

# Modeling the pion Generalized Parton Distribution

C. Mezrag

CEA Saclay - IRFU/SPhN

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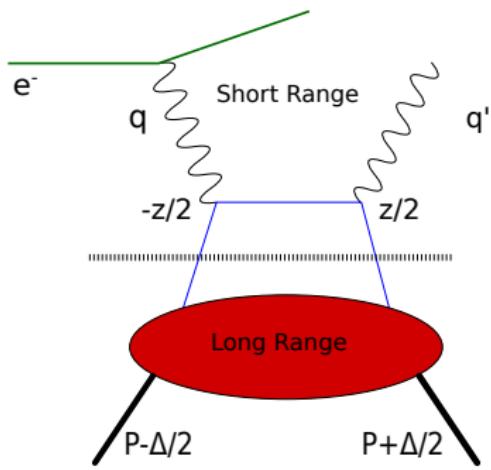
In collaboration with:

L. Chang, H. Moutarde, C. Roberts,  
J. Rodriguez-Quintero, F. Sabatié,

- 1 GPDs : generalities, properties, and models
- 2 GPD: the pion case
- 3 Focus: the  $\xi = 1$  region
- 4 Conclusions and outlooks

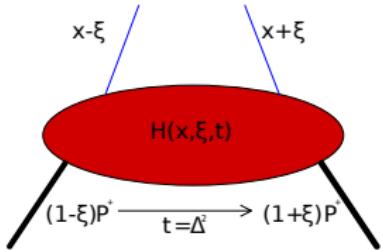
# The Golden Channel: Deep-Virtual Compton Scattering

## Deep-Virtual Compton Scattering (DVCS)



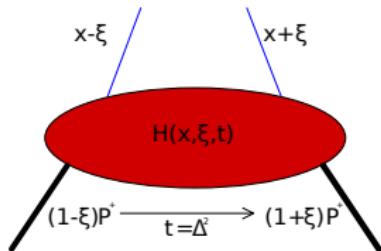
- Short range  $\rightarrow$  perturbation theory.
- Long range  $\rightarrow$  non perturbative objects: GPDs.
- Encode the hadrons 3D partonic structure and the spin structure.
- *Universality*

# Kinematic Variables

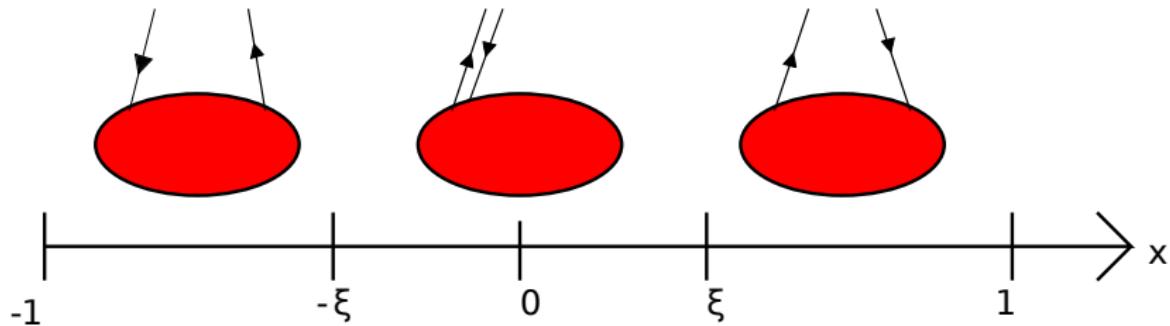


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- depending on 3 variables:  $x$ ,  $\xi$ ,  $t$ .

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# Formal Definition

- Proton case:

$$\begin{aligned} & \frac{1}{2} \int \frac{e^{ixP^+z^-}}{2\pi} \langle P + \frac{\Delta}{2} | \bar{q}(-\frac{z}{2}) \gamma^+ q(\frac{z}{2}) | P - \frac{\Delta}{2} \rangle dz^-|_{z^+=0,z=0} \\ = & \frac{1}{2P^+} [H^q(x, \xi, t) \bar{u}(P + \frac{\Delta}{2}) \gamma^+ u(P - \frac{\Delta}{2}) \\ & + E^q(x, \xi, t) \bar{u}(P + \frac{\Delta}{2}) \frac{i\sigma^{+\alpha} \Delta_\alpha}{2M} u(P - \frac{\Delta}{2})]. \end{aligned}$$

X. Ji, 1997

D. Müller et al., 1994

A. Radyushkin, 1997

# Formal Definition

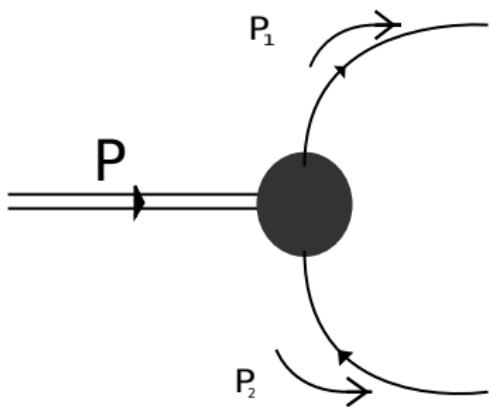
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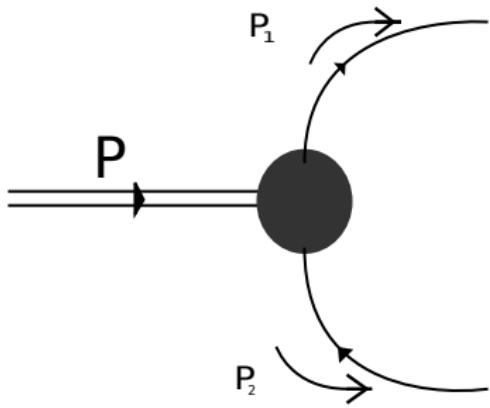
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# The pion? But why?



- Advantages :
  - ▶ Two-body system.
  - ▶ Pseudo-scalar meson.
  - ▶ Valence quarks u and d.
  - ▶ Isospin symmetry.
- Drawbacks
  - ▶ Very few experimental data available.
  - ▶ No data at  $\xi \neq 0$   
→ The model can be compared only at  $\xi = 0$ , i.e. to the Parton Distribution Function (PDF) and to the form factor.
  - ▶ Amrath et al., Eur. Phys. J. C58 179

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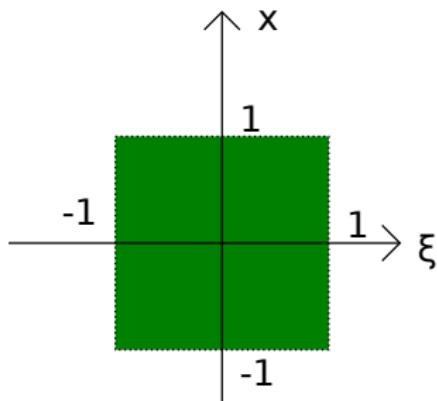
Good starting point before dealing with more complicated objects.

