The Field Theory of Avalanches

Kay Wiese

LPT-ENS, Paris

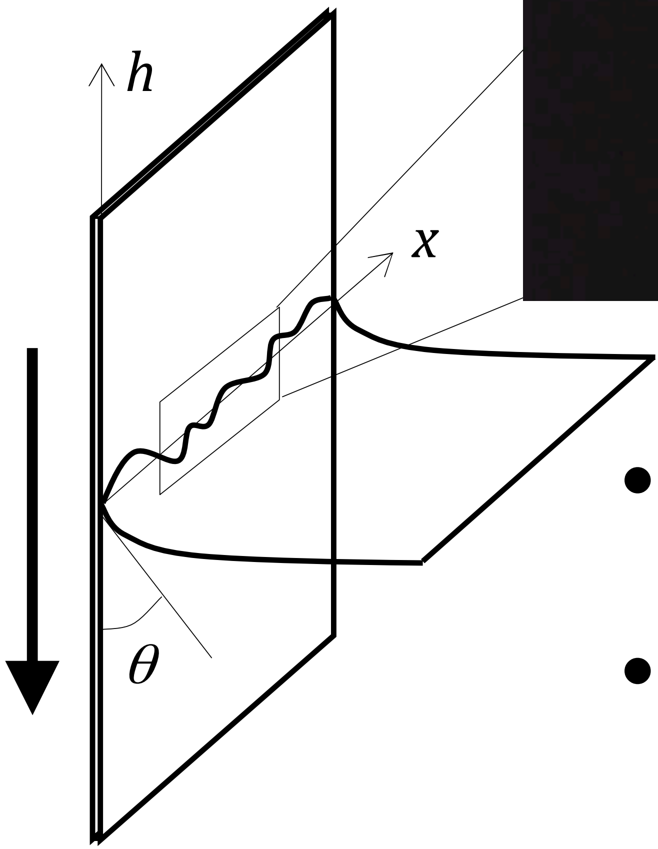
with Pierre Le Doussal,
Alberto Rosso, Alain Middleton, Alejandro Kolton,
Sébastien Moulinet, Etienne Rolley, Gianfranco Durin,
Alexander Dobrinevski, Mathieu Delorme, Thimotée
Thiery, Andrei Fedorenko, Markus Mueller

Conférence Itzykson, IPHT, June 2015

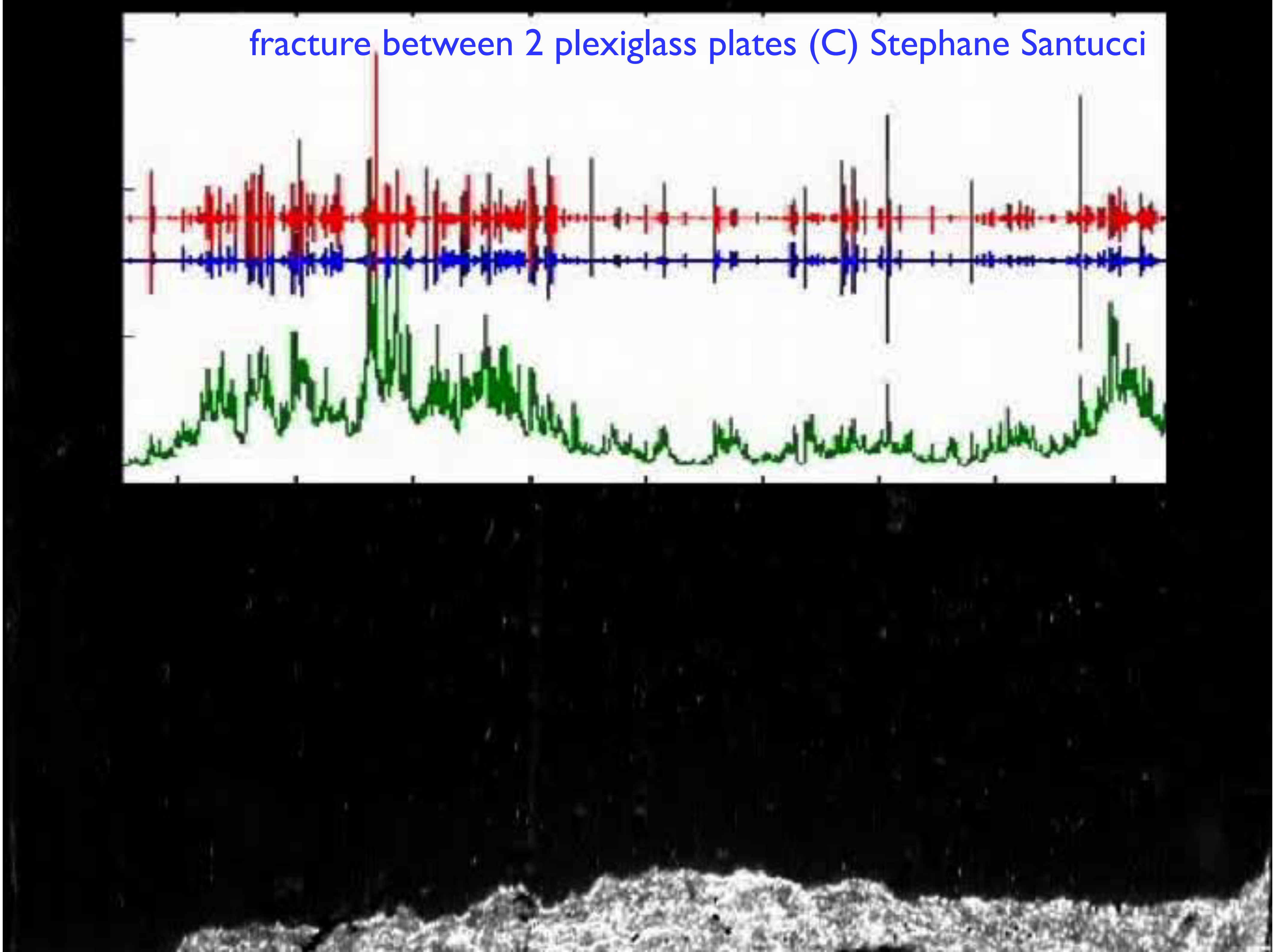
http://www.phys.ens.fr/~wiese/

Contact line wetting

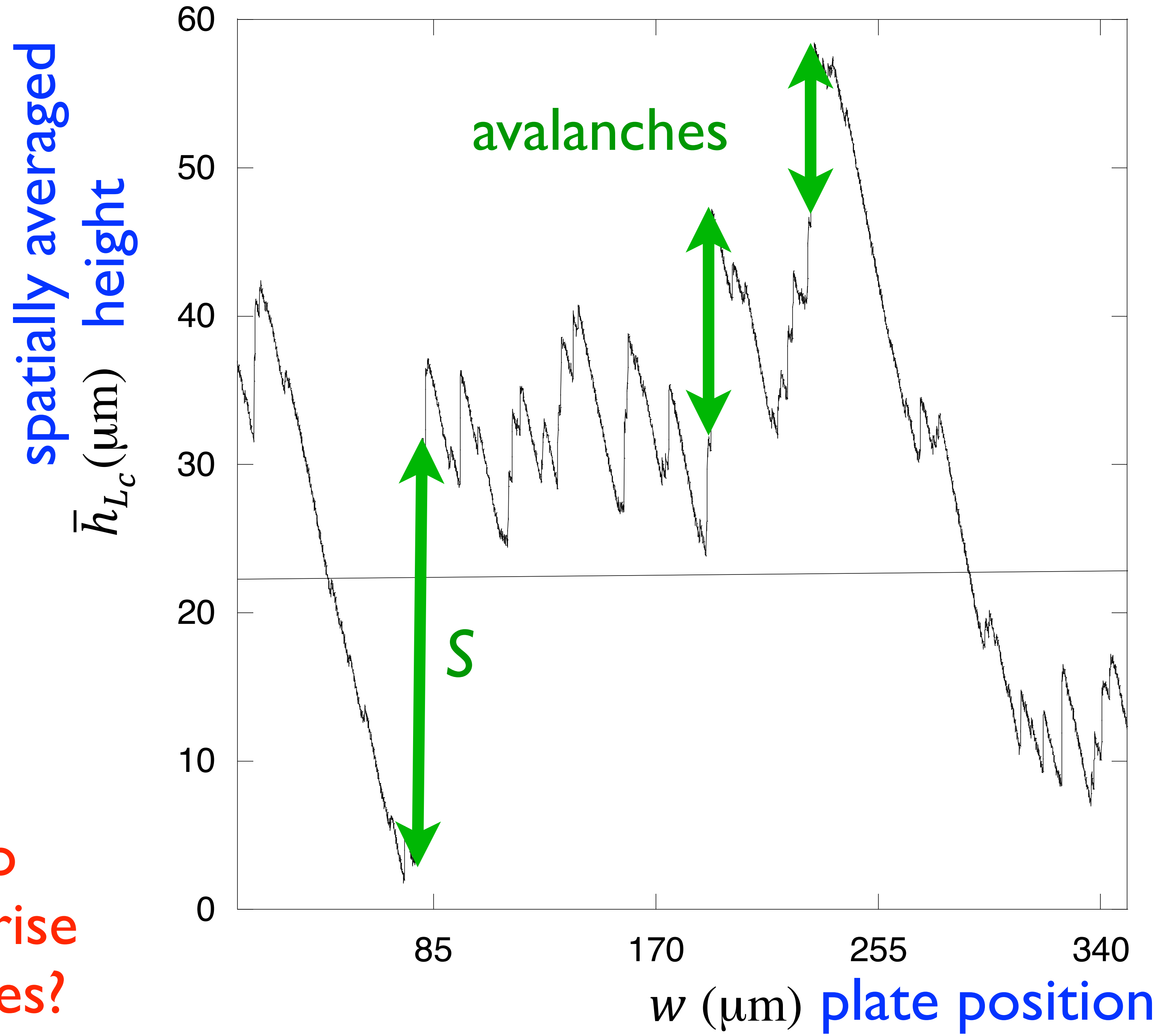
(C) E. Rolley



- isobutanol on a randomly silanized silicon wafer
- hydrogen on disordered Cesium substrate


fracture between 2 plexiglass plates (C) Stephane Santucci

height jumps = avalanches



how to characterise avalanches?

The model

Displacement field $\quad x \in \mathbb{R} \quad \longrightarrow u(x) \in \mathbb{R}$

Elastic energy:

$$\mathcal{H}_{\mathrm{el}} = \frac{1}{2}\int \frac{\mathrm{d}^d k}{2\pi} \, |\tilde{u}_k|^2 \, \varepsilon_k + \int_x \frac{m^2}{2} [u(x) - w]^2$$

$$w = vt$$

for contact angle $\theta = 90^\circ$: $\quad \varepsilon_k \approx \sqrt{k^2 + \kappa^2} - \kappa$

$\kappa^{-1} = m^{-2}$ kapillary length $\quad$ (instead of $\varepsilon_k = k^2$)

Disorder energy

$$\mathcal{H}_{\mathrm{DO}} = \int \mathrm{d}^d x \, V(x, u(x))$$

with correlations

$$\overline{V(x,u)V(x',u')} = \delta^d(x - x') R(u - u')$$

Functional renormalization group (FRG)

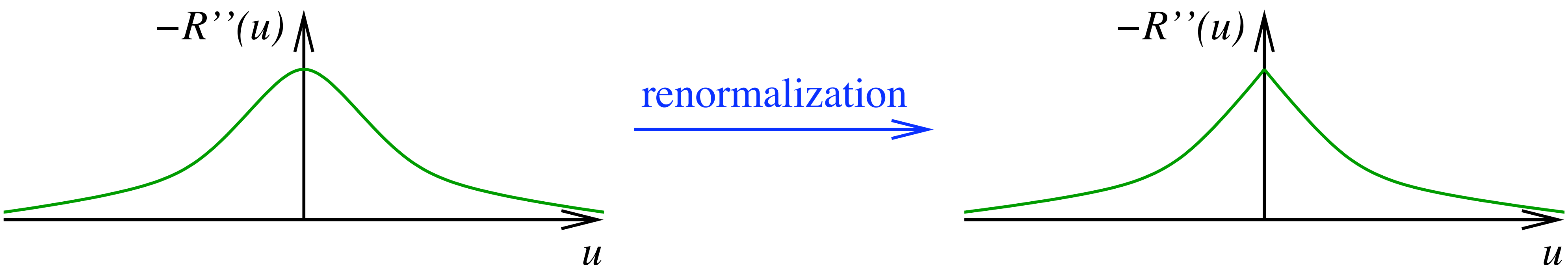
(D. Fisher 1986)

$$\frac{\mathcal{H}[u]}{T} = \frac{1}{2T}\sum_{\alpha=1}^{n}\left[\int_k \varepsilon_k |\tilde{u}_k^\alpha|^2 + \int_x m^2(u^\alpha(x)-w)^2\right]$$
$$-\frac{1}{2T^2}\int_x \sum_{\alpha,\beta=1}^{n} R\big(u^\alpha(x)-u^\beta(x)\big)$$

Functional renormalization group equation (FRG) for the disorder correlator $R(u)$ at 1-loop order:

$$-\frac{m\mathrm{d}}{\mathrm{d}m}R(u) = (\varepsilon-4\zeta)R(u) + \zeta u R'(u) + \frac{1}{2}R''(u)^2 - R''(u)R''(0)$$

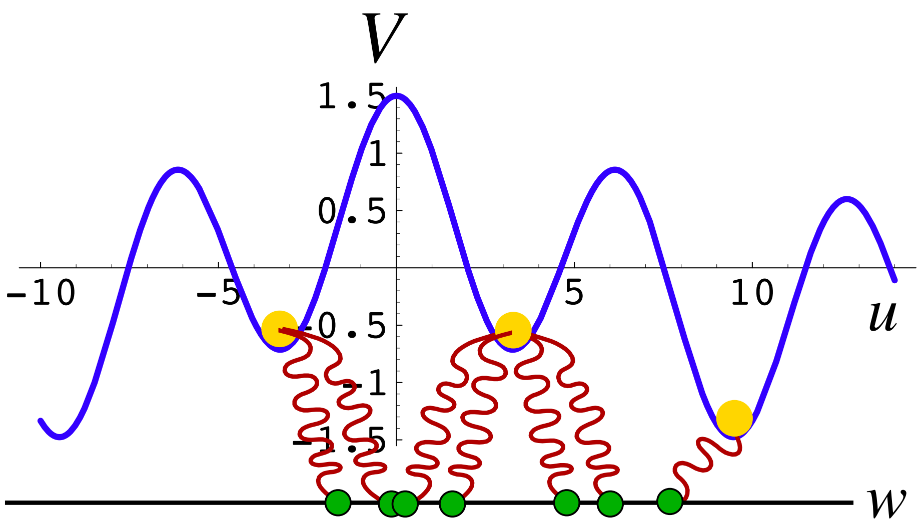
Solution for force-force correlator $-R''(u)$:



