Gluon saturation in Heavy Ion Collisions

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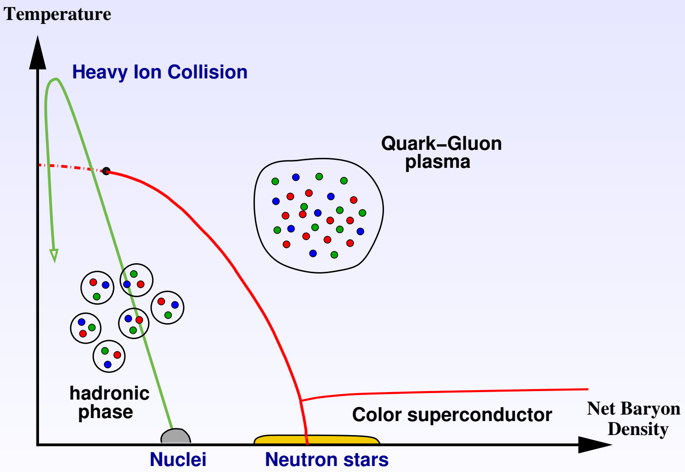
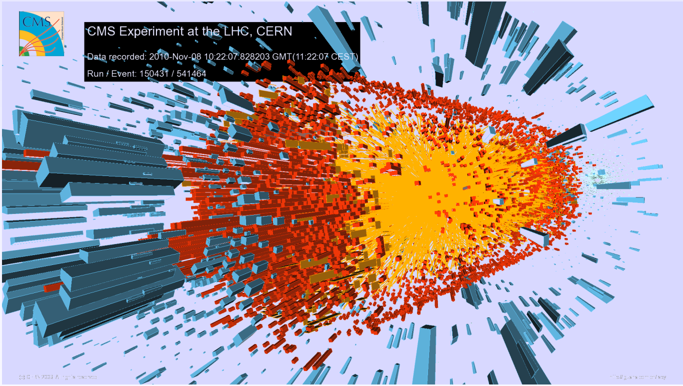
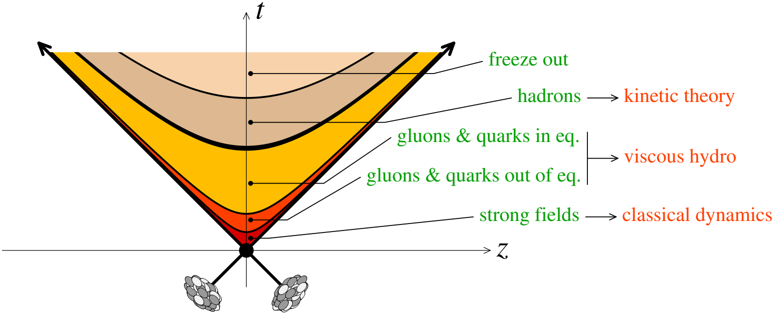
Heavy ion collisions



Heavy ion collision at the LHC



Stages of a nucleus-nucleus collision



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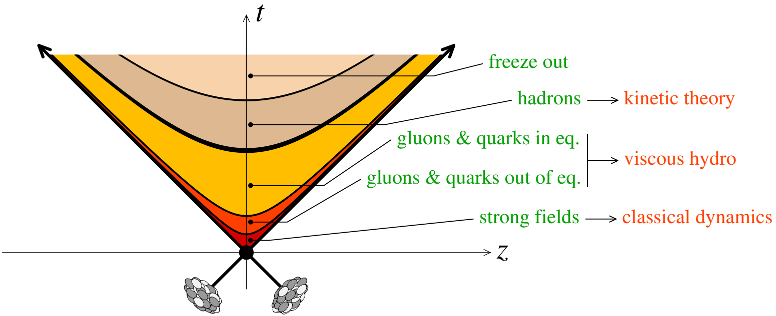

- Well described as a fluid expanding into vacuum according to relativistic hydrodynamics

What is hydrodynamics?

- Hydrodynamics is a macroscopic description based on energy-momentum conservation :

$$\partial_\mu T^{\mu\nu} = 0$$

True in any quantum field theory

Not closed : 4 equations, 10 independent components in $T^{\mu\nu}$

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- For a frictionless fluid : $T^{\mu\nu}_{\text{ideal}} = (\epsilon + P)\, u^\mu u^\nu - P\, g^{\mu\nu}$

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- In general : $T^{\mu\nu} = T^{\mu\nu}_{\rm ideal} \oplus \underbrace{\eta\, \nabla^\mu u^\nu \oplus \zeta\, (\nabla_\rho u^\rho) \oplus \cdots}_{\Pi^{\mu\nu} \equiv \text{ deviation from ideal fluid}}$

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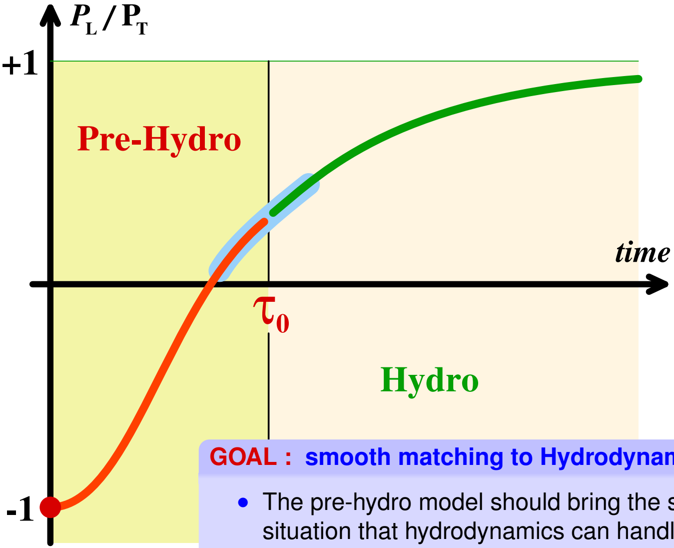
- Microscopic inputs : $\epsilon = f(P)$ (EoS), $\eta, \zeta, \cdots$ (transport coeff.)

