Quantum Space Time Engineering

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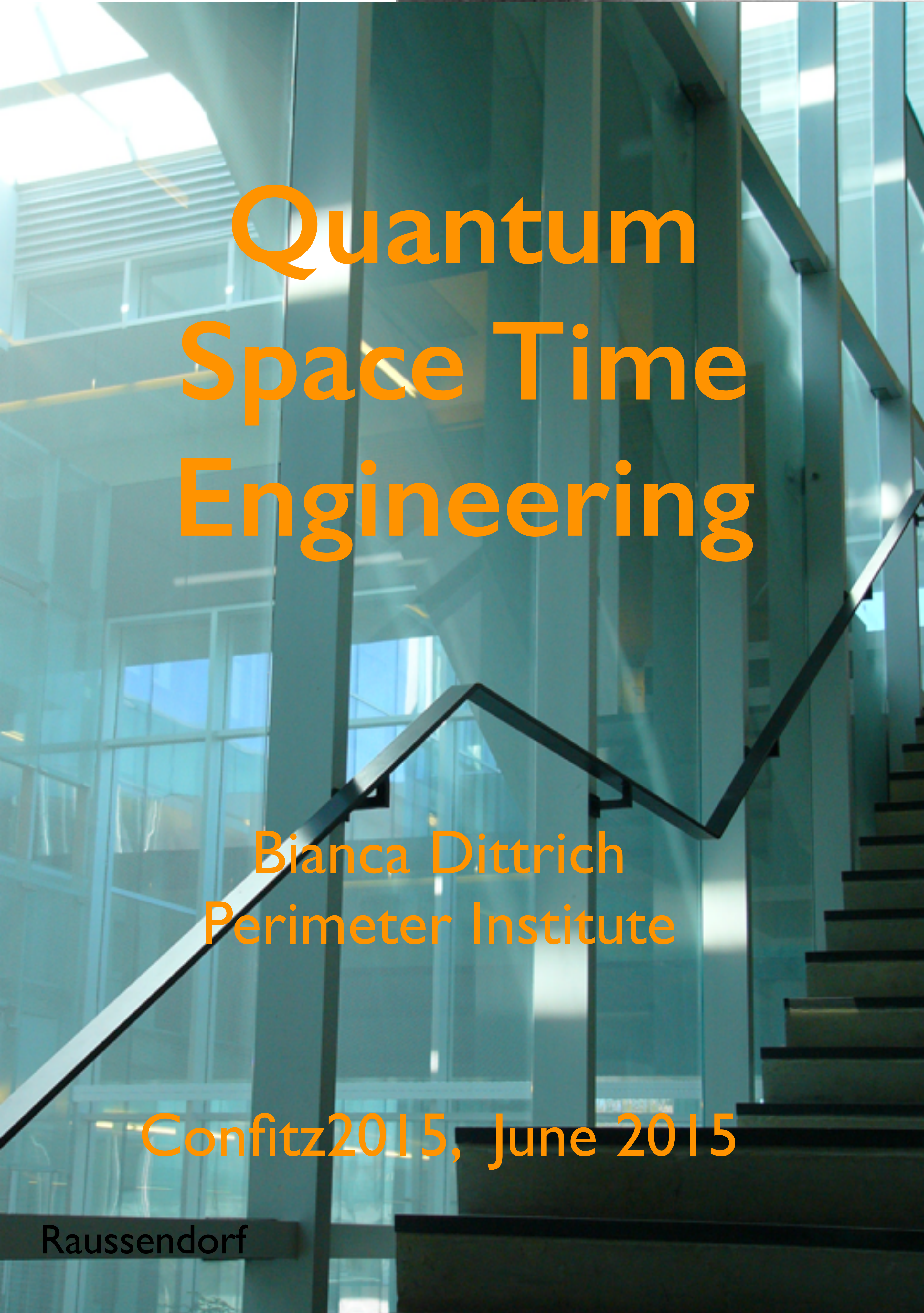


Raussendorf

What is
Quantum Space Time?

Overview

How to construct quantum geometry:

Topological Field Theory with (continuum limit of) defects

How to construct quantum space time?

Phase structure of quantum space time models

Consistent boundary formulation

Quantum geometry

Encoded in
combinatorics of
triangulation.
Path integral: sum over
weighted (equilateral)
triangulations

continuum
limit:
lattice constant
goes to zero

Encoded in
data assigned to a
triangulation.
Path integral: sum over
weighted
data

continuum
limit:
no lattice constant.
Expressed via consistent
boundary formulation.

[BD NJP 12, 14]

Choosing amplitudes(data):

More freedom to design quantum geometries.

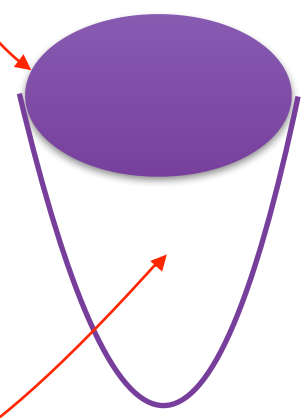
Vacuum state(amplitudes):

characterizes the quantum geometry encoded in the amplitudes

What is vacuum?

vacuum

ψ_{vac}

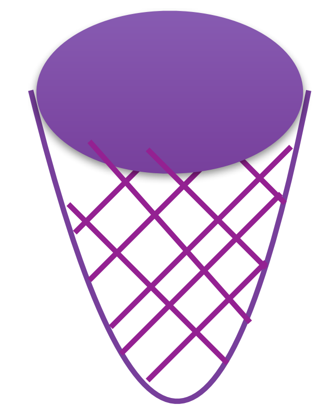


sum over
weighted
geometries

(Hartle-Hawking)
no-boundary wave
function defined by:
path integral
with one
vanishing boundary

regularize
by (fixed)
discretization

ψ



in general problem:
-wave function depends on dizcretization
-breaking of diffeomorphism symmetry

[2009: Bahr, BD]

But not if
weights
define a topological QFT
(by definition a discretization
independent theory)

General case:
continuum limit can be formulated via
consistent boundary formulation

[20012/2014: BD]

Topological QFT

- local field theory with gauge symmetries (eg diffeomorphism) eliminating kinematical degrees of freedom
- no propagating degrees of freedom
- only one physical state (spherical topology): vacuum state
- 3D gravity is a topological QFT
- 4D gravity is not topological: need more (excited) states

One strategy: TQFT with defects

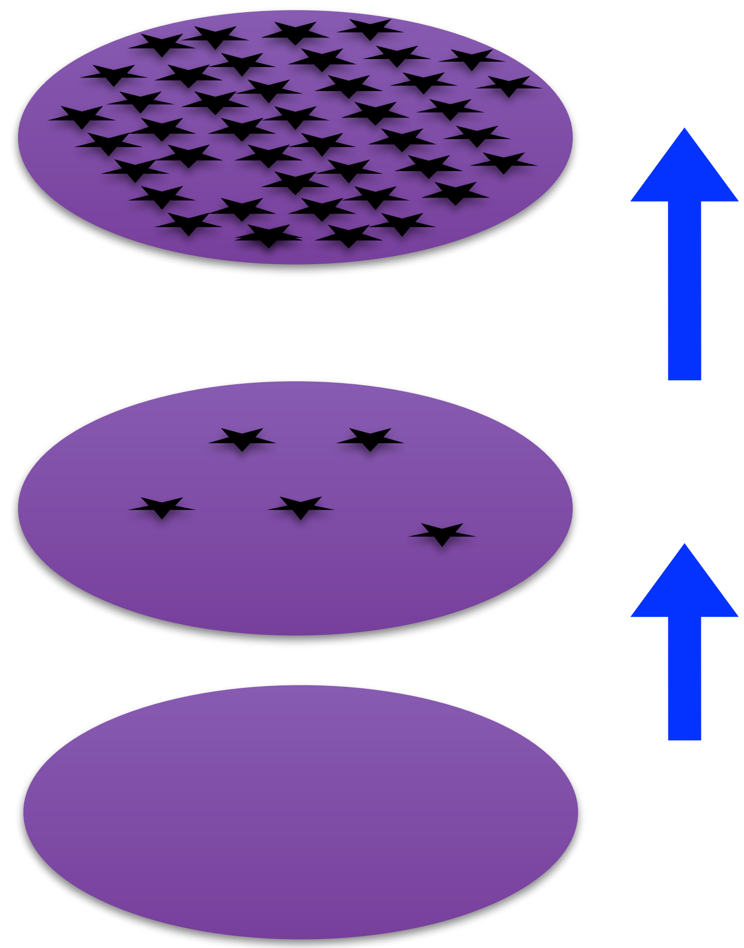
[2013: BD, Steinhaus]

Describe more general wave functions as perturbed vacuum, that is as vacuum with defects.