# GPD properties

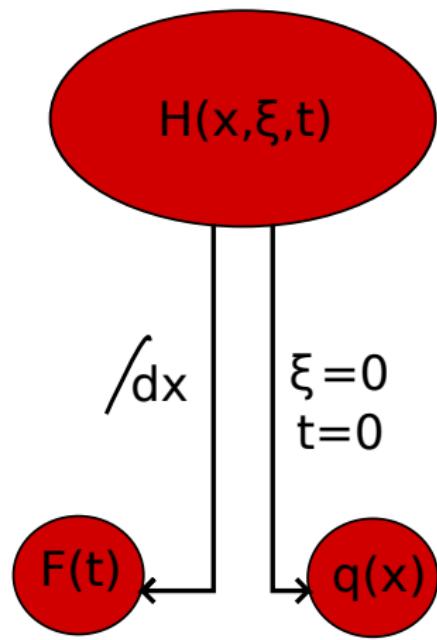
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$$|x| \leq 1 \text{ and } |\xi| \leq 1$$

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# GPD properties

$$\begin{aligned}\mathcal{M}_m(\xi, t) = & \sum_{i=0}^{\frac{m}{2}} c_{2i}(t) \xi^{2i} \\ & + \text{mod}(m, 2) c_{m+1} \xi^{m+1}\end{aligned}$$

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Those properties make GPD modeling a challenge.

# Models of GPDs

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- Mellin-Barnes approach:
  - ▶ D. Müller and A. Schäfer (2006), K. Kumericki and D. Müller (2010).

## Alternative ideas

- Lattice QCD:
  - ▶ Computations of Mellin Moments.
  - ▶ Until now, only the very first Mellin moments have been computed.
  - ▶ Still, new proposals done by X. Ji (*X. Ji, 2013*).

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Dyson-Schwinger equations seems to be a very promising approach to model GPDs!

# Dyson-Schwinger Equations

- Equations between non perturbative Green functions.
- Infinite number of coupled equations → no one has solved it until now!
- This requires approximations. In QCD, there are mainly two:
  - ▶ Rainbow Ladder (RL), resumming over a certain class of diagrams,
  - ▶ Dynamical Chiral Symmetry Breaking (DCSB).

See for instance *L.Chang et al.*, PRC87,2013 for details about truncation schemes.

# Example: the quark propagator

Perturbative case:

$$\text{---} \bullet \text{---} = \text{---} + \text{---} \text{---} + \dots$$

The diagram shows the perturbative expansion of a quark propagator. On the left, a horizontal line with a black dot at its center represents the full propagator. An equals sign follows, followed by a plus sign. To the right of the plus sign is a diagram showing a horizontal line with a wavy loop attached to it, representing a one-loop correction to the propagator.

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$$\text{---} \bullet \text{---} = \text{---} + \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} + \dots$$

The diagram shows a quark loop (a horizontal line with a black dot) equated to a bare quark line plus a quark loop with a gluon vertex and a quark-gluon vertex insertion.

Dyson-Schwinger case:

$$(\text{---} \bullet \text{---})^{-1} = (\text{---})^{-1} - \text{---} \bullet \text{---}$$

The diagram shows the inverse quark propagator as the inverse of the bare quark line minus the quark loop with a gluon vertex and a quark-gluon vertex insertion.

# Solutions of the BSE-DSE

DSE-BSE equations have been solved numerically and solutions have been fitted on specific parametrizations (*L. Chang et al., 2013*).

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- Propagator → linear combination of free propagators using complex conjugate poles:

$$S(k) = \sum_{j=1}^m \left( \frac{z_j}{i\cancel{k} + m_j} + \frac{z_j^*}{i\cancel{k} + m_j^*} \right)$$

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- Pion Bethe-Salpeter amplitude → use the Nakanishi representation:

$$\Gamma_\pi(k, P) = c_j \int_{-1}^1 dz \frac{\rho_\nu(z) \Lambda_j^{2\nu}}{\left[ \left( k - \frac{1-z}{2} P \right)^2 + \Lambda_j^2 \right]^\nu} + \dots$$

# Analytic model for pion GPD

Propagator:

$$S(p^2) = \frac{-ip \cdot \gamma + M}{p^2 + M^2}$$

Vertex:

$$\Gamma_\pi \propto i\gamma_5 \int \frac{dz \ M^2 \rho_\nu(z)}{(q(k, \Delta, P)^2 + M^2)^\nu}$$

- $p$  is the quark momentum,
- $M$  is the effective mass of the constituent quark.

- $\rho_\nu(z) \propto (1 - z^2)^\nu$  is the  $z$  distribution.
- $q(k, \Delta, P) = k - \frac{1-z}{2} (P \pm \frac{\Delta}{2})$  deals with the momentum fraction carried by the quark.

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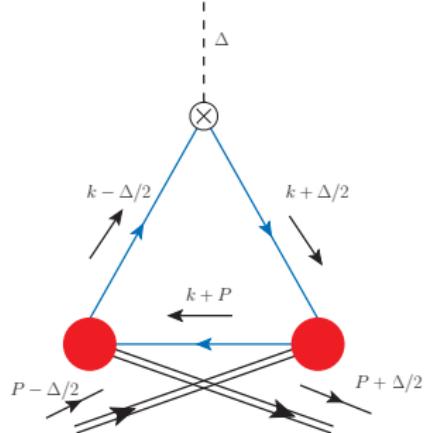
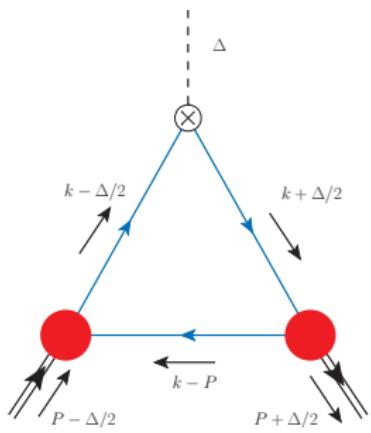
Those functions are building blocks of the realistic Bethe-Salpeter computations.