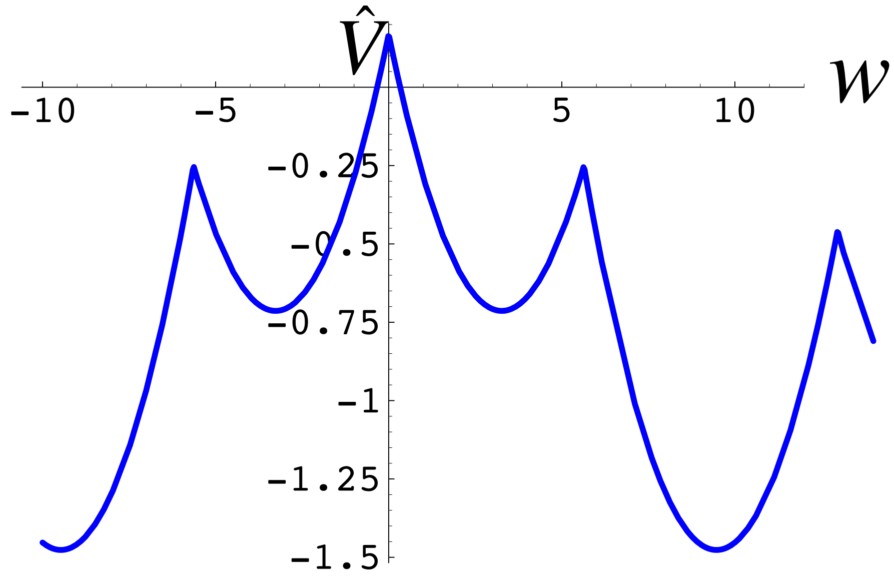
Cusp: $R''''(0)=\infty$ appears after finite RG-time (at Larkin-length)

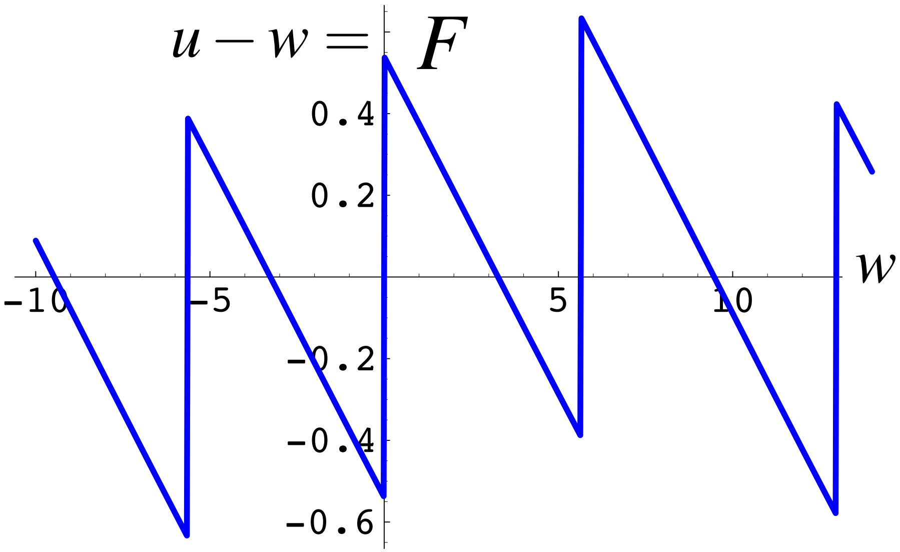
Why is a cusp necessary?

…calculate effective action for single degree of freedom…



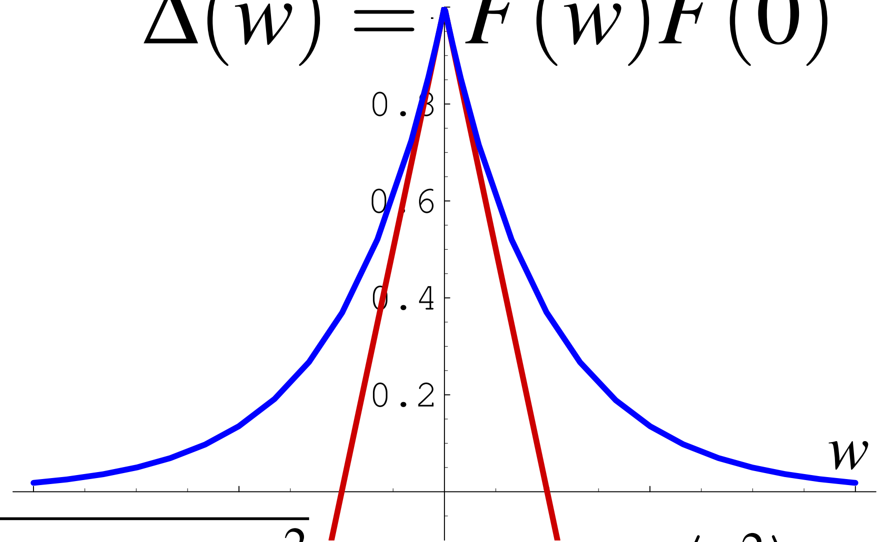
$\xrightarrow{\text{Min}}$





$\xrightarrow{\text{average}}$

$$\Delta(w) = \overline{F(w)F(0)}$$



$$\overline{[F(w) - F(0)]^2} = \underbrace{\rho_{\text{shock}} |w|}_{p_{\text{shock}}} \langle S^2 \rangle$$

Renormalized Disorder Correlator in FRG

$$\mathcal{H}^{w}[u] = \int \frac{1}{2}[\nabla u(x)]^2 + V\big(x, u(x)\big) + \frac{m^2}{2}[u(x) - w]^2 \ \mathrm{d}^d x$$

Local minimum $u_w(x)$ satisfies:

$$0 = \frac{\delta \mathcal{H}^{w}[u]}{\delta u_w(x)} = -\nabla^2 u_w(x) - F\big(x, u_w(x)\big) + m^2[u_w(x) - w]$$

Center-of-mass u_w fluctuates around w

$$u_w - w := \frac{1}{L^d}\int [u_w(x) - w]\,\mathrm{d}^d x = \frac{1}{L^d m^2}\int F\big(x, u_w(x)\big)\ \mathrm{d}^d x$$

Thus naively

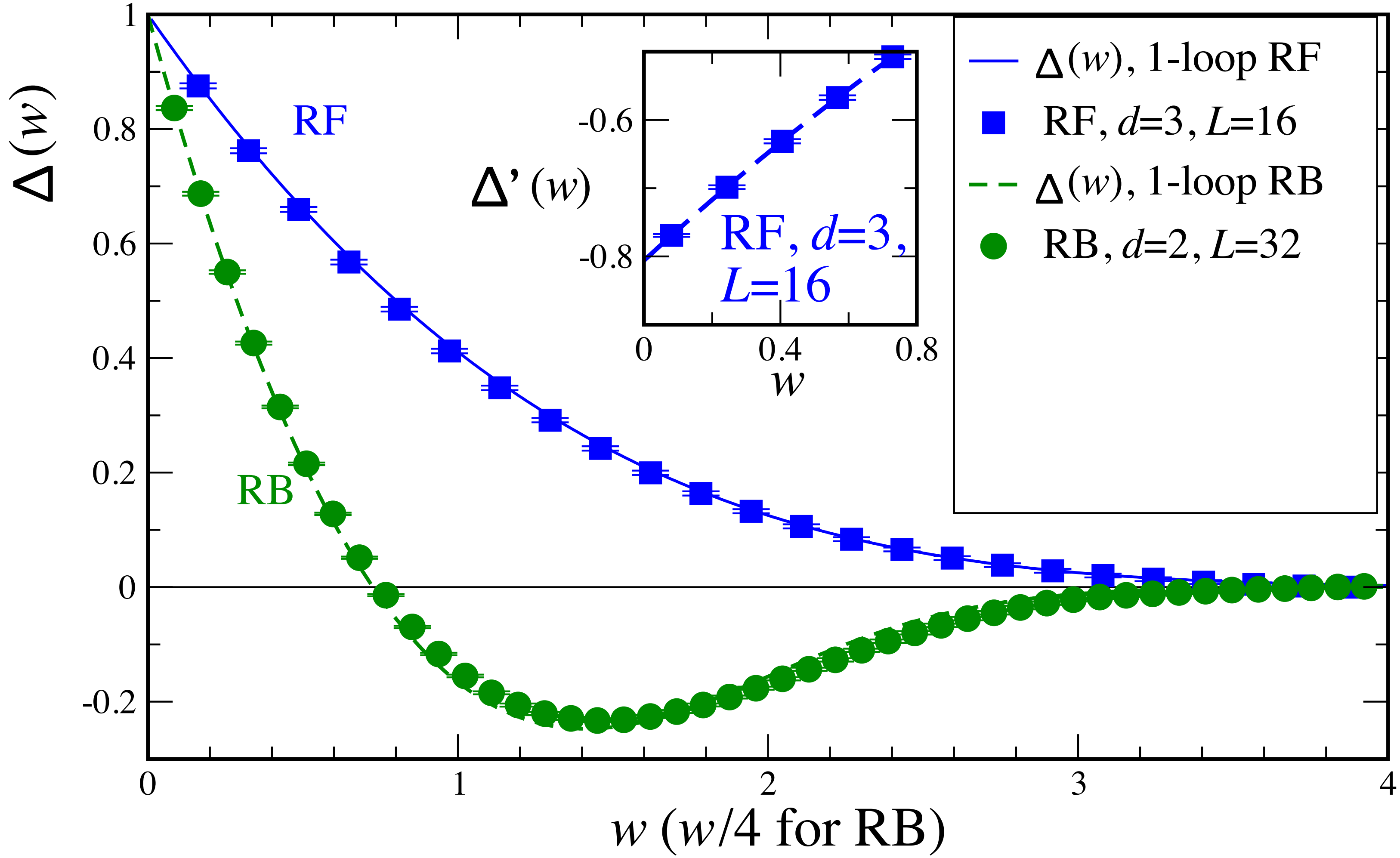
$$\boxed{\overline{h_w h_{w'}} = \overline{[u_w - w]\,[u_{w'} - w']} = \frac{\Delta(w - w')}{L^d m^4}}$$

FRG - Legendre-transform ... confirm this picture !

