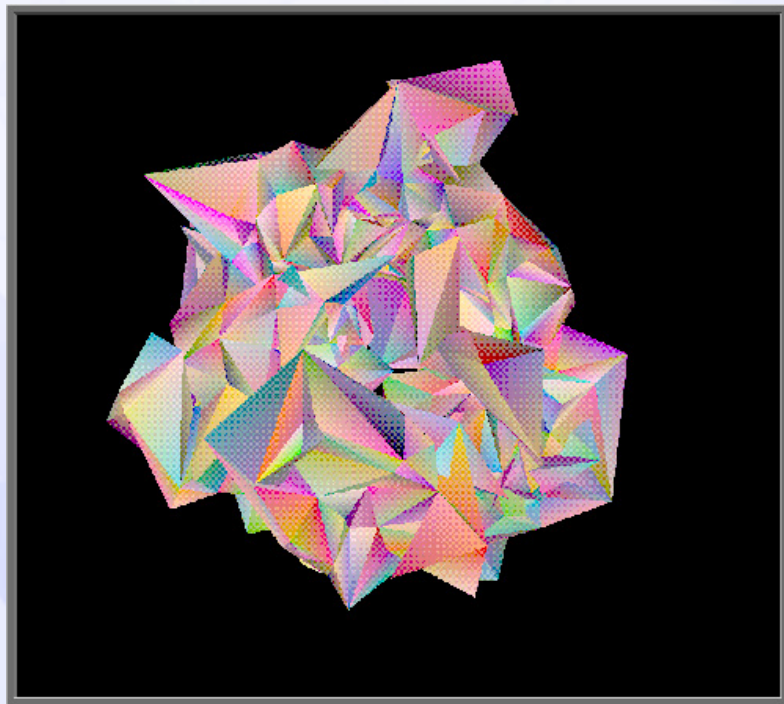


Causal Dynamical Triangulations in 4D - the plot thickens



triangulated model of quantum space

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20th Itzykson Conference, Saclay

12 Jun 2015

Going back in history ...

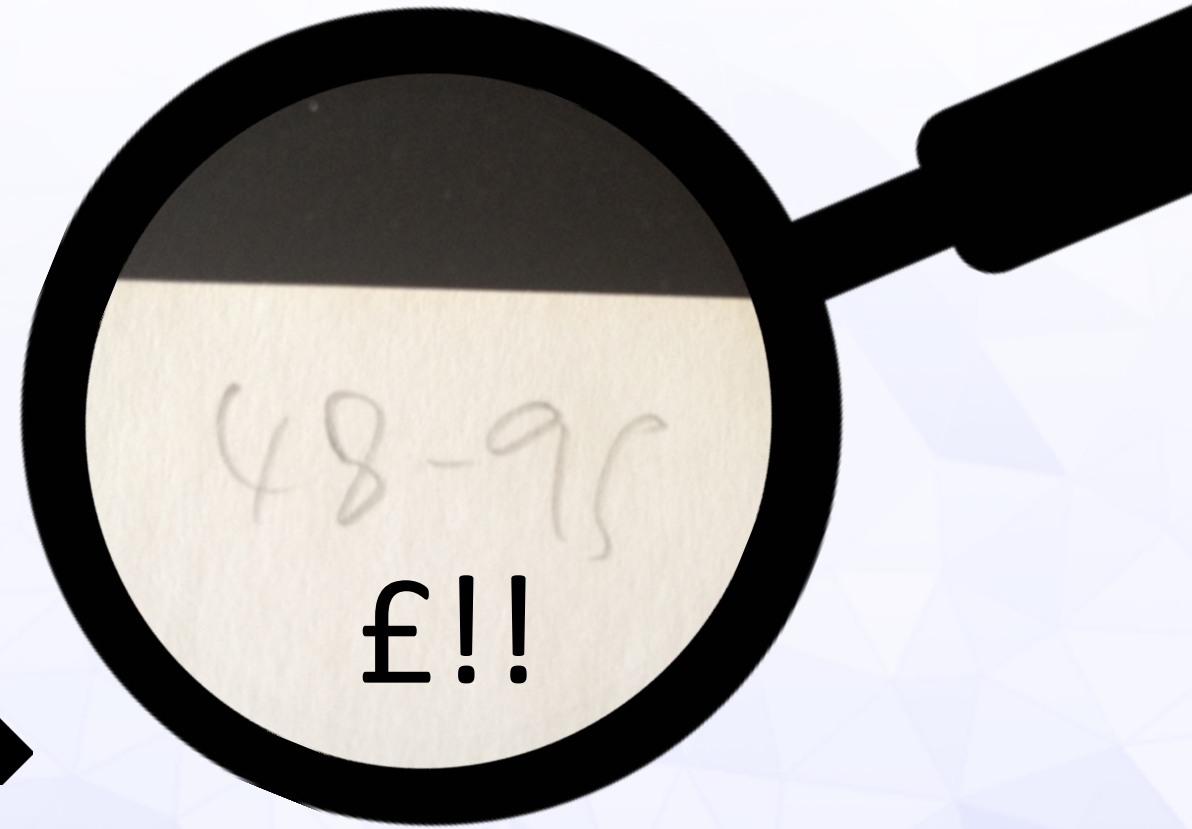
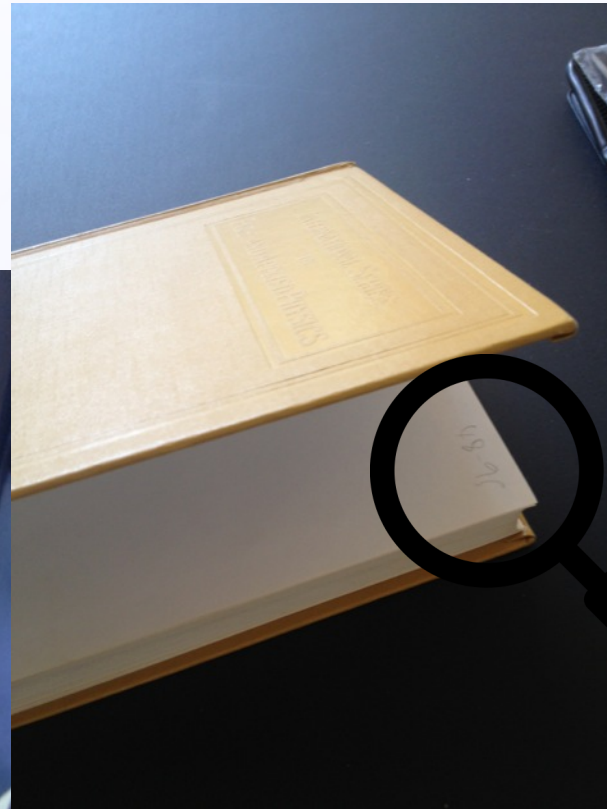
London 1985:
a great book ...



however ...

Going back in history ...

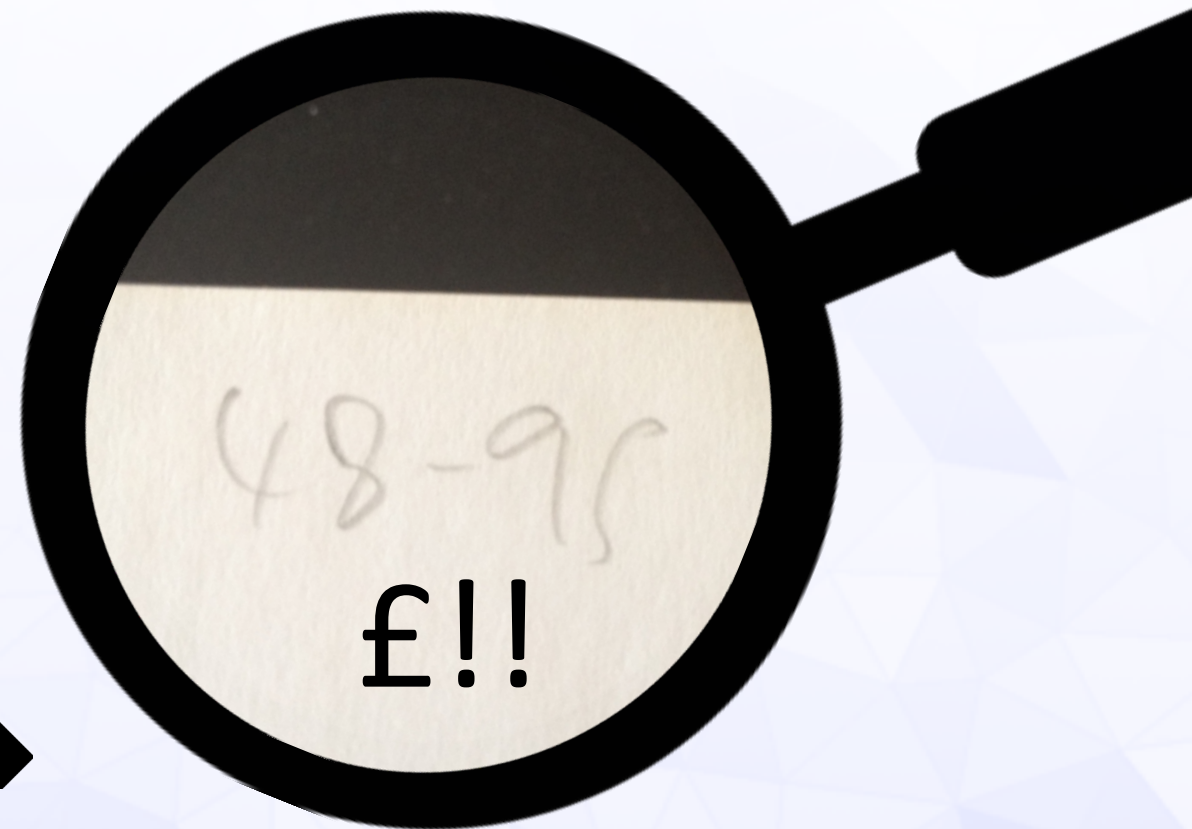
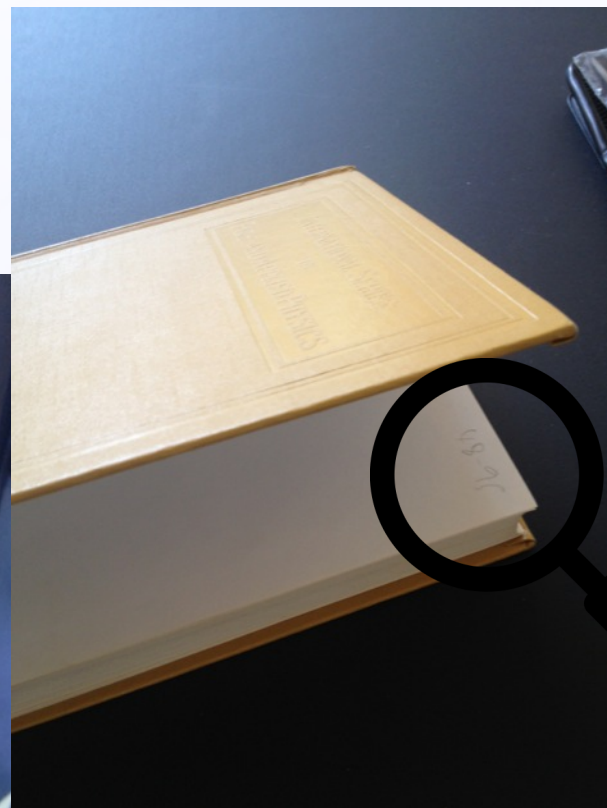
London 1985:
a great book ...



however ... two weeks' rent!