Conditions for hydrodynamics

- The difference between P_L and P_T should not be too large (for the stability of hydro codes)

- The ratio η/s should be small enough (for an efficient transfer from spatial to momentum anisotropy)

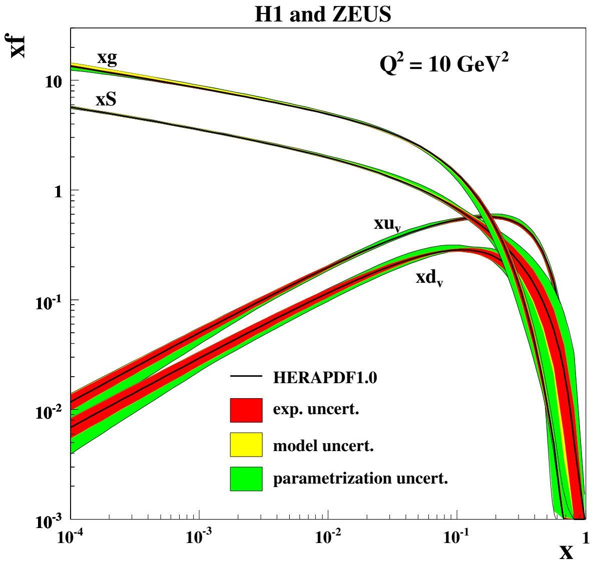


GOAL : smooth matching to Hydrodynamics

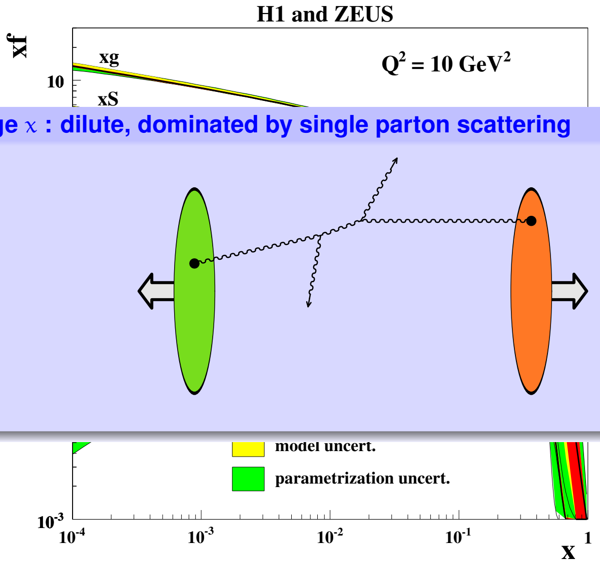
- The pre-hydro model should bring the system to a situation that hydrodynamics can handle
- Pre-hydro and hydro should agree over some range of time $\Rightarrow$ no τ_0 dependence

Color Glass Condensate in Heavy Ion Collisions

Parton distributions in a nucleon

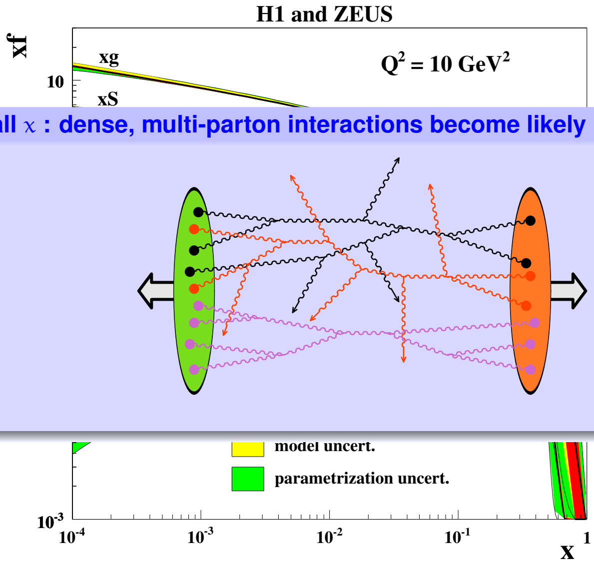


Parton distributions in a nucleon



Large x : dilute, dominated by single parton scattering

Parton distributions in a nucleon



Small x : dense, multi-parton interactions become likely

- When their occupation number becomes large, gluons can recombine :

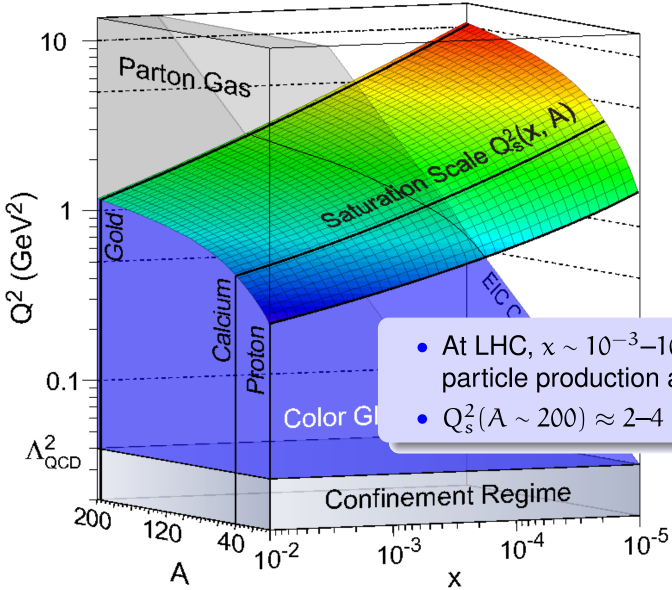
Gluon Saturation

Saturation criterion **[Gribov, Levin, Ryskin (1983)]**

$$\underbrace{\alpha_s Q^{-2}}_{\sigma_{gg\to g}} \times \underbrace{A^{-2/3} xG(x,Q^2)}_{\text{surface density}} \geq 1$$

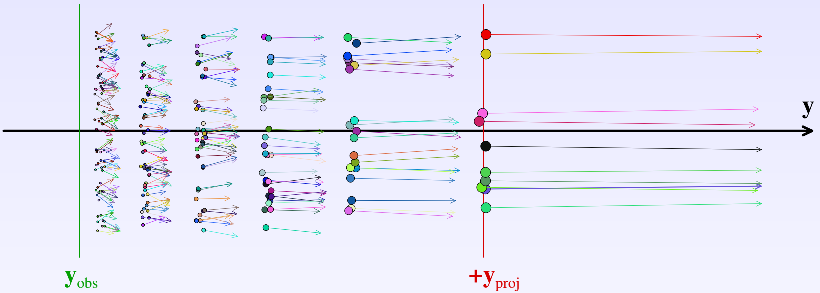
$$\underbrace{Q^2 \leq Q_s^2 \equiv \frac{\alpha_s xG(x,Q_s^2)}{A^{2/3}}}_{\text{(saturation momentum)}^2} \sim A^{1/3} x^{-0.3}$$

