Random Tensors

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Random Geometry

We expect random geometry to follow the same development path than ordinary geometry, that is from lower towards higher dimensions, and from embedded, or extrinsic aspects towards intrinsic aspects (Gromov-Hausdorff).

Interesting random geometries should neither give all (or most of) the weight to too trivial nor to too complicated geometries.

Among physical motivations:

Quantizing Gravity $\simeq$ Randomizing Geometry

$$Z \simeq \int Dg \quad e^{\int_S A_{EH}(g)}$$

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Intrinsic 1D and 2D Random Geometry

- A simple intrinsic random geometry is the CRT (branched polymers). It has Hausdorff dimension 2, spectral dimension 4/3. In physics it corresponds to the $1/N$ limit of vector models.
- The next typical intrinsic random geometry is the Brownian sphere. It has Hausdorff dimension 4, very probably spectral dimension 2. In physics it corresponds to the $1/N$ limit of matrix models. It can be viewed as a CRT equipped with extra labels defining the shortcuts. It is linked to 2d gravity in particular through the many inspiring works of the IphT school (Bouttier, David, Duplantier, Eynard, Di Francesco, Guitter, Itzykson, Zuber...)
- These geometries have universality properties. Essential for their definition are the exact counting of the graphs involved (Catalan, Tutte) and interesting one-to-one maps (Dyck, Schaeffer) to explore the metric aspects.
- What about higher dimensions?

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We would like to handle *sums over random three-dimensional (and higher-dimensional) objects*, hence create a theory of random knots, random manifolds, etc.. but

- it is difficult to classify all geometries in dimension 3
- it is essentially impossible to classify all (smooth) geometries in dimension ≥ 4.

Mathematicians are developing proposals for random 3d geometry, eg petal model of random knots (Adams et al., 2012), random 3-manifolds based on random mapping class group gluing for Heegaard splitting into handlebodies (J. Maher et al.). However they may benefit from physicists input (formalism that extends to any dimension, $1/N$ expansion, connection to gravity...).

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Higher Dimensional Triangulations

- It is difficult to decide whether a general triangulation in 3D is homeomorphic to the sphere S_3
- It is essentially impossible (through a single algorithm) to decide whether a general triangulation in 4D is homeomorphic to the sphere S_4

We should distinguish $ST(v)$, the number of spherical triangulations with v vertices, from $ST(t)$, the number of spherical triangulations with t tetrahedra. In particular one can have $v << t$.

T. Jonsson's talk: LC =locally constructible, CDT = causal triangulations: exponential growth

$$LC(t) \leq C^t, \quad CDT(t) \leq C^t$$

Open, difficult: Is the number $ST(t)$ of triangulations of the 3-sphere with t tetrahedra exponentially bounded in t?

- Lower bounds (super-exponential growth) on $ST(v)$: J. Pfeiffe and G. Ziegler $ST(v) \geq e^{v^{5/4}}$ (2004) E. Nevo and S. Wilson: $\log ST_v \geq e^{v^2}$ (2013).
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Random Tensors as Symmetry Breaking

$\exists!$ Hilbert space $\ell_2(\mathbb{Z})$. $U(N)$ invariance can be broken.

vector models => matrix models => tensor models

Smaller symmetry means there are more invariants available for interactions

Random vectors have exactly one connected invariant interaction, of degree 2 namely the scalar product $\bar{\phi} \cdot \phi$.

Random matrices: $N = N_1 N_2$, => $U(N_1 N_2)$ symmetry can break to $U(N_1) \otimes U(N_2)$ giving rise to infinitely many connected invariant interactions, one at every (even) degree, namely $\mathrm{Tr}\,(MM^\dagger)^p$.

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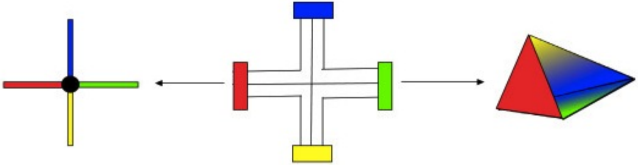
Colored Triangulations and Edge Colored Graphs

Italian school, Lins, crystallization theory: D-dimensional colored triangulations are simpler than general triangulations. They triangulate pseudo-manifolds with a well defined D-homology and they are dual to $(D+1)$-edge-colored graphs.