Going back in history ...

London 1985:
a great book ...



however ... two weeks' rent!

At roughly the same time, fruitful ideas involving two-dimensional random geometry emerged and found many applications, starting with the attempt to describe nonperturbative dynamics of 2D world sheets of bosonic strings. Today's story: a further development of the idea of random geometry “with a twist”, leading to a candidate theory of quantum gravity in 4D in terms of Causal Dynamical Triangulations (CDT).

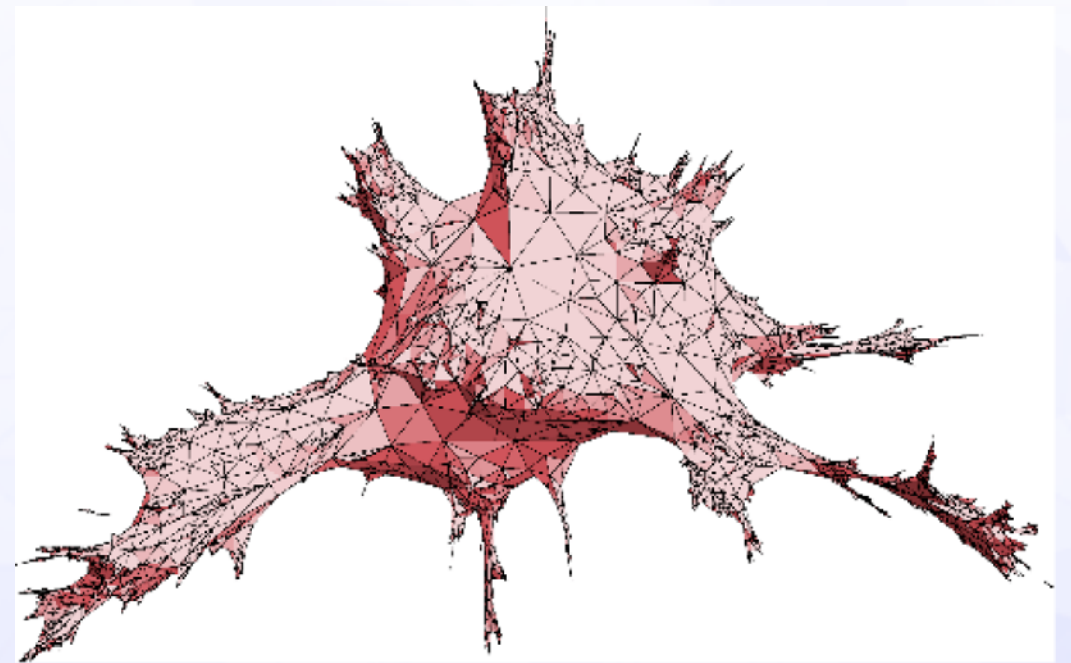
What is the overall outlook of CDT quantum gravity?

- we find many new results by looking at a *concrete, specific* model of quantum gravity; scarcity of free parameters considered an asset
- health warning: my primary motivation is physics
- explicit framework; concrete, quantitative results: falsifiable!

(J. Ambjørn, A. Görlich, J. Jurkiewicz & RL, “Nonperturbative Quantum Gravity”, Physics Report 519 (2012) 127 [arXiv: 1203.3591])

Which insights (in part empirical) are behind its success so far?

- For the purposes of the nonperturbative quantum theory, assembling (piecewise flat) curved manifolds from identical triangular building blocks explores the space of all geometries very effectively.
- (C)DT is very close in spirit to Einstein's "rods and clocks": intrinsic geometry of spacetime is determined by measuring space- and timelike distances (like we understand the curvature of the universe at large) - the local metric is a derived object.
- CDT is "as simple as it can be, but not simpler": random geometry must be replaced by *causal* random geometry to obtain a good classical limit in four dimensions (DT \rightarrow CDT).



Typical path integral history (glued from triangles in 2d quantum gravity)

Quantum Gravity from CDT★

is a *nonperturbative* implementation of the gravitational path integral,

$$Z(G_N, \Lambda) = \int_{g \in \mathcal{G}} \mathcal{D}g \, e^{iS_{G_N, \Lambda}^{\text{EH}}[g]}$$

Newton's constant $\rightarrow G_N$
cosmological constant $\rightarrow \Lambda$
spacetimes $g \in \mathcal{G}$
Einstein-Hilbert action $\rightarrow S_{G_N, \Lambda}^{\text{EH}}[g]$

much in the spirit of lattice quantum field theory, but based on *dynamical* triangular lattices, reflecting the dynamical nature of spacetime geometry:

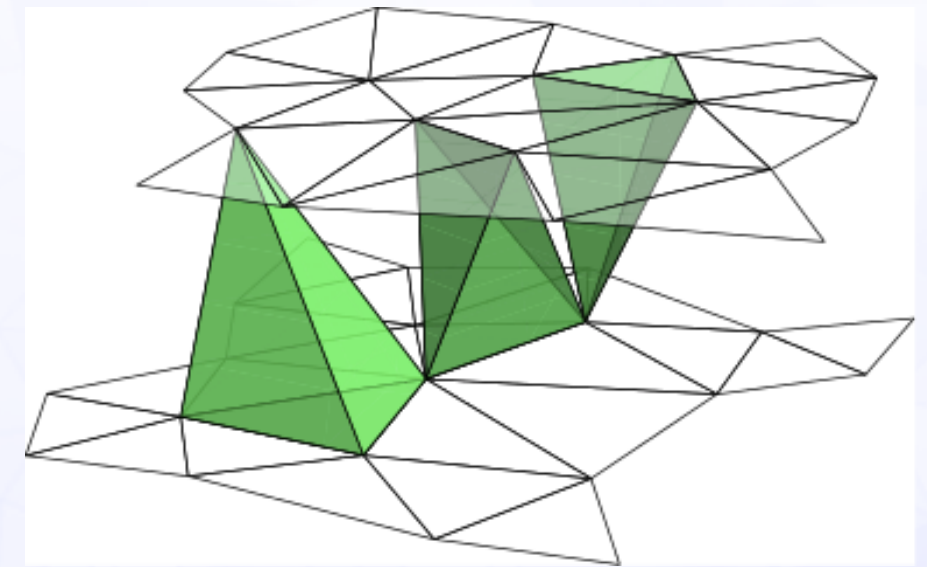
$$Z(G_N, \Lambda) := \lim_{\substack{a \rightarrow 0 \\ N \rightarrow \infty}} \sum_{\substack{\text{inequiv.} \\ \text{triangul.s} \\ T \in \mathcal{G}_{a, N}}} \frac{1}{C(T)} e^{iS_{G_N, \Lambda}^{\text{Regge}}[T]}$$

UV cutoff $\rightarrow a$
building blocks $\rightarrow N$
inequiv. triangul.s $T \in \mathcal{G}_{a, N}$
 $C(T) \rightarrow |\text{Aut}(T)|$

★ some recent contributors: J. Ambjørn, D. Benedetti, T. Budd, J. Cooperman, D. Coumbe, B. Durhuus, J. Gizbert-Studnicki, L. Glaser, A. Görlich, J. Henson, A. Ipsen, T. Jonsson, S. Jordan, J. Jurkiewicz, A. Kreienbuehl, J. Laiho, B. Ruijl, Y. Sato, Y. Watabiki, J. Wheeler, H.-G. Zhang ...

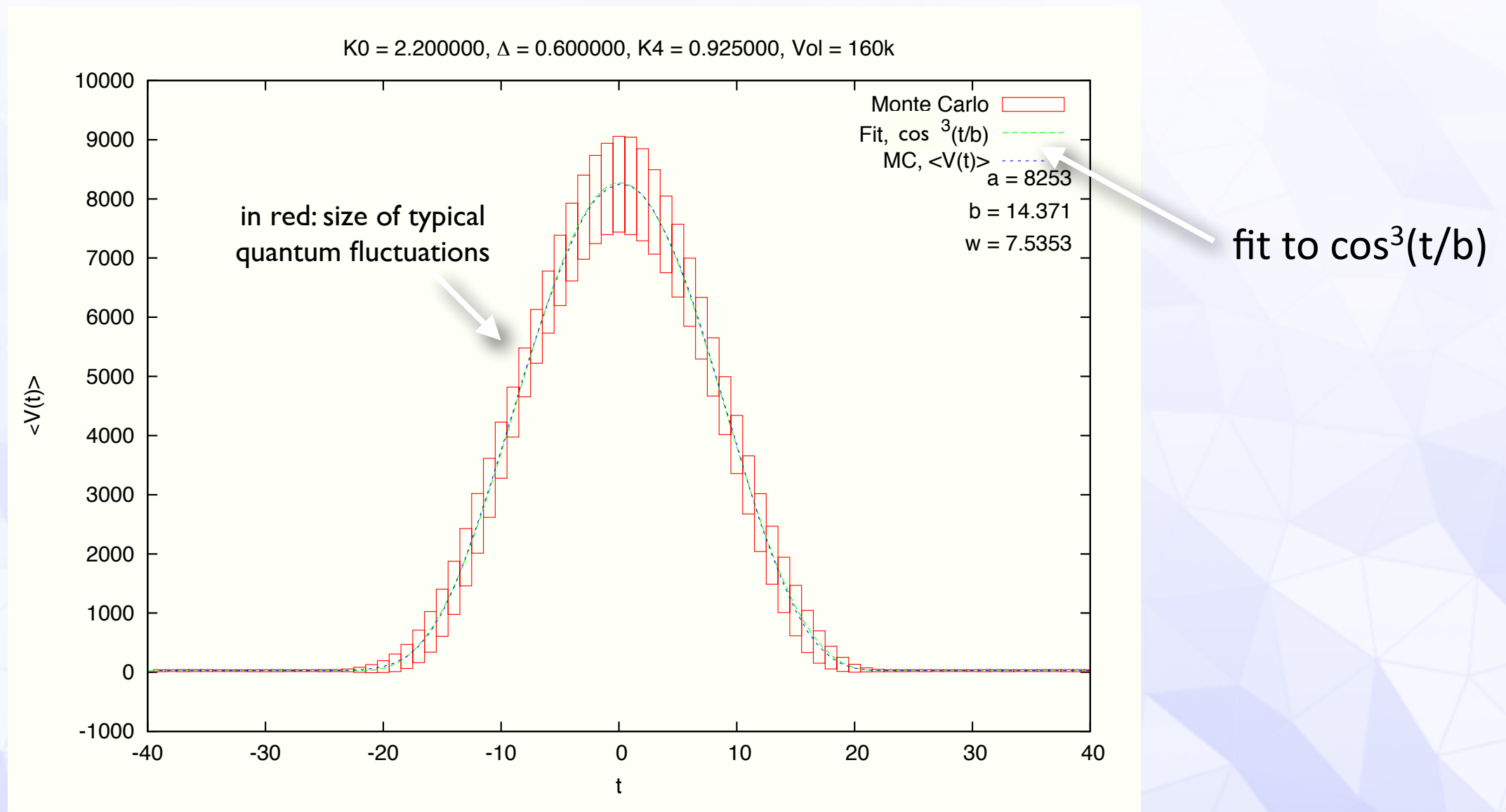
Key properties and ingredients of the CDT approach to quantum gravity

