Ab Initio Holography

Sung-Sik Lee
McMaster University
Perimeter Institute

Can one construct holographic duals for general QFTs?

- Answer : Yes, in principle (Quantum RG)
- This talk : a concrete application

Outline

- Review of quantum renormalization group
- Holographic dual for the U(N) vector mode
- (non-) locality as holographic order parameter

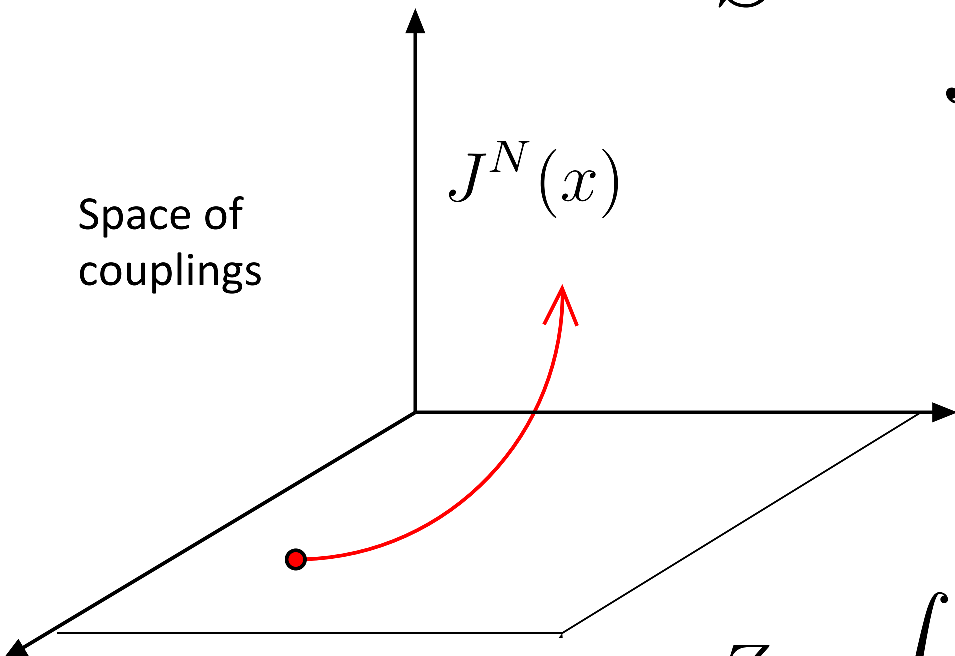
Holographic RG / Alternative approaches :

- E. T. Akhmedov, Phys. Lett. B 442 (1998) 152
- J. de Boer, E. Verlinde and H. Verlinde, J. High Energy Phys. 08, 003 (2000)
- K. Skenderis, Class. Quant. Grav. 19, 5849 (2002)
- S. R. Das and A. Jevicki, Phys. Rev. D 68 (2003) 044011.
- R. Gopakumar, Phys. Rev. D 70 (2004) 025009; ibid. 70 (2004) 025010.
- I. Heemskerk, J. Penedones, J. Polchinski and J. Sully, J. High Energy Phys. 10 (2009) 079.
- R. Koch, A. Jevicki, K. Jin and J. P. Rodrigues, arXiv:1008.0633.
- M. Douglas, L. Mazzucato, and S. Razamat, Phys. Rev. D 83 (2011) 071701.
- I. Heemskerk and J. Polchinski, arXiv:1010.1264
- T. Faulkner, H. Liu and M. Rangamani, arXiv:1010.4036.

Conventional (Classical) RG

$$S = \int dx J^N(x) O_N$$

Space of couplings



Beta function

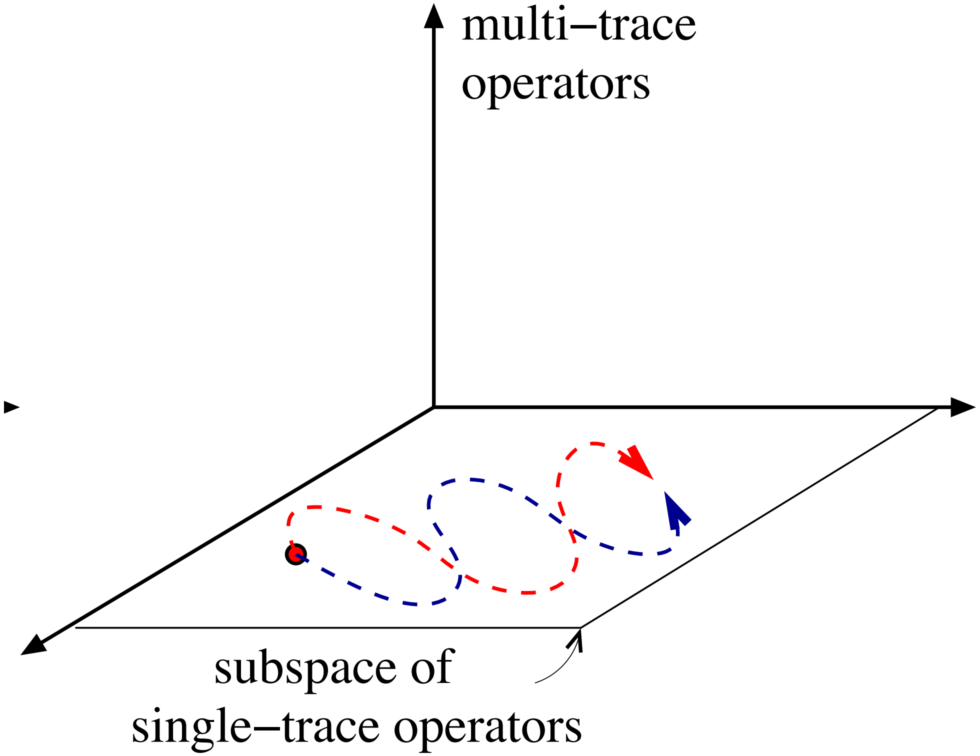
$$\frac{dJ^N(x,z)}{d\ z} = -\beta^N(x, J]$$

$$Z = \int Dj(x,z) Dp(x,z) \quad e^{iS^{D+1}[j,p]}$$

$$S^{D+1} = N^2 \int dz \int d^D x \left\{ p_N(\partial_z j^N) + \mathcal{L}_c(x;j] + \beta^N(x;j] p_N \right\}$$

[B. Dolan]

Quantum RG



- Only single-trace operators are included
- Generating function is given by a sum over all RG paths

Quantum RG

$$\Psi[J(x)] = \int D\phi \; e^{i \int dx \mathcal{L}}$$

$$\mathcal{L} = J^n(x) O_n$$

- O_n : a set of **single-trace operators** : a minimal set of operators where all other operators can be written as composite of O_n's

e.g. $\mathrm{tr}[\phi^n], \; \mathrm{tr}[\phi \partial_\mu \partial_\nu \phi], \; \mathrm{tr}[\phi(\partial_{\mu_1}\partial_{\mu_2}...\partial_{\mu_i}\phi)...(\partial_{\nu_1}\partial_{\nu_2}...\partial_{\nu_i}\phi)], ..$

Coarse graining

$$a \to a e^{dz}$$

$$\mathcal{L} \to \mathcal{L}' = \mathcal{L} + \delta\mathcal{L},$$

$$\delta\mathcal{L} = dz \left\{ \mathcal{L}_c[J,x) - \beta^n[J,x)O_n + G^{mn}[J,x)O_m O_n \right\}$$