Why (only) defects?

Only framework so far that allows (spatial) diffeomorphism invariant formulation.

Physically:
distances between defects do not matter.
Reflects background (geometry) independence.



Condensation of defects to a new vacuum / phase transition (leading to unitary inequivalent representation of observable algebra)

Defects generated by observable (algebra)

- Approximating GR dynamics via (curvature) defects: Regge 60's, t' Hooft '00s , …
- a posteriori interpretation of Loop Quantum Gravity: (in Ashtekar Lewandowski rep.): geometry is a defect
- New representation of Loop Quantum Gravity: [BD, Geiller 14a,14b; Bahr, BD, Geiller 15] back to curvature defects

Quantum Geometry

My definition:

Construct Hilbert space
supporting diffeomorphism invariant excitations
and (geometric) operators to extract quantum geometry.

Examples: loop quantum gravity* ,
(causal) dynamical triangulations, group field theories, …

* approach which most explicitly constructs
such a Hilbert space and quantum geometry

Progress

- 1990's: Ashtekar, Isham, Lewandowski:

  First construction of a spatially diffeomorphism invariant Hilbert space

  supporting the (kinematical) observable algebra of general relativity and matter.

  Based on a no-spatial-geometry vacuum. 2005: F-LOST uniqueness theorem.

  Fleischhack - Lewandowski, Okolov, Sahlmann, Thiemann

  There is only one Hilbert space representation, so we have to stick with it . It has to be the correct one. Even if it is a pain.

- 2014: BD, Geiller (2015: Bahr, BD, Geiller):

  Construction of an alternative spatially diffeomorphism invariant Hilbert space

  based on a no-curvature vacuum.

  There are several Hilbert spaces. Can choose the most convenient one.

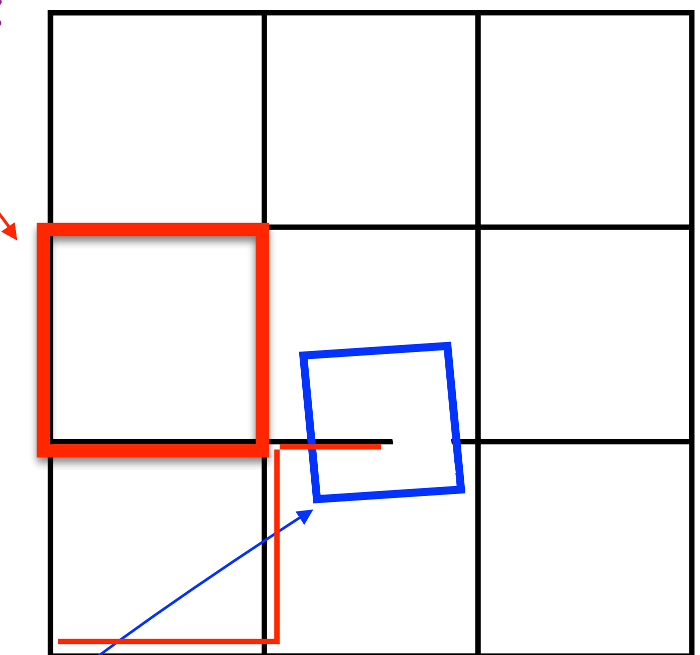
- 2013: BD, Steinhaus:

  Topological field theories (topological phases) give rise to

  spatially diffeomorphism invariant Hilbert spaces.

Loop quantum gravity: Lattice theory without a lattice

(Canonical)
Lattice gauge
theory variables:



Wilson loop
measures
curvature

electric
flux through
surface (in (3+1)D)

Using a lattice allows
formulation of non-perturbative physics.

Problem: breaks diffeomorphism symmetry.

Key: use the same variables but
do not restrict to the lattice.

Challenge: specifying a diffeomorphism
invariant vacuum state ‘everywhere’.

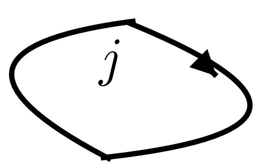
Quantum geometry operators

Ashtekar variables (1986)
Geometric variables (metric, curvature) can be encoded into variables of electro-magnetism (generalized to SU(2) Yang Mills).

magnetic field observable:
Wilson loop operator associated to a curve.

measures magnetic field integrated over enclosed surface
generates an electrical flux line

h^j_{curve}

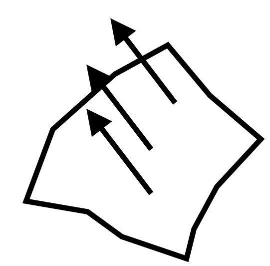


gravity context:
measures (extrinsic) curvature,
generates 'quanta of spatial geometry'

electric field observable:
Electric flux associated to a surface.

measures electric field flux through surface
generates magnetic field

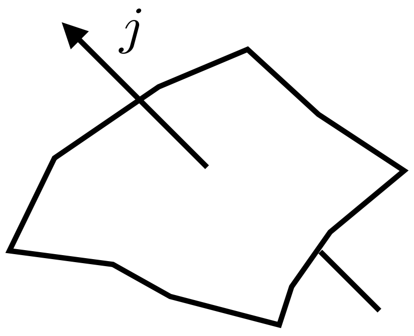
E_{surf}



gravity context:
measures (spatial) areas, angles, volumes
generates (extrinsic) curvature

Algebra of quantum geometry operators

$$\left[E_{\text{surf}}, h^j_{\text{curve}}\right] = h^j_{\text{curve}} \circ \underbrace{(\tau)}_{\text{Lie algebra generator}}{}^j$$



Only non-vanishing if holonomy curve (electric flux line) cuts through surface.

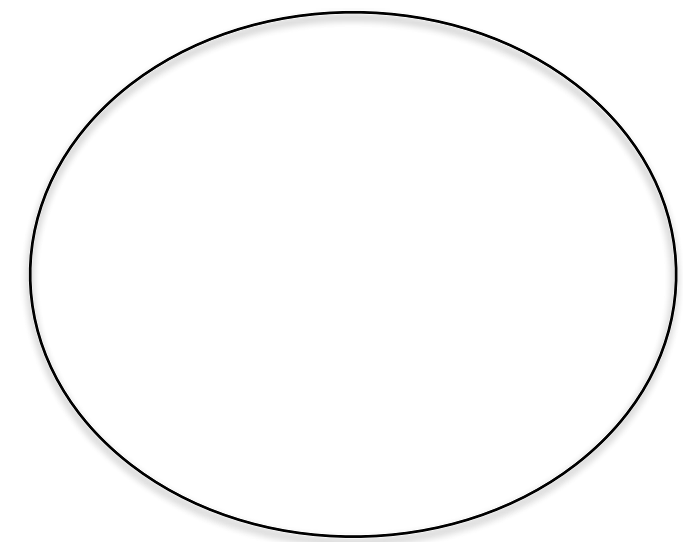
Is of topological nature.
(Does not need background metric.)