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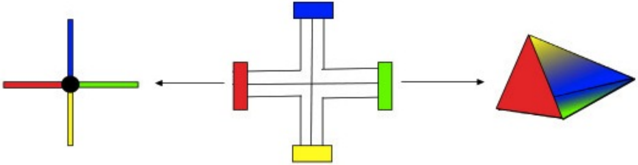

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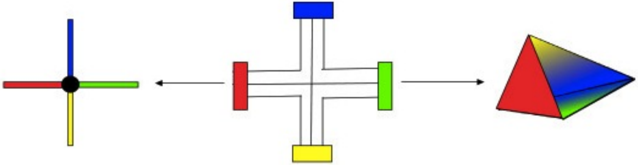

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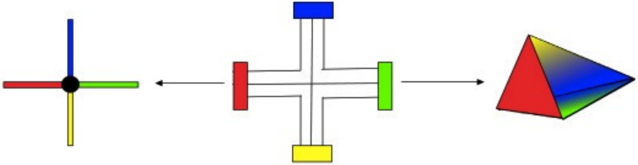
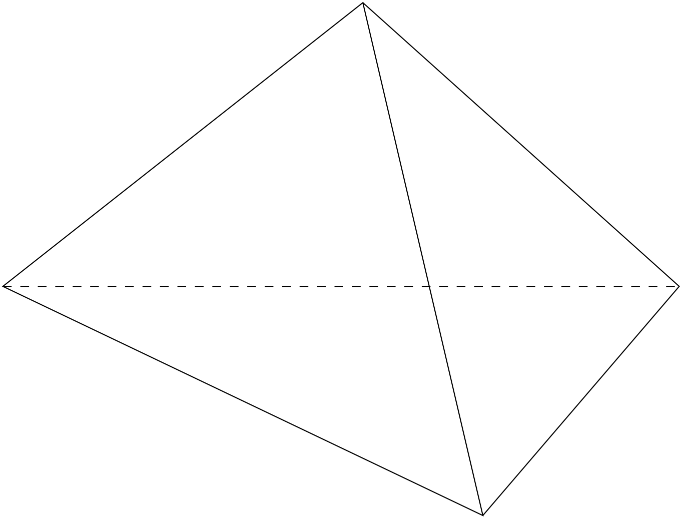


Are colored triangulations general enough for random geometry? Yes, since any D-dimensional triangulation uniquely defines a D dimensional colored triangulation, its barycentric subdivision.

Barycentric Colored Triangulations



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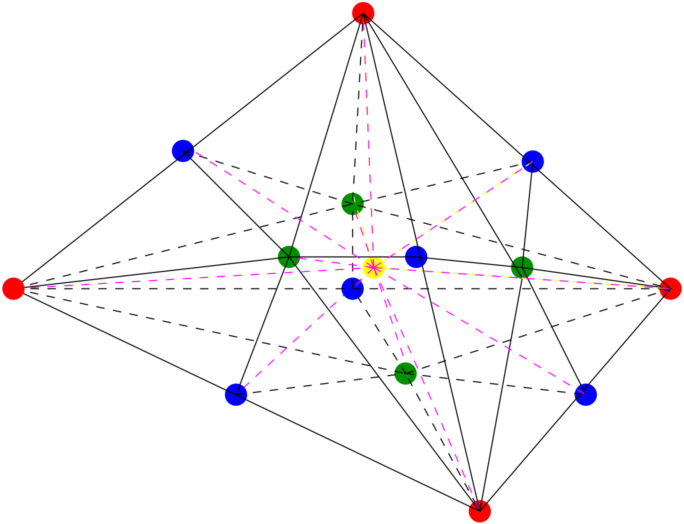

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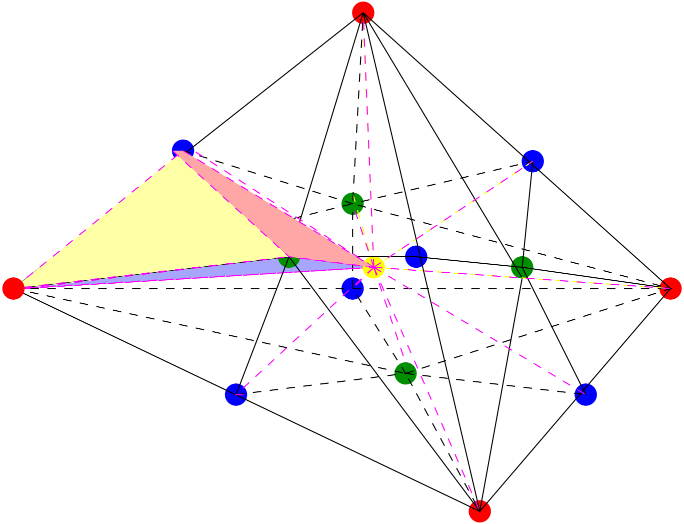

Tensor Models

R. Gurau found in 2009 that crystallization theory is dual to a quantum field theory and in 2010 that this field theory admits a $1/N$ expansion.