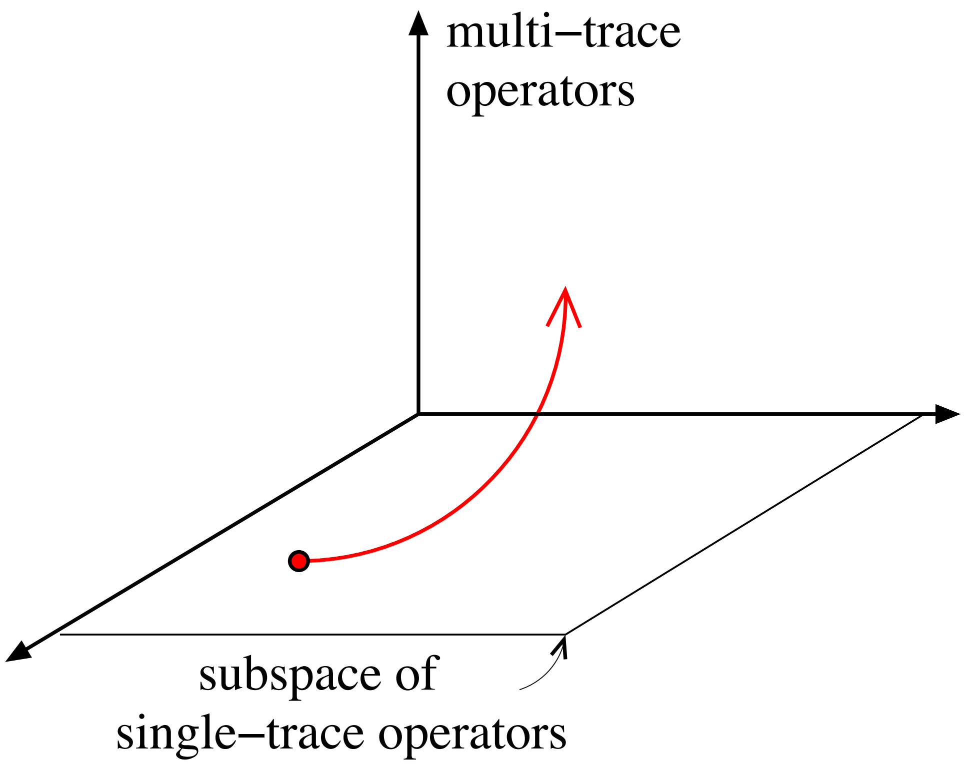
- Under coarse graining, the original theory is mapped into another theory
- Specifically, double-trace operators are generated out of single-trace operators to the linear order of dz

[Becchi, Guisto, Imbimbo (02)]

[Heemskerk and Polchinski, arXiv:1010.1264]

[Faulkner, Liu and Rangamani, arXiv :1010.4036.]

Conventional (Classical) RG



Beta function

$$\frac{dJ^{nm\dots}(x,z)}{d\ z} = -\beta[J^n, J^{nm}, \dots]$$

Dynamical source and operator fields

$$Z = \int D\Phi e^{i\int \mathcal{L}'}$$

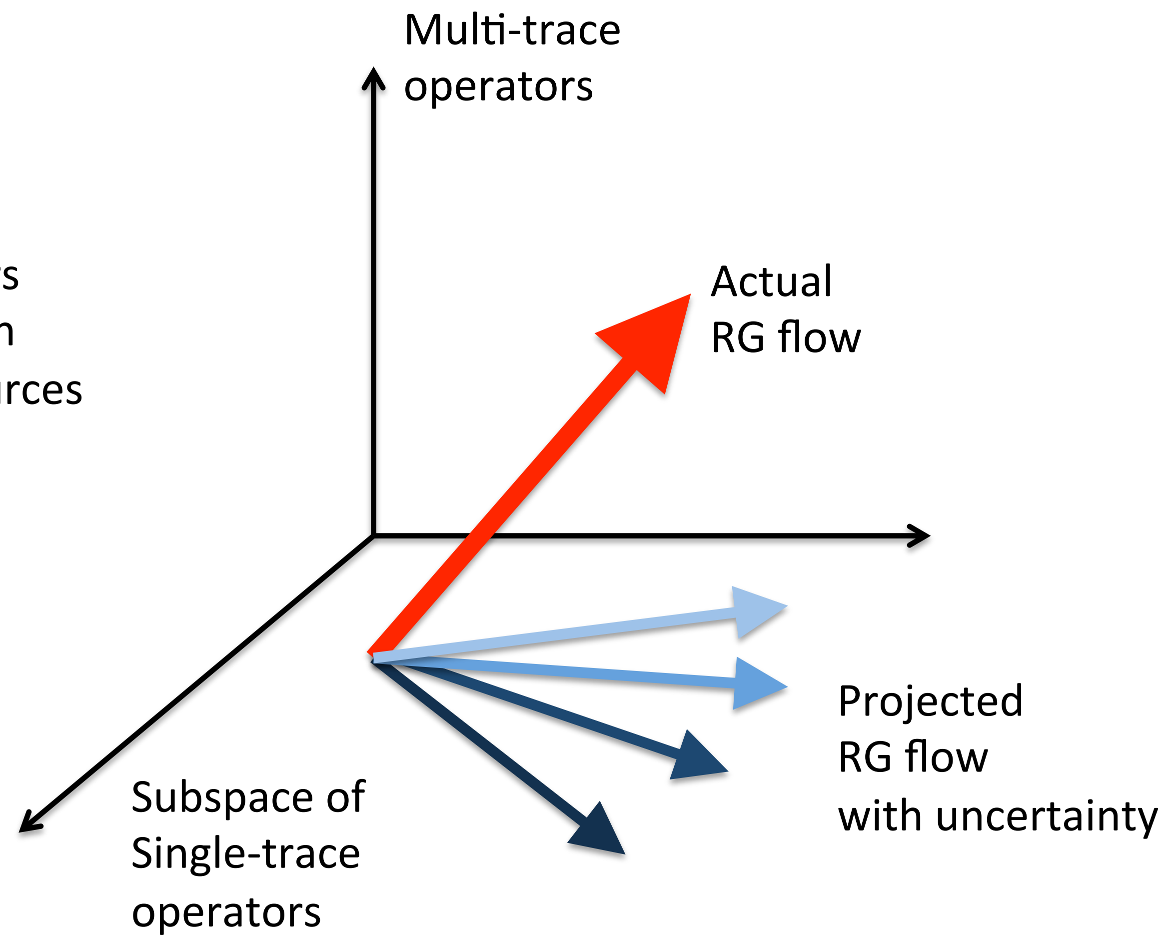
$$\mathcal{L}' = dz\mathcal{L}_c[J,x) + (J^n - dz\beta^n[J,x))\,O_n + dzG^{mn}[J,x)O_mO_n$$

$$Z = \int D\Phi D j^{(1)n} D p_n^{(1)} e^{i\int \mathcal{L}''}$$

$$\mathcal{L}'' = dz\mathcal{L}_c[J,x) + j^{(1)n}O_n + p_n^{(1)}(j^{(1)n} - J^n) + dz\beta^n[J,x)p_n^{(1)} + dzG^{mn}[J,x)p_m^{(1)}p_n^{(1)}$$