# Pion GPD model

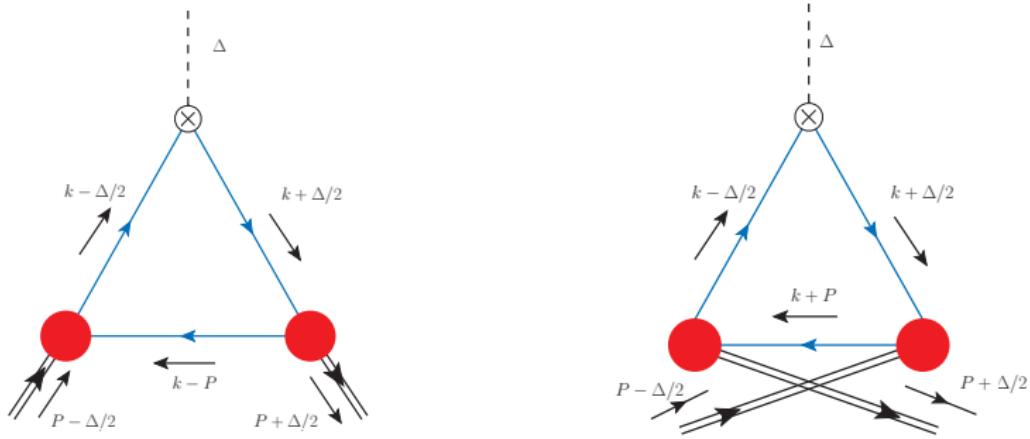
$$\begin{aligned}\mathcal{M}_m(\xi, t) &= \int_{-1}^1 dx \ x^m \ H(x, \xi, t) \\ &= \frac{1}{2(P \cdot n)^{m+1}} \left\langle \pi, P + \frac{\Delta}{2} \left| \bar{\psi}(0) \gamma \cdot n (i \not{D} \cdot n)^m \psi(0) \right| \pi, P - \frac{\Delta}{2} \right\rangle.\end{aligned}$$

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# Pion GPD model



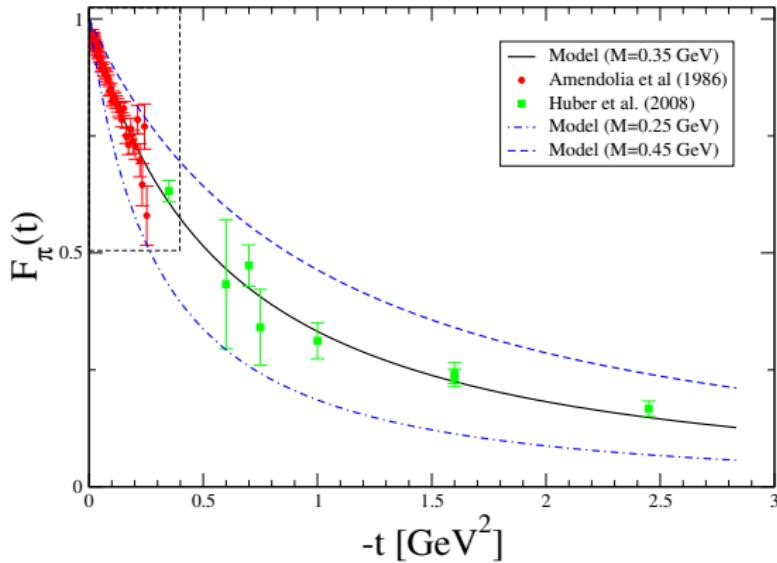
$$2(P \cdot n)^{m+1} \mathcal{M}_m(\xi, t) = \text{tr}_{CFD} \int \frac{d^4 k}{(2\pi)^4} (k \cdot n)^m i\Gamma_\pi(k - \frac{\Delta}{2}, P - \frac{\Delta}{2}) S(k - \frac{\Delta}{2}) \\ i\gamma \cdot n S(k + \frac{\Delta}{2}) i\bar{\Gamma}_\pi(k + \frac{\Delta}{2}, P + \frac{\Delta}{2}) S(k - P)$$

# Form factor

$$\mathcal{F}_\pi^q(t) = \mathcal{M}_0(t) = \int_{-1}^1 dx \ H^q(x, \xi, t)$$

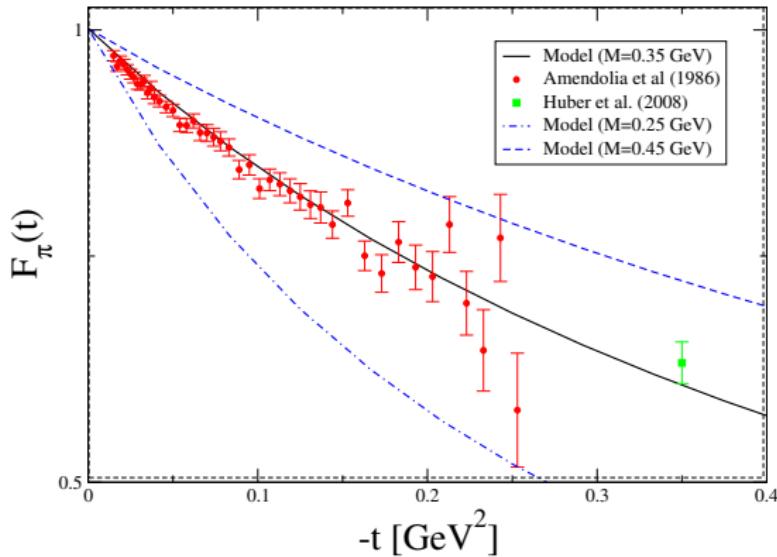
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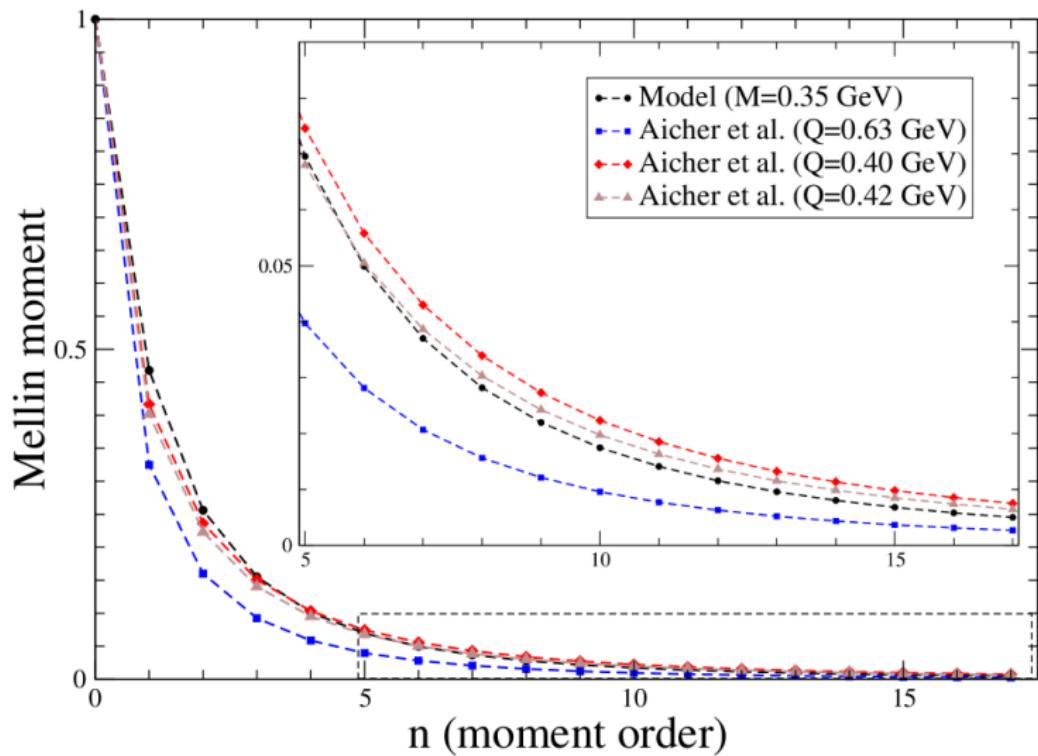
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C. Mezrag et al., arXiv 1406.7425

