Behind the geon horizon

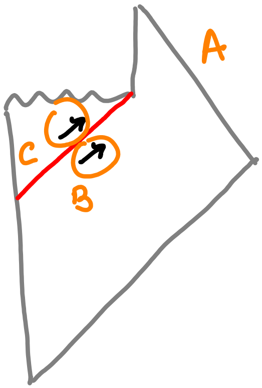
Monica Guică

Uppsala University & Nordita

· based on 1412.1084, w/ Simon Ross

Motivation

- Black hole information paradox



- recovery of information $\Rightarrow$ $S_{AB} < S_A$
- smoothness of horizon $\Rightarrow$ $S_{BC} \simeq 0$
- strong subadditivity $S_A + S_C \leq S_{AB} + S_{BC}$

$\downarrow$

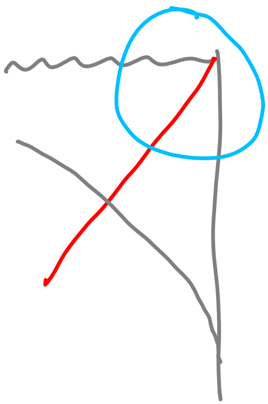
contradiction!

- possible way out : $C \subset A$ $\rightarrow$ black hole complementarity

$\rightarrow$ How is the black hole interior encoded outside?

The black hole interior in AdS/CFT

- large black hole in AdS w/ a single exterior



- dual to pure state $|\psi\rangle \subset CFT$ thermalises

- i.e. when probed by a small algebra of observables $\mathcal{O}_i$:

$$\langle \psi | \mathcal{O}_i .. | \psi \rangle = \mathrm{Tr}(\rho_{th} \mathcal{O}_i ..) + \mathcal{O}(e^{-S})$$

- Papadodimas - Raju (PR) → quantitative proposal for reconstructing the b.h. interior

- this talk → concrete example of the PR construction ($\mathbb{R}P^2$ geon black hole)

Plan

- review : reconstruction of bulk from the boundary
    - the PR proposal
    - the $\mathbb{RP}^2$ geon & properties
- construction of mirror operators
- modifications of the geon state
- future directions

Reconstructing the bulk from the boundary in AdS/CFT

- CFT → large N, sparse light spectrum

- correlation functions of single-trace operators factorize

$$\langle \mathcal{O}\,\mathcal{O}\,\mathcal{O}\,\mathcal{O} \rangle = \langle \mathcal{O}\,\mathcal{O} \rangle \langle \mathcal{O}\,\mathcal{O} \rangle + \text{perm.} + \mathcal{O}(1/N)$$

- generalized free field operators → free scalar in AdS

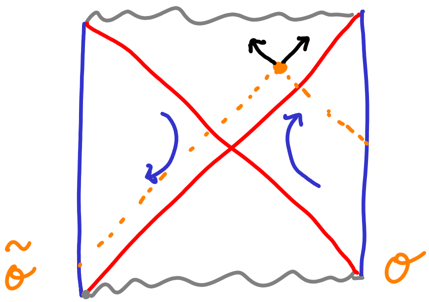
$$(\Box_{AdS} - m^2)\Phi = 0$$

$$\Phi(z, x^\mu) = \int d^d x' \, \mathcal{K}(z, x; x') \, \mathcal{O}(x')$$

- reproduces local EFT in the bulk, pert. in $1/N$

- breaks down if we compute very "long" correlators

Reconstructing the black hole interior : eternal b.h



- entangled state in 2 copies of CFT

$$\psi_{tfd} = \sum_i e^{-\frac{\beta E_i}{2}} |E_i\rangle \widetilde{|E_i\rangle}$$

- $[O, \tilde{O}] = 0$ $\qquad\qquad \langle O O \ldots \rangle$ - thermal

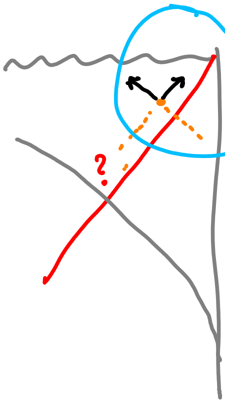
- $\langle \psi_{tfd} | O(t_1, x_1) \ldots \tilde{O}(t_n, x_n) \ldots | \psi_{tfd} \rangle = Z_\beta^{-1} \, \mathrm{Tr}\left[e^{-\beta H} O(t_1, x_1) \ldots O(t_n + \frac{i\beta}{2}, x_n) \right]$