P. Le Doussal, EPL 76 (2006), 457; Annals of Physics 325 (2009) 49

Measuring the cusp = effective action

A. Middleton+PLD+KW, PRL 98 (2007) 155701

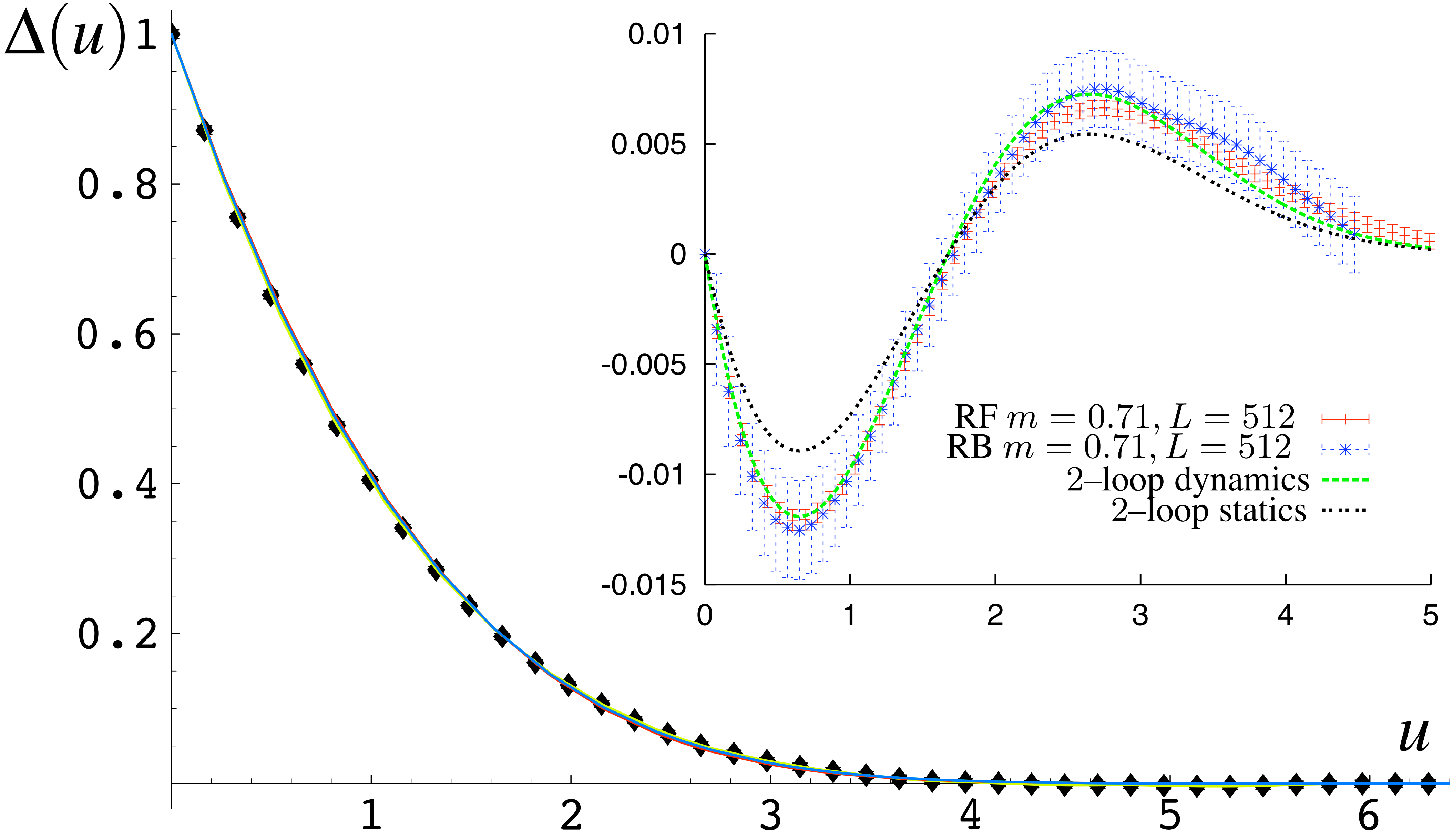


$$\Delta(w-w') = m^4 L^d \overline{[u_w - w]\,[u_{w'} - w']}$$

Δ = renormalized disorder correlator

Depinning in 1+1 dimensions

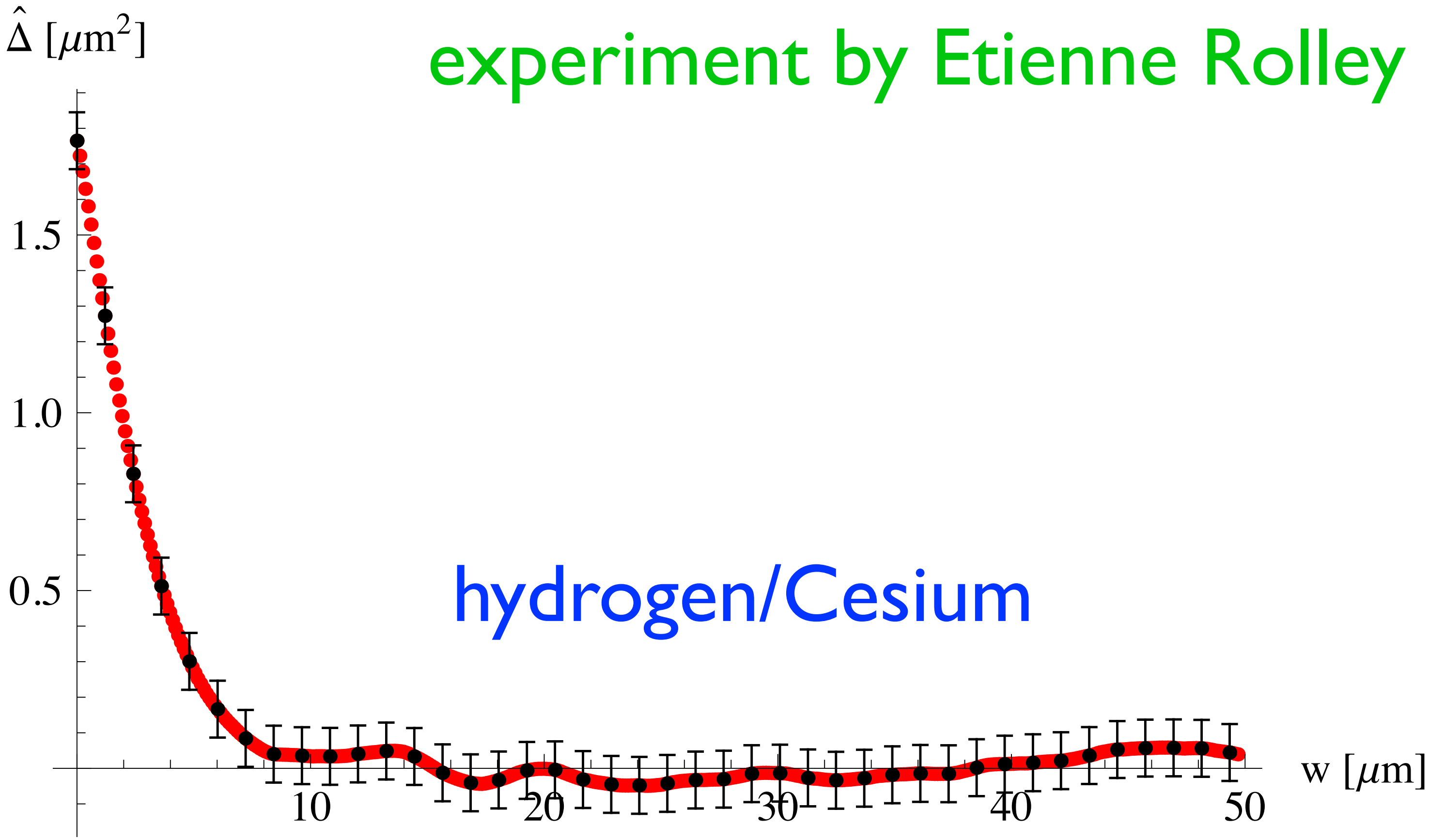
$\zeta = \frac{\varepsilon}{3} + 0.04777\varepsilon^2$: 1.0 (1 loop), 1.2 ± 0.2 (2 loop), 1.25 (numerics).

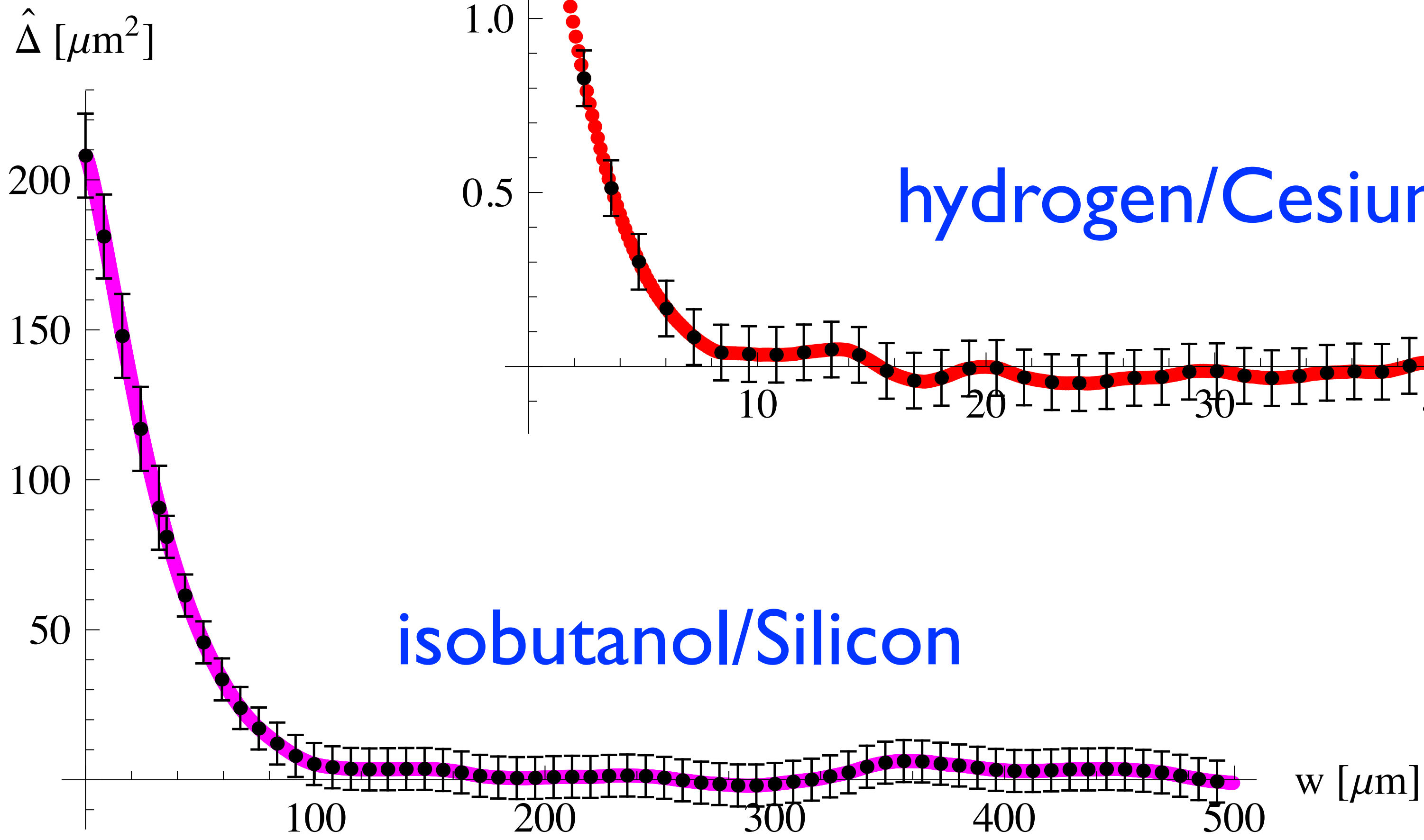


A. Rosso, P. Le Doussal, KW, PRB 75 (2007) 220201

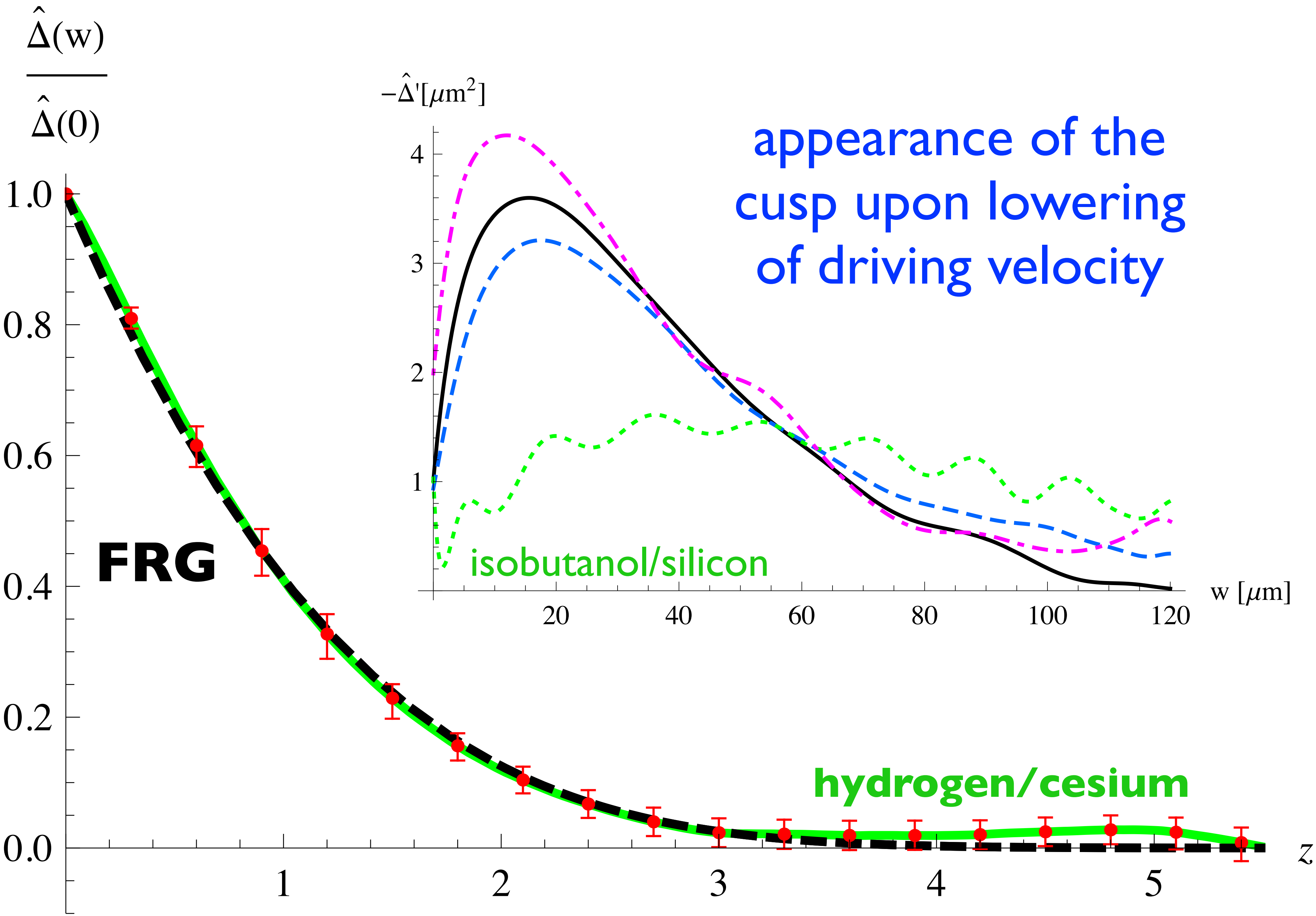
Experiments on contact line

experiment by Etienne Rolley



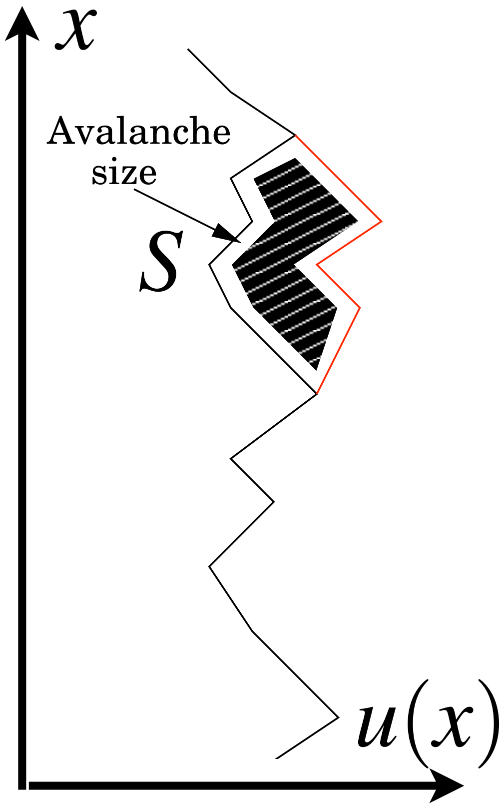


The renormalized force-force correlator



P. Le Doussal, KW, S. Moulinet, E. Rolley, EPL 87 (2009) 56001

Slope at the cusp and avalanche size moments



$$\rho \left\langle S \right\rangle |w - w'| = L^d \overline{|u_w - u_{w'}|} = L^d |w - w'|$$

#avalanches/unit length

$$\rho \left\langle S^2 \right\rangle |w - w'| \approx L^{2d} \overline{|u_w - u_{w'}|^2}$$
$$\approx 2L^d \frac{|\Delta'(0^+)|}{m^4} |w - w'|$$

together:
(exact)

$$\boxed{S_m := \frac{\left\langle S^2 \right\rangle}{2 \left\langle S \right\rangle} = \frac{|\Delta'(0^+)|}{m^4}}$$

Avalanches

- avalanches appear in many systems: contact-lines, vortex lattices, domain walls, earthquakes, etc.