- CDT uses few ingredients/priors:
 - path integral/quantum superposition principle
 - locality and causal structure (this is *not* Euclidean quantum gravity)
 - notion of (proper) time; can be relaxed
 - Wick rotation
 - standard tools of quantum field theory (and standard QT!)
- “conservative” configuration space: curved spacetimes of GR are represented by piecewise flat geometries (the triangulations)
- phase space spanned by few free parameters (Λ , G_N , Δ)
- universal properties (contributes to uniqueness!)
- Crucial: nonperturbative computational tools to extract quantitative results



piece of causal triangulation

“CDT Classic”: universal de Sitter-like volume profile



The average volume profile $\langle V_3(i) \rangle$ of the universe, as function of Euclidean proper time t , matches to great accuracy a corresponding GR minisuperspace calculation. (N.B.: also compatible with HL gravity).

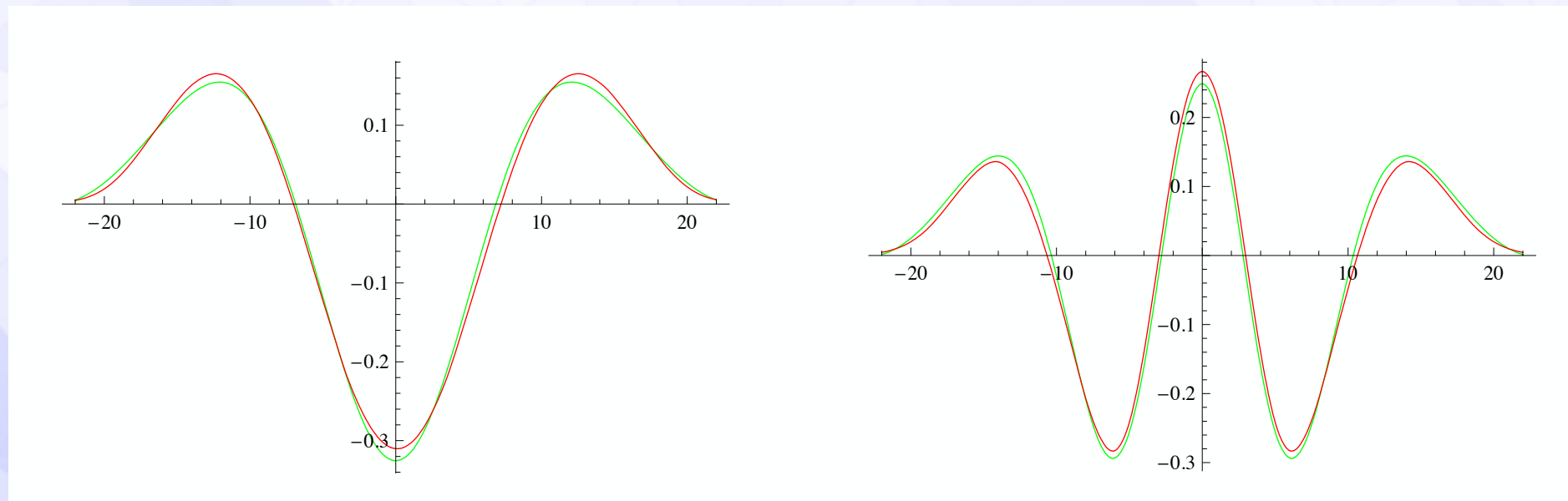
The classical line element of Euclidean de Sitter space, derived by *assuming homogeneity and isotropy a priori*, as function of Euclidean proper time $t=i\tau$, is

$$ds^2 = dt^2 + \underset{\substack{\uparrow \\ \text{scale factor}}}{a(t)^2} d\Omega_{(3)}^2 = dt^2 + c^2 \cos^2 \left(\frac{t}{c} \right) d\Omega_{(3)}^2 \longleftarrow \text{volume el. } S^3$$

In addition, expanding the minisuperspace action around the de Sitter solution,

$$S_{\text{eu}}(V_3) = S(V_3^{\text{dS}}) + \kappa \int dt \delta V_3(t) \hat{H} \delta V_3(t)$$

the eigenmodes of \hat{H} match well with those extracted from the simulations:

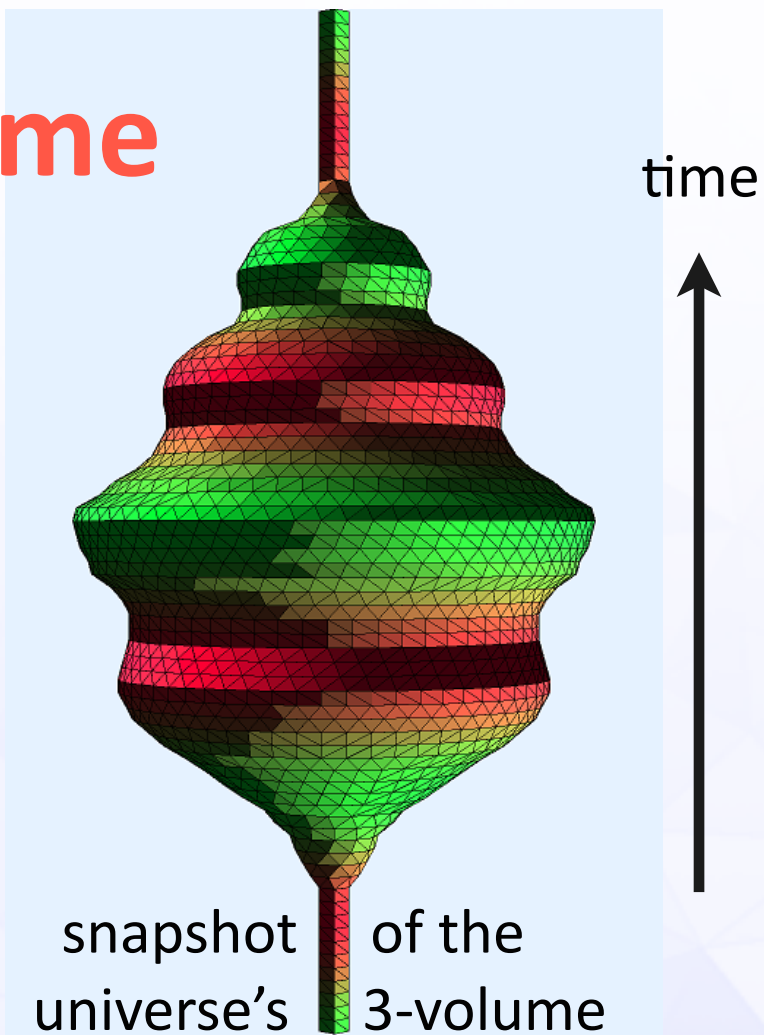


(N.B.: no further fitting necessary)

(J. Ambjørn, A. Görlich, J. Jurkiewicz, RL, PRL 100 (2008) 091304, PRD 78 (2008) 063544, NPB 849 (2011) 144 (with J. Gizbert-Studnicki, T. Trzesniewski))

Dynamical emergence of spacetime (out of 'quantum foam')

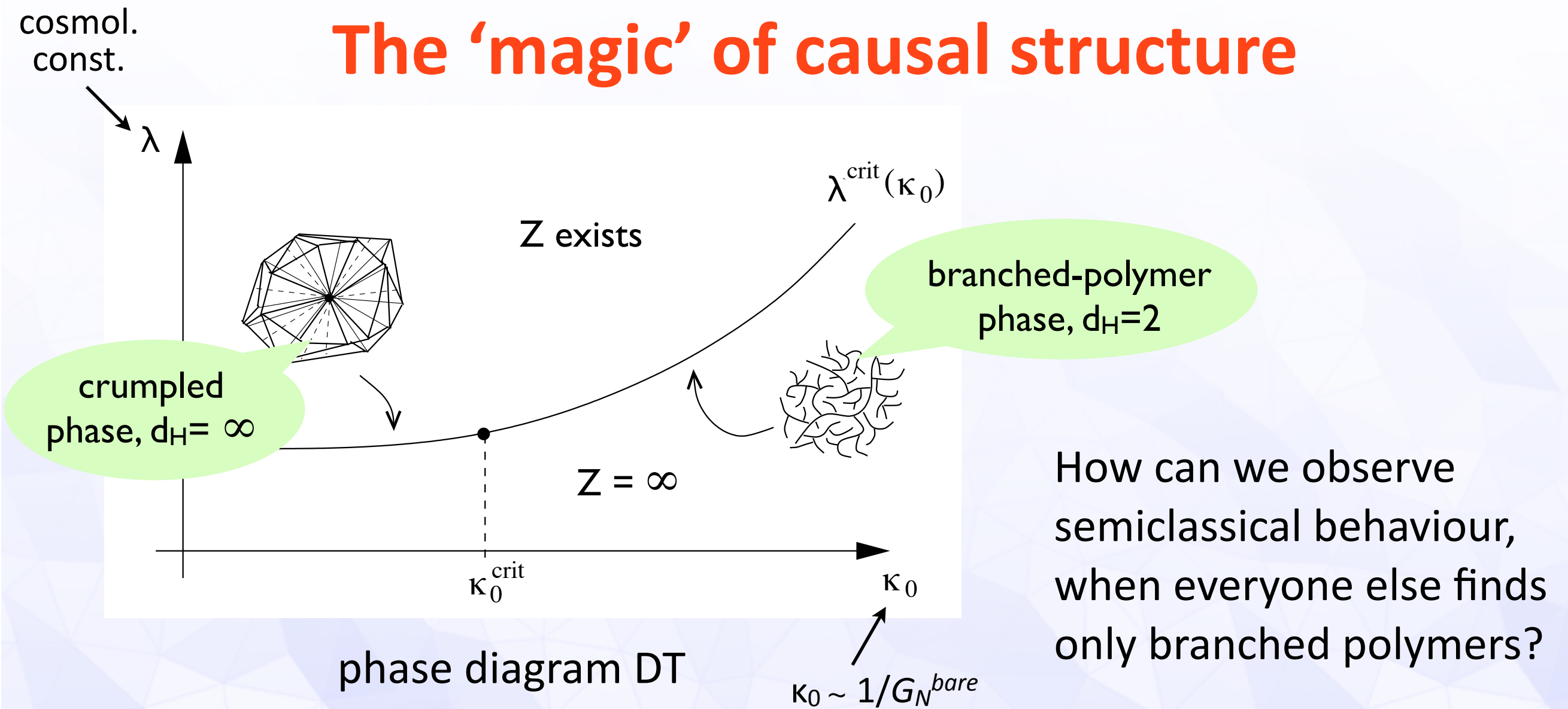
We conclude that for suitable bare coupling constants, CDT quantum gravity dynamically produces a “quantum spacetime”, that is, a ground state (“vacuum”), whose macroscopic scaling properties are ***four-dimensional*** and whose macroscopic shape is that of a well known cosmology, ***de Sitter space***.