- bulk field in interior

$$\Phi(x, z) = \int_{\omega > 0} d\omega \, d^{d-1}k \left[O_{\omega,k} K^{(1)}(x, z) + \tilde{O}_{\omega k} K^{(2)}(x, z) + h.c \right]$$

- reproduces local EFT in the eternal black hole

Reconstruction of the black hole interior – single-sided b.h



- $|\psi\rangle$ – pure state that thermalizes
- need right-moving modes!
- PR proposal:

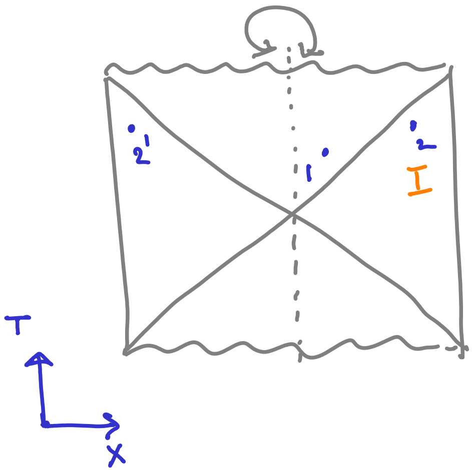
$$\Phi(z,x) = \int d\omega\, d^{d-1}k \left[O_{\omega k}\, K^{(1)}_{BTZ}(z,x) + \tilde{O}_{\omega k}\, K^{(2)}_{BTZ}(z,x) + h.c. \right]$$

Conditions on $\tilde{O}$:

- "correctly" entangled $\quad \tilde{O}|\psi\rangle = e^{-\frac{\beta H}{2}}\, O^{\dagger}\, e^{\frac{\beta H}{2}}|\psi\rangle$ $\quad$ state dependent!
- commute inside correlation functions $\quad [\tilde{O}, O]\, O \ldots |\psi\rangle = 0$

The $\mathbb{RP}^2$ geon example

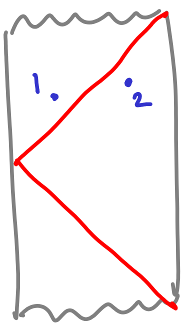
Definition



- BTZ: $$ds^2 = -\frac{r^2 - r_+^2}{\ell^2}\, dt^2 + \frac{\ell^2 dr^2}{r^2 - r_+^2} + r^2 d\varphi^2 \qquad (\mathrm{I})$$

- quotient : $\left.\begin{array}{l} X \to -X \\ \varphi \to \varphi + \pi \end{array}\right\} \Rightarrow \mathbb{RP}^2$ geon

- correlators obtained via method of images:



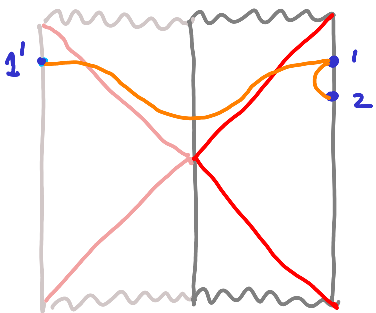
$$\langle \phi(P_1)\, \phi(P_2) \rangle_{\text{geon}} = \langle \phi(P_1)\, \phi(P_2) \rangle_{\text{BTZ}}$$

$$+ \langle \phi(P_1)\, \phi(P_2') \rangle_{\text{BTZ}}$$

- analyticity → geodesic approximation

Thermality

- $|\Psi_g\rangle \rightarrow$ pure state that thermalizes at late times



- late-time correlators

$$\langle O(t_1)\, O(t_2)\rangle_{geon} = \underbrace{\langle O(t_1)\, O(t_2)\rangle_{BTZ}}_{\text{thermal}}$$