This expansion is not topological !

Basic objects: $U(N)^{\otimes D}$ tensor invariants = regular D-edge-colored connected bipartite graphs

- are dual to colored triangulations
- are the interactions (vertices) of rank-D random tensors
- are the observables of rank-D random tensors
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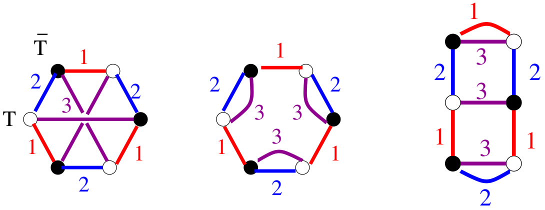

Tensor invariants can be counted as equivalence classes of permutations, in the style of J.B. Zuber's talk on doodles (J. Ben Geloun and S. Ramgoolam)

$$Z_1^c(n) = 1, 0, 0, 0, 0, \ldots \qquad \bar{\Phi} \cdot \Phi$$

$$Z_2^c(n) = 1, 1, 1, 1, 1, 1, 1 \ldots \qquad \mathrm{Tr}(MM^\dagger)^n$$

$$Z_3^c(n) = 1, 3, 7, 26, 97, 624, 4163 \ldots$$

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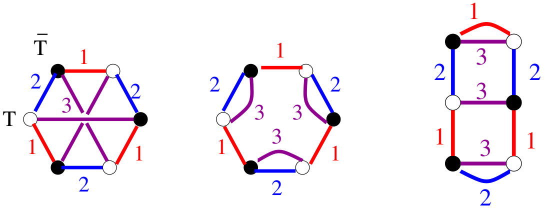

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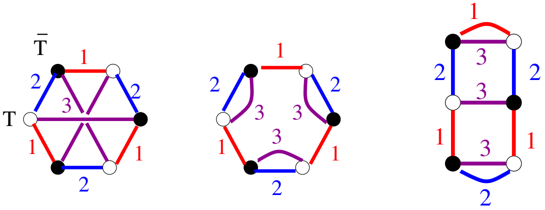

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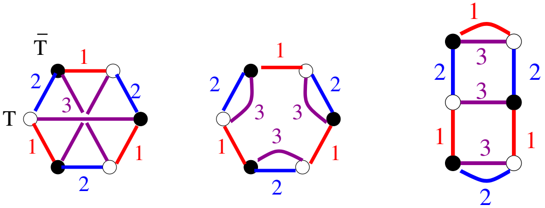

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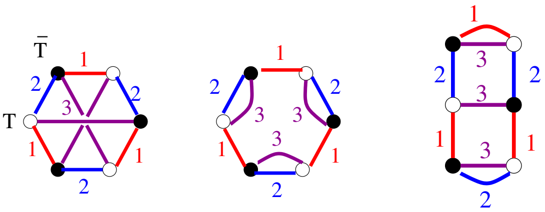

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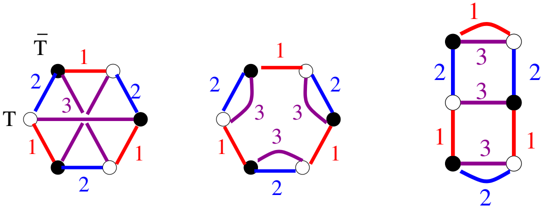

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A general tensor model (with polynomial interactions) is

$$S(T,\bar{T}) = T\cdot\bar{T} + \sum_{\mathcal{B}} t_{\mathcal{B}} \mathrm{Tr}_{\mathcal{B}}(\bar{T},T)$$
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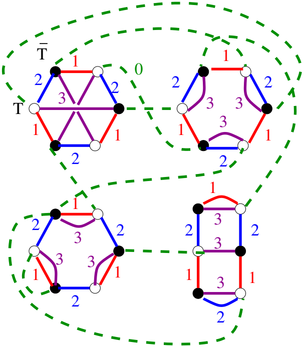
Feynman graphs: "vertices" $\mathcal{B}$. Gaussian integral: Wick contractions of T and $\bar{T}$ $\to$ dashed edges to which we assign the index 0 (here green color).



