Symmetry, Physical Theories and Theory Change

M. Lachièze-Rey
APC

3 de março de 2015

In collaboration with:

A. Queiroz (UnBrasilia) and

S. Simon (UnBrasilia)

Aims and Tasks: logic of discovery

A task in philosophy of science: to characterize a scientific theory.

There are many distinct approaches: syntatic, semantic and structuralist.

We propose a new perspective to such characterization in order to understand the problem of theory change.

The proposed characterization is based on the theory of groups of symmetry. Theory change can be seen in terms of Inönü-Wigner contraction/extension.

Charaterizing Some Physical Theories

Contractions and Extensions of Groups

From Newton Physics to Special relativity

Discussion

We consider a minimal characterization of the nature of a physical theory through a set of labels attached to it.

The landscape consists of physical theories of non-interacting particles moving as geodesics on some geometric background.

Label 1: Symmetries
The first label of a physical theory consists of its symmetry: Galilean (Newtonian), Poincaré (special relativity) and deSitter/anti-deSitter (cosmological).

Label 2: Domains
The second label – referred to as the domain – refers to the type of representation one considers for the symmetries of the theory: the classical or the quantum regime.

Charaterizing Some Physical Theories

Contractions and Extensions of Groups

From Newton Physics to Special relativity

Discussion

Lie groups and algebras

The symmetries of a physical or mathematical object or theory are described by a *Lie group*.
Many properties of a Lie group are described in terms of its associate *Lie algebra*.

A Lie algebra is described by a set of linear operators – generators – $J_i, \; i = 1, ..., N$, satisfying commutation relations (= Lie algebra product)

$$[J_i, J_j] = f_{ijk} \; J_k.$$

Each Lie group element $g \in G$ is obtained from the generators J_i through a exponential map $g = \exp(a^i \; J_i)$.
The Inönü-Wigner contraction $G \to G_0$ of a Lie group is best described in terms of its associated Lie algebra: $\underline{\mathfrak{g}} \to \underline{\mathfrak{g}}_0$.

Inönü-Wigner Group Contractions

(canonical procedure !)
Start with a Lie algebra $\mathfrak{g}$.

Construct a parameterized family of new Lie algebras $\mathfrak{g}_\varepsilon$ isomorphic to the initial Lie algebra for $\varepsilon \neq 0$ but not for the singular value $\varepsilon = 0$. This defines a new Lie algebra.

$$\mathfrak{g} \to (\mathfrak{g}_\epsilon)$$

$$\epsilon \neq 0 : \ \mathfrak{g}_\epsilon \equiv \mathfrak{g}$$

$$\epsilon = 0 : \ \mathfrak{g}_0 \text{ new algebra}$$

This works if $\mathfrak{g}$ contains a Lie subalgebra $\mathfrak{h} \subset \mathfrak{g}$.
Then call $\mathfrak{p}$ the complement of $\mathfrak{h}$ in $\mathfrak{g}$:

$$\mathfrak{g} = \mathfrak{h} \oplus \mathfrak{p} \text{ (direct sum of vector spaces).}$$

Schematically, the defining commutators write as

$$[\mathfrak{h}, \mathfrak{h}] \subset \mathfrak{h}, \qquad [\mathfrak{h}, \mathfrak{p}] \subset \mathfrak{p}, \qquad [\mathfrak{p}, \mathfrak{p}] \subset \mathfrak{h} + \mathfrak{p}.$$

Inönü-Wigner Group Contractions

Reparametrize each generator $J \in \mathfrak{p}$ by $J' = \epsilon\ J$, with $\epsilon \neq 0$, so that $\mathfrak{p}$ becomes $\mathfrak{p}' = \epsilon\ \mathfrak{p}$.

This changes nothing : this is just a relabelling and the algebra (now written $\mathfrak{g}_\epsilon$) remains the same, with commutation relations now written

$$[\mathfrak{h}, \mathfrak{h}] \subset \mathfrak{h}, \quad [\mathfrak{h}, \mathfrak{p}'] = [\mathfrak{h}, \epsilon\ \mathfrak{p}] \subset \epsilon\ \mathfrak{p} = \mathfrak{p}', \quad [\mathfrak{p}', \mathfrak{p}'] = [\epsilon\ \mathfrak{p}, \epsilon\ \mathfrak{p}] \subset \epsilon^2\ (\mathfrak{h} + \mathfrak{p}).$$

But the singular limit $\epsilon \to 0$ gives a (well-defined but) different Lie algebra obeying

$$[\mathfrak{h}, \mathfrak{h}] \subset \mathfrak{h}, \qquad [\mathfrak{h}, \mathfrak{p}'] \subset \mathfrak{p}', \qquad [\mathfrak{p}', \mathfrak{p}'] = 0.$$

Observe that $\mathfrak{p}'$ is now an Abelian ($=$ commutative) algebra.

