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TMDs, offering opportunities at small k_T and small x!

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Outline

- TMD's: color gauge invariant correlators, describing distribution and fragmentation functions including partonic transverse momentum.
  - Taking small p_T out of the perturbative regime

- What are we talking about (definitions): gauge invariant description involves parton fields at different points, color-connected with gauge links/Wilson lines

- They can be measured in azimuthal and spin asymmetries provided one carefully accounts for right factors/signs etc.

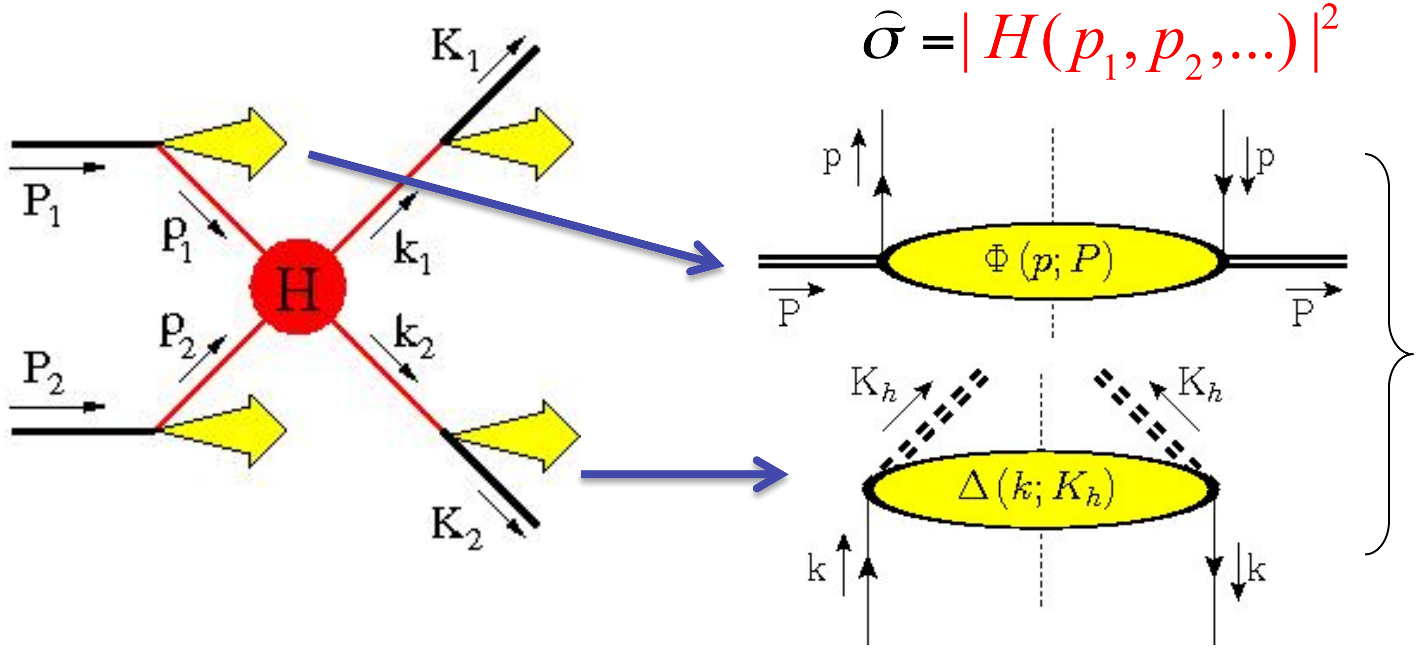
- High/low energy, relations with small-x physics and diffraction

PDFs and PFFs

Basic idea of PDFs and PFFs (also for TMDs) is to obtain a full factorized description of high energy scattering processes

$$\hat{\sigma} = | H(p_1, p_2, \ldots) |^2$$

calculable



defined (!)
&
portable

$$\sigma(P_1, P_2, \ldots) = \iiint dp_1 dp_2 \ldots \; \Phi_a(p_1, P_1; \mu) \otimes \Phi_b(p_2, P_2; \mu)$$

$$\otimes \hat{\sigma}_{ab,c\ldots}(p_1, p_2, \ldots; \mu) \otimes \Delta_c(k_1, K_1; \mu) \ldots$$

Give a meaning to integration variables, including e.g. $q_T = p_{1T} + p_{2T}$!

(Un)integrated correlators

$$\Phi(x,p_T,p.P) = \int \frac{d^4\xi}{(2\pi)^4}\, e^{i\,p.\xi} \left\langle P \middle| \bar{\psi}(0)\,\psi(\xi) \middle| P \right\rangle$$

- unintegrated

$$\Phi(x,p_T;n) = \int \frac{d(\xi.P)\,d^2\xi_T}{(2\pi)^3}\, e^{i\,p.\xi} \left\langle P \middle| \bar{\psi}(0)\,\psi(\xi) \middle| P \right\rangle_{\boxed{\xi.n=\xi^+=0}}$$

- TMD (light-front)

- σ = p^- integration makes time-ordering automatic. The soft part is simply sliced at the light-front

$$\Phi(x) = \int \frac{d(\xi.P)}{(2\pi)}\, e^{i\,p.\xi} \left\langle P \middle| \bar{\psi}(0)\,\psi(\xi) \middle| P \right\rangle_{\boxed{\xi.n=\xi_T=0}\ or\ \boxed{\xi^2=0}}$$

- Is already equivalent to a point-like interaction

- collinear (light-cone)

$$\Phi = \left\langle P \middle| \bar{\psi}(0)\,\psi(\xi) \middle| P \right\rangle_{\boxed{\xi=0}}$$

- local

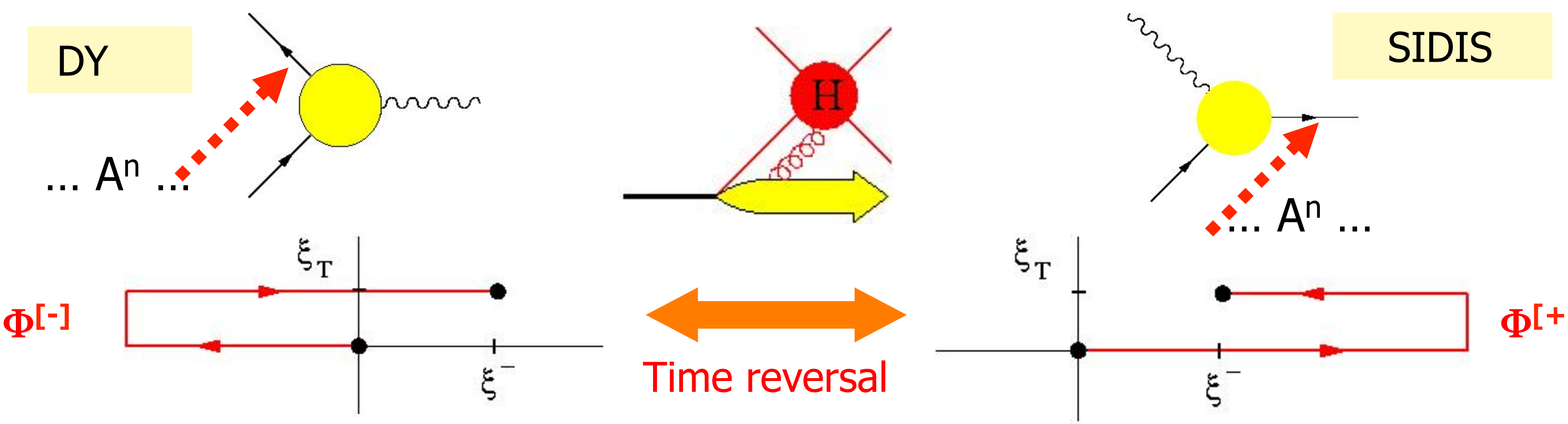
- Local operators with calculable anomalous dimension