# PDF's Mellin moments



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# Double Distributions

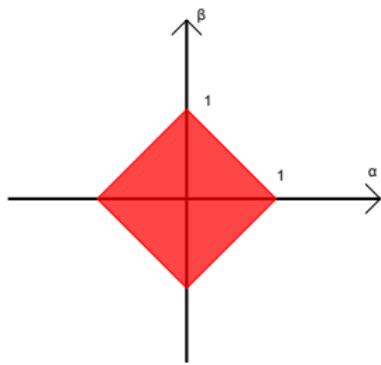
Double Distributions are formally the Radon transform of the GPDs.

$$H(x, \xi) = \int_{\Omega} d\alpha d\beta (F(\beta, \alpha) + \xi G(\beta, \alpha)) \delta(x - \beta - \xi\alpha)$$

$$\Omega = \{(\alpha, \beta) | |\alpha| + |\beta| \leq 1\}$$

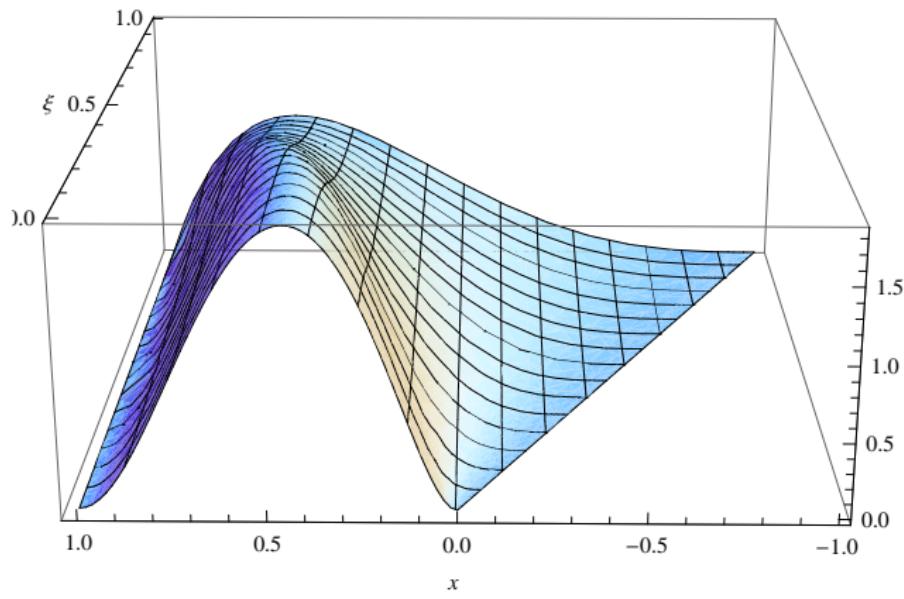
**Advantage:**

Easy way to respect the polynomiality in  $\xi$



$$\begin{aligned} & \int_{-1}^1 x^n H(x, \xi) dx \\ &= \int_{\Omega} (\beta + \xi\alpha)^n (F(\beta, \alpha) + \xi G(\beta, \alpha)) d\Omega \end{aligned}$$

## Reconstruction ( $t = 0$ )



We get back the support properties!

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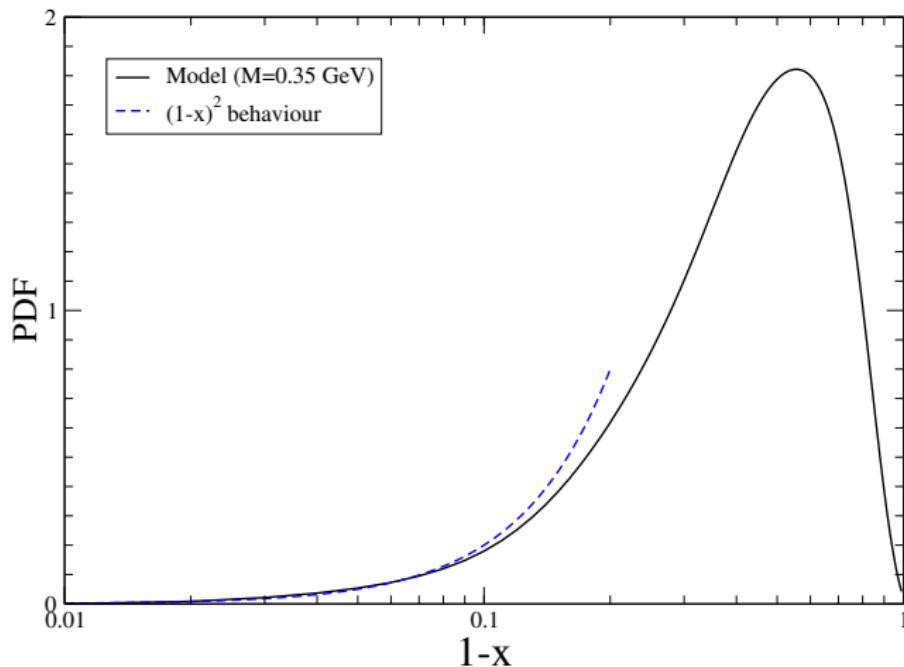
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- Prove the continuity at  $x = \xi$ .
- Double Distributions ensure polynomiality and parity in  $\xi$
- We can get analytic expressions. For the PDF( $\nu = 1$ ):

$$\begin{aligned} q(x) = & \frac{72}{25} (x^3(x(-2(x - 4)x - 15) + 30) \log(x) \\ & + (2x^2 + 3)(x - 1)^4 \log(1 - x) \\ & + x(x(x(2x - 5) - 15) - 3)(x - 1)) \end{aligned}$$

## Large $x$ behavior



At large  $x$ :

$$q(x) \approx (1 - x)^2$$

# Soft Pion Theorem

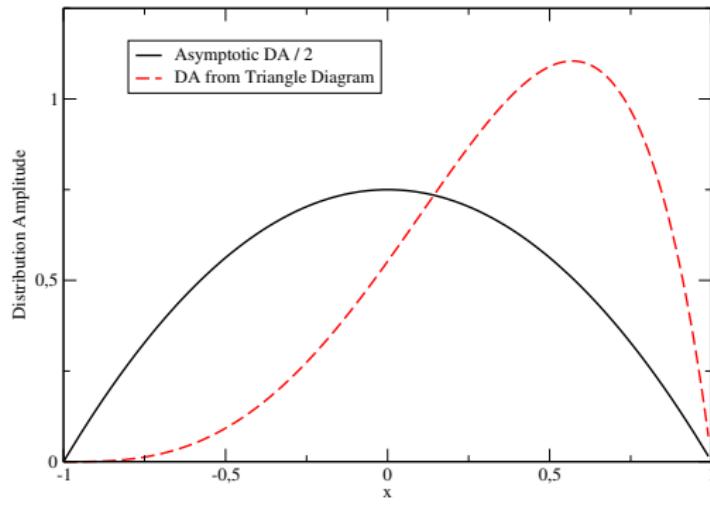
Polyakov soft pion theorem: if  $\xi = 1$  and  $t = 0$  then  $H \propto$  Pion DA.