Building quantum geometry states: version I

[Ashtekar-Lewandowski-Isham representation, 90’s]

the vacuum state:
all flux operators have vanishing expectation values and vanishing fluctuations

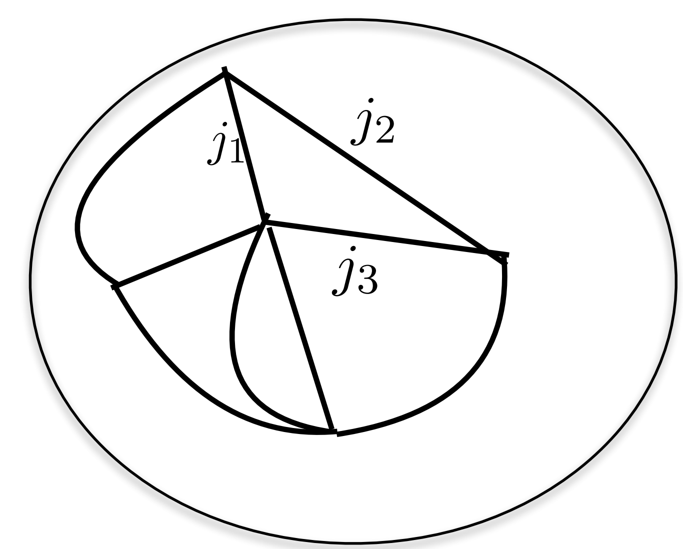
$$|0\rangle$$



excited states:
by applying Wilson loop operators,
some fluxes get non-vanishing expectation values.

encode spatial geometry!

$$h^{j_1}_{\text{curve}_1} h^{j_2}_{\text{curve}_2} h^{j_3}_{\text{curve}_3} |0\rangle$$

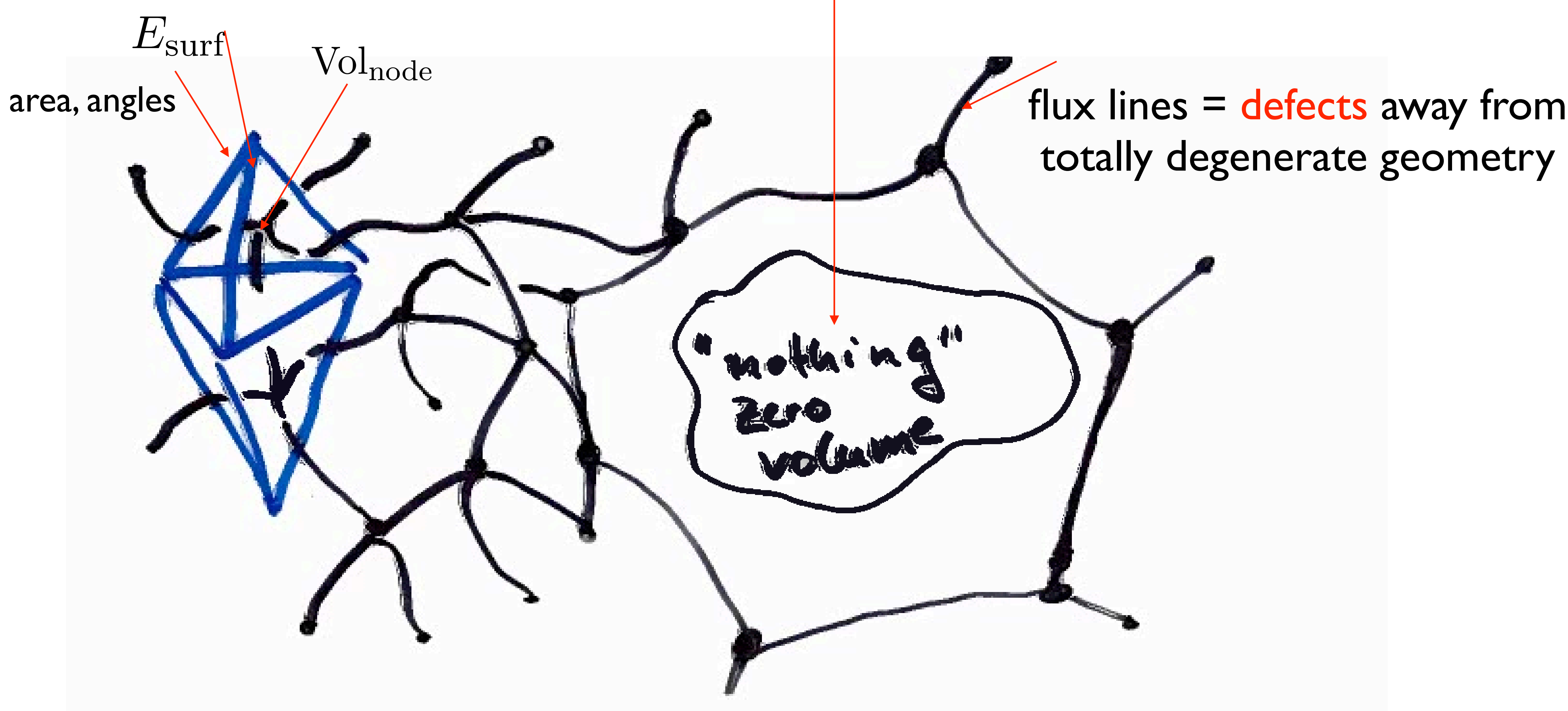


First rigorous realization of quantum geometry. (Technically: inductive limit of Hilbert spaces)

[Ashtekar-Lewandowski-Isham representation, 90’s]

Building quantum geometry states: version 1

vacuum = state where spatial geometry is totally degenerate



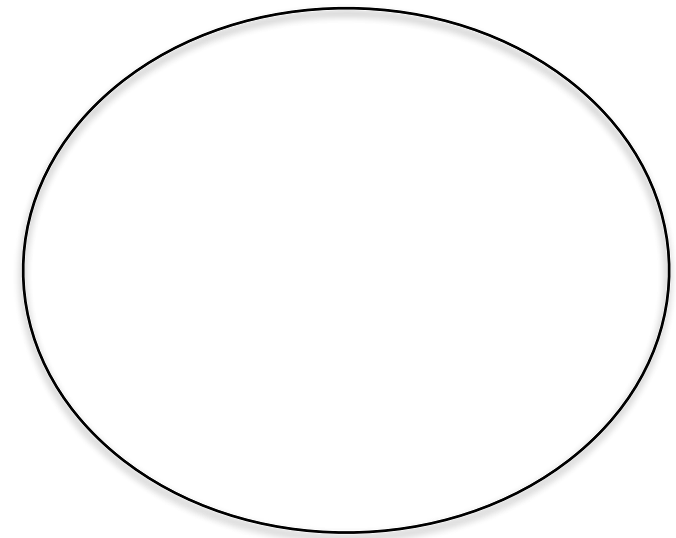
Quantum state determines quantum geometry
(in a spatial diffeomorphism invariant way).

Building quantum geometry states: version II

[BD, Geiller 14a,14b; Bahr, BD, Geiller 15]

the vacuum state:
all curvature operators have vanishing expectation values and vanishing fluctuations

$$|0\rangle$$

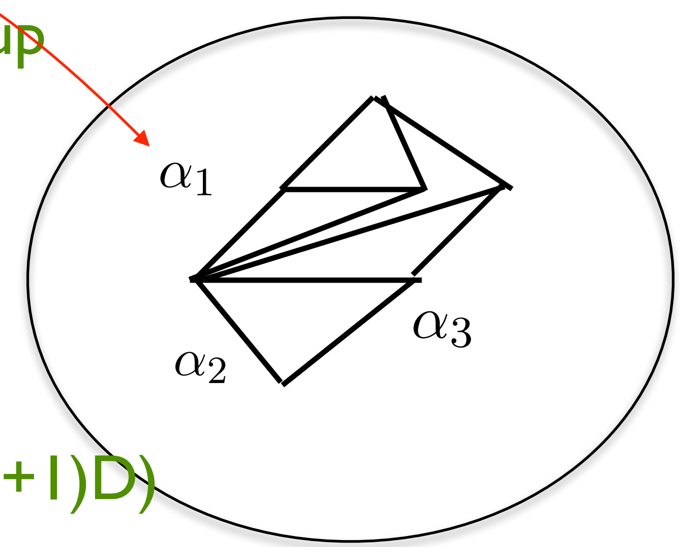


excited states:
by applying flux operators, some curvature operators get non-vanishing expectation values.

only exponentiated fluxes exist as operators

$$\exp(\alpha_3 i E_{\mathrm{surf}_3})\,\exp(\alpha_2 i E_{\mathrm{surf}_2})\,\exp(\alpha_1 i E_{\mathrm{surf}_1})\,|0\rangle$$

discrete topology for group labels - due to (Bohr) compactification of Pontryagin dual