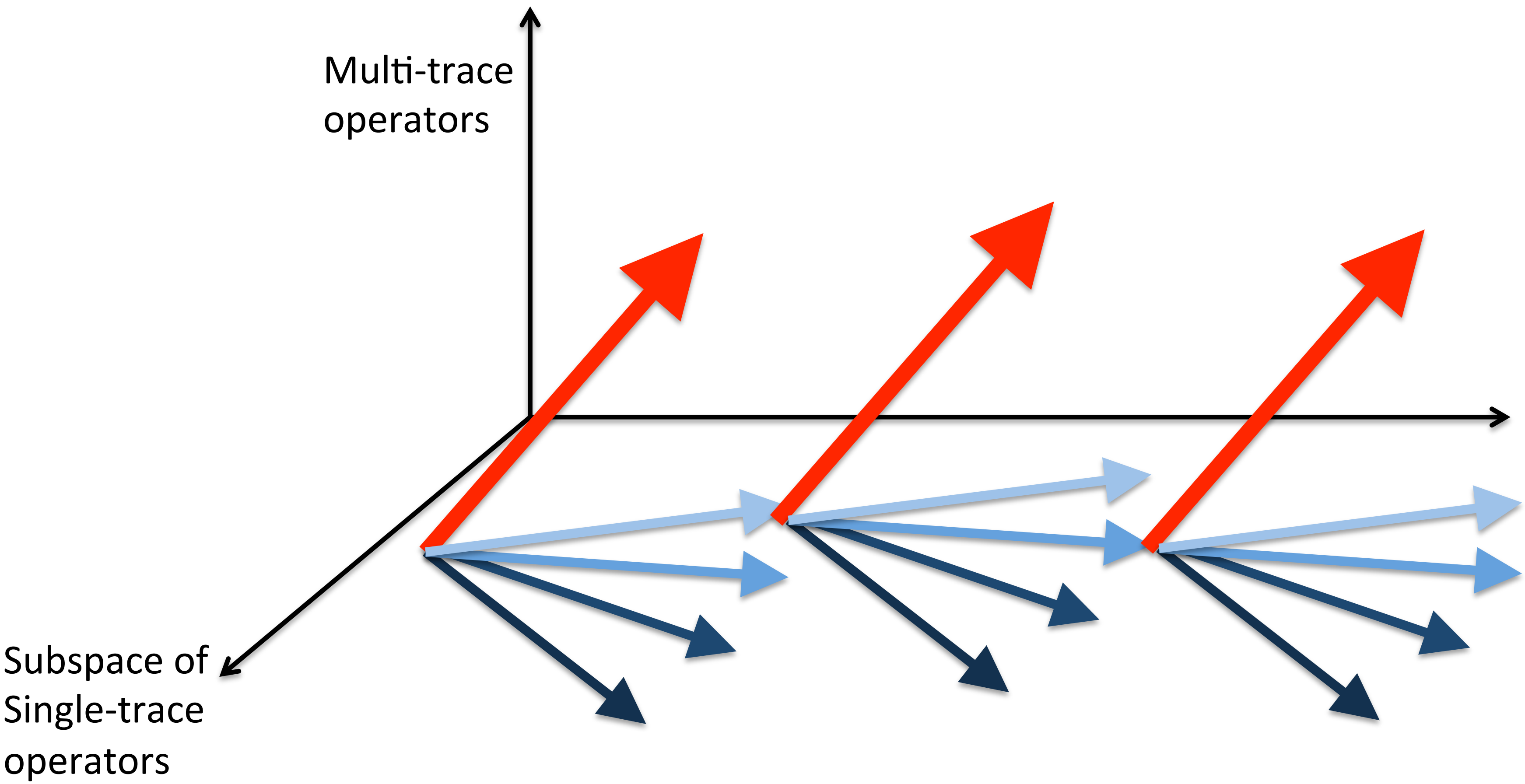
Quantum RG : projection creates uncertainty

- Theory with multi-trace operators can be mapped into a theory with single-trace operators whose sources are dynamical

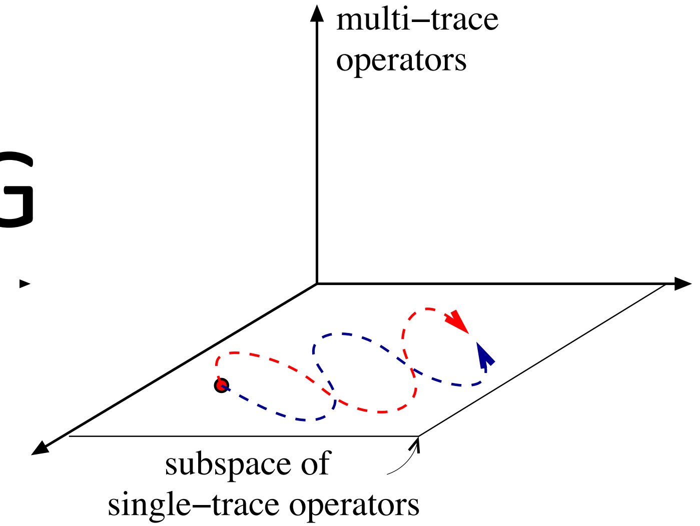


Quantum RG

- Iterate these steps
  1) Coarse grain starting from single-trace action
  2) Project to the subspace



Action for quantum RG



$$Z = \int Dj(x,z) Dp(x,z) \quad e^{iS^{D+1}[j,p]}$$

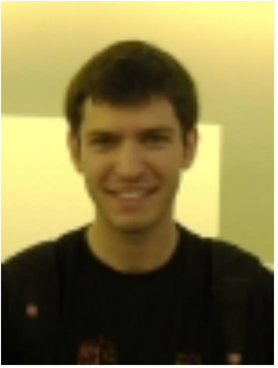
$$S^{D+1} = N^2 \int dz \int d^D x \left\{ p_n(\partial_z j^n) + \mathcal{L}_c(x;j] + \beta^m(x;j] p_m + \frac{G^{mn}(x;j]}{2} p_m p_n \right\}$$

- Casimir energy, beta functions on the subspace of single-trace operators completely specify the (D+1)-dimensional action
- The (D+1)-dimensional theory includes quantum gravity
- Freedom to choose different local RG prescription gives rise to the (D+1)-dimensional diffeomorphism [SL, (2012,2013)]
- In most cases, the bulk theory constructed from QRG is intractable due to a large number of bulk fields
- Concrete application ?

Vector model

$$\mathcal{S} = \int d^D x \left[|\nabla \vec{\phi}|^2 + m^2 |\vec{\phi}|^2 + \frac{\lambda}{N} (|\vec{\phi}|^2)^2 \right]$$

- Exactly solvable in the large N limit
- Believed to be dual to Vasiliev's higher-spin gauge theory [Polyakov-Klebanov, Giombi-Yin, ..]



Lattice regularization

[**Peter Lunts,** S. Bhattacharjee, E. Schnetter, Y.B. Kim, SL, to appear]

$$\mathcal{S}_0 = m^2 \sum_i (\phi_i^* \cdot \phi_i) + \frac{\lambda}{N} \sum_i (\phi_i^* \cdot \phi_i)^2 - \sum_{ij} t_{ij}^{(0)} \left(\phi_i^* \cdot \phi_j\right)$$

- U(N) vector model (N complex bosons)
- Goal
  - Derive holographic dual
  - Solve the bulk EOM without truncation

Related works :

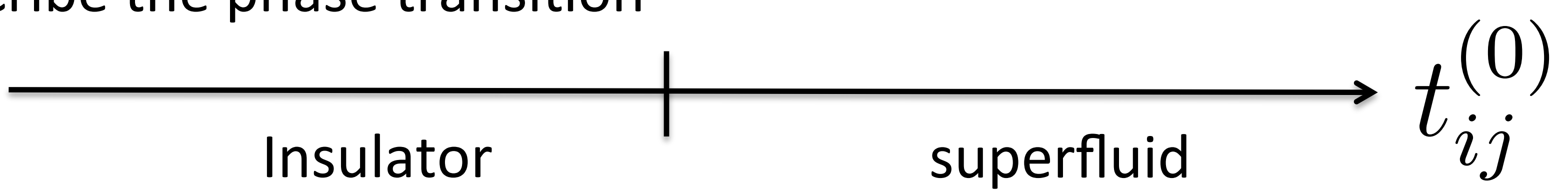
- S. R. Das and A. Jevicki, Phys. Rev. D 68 (2003) 044011.
- R. Koch, A. Jevicki, K. Jin and J. P. Rodrigues, arXiv:1008.0633.
- M. Douglas, L. Mazzucato, and S. Razamat, Phys. Rev. D 83 (2011) 071701.
- R. Leigh, O. Parrikar, A. Weiss, arXiv:1402.1430
- E. Mintun and J. Polchinski, arXiv:1411.3151

Real Space RG

$$\mathcal{S}_0 = m^2 \sum_i (\phi_i^* \cdot \phi_i) + \frac{\lambda}{N} \sum_i (\phi_i^* \cdot \phi_i)^2 - \sum_{ij} t_{ij}^{(0)} \left(\phi_i^* \cdot \phi_j\right)$$