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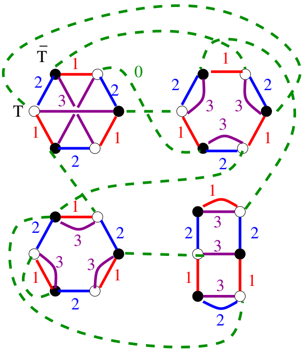

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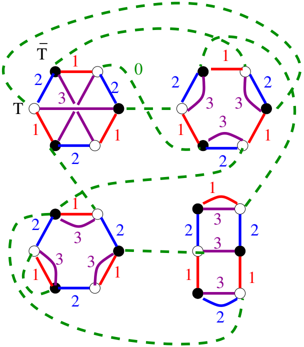

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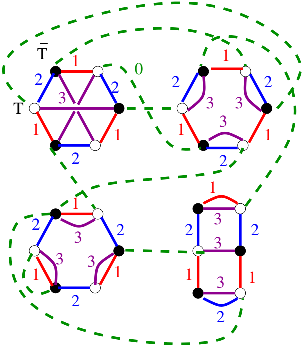

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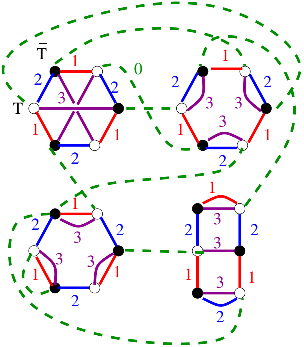

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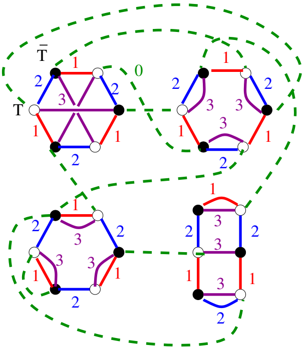

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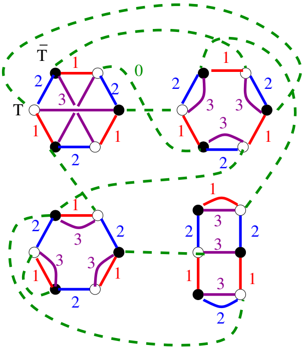

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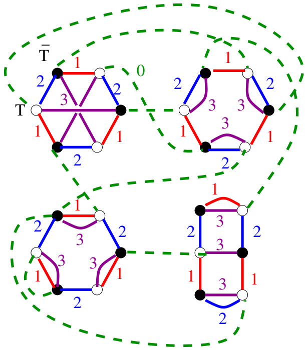

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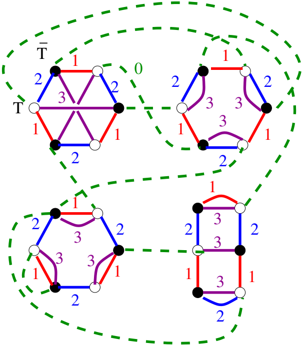

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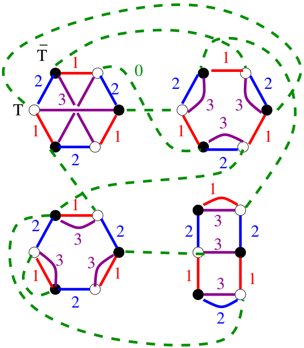
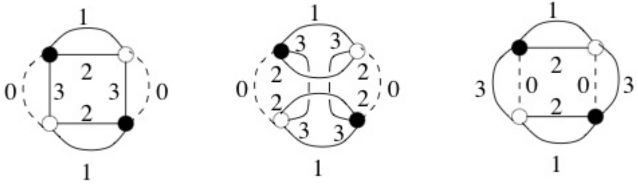


Jackets, Degree, $1/N$ Expansion

Jackets = color cycle up to orientation ($D!/2$ at rank D)
= canonical system of $D!/2$ globally defined Heegaard surfaces in the dual triangulation