Simplest gauge links for quark TMDs

$$\Phi_{ij}^{q[C]}(x, p_T; n) = \int \frac{d(\xi.P)\, d^2\xi_T}{(2\pi)^3}\, e^{i\,p.\xi} \left\langle P \left| \bar{\psi}_j(0)\, U^{[C]}_{[0,\xi]}\, \psi_i(\xi) \right| P \right\rangle_{\xi.n=0}$$

TMD

- Essential feature is the color connection of parton fields: gauge links originating from dimension zero (not suppressed!) A.n gluons (gluons polarized along parton momentum), but this is ambiguous for TMDs leading to process-dependence:



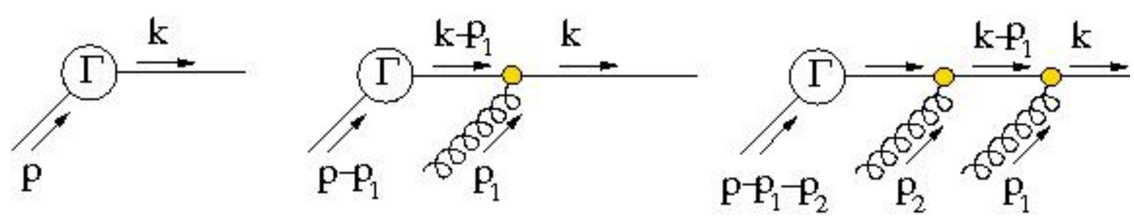
AV Belitsky, X Ji and F Yuan, NP B 656 (2003) 165
D Boer, PJM and F Pijlman, NP B 667 (2003) 201

Some details on the gauge links (1)

- Proper gluon fields (F rather than A, Wilson lines and boundary terms)

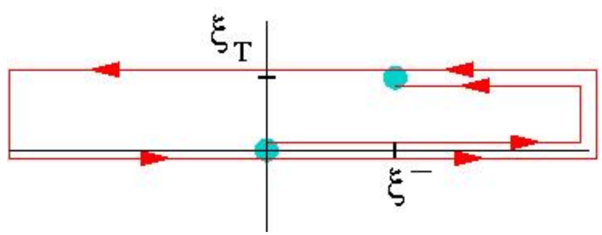
$$A^{\mu}(p_1) = n.A(p_1)\,\frac{P^{\mu}}{n.P} + i\,A_T^{\mu}(p_1) + \ldots \;=\; \frac{1}{p_1.n}\left[n.A(p_1)\;p_1^{\mu} + iG_T^{n\mu}(p_1) + \ldots\right]$$

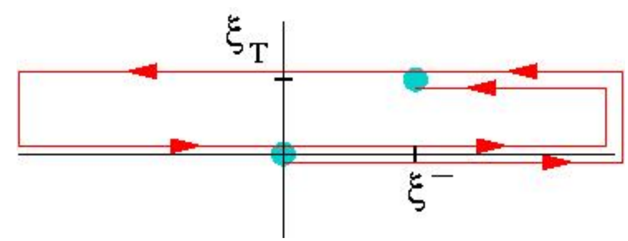
- Resummation of soft n.A gluons (coupling to outgoing color-line) for one correlator produces a gauge-line (along n)



- Boundary terms give transverse pieces
- TMD's may have complex gauge links involving loops (linked to color flow)

$\mathbf{U^{\square} = U^{+}U^{-\dagger}}$





Simplest gauge links for gluon TMDs

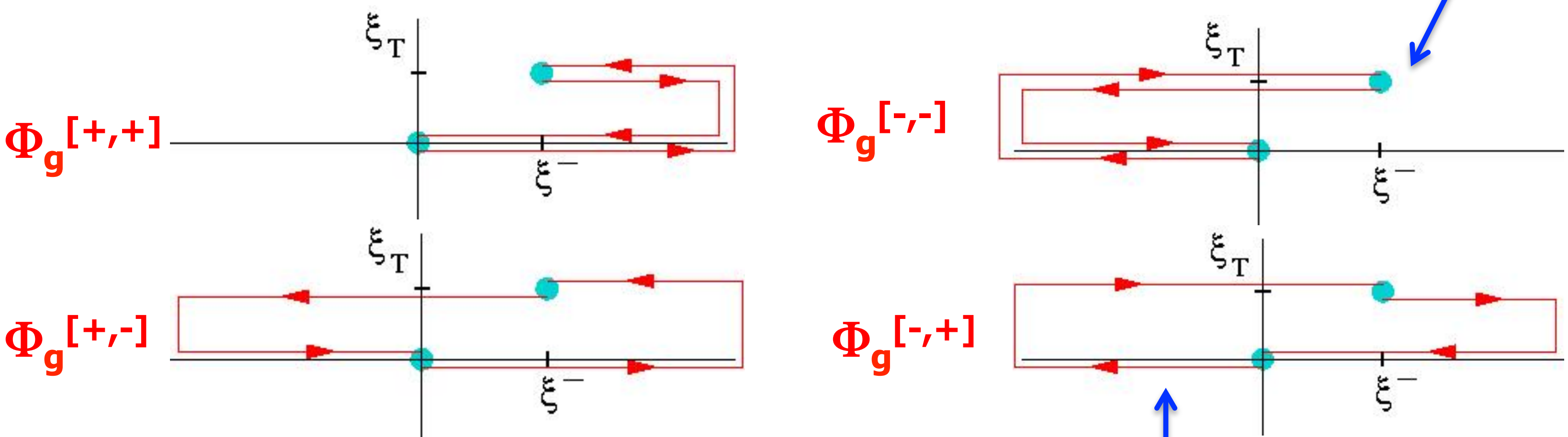
$$\Phi_g^{\alpha\beta[C,C']}(x,p_T;n) = \int \frac{d(\xi.P)d^2\xi_T}{(2\pi)^3} e^{ip.\xi} \left\langle P \left| U_{[\xi,0]}^{[C]} F^{n\alpha}(0) U_{[0,\xi]}^{[C']} F^{n\beta}(\xi) \right| P \right\rangle_{\xi.n=0}$$