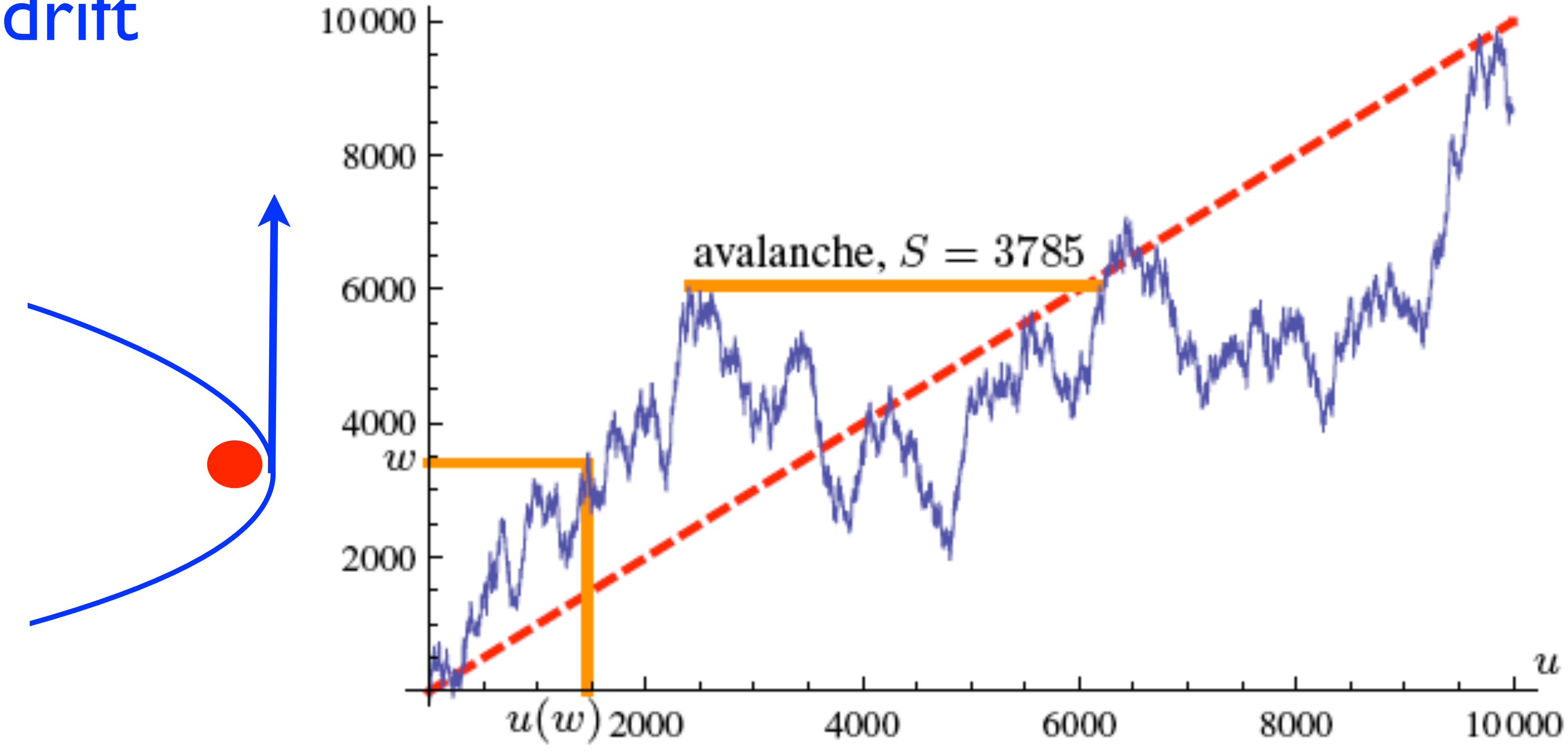
- Oldest example: Galton process
- Galton process = Mean Field (MF) = ABBM model
- Brownian force model (BFM) = starting point for field theory
- center-of-mass mode of BFM = ABBM

- avalanches in SK model are different ($\tau = 1$) (M. Mueller, PLD, KW)

- Self-Organized Criticality (SOC)
- Manna model: mapping on disordered elastic manifolds

The Galton process

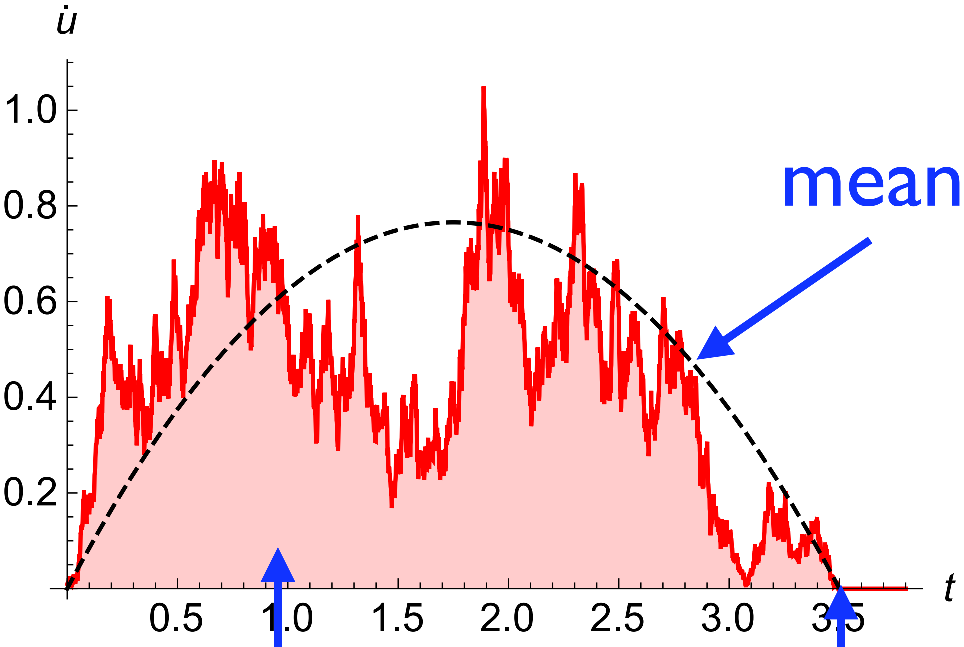
- old quesstion: survival probability of male line (Galton, Watson1873)
- equivalent: driven particle in random force landscape which itself is a Brownian = records with drift



$$P(S) \sim S^{-3/2} \mathrm{e}^{-S/S_m}$$

Avalanche observables

velocity



mean shape at fixed duration and its fluctuations

avalanche duration T

avalanche size S = area under curve

The ABBM model

B. Alessandro, C. Beatrice, G. Bertotti and A. Montorsi, J. Applied Phys. 68 (1990) 2901; ibid, 2908

A particle subjected to force which is a random walk:

$$\partial_t \dot{u}(t) = m^2 [v - \dot{u}(x,t)] + \partial_t F(u(t)) \qquad \langle [F(u) - F(u')]^2 \rangle = |u - u'|$$

$$\partial_t F(u(t)) = \sqrt{\dot{u}(t)}\,\xi(t) \;, \qquad \langle \xi(t)\xi(t') \rangle = \delta(t - t')$$

The Brownian force model (BFM) PLD+KW

$$\partial_t \dot{u}(x,t) = \nabla^2 \dot{u}(x,t) + m^2 [v - \dot{u}(x,t)] + \partial_t F(u(x,t),x)$$

$$\partial_t F(u(x,t),x) = \sqrt{\dot{u}(x,t)}\,\xi(x,t) \qquad \langle \xi(x,t)\xi(x't') \rangle = \delta^d(x - x')\delta(t - t')$$

Short-ranged rough disorder A. Dobrinevski, PLD+KW

$$\partial_t F(u(x,t),x) = -\gamma \dot{u}(x,t) F(u(x,t),x) + \sqrt{\dot{u}(x,t)}\,\xi(x,t)$$

$$\overline{F(u,x)F(u',x')} = \delta^d(x - x') \frac{\mathrm{e}^{-\gamma|u-u'|}}{2\gamma}$$

disorder correlator in steady state

The ABBM model

B. Alessandro, C. Beatrice, G. Bertotti and A. Montorsi, J. Applied Phys. 68 (1990) 2901; ibid, 2908

A particle subjected to force which is a random walk:

$$\partial_t \dot u(t) = m^2 [v - \dot u(x,t)] + \partial_t F(u(t)) \qquad \langle [F(u) - F(u')]^2 \rangle = |u - u'|$$

$$\partial_t F(u(t)) = \sqrt{\dot u(t)} \xi(t) , \quad \langle \xi(t) \xi(t') \rangle = \delta(t - t')$$

MF = model for 1 degree of freedom = ABBM

Key Results

size and duration distributions

$$\mathscr{P}(S) \simeq S^{-3/2} \mathrm{e}^{-\frac{S}{4S_m}} \qquad \mathscr{P}(T) \simeq 1/\sinh^2\left(\frac{T}{2T_m}\right) \sim T^{-2}$$

steady state velocity distribution

$$\mathscr{P}(\dot u) \simeq \dot u^{v-1} \mathrm{e}^{-\dot u / \dot u_m}$$

shape at fixed duration T (small durations):

$$\langle \dot u(t) \rangle_T = t(1 - t/T)$$

shape at fixed size S (any size) $\langle \dot u(t) \rangle_S = \sqrt{S} \mathrm{e}^{-t^2/S}$

The Brownian force model (BFM)

PLD+KW, EPL 97 (2012) 46004; Phys. Rev. E 88 (2013) 022106

$$\partial_t \dot{u}(x,t) = \nabla^2 \dot{u}(x,t) + m^2 [v - \dot{u}(x,t)] + \partial_t F(u(x,t),x)$$

$$\partial_t F(u(x,t),x) = \sqrt{\dot{u}(x,t)}\xi(x,t) ,$$

$$\langle \xi(x,t)\xi(x't')\rangle = \delta^d(x-x')\delta(t-t')$$

(space dependent) field theory formulation for dynamics

THEOREM 1

the zero mode of the field theory is the same random process as ABBM

THEOREM 2

The field theory of this process = sum of all tree diagrams

Short-ranged rough disorder

AD+PLD+KW

$$\partial_t \dot{u}(x,t) = \nabla^2 \dot{u}(x,t) + m^2 [v - \dot{u}(x,t)] + \partial_t F(u(x,t),x)$$

force is an Ornstein-Uhlenbeck process

$$\partial_t F(u(x,t),x) = -\gamma \dot{u}(x,t) F(u(x,t),x) + \sqrt{\dot{u}(x,t)}\,\xi(x,t)$$

$$\langle \xi(x,t)\xi(x't')\rangle = \delta^d(x-x')\delta(t-t')$$

equivalent to (we use $\dot{u}(x,t) \geq 0$)

$$\partial_u F(u,x) = -\gamma F(u,x) + \tilde{\xi}(u,x)$$

$$\left\langle \tilde{\xi}(u,x)\tilde{\xi}(u',x') \right\rangle = \delta(u-u')\delta(x-x')$$

disorder correlator in steady state is short-ranged

$$\overline{F(u,x)F(u',x')} = \delta^d(x-x')\frac{\mathrm{e}^{-\gamma|u-u'|}}{2\gamma}$$

A tiny little bit of field theory...

Langevin equation

$$\eta \partial_t u(x,t) = \nabla^2 u(x,t) + m^2 \left[w - u(x,t)\right] + F\left(x, u(x,t)\right)$$

this is now a theory of the velocity, not of the position:

$$S = \int_{x,t} \tilde{u}(x,t)\left[\eta \partial_t \dot{u}(x,t) - \nabla^2 \dot{u}(x,t) + m^2\left(\dot{w} - \dot{u}(x,t)\right)\right] - \lambda(x,t)\dot{u}(x,t)$$

$$- \int_{x,t,t'} \tilde{u}(x,t)\tilde{u}(x,t')\partial_t \partial_{t'} \Delta\left(u(x,t) - u(x,t')\right)$$

Disorder Vertex:

$$\begin{aligned} &\partial_t \partial_{t'} \Delta(v(t-t') + u_{xt} - u_{xt'}) \\ &= (v + \dot{u}_{xt})\partial_{t'} \Delta'(v(t-t') + u_{xt} - u_{xt'}) \\ &= (v + \dot{u}_{xt})\Delta'(0^+)\partial_{t'} \mathrm{sgn}(t-t') + \ldots \end{aligned}$$

simplifies to

$$S_{\mathrm{dis}}^{\mathrm{tree}} = \Delta'(0^+) \int_{xt} \tilde{u}_{xt}\tilde{u}_{xt}(v + \dot{u}_{xt})$$

simple local cubic theory = Brownian Force model (BFM)

Avalanche Instanton

Since the action is linear in $\dot{u}(x,t)$, the instanton equation

$\frac{\delta \mathscr{S}[\dot{u},\tilde{u}]}{\dot{u}(x,t)} = 0$ is exact:

$$(\partial_t - m^2 + \nabla^2)\tilde{u}(x,t) + |\Delta'(0^+)|\tilde{u}(x,t)^2 = -\lambda(x,t)$$

For $\lambda(x,t) = \lambda\delta(t)$ and setting $m^2 = |\Delta'(0^+)| = 1$:

$$(\partial_t - 1)\tilde{u}_t + \tilde{u}_t^2 = -\lambda\delta(t)$$

Solution $\tilde{u}_t = \frac{\lambda}{\lambda + (1-\lambda)e^{-t}}\theta(-t)$

$$Z_{\text{tree}}(\lambda) = \left\langle \mathrm{e}^{\lambda \dot{u}(t)} - 1 \right\rangle\Big|_{t=0} = \int_{t<0} \tilde{u}_t = -\ln(1-\lambda)$$

$$\mathscr{P}_{\text{tree}}(\dot{u}) = \frac{\mathrm{e}^{-\dot{u}}}{\dot{u}}$$

MF
= ABBM
for COM
observables

higher-point functions also possible.