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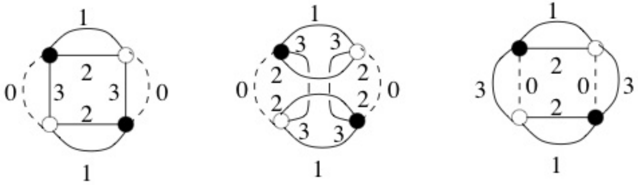

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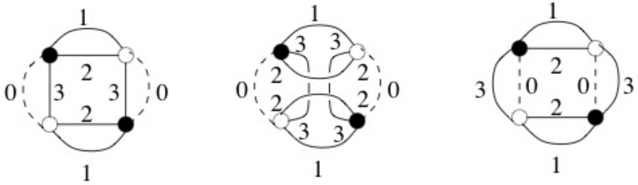

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$$S_{\mathrm{Regge}} = \Lambda \sum_{\sigma_D} vol(\sigma_D) - \frac{1}{16\pi G} \sum_{\sigma_{D-2}} vol(\sigma_{D-2})\, \delta(\sigma_{D-2})$$

Discretized Einstein Hilbert action on a triangulation with Q_D equilateral D-simplices and Q_{D-2} $(D-2)$-simplices:

$$A_G(N) = e^{\kappa_1 Q_{D-2} - \kappa_2 Q_D}$$

On the Feynman dual graph: $Q_D \to n$, number of vertices; $Q_{D-2} \to F$, number of faces, hence Regge action for equilateral simplices becomes

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the natural amplitudes of tensor models. The exact correspondence is

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Tensor Models and Quantum Gravity

The Feynman graphs of tensor models can be considered an equilateral (F. David) version of Regge calculus (1962):

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Discretized Einstein Hilbert action on a triangulation with Q_D equilateral D-simplices and Q_{D-2} $(D-2)$-simplices:

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Random tensors therefore provide a new approach, nicknamed the tensor track, to the quantization of gravity in dimension ≥ 3.

It is based on quantum field theories of space time, not on space-time, with several nice features

- background independence
- sum over all topologies
- renormalizability
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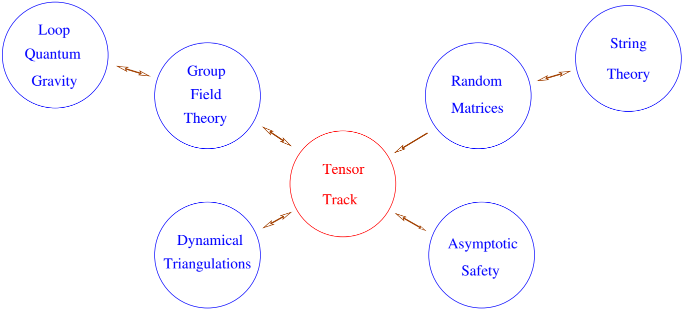

The tensor track is related to many of the main approaches to quantum gravity.

It suggests a cosmological scenario, of course highly speculative: condensation of space-time and our universe through a sequence of phase transitions, starting from a pregeometric, transplanckian combinatorial phase. Tensor renormalization group flows hopefully can provide mathematical modeling of such a scenario.

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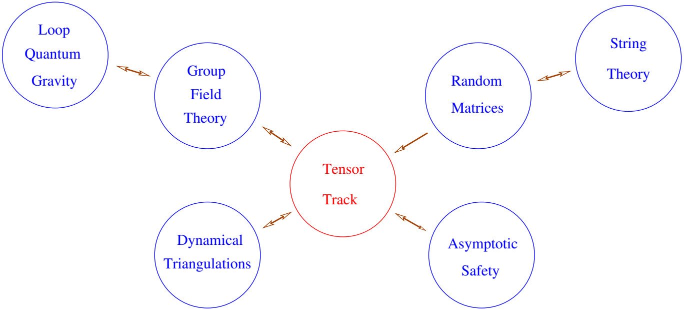

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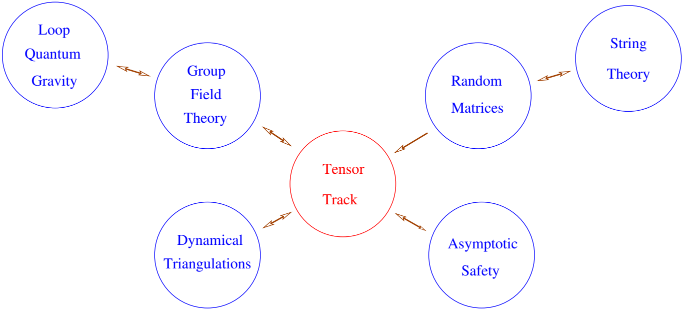