Simple Example: $\mathfrak{so}(3) \to \mathfrak{e}(2)$

The $\mathfrak{so}(3)$ Lie algebra is generated by J_i, $i = 1, 2, 3$, with commutation relations

$$[J_1, J_2] = J_3, \qquad [J_2, J_3] = J_1, \qquad [J_3, J_1] = J_2.$$

Nothing commutes : simple algebra

Set $\mathfrak{h}$ as the subalgebra generated by only J_3.
So, J_1, J_2 generate $\mathfrak{p}$. Rescale them as

$$j_1 = \Lambda\ J_1, \qquad j_2 = \Lambda\ J_2, \qquad j_3 = J_3 \ \text{(fixed)}.$$

For $\Lambda \neq 0$, the algebra remains the same with commutators

$$[j_1, j_2] = \Lambda^2\ j_3, \qquad [j_2, j_3] = j_1, \qquad [j_3, j_1] = j_2.$$

Contraction: the singular limit $\Lambda \to 0$, leads to a new Lie algebra

$$[j_1, j_2] = 0, \qquad [j_2, j_3] = j_1, \qquad [j_3, j_1] = j_2.$$

They characterize the Lie algebra $\mathfrak{e}(2) = \mathfrak{so}(2) \ltimes \mathbb{R}^2$ (not simple).

Group Extensions

A Lie algebra that cannot be written as a (semi-)direct product of Lie subalgebras is called a *simple Lie algebra*.
As illustrated by the example above, *the Inönü-Wigner contraction diminishes the simplicity* of the Lie algebra.

There is an inverse procedure, called *the Inönü-Wigner extension, which achieves simplicity*.

It extends a Lie algebra, composed as (semi-)direct product of two (or more) other Lie algebra, towards a simpler algebra: a algebra with less (semi-)direct products of algebras.

Charaterizing Some Physical Theories

Contractions and Extensions of Groups

From Newton Physics to Special relativity

Discussion

The group $ISO(3)$

The group $ISO(3)$ is characteristic of Newtonian (or Galilean) physics.
This is a subgroup of the Galilei group $\mathcal{G}$ (see below), generated by
- the *spatial rotations* $R \in SO(3)$ of Euclidean space $\mathbb{R}^3$
- and the *Galilean boosts* $\vec{v} = (v_x, v_y, v_z) \in \mathbb{R}^3$.

general element : $g = (\vec{v}, R)$.
Multiplication rule: $(\vec{v}', R')\ (\vec{v}, R) = (\vec{v}' + R'\vec{v}, R'R)$.
Unit: $e = (0, 1)$.
Inverse of g: $g^{-1} = (-R^{-1}\ \vec{v}, R^{-1})$.
characteristic of Galilean kinematics (we will add translation to complete...)

Extension of the group $ISO(3)$

The group $ISO(3)$ is not simple since the boosts commute.
It admits a natural Inönü-Wigner extension, with parameter $1/c$: this gives the Lorentz group $SO(3,1)$:

$$ISO(3) \stackrel{1/c}{\rightarrow} SO(3,1)$$

The SO(3) part remains unchanged.
The new boosts (Lorentz boosts instead of Galilean boosts) do not commute.

The Lorentz group is the rotation group of the Minkowski spacetime $\mathbb{M}_k$, the space-time of *special relativity*.
It is stable, i.e., admits no further similar extension.

Physical Interpretation

The following discussion is based on work by Lévy-Leblond and collaborators on the classification of the kinematic groups.
A theory is characterized by its *kinematic group*: the group of isometries of its space-time: the set of transformations preserving its metric, with the composition of transformations.