$$+ \langle O(t_1')\, O(t_2)\rangle_{BTZ}$$

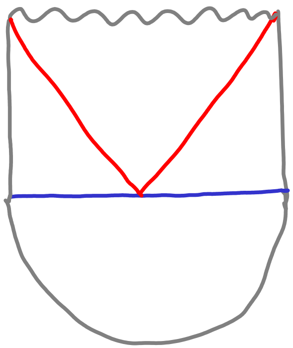
$$\propto e^{-(t_1+t_2)\Delta/\beta}$$

- for $t > t_* = \frac{\beta}{2\pi} \ln S_{BH}$ scrambling time supressed

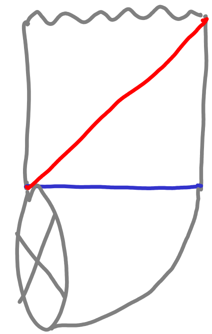
- geon correlators are not thermal for $t \simeq 0$

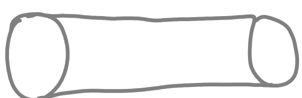
Path integral construction

- eternal BTZ



- $\mathbb{R}P^2$ geon



$Z_{CFT}\big[$  $\big]$

$\beta/2$

$Z_{CFT}\big[$  $\big]$

$\beta/4$

$$\Psi_{tfd} = \sum_E e^{-\frac{\beta E}{2}} |E\rangle |E\rangle$$

$$|\Psi_g\rangle = e^{-\frac{\beta H}{4}} |C\rangle$$

crosscap

Properties of the geon state

- cross cap : $\quad \left(L_n - (-1)^n \bar{L}_{-n}\right)|C\rangle = 0$

  $\rightarrow$ entangled state between LM & RM $\quad$ (in single CFT)

  $$|C\rangle = \sum_i C_{i,m_i} \, |i, m_i\rangle_L \, |i, m_i\rangle_R \implies |\psi_g\rangle \sim \sum_i e^{-\frac{\beta E_i}{4}} |i\rangle_L \, |i\rangle_R$$

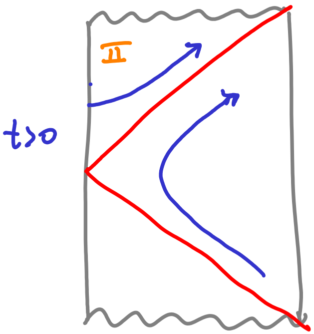
- Cardy growth of degeneracy @ high T 

  $$Z_K = \langle \Theta C | e^{-\frac{\beta H}{2}} | C\rangle = \sum_i e^{-\frac{\beta E_i}{2}} d_c(E_i) \sim e^{\pi\sqrt{\frac{cE}{3}}}$$

- entanglement structure $\quad A^\dagger(t,\varphi)|C\rangle = A(-t, \varphi+\pi)|C\rangle$

  $$\boxed{e^{-\frac{\beta H}{2}} \, O^\dagger(t,\varphi) \, e^{\frac{\beta H}{2}} |\psi_g\rangle = O(-t, \varphi+\pi)|\psi_g\rangle}$$

Mirror operators – method I



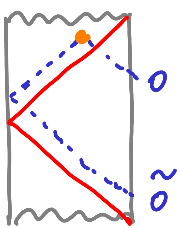
- $\Phi^{\mathrm{II}}_{\text{geon}}(t,r,\varphi) = \sum_m \int d\omega \left[\mathcal{O}_{\omega,m} \left(e^{-i\omega t + im\varphi} + (-1)^m e^{i\omega t + im\varphi} \right) K^{(1)}_{BTZ} + h.c \right]$

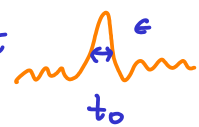
- mirror op : $\tilde{\mathcal{O}}^{g}_{\omega,m} = (-1)^m \mathcal{O}_{\omega,-m}$

$$\tilde{\mathcal{O}}_g(t,\varphi) = \mathcal{O}(-t, \varphi+\pi)$$

- how to distingush $\mathcal{O}$ & $\tilde{\mathcal{O}}$? → smearing

$$\Phi_{\omega_0,m_0}(t_0,r) = \int dt\, d\varphi \left[\xi^*_{\omega_0 t_0}(t)\, \Phi^\dagger(t,r,\varphi) + h.c \right]$$