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- Tensor field theories extend non-commutative field theory just as tensors extend matrices. They have tensor interactions and propagators of the inverse Laplacian type. They can be renormalized in many cases (up to rank/dimension 6) (Ben Geloun, R.) and can also incorporate spin-foam data such as Boulatov-type projectors (Carrozza, Lahoche, Oriti, R....).
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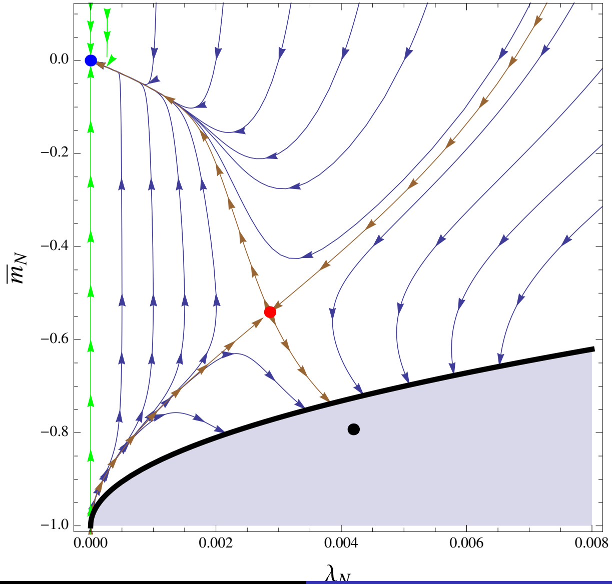

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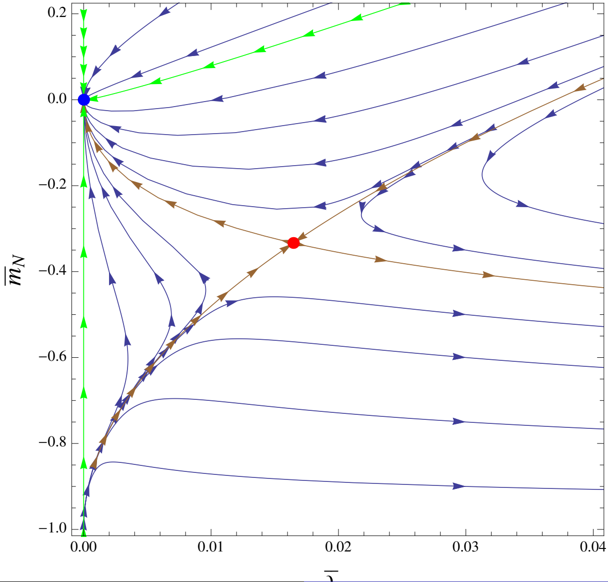

Enhanced Rank Four Quartic Tensor Models

(joint work V. Bonzom and T. Delepouve, arXiv:1502.01365)

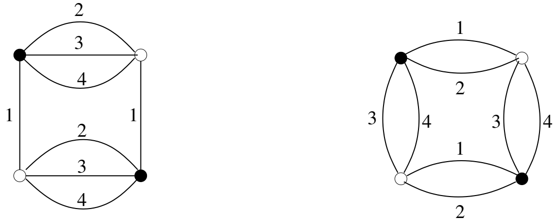

Figure: The quartic invariants at rank 4.

$$B_{\mathcal{C}_1}(\overline{\mathbf{T}},\mathbf{T}) = \sum_{n_1,\dots,n_4,n'_1,\dots,n'_4} \overline{T}_{n_1 n_2 n_3 n_4} T_{n_1 n'_2 n'_3 n'_4} \overline{T}_{n'_1 n'_2 n'_3 n'_4} T_{n'_1 n_2 n_3 n_4}$$

and three similar formulae for $B_{\mathcal{C}_2}$, $B_{\mathcal{C}_3}$ and $B_{\mathcal{C}_4}$. Also

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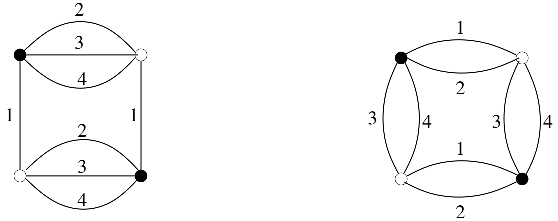