A concrete examples of theory change, through extensions:

$$\text{Galilei} \to \text{Einstein} \to \text{(anti-)deSitter} \to \text{conformal}.$$

Newtonian Kinematic-1

The kinematic group of Newtonian theory is the *Galileo group* $\mathcal{G}$.
First, the Galileo (or Newton) space = Euclidean space $\mathbb{R}^3$.
group of isometries : Euclide group

- $R \in \mathsf{SO}(3)$: three-dimensional rotation
- $\vec{a} = (a_x, a_y, a_z) \in \mathbb{R}^3$: three-dimensional spatial translation,

Natural Prolongation: to Galileo (or Newton) space-time $\mathbb{R} \times \mathbb{R}^3$.
group of isometries : R and $\vec{a}$ plus

- b: time translation,
- $\vec{v} = (v_x, v_y, v_z)$: Galilei boost.

This forms the (proper) Galilei group $\mathcal{G}$, with general element :

$$g = (b, \vec{a}, \vec{v}, R) :$$

Newtonian Kinematic-2: **Galileo group**

general element $g = (b, \vec{a}, \vec{v}, R)$
Multiplication rule:

$$(b', \vec{a}', \vec{v}', R')\ (b, \vec{a}, \vec{v}, R) = (b' + b, \vec{a}' + R'\vec{a} + b\vec{v}', \vec{v}' + R'\vec{v}, R'R).$$

Unit: $e = (0, 0, 0, 1)$
Inverse of g: $g^{-1} = (-b, -R^{-1}(\vec{a} - b\vec{v}), -R^{-1}\vec{v}, R^{-1})$.

The transformation g acts on a space-time point $(t, \vec{x})$ as

$$(t, \vec{x}) \mapsto g \cdot (t, \vec{x}) \equiv (t', \vec{x}'),$$
$$\vec{x}' = R \cdot x + \vec{v}t,$$
$$t' = t.$$

(Newtonian Kinematic-3)

The Galilei group depicts the kinematic symmetries of the non-relativistic free particle. The e.o.m. are obtained from a variational principle involving an action invariant under the Galilei group of transformations.

The action is $S = \int dtL$, with Lagrangian $L = \frac{m}{2}\dot{x}_i^2$.

The action is invariant if and only if the Lagrangian is invariant up to a total derivative. Indeed,

$$L = \frac{m}{2}\dot{x}_i^2 \rightarrow L' = L + \frac{d}{dt}\left(mx_iv_i + \frac{m}{2}v_i^2 t\right).$$

From Galileo group to Poincaré group

The Galilei group $\mathcal{G}$ is the *natural prolongation* of its subgroup $ISO(3)$, generated by the spatial rotations $SO(3)$ and the boosts $\mathbb{R}^3$:

(like $ISO(3)$ itself is the *NP* of SO(3)).

We have seen above the natural Inönü-Wigner extension to Lorentz group

$$ISO(3) \stackrel{1/c}{\to} \mathcal{L} = SO(3,1).$$

This implies the extension of their prolongations:

$\mathcal{G} \to \mathcal{P} \stackrel{def}{=} ISO(3,1)$

Galilei group $\to$ Poincaré group.

The *natural prolongation* of the Lorentz group is the Poincaré group .
The Poincaré group is the kinematical group of Minkowski spacetime.
Newtonian physics $\to$ special relativity

$$
\begin{tikzcd}
SO(2) \arrow[d, "\text{augm}"] \arrow[dr, "\text{sg}"] & & & & \\
ISO(2) \arrow[r, "\text{ext}"] & SO(3) \arrow[d, "\text{augm}"] \arrow[dr, "\text{sg}"] & & & \\
& ISO(3) \arrow[r, "\frac{1}{c}"] \arrow[d, "\text{augm}"] & \mathcal{L} = SO(3,1) \arrow[d, "\text{augm}"] \arrow[dr, "\text{sg}"] & & \\
& \mathcal{G} \arrow[r, "\frac{1}{c}"] & \mathcal{P} = ISO(3,1) \arrow[r, "\Lambda"] & dS = SO(4,1) \arrow[d, "\text{augm}"] \arrow[dr, "\text{sg}"] & \\
& & & ISO(4,1) \arrow[r, "\text{ext}"] & SO(4,2)\ldots
\end{tikzcd}
$$

Figura: The chain

Poincaré group: Special Relativity-1

The Poincaré group is the group of isometries of the Minkowski space-time with metric $ds^2 = -c^2dt^2 + dx^2 + dy^2 + dz^2$.
It includes
- the space-time translations $a = (a_0, a_1, a_2, a_3) \in \mathbb{R}^4$
- and the four-dimensional rotations $R \pm in\mathrm{SO}(3,1)$, the Lorentz group.