Fixed point action — Deformation

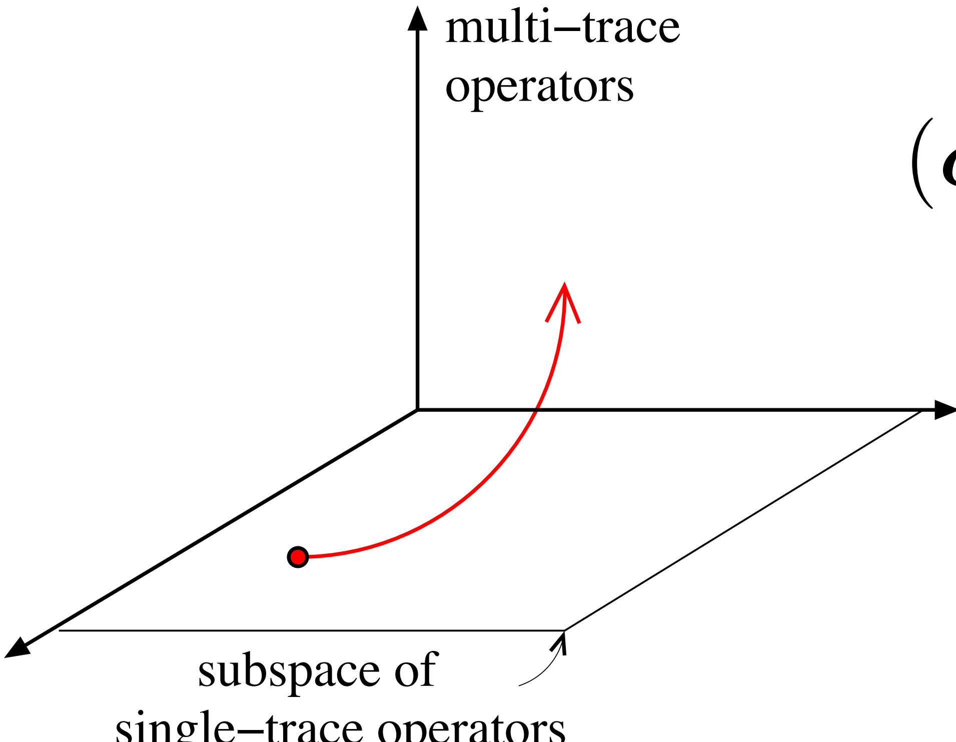
- The on-site term describes the insulating fixed point
- Hopping term is treated as deformation to the fixed point
- If the hopping is large, it flows to superfluid
- Although the quadratic (single-trace) term is enough to describe the Insulating fixed point, quartic (double-trace) term is needed to describe the phase transition



Renormalized Action

$$
\begin{aligned}
\tilde{\mathcal{S}}_1 \;=\; & 2Ndz\; \left\{ -\frac{1}{m^2}\sum_i t_{ii}^{(0)} \right\} \\
& +2dz\left\{ \frac{2\lambda\left(1+\frac{1}{N}\right)}{m^2}\sum_i \left(\boldsymbol{\phi}_i^* \cdot \boldsymbol{\phi}_i\right) - \frac{4\lambda^2}{m^2N^2}\sum_i \left(\boldsymbol{\phi}_i^* \cdot \boldsymbol{\phi}_i\right)^3 \right\} \\
& +2dz\left\{ \frac{2\lambda}{m^2N}\sum_{ij} t_{ij}^{(0)} \left(\boldsymbol{\phi}_i^* \cdot \boldsymbol{\phi}_j\right) \left\{ \left(\boldsymbol{\phi}_i^* \cdot \boldsymbol{\phi}_i\right) + \left(\boldsymbol{\phi}_j^* \cdot \boldsymbol{\phi}_j\right) \right\} \right\} \\
& +2dz\left\{ -\frac{1}{m^2}\sum_{ijk} t_{ik}^{(0)} t_{kj}^{(0)} \left(\boldsymbol{\phi}_i^* \cdot \boldsymbol{\phi}_j\right) - 2\frac{\lambda}{N}\sum_i \left(\boldsymbol{\phi}_i^* \cdot \boldsymbol{\phi}_i\right)^2 + \sum_{ij} t_{ij}^{(0)} \left(\boldsymbol{\phi}_i^* \cdot \boldsymbol{\phi}_j\right) \right\} \\
& -\sum_{ij} t_{ij}^{(0)} \left(\boldsymbol{\phi}_i^* \cdot \boldsymbol{\phi}_j\right) + m^2 \sum_i \left(\boldsymbol{\phi}_i^* \cdot \boldsymbol{\phi}_i\right) + \frac{\lambda}{N}\sum_i \left(\boldsymbol{\phi}_i^* \cdot \boldsymbol{\phi}_i\right)^2
\end{aligned}
$$

Conventional RG



$$\left(\phi_i^* \cdot \phi_j\right)$$

$$\left(\phi_i^* \cdot \phi_i\right)^n ,$$

$$\left(\phi_i^* \cdot \phi_i\right)^n \left(\phi_i^* \cdot \phi_j\right) , \ldots$$

- Multi-trace deformations are generated

Multi-trace operator = Single-trace operator with dynamical sources

$$\mathcal{Z} = \int \mathcal{D}\phi\, \mathcal{D}\phi^* \boxed{\mathcal{D}t_{ij}^{(1)}\ \mathcal{D}t_{ij}^{*(1)}} e^{-\mathcal{S}_2},$$

$$\begin{aligned}
\mathcal{S}_2 = N & \left\{ -\sum_{ij} \left[\left(t_{ij}^{(0)} - t_{ij}^{(1)}\right) t_{ij}^{*(1)} \right] \right\} \\
&+2N\alpha dz \left\{ -\frac{1}{m^2} \sum_i t_{ii}^{(0)} + \frac{2\lambda\left(1+\frac{1}{N}\right)}{m^2} \sum_i t_{ii}^{*(1)} - \frac{4\lambda^2}{m^2} \sum_i \left(t_{ii}^{*(1)}\right)^3 \right\} \\
&+2N\alpha dz \left\{ \frac{2\lambda}{m^2} \sum_{ij} t_{ij}^{(0)} t_{ij}^{*(1)} \left[t_{ii}^{*(1)} + t_{jj}^{*(1)} \right] - \frac{1}{m^2} \sum_{ijk} t_{ik}^{(0)} t_{kj}^{(0)} t_{ij}^{*(1)} - 2\lambda \sum_i \left(t_{ii}^{*(1)}\right)^2 + \sum_{ij} t_{ij}^{(0)}\ t_{ij}^{*(1)} \right\} \\
&\boxed{-\sum_{ij} t_{ij}^{(1)} (\phi_i^* \cdot \phi_j) + m^2 \sum_i (\phi_i^* \cdot \phi_i) + \frac{\lambda}{N} \sum_i (\phi_i^* \cdot \phi_i)^2}
\end{aligned}$$