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Standard general (color-symmetric) quartic tensor model at rank 4

$$d\mu_{standard} = d\mu_0 \; e^{-N^3 \lambda \sum_{i=1}^4 B_{\mathcal{C}_i}(\overline{\mathbf{T}},\mathbf{T}) \, - \lambda' N^3 \sum_{i=2}^4 B_{\mathcal{C}_{1i}}(\overline{\mathbf{T}},\mathbf{T})} \; .$$

Borel summable uniformly in N for λ, λ' in cardioid domains (Delepouve, Gurau, R.).

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Intermediate Field Representation

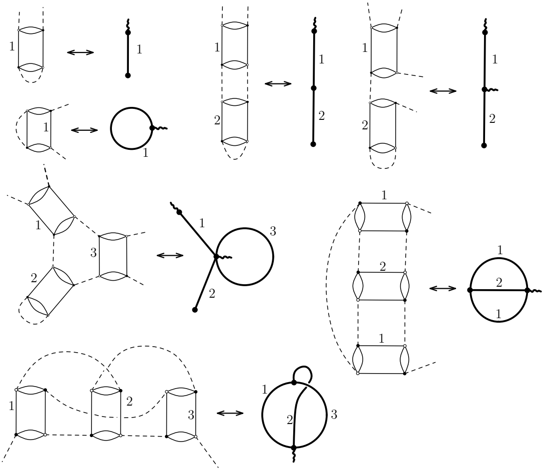

Figure: Intermediate Field Maps (courtesy L. Lionni)

Leading Order Maps

The leading order maps (in the IF representation) are planar, and made of trees of unicolored edges which connect bicolored connected objects. The latter can touch one another at a single vertex at most and do not form closed chains, thus displaying a "cactus" structure.

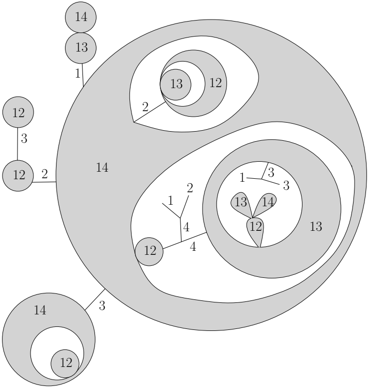

Figure: Grey areas are connected components of given color types. A bicolored connected component can be attached to another one on a single vertex, without forming cycles of such components.

Universality

Induction: A *tree of necklaces* of type $\{p_1, \dots, p_n, p_{n+1}\}$ is obtained from a tree of necklaces of type $\{p_1, \dots, p_n\}$ by removing any edge of color i and replacing it with the necklace of size p_{n+1} open on an edge of color i (preserving bipartite character).

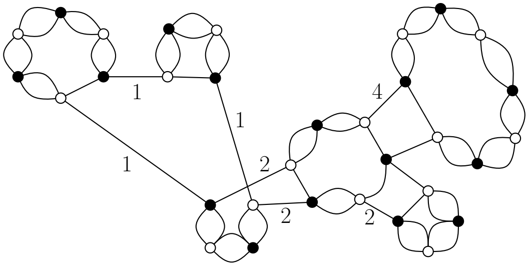

Figure: Trees of necklaces

The data $\{p_1, \dots, p_n\}$ does not capture the full structure of the observable. It only records the sizes of the necklaces which are inserted one after the other one. It is sufficient for enumeration of the leading order contributions.

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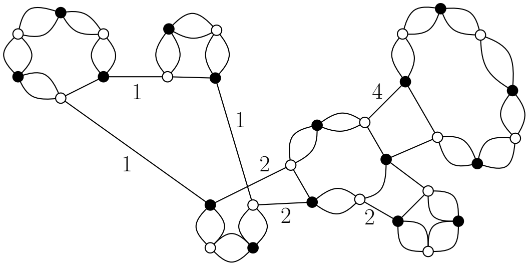

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Enhancement of trees of necklaces

Let us denote a generic tree of necklaces by $\mathcal{L}$. If it is of type $\{p_1, \ldots, p_n\}$, the enhancement it requires to contribute at large N is

$$\omega(\mathcal{L}) = \sum_{k=1}^{n} (2 + p_k) - 3(n-1) = 3 - n + \sum_{k=1}^{n} p_k .$$