Scaling laws

suppose that there is a small-*m* limit of response to kick

$$\lim_{m\to 0} \frac{\delta u(x,t)}{\delta f} = \text{finite} \Leftrightarrow \tilde{u}(x,t) \text{ unrenormalized}$$

This implies a plethora of scaling laws:

	$\mathscr{P}(S)$	$\mathscr{P}(S_\phi)$	$\mathscr{P}(T)$	$\mathscr{P}(\dot{u})$	$\mathscr{P}(\dot{u}_\phi)$
	$S^{-\tau}$	$S_\phi^{-\tau_\phi}$	$T^{-\alpha}$	$\dot{u}^{-\mathsf{a}}$	$\dot{u}_\phi^{-\mathsf{a}_\phi}$
SR	$\tau = 2 - \frac{2}{d+\zeta}$	$\tau_\phi = 2 - \frac{2}{d_\phi+\zeta}$	$\alpha = 1 + \frac{d-2+\zeta}{z}$	$\mathsf{a} = 2 - \frac{2}{d+\zeta-z}$	$\mathsf{a}_\phi = 2 - \frac{2}{d_\phi+\zeta-z}$
LR	$\tau = 2 - \frac{1}{d+\zeta}$	$\tau_\phi = 2 - \frac{1}{d_\phi+\zeta}$	$\alpha = 1 + \frac{d-1+\zeta}{z}$	$\mathsf{a} = 2 - \frac{1}{d+\zeta-z}$	$\mathsf{a}_\phi = 2 - \frac{1}{d_\phi+\zeta-z}$

	d	ζ	z	τ	τ_ϕ	α	a	γ
SR	1	1.25	1.433	1.11	0.4	1.17	−0.45	1.57
	2	0.75	1.56	1.27	−0.67	1.48	0.32	1.76
	3	0.35	1.75	1.40	−3.71	1.77	0.75	1.91
LR	1	0.39	0.77	1.28	−0.56	1.51	0.39	1.81

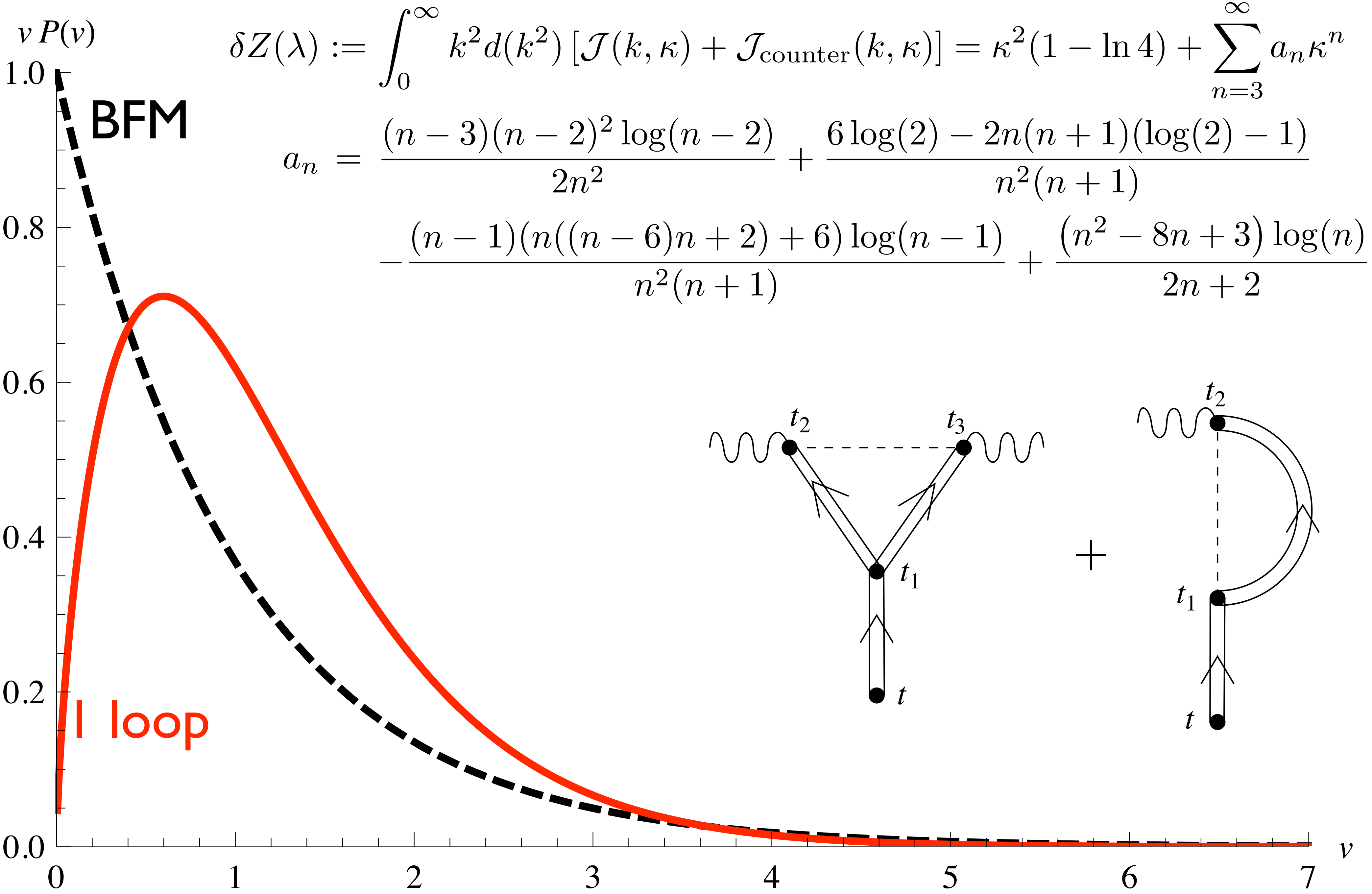
$$S \sim_{S \ll 1} T^\gamma$$

$$\gamma = \frac{d+\zeta}{z}$$

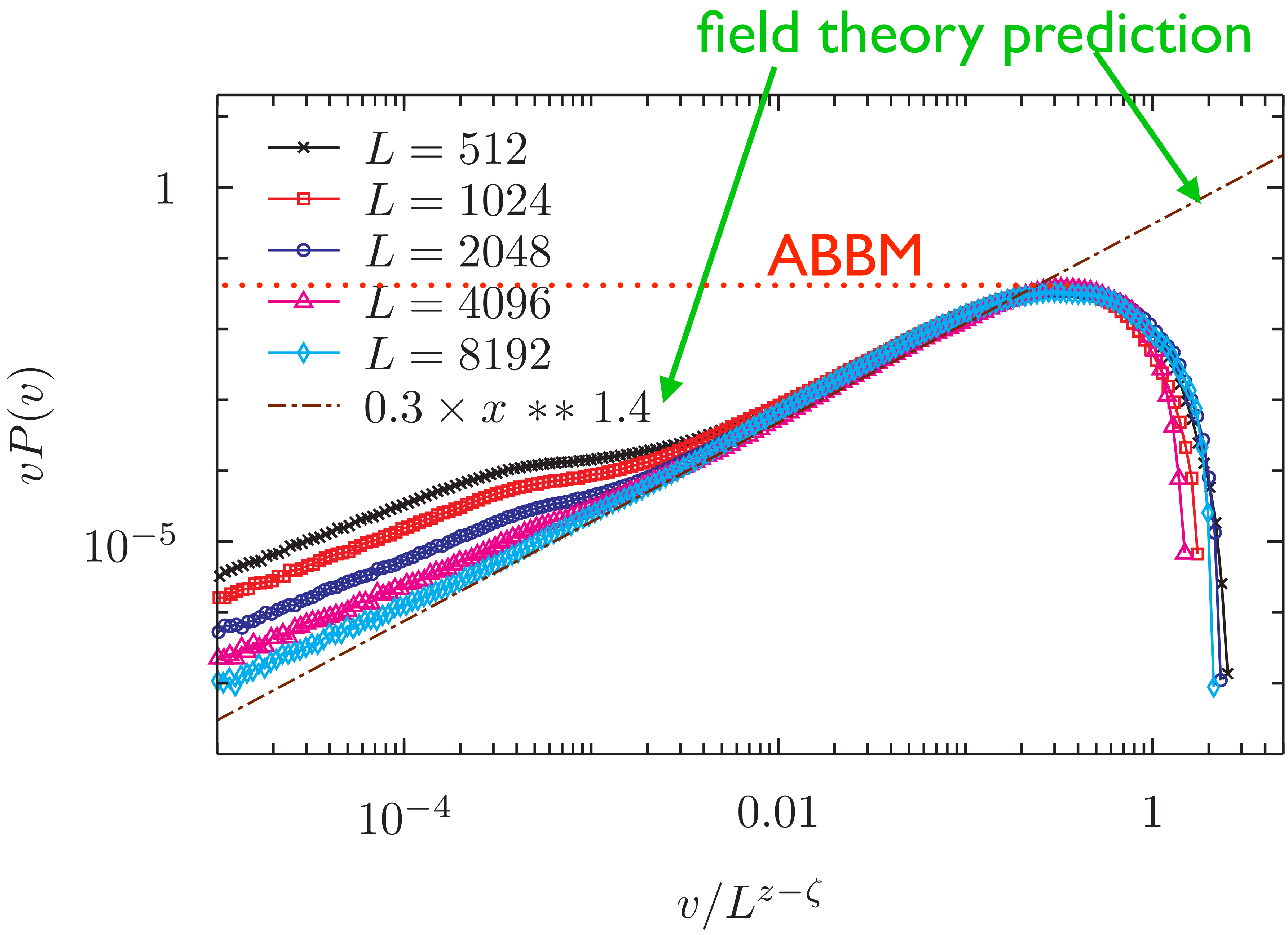
Velocity distribution in avalanche: tree + loops

$$\delta Z(\lambda) := \int_0^\infty k^2 d(k^2)\left[\mathcal{J}(k,\kappa) + \mathcal{J}_{\text{counter}}(k,\kappa)\right] = \kappa^2(1 - \ln 4) + \sum_{n=3}^{\infty} a_n \kappa^n$$

$$a_n = \frac{(n-3)(n-2)^2 \log(n-2)}{2n^2} + \frac{6\log(2) - 2n(n+1)(\log(2)-1)}{n^2(n+1)} - \frac{(n-1)(n((n-6)n+2)+6)\log(n-1)}{n^2(n+1)} + \frac{\left(n^2 - 8n + 3\right)\log(n)}{2n+2}$$



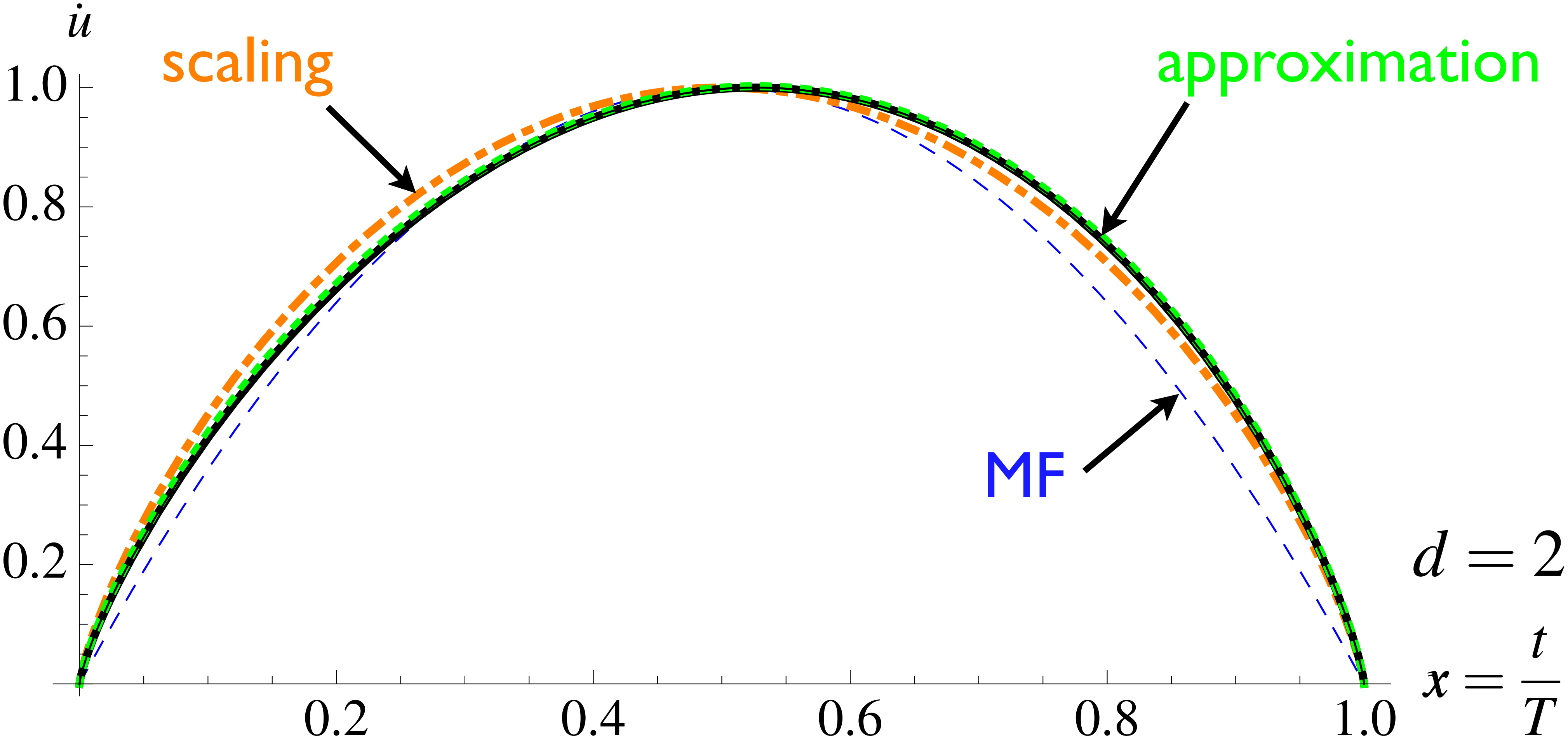
Preliminary data by Alejandro Kolton



Shape at fixed duration

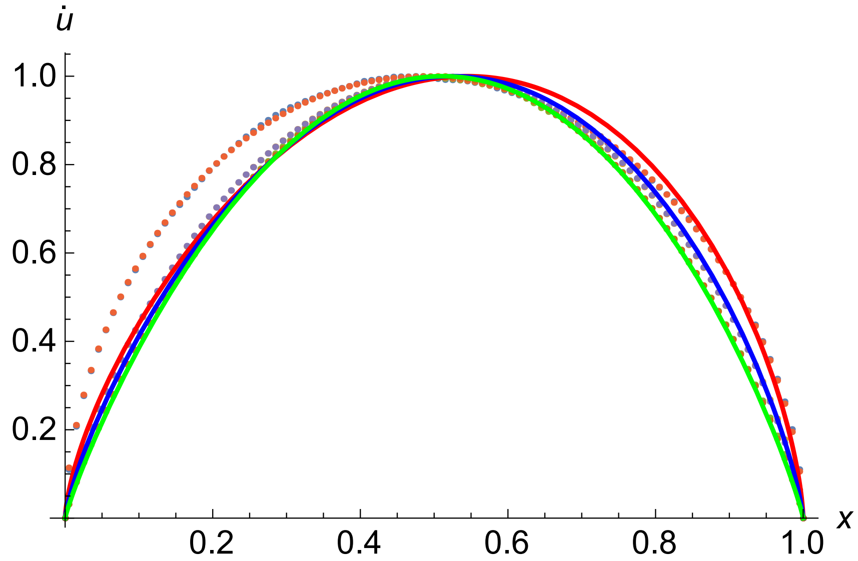
$$\left\langle \dot{u}\left(x=\frac{t}{T}\right)\right\rangle = \mathcal{N}\Big[Tx(1-x)\Big]^{1+\frac{2\alpha}{d_c}} \exp\left(\frac{8\alpha}{d_c}\left[\mathrm{Li}_2(1-x)-\mathrm{Li}_2\left(\frac{1-x}{2}\right)+\frac{x\log(2x)}{x-1}+\frac{(x+1)\log(x+1)}{2(1-x)}\right]\right)$$