- The TMD gluon correlators contain two links, which can have different paths. Note that standard field displacement involves C = C'

$$F^{\alpha\beta}(\xi) \to U_{[\eta,\xi]}^{[C]} F^{\alpha\beta}(\xi) U_{[\xi,\eta]}^{[C]}$$

- Basic (simplest) gauge links for gluon TMD correlators:

gg → H



in gg → $Q\bar{Q}$

C Bomhof, PJM, F Pijlman; EPJ C 47 (2006) 147
F Dominguez, B-W Xiao, F Yuan, PRL 106 (2011) 022301

Color gauge invariant correlators

- Including gauge links well-defined matrix elements for TMDs but this implies multiple possiblities for gauge links depending on the process and the color flow
- **Leading quark TMDs**

$$\Phi^{[U]}(x,p_T;n) = \Big\{ f_1^{[U]}(x,p_T^2) - f_{1T}^{\perp[U]}(x,p_T^2)\,\frac{\epsilon_T^{p_T S_T}}{M} + g_{1s}^{[U]}(x,p_T)\gamma_5 + h_{1T}^{[U]}(x,p_T^2)\,\gamma_5\,\not{S}_T + h_{1s}^{\perp[U]}(x,p_T)\,\frac{\gamma_5\,\not{p}_T}{M} + i h_1^{\perp[U]}(x,p_T^2)\,\frac{\not{p}_T}{M}\Big\}\frac{\not{P}}{2},$$

- **Leading gluon TMDs:**

$$2x\,\Gamma^{\mu\nu[U]}(x,p_T) = -g_T^{\mu\nu}\, f_1^{g[U]}(x,p_T^2) + g_T^{\mu\nu}\,\frac{\epsilon_T^{p_T S_T}}{M}\, f_{1T}^{\perp g[U]}(x,p_T^2) + i\epsilon_T^{\mu\nu}\; g_{1s}^{g[U]}(x,p_T) + \left(\frac{p_T^\mu p_T^\nu}{M^2} - g_T^{\mu\nu}\,\frac{p_T^2}{2M^2}\right) h_1^{\perp g[U]}(x,p_T^2) - \frac{\epsilon_T^{p_T\{\mu}\,p_T^{\nu\}}}{2M^2}\, h_{1s}^{\perp g[U]}(x,p_T) - \frac{\epsilon_T^{p_T\{\mu}\,S_T^{\nu\}} + \epsilon_T^{S_T\{\mu}\,p_T^{\nu\}}}{4M}\, h_{1T}^{g[U]}(x,p_T^2).$$

Studying dependence on gauge links

Operator structure in TMD case

- Operator product expansion takes you from local to nonlocal operator combinations.
- For TMD functions one can investigate this through transverse moments

$$\Phi(x,p_T;n) = \int \frac{d(\xi.P)d^2\xi_T}{(2\pi)^3} e^{i\,p.\xi} \left\langle P \middle| \bar{\psi}(0)\, U^{[\pm]}_{[0,\xi]}\, \psi(\xi) \middle| P \right\rangle_{\xi.n=0}$$

$$p_T^{\alpha}\Phi^{[\pm]}(x,p_T;n) = \int \frac{d(\xi.P)d^2\xi_T}{(2\pi)^3} e^{i\,p.\xi} \left\langle P \middle| \bar{\psi}(0) U_{[0,\pm\infty]} D_T^{\alpha} U_{[\pm\infty,\xi]} \psi(\xi) \middle| P \right\rangle_{\xi.n=0}$$

$$p_T^{\alpha_1} p_T^{\alpha_2}\Phi^{[\pm]}(x,p_T;n) = \int \frac{d(\xi.P)d^2\xi_T}{(2\pi)^3} e^{i\,p.\xi} \left\langle P \middle| \bar{\psi}(0) U_{[0,\pm\infty]} D_T^{\alpha_1} D_T^{\alpha_2} U_{[\pm\infty,\xi]} \psi(\xi) \middle| P \right\rangle_{\xi.n=0}$$

- Upon integration, these involve collinear twist-3 multi-parton correlators, they remain highly non-local and are U-dependent

MGA Buffing, A Mukherjee, PJM, PRD 86 (2012) 074030 , Arxiv: 1207.3221 [hep-ph]

Operator structure in TMD case

- For first transverse moment one needs (both) twist-3 quark-gluon correlators

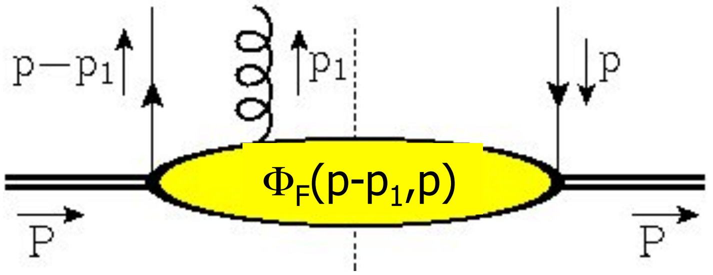
$$\Phi_D^\alpha(x-x_1,x_1\,|\,x) = \int \frac{d\xi.P\,d\eta.P}{(2\pi)^2}\,e^{i(p-p_1).\xi+ip_1.\eta}\left\langle P\middle|\bar{\psi}(0)D_T^\alpha(\eta)\,\psi(\xi)\middle|P\right\rangle_{\xi.n=\xi_T=0}$$

$$\Phi_F^\alpha(x-x_1,x_1\,|\,x) = \int \frac{d\xi.P\,d\eta.P}{(2\pi)^2}\,e^{i(p-p_1).\xi+ip_1.\eta}\left\langle P\middle|\bar{\psi}(0)F^{n\alpha}(\eta)\,\psi(\xi)\middle|P\right\rangle_{\xi.n=\xi_T=0}$$

- In principle multi-parton, but we need 'local'

$$\Phi_D^\alpha(x) = \int dx_1\,\Phi_D^\alpha(x-x_1,x_1\,|\,x)$$

$$\Phi_A^\alpha(x) = PV\int dx_1\,\frac{1}{x_1}\,\Phi_F^{n\alpha}(x-x_1,x_1\,|\,x)$$



$$\boxed{\tilde{\Phi}_\partial^\alpha(x) = \Phi_D^\alpha(x) - \Phi_A^\alpha(x)}$$

T-even (gauge-invariant derivative)

$$\boxed{\Phi_G^\alpha(x) = \pi\,\Phi_F^{n\alpha}(x,0\,|\,x)}$$

T-odd (soft-gluon or gluonic pole)

Efremov, Teryaev; Qiu, Sterman; Brodsky, Hwang, Schmidt; Boer, Teryaev, M; Bomhof, Pijlman, M

Operator structure in TMD case

- Transverse moments can be expressed in these particular collinear multi-parton twist-3 (and higher) correlators (which are not suppressed!)