Generalized model has measure

$$d\mu(\mathbf{T}, \overline{\mathbf{T}}) = \exp\Big(- \sum_{\mathcal{L}} N^{\omega(\mathcal{L})} \, t_{\mathcal{L}} \, B_{\mathcal{L}}(\mathbf{T}, \overline{\mathbf{T}}) \Big) \; d\mu_0(\mathbf{T}, \overline{\mathbf{T}}).$$

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Factorization

Theorem

Let us denote the expectation of the necklace of size p as

$$C_p = \frac{N^{2+p}}{N^4} \left\langle B^{(p)}_{12}(\mathbf{T}, \overline{\mathbf{T}}) \right\rangle = \frac{N^{2+p}}{N^4} \frac{\int d\mu(\mathbf{T}, \overline{\mathbf{T}}) B^{(p)}_{12}(\mathbf{T}, \overline{\mathbf{T}})}{\int d\mu(\mathbf{T}, \overline{\mathbf{T}})}.$$

Then the expectation of any tree of necklaces $\mathcal{L}_{\{p_1,\dots,p_n\}}$ factorizes in the large N limit like

$$\frac{N^{\omega(\mathcal{L}_{\{p_1,\dots,p_n\}})}}{N^4} \langle \mathcal{L}_{\{p_1,\dots,p_n\}}(\mathbf{T}, \overline{\mathbf{T}}) \rangle = \prod_{k=1}^{n} C_{p_k}.$$

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$$C_p = \frac{N^{2+p}}{N^4} \left\langle B_{12}^{(p)}(\mathbf{T}, \overline{\mathbf{T}}) \right\rangle = \frac{N^{2+p}}{N^4} \frac{\int d\mu(\mathbf{T}, \overline{\mathbf{T}}) B_{12}^{(p)}(\mathbf{T}, \overline{\mathbf{T}})}{\int d\mu(\mathbf{T}, \overline{\mathbf{T}})}.$$

Then the expectation of any tree of necklaces $\mathcal{L}_{\{p_1,\dots,p_n\}}$ factorizes in the large N limit like

$$\frac{N^{\omega(\mathcal{L}_{\{p_1,\dots,p_n\}})}}{N^4} \left\langle \mathcal{L}_{\{p_1,\dots,p_n\}}(\mathbf{T}, \overline{\mathbf{T}}) \right\rangle = \prod_{k=1}^{n} C_{p_k}.$$

Factorization

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Schwinger-Dyson equations at leading order

Schwinger-Dyson equation

$$C_p = \sum_{k=0}^{p-1} C_k \, C_{p-k-1} + \sum_{j\geq 1} j \, \partial_j V(C_1, C_2, C_3, \dots) \; C_{j+p-1}$$

where V is some polynomial, and C_p is the number of maps with root vertex of degree p. The quadratic term corresponds, as usual for equations *à la Tutte*, to the case where the root edge is a bridge.

The second term extends the length of the boundary face from p to $p+j-1$, which is also usual for planar maps. However, it here comes with a more complicated weight $j\partial_j V(C_1, C_2, \dots)$, due to the *branching process*.

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This equation was analyzed in the 90's in the context of multi-trace matrix models (Das, Dhar, Sengupta, Wadia, Korchemsky, Klebanov et al...). Free energy behaves like $(g - g_c)^{2-\gamma}$, where γ is the *entropy exponent*.

- Critical maps, non-critical trees => $\gamma = -1/2$ (pure 2D gravity).
- Non-critical maps, critical trees => $\gamma = 1/2$ (branched polymers)
- Both simultaneously critical => $\gamma = 1/3$ (proliferation of baby universes)
- Tuning more couplings => $\gamma = p/(n + m + 1)$, $p \leq n$ and m integers.
- Any tensor invariant interaction can be enhanced. Recently, leading order computed for rank-3 invariants up to order 6 (V. Bonzom, L. Lionni, R.) but still no general description yet.

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Conclusion

- Simple quartic tensor models at rank 4 can interpolate between branched polymers and brownian sphere behavior.
- The intermediate field representation provides a relationship between matrix and tensor models. Tensor models are multi-matrix models, but coupled in a new way.
- The numerical exploration of renormalization group flows in the tensor theory space has started and confirms their asymptotic freedom.
- Hopefully further analysis of random tensor models might lead to the discovery of new universal phases of random geometry in higher dimension.

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