Bulk Action

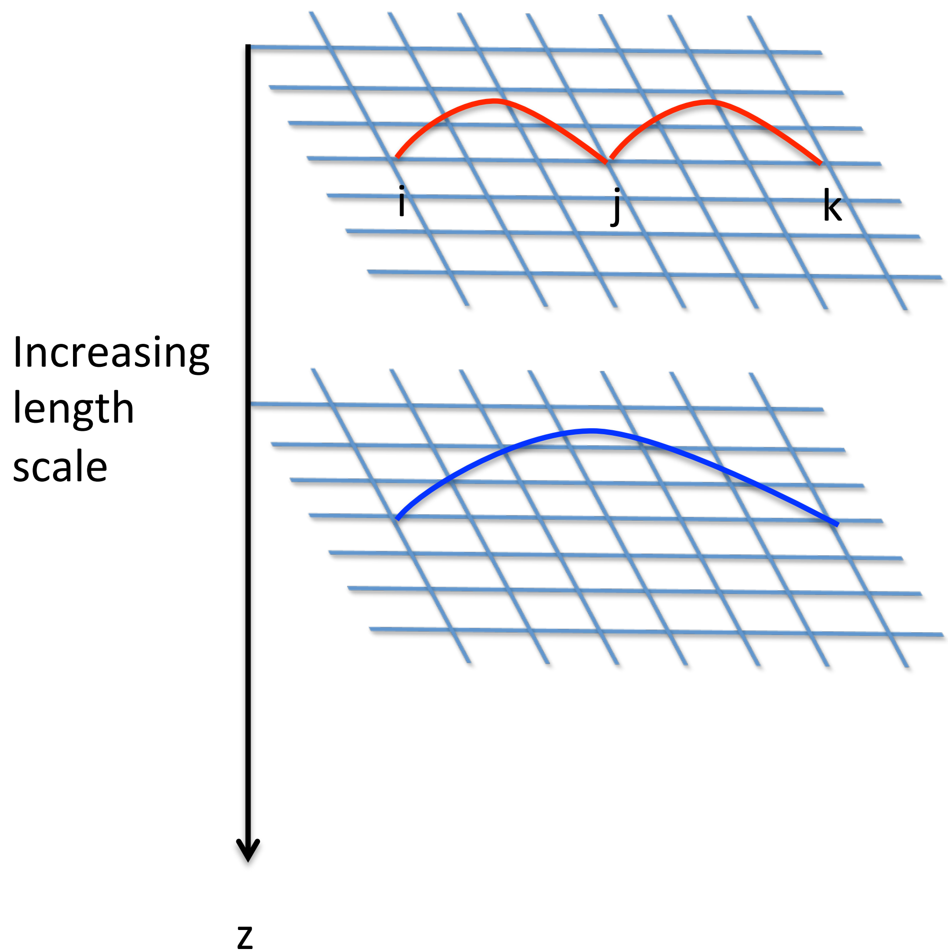
$$Z = \int Dt_{ij}(z) Dt^*_{ij}(z) e^{-NS_{bulk}[t,t^*]}$$

$$\begin{aligned}\mathcal{S}_{Bulk} \;=\; & \int_0^\infty dz \left\{ \sum_{ij} t^*_{ij}(z)\, \partial_z t_{ij}(z) \right. \\ & + \sum_i \left[-\frac{2}{m^2} t_{ii}(z) + \frac{4\lambda\left(1+\frac{1}{N}\right)}{m^2} t^*_{ii}(z) - 4\lambda \left(t^*_{ii}(z)\right)^2 - \frac{8\lambda^2}{m^2}\left(t^*_{ii}(z)\right)^3 \right] \\ & \left. + \sum_{ij} \left[2t_{ij}(z)\; t^*_{ij}(z) + \frac{4\lambda}{m^2} t_{ij}(z) t^*_{ij}(z) \left(t^*_{ii}(z) + t^*_{jj}(z)\right) \right] - \frac{2}{m^2} \sum_{ijk} \left[t_{ik}(z) t_{kj}(z) t^*_{ij}(z) \right] \right\}\end{aligned}$$

Continuum limit : $$t_{ij}(z) \rightarrow t(r_1, r_2, z)$$

$$t(r_1, r_2, z) \sim \sum_n \sum_{\mu_1, \mu_2, .., \mu_n} \left[(r_1 - r_2)_{\mu_1} (r_1 - r_2)_{\mu_2} .. (r_1 - r_2)_{\mu_n} \right] t_{\mu_1, \mu_2, .., \mu_n}((r_1 + r_2)/2, z)$$

Sources as quantum operator



$$t^\dagger_{ik} t_{ij} t_{jk}$$

- Long-range hoppings are generated as short-range hoppings merge

Saddle point approximation

$$\begin{aligned}\partial_z t_{ij} \quad &= -2\left\{\frac{2\lambda\,\delta_{ij}}{m^2} - \delta_{ij}\left[4\lambda + \frac{12\lambda^2}{m^2}t^*_{ii}\right]t^*_{ii} + \frac{2\lambda\,\delta_{ij}}{m^2}\sum_k\left(t_{ik}t^*_{ik} + t_{ki}t^*_{ki}\right)\right.\\ &\qquad \left. + \left[1 + \frac{2\lambda}{m^2}\left(t^*_{ii} + t^*_{jj}\right)\right]t_{ij} - \frac{1}{m^2}\sum_k t_{ik}t_{kj}\right\}\\ \partial_z t^*_{ij} \quad &= 2\left\{-\frac{\delta_{ij}}{m^2} + \left[1 + \frac{2\lambda}{m^2}\left(t^*_{ii} + t^*_{jj}\right)\right]t^*_{ij} - \frac{1}{m^2}\sum_k\left(t^*_{ik}t_{jk} + t_{ki}t^*_{kj}\right)\right\}\end{aligned}$$

- In the large N limit, semi-classical RG path dominates the partition function

Bulk equation of motion

- Saddle point solution determines the geometry on which fluctuations propagate in the bulk
- Kinetic term of bulk degrees of freedom is generated from the condensate, $<t_{ij}>$

$$t_{ij} =< t_{ij} > +\delta t_{ij}$$

$$t^*_{ik} t_{ij} t_{jk} \rightarrow \delta t^*_{ik} \delta t_{ij} < t_{jk} >$$

Gauge symmetry in the bulk

$$\mathcal{S}_{Bulk} = \int_0^\infty dz \Bigg\{ \sum_{ij} t^*_{ij}(z)\,\partial_z t_{ij}(z)$$

in the radial gauge

$$+ \sum_i \left[-\frac{2}{m^2} t_{ii}(z) + \frac{4\lambda\left(1+\frac{1}{N}\right)}{m^2} t^*_{ii}(z) - 4\lambda\left(t^*_{ii}(z)\right)^2 - \frac{8\lambda^2}{m^2}\left(t^*_{ii}(z)\right)^3 \right]$$

$$+ \sum_{ij}\left[2t_{ij}(z)\; t^*_{ij}(z) + \frac{4\lambda}{m^2} t_{ij}(z) t^*_{ij}(z)\left(t^*_{ii}(z) + t^*_{jj}(z)\right)\right] - \frac{2}{m^2}\sum_{ijk}\left[t_{ik}(z) t_{kj}(z) t^*_{ij}(z)\right] \Bigg\}$$

Higher spin symmetry :
(λ=0)

$$t_{ij}(z) \to \sum_k A^{-1}_{ii'} t_{i'j'}(z) A_{j'j},$$

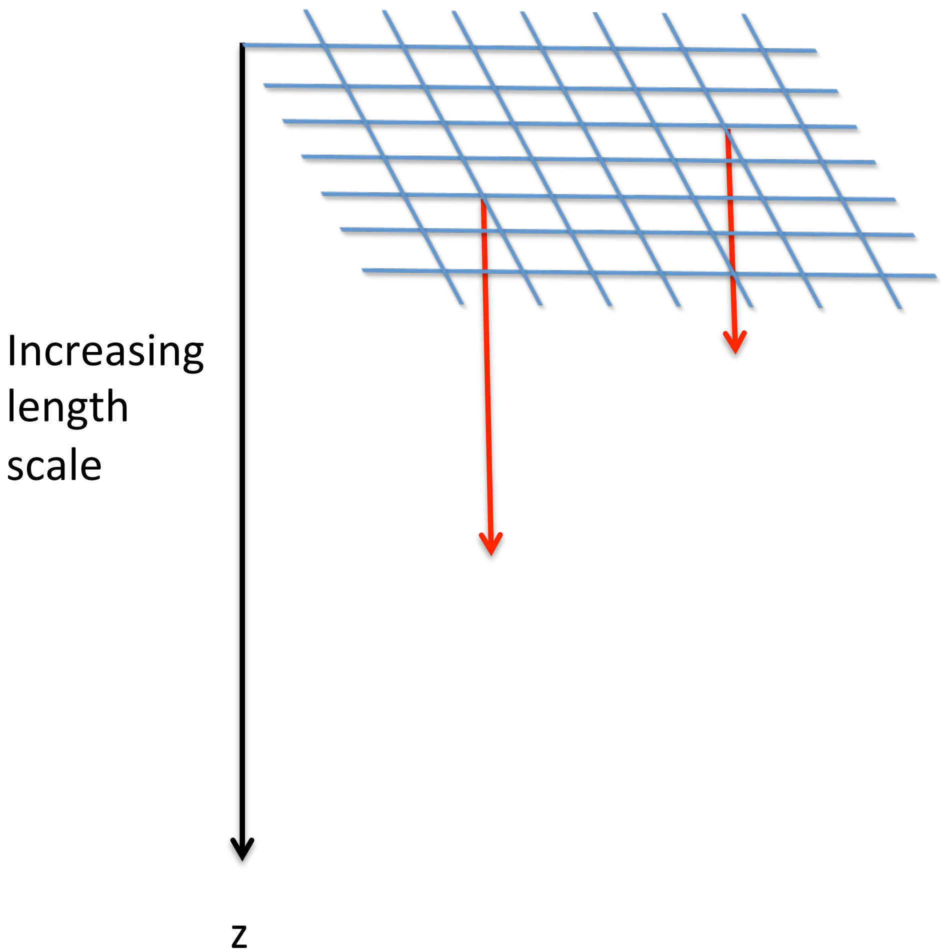
$$t^*_{ij}(z) \to \sum_k A^{-1}_{ii'} t^*_{i'j'}(z) A_{j'j}$$