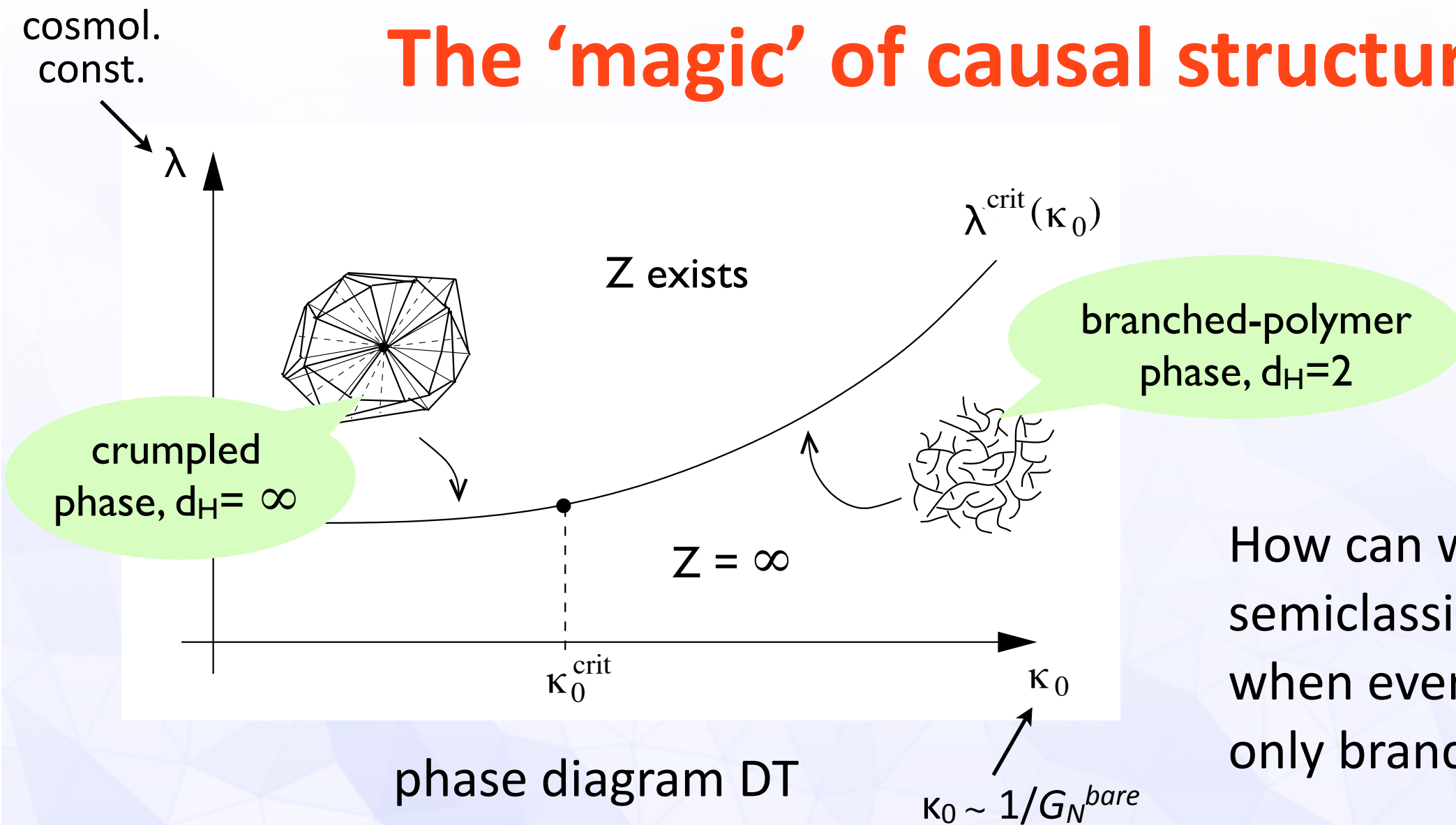
This is brought about by a ***nonperturbative*** mechanism, with “energy” (the bare action) and “entropy” (the measure, i.e. number of microscopic spacetime configurations) contributing in equal measure.

The region in coupling constant space where we see interesting physics is far away from the perturbative regime; quantum fluctuations are large and entropy matters; impossible to obtain from quantum cosmology.

The 'magic' of causal structure



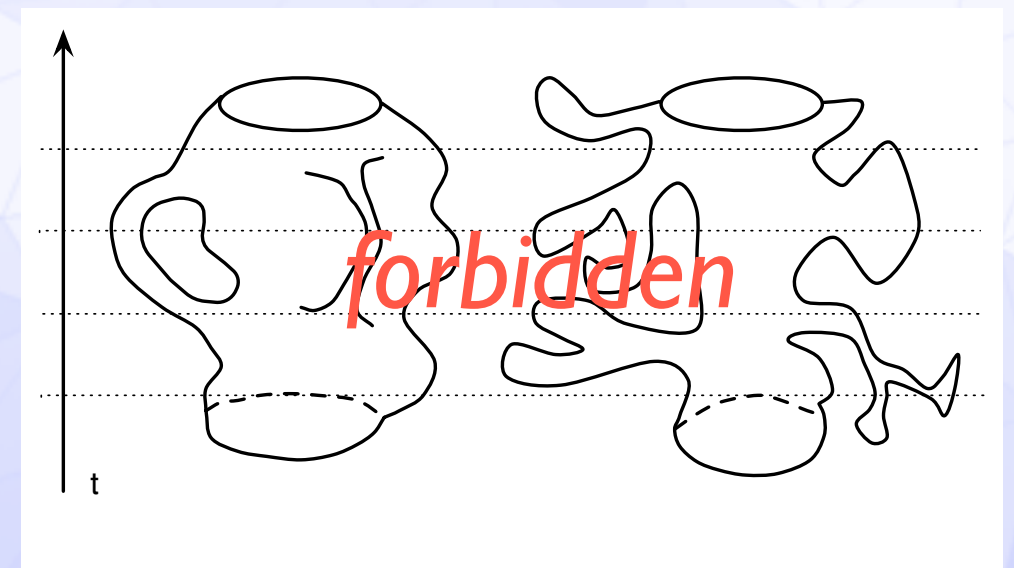
The 'magic' of causal structure



How can we observe semiclassical behaviour, when everyone else finds only branched polymers?

It's the causal structure, stupid!

Elementary building blocks are given a Lorentzian (=light cone) structure, and gluing rules ensure a *well-behaved causal structure* overall. "Baby universes" are forbidden.

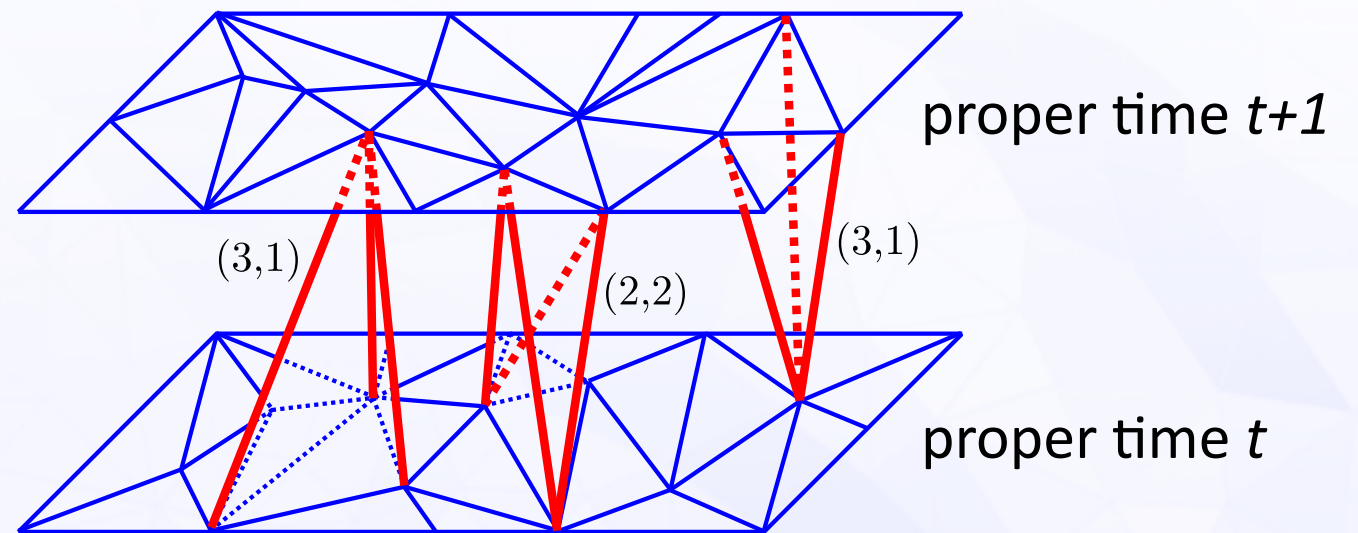


N.B.: singular "trouser points"

Causal structure vs. proper time foliation

Which is responsible for the “good” behaviour of CDT?

Does the preferred time/foliation affect the results?