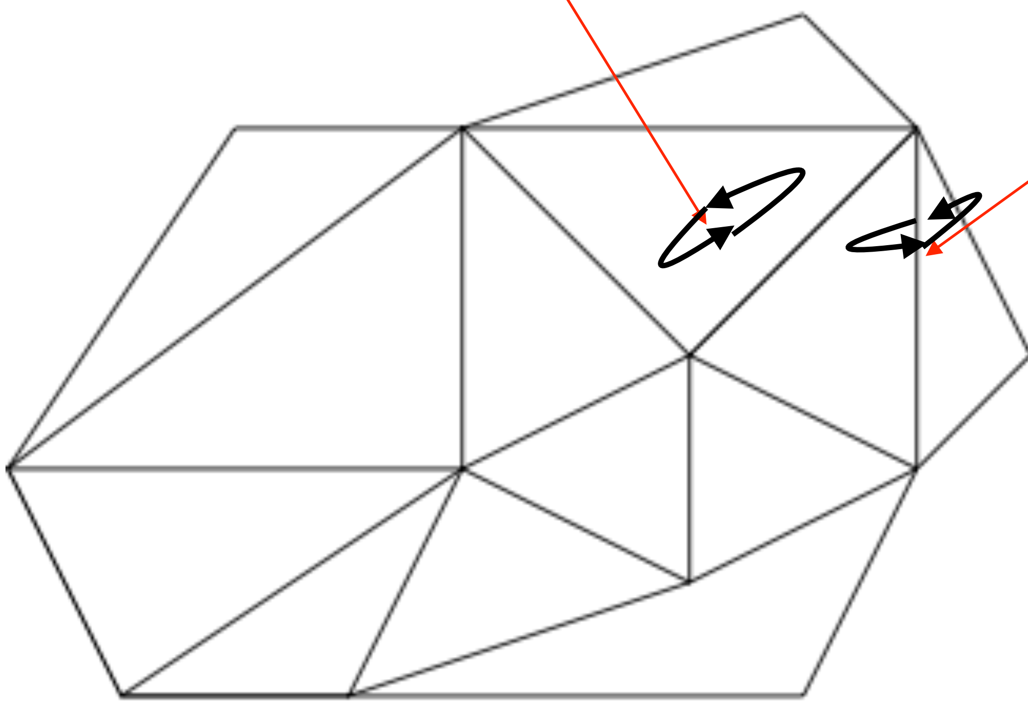


curvature defects along curves (in (3+1)D)

Building quantum geometry states: version II

[BD, Geiller 14a,14b; Bahr, BD, Geiller to appear 15]

vacuum peaked on vanishing curvature,
flux variables (spatial geometry) maximally uncertain



lines = curvature defects

state of
BF topological theory
with defects (= excitations)

Remark: Gives solution of
(2+1)D gravity
(with point particles).

Quantum state determines a (very different) quantum geometry
(in a spatial diffeomorphism invariant way).

Two vacua (and quantum-geometry representations)

[BD, Geiller 14a, 14b]

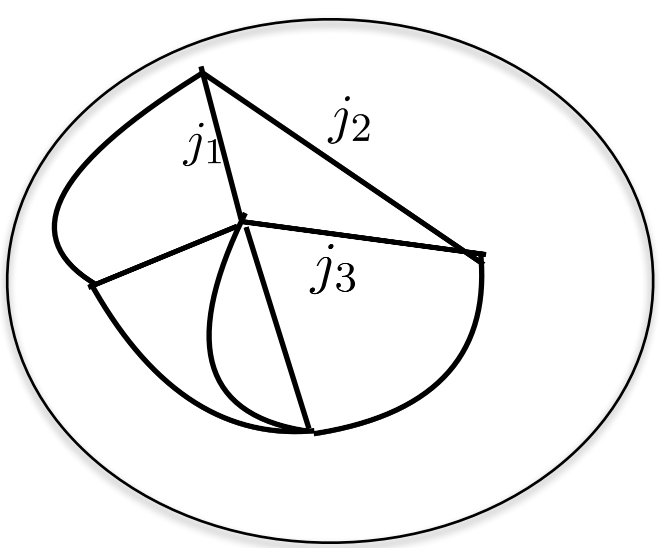
Phase transition

Ashtekar - Lewandowski - Isham vacuum (90's)

$$\psi_{\rm vac}(\{h_{\rm curve}\}) \; = \; 1$$

peaked on degenerate (spatial) geometry
maximal uncertainty in
(extrinsic) curvature

excitations:
spin network states supported on graphs
describing spatial geometry defects

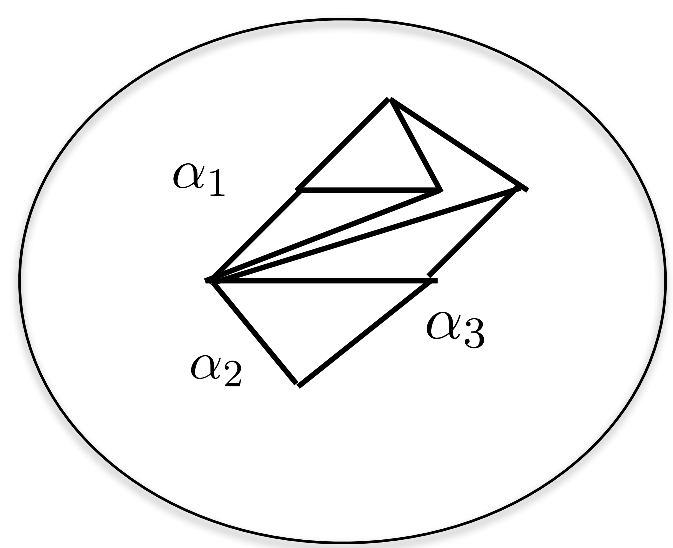


BF (topological) theory vacuum

$$\psi_{\rm vac}(\{h_{\rm loops}\}) \; = \; \prod_{\rm loops} \delta(h_{\rm loops})$$

peaked on vanishing
(Ashtekar connection) curvature
maximal uncertainty in spatial geometry

excitations:
curvature defects on edge network
(triangulation)



Why different (kinematical) vacua?

In standard qft: needed to describe symmetry breaking / condensation processes.

In q-gravity: need states satisfying the quantum equations of motions (physical states).

This is like asking for the energy eigenstates of an interacting quantum field theory: solving the theory.

Such states will not be (normalizable) in the Hilbert space we started with (kinematical Hilbert space).

Nevertheless some kinematical vacua might give easier access to physical states than other kinematical vacua.

Quantum space time:

dynamics of quantum geometry

What is a good vacuum (physical) state?

Should be adjusted to the dynamics of the system.

Time-evolution = applying path integral.

Usually:
Vacuum state should be invariant under time evolution.

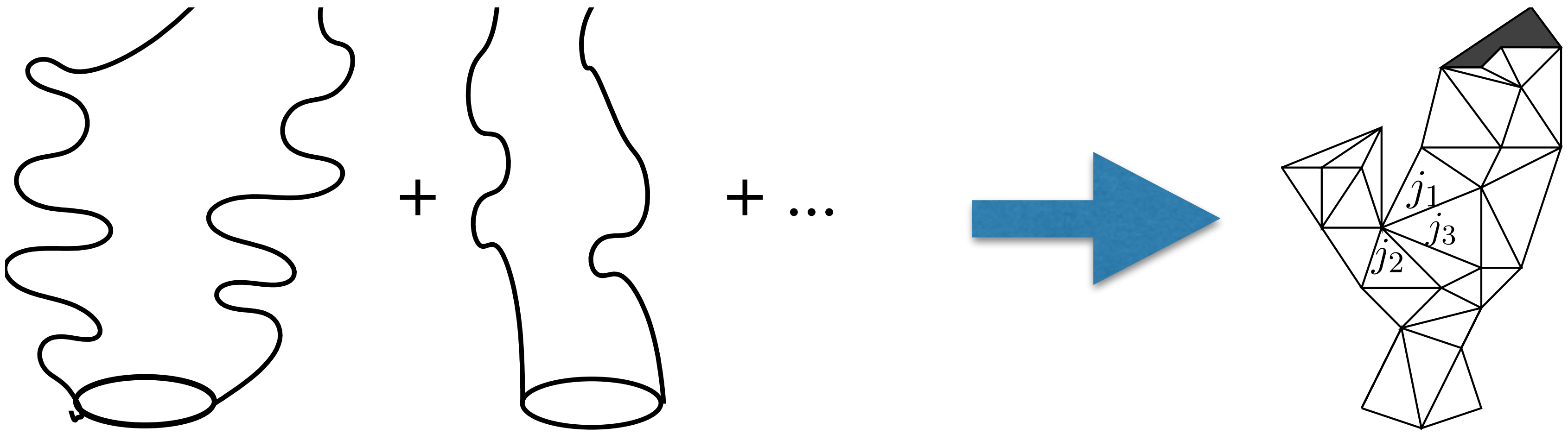
In diff-invariant systems:
All physical states should be invariant under time evolution.

