20ème conférence Claude Itzykson - Random Surfaces and Random Geometry



ID de Contribution: 2

Type: Invited talk

The Potts model on planar maps

mercredi 10 juin 2015 11:30 (45 minutes)

(joint work with Olivier Bernardi, Brandeis University)

Let q be an integer. We address the enumeration of q-colored planar maps (planar graphs embedded in the sphere), counted by the total number of edges and the number of monochromatic edges (those that have the same colour at both ends). In physics terms, we are averaging the partition function of the Potts model over all maps of a given size. We prove that the associated generating function is algebraic when q is of the form 2 + $2 \cos(j\pi/m)$, for integers j and m (but distinct from 0 and 4). This includes the two integer values q = 2 and q = 3, for which we give explicit algebraic equations.

For a generic value of q, we prove that the generating function satisfies an explicit system of differential equations. Both results hold as well for planar triangulations, with a strikingly similar system of differential equations.

The starting point of our approach is a recursive construction of q-coloured maps, in the spirit of what W. Tutte did in the seventies and eighties for properly coloured triangulations. This model has also been addressed by other authors and other methods (Bonnet & Eynard in 1999, and more recently Guionnet, Jones, Shlyakhtenko & Zinn-Justin, and Borot, Bouttier & Guitter), but our results are of a different nature and seem more explicit.

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