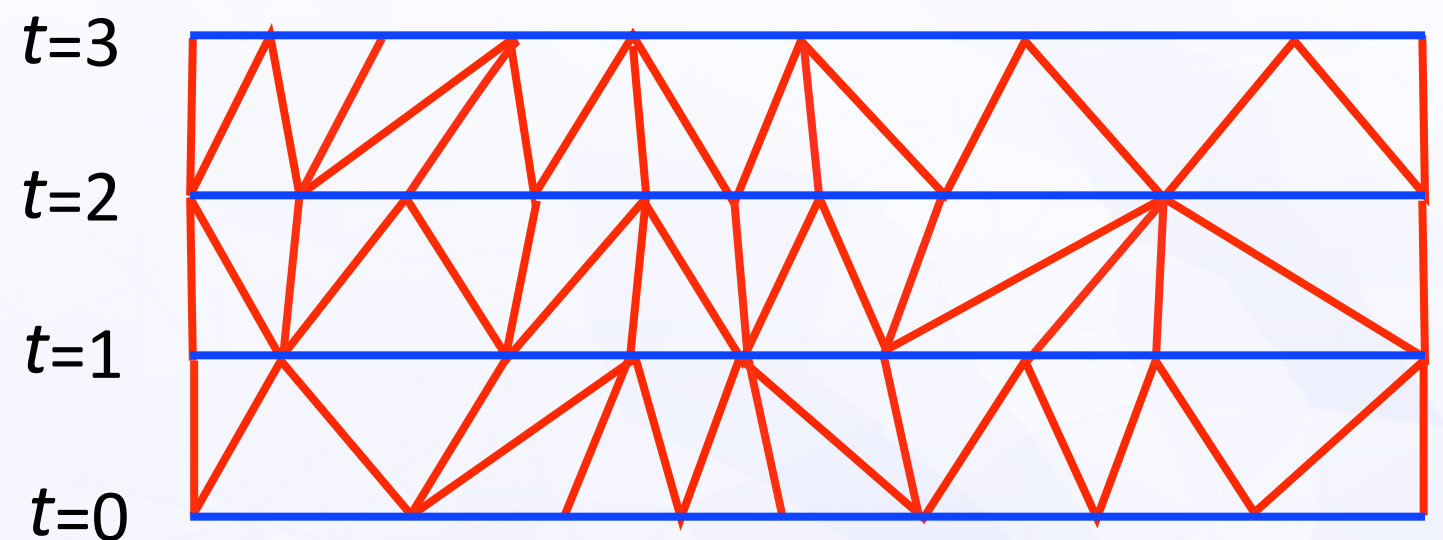
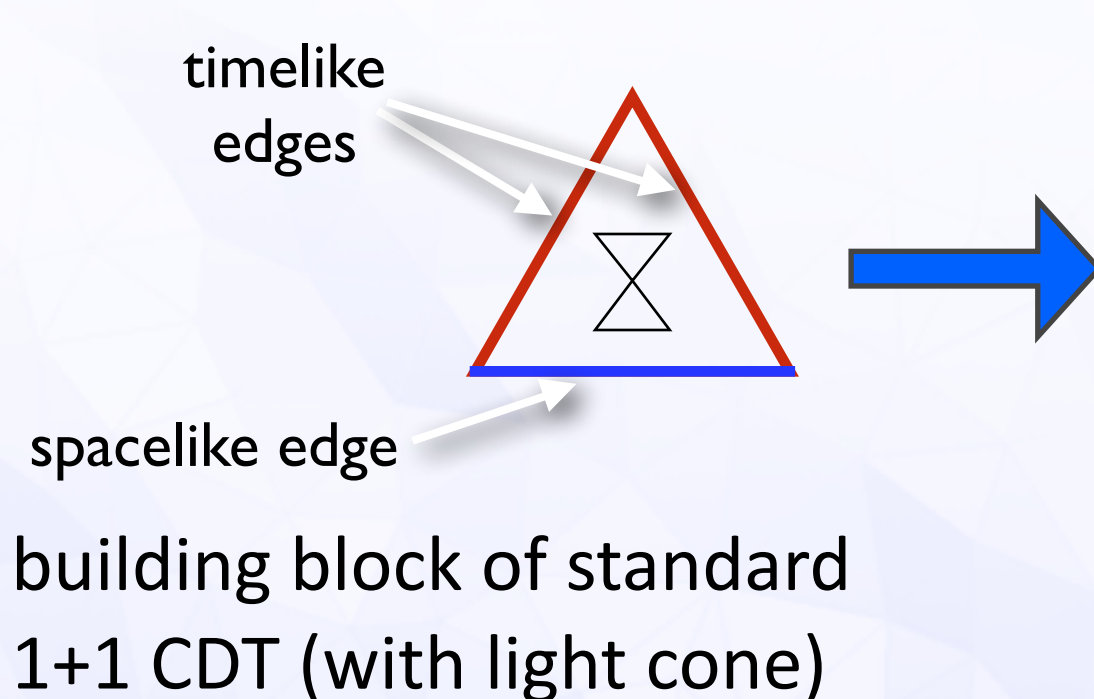


sliced structure of 2+1 CDT

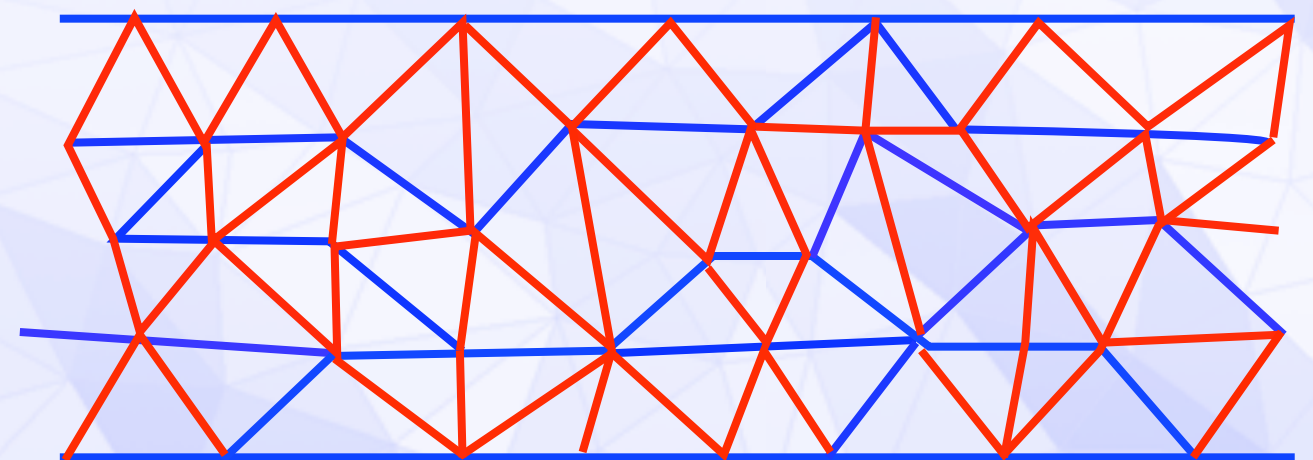
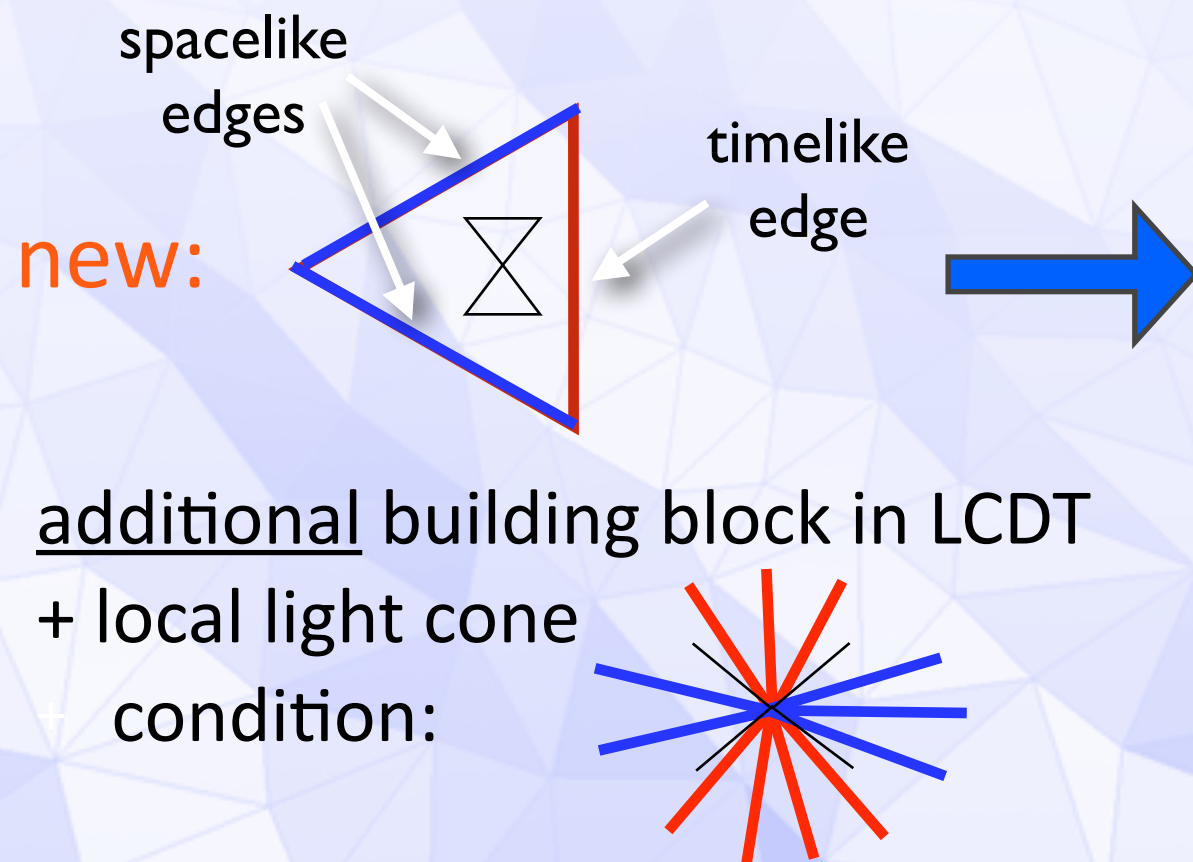
We have introduced a new version of CDT quantum gravity, “*Locally Causal Dynamical Triangulations (LCDT)*”, where the causal structure and the preferred time are dissociated (in fact, there is no preferred time), and have repeated the standard analysis of the phase structure and the volume profiles for 2+1 CDT quantum gravity. Key CDT findings appear unaltered!