Ten parameters to describe the Poincaré group: four for the space-time translations and six for the Lorentz group (three for spatial rotations and three for the Lorentz boosts).
General element of the *proper orthochronous* Poincaré group $\mathcal{P}$:
$g = (a, R) \in \mathcal{P}$;
Multiplication rule: $g'g = (a', R')(a, R) = (a' + R'a, R'R)$;
Unit: $e = (0, 1)$;
inverse of an element g: $g^{-1} = (-R^{-1}a, R^{-1})$.

Special Relativity-2

The action of $g \in \mathcal{P}$ on a point $x = (x_0, x_1, x_2, x_3)$ of Minkowski space-time:

$$x \mapsto g \cdot x = x' = R \cdot x + a.$$

Lorentz group $\mathcal{L} = \mathrm{SO}(3,1)$ acts on $\mathbb{M}_k$ exactly like $\mathrm{SO}(3)$ acts on $\mathbb{R}^3$.
Poincaré group $\mathcal{P}$ acts on $\mathbb{M}_k$ exactly like Euclide group acts on $\mathbb{R}^3$.

The (homogeneous) Lorentz group is a simple group and admits no extension.
The Poincaré group is not a simple group: its associated Lie algebra $\mathfrak{p}$ of $\mathcal{P}$ factorizes as

$$\mathfrak{p} = \mathfrak{l} \ltimes \mathbb{R}^{1,3},$$

where $\mathfrak{l}$ stands for the Lie algebra of $\mathcal{L}$.

Cosmology: $SO(3,2)$ and $SO(4,1)$

The Poincaré group is not simple and the process may be continued:
$\underline{\mathfrak{h}}$ =Lorentz algebra (space-time rotations);
p=commuting space-time translations ($\mathbb{R}^4$).

Two possible extensions (exactly same process) with parameter Λ. They lead to two possible simple groups:
- the *anti-deSitter group* $AdS = SO(3,2)$ ($\Lambda < 0$)
- and the *deSitter group* $S = SO(4,1)$ ($\Lambda > 0$).

(The limit value $\Lambda \to 0$ corresponds to their common contraction to the Poincaré group $\mathcal{P}$.)

They act as kinematic groups (= groups of isometries) of the four-dimensional space-times of constant but not zero curvatures (associated with the symm. of the relativistic particle moving in them):
- the *deSitter space-time*, with a constant positive curvature.
- the *anti-deSitter space-time*, with a constant negative curvature.

Beyond...

The groups dS and AdS are simple
(they are not a semi-direct product of other subgroups);
thus stable: they admit no further similar extension.
However, they admit *natural augmentations* (adding "translations "):
dS $\rightsquigarrow ISO(4,1)$ and AdS $\rightsquigarrow ISO(3,2)$.
(Remember the natural augmentation $SO(3) \rightsquigarrow ISO(3)$; and then $ISO(3)$ admits natural extensions.)

The two can be extended by the same process: they admit a common extension under the form of the *conformal group* $SO(4,2)$.

The conformal group admits the ten generators of the kinematic group, augmented by five new generators, one for *scaling transformations*, and four generating the so called *special conformal transformations*.

Charaterizing Some Physical Theories

Contractions and Extensions of Groups

From Newton Physics to Special relativity

Discussion

Summary and Discussion

The *Inönü-Wigner extension* generates a progression from Newton kinematics to (anti-)deSitter and conformal kinematics.
The opposite *Inönü-Wigner contraction* provides the way back.

This mathematical procedure describes *theory change* (theory incorporation) via an *improvement of their symmetries*.
This enlightens the problem of scientific discovery.
This precise characterization of a physical theory is not achieved in other exclusively logic approaches like for instance the semantic approach or the logic approaches (Frigg-2006), since they are too general to usefully characterize specific disciplines (Halvorson-2012).

This work has shown the heuristic power embodied by the concept of symmetries – here presented in the framework of group theory – for the discovery of new (physical) theories and the setting of their validity boundaries.

Merci beaucoup!