[R. Leigh, O. Parrikar, A. Weiss, arXiv:1402.1430]

Only diffeomorphism survives :
for λ≠0

$$A_{ij} = \delta_{ij+N_i}$$

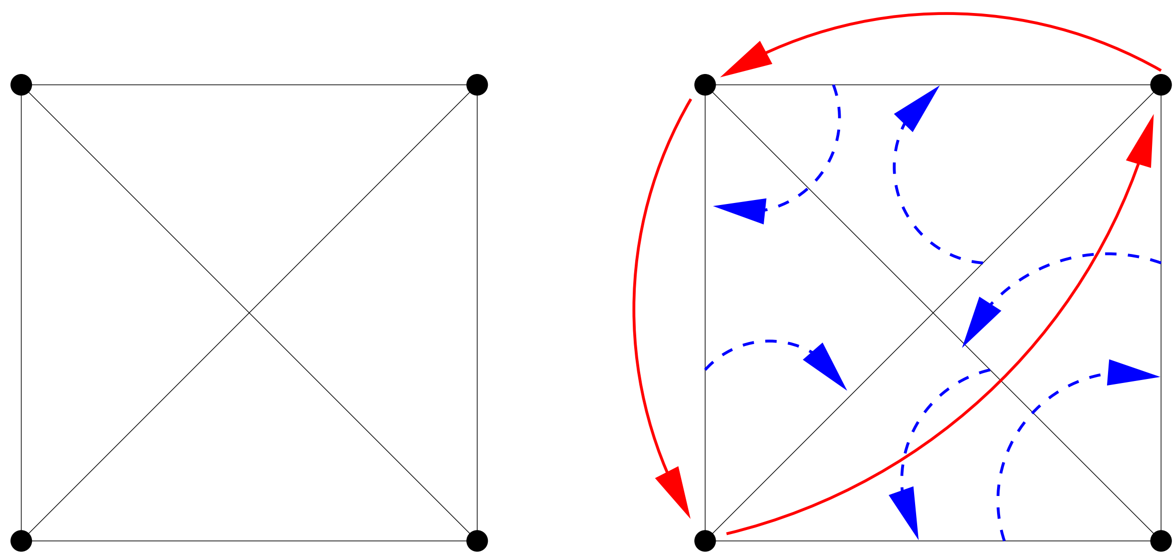
(D+1)-dim diffeomorphism inv.



spacetime dependent speed of RG
[Osborn(94); SL(12)]

(D+1)-dim diffeomorphism inv.

$$\mathcal{S}_0 = m^2 \sum_i (\phi_i^* \cdot \phi_i) + \frac{\lambda}{N} \sum_i (\phi_i^* \cdot \phi_i)^2 - \sum_{ij} t_{ij}^{(0)} \left(\phi_i^* \cdot \phi_j\right)$$



(a) (b)

Permutation (event symmetry) in D-dimensional network

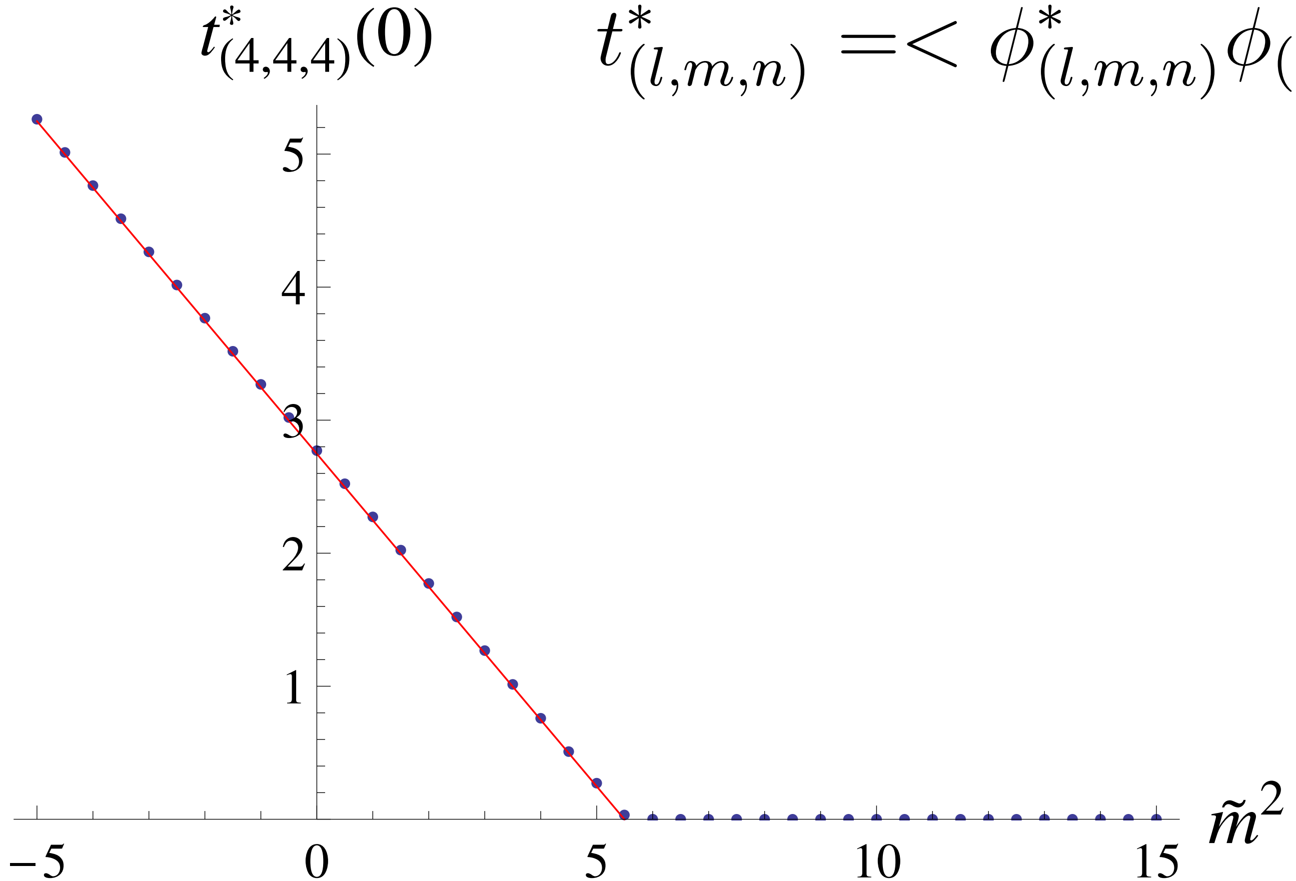
$$
\begin{aligned}
\partial_z t_{ij} &= -2\left\{ \tfrac{2\lambda\,\delta_{ij}}{m^2} - \delta_{ij}\left[4\lambda + \tfrac{12\lambda^2}{m^2} t^*_{ii}\right] t^*_{ii} + \tfrac{2\lambda\,\delta_{ij}}{m^2}\sum_k \left(t_{ik}t^*_{ik} + t_{ki}t^*_{ki}\right) \right. \\
&\qquad \left. + \left[1 + \tfrac{2\lambda}{m^2}\left(t^*_{ii} + t^*_{jj}\right)\right] t_{ij} - \tfrac{1}{m^2}\sum_k t_{ik}t_{kj} \right\} \\
\partial_z t^*_{ij} &= 2\left\{ -\tfrac{\delta_{ij}}{m^2} + \left[1 + \tfrac{2\lambda}{m^2}\left(t^*_{ii} + t^*_{jj}\right)\right] t^*_{ij} - \tfrac{1}{m^2}\sum_k \left(t^*_{ik}t_{jk} + t_{ki}t^*_{kj}\right) \right\}
\end{aligned}
$$