⇒ The causal structure is responsible; the time foliation is very convenient, but not strictly necessary. (S. Jordan and R.L., *Phys. Lett. B* 724 (2013) 155; *Phys. Rev. D* 88 (2013) 044055)

Causal DT vs. locally causal DT, in 1+1D



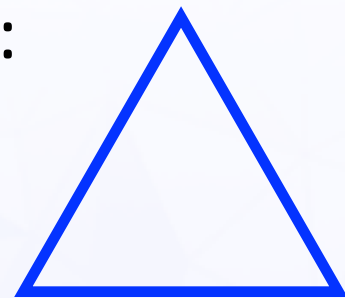
building causal spacetimes from proper-time strips in standard CDT quantum gravity



building causal spacetimes in nonfoliated LCDT from these two building blocks

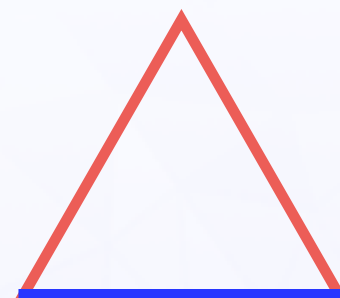
3 versions of Dynamical Triangulations, in 2D

building blocks:



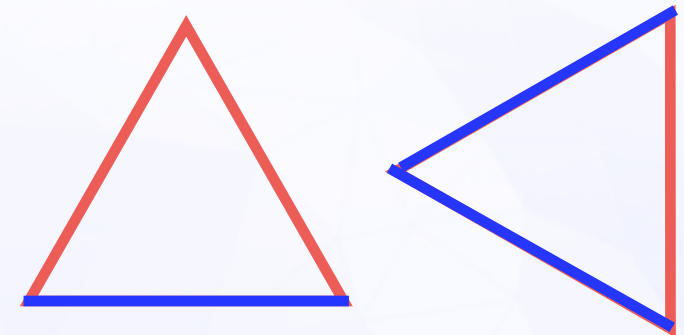
DT

$d_S=2, d_H=4$



CDT

$d_S=2, d_H=2$



LCDT

— spacelike edge of length a^2
— timelike edge of length $-\alpha a^2, \alpha > 0$

- numerical evidence that CDT and LCDT in 2D lie in the same universality class(?) $d_S=2.02 \pm 0.02$, $d_H=2.71 \pm 0.25$ for LCDT (B. Ruijl and R.L., to appear) - can this model be solved exactly?
- in 3D, LCDT is computationally *very* intensive (many new building blocks); good numerical evidence that in a region of phase space CDT results are reproduced: w.r.t. a notion of “cosmological” proper time introduced *a posteriori* on the geometries we find an excellent matching of volume profiles to a Euclidean de Sitter universe. (papers with S. Jordan)

Phase diagram of CDT quantum gravity in 4D

The CDT gravitational action is *simple*:

$$S_{\text{eu}}^{\text{Regge}} = -\kappa_0 N_2 + N_4 (c\kappa_0 + \lambda) + \Delta (2N_4^{(4,1)} + N_4^{(3,2)})$$

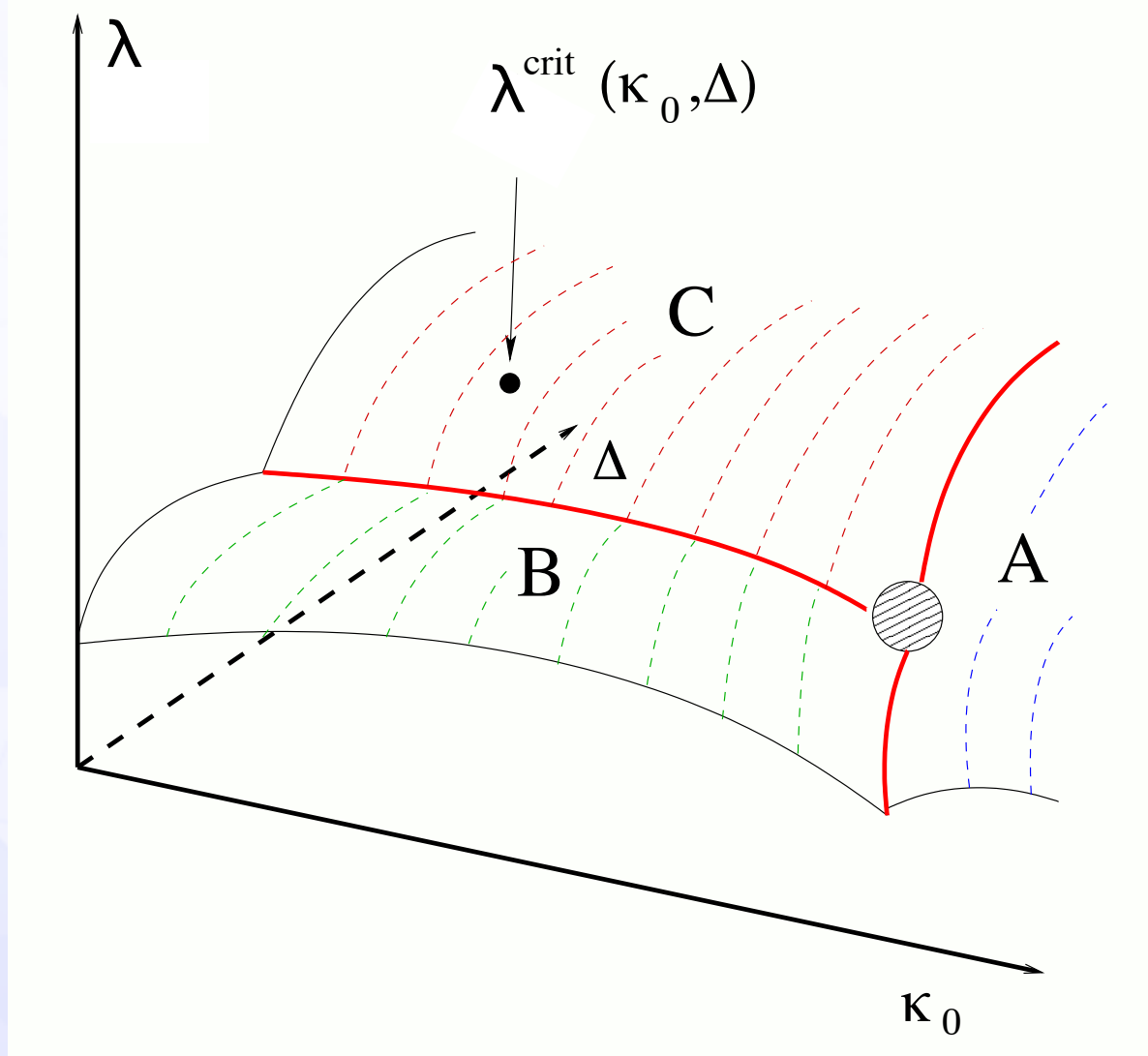
$\lambda \sim$ cosmological constant

$\kappa_0 \sim 1/G_N$ inverse Newton's constant

$\Delta \sim$ relative time/space scaling

$c \sim$ numerical constant, >0

$N_i \sim$ # of triangular building blocks of dimension i

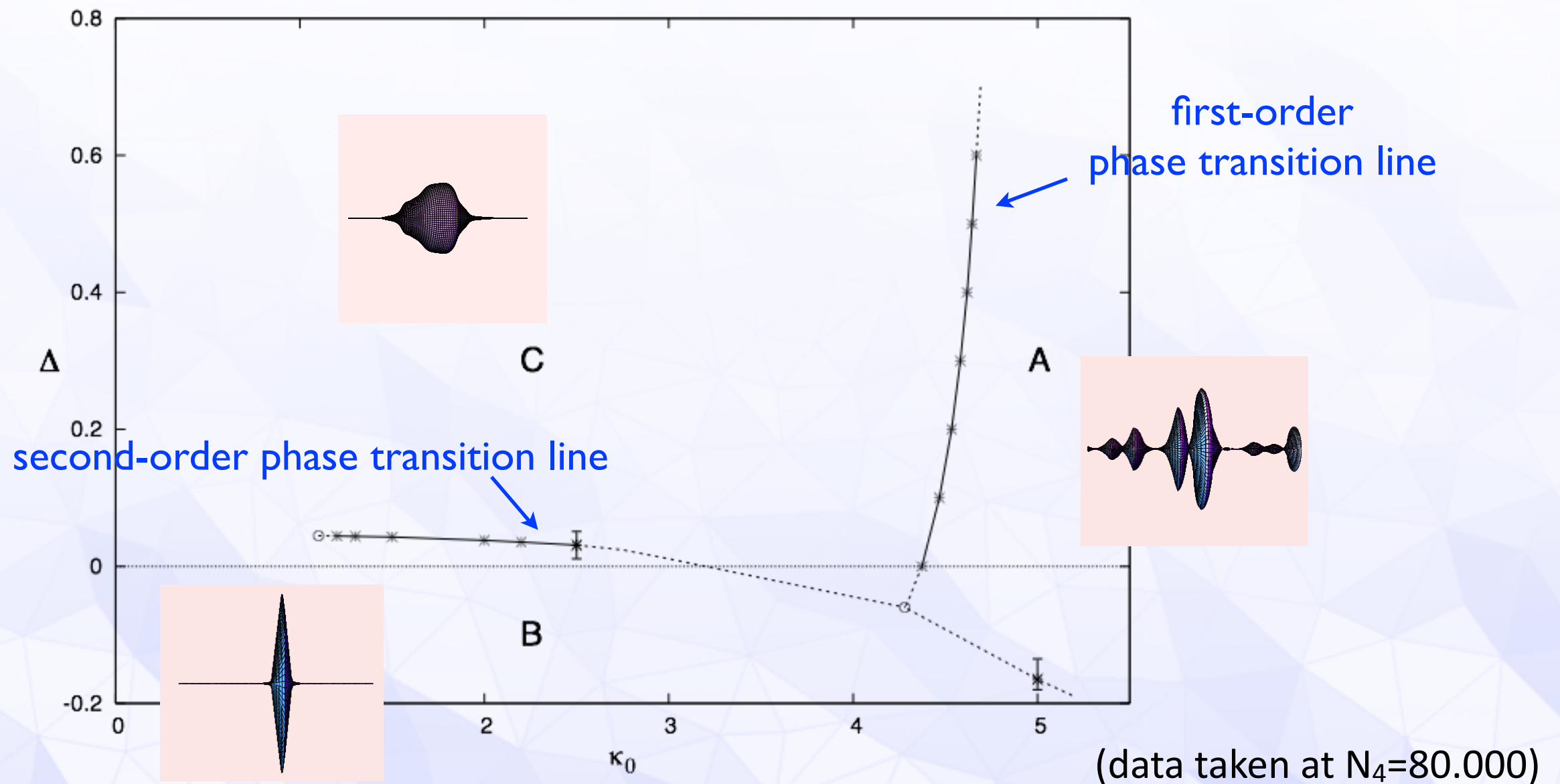


The partition function is defined for $\lambda > \lambda^{\text{crit}}(\kappa_0, \Delta)$;
approaching the critical surface from above = taking infinite-volume limit.
red lines \sim phase transitions

(J. Ambjørn, J. Jurkiewicz, RL, PRD 72 (2005) 064014;

J. Ambjørn, A. Görlich, S. Jordan, J. Jurkiewicz, RL, PLB 690 (2010) 413)

4D CDT phase diagram in the κ_0 - Δ plane



The *average* geometry in phases A and B is degenerate and does *not* have a classical, four-dimensional limit. The interesting physics happens in phase C.

The B-C transition appears to be of second order; unprecedented! (J. Ambjørn, S. Jordan, J. Jurkiewicz, R.L., PRL 107 (2011) 211303; PRD 85 (2012) 124044)

Standard renormalization *can* be applied!