Path integral is a projector onto physical states.

Need to construct the gravitational path integral.

Changing coupling constants and thus adjusting the the dynamics.

Discretization and spin foam models



sum over geometries = sum over labels associated to the triangulation

construction of amplitudes from GR action

spin foam model

[Reisenberger, Rovelli, Barrett, Crane, Freidel, Krasnov, Livine, Speziale…]

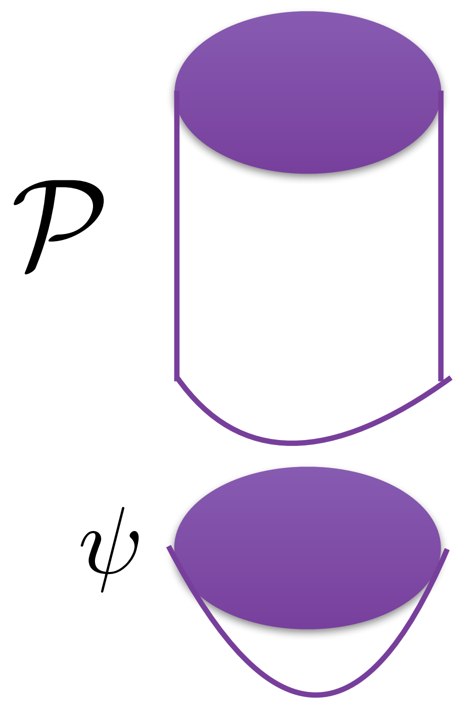
However the projector property can be expected to hold only in the refinement limit.

[Bahr, BD, Steinhaus 09 …11]

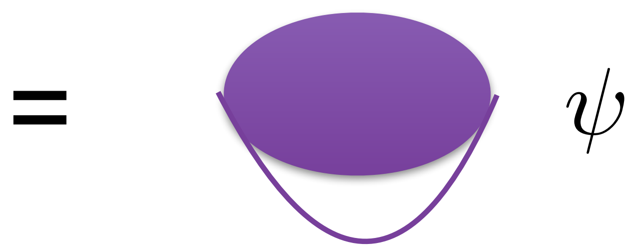
Do we know states with $\psi = \mathcal{P}\psi$?

- In 3D: yes, the BF vacuum state
- In 4D: (apart from BF state) not yet for 'gravitational' spin foam models
- This is actually the key problem: equivalent to solving QG dynamics

How can we construct physical states?



path integral
with one
vanishing boundary
(with kinematical
vacuum state)

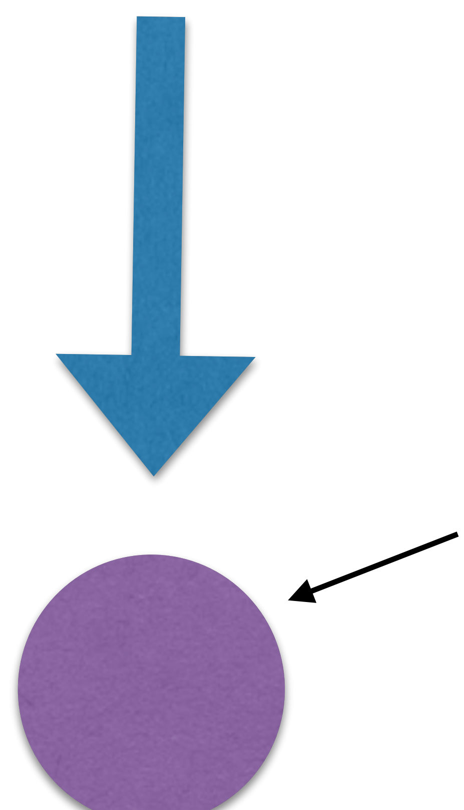


(version of)

Hartle Hawking

no boundary wave function.

Is a physical state.



Path integral over a disk gives

vacuum functional for boundary wave functions

$$\mathcal{A}_{vac}(\psi_{out}) = \langle \psi_{out} | \mathcal{P} | \emptyset \rangle$$

<u>**encodes (continuum) dynamics.**</u>

Need to compute the path integral in the refinement limit.

Problem: Extremely difficult for 4D (gravitational) spin foams.

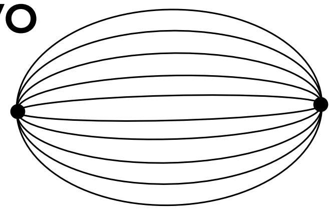
- cannot apply Monte Carlo simulations, due to complex amplitudes
- additional difficulties: infinite summations and (emerging) divergencies due to diffeomorphism symmetry
- so far no real space coarse graining method for 4D spin foam models available [BD, Mizera, Steinhaus14]
- but now: tensor network method for 3D lattice gauge models is working [BD, Delcamp wip]

Devised 2D 'analogue models' capturing key dynamical ingredients of spin foams.

- mimics a 2D-4D duality of lattice gauge theory to spin systems
- hope that phase structure is similar [BD, Eckert, Martin-Benito, Schnetter, Steinhaus, 11-13]
- path integral / refinement limit can be computed via tensor network renormalization [Vidal, Levin-Nave, Gu-Wen, ...]

Phase diagram for spin foam analogues

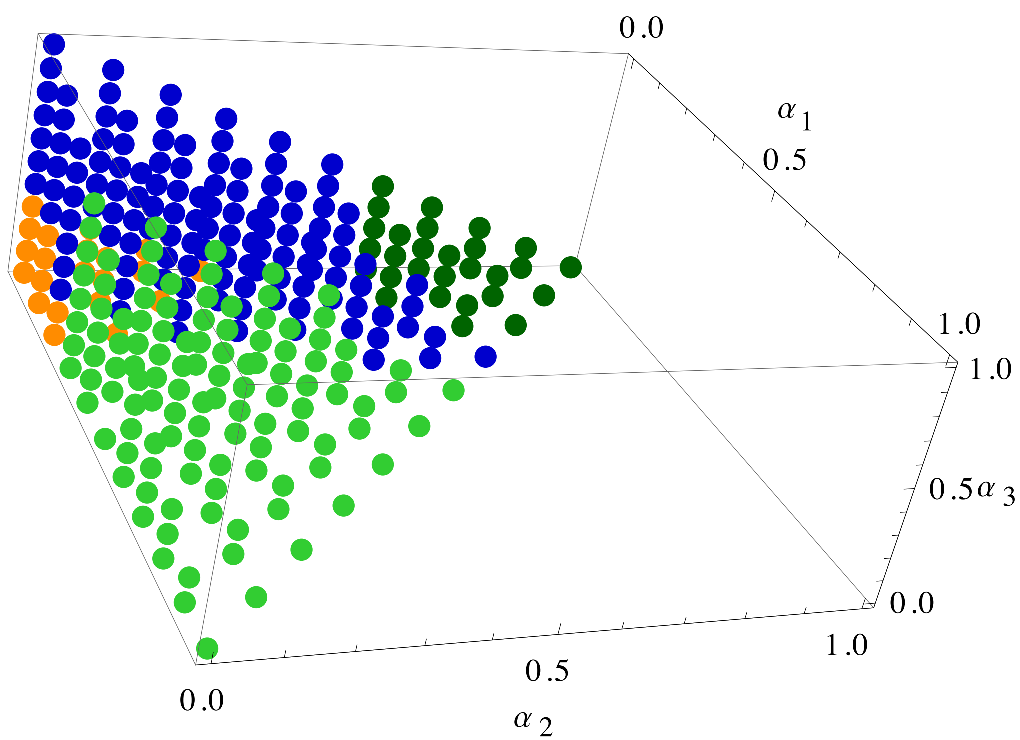
- models are similar to anyonic spin chains [Feiguin et al 06]
- but can be also interpreted as particular spin foams describing the gluing of two space time atoms



- changing certain parameters in initial model: changes how the atoms glue

  (technically: changes implication of simplicity constraints)

- anyonic spin chains support very rich phase structure, classification in [BD, Kaminski 13 and to appear]



Interpretation: different phases describe uncoupled space time atoms (green) and coupled space time atoms (orange,blue).