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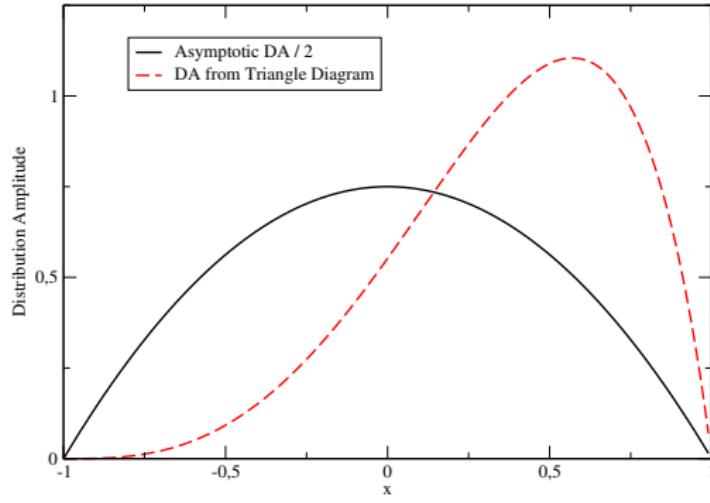
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Here the soft pion theorem is violated. Why?

# Definitions

- Axial-Vector vertex:

$$S(\ell)\Gamma_5^\mu(\ell, P)S(P - \ell) = \text{FT} [\langle 0 | T \{ j_5^\mu(x) \Psi(y) \bar{\Psi}(z) \} | 0 \rangle]$$

where  $j_5^\mu(x)$  is the axial vector current.

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- Pseudo-scalar vertex:

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# Axial-Vector Ward-Takahashi Identities

- Propagator:

$$S^{-1}(k) = -i\gamma \cdot k A(k^2) + B(k^2)$$

- Vertex:

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- “Pole part”:

$$\begin{aligned} 2f_\pi \Gamma_\pi(k; p) &\stackrel{p \simeq 0}{\approx} p^\mu \Gamma_{5\mu}(k, p), \\ 2r_\pi \Gamma_\pi(k; p) &\stackrel{p^2 \simeq 0}{\approx} p^2 \Gamma_5(k, p), \end{aligned}$$

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Freezing the mass leads to a violation of AWWI!

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Question?

Is it sufficient to respect the AVWTI to get back the soft pion theorem?

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$$M \frac{M^2}{k^2 + M^2} = M$$

Problem!

Freezing the mass leads to a violation of AVWTI!

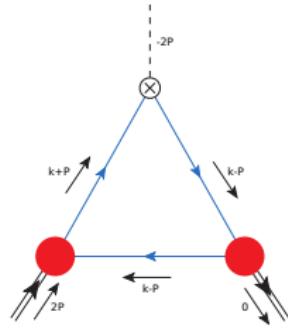
Question?

Is it sufficient to respect the AVWTI to get back the soft pion theorem?

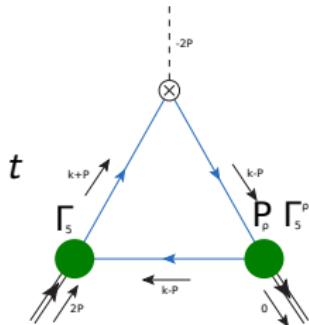
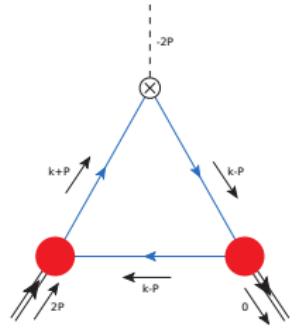
Answer

Yes, providing that the truncation scheme is consistent enough.

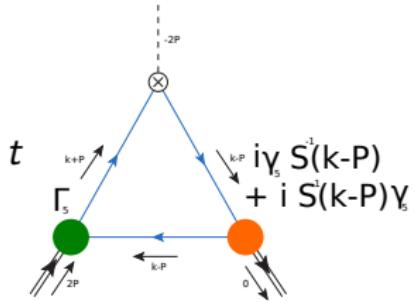
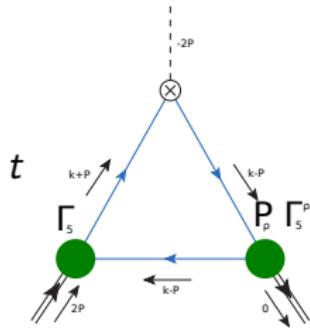
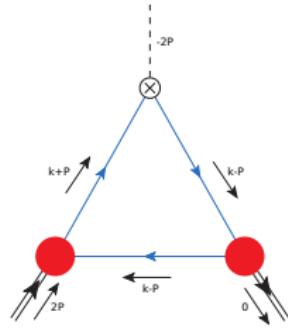
# Soft Pion Theorem: guidelines



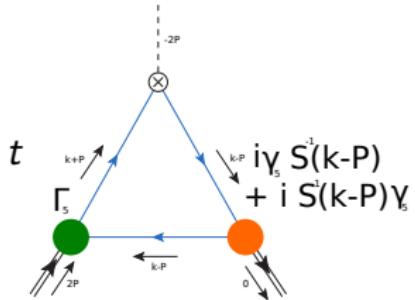
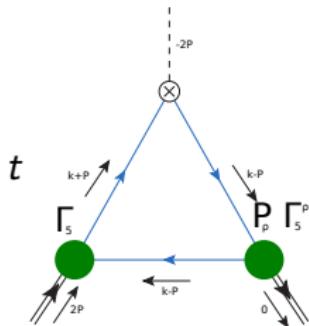
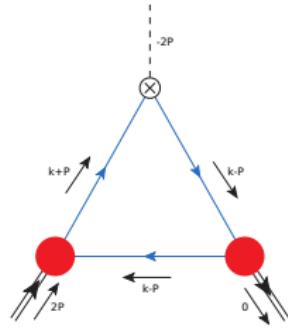
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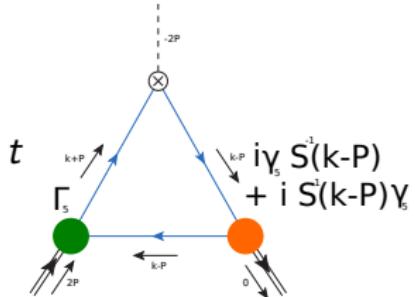
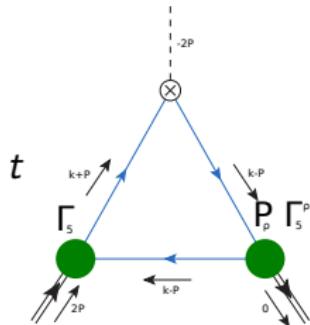
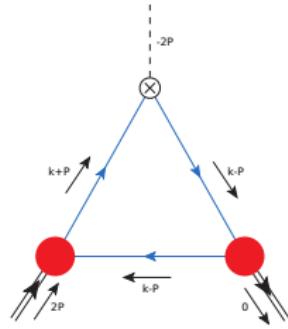
# Soft Pion Theorem: guidelines