We fix gauge and solve the EOM numerically for D=3 (boundary dim)

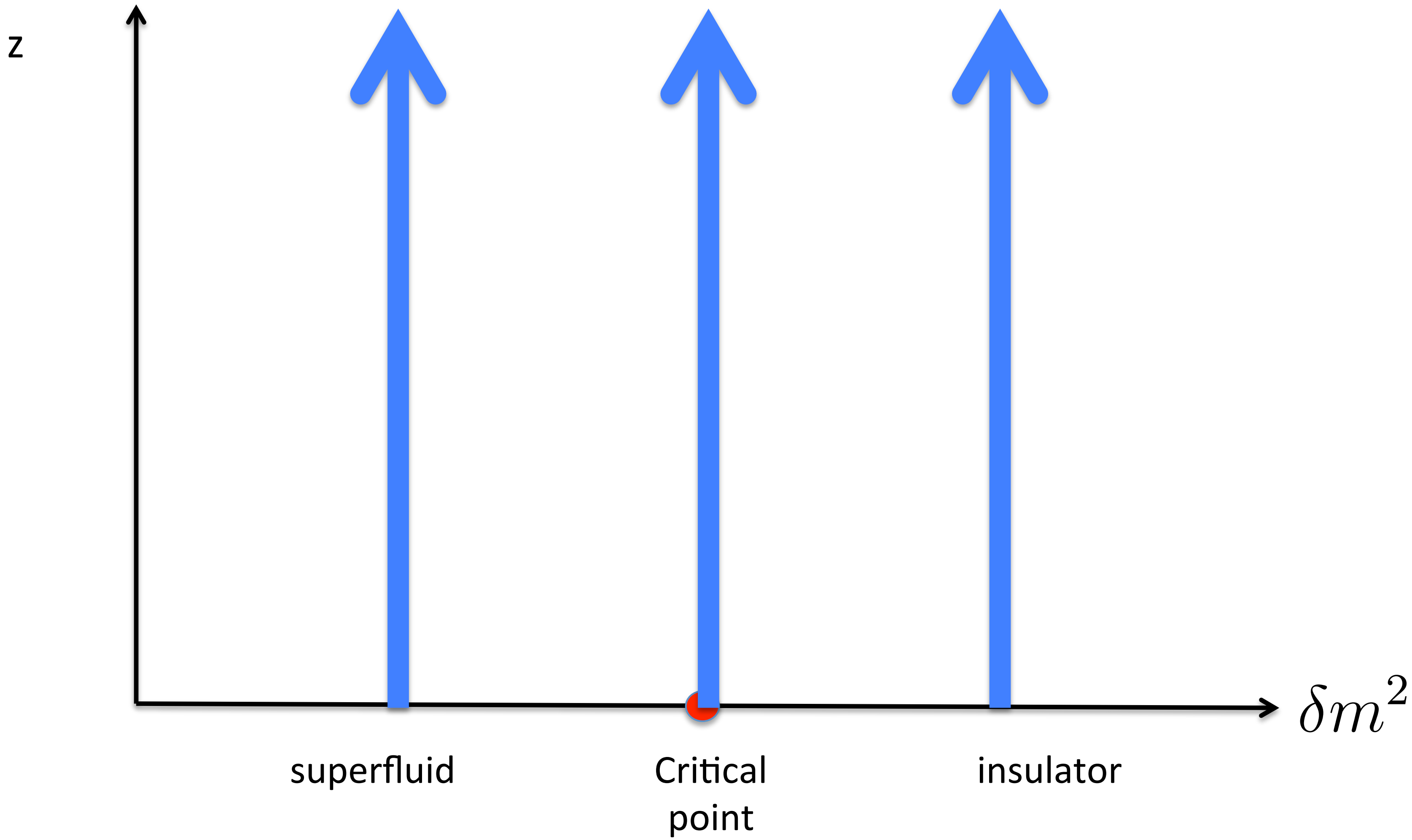
Field theory observable at the UV boundary