$$\boxed{\Phi_{\partial}^{\alpha[U]}(x) = \int d^2p_T\; p_T^{\alpha}\,\Phi^{[U]}(x,p_T;n) = \tilde{\Phi}_{\partial}^{\alpha}(x) + C_G^{[U]}\,\Phi_G^{\alpha}(x)}$$

T-even

T-odd

T-even

T-even

T-odd

$$\boxed{\Phi_{\partial\partial}^{\alpha\beta[U]}(x) = \tilde{\Phi}_{\partial\partial}^{\alpha\beta}(x) + C_{GG,c}^{[U]}\,\Phi_{GG,c}^{\alpha\beta}(x) + C_G^{[U]}\Big(\tilde{\Phi}_{\partial G}^{\alpha\beta}(x) + \tilde{\Phi}_{G\partial}^{\alpha\beta}(x)\Big)}$$

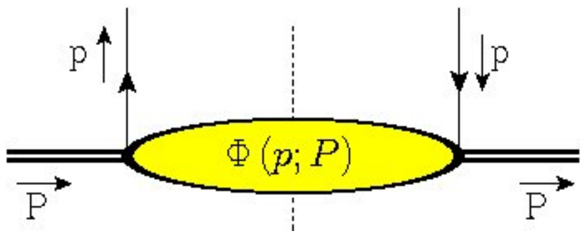
$Tr_c(GG\,\psi\bar{\psi})$

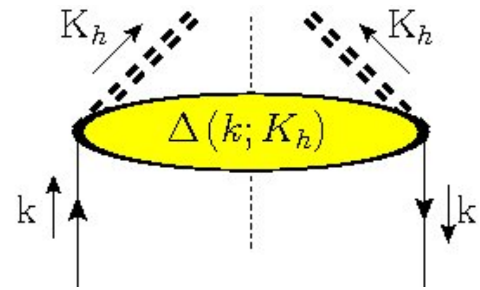
$Tr_c(GG)\;Tr_c(\psi\bar{\psi})$

- $C_G^{[U]}$ calculable gluonic pole factors

U	$U^{[\pm]}$	$U^{[+]}\,U^{[\Box]}$	$\frac{1}{N_c}\,\mathrm{Tr}_c(U^{[\Box]})\,U^{[+]}$
$\Phi^{[U]}$	$\Phi^{[\pm]}$	$\Phi^{[+\Box]}$	$\Phi^{[(\Box)+]}$
$C_G^{[U]}$	± 1	3	1
$C_{GG,1}^{[U]}$	1	9	1
$C_{GG,2}^{[U]}$	0	0	4

Distribution versus fragmentation functions





- Operators:

$$\Phi^{[U]}(p\,|\,p) \sim \langle P\,|\,\bar{\psi}(0)U_{[0,\xi]}\psi(\xi)\,|\,P\rangle$$

$$\Phi_{\partial}^{\alpha[U]}(x) = \tilde{\Phi}_{\partial}^{\alpha}(x) + C_G^{[U]}\,\Phi_G^{\alpha}(x)$$

T-even T-odd (gluonic pole)

$$\boxed{\Phi_G^{\alpha}(x) = \pi\Phi_F^{n\alpha}(x,0\,|\,x) \neq 0}$$

- Operators:

out state

$$\Delta(k\,|\,k) \sim \sum_X \langle 0\,|\,\psi(\xi)\,|\,K_h X\rangle\langle K_h X\,|\,\bar{\psi}(0)\,|\,0\rangle$$

$$\boxed{\Delta_G^{\alpha}(x) = \pi\Delta_F^{n\alpha}(\tfrac{1}{z},0\,|\,\tfrac{1}{z}) = 0}$$

$$\Delta_{\partial}^{\alpha[U]}(x) = \tilde{\Delta}_{\partial}^{\alpha}(x)$$

T-even operator combination,
but still T-odd functions!

Collins, Metz; Meissner, Metz; Gamberg, M, Mukherjee, PR D 83 (2011) 071503

Classifying Quark TMDs

- Collecting the right moments gives expansion into full TMD PDFs of definite rank

$$\Phi^{[U]}(x,p_T) = \Phi(x,p_T^2) + p_{Ti}\tilde{\Phi}_{\partial}^{i}(x,p_T^2) + p_{Tij}\tilde{\Phi}_{\partial\partial}^{ij}(x,p_T^2) + \ldots$$
$$+\sum_c C_{G,c}^{[U]}\left[\qquad p_{Ti}\Phi_{G,c}^{i}(x,p_T^2) + p_{Tij}\tilde{\Phi}_{\{\partial G\},c}^{ij}(x,p_T^2) + \ldots\right]$$
$$+\sum_c C_{GG,c}^{[U]}\left[p_T^2\Phi_{G.G,c}(x,p_T^2) + \quad\ldots\quad + p_{Tij}\Phi_{GG,c}^{ij}(x,p_T^2) + \ldots\right]$$

- While for TMD PFFs

$$\Delta^{[U]}(z^{-1},k_T) = \Delta(z^{-1},k_T^2) + k_{Ti}\tilde{\Delta}_{\partial}^{i}(z^{-1},k_T^2) + k_{Tij}\tilde{\Delta}_{\partial\partial}^{ij}(z^{-1},k_T^2) + \ldots$$

Classifying Quark TMDs

factor	TMD PDF RANK			
	0	1	2	3
1	$\Phi(x,p_T^2)$	$\tilde{\Phi}_{\partial}(x,p_T^2)$	$\tilde{\Phi}_{\partial\partial}(x,p_T^2)$	$\tilde{\Phi}_{\partial\partial\partial}(x,p_T^2)$
$C_{G,c}^{[U]}$		$\Phi_{G,c}(x,p_T^2)$	$\tilde{\Phi}_{\{G\partial\},c}(x,p_T^2)$	$\tilde{\Phi}_{\{G\partial\partial\},c}(x,p_T^2)$
$C_{GG,c}^{[U]}$			$\Phi_{GG,c}(x,p_T^2)$	$\tilde{\Phi}_{\{GG\partial\},c}(x,p_T^2)$
$C_{GGG,c}^{[U]}$				$\Phi_{GGG,c}(x,p_T^2)$

- Only a finite number needed: rank up to $2(S_{hadron}+s_{parton})$
- Rank m shows up as cos(mϕ) and sin(mϕ) azimuthal asymmetries
- No gluonic poles for PFFs

factor	TMD PFF RANK			
	0	1	2	3
1	$\Delta(z^{-1},k_T^2)$	$\tilde{\Delta}_{\partial}(z^{-1},k_T^2)$	$\tilde{\Delta}_{\partial\partial}(z^{-1},k_T^2)$	$\tilde{\Delta}_{\partial\partial\partial}(z^{-1},k_T^2)$

MGA Buffing, A Mukherjee, PJM, PRD2012 , Arxiv: 1207.3221 [hep-ph]

Explicit classification quark TMDs

factor	QUARK TMD PDF RANK UNPOLARIZED HADRON			
	0	1	2	3
1	f_1			
$C_G^{[U]}$		$h_1^\perp$		
$C_{GG,c}^{[U]}$				