Positive indication for finding a geometric phase in spin foams.

[BD, Martin-Benito, Schnetter NJP 13]

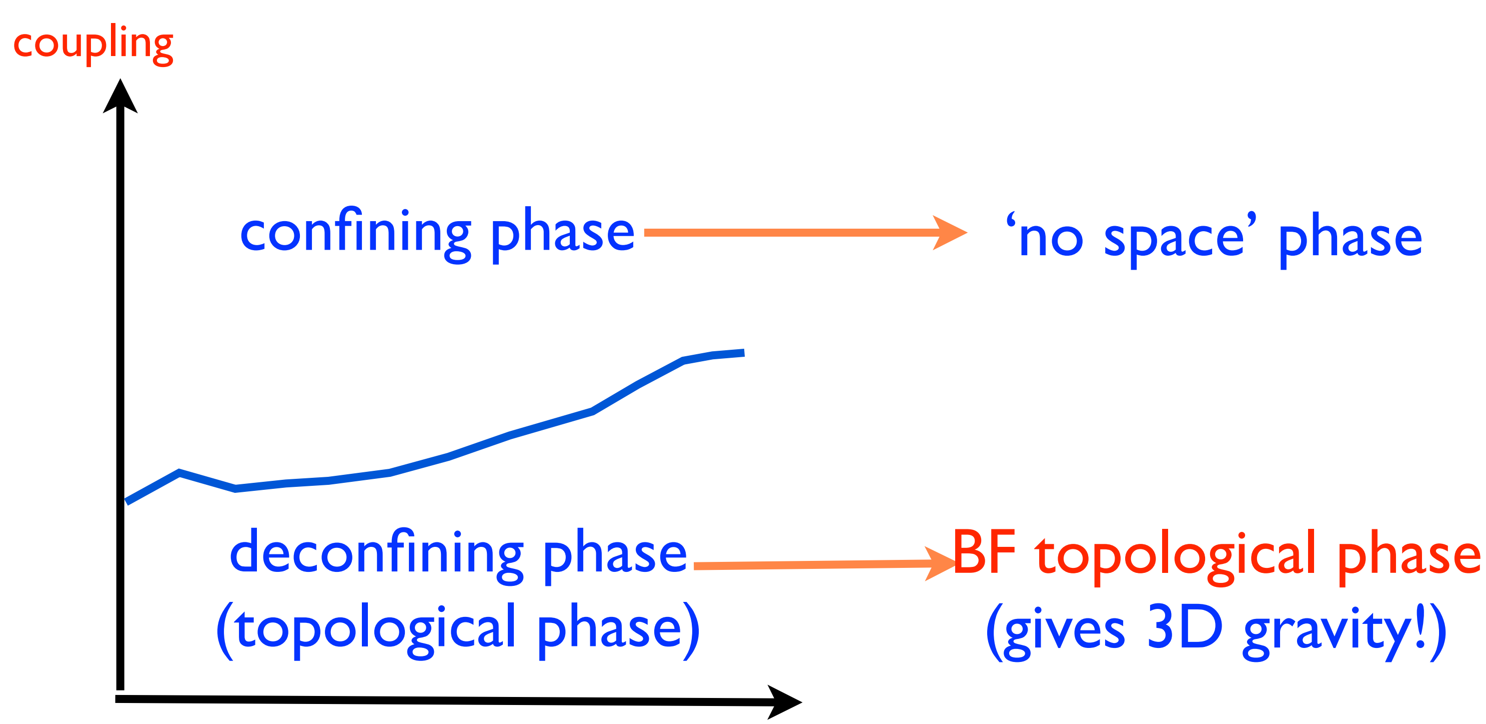
BD, Martin-Benito, Steinhaus PRD 13]

Phase diagram for spin foams ?

- need to develop (tensor network) coarse graining algorithms for

  spin foams = generalized lattice gauge theories
- first algorithm for 3D Abelian lattice gauge theories: decorated tensor networks [BD, Mizera, Steinhaus 14]
- 3D Non-Abelian lattice gauge theories [Delcamp, BD wip]

Phases in lattice gauge theory



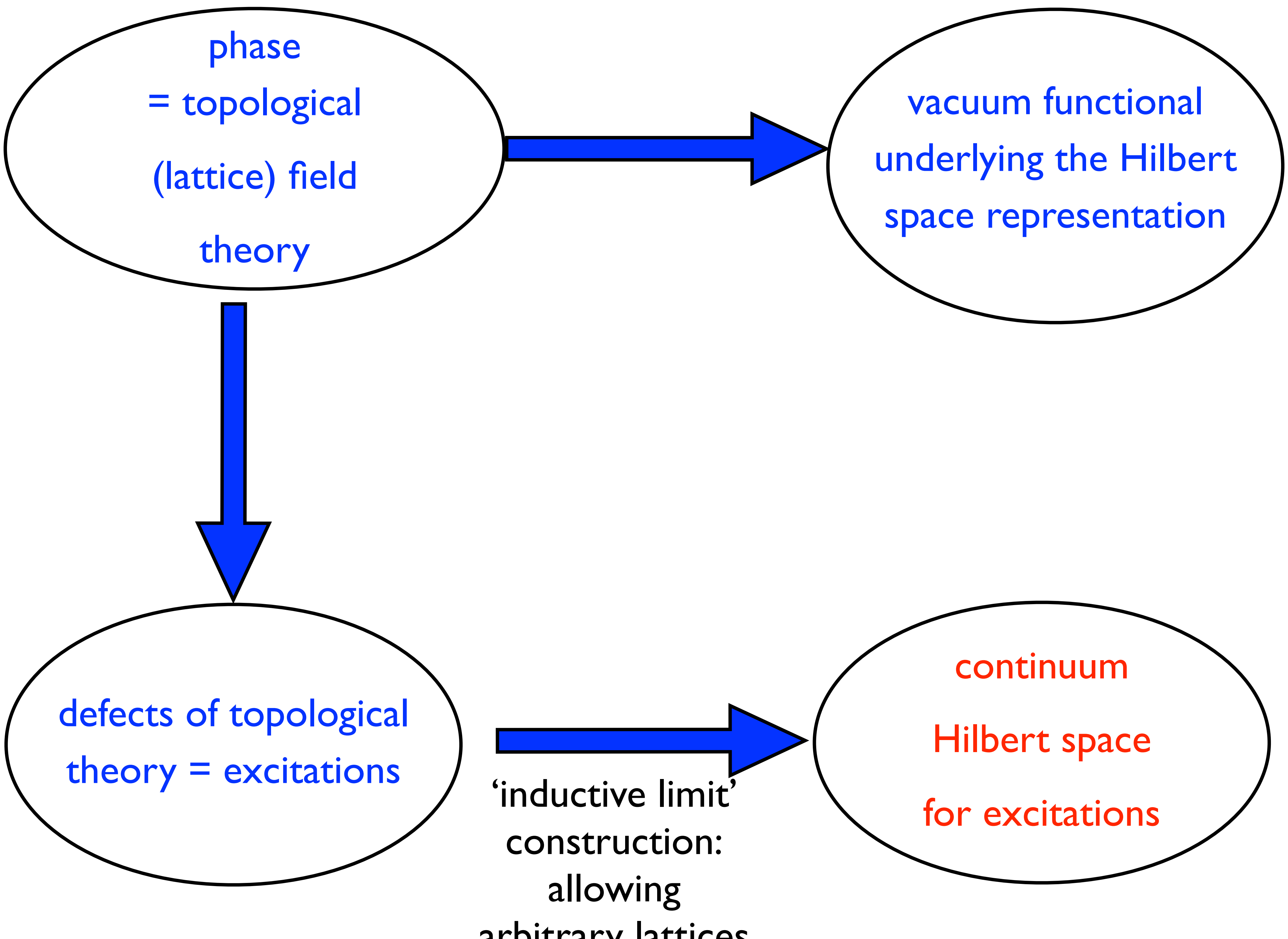
Are there more phases in spin foams?