$$t \sum_{n=0}^{\infty} \sum_{j=0}^n$$

The equation is represented by two Feynman diagrams. The first diagram shows a vertical stack of  $n$  horizontal lines. A shaded circle is at position  $j+1$  from the top. The second diagram shows a similar stack of  $n$  horizontal lines, but the shaded circle is at position  $j$  from the top. A plus sign is between the two diagrams.

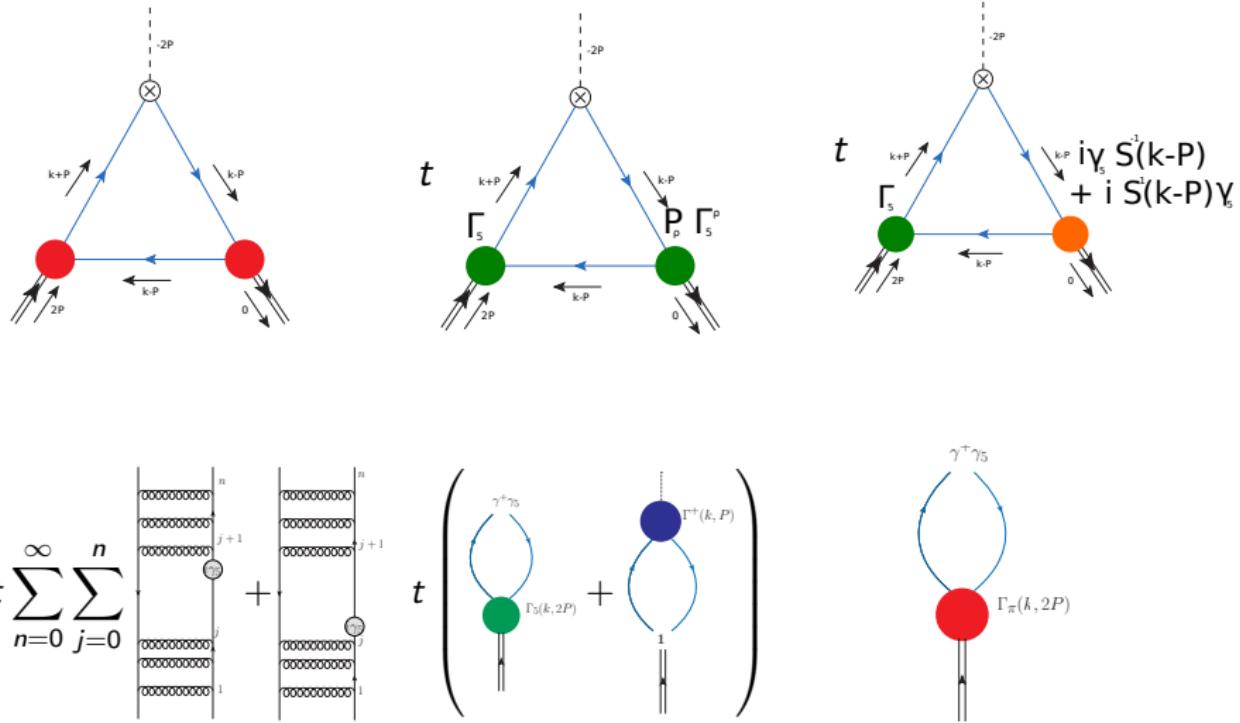
# Soft Pion Theorem: guidelines



$$t \sum_{n=0}^{\infty} \sum_{j=0}^n$$

$t \left( \Gamma_5(k, 2P) + \Gamma^+(k, P) \right)$

# Soft Pion Theorem: guidelines



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The soft pion theorem will be automatically implemented when modeling the pion GPD from the full solutions of the BSE-DSE.

(C. Mezrag et al., arXiv 1411.6634 to appear in PLB).

## Summary and conclusions

- We presented a new model for pion GPD which fulfills most of the required symmetry properties.
- Double Distributions make the full problem analytic.
- Our comparisons with available experimental data are very encouraging.
- Limitations highlight physics key points (gluons, AVWTI...).

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- Limitations highlight physics key points (gluons, AVWTI...).

If the GPDs remain the good objects to understand the physics, DDs are the good objects to deal with support properties and full reconstruction.

## Outlooks

- Going beyond the triangle diagram approximation in the non-forward case.
- We want to reconstruct the GPD thanks to DD in the realistic case, *i.e.* with vertices and propagators coming from numerical solutions of the Dyson-Schwinger equations.
- Compare our model with the existing phenomenological DD models, *i.e.* Radyushkin Ansatz.
- The proton case remains the Holy Grail...

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- Compare our model with the existing phenomenological DD models, *i.e.* Radyushkin Ansatz.
- The proton case remains the Holy Grail... which may be reached in the valence region using a quark-diquark model.

# Thank You!

# Back up

## More realistic model

- Change the propagator (fitted on DSE numerical solutions in *L. Chang et al.*, 2013):

$$S(k) = \sum_{j=1}^m \left( \frac{z_j}{i\cancel{k} + m_j} + \frac{z_j^*}{i\cancel{k} + m_j^*} \right)$$

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- Change the Pion amplitude (also fitted on DSE numerical solutions):

$$\Gamma_\pi(k, P) = c_j \int_{-1}^1 dz \rho_\nu(z) \Lambda^2 \Delta^2(k, P, \Lambda) + \dots$$

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- New Ansatz for the inserted operator.