- Example: quarks in an unpolarized target are described by just 2 TMD structures

$$\tilde{\Phi}(x,p_T^2) = \left(f_1\,(x,p_T^2)\right)\frac{\not{P}}{2} \qquad \tilde{\Phi}_G^\alpha(x,p_T^2) = \left(i h_1^\perp(x,p_T^2)\frac{\gamma_T^\alpha}{M}\right)\frac{\not{P}}{2}$$

T-even

T-odd

[B-M function]

- Gauge link dependence: $h_1^{\perp[U]}(x,p_T^2) = C_G^{[u]} h_1^\perp(x,p_T^2)$

Explicit classification quark TMDs

factor	QUARK TMD PDFs RANK SPIN ½ HADRON			
	0	1	2	3
1	$f_1,\ g_1,\ h_{1T}$	$g_{1T},\ h_{1L}^{\perp}$	$h_{1T}^{\perp(A)}$	
$C_G^{[U]}$		$h_1^{\perp},\ f_{1T}^{\perp}$		
$C_{GG,c}^{[U]}$	$\delta f_1,\ \delta g_1,\ \delta h_{1T}$		$h_{1T}^{\perp(B1)},\ h_{1T}^{\perp(B2)}$	

trace terms

Multiple color possibilities

$$A:\ \ \bar{\psi}\partial\partial\psi = Tr_c\left[\partial\partial\psi\bar{\psi}\right]$$

$$B1:\ Tr_c\left[GG\psi\bar{\psi}\right]$$

$$B2:\ Tr_c\left[GG\right]Tr_c\left[\psi\bar{\psi}\right]$$

Explicit classification quark TMDs

factor	QUARK TMD PDFs RANK SPIN ½ HADRON			
	0	1	2	3
1	$f_1,\ g_1,\ h_{1T}$	$g_{1T},\ h_{1L}^{\perp}$	$h_{1T}^{\perp(A)}$	
$C_G^{[U]}$		$h_1^{\perp},\ f_{1T}^{\perp}$		
$C_{GG,c}^{[U]}$	$\delta f_1,\ \delta g_1,\ \delta h_{1T}$		$h_{1T}^{\perp(B1)},\ h_{1T}^{\perp(B2)}$	

Three pretzelocities:

$$A:\ \ \bar{\psi}\partial\partial\psi = Tr_c\left[\partial\partial\psi\bar{\psi}\right]$$

$$B1:\ Tr_c\left[GG\psi\bar{\psi}\right]$$

$$B2:\ Tr_c\left[GG\right]Tr_c\left[\psi\bar{\psi}\right]$$

Process dependence in f_1, g_1 and h_1
(U-dependent broadening made explicit)

$$f_1^{[U]} = f_1 + C_{GG,c}^{[U]}\,\delta f_1^{(Bc)}$$

$$h_1^{[U]} = h_{1T} + h_{1T}^{\perp(1)(A)} + C_{GG,c}^{[U]}\left(\delta h_{1T}^{\perp(Bc)} + h_{1T}^{\perp(1)(Bc)}\right)$$

B Boer, MGA Buffing, PJM, work in progress

Explicit classification quark TMDs

factor	QUARK TMD PDFs RANK SPIN ½ HADRON			
	0	1	2	3
1	$f_1,\ g_1,\ h_{1T}$	$g_{1T},\ h_{1L}^{\perp}$	$h_{1T}^{\perp(A)}$	
$C_G^{[U]}$		$h_1^{\perp},\ f_{1T}^{\perp}$		
$C_{GG,c}^{[U]}$	$\delta f_1,\ \delta g_1,\ \delta h_{1T}$		$h_{1T}^{\perp(B1)},\ h_{1T}^{\perp(B2)}$	

factor	QUARK TMD PFFs RANK SPIN ½ HADRON			
	0	1	2	3
1	$D_1,\ G_1,\ H_{1T}$	$D_{1T}^{\perp}, G_{1T}, H_1^{\perp}, H_{1L}^{\perp}$	$H_{1T}^{\perp}$	

Just a single 'pretzelocity' PFF

Explicit classification gluon TMDs

factor	GLUON TMD PDF RANK UNPOLARIZED HADRON			
	0	1	2	3
1	f_1		$h_1^{\perp(A)}$	
$C_{GG,c}^{[U]}$	$\delta f_1^{(Bc)}$		$h_1^{\perp(Bc)}$	

- Note process dependence of unpolarized gluon TMD:

$$f_1^{g[U]} = f_1^g + C_{GG,c}^{[U]}\delta f_1^{g(Bc)}$$

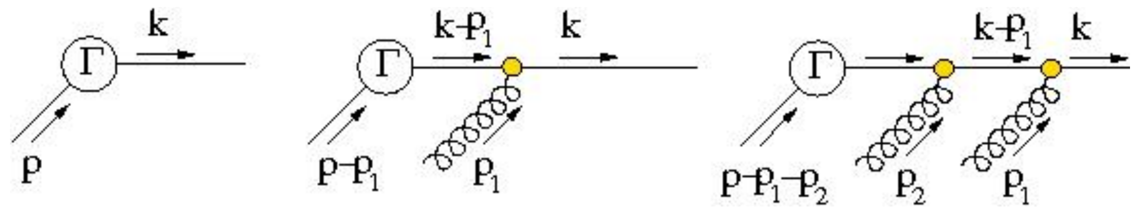
$$h_1^{g\perp[U]} = h_1^{\perp(A)} + C_{GG,c}^{[U]} h_1^{\perp(Bc)}$$

D. Boer (talk spin 20014): B Boer, MGA Buffing, PJM, work in progress

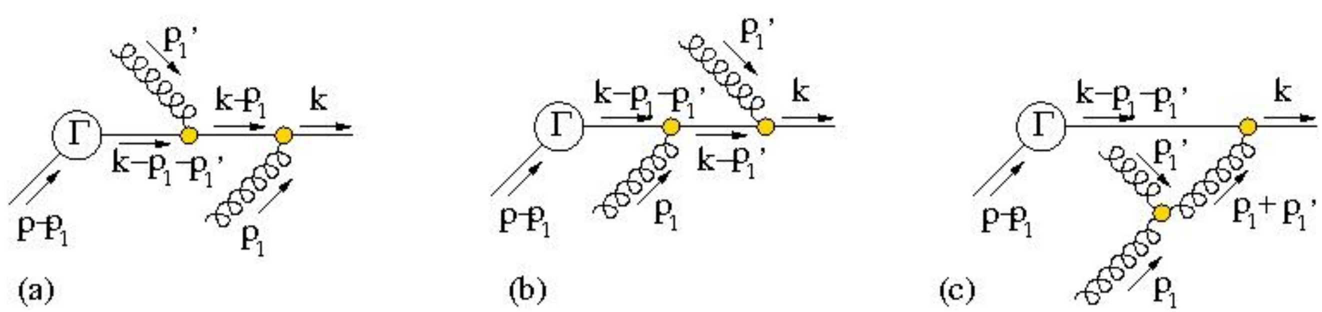
Multiple TMDs in cross sections

Some details on the gauge links (2)