$$t^*_{(l,m,n)} = < \phi^*_{(l,m,n)} \phi_{(0,0,0)} >$$

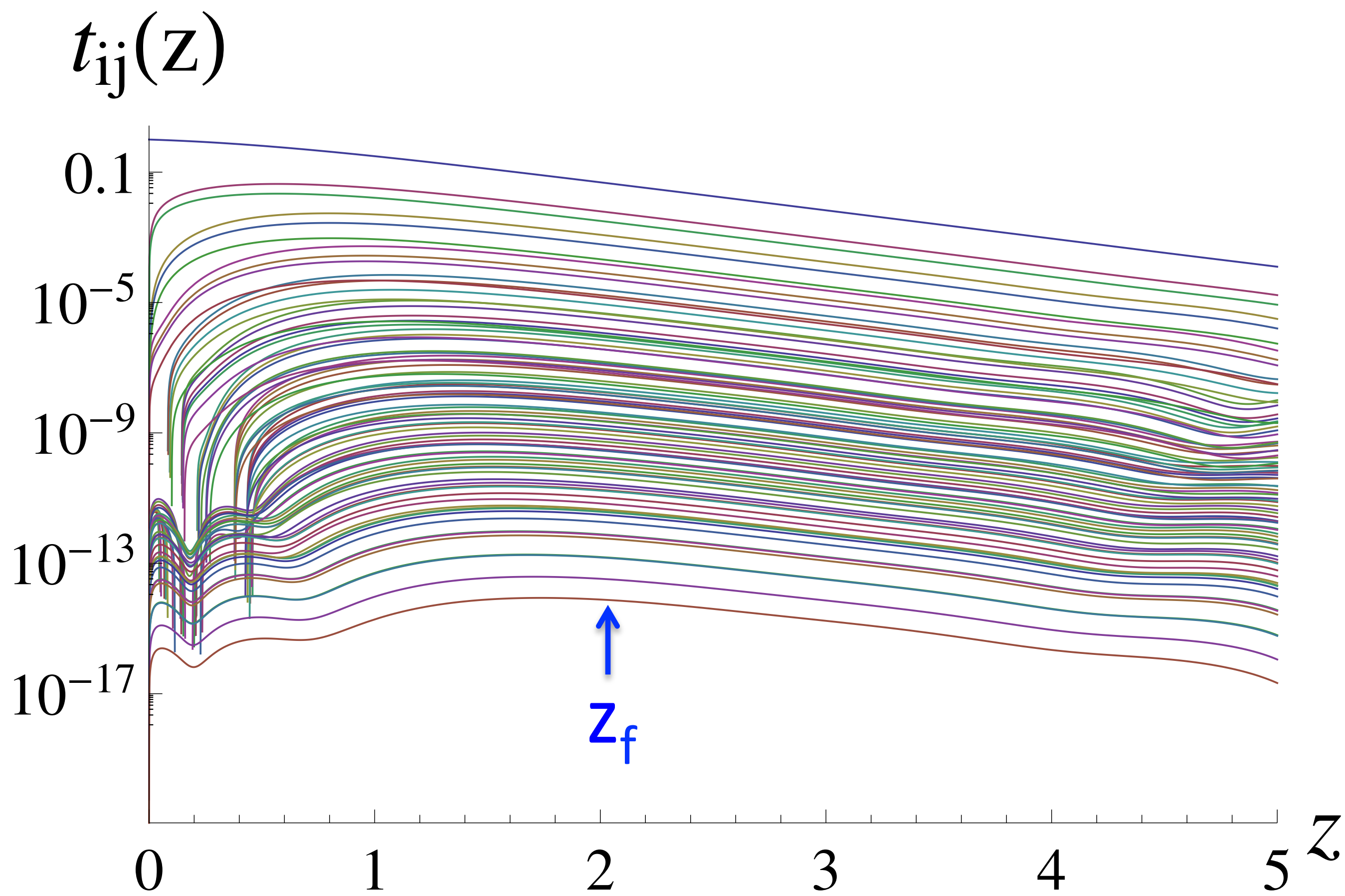
L=9



Bulk fields

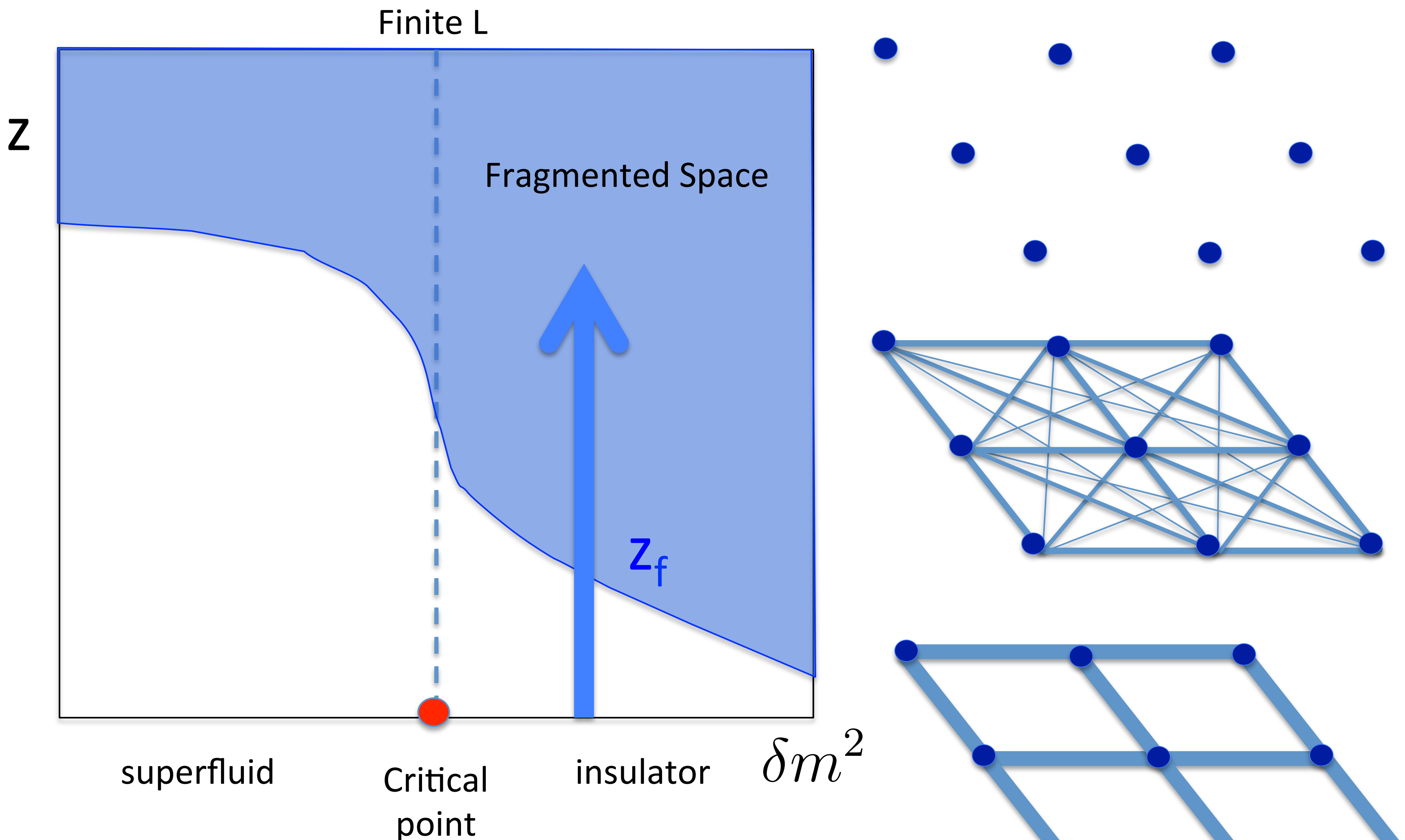


Mott Insulating Phase

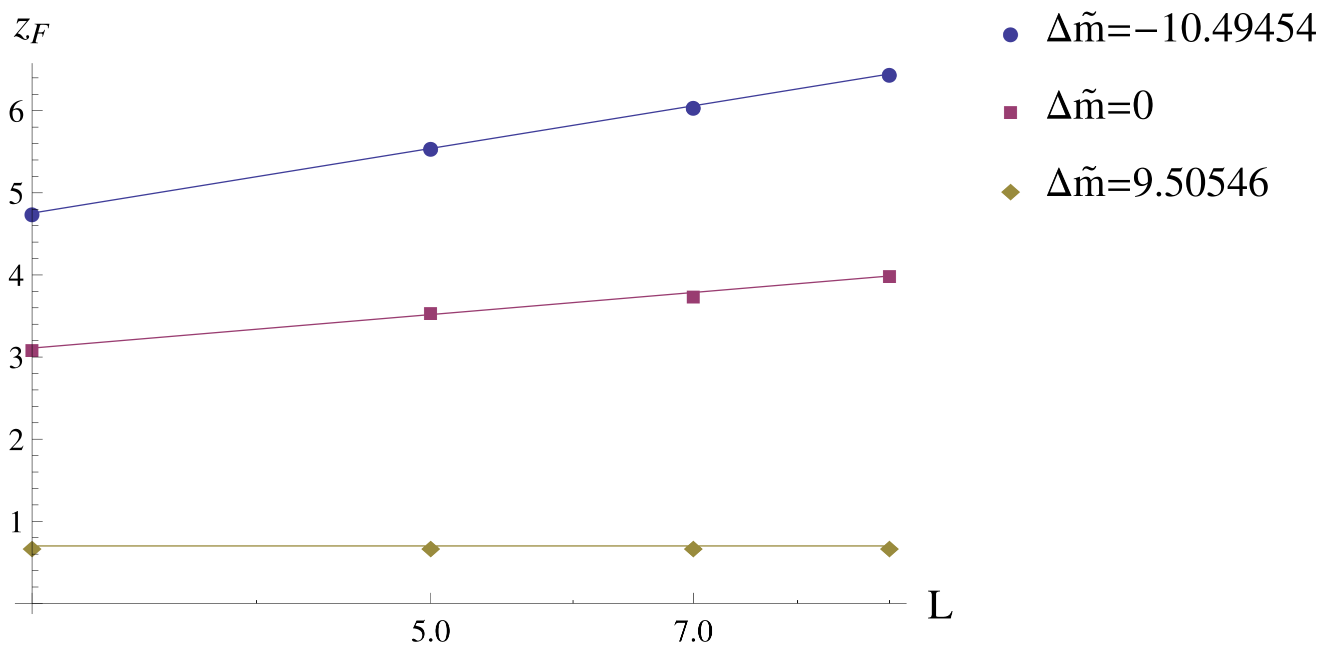


- $t_{ij}(z)$ decay exponentially both in $|i-j|$ and z
- For $z>z_f$, correlation length of $t_{ij}^*(z) \sim e^{-|i-j|/\xi}$ becomes much less than the lattice spacing (not a sharp transition)

Fragmentation in insulating phase