Saturation domain



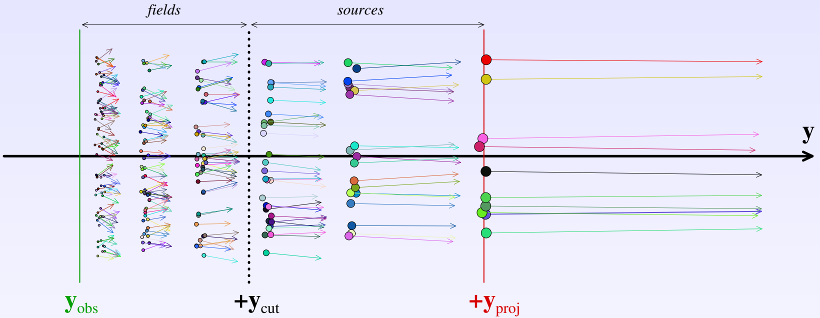
- At LHC, $x \sim 10^{-3}$–10^{-4} (for bulk particle production at mid-rapidity)
- $Q_s^2(A \sim 200) \approx 2$–$4$ GeV^2

Degrees of freedom [McLerran, Venugopalan (1994)]



- $p_\perp^2 \sim Q_s^2 \sim \Lambda_{QCD}\, e^{\lambda(y_{proj}-y)}$, $p_z \sim Q_s\, e^{y-y_{obs}}$
- Fast partons : frozen dynamics, negligible $p_\perp$ $\Rightarrow$ classical sources
- Slow partons : evolve with time $\Rightarrow$ gauge fields

Degrees of freedom [McLerran, Venugopalan (1994)]



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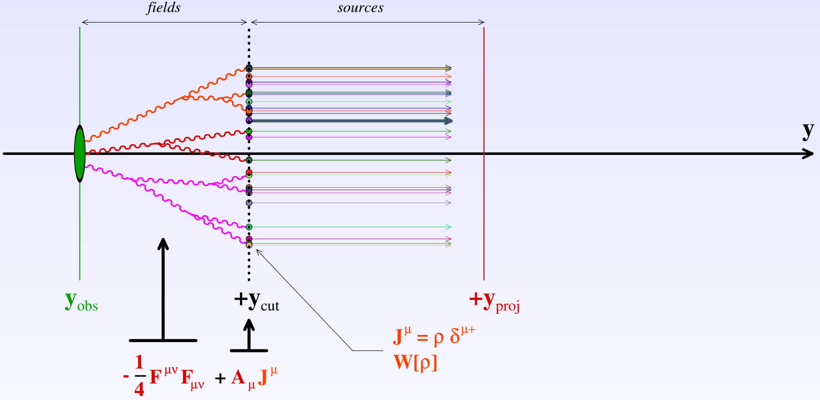

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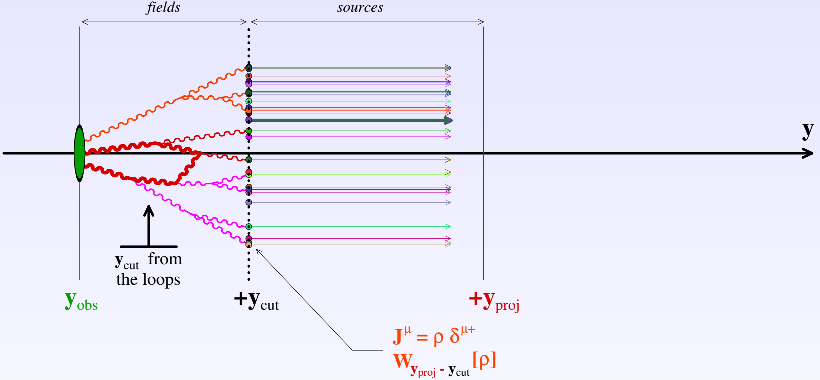
Cancellation of the cutoff dependence



- The cutoff y_{cut} is arbitrary and should not affect the result
- The probability density $W[\rho]$ changes with the cutoff
- Loop corrections cancel the cutoff dependence from $W[\rho]$

B-JIMWLK evolution equation

[Jalilian-Marian, Kovner, Leonidov, Weigert (1998)]
[Balitsky (1996)] [Iancu, Leonidov, McLerran (2001)]

B-JIMWLK equation at Leading Log

$$\frac{\partial W_Y[\rho]}{\partial Y} = \underbrace{\frac{1}{2}\int_{\vec{x}_\perp,\vec{y}_\perp} \frac{\delta}{\delta\rho_a(\vec{x}_\perp)} \chi_{ab}(\vec{x}_\perp,\vec{y}_\perp) \frac{\delta}{\delta\rho_b(\vec{y}_\perp)}}_{\mathcal{H}\quad\text{(JIMWLK Hamiltonian)}} W_Y[\rho]$$

- Mean field approx. (BK equation) : **[Kovchegov (1999)]**
- Langevin form of B-JIMWLK : **[Blaizot, Iancu, Weigert (2003)]**
- First numerical solution : **[Rummukainen, Weigert (2004)]**

B-JIMWLK evolution equation

[Jalilian-Marian, Kovner, Leonidov, Weigert (1998)]
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B-JIMWL

Recent developments :

Running coupling correction

[Lappi, Mäntysaari (2012)]

B-JIMWLK equation at Next to Leading Log

[Kovner, Lublinsky, Mulian (2013)]

[Caron-Huot (2013)][Balitsky, Chirilli (2013)]

- Me
- Langevin form of B-JIMWLK : **[Blaizot, Iancu, Weigert (2003)]**
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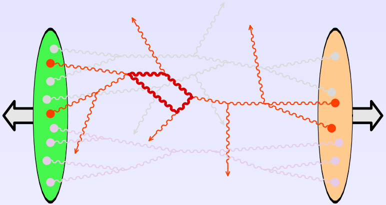
Power counting in the saturated regime

$$\mathcal{S} = \underbrace{-\frac{1}{4}\int F_{\mu\nu}F^{\mu\nu}}_{\text{slow gluons}} + \int \underbrace{(J_1^\mu + J_2^\mu)}_{\text{fast partons}} A_\mu$$



In the saturated regime: $J^\mu \sim g^{-1}$

$$g^{-2}\; g^{\#\text{ of external gluons}}\; g^{2\times(\#\text{ of loops})}$$

- No dependence on the number of sources J^μ
  - ▷ infinite number of graphs at each order in g^2

Example : expansion of $T^{\mu\nu}$ in powers of g^2

$$T^{\mu\nu} \sim \frac{1}{g^2}\left[c_0 + c_1\, g^2 + c_2\, g^4 + \cdots\right]$$

Inclusive observables at Leading Order

[FG, Venugopalan (2006)]

- The Leading Order is the sum of all the tree diagrams

  Expressible in terms of classical solutions of Yang-Mills equations :

$$\mathcal{D}_\mu \mathcal{F}^{\mu\nu} = J_1^\nu + J_2^\nu$$

- Boundary conditions : $\lim_{x^0 \to -\infty} \mathcal{A}^\mu(x) = 0$

  (WARNING : this is not true for exclusive observables!)