- Resummation of soft n.A gluons (coupling to outgoing color-line) for one correlator produces a gauge-line (along n)

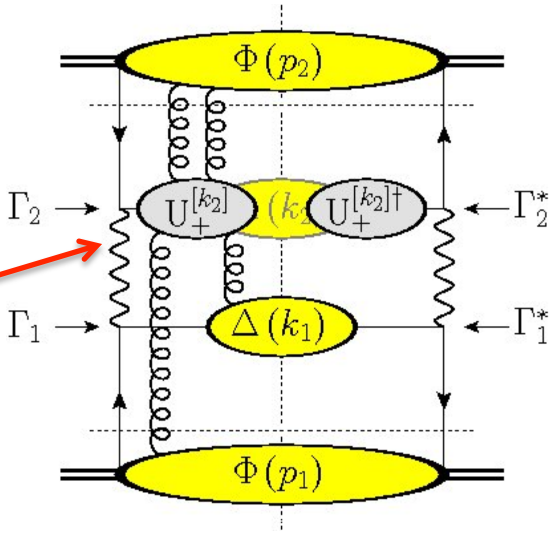


- The lowest order contributions for soft gluons from two different correlators coupling to outgoing color-line resums into gauge-knots: shuffle product of all relevant gauge-lines from that (external initial/final state) line.



Which gauge links?

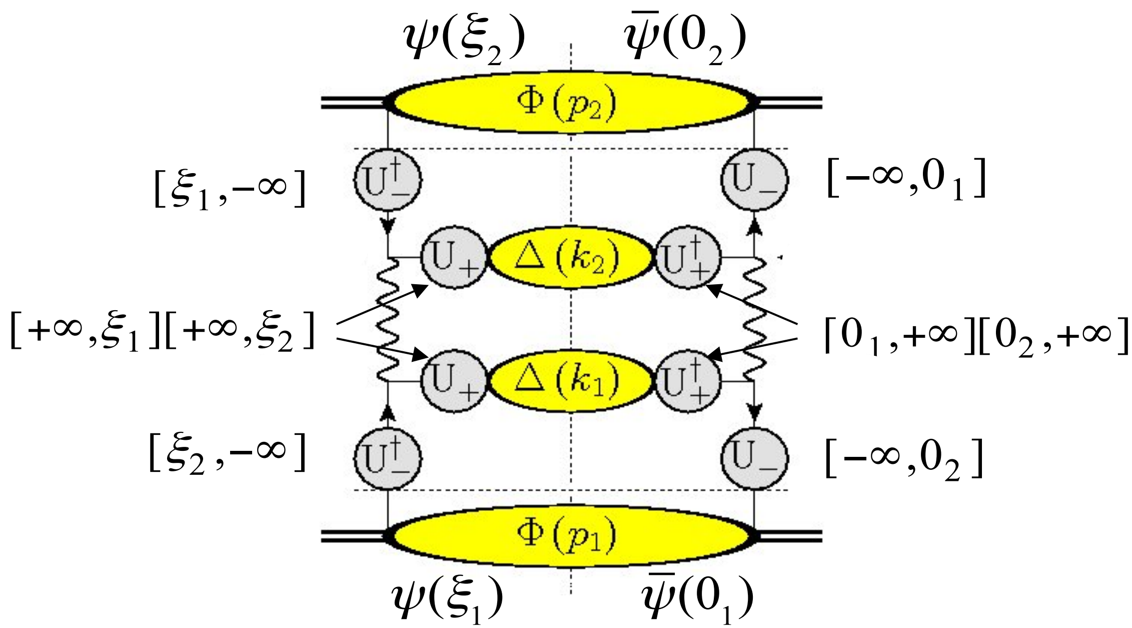
- With more (initial state) hadrons color gets entangled, e.g. in pp
- Gauge knot $U_+[p_1,p_2,\ldots]$



- Outgoing color contributes to a future pointing gauge link in $\Phi(p_2)$ and future pointing part of a gauge loop in the gauge link for $\Phi(p_1)$
- This causes trouble with factorization

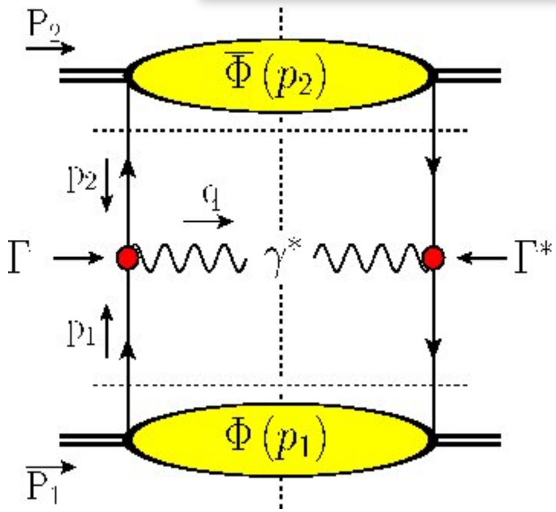
T.C. Rogers, PJM, PR D81 (2010) 094006

Which gauge links?

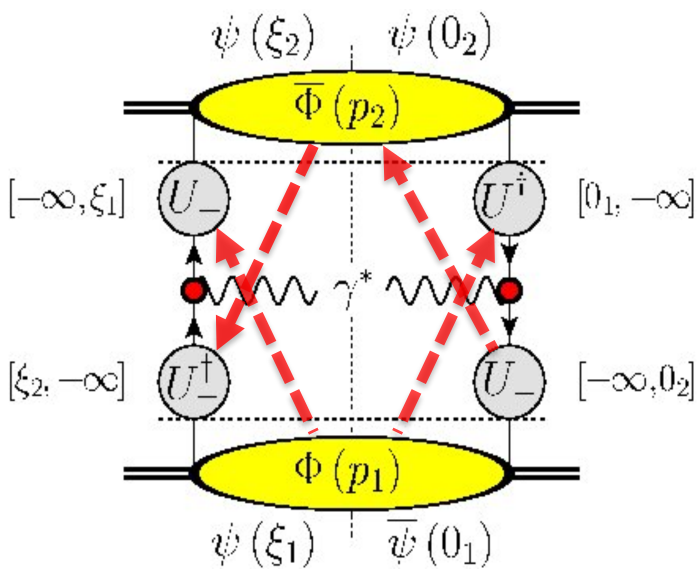


- Can be color-detangled if only p_T of single correlator is relevant (using polarization, …) but must include Wilson loops in final U

Correlators in description of hard process (e.g. DY)



$$\xcancel{\begin{aligned} d\sigma_{\rm DY} &\sim \mathrm{Tr}_c\left[\Phi(x_1,p_{1T})\Gamma^*\overline{\Phi}(x_2,p_{2T})\Gamma\right] \\ &= \frac{1}{N_c}\Phi(x_1,p_{1T})\Gamma^*\overline{\Phi}(x_2,p_{2T})\Gamma, \end{aligned}}$$