## Kroll - Goloskokov model.

- Factorized Ansatz. For  $i = g$ , sea or val :

$$\begin{aligned} H_i(x, \xi, t) &= \int_{|\alpha|+|\beta| \leq 1} d\beta d\alpha \delta(\beta + \xi\alpha - x) f_i(\beta, \alpha, t) \\ f_i(\beta, \alpha, t) &= e^{b_i t} \frac{1}{|\beta|^{\alpha' t}} h_i(\beta) \pi_{n_i}(\beta, \alpha) \\ \pi_{n_i}(\beta, \alpha) &= \frac{\Gamma(2n_i + 2)}{2^{2n_i+1}\Gamma^2(n_i + 1)} \frac{(1 - |\beta|)^2 - \alpha^2]{n_i}}{(1 - |\beta|)^{2n_i+1}} \end{aligned}$$

- Expressions for  $h_i$  and  $n_i$  :

$$\begin{array}{llll} h_g(\beta) &= |\beta|g(|\beta|) & n_g &= 2 \\ h_{\text{sea}}^q(\beta) &= q_{\text{sea}}(|\beta|)\text{sign}(\beta) & n_{\text{sea}} &= 2 \\ h_{\text{val}}^q(\beta) &= q_{\text{val}}(\beta)\Theta(\beta) & n_{\text{val}} &= 1 \end{array}$$

Goloskokov and Kroll, Eur. Phys. J. C42, 281 (2005)

- Comparison to existing DVCS measurements at LO.

Kroll et al., Eur. Phys. J. C73, 2278 (2013)

# Double Distribution Ambiguity

Teryaev Phys. Lett. B **510** (2001) 125

Tiburzi Phys. Rev. D **70** (2004) 057504

Rewrite the non forward matrix element in terms of DD :

$$\begin{aligned} & \langle P - \frac{r}{2} | \bar{\psi}(-\frac{z}{2}) \gamma_2 \psi(\frac{z}{2}) | P + \frac{r}{2} \rangle \\ &= \int_{\Omega} e^{-i\beta(Pz) - i\alpha(\frac{rz}{2})} (2(Pz)F(\beta, \alpha) + (rz)G(\beta, \alpha)) d\alpha d\beta \end{aligned}$$

Matrix element **invariant** under the following transformation :

$$F(\beta, \alpha) \rightarrow F(\beta, \alpha) + \frac{\partial \sigma}{\partial \alpha}$$

$$G(\beta, \alpha) \rightarrow G(\beta, \alpha) - \frac{\partial \sigma}{\partial \beta}$$

$$\sigma(\beta, \alpha) = -\sigma(\beta, -\alpha)$$

This invariance allows for **different** methods to parametrise GPDs.

# Positivity

- Positivity condition in the DGLAP region:

$$|H(x, \xi, t)| \leq \sqrt{q\left(\frac{x-\xi}{1-\xi}\right)q\left(\frac{x+\xi}{1+\xi}\right)}$$

Pire, Soffer, Teryaev, 1999

- In our two-body problem,  $q(x) \propto x^2$  at small  $x$ .
- Consequently  $H(x, \xi, t)$  should vanish on the line  $x = \xi$ .
- We'll see how the more realistic model behaves.

# Double Distributions

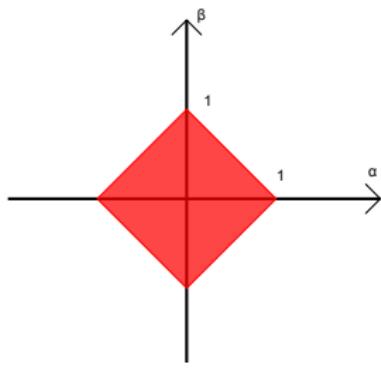
Double Distributions are formally the Radon transform of the GPDs.

$$H(x, \xi) = \int_{\Omega} d\alpha d\beta (F(\beta, \alpha) + \xi G(\beta, \alpha)) \delta(x - \beta - \xi\alpha)$$

$$\Omega = \{(\alpha, \beta) | |\alpha| + |\beta| \leq 1\}$$

**Advantage:**

Easy way to respect the polynomiality in  $\xi$



$$\begin{aligned} & \int_{-1}^1 x^n H(x, \xi) dx \\ &= \int_{\Omega} (\beta + \xi\alpha)^n (F(\beta, \alpha) + \xi G(\beta, \alpha)) d\Omega \end{aligned}$$

# From Mellin moments to Double Distributions (DD)

$$H(x, \xi, t) = \int_{-1}^1 d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha (F(\beta, \alpha, t) + \xi G(\beta, \alpha, t)) \delta(x - \beta - \alpha \xi)$$

- Time reversal invariance is encoded in the parity in  $\alpha$ :
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- PDF case:

$$q(x) = H(x, 0, 0) = \int_{-1}^1 d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha F(\beta, \alpha, t) \delta(x - \beta)$$

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- Form Factor case:

$$\mathcal{F}(t) = \int_{-1}^1 dx H(x, \xi, t) = \int_{-1}^1 d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha F(\beta, \alpha, t)$$

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$G(\beta, \alpha)$  does not play any role in those cases.

# Properties of Mellin moments

Polynomiality:

$$\begin{aligned} & \mathcal{M}_m(\xi, t) \\ = & \frac{1}{2(P \cdot n)^{m+1}} \left\langle \pi, P + \frac{\Delta}{2} \left| \bar{\psi}(0) \gamma \cdot n (i \not{D} \cdot n)^m \psi(0) \right| \pi, P - \frac{\Delta}{2} \right\rangle \\ = & \frac{n_\mu n_{\mu_1} \dots n_{\mu_m}}{(P \cdot n)^{m+1}} P^{\{\mu} \sum_{j=0}^m \binom{m}{j} F_{m,j}(t) P^{\mu_1} \dots P^{\mu_j} \left(-\frac{\Delta}{2}\right)^{\mu_{j+1}} \dots \left(-\frac{\Delta}{2}\right)^{\mu_m}\} \\ & - n_\mu n_{\mu_1} \dots n_{\mu_m} \frac{\Delta}{2} \sum_{j=0}^m \binom{m}{j} G_{m,j}(t) P^{\mu_1} \dots P^{\mu_j} \left(-\frac{\Delta}{2}\right)^{\mu_{j+1}} \dots \left(-\frac{\Delta}{2}\right)^{\mu_m}\} \end{aligned}$$

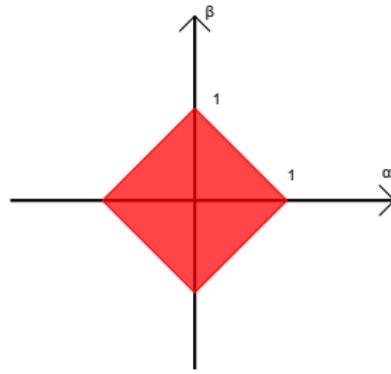
$\xi = -\frac{\Delta \cdot n}{2P \cdot n} \Rightarrow \mathcal{M}_m(\xi, t)$  is a polynomial in  $\xi$  of order  $m+1$ .

