they will interact via Z,W,Higgs, squark and slepton exchange.
- Due to LHC bounds, we expect that the squarks must be heavy, too heavy to provide sufficient annihilation.
- If we assume that it is entirely bino, then selectron exchange leads to

$$\Omega h^2 \sim 1.3 \times 10^{-2} \left(\frac{m_{\tilde{e}_R}}{100 \text{ GeV}} \right)^2 \frac{(1+r)^4}{r(1+r^2)}, \qquad r \equiv \frac{M_1^2}{m_{\tilde{e}_R}^2}$$

- If it is mostly wino, then it interacts via W-bosons and

$$\Omega h^2 \sim 0.13 \left(\frac{M_2}{2.5 \text{ TeV}} \right)^2$$

- If it is mostly Higgsino, then it behaves like Dirac fermion during the early universe! We find

$$\Omega h^2 \sim 0.1 \left(\frac{\mu}{\text{TeV}} \right)^2$$

- Hence the best combination for a relatively light neutralino has long been thought to be a mixture of the bino and higgsinos.

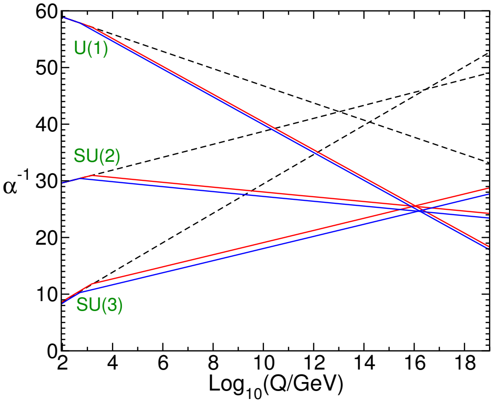
The case for SUSY

Having looked a little at the properties of low-energy SUSY models, I can now start to make a case for SUSY as a part of nature:

- We have seen on the theoretical side that it allows a solution of the hierarchy problem – or at least a framework in which to address it.
- Related to this: it allows us to calculate the cosmological constant, even if the result is a mystery.
- In addition, it seems to be necessary for the consistency of string theory – so we expect it to be present at some scale.
- It provides us with WIMP dark matter candidates that do not need the Z, W portal!

Unification

- In the standard model, it was noticed that all of the matter fields could be fit into representations of $SU(5)$ or $SO(10)$ (or even E_6 ...).
- However, when the gauge couplings were extrapolated to high energies, they did not meet at a point – arguing against non-supersymmetric grand unified theories.
- One of the major interesting discoveries about the MSSM is that the correction to the running from the gauginos and higgsinos (the scalars in the matter fields, sitting in complete $SU(5)$ representations, contribute equally to all gauge groups) causes the groups to unify!



This quite startling result is one of the most compelling reasons to keep searching for

Split SUSY

- One rather radical idea is to abandon the hierarchy problem: imagine that all of the SUSY scalars except for the SM Higgs are at a scale M_S.
- Keep the gauginos and higgsinos light, at the weak – TeV scale; this preserves unification!
- Requires an approximate R-symmetry.
- We must invoke anthropic tuning of the electroweak scale. This might not be so crazy, since only one parameter must be adjusted in the Higgs mass matrix

$$\det \begin{pmatrix} m_{H_u}^2 & B_\mu \\ B_\mu & m_{H_d}^2 \end{pmatrix} \simeq 0 \to m_{H_u}^2 m_{H_d}^2 = B_\mu^2, \tan\beta = \frac{m_{H_u}^2}{m_{H_d}^2}$$

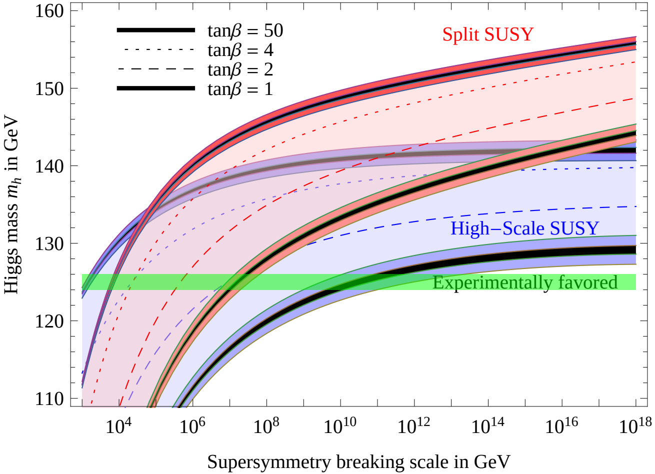
- Still have neutralino dark matter!
- But greatly ameliorate the flavour problem!

Higgs mass

- Split SUSY makes a prediction for the Higgs mass! The SM Higgs quartic coupling becomes

$$\lambda(M_S) = \frac{1}{4}(g^2 + (g')^2)\cos^2 2\beta + \ldots$$

Predicted range for the Higgs mass



Non-minimal models of low-energy SUSY

- On the other hand, maybe we are not ready to abandon low energy SUSY yet: perhaps the MSSM is too restrictive.
- Indeed, in the NMSSM, where we add a new singlet S, we modify the superpotential to

$$W = \lambda_S S H_u \cdot H_d + \frac{k}{3} S^3 + W_{Yukawa}$$

- This gives a new quartic coupling $\lambda_S^2 |H_u \cdot H_d|^2 \rightarrow$ boosts the Higgs mass at tree level.
- This allows more compressed spectra of sparticles which may have evaded searches so far.

Alternatively, the NMSSM shares many of the advantages of Dirac gaugino models, except:

- Recall the Dirac gaugino mass is <u>supersoft</u> $\rightarrow$ makes only finite corrections to stop and Higgs masses.
- Can therefore have a heavy gluino compared to stops
- Lack of chirality-flip processes weakens bounds on light squarks and alleviates flavour problem!

⋮

Can consider many more. Fortunately there is now a tool to tackle generic models: `SARAH`.

Summary

- Supersymmetry is the only fundamental symmetry that can extend spacetime symmetries.
- It seems to be necessary for the consistency of high-energy theories that we wish to descend to the Standard Model.
- It provides us with WIMP dark matter.
- Unification is perhaps the most compelling motivation.
- The Higgs mass is calculable in SUSY and can be used as a precision observable.
- It is time to consider non-minimal models!