- Complications if the transverse momentum of two initial state hadrons is involved, resulting for DY at measured Q_T in

$$\begin{aligned} d\sigma_{\rm DY} &= \mathrm{Tr}_c\Big[U_-^\dagger[p_2]\Phi(x_1,p_{1T})U_-[p_2]\Gamma^* \\ &\qquad \times U_-^\dagger[p_1]\overline{\Phi}(x_2,p_{2T})U_-[p_1]\Gamma\Big] \\ &\neq \frac{1}{N_c}\Phi^{[-]}(x_1,p_{1T})\Gamma^*\overline{\Phi}^{[-^\dagger]}(x_2,p_{2T})\Gamma, \end{aligned}$$

Just as for twist-3 squared in collinear DY

Classifying Quark TMDs

factor	TMD RANK			
	0	1	2	3
1	$\Phi(x,p_T^2)$	$\tilde{\Phi}_{\partial}(x,p_T^2)$	$\tilde{\Phi}_{\partial\partial}(x,p_T^2)$	$\tilde{\Phi}_{\partial\partial\partial}(x,p_T^2)$
$C_{G,c}^{[U]}$		$\Phi_{G,c}(x,p_T^2)$	$\tilde{\Phi}_{\{G\partial\},c}(x,p_T^2)$	$\tilde{\Phi}_{\{G\partial\partial\},c}(x,p_T^2)$
$C_{GG,c}^{[U]}$			$\Phi_{GG,c}(x,p_T^2)$	$\tilde{\Phi}_{\{GG\partial\},c}(x,p_T^2)$
$C_{GGG,c}^{[U]}$				$\Phi_{GGG,c}(x,p_T^2)$

$$\sigma(x_1,x_2,q_T) \sim \frac{1}{N_c} f^{[U_1,U_2]}_{R_{G1}R_{G2}}\, \Phi^{[U_1]}(x_1,p_{1T}) \otimes \overline{\Phi}^{[U_2]}(x_2,p_{2T})\,\hat{\sigma}(x_1,x_2),$$

R_G for $\overline{\Phi}^{[-^\dagger]}$	R_G for $\Phi^{[-]}$: 0	1	2
0	1	1	1
1	1	$-\frac{1}{N_c^2-1}$	$\frac{N_c^2+2}{(N_c^2-2)(N_c^2-1)}$
2	1	$\frac{N_c^2+2}{(N_c^2-2)(N_c^2-1)}$	$\frac{3N_c^4-8N_c^2-4}{(N_c^2-2)^2(N_c^2-1)}$

$$\frac{\mathrm{Tr}_c[T^aT^bT^aT^b]}{\mathrm{Tr}_c[T^aT^a]\,\mathrm{Tr}_c[T^bT^b]} = -\frac{1}{N_c^2-1}\frac{1}{N_c}.$$

MGA Buffing, PJM, PRL (2014), Arxiv: 1309.4681 [hep-ph]

Remember classification of Quark TMDs

factor	QUARK TMD RANK UNPOLARIZED HADRON			
	0	1	2	3
1	f_1			
$C_G^{[U]}$		$h_1^\perp$		
$C_{GG,c}^{[U]}$				

- Example: quarks in an unpolarized target needs only 2 functions
- Resulting in cross section for unpolarized DY at measured Q_T

$$\sigma_{DY}(x_1,x_2,q_T) = \frac{1}{N_c}\Phi(x_1,p_{1T})\otimes\bar{\Phi}(x_2,p_{2T}) \qquad \text{contains } f_1$$

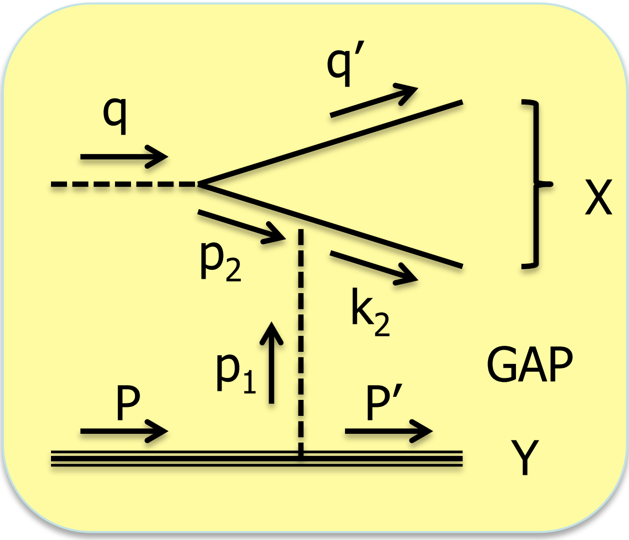
$$-\frac{1}{N_c}\frac{1}{N_c^2-1}\,q_T^{\alpha\beta}\Phi_G^{\alpha}(x_1,p_{1T})\otimes\bar{\Phi}_G^{\beta}(x_2,p_{2T}) \qquad \text{contains } h_1^{perp}$$

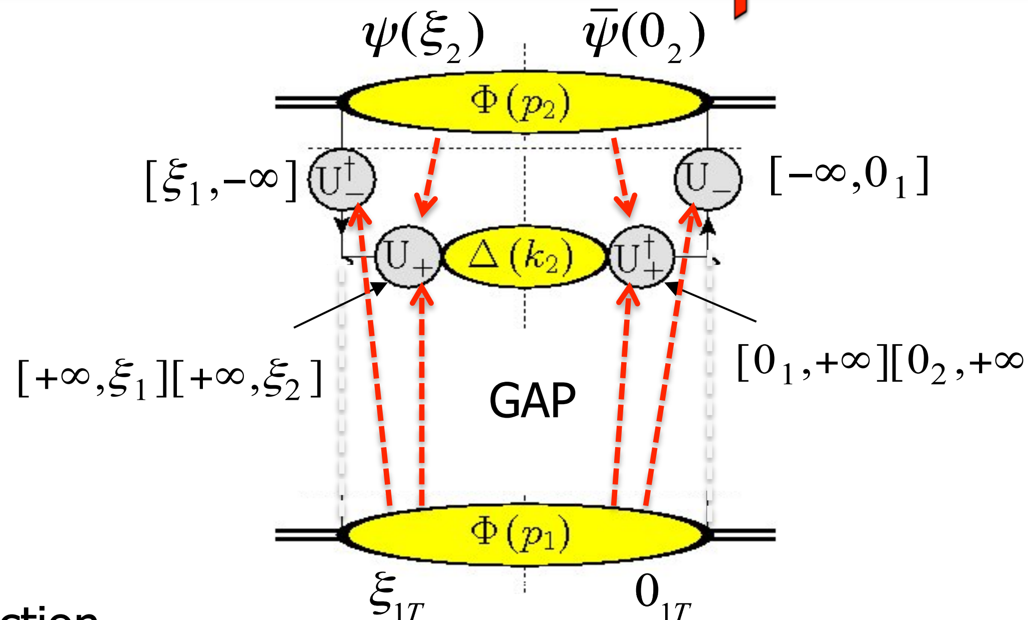
D. Boer, PRD 60 (1999) 014012; MGA Buffing, PRL (2014) PJM, Arxiv: 1309.4681 [hep-ph]