# Properties of Mellin moments

Double distributions:

$$F_{m,j}(t) = \int_{-1}^1 d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha \beta^{m-j} \alpha^j F(\beta, \alpha, t)$$

$$G_{m,j}(t) = \int_{-1}^1 d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha \beta^{m-j} \alpha^j G(\beta, \alpha, t)$$



# Properties of Mellin moments

$$\begin{aligned}\mathcal{M}_m(\xi, t) &= n_\mu n_{\mu_1} \dots n_{\mu_m} \sum_{j=0}^m \binom{m}{j} \int_{-1}^1 d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha \beta^{m-j} \alpha^j \\ &\quad F(\beta, \alpha, t) P^{\{\mu} P^{\mu_1} \dots P^{\mu_j} \left(-\frac{\Delta}{2}\right)^{\mu_{j+1}} \dots \left(-\frac{\Delta}{2}\right)^{\mu_m}\}} \\ &\quad - G(\beta, \alpha, t) \frac{\Delta}{2}^{\{\mu} P^{\mu_1} \dots P^{\mu_j} \left(-\frac{\Delta}{2}\right)^{\mu_{j+1}} \dots \left(-\frac{\Delta}{2}\right)^{\mu_m}\}}\end{aligned}$$

# Analytic Results

$$F^u(\beta, \alpha, t) = \frac{48}{5} \left\{ -\frac{18M^4 t(\beta-1)(\alpha-\beta+1)(\alpha+\beta-1) \left( (\alpha^2 - (\beta-1)^2) \tanh^{-1} \left( \frac{2\beta}{-\alpha^2 + \beta^2 + 1} \right) + 2\beta \right)}{(4M^2 + t((\beta-1)^2 - \alpha^2))^3} \right. \\ + \frac{9M^4(\alpha-\beta+1) \left( -4\beta(-\alpha^2 + \beta^2 + 1) + 2 \tanh^{-1} \left( \frac{2\beta}{-\alpha^2 + \beta^2 + 1} \right) \right)}{4(\alpha-\beta-1)(4M^2 + t((\beta-1)^2 - \alpha^2))^2} \\ + \frac{9M^4(\alpha-\beta+1) \left( (\alpha^4 - 2\alpha^2(\beta^2 + 1) + \beta^2(\beta^2 - 2)) \log \left( \frac{(\alpha-\beta-1)(\alpha+\beta+1)}{\alpha^2 - (\beta-1)^2} \right) \right)}{4(\alpha-\beta-1)(4M^2 + t((\beta-1)^2 - \alpha^2))^2} \\ + \frac{9M^4(\alpha+\beta-1) \left( -4\beta(-\alpha^2 + \beta^2 + 1) + 2 \tanh^{-1} \left( \frac{2\beta}{-\alpha^2 + \beta^2 + 1} \right) \right)}{4(\alpha+\beta+1)(4M^2 + t((\beta-1)^2 - \alpha^2))^2} \\ + \frac{9M^4(\alpha+\beta-1) \left( (\alpha^4 - 2\alpha^2(\beta^2 + 1) + \beta^4 - 2\beta^2) \log \left( \frac{(\alpha-\beta-1)(\alpha+\beta+1)}{\alpha^2 - (\beta-1)^2} \right) \right)}{4(\alpha+\beta+1)(4M^2 + t((\beta-1)^2 - \alpha^2))^2} \\ + \frac{9M^4 \beta(\alpha-\beta+1)^2(\alpha+\beta-1)^2 \left( \frac{2(\alpha^2\beta - \beta^3 + \beta)}{\alpha^4 - 2\alpha^2(\beta^2 + 1) + (\beta^2 - 1)^2} \right)}{(4M^2 + t((\beta-1)^2 - \alpha^2))^2} \\ \left. + \frac{9M^4 \beta(\alpha-\beta+1)^2(\alpha+\beta-1)^2 \left( -\tanh^{-1}(\alpha-\beta) + \tanh^{-1}(\alpha+\beta) \right)}{(4M^2 + t((\beta-1)^2 - \alpha^2))^2} \right\},$$

# Analytic Results

$$\begin{aligned} H_{x \geq \xi}^u(x, \xi, 0) = & \frac{48}{5} \left\{ \frac{3 \left( -2(x-1)^4 (2x^2 - 5\xi^2 + 3) \log(1-x) \right)}{20(\xi^2 - 1)^3} \right. \\ & \frac{3 \left( +4\xi \left( 15x^2(x+3) + (19x+29)\xi^4 + 5(x(x(x+11)+21)+3)\xi^2 \right) \tanh^{-1} \left( \frac{(x-1)\xi}{x-\xi^2} \right) \right)}{20(\xi^2 - 1)^3} \\ & + \frac{3 \left( x^3(x(2(x-4)x+15)-30) - 15(2x(x+5)+5)\xi^4 \right) \log(x^2 - \xi^2)}{20(\xi^2 - 1)^3} \\ & + \frac{3 \left( -5x(x(x(x+2)+36)+18)\xi^2 - 15\xi^6 \right) \log(x^2 - \xi^2)}{20(\xi^2 - 1)^3} \\ & + \frac{3 \left( 2(x-1) \left( (23x+58)\xi^4 + (x(x(x+67)+112)+6)\xi^2 + x(x((5-2x)x+15)+3) \right) \right)}{20(\xi^2 - 1)^3} \\ & + \frac{3 \left( (15(2x(x+5)+5)\xi^4 + 10x(3x(x+5)+11)\xi^2) \log(1-\xi^2) \right)}{20(\xi^2 - 1)^3} \\ & \left. + \frac{3 \left( 2x(5x(x+2)-6) + 15\xi^6 - 5\xi^2 + 3 \right) \log(1-\xi^2) \right\}, \end{aligned}$$

# Analytic Results

$$H_{|x| \leq \xi}^u(x, \xi, 0) = \frac{48}{5} \left\{ \frac{6\xi(x-1)^4 \left( - (2x^2 - 5\xi^2 + 3) \right) \log(1-x)}{40\xi(\xi^2 - 1)^3} \right.$$
$$+ \frac{6\xi \left( -4\xi \left( 15x^2(x+3) + (19x+29)\xi^4 + 5(x(x(x+11)+21)+3)\xi^2 \right) \log(2\xi) \right)}{40\xi(\xi^2 - 1)^3}$$
$$+ \frac{6\xi(\xi+1)^3 \left( (38x+13)\xi^2 + 6x(5x+6)\xi + 2x(5x(x+2)-6) + 15\xi^3 - 9\xi + 3 \right) \log(\xi+1)}{40\xi(\xi^2 - 1)^3}$$
$$+ \frac{6\xi(x-\xi)^3 \left( (7x-58)\xi^2 + 6(x-4)x\xi + x(2(x-4)x+15) + 15\xi^3 + 75\xi - 30 \right) \log(\xi-x)}{40\xi(\xi^2 - 1)^3}$$
$$+ \frac{3(\xi-1)(x+\xi) \left( 4x^4\xi - 2x^3\xi(\xi+7) + x^2(\xi((119-25\xi)\xi-5)+15) \right)}{40\xi(\xi^2 - 1)^3}$$
$$\left. + \frac{3(\xi-1)(x+\xi) (x\xi(\xi(\xi(71\xi+5)+219)+9) + 2\xi(\xi(2\xi(34\xi+5)+9)+3))}{40\xi(\xi^2 - 1)^3} \right\}.$$