Positive indication from

2D analogue models.

New phases give rise to new vacua and new quantum geometry realizations

[BD, Steinhaus NJP 13]

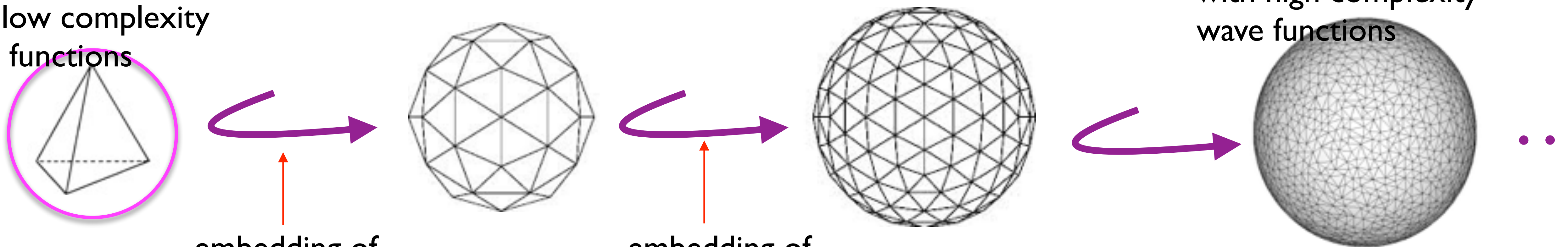


Consistent boundary formulation for the continuum dynamics

[BD NJP 12, 14]

Boundary Hilbert space with low complexity wave functions

Boundary Hilbert space with high complexity wave functions



embedding of boundary Hilbert spaces

embedding of boundary Hilbert spaces

initial discrete theory gives approximation to

$\mathcal{A}_{vac}^{\text{low com}}(\psi_{\text{low com}}) \longleftarrow \mathcal{A}_{vac}^{\text{med com}}(\psi_{\text{med com}}) \xleftarrow{\text{restricts to}} \mathcal{A}_{vac}^{\text{high com}}(\psi_{\text{high com}}) \cdots$

A (complete) family of consistent amplitudes defines a theory* of quantum gravity.

* Corresponds to a complete renormalization trajectory,

with scale given by complexity parameter.

Amplitudes can be computed iteratively in an approximation scheme.

Least effort necessary for low complexity = homogeneous 'cosmology' configurations.

[BD NJP 12, 14]

Summary

Quantum gravity models
as many body system

- tensor network algos
- categorification

Identify phases
and transitions

- (modified) inductive limit construction

New quantum geometry realizations

- computing refinement limit with tensor network algos

continuum limit:
consistent family of amplitudes

Quantum
Space Time

Many things to happen in the near future!

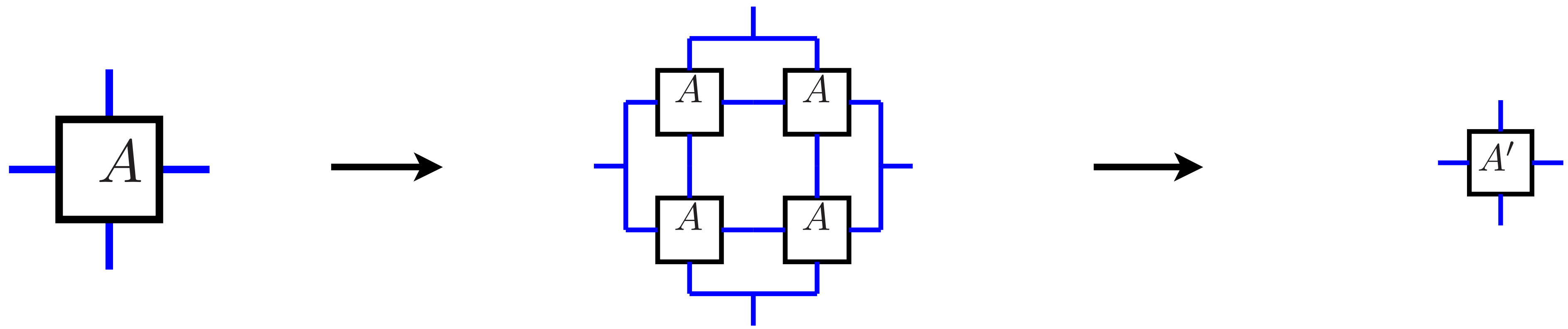
- Review:

Bianca Dittrich, *The continuum limit of loop quantum gravity - a framework for solving the theory*

arXiv:1409.1450

Tensor network renormalization

[Levin-Nave, Gu-Wen, …]



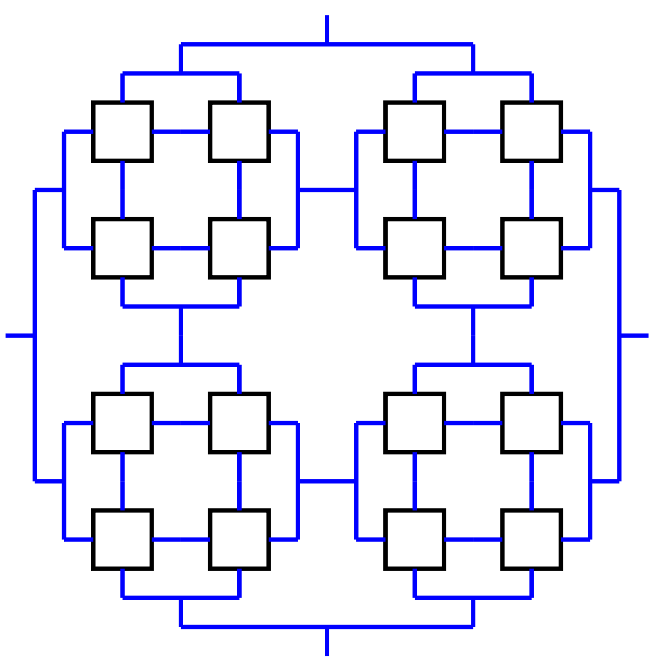
Amplitude of a disk region with edges representing boundary data.

Path integral for a disk region by gluing disks.

Neglect dependence on 'finer boundary data' to find effective amplitude.

Related to identifying vacuum.

[BD 12, BD 15]



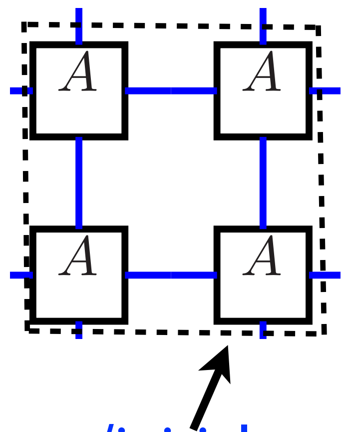
Iterate to find effective amplitude

incorporating many 'bare building blocks'.

Tensor network renormalization methods

(using local truncation method)

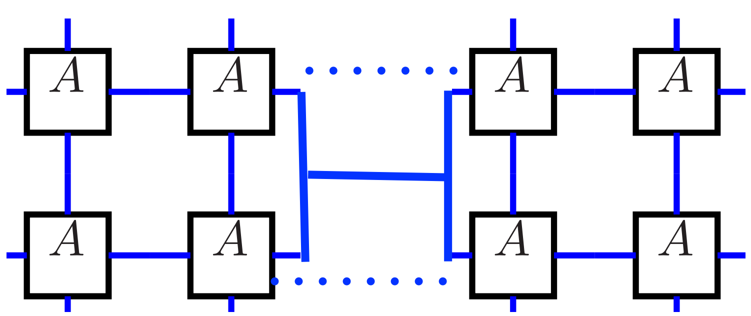
Coarse grain



bare/initial amplitude
depending on four variables

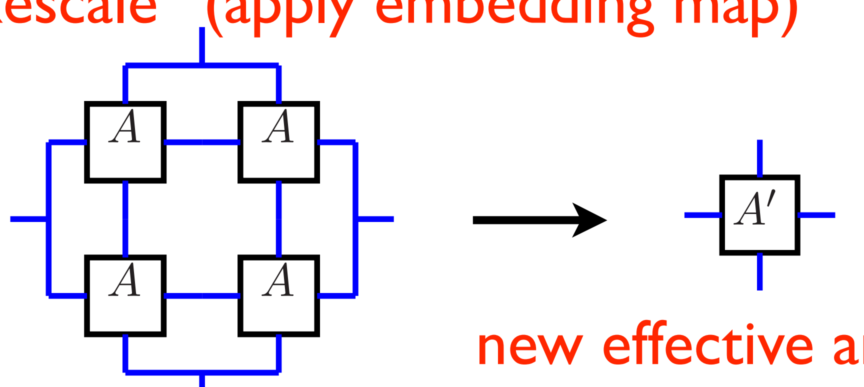
Contract initial amplitudes (sum over bulk variables). Obtain “effective amplitude” with more boundary variables.