TMDs and diffraction

A TMD picture for diffractive scattering

Preliminary



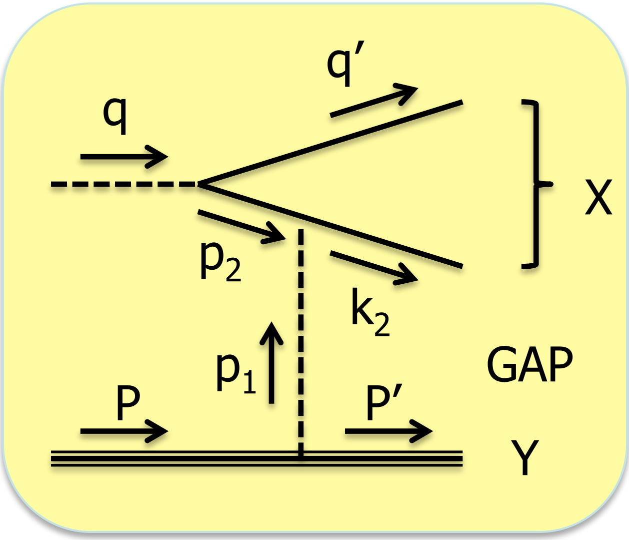


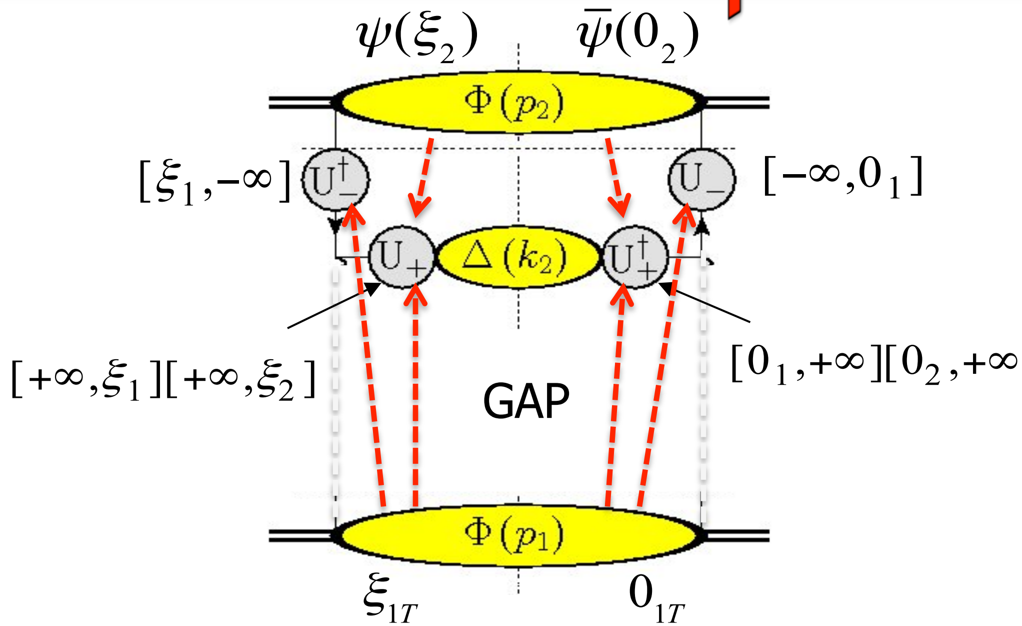
- Momentum flow in case of diffraction
  $x_1 \rightarrow M_X^2/W^2 \rightarrow 0$ and $t \rightarrow p_{1T}^2$
- Picture in terms of TMDs and inclusion of gauge links
  (including gauge links/collinear gluons in M ~ S – 1)

- (Work in progress: Hoyer, Kasemets, Pisano, Zhou, M)
- (Another way of looking at diffraction, cf Dominguez, Xiao, Yuan 2011 or older work of Gieseke, Qiao, Bartels 2000)

A TMD picture for diffractive scattering

Preliminary





- Cross section

$$d\sigma = \Phi(p_{1T};P)\,\mathrm{Tr}_c\left[U_-^\dagger[p_1]U_+[p_1,p_2]\,U_+^\dagger[p_1,p_2]U_-[p_1]\Phi(x_2,p_{2T};q)\right]$$

- involving correlators for proton and photon/…

$$\Phi^{q/\gamma[+]}(x_2,p_{2T};q) = \int \frac{d(\xi.q)d^2\xi_T}{(2\pi)^3} e^{i\,p_2.\xi_2}\left\langle \gamma^*(q)\middle|\bar\psi(0)U^{[+]}_{[0,\xi]}\psi(\xi)\middle|\gamma^*(q)\right\rangle_{\xi.n=0}$$

$$\Phi^{[loop]}_{DIF}(x_1,p_{1T};P) = \delta(x_1)\int \frac{d^2\xi_T}{(2\pi)^2} e^{i\,p_{1T}.\xi}\left\langle P\middle|U^{[loop]}-1\middle|P\right\rangle_{\xi.n=0}$$

Diffractive TMD correlator

Preliminary

- no partons! $\Phi^{[loop]}_{DIF}(x,p_T;P) = \delta(x)\int \frac{d^2\xi_T}{(2\pi)^2} e^{i\,p_T\cdot\xi} \left\langle P \middle| U^{[loop]} - 1 \middle| P \right\rangle_{\xi.n=0}$
- Simplest transverse moment is rank 2

$$\Phi^{\alpha\beta}_{0GG}(x,p_T;P) = \pi\Phi^{\alpha\beta}_{FF}(0,0)$$

- Leading to $\Phi^{[U]}_{DIFF}(x,p_T) \;=\; C^{[U]}_{GG}\left[p_T^2\,\tilde{\Phi}_{G.G}(p_T^2) + p_{Tij}\,\tilde{\Phi}^{ij}_{GG,c}(p_T^2) + \ldots\right]$

Conclusion with (potential) rewards

- We study the (generalized) universality of TMDs via operator product expansion, extending the well-known collinear distributions (including polarization 3 for quarks and 2 for gluons) to TMD PDF and PFF functions, ordered into functions of definite rank.
- Theoretical and experimental relevance

- ! Multiple operator possibilities for pretzelocity/transversity and linearly polarized gluons
- ! The TMD PDFs appear in cross sections with specific calculable factors that deviate from (or extend on) the naïve parton universality for hadron-hadron scattering.
- ! Applications in polarized high energy processes, but also in unpolarized situations (linearly polarized gluons) and possibly diffractive processes.

Thank you