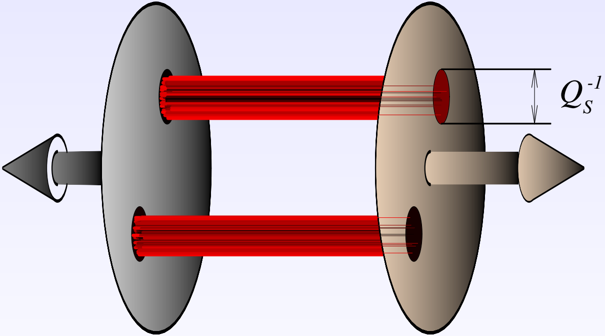
Components of the energy-momentum tensor at LO :

$$T^{00}_{\rm LO} = \frac{1}{2}\big[\underbrace{\mathbf{E}^2 + \mathbf{B}^2}_{\text{class. fields}}\big] \qquad T^{0i}_{\rm LO} = \big[\mathbf{E} \times \mathbf{B}\big]^i$$

$$T^{ij}_{\rm LO} = \frac{\delta^{ij}}{2}\big[\mathbf{E}^2 + \mathbf{B}^2\big] - \big[\mathbf{E}^i\mathbf{E}^j + \mathbf{B}^i\mathbf{B}^j\big]$$

Field lines at $\tau \ll Q_s^{-1}$: Glasma flux tubes

[McLerran, Lappi (2006)]



- E parallel to B : $P_T = -P_L = \epsilon$

Field lines at $\tau \ll Q_s^{-1}$: Glasma flux tubes

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[Dumitru, Nara, Petreska (2013)]
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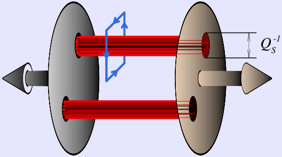
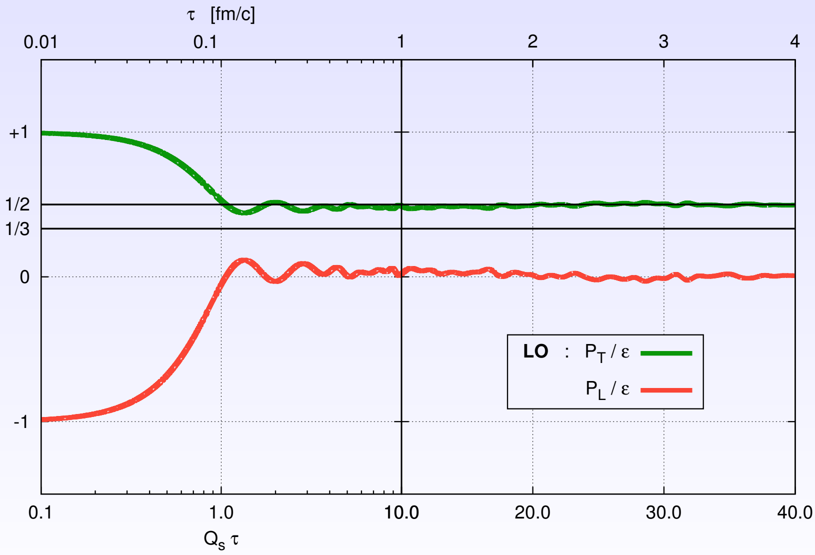


$$W \equiv \left\langle \mathrm{P} \exp ig \int_\gamma dx^i \mathcal{A}^i \right\rangle$$

$W \sim \exp(-\text{Area})$ for Area $\times Q_s^2 \gtrsim 1$

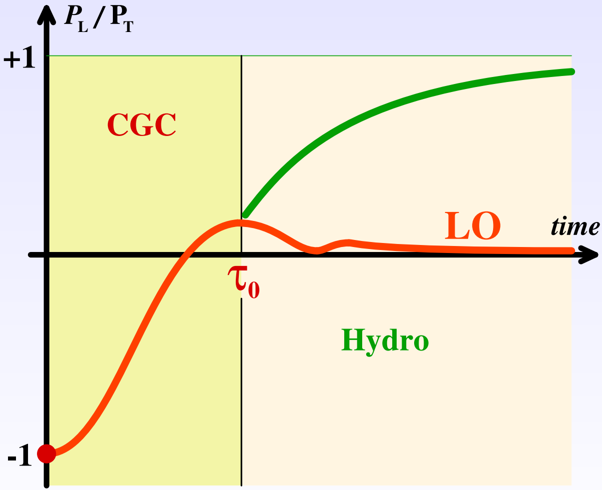
$\Rightarrow$ magnetic flux bundled in domains of area $\sim Q_s^{-2}$

- $\mathbf{E}$ parallel to $\mathbf{B}$: $\quad P_T = -P_L = \epsilon$

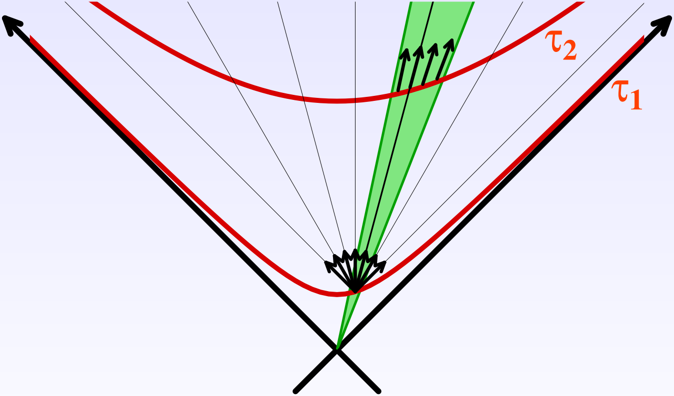


τ [fm/c]
0.01
0.1
1
2
3
4
+1
1/2
1/3
0
-1
LO : P_T / ε
P_L / ε
0.1
1.0
10.0
20.0
30.0
40.0
$Q_s \tau$

CGC at LO : unsatisfactory matching to hydrodynamics



Competition between Expansion and Isotropization



Does it get better at Next-to-Leading Order?