Truncate /determine embedding map



Find an approximation (embedding map) that would minimize the error as compared to full summation (dotted lines). For instance using singular value decomposition, keeping only the largest ones.
Leads to field redefinition, and ordering of fields into more and less relevant.

“Rescale” (apply embedding map)

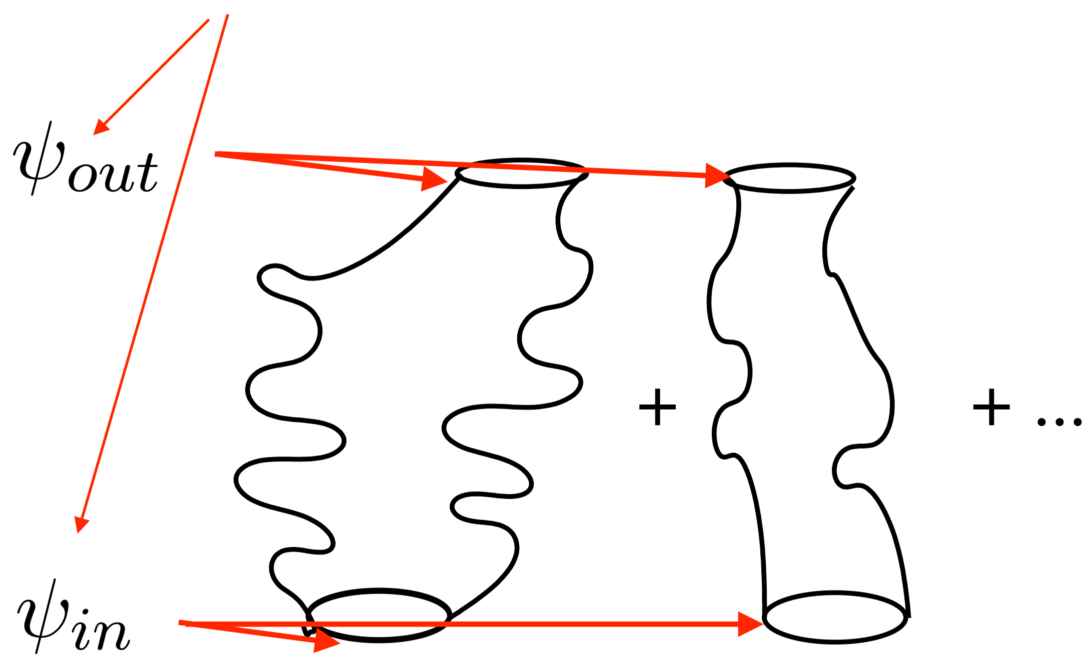


new effective amplitude

Use embedding maps to define coarse grained amplitude with the same (as initial) number of boundary variables.

Path integral = sum over spacetime geometries

'boundary states' encode (actually 4D) quantum geometry



'sum' over quantum space time geometries

need to define how to sum

quantum amplitude

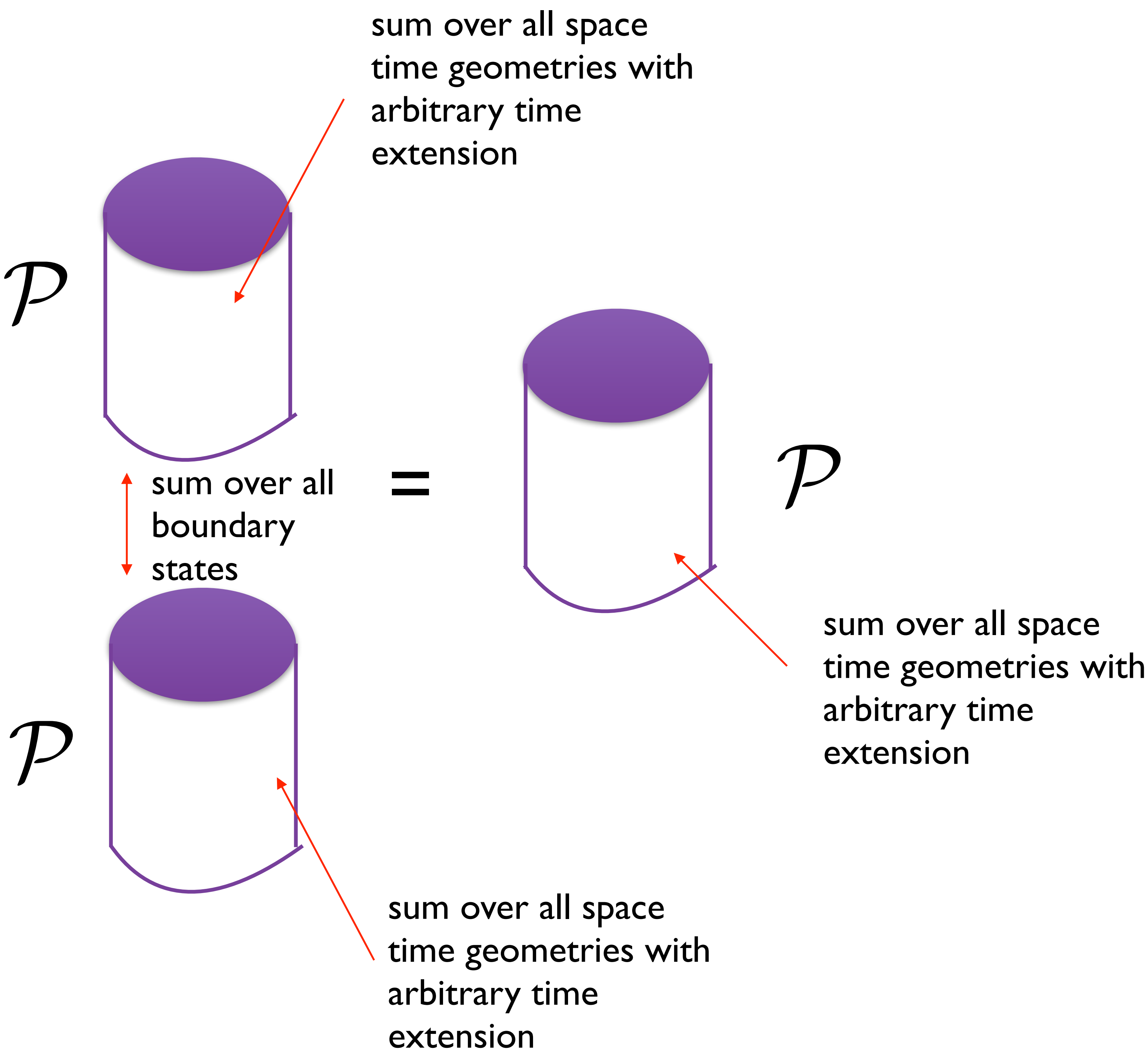
$$\langle \psi_{out} | \mathcal{P} | \psi_{in} \rangle \;=\; \overline{\psi_{out}}(\partial_{out}\mathrm{conf}\,1)\, \exp(\frac{i}{\hbar} S(\mathrm{conf}\,1))\, \psi_{in}(\partial_{in}\mathrm{conf}\,1) \;+$$
$$\overline{\psi_{out}}(\partial_{out}\mathrm{conf}\,2)\, \exp(\frac{i}{\hbar} S(\mathrm{conf}\,2))\, \psi_{in}(\partial_{in}\mathrm{conf}\,2) \;+\ldots$$

path integral matrix element

Physical states: $\psi \;=\; \mathcal{P}\psi$

Path integral is a projector

[Halliwell, Hartle 91]



$$\mathcal{P} \circ \mathcal{P} = \mathcal{P}$$

projector property