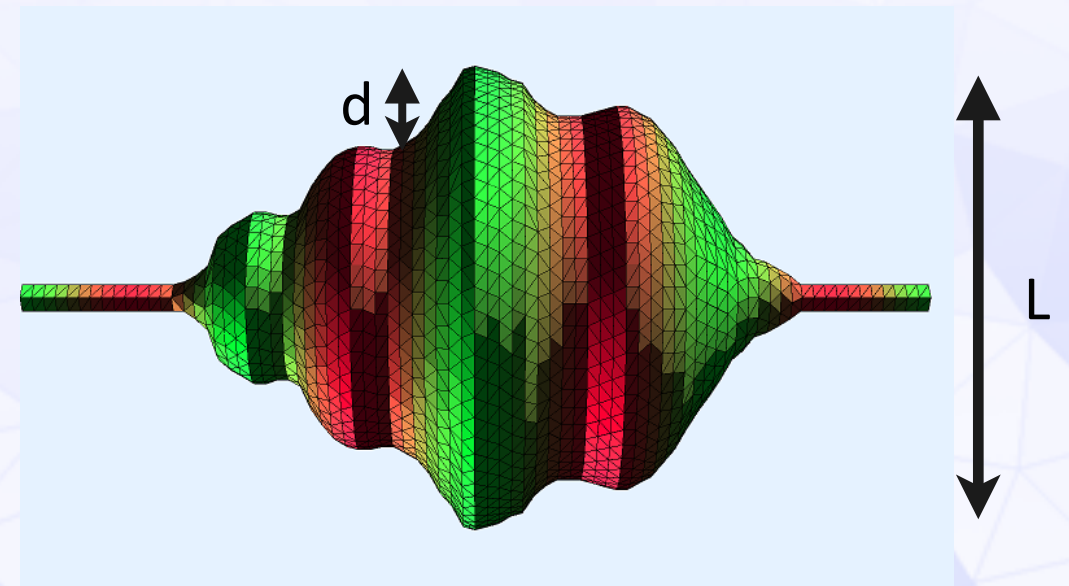
Having located a line of second-order transition points, we want to investigate the scaling behaviour of the theory in their vicinity.

We are interested in renormalization group flows probing ever shorter distances. Since there is no correlation length immediately available, we let the linear lattice size $N_4^{1/4} \rightarrow \infty$ while keeping physics constant.

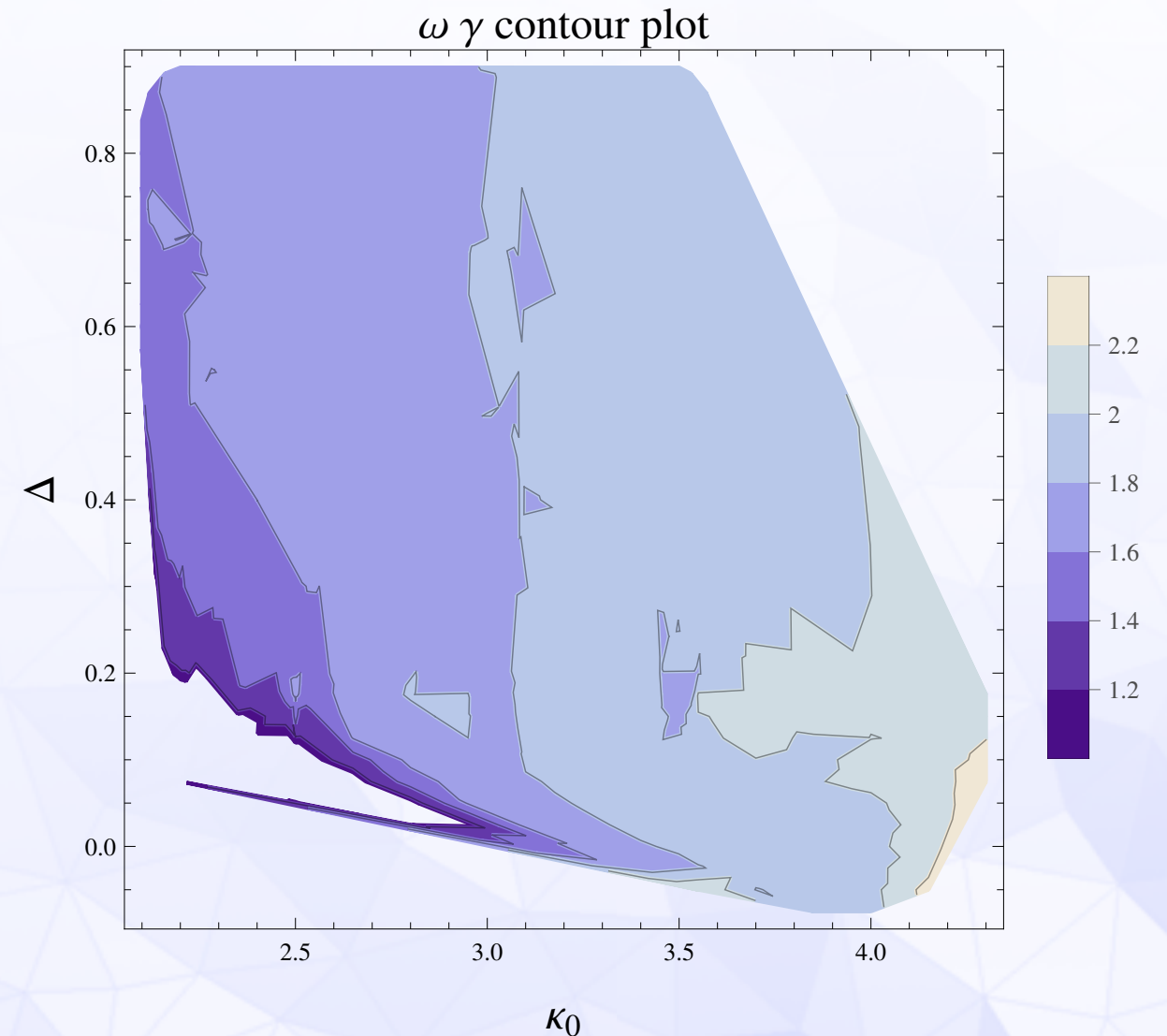
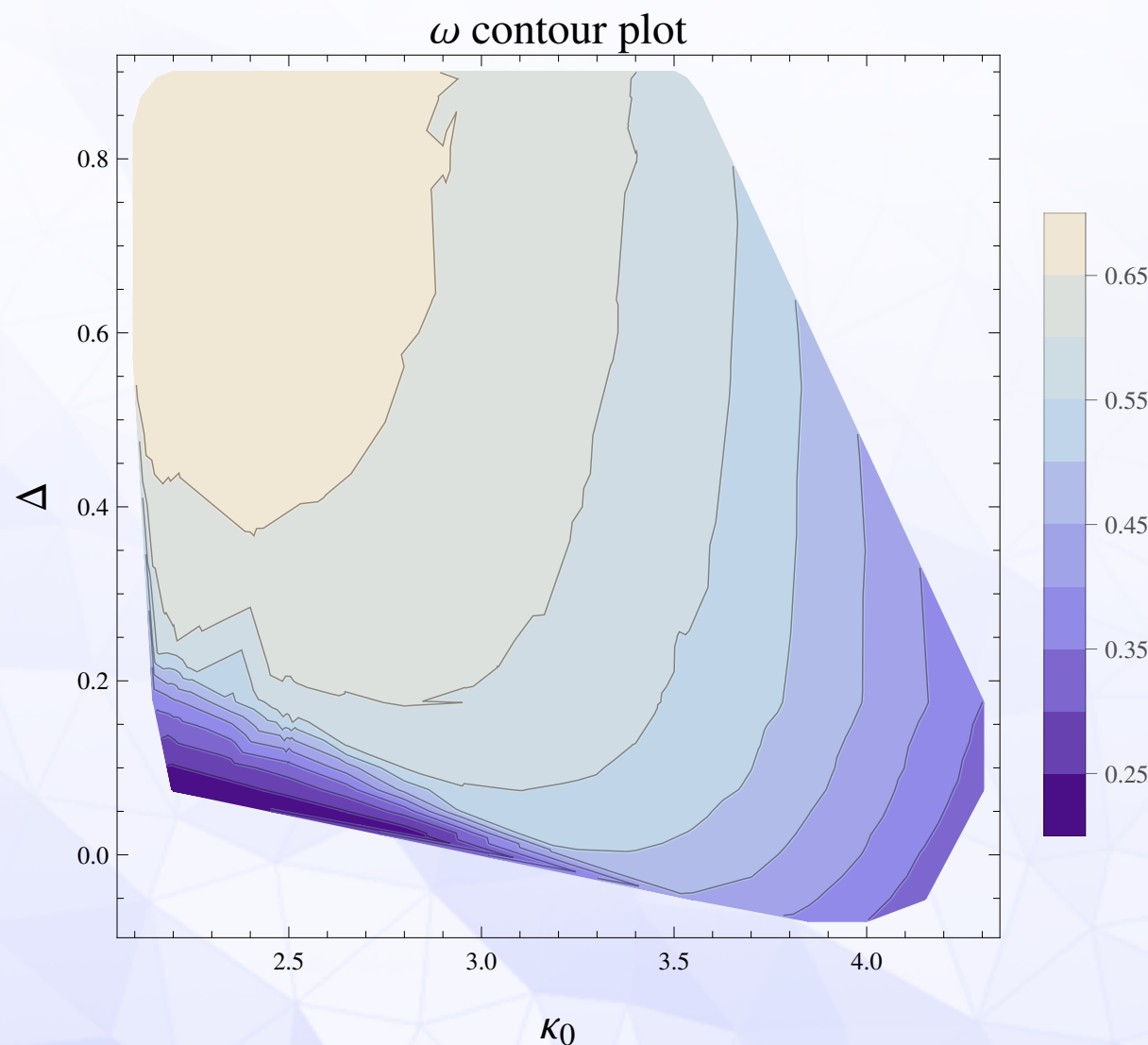
Idea: use the length scales associated with the dynamically generated de Sitter universe in CDT to define physical ‘yardsticks’.

Under simplifying assumptions this has enabled us to perform a first explicit study of such RG flows.

(J. Ambjørn, A. Görlich, J. Jurkiewicz, A. Kreienbühl, RL, CQG 31 (2014) 165003, and work in progress)



Computing RG flow lines in 4d CDT



Taking $N_4^{1/4} \rightarrow \infty$ translates to following $\omega = \text{const.}$ flow lines in the direction of growing $\gamma\omega$ (second-order transition line lies along bottom of plots).

(Our assumptions: everywhere in phase C,

- (i) the semiclassical interpretation in terms of minisuperspace is valid, and
- (ii) lattice units correspond to the same physical proper times and distances.)

Wilson loop observables in CDT

Our RG study highlights the need for more *observables*.