Next to Leading Order in g^2 : 1-loop diagrams

[FG, Lappi, Venugopalan (2008)]

Getting the NLO from tree graphs...

$$\mathcal{O}_{\text{NLO}} = \left[\frac{1}{2} \int_{\mathbf{u},\mathbf{v}} \mathbf{\Gamma_2(u,v)}\, \mathbb{T}_{\mathbf{u}} \mathbb{T}_{\mathbf{v}} + \int_{\mathbf{u}} \boldsymbol{\alpha}(\mathbf{u})\, \mathbb{T}_{\mathbf{u}} \right] \mathcal{O}_{\text{LO}}$$

- $\mathbb{T}$ is the generator of the shifts of the initial value of the field :

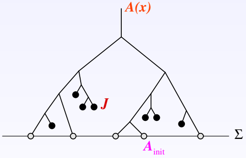
$$\mathbb{T}_{\mathbf{u}} \sim \frac{\partial}{\partial \mathcal{A}_{\text{init}}}$$

$$\exp\left[\int_{\mathbf{u}} \boldsymbol{\alpha}_{\mathbf{u}}\, \mathbb{T}_{\mathbf{u}}\right] \mathcal{O}\Big[\overbrace{\mathcal{A}_\tau(\underbrace{\mathcal{A}_{\text{init}}}_{\text{init. value}})}^{\text{class. field at }\tau}\Big] = \mathcal{O}\Big[\mathcal{A}_\tau(\underbrace{\mathcal{A}_{\text{init}} + \boldsymbol{\alpha}}_{\text{shifted init. value}})\Big]$$

Shift operator $\mathbb{T}$ – Definition

Equations of motion for a field $\mathcal{A}$ and a small perturbation α

$$\begin{aligned}\square\mathcal{A} + V'(\mathcal{A}) &= J \\ \left[\square + V''(\mathcal{A})\right]\ \alpha &= 0\end{aligned}$$



- Getting the perturbation by shifting the initial condition of $\mathcal{A}$ at one point :

$$\alpha(x) = \int_u \alpha_u\, \mathbb{T}_u\ \mathcal{A}(x)$$

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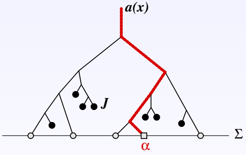

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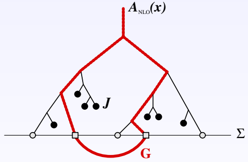


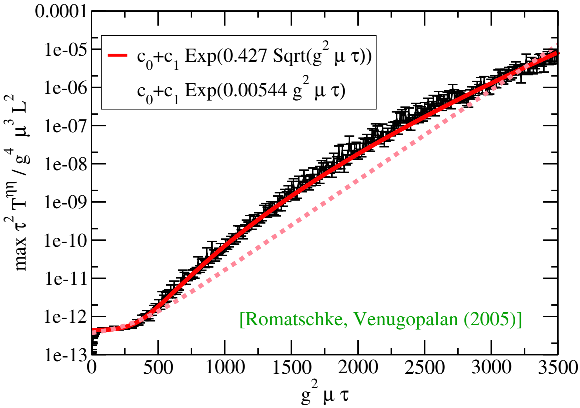
- Getting the perturbation by shifting the initial condition of $\mathcal{A}$ at one point :

$$\alpha(x) = \int_{\mathbf{u}} \alpha_{\mathbf{u}}\, \mathbb{T}_{\mathbf{u}}\ \mathcal{A}(x)$$

- A loop is obtained by shifting the initial condition of $\mathcal{A}$ at two points

CGC at NLO : instabilities

[Mrowczynski (1988), Romatschke, Strickland (2003), Arnold, Lenaghan, Moore (2003), Rebhan, Romatschke, Strickland (2005), Arnold, Lenaghan, Moore, Yaffe (2005), Romatschke, Rebhan (2006), Bodeker, Rummukainen (2007), Fujii, Itakura (2008),...,**Attems, Rebhan, Strickland (2012)**, **Fukushima (2013)**]



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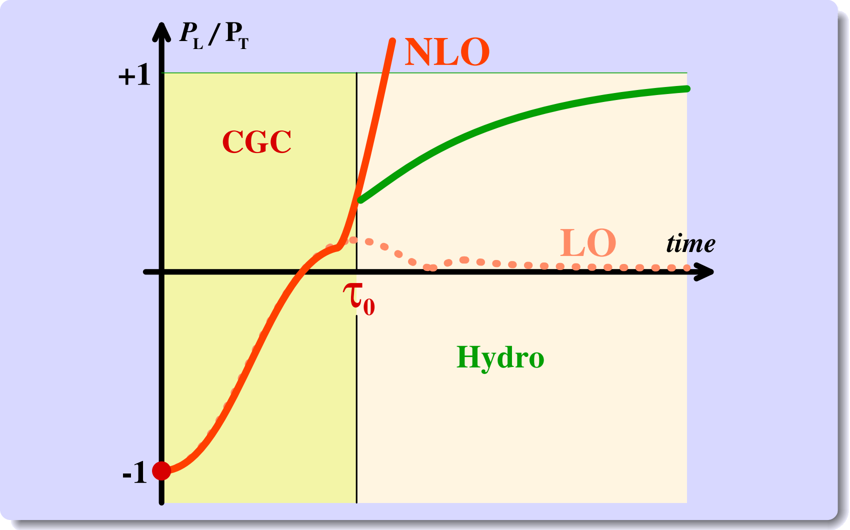


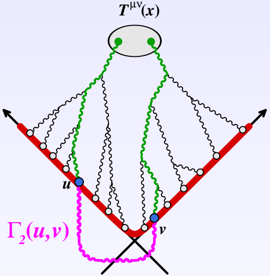
$g^2 \mu \tau$

Beyond NLO :

Classical Statistical Approximation

Improved power counting and resummation

$$\text{Loop} \sim g^2 \qquad , \qquad \mathbb{T} \sim e^{\sqrt{\mu\tau}}$$



- 1 loop :
  $(g e^{\sqrt{\mu\tau}})^2$

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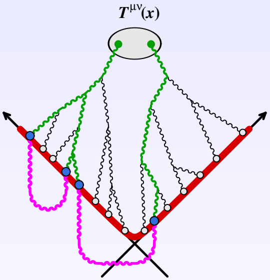

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- 2 disconnected loops :
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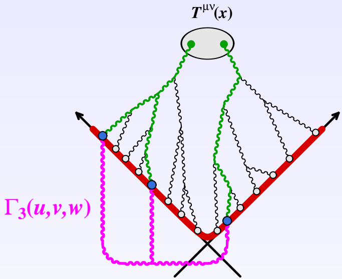