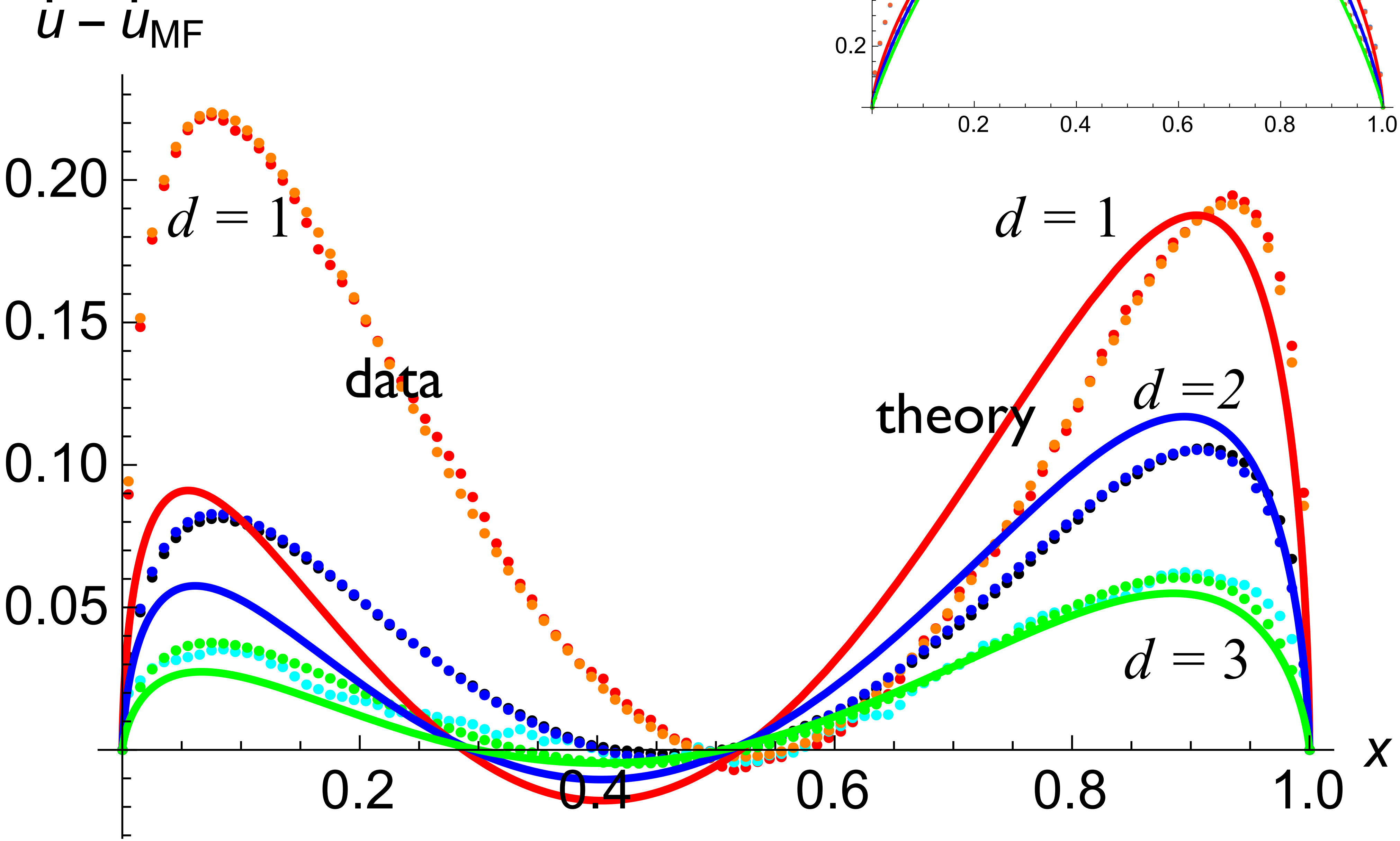
$$\langle \dot{u}(x)\rangle \simeq \Big[Tx(1-x)\Big]^{\gamma-1}\exp\left(\mathcal{A}\left[\frac{1}{2}-x\right]\right) \qquad \mathcal{A}\approx -0.336\left(1-\frac{d}{d_c}\right)$$



The shape at fixed duration

data by Lasse Laurson





The shape at fixed duration

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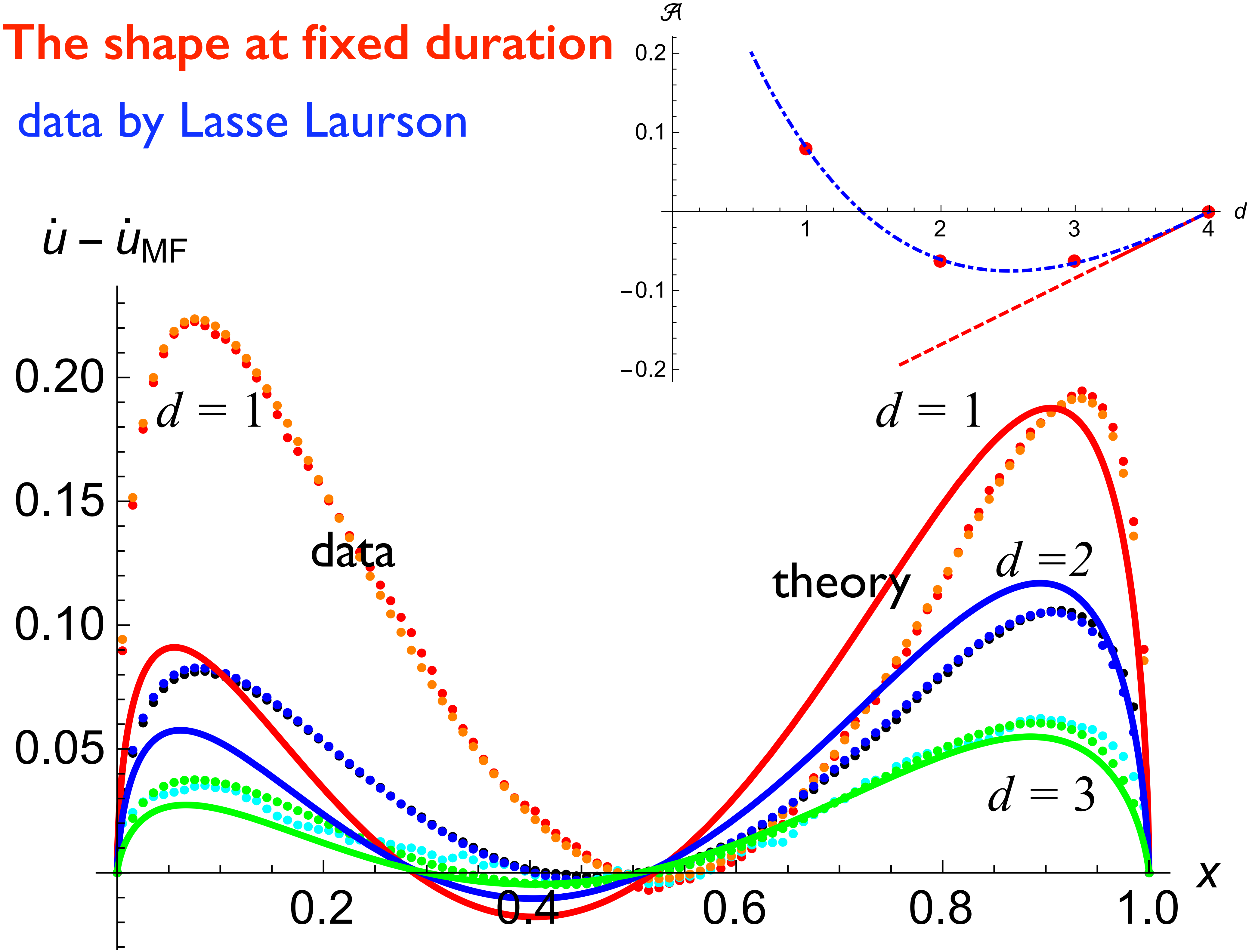
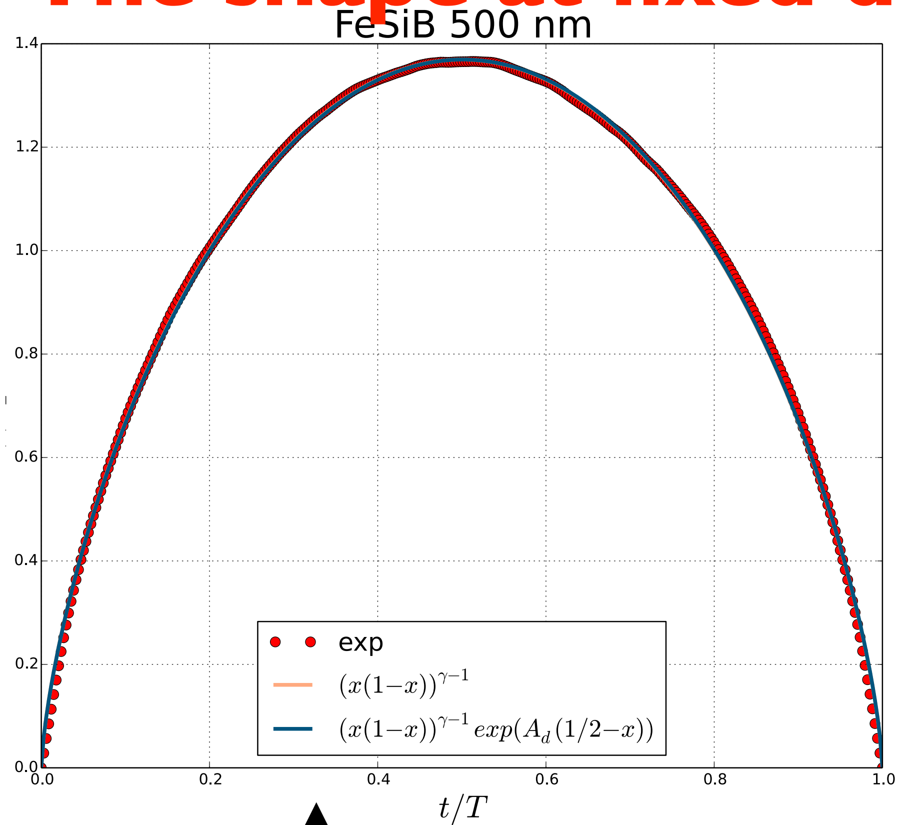
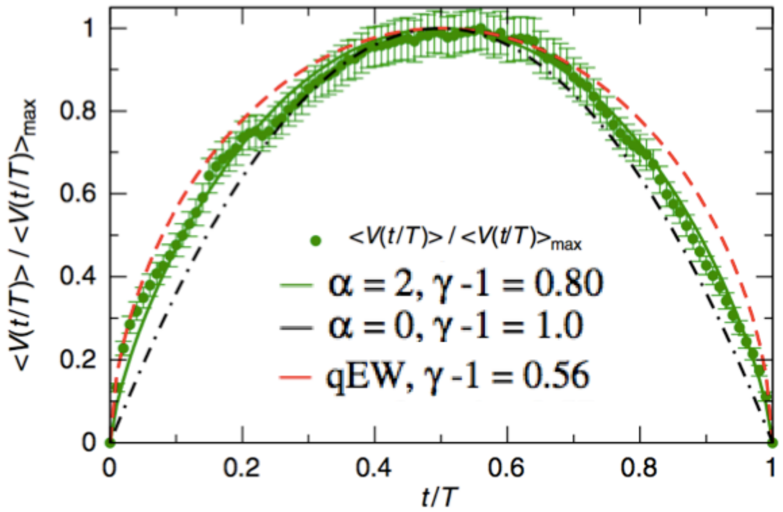