In nonperturbative quantum gravity, observables must be invariantly defined, without reference to coordinates or any background (unless obtained dynamically). Standard QFT observables can sometimes be adapted to be meaningful in the functional integral over geometry.

Example: a two-point function $G_2(x,y)$ is not a good observable, since we cannot fix specific points x and y in the path integral, but

$$G_2(r) = \int \mathcal{D}[g_{\mu\nu}] e^{-S[g_{\mu\nu}]} \int dx dy \sqrt{g(x)g(y)} G_2(x, y) \delta(r - d_{g_{\mu\nu}}(x, y)) \text{ is.}$$

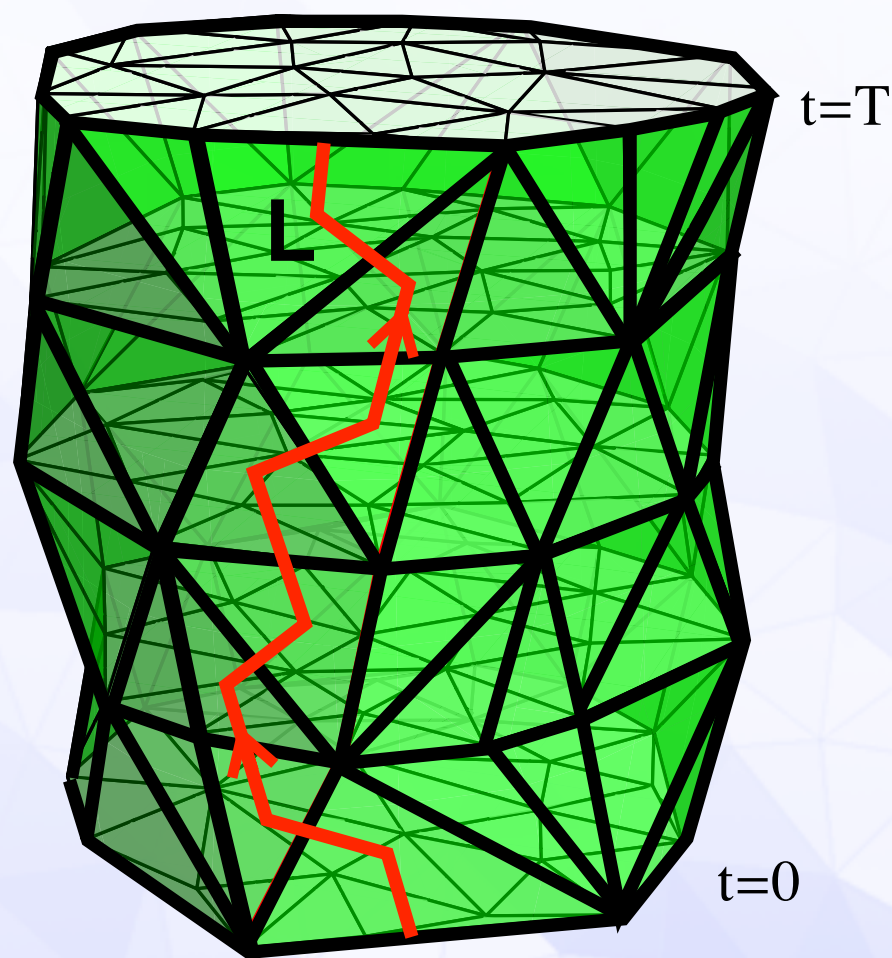
geodesic distance

Idea: can we define observables involving gravitational Wilson loops

$$W_\gamma(\Gamma) = \text{Tr } \mathcal{P} \exp \oint_\gamma \Gamma \quad \leftarrow \text{Levi-Civita connection}$$

(referring to entire curves γ , not just points) in a similar way, to obtain curvature information about the underlying quantum spacetime?

Defining physical Wilson loops in CDT



We let the loop γ coincide with the world line L of a particle moving forward in time. The loops wind once around the compactified time direction of the triangulated spacetimes, which have topology $S^1 \times S^3$, as usual.

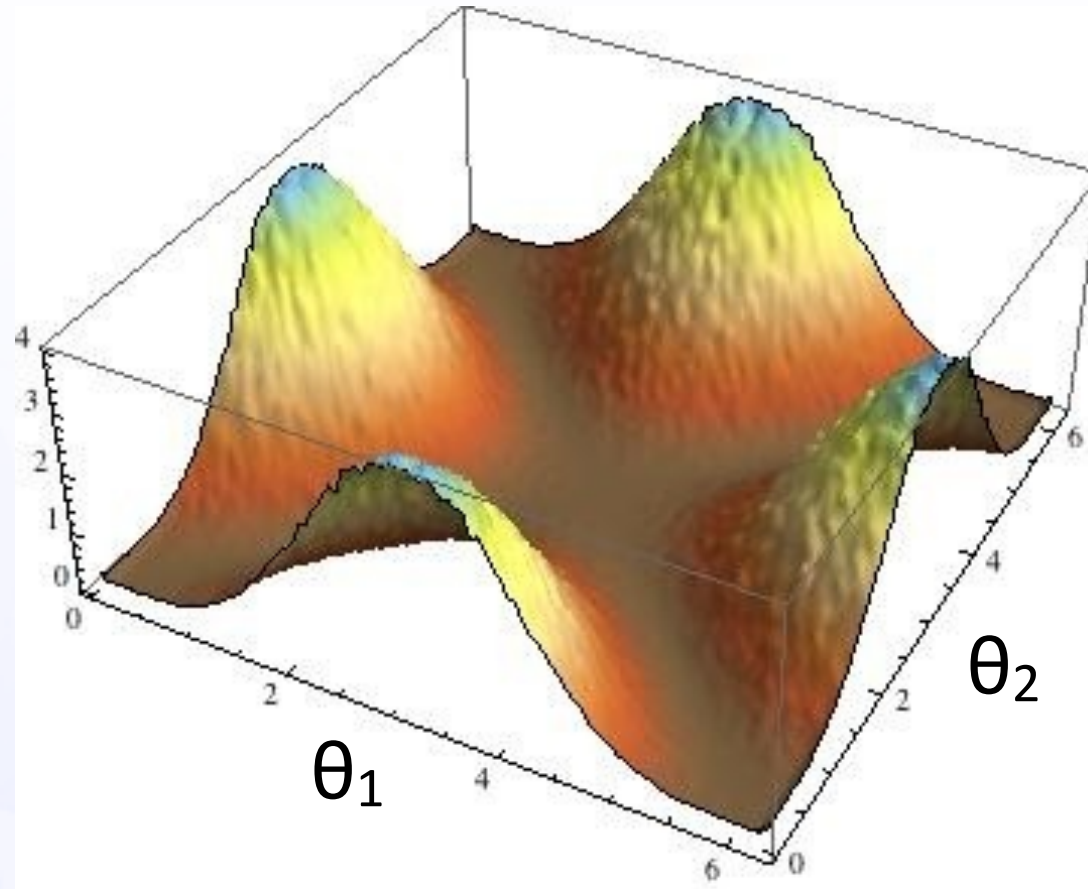
Correspondingly, we add to the pure-gravity action a term for a free massive point particle

$$S^{\text{p.p.}} = m \int dl \quad \rightarrow \quad S_{\text{CDT}}^{\text{p.p.}} = m_0 N_L$$

where N_L = number of four-simplices along L .

In Monte Carlo simulations for the combined gravity-particle system it is straightforward to compute Wilson lines and extract two coordinate-independent trace invariants, angles θ_1, θ_2 labelling $\text{SO}(4)$ -conjugacy classes.

(J. Ambjørn, A. Görlich, J. Jurkiewicz, RL, [arXiv:1504.01065](https://arxiv.org/abs/1504.01065))



The measured distribution of the invariant angles θ_i shown here is in almost perfect agreement with the theoretical result one obtains from assuming a uniform distribution of the holonomy matrices over the group manifold $SO(4)$,

$$P(\theta_1, \theta_2) = \frac{1}{\pi^2} \sin^2 \left(\frac{\theta_1 + \theta_2}{2} \right) \sin^2 \left(\frac{\theta_1 - \theta_2}{2} \right).$$

Despite being a coordinate-free approach, holonomies and Wilson loops are straightforward to define and implement in CDT, without significant discretization effects, but do not tell us much about quantum curvature yet.

Summary and conclusions

We have looked at a cross section of properties of CDT quantum gravity that have been investigated, with highly nontrivial results:

- emergence of macroscopic 4D geometry (crucial: causal structure)
- interesting variant of CDT: locally causal dynamical triangulations
- dynamical dimensional reduction to $d_S \approx 2$ near Planck scale
- phase diagram with line of second-order phase transitions: natural candidates for scaling limit
- concrete realization of renormalization group flows in a background-independent, nonperturbative setting
- *observables*: can measure Wilson loops, but need many more

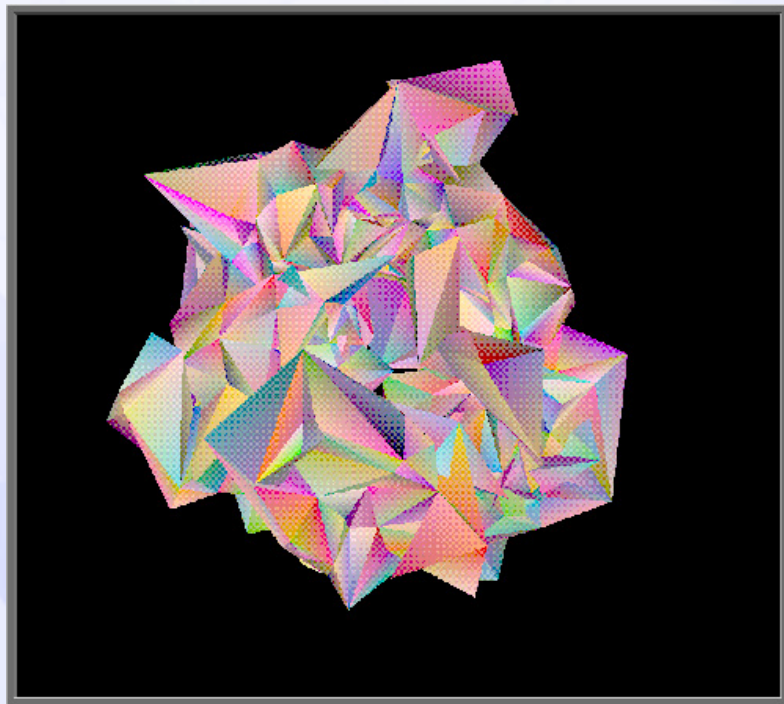
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- concrete realization of renormalization group flows in a background-independent, nonperturbative setting
- *observables*: can measure Wilson loops, but need many more

CDT quantum gravity is an exciting story that keeps going!

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triangulated model of quantum space

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Thank you!

20th Itzykson Conference, Saclay

12 Jun 2015