- 1 loop : $(ge^{\sqrt{\mu\tau}})^2$
- 2 disconnected loops : $(ge^{\sqrt{\mu\tau}})^4$
- 2 entangled loops : $g(ge^{\sqrt{\mu\tau}})^3$ $\triangleright$ subleading

Leading terms

- All disconnected loops to all orders
  - $\triangleright$ exponentiation of the 1-loop result

Resummation of the leading secular terms

$$
\begin{aligned}
T^{\mu\nu}_{\text{resummed}} &= \exp\left[\frac{1}{2}\int_{u,v} \Gamma_2(u,v)\,\mathbb{T}_u\mathbb{T}_v\right] T^{\mu\nu}_{\text{LO}}[\mathcal{A}_{\text{init}}] \\
&= \underbrace{T^{\mu\nu}_{\text{LO}} + T^{\mu\nu}_{\text{NLO}}}_{\text{in full}} + \underbrace{T^{\mu\nu}_{\text{NNLO}} + \cdots}_{\text{partially}}
\end{aligned}
$$

- The exponentiation of the 1-loop result collects all the terms with the worst time behavior

$$e^{\frac{\alpha}{2}\partial_x^2}\, f(x) = \int_{-\infty}^{+\infty} dz\, \frac{e^{-z^2/2\alpha}}{\sqrt{2\pi\alpha}}\, f(x+z)$$

Resummation of the leading secular terms

$$T^{\mu\nu}_{\text{resummed}} = \exp\left[\frac{1}{2}\int_{u,v} \Gamma_2(u,v)\mathbb{T}_u\mathbb{T}_v\right] T^{\mu\nu}_{\text{LO}}[\mathcal{A}_{\text{init}}]$$

$$= \int[Da]\,\exp\left[-\frac{1}{2}\int_{u,v} a(u)\Gamma_2^{-1}(u,v)a(v)\right] T^{\mu\nu}_{\text{LO}}[\mathcal{A}_{\text{init}}+a]$$

- The exponentiation of the 1-loop result collects all the terms with the worst time behavior
- Equivalent to Gaussian fluctuations of the initial field
  + classical time evolution
- Wigner distribution of a coherent state

Classical Statistical Approximation (CSA)

- Classical time evolution
- Quantum initial condition

- Dynamics fully non-linear $\Rightarrow$ no unbounded growth
- Individual classical trajectories are chaotic $\Rightarrow$ a small initial ensemble can span a large phase space volume

CSA in Quantum Mechanics

- Consider the von Neumann equation for the density operator :

$$\frac{\partial \widehat{\rho}_\tau}{\partial \tau} = i\hbar \left[\widehat{H}, \widehat{\rho}_\tau\right]$$

- Introduce the Wigner transforms :

$$\begin{aligned} W_\tau(\mathbf{x},\mathbf{p}) &\equiv \int d\mathbf{s}\; e^{i\mathbf{p}\cdot\mathbf{s}} \left\langle \mathbf{x}+\frac{\mathbf{s}}{2}\middle|\widehat{\rho}_\tau\middle|\mathbf{x}-\frac{\mathbf{s}}{2}\right\rangle \\ \mathcal{H}(\mathbf{x},\mathbf{p}) &\equiv \int d\mathbf{s}\; e^{i\mathbf{p}\cdot\mathbf{s}} \left\langle \mathbf{x}+\frac{\mathbf{s}}{2}\middle|\widehat{H}\middle|\mathbf{x}-\frac{\mathbf{s}}{2}\right\rangle \quad \text{(classical Hamiltonian)} \end{aligned}$$

- The von Neumann eq. is equivalent to

$$\begin{aligned} \frac{\partial W_\tau}{\partial \tau} &= \mathcal{H}(\mathbf{x},\mathbf{p})\,\frac{2}{i\hbar}\,\sin\left(\frac{i\hbar}{2}\left(\overleftarrow{\partial}_{\mathbf{p}}\overrightarrow{\partial}_{\mathbf{x}} - \overleftarrow{\partial}_{\mathbf{x}}\overrightarrow{\partial}_{\mathbf{p}}\right)\right) W_\tau(\mathbf{x},\mathbf{p}) \\ &= \underbrace{\left\{\mathcal{H}, W_\tau\right\}}_{\text{Poisson bracket}} + \mathcal{O}(\hbar^2) \end{aligned}$$

CSA in Quantum Mechanics

- Approximating the right hand side by the Poisson bracket
$\Longleftrightarrow$ classical time evolution instead of quantum
$\Longrightarrow$ $\mathcal{O}(\hbar^2)$ error

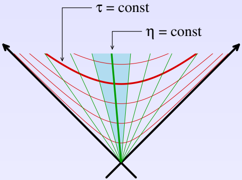
- In addition : $\hbar$ dependence in the initial state
Uncertainty principle, $\Delta x \cdot \Delta p \geq \hbar$
$\Longrightarrow$ the Wigner distribution $W_{\tau=0}(x,p)$ must have a width $\gtrsim \hbar$

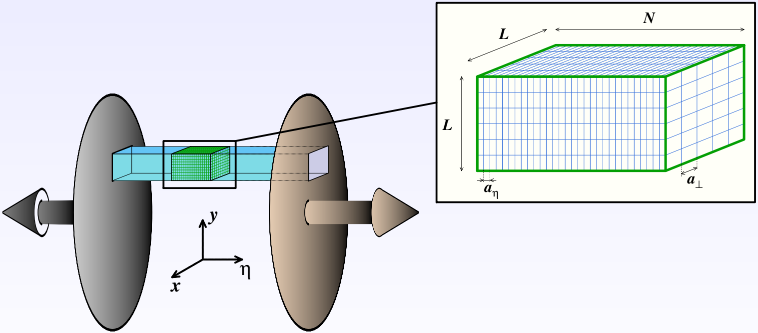
- All the $\mathcal{O}(\hbar)$ effects can be accounted for by a Gaussian initial distribution $W_{\tau=0}(x,p)$

Numerical implementation and Results

Discretization of the expanding volume

- Comoving coordinates : $\tau, \eta, \boldsymbol{x}_\perp$
- Simulation of a sub-volume
  + periodic boundary conditions
- $\mathrm{L}^2 \times \mathrm{N}$ lattice