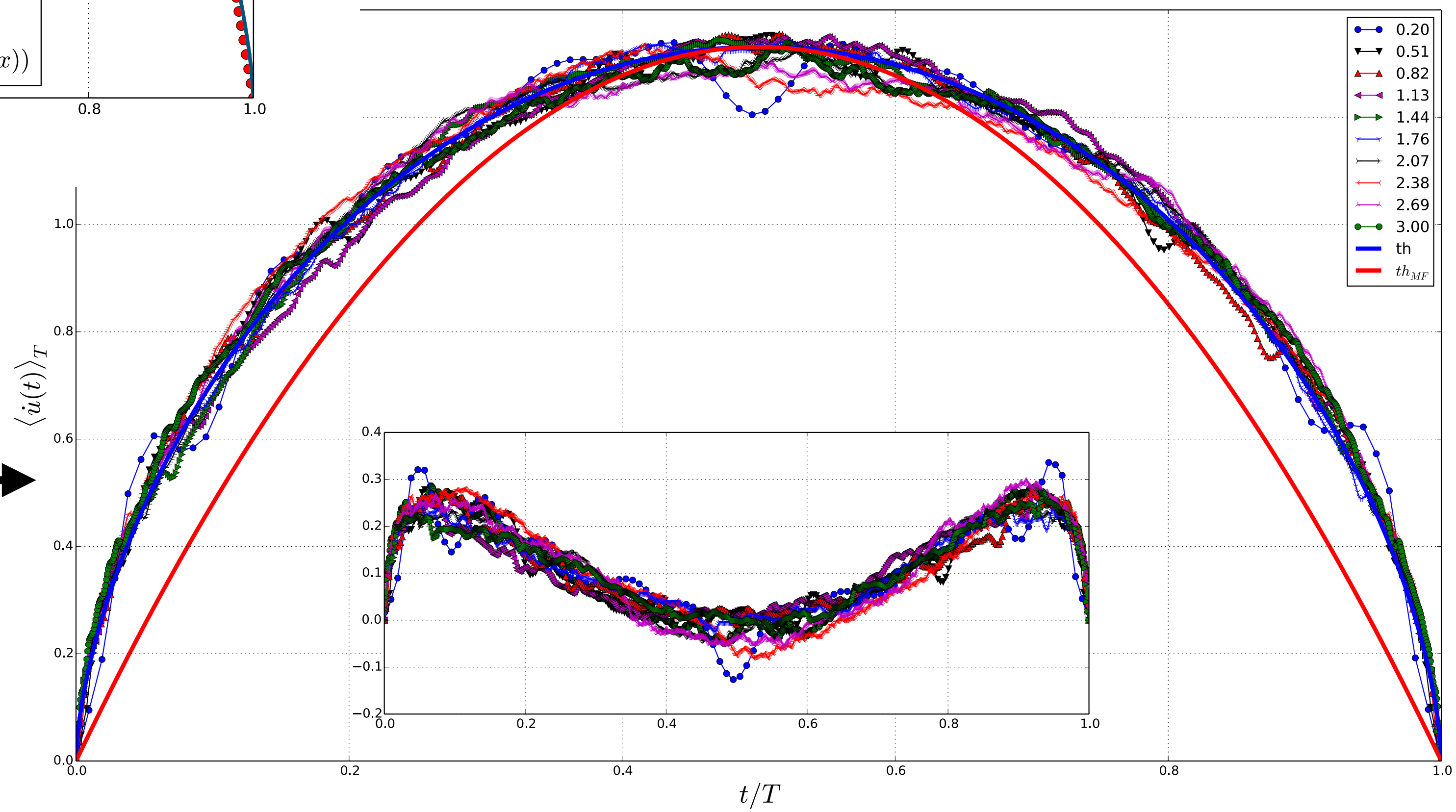
The shape at fixed duration





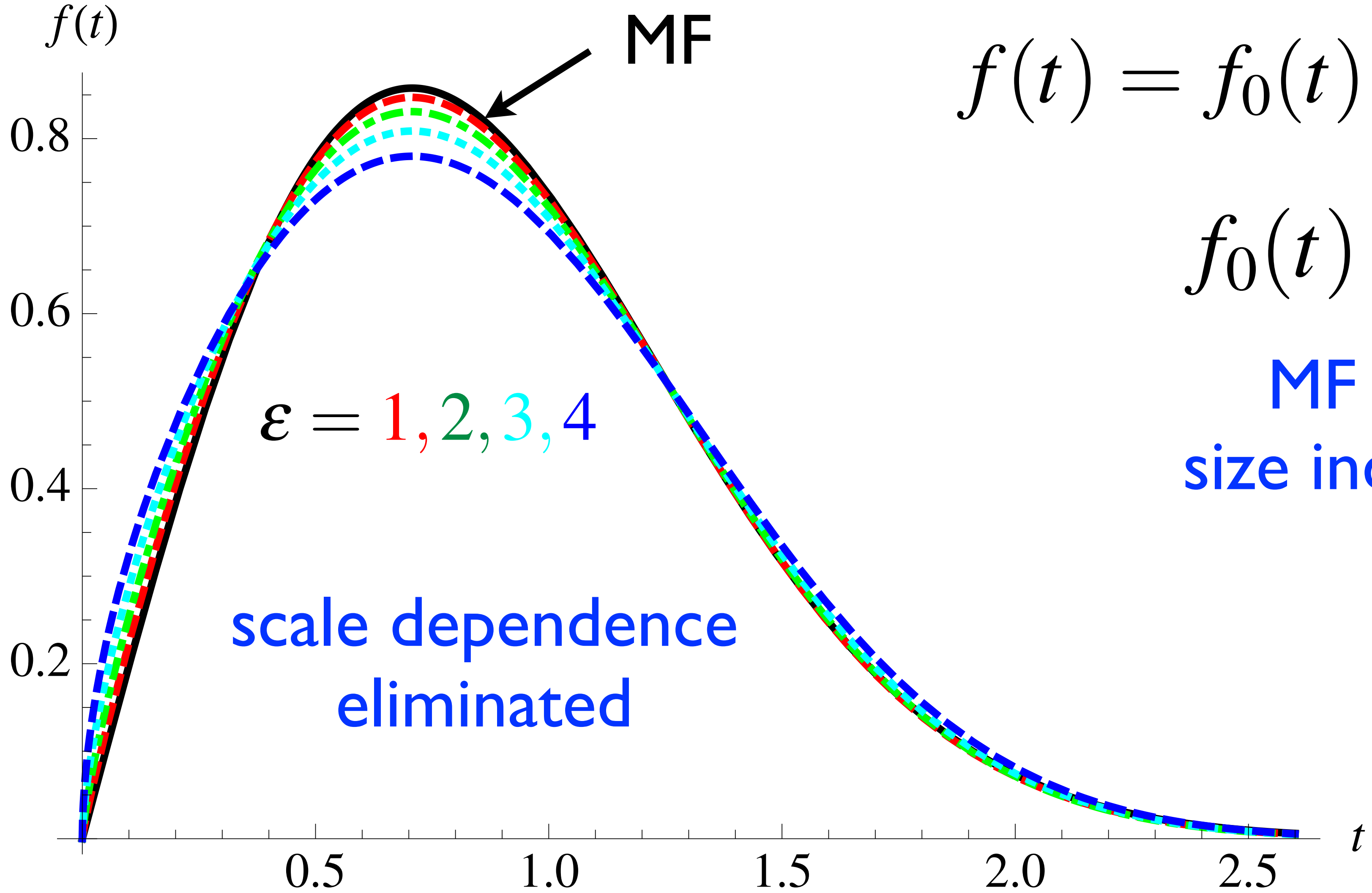
fracture: S. Santucci

data: G. Durin, F. Bohn Barkhausen results



Shape at fixed (small) size

$$\dot{u}(t,S) = S\left(\frac{S}{S_m}\right)^{-\frac{1}{\gamma}} f\left(\frac{t}{\tau_m} \Big/ \left(\frac{S}{S_m}\right)^{\frac{1}{\gamma}}\right)$$



$$f(t) = f_0(t) + \frac{\alpha}{2}\delta f(t)$$

$$f_0(t) = 2te^{-t^2}$$

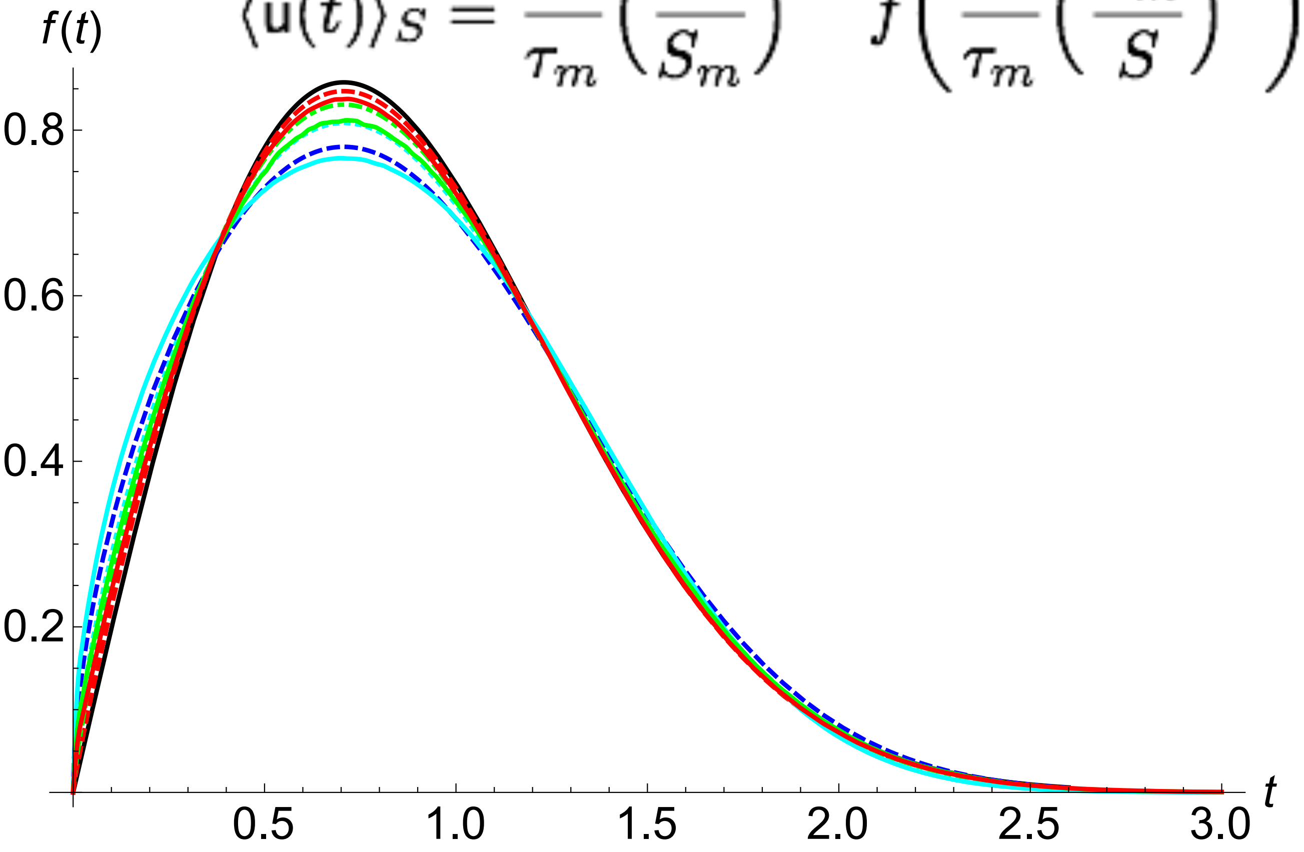
MF shape is size independent!