$e^{-i\omega_0 t}$ 

$\omega_0 \gg \epsilon^{-1}, M$

Mirror operators – method II

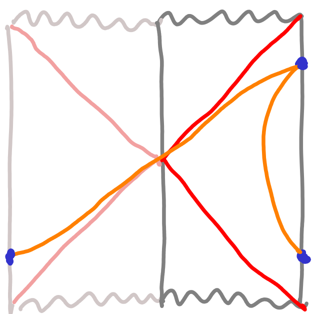
- PR conditions:
$$\begin{cases} \tilde{O}\,|\psi_g\rangle = e^{-\beta H/2}\, O\, e^{\beta H/2}\,|\psi_g\rangle \\ [\tilde{O}, O]\,|\psi_g\rangle = 0 \end{cases}$$

stays

bks dn

- HH-like state $d^g_{\omega,m}|\psi_g\rangle \propto \left(O_{\omega,m} - e^{-\beta\omega/2}(-1)^m O^+_{\omega,-m}\right)|\psi_g\rangle = 0$

↳ consistent w/ $\tilde{O} = O(-t, \varphi+\pi)$



- $\langle O\,\tilde{O}\rangle_{geon} = \underbrace{\langle O(t)\, O(-t)\rangle_{BTZ}}_{\sim e^{-t/\beta}} +$

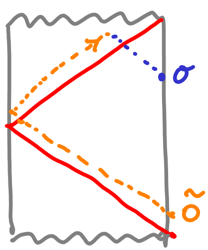
$+ \langle O(t)\,\tilde{O}(t)\rangle_{BTZ}$

↑ time-indep, $[O, \tilde{O}]_{BTZ} = 0$

- $\langle [O, \tilde{O}]\rangle_{geon} \sim e^{-t/\beta}$

Modifications of the geon state

- PR prescription → assumes smooth horizon → can one predict when horizon not smooth?
- prescription only applies to equilibrium states



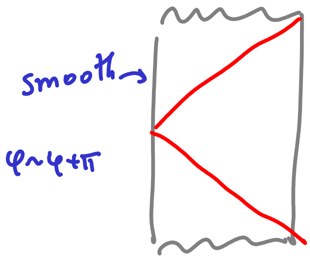
- $O|\psi_g\rangle$ → out of equilibrium
- $\tilde{O}|\psi_g\rangle \to \mathcal{H}$. should horizon still be smooth?
- $U(\tilde{O})|\psi_g\rangle$, e.g. $e^{i\alpha \tilde{O}^\dagger \tilde{O}}|\psi_g\rangle$

  $d^0_{wm}|\tilde{U}\psi_g\rangle \neq 0 \Rightarrow$ horizon not smooth?

- $e^{i\alpha O_w^\dagger O_w}|\psi_g\rangle$ → most perverse ↦ ambiguity of the PR proposal?

A simple example

GEON

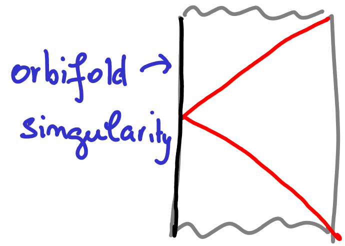


$$\psi_g = e^{-\frac{\beta H}{4}} |C\rangle$$

Another $\mathbb{Z}_2$ BTZ quotient

$X \to -X$

$\varphi \to \varphi$



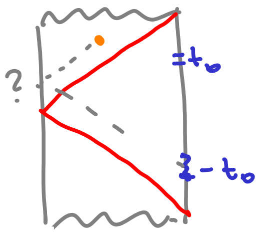
$$\psi_B = e^{-\frac{\beta H}{4}} |B\rangle$$

$|\psi_B\rangle = \mathcal{U}\, |\psi_g\rangle$, however : horizon is smooth!

- only interior changes

Future directions

- $|\Psi_g\rangle \sim \prod_{\omega_0, t_0} \exp\left[\alpha_{\omega_0} \mathcal{O}^{\dagger}_{\omega_0, t_0} \mathcal{O}^{\dagger}_{\omega_0, -t_0}\right] |0\rangle$ + unitary rot $\alpha_{\omega_0} \to e^{i\varepsilon}\alpha_{\omega_0}$



what happens to the geometry?

- back reaction $\tilde{\delta}|\Psi_g\rangle$

- more general geons, w/ non-trivial topology behind the horizon?

Thank you!