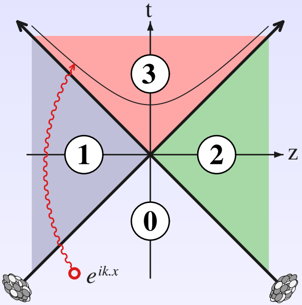
Gaussian spectrum of fluctuations

CGC at $\tau \ll Q_s^{-1}$ (1-loop accurate)

$$\langle \mathcal{A}^\mu \rangle = \mathcal{A}^\mu_{LO} \qquad \text{Var.} = \int_{\text{modes } \mathbf{k}} \frac{1}{2}\, a_{\mathbf{k}}(u) a^*_{\mathbf{k}}(v)$$

$$\left[\mathcal{D}_\rho \mathcal{D}^\rho \delta^\nu_\mu - \mathcal{D}_\mu \mathcal{D}^\nu + ig\,\mathcal{F}_\mu{}^\nu\right] a^\mu_{\mathbf{k}} = 0$$

$$\lim_{x^0 \to -\infty} a_{\mathbf{k}}(x) = e^{ik\cdot x}$$



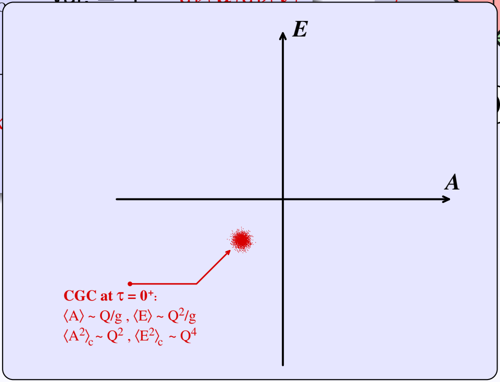
Gaussian spectrum of fluctuations

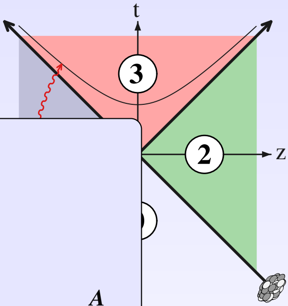
CGC at $\tau \ll Q_s^{-1}$ (1-loop accurate)

$\langle \mathcal{A}^\mu \rangle = \mathcal{A}^\mu_{\rm LO}$ Var. $= \int \frac{1}{\ }\, a_k(u) a_k^*(v)$

$\big[\mathcal{D}_\rho \mathcal{D}^\rho \delta^\nu_\mu -$

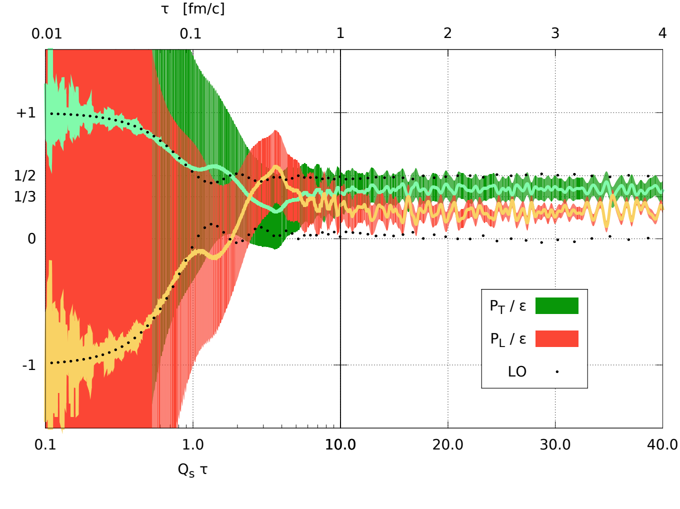
$\lim_{x^0 \to -\infty} a_k(x$





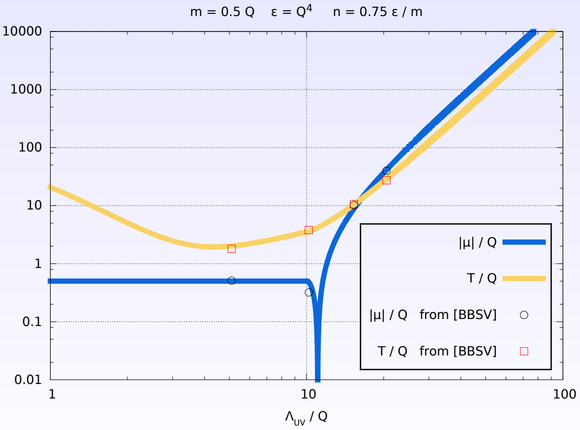
Time evolution of P_T/ϵ and P_L/ϵ $(64 \times 64 \times 128$ lattice)

$g = 0.5$ ($N_{\text{confs}} = 2000$)



UV cutoff dependence of the asymptotic distribution

- At late times, $f(\mathbf{p}) \approx \dfrac{T}{\omega_{\mathbf{p}} - \mu} - \dfrac{1}{2}$, but T and μ depend on Λ_{UV}



(points : classical statistical simulations, curves : Boltzmann eq.)

CSA with other Initial Conditions

Particle-like initial conditions

- The UV divergences in the CSA are due to the type of fluctuations of the initial conditions
- One can make up other ensembles of (non CGC) initial fields that do not lead to UV problems
- Possible sources of fluctuations of the initial fields :

$$G_{22}(p) \sim \Big(\underbrace{f_0(\mathbf{p})}_{\text{quasiparticles}} + \underbrace{\frac{1}{2}}_{\text{vacuum fluctuations}}\Big)\,\delta(p^2)$$

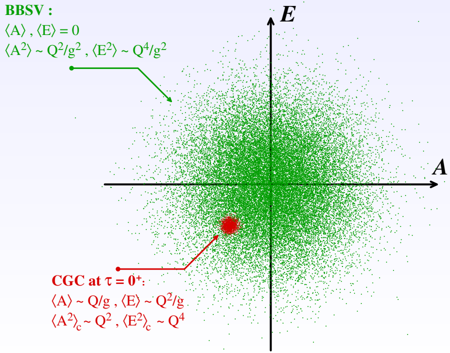
- **Vacuum fluctuations** make the CSA non-renormalizable because they have a flat spectrum up to the UV cutoff
- With only **quasiparticle**-induced fluctuations :
  - Finite if $f_0(p)$ falls faster than p^{-1}
  - Super-renormalizable if $f_0(p) \sim p^{-1}$ [Aarts, Smit (1997)]

Berges Boguslavski Schlichting Venugopalan (2013) :