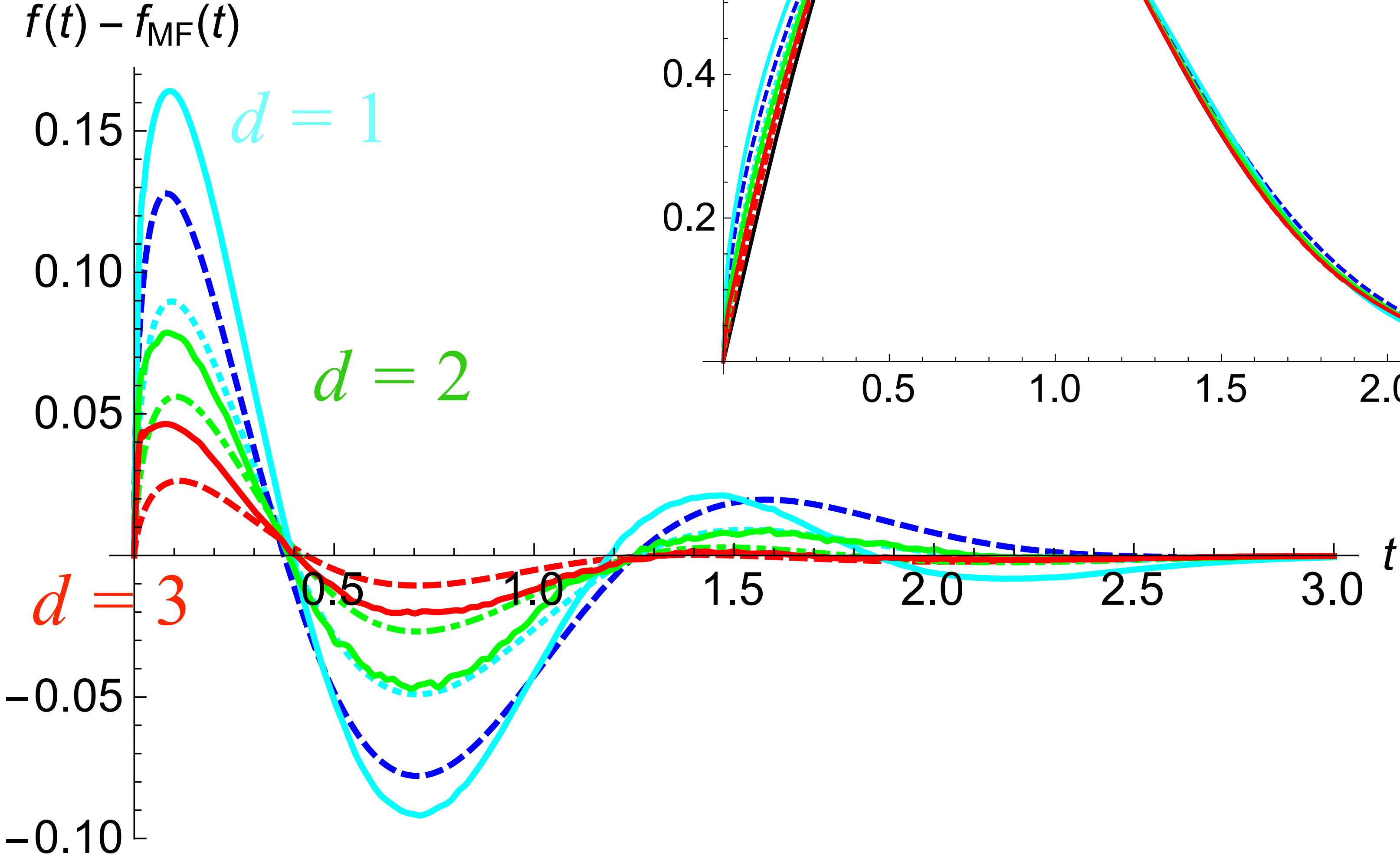
The shape at fixed size

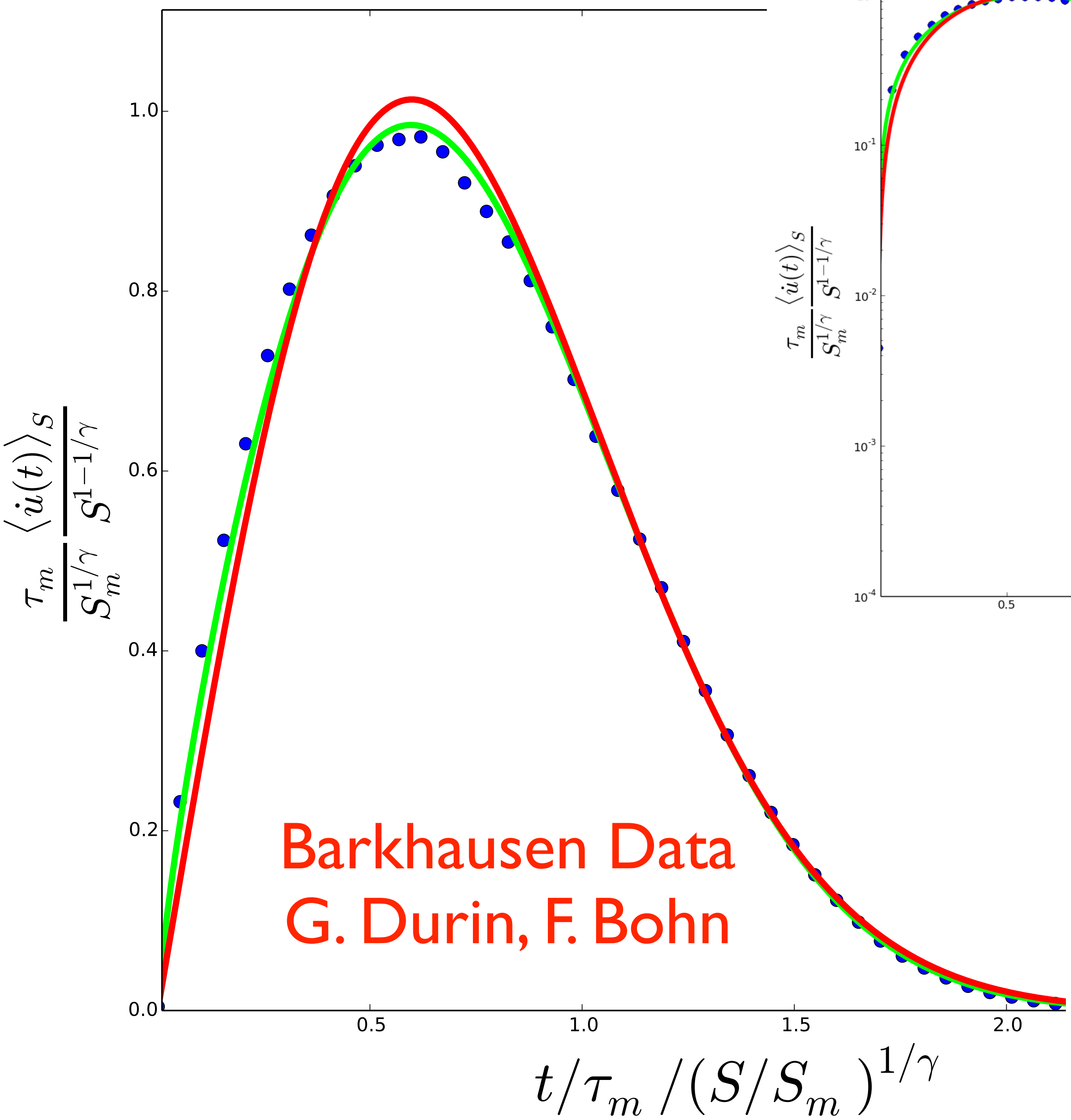
data (Lasse) = solid

theory = dashed

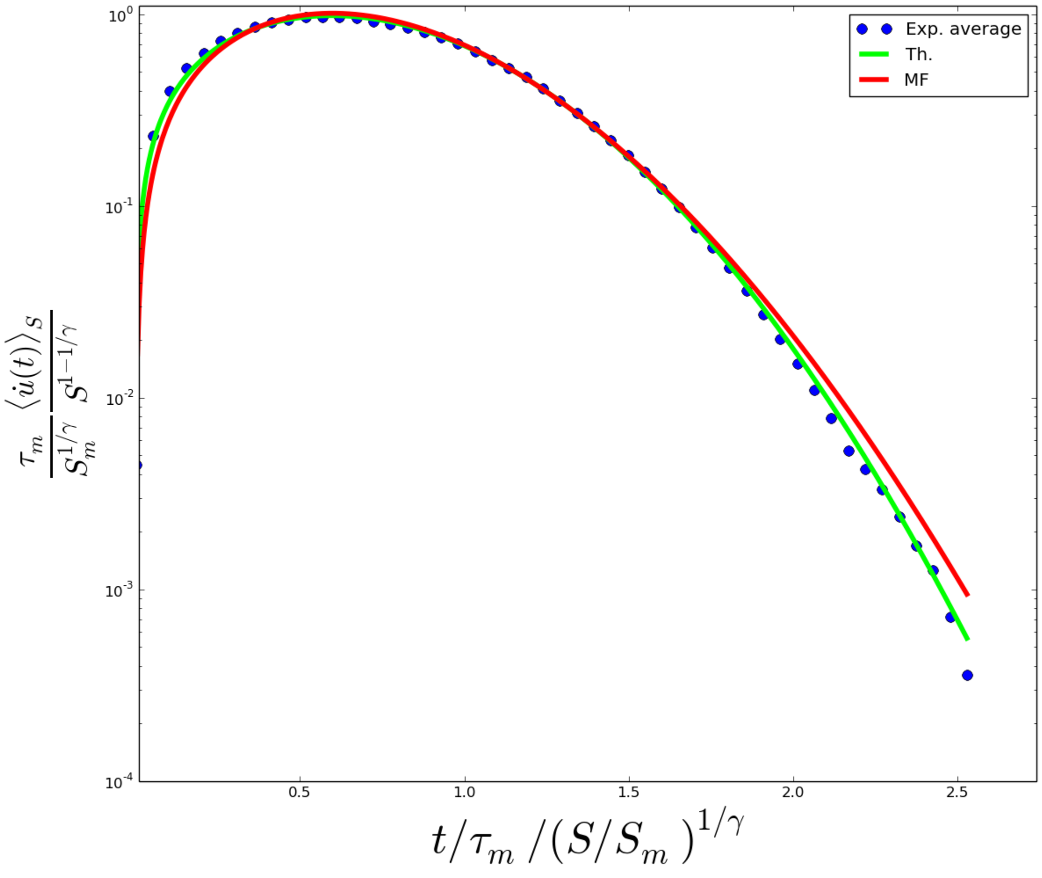
$$\langle \dot{u}(t) \rangle_S = \frac{S}{\tau_m} \left(\frac{S}{S_m} \right)^{-\frac{1}{\gamma}} f\left(\frac{t}{\tau_m} \left(\frac{S_m}{S} \right)^{\frac{1}{\gamma}} \right)$$



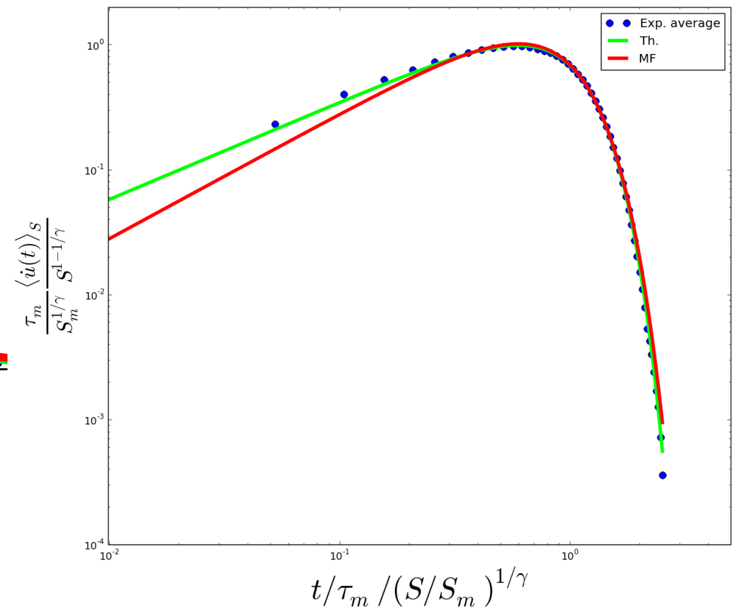




Barkhausen Data
G. Durin, F. Bohn
$\frac{\tau_m}{S_m^{1/\gamma}} \frac{\langle \dot{u}(t) \rangle_S}{S^{1-1/\gamma}}$
$t/\tau_m/(S/S_m)^{1/\gamma}$
0.0
0.2
0.4
0.6
0.8
1.0
0.5
1.0
1.5
2.0

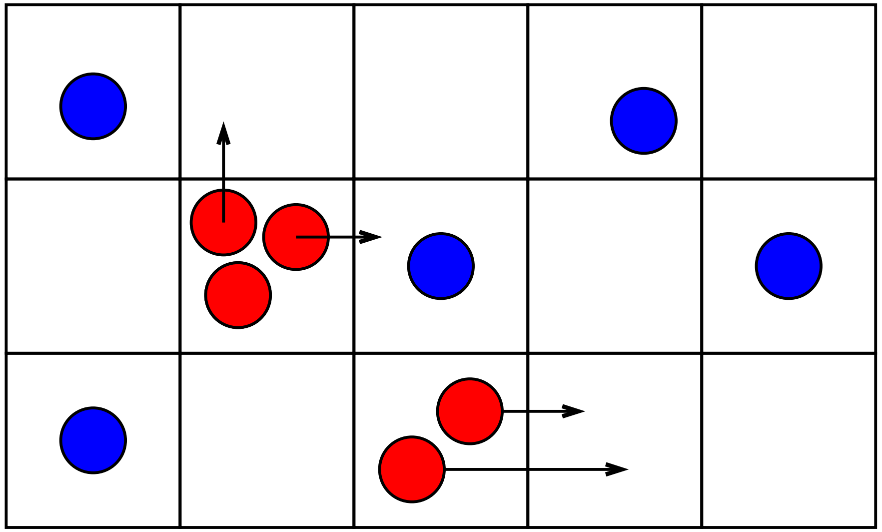


Exp. average
Th.
MF
$\frac{\tau_m}{S_m^{1/\gamma}} \frac{\langle \dot{u}(t) \rangle_S}{S^{1-1/\gamma}}$
$t/\tau_m/(S/S_m)^{1/\gamma}$
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
0.5
1.0
1.5
2.0
2.5



Exp. average
Th.
MF
$\frac{\tau_m}{S_m^{1/\gamma}} \frac{\langle \dot{u}(t) \rangle_S}{S^{1-1/\gamma}}$
$t/\tau_m/(S/S_m)^{1/\gamma}$

Manna sandpiles to C-DP



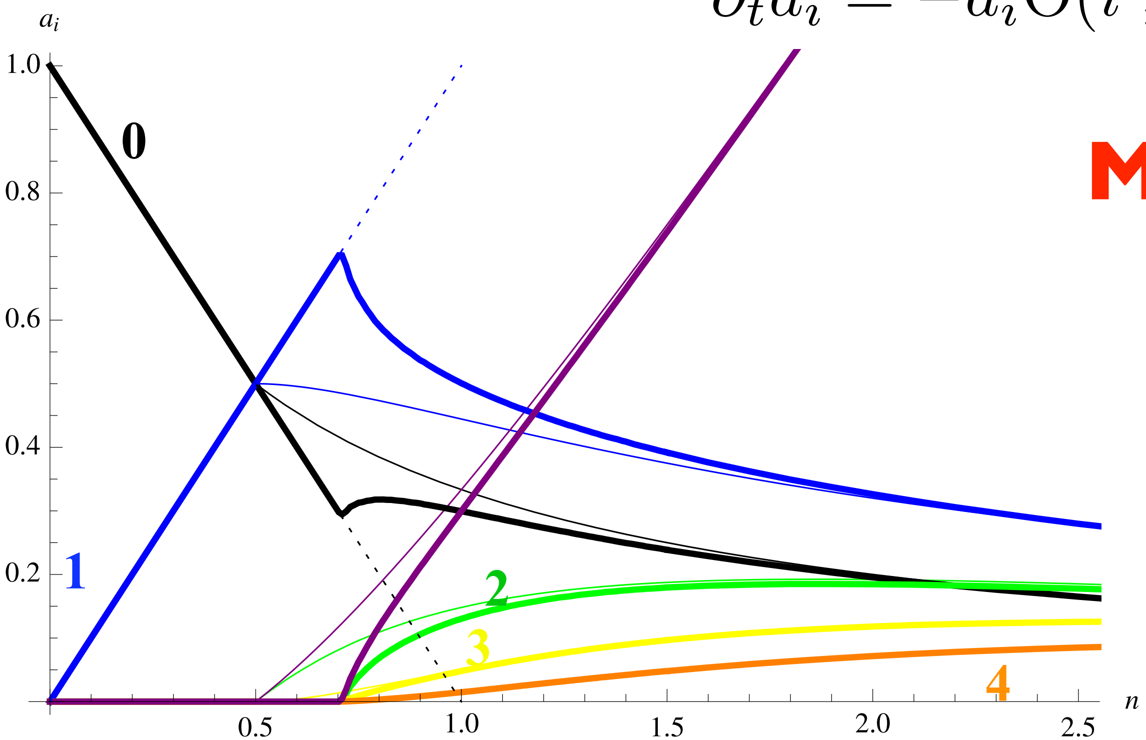
KW, arXiv:1501.06514

Manna sandpile rule: If 2 or more grains are on a site, topple them to randomly chosen neighbours.

2 grains can end up on same site.

Mean-Field Equations

$$\partial_t a_i = -a_i \Theta(i \geq 2) + a_{i+2} + 2\Big[\sum_{j\geq 2} a_j\Big](a_{i-1} - a_i)$$



Mean-Field Solution

$$a_0 = \frac{1}{1+2n},$$

$$a_{i>0} = \frac{4n\left(\frac{2n-1}{2n+1}\right)^i}{4n^2-1}.$$

fraction of sites with ***i*** grains

Beyond Mean-Field

number of grains $n := \sum_i a_i\, i$

empty sites $e := a_0$ **activity** $\rho := \sum_{i \geq 2} a_i (i-1)$

sum rules $\sum_i a_i = 1$ **and** $n - \rho + e = 1$

The CDP field theory

MF

activity

$$\partial_t \rho(x,t) = \frac{1}{d} \nabla^2 \rho(x,t) + \left[2n(x,t) - 1\right] \rho(x,t)$$

$$- 2\rho(x,t)^2 + \sqrt{2\rho(x,t)}\, \xi(x,t)$$

number of grains

$$\partial_t n(x,t) = \frac{1}{d} \nabla^2 \rho(x,t)$$

noise

$$\langle \xi(x,t) \xi(x',t') \rangle = \delta^d (x - x') \delta(t - t') \, .$$

The C-DP field theory

PLD+KW, PRL arXiv:1410.1930

activity

$$\partial_t \rho(x,t) = [n(x,t)-1]\rho(x,t) - \rho(x,t)^2 + \nabla^2 \rho(x,t) + \sqrt{\rho(x,t)}\,\xi(x,t)$$

number of grains

noise

$$\partial_t n(x,t) = (\nabla^2 - m^2)\rho(x,t)$$

"dissipation"

$$\langle \xi(x,t)\xi(x',t')\rangle = \delta^d(x-x')\delta(t-t')$$

Change of variables to random manifold:

$$\dot{u}(x,t) := \rho(x,t)\ , \qquad F(x,t) = \rho(x,t) - n(x,t) + 1$$

$= e(x,t)$

leads to

$$\partial_t \dot{u}(x,t) = (\nabla^2 - m^2)\dot{u}(x,t) + \partial_t F(x,t)$$

$$\partial_t F(x,t) = -F(x,t)\dot{u}(x,t) + \sqrt{\dot{u}(x,t)}\,\xi(x,t)$$

an interface in a short-range correlated disorder!

Some references for our work on Avalanches

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Conclusions

- ABBM model = MF model for avalanches
- Brownian force model (BFM) = field theory
- zero-mode of BFM equivalent to ABBM = MF
- field theory can be constructed in an expansion around the upper critical dimension
- non-trivial scaling relations and functions in all dimensions
- Manna sandpile = CDP = disordered elastic manifolds
- many theoretical results in search for high-precision experiments

Title: The Field theory of avalanches

Abstract: When elastic systems like contact lines on a rough substrate, domain walls in disordered magnets, or tectonic plates are driven slowly, they remain immobile most of the time, before responding with strong intermittent motion, termed avalanche. I will describe the field theory behind these phenomena, explain why its effective action has a cusp, and how such intricate objects as the temporal shape of an avalanche can be obtained.