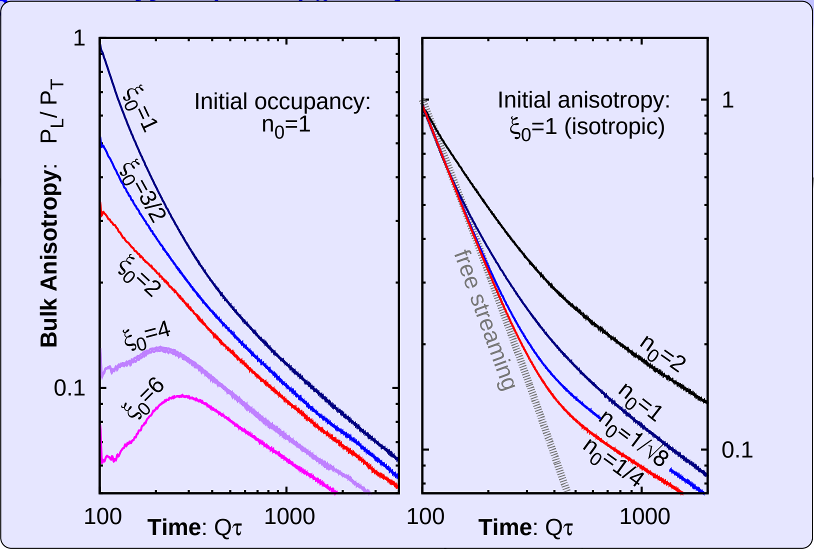
Dense gas of free gluons at $Q_s\tau \gg 1$

$$\langle \mathcal{A}^\mu \rangle = 0 \qquad \text{Var.} = \int_{\text{modes } \mathbf{k}} f_0(\mathbf{k})\; a_{\mathbf{k}}(\mathbf{u}) a^*_{\mathbf{k}}(\mathbf{v}) \qquad a_{\mathbf{k}}(x) \equiv e^{ik\cdot x}$$

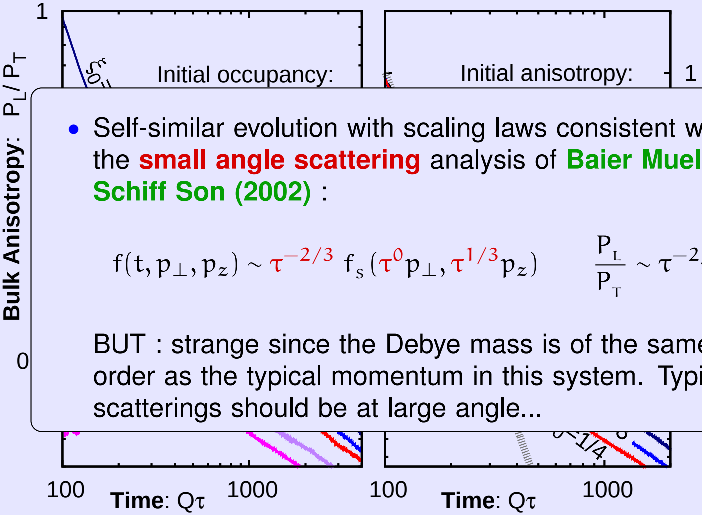
$$f_0(\mathbf{k}) \sim g^{-2} \times \theta(Q_s - k)$$



Berges Boguslavski Schlichting Venugopalan (2013) :



Berges Boguslavski Schlichting Venugopalan (2013) :

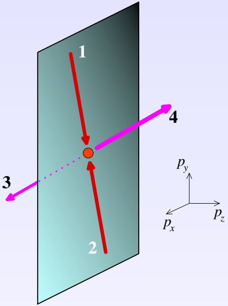


- Self-similar evolution with scaling laws consistent with the **small angle scattering** analysis of **Baier Mueller Schiff Son (2002)** :

$$f(t, p_\perp, p_z) \sim \tau^{-2/3}\, f_s(\tau^0 p_\perp, \tau^{1/3} p_z) \qquad \frac{P_L}{P_T} \sim \tau^{-2/3}$$

BUT : strange since the Debye mass is of the same order as the typical momentum in this system. Typical scatterings should be at large angle...

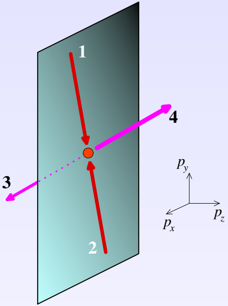
Can we understand this with the Boltzmann equation?



- CSA without vacuum fluctuations
  $\Leftrightarrow$ keep only f^3 terms in the collision term :

$$\partial_t f_4 \quad \sim \quad g^4 \int_{123} \cdots \left[f_1 f_2 (f_3 + f_4) - f_3 f_4 (f_1 + f_2)\right] + \cdots \left[\cancel{f_1 f_2 - f_3 f_4}\right]$$

Can we understand this with the Boltzmann equation?



- CSA without vacuum fluctuations
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$$\partial_t f_4 \sim g^4 \int_{123} \cdots \left[\cancel{f_1 f_2 (f_3 + f_4) - f_3 f_4 (f_1 + f_2)}\right] + \cdots \left[\cancel{f_1 f_2 - f_3 f_4}\right]$$

- If the distribution becomes very anisotropic, trying to produce the particle 4 at large angle results in $f_3 \approx f_4 \approx 0 \quad \Rightarrow$ nothing left !

The CSA with no vacuum fluctuations underestimates large angle scatterings when the distribution is anisotropic, and may lead to wrong conclusions regarding isotropization...

Can we understand this with the Boltzmann equation?

- CSA without vacuum fluctuations
  $\Leftrightarrow$ keep only f^3 terms in the collision term :

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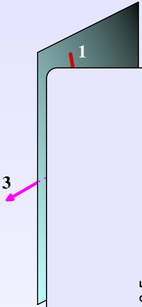

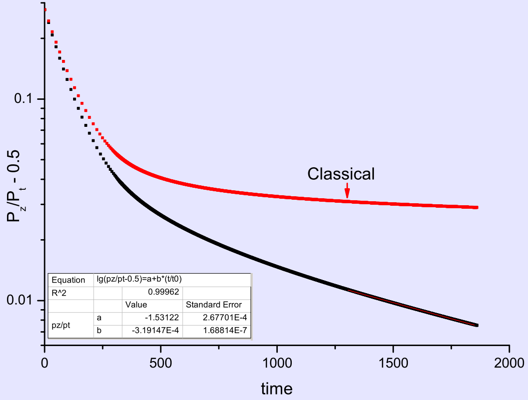

[Plot from **Blaizot, Liao**]

Summary

Summary

- LO : no pressure isotropization, NLO : instabilities
- Resummation beyond NLO : Classical statistical approximation
- Two implementations... and two different results :
  - **a.** one with CGC (vacuum-like) initial conditions
  - **b.** one with particle-like initial conditions
- The reason of the UV complications encountered in **(a)** is the very nature of the CGC initial conditions...

  Highly needed : ways to overcome the problems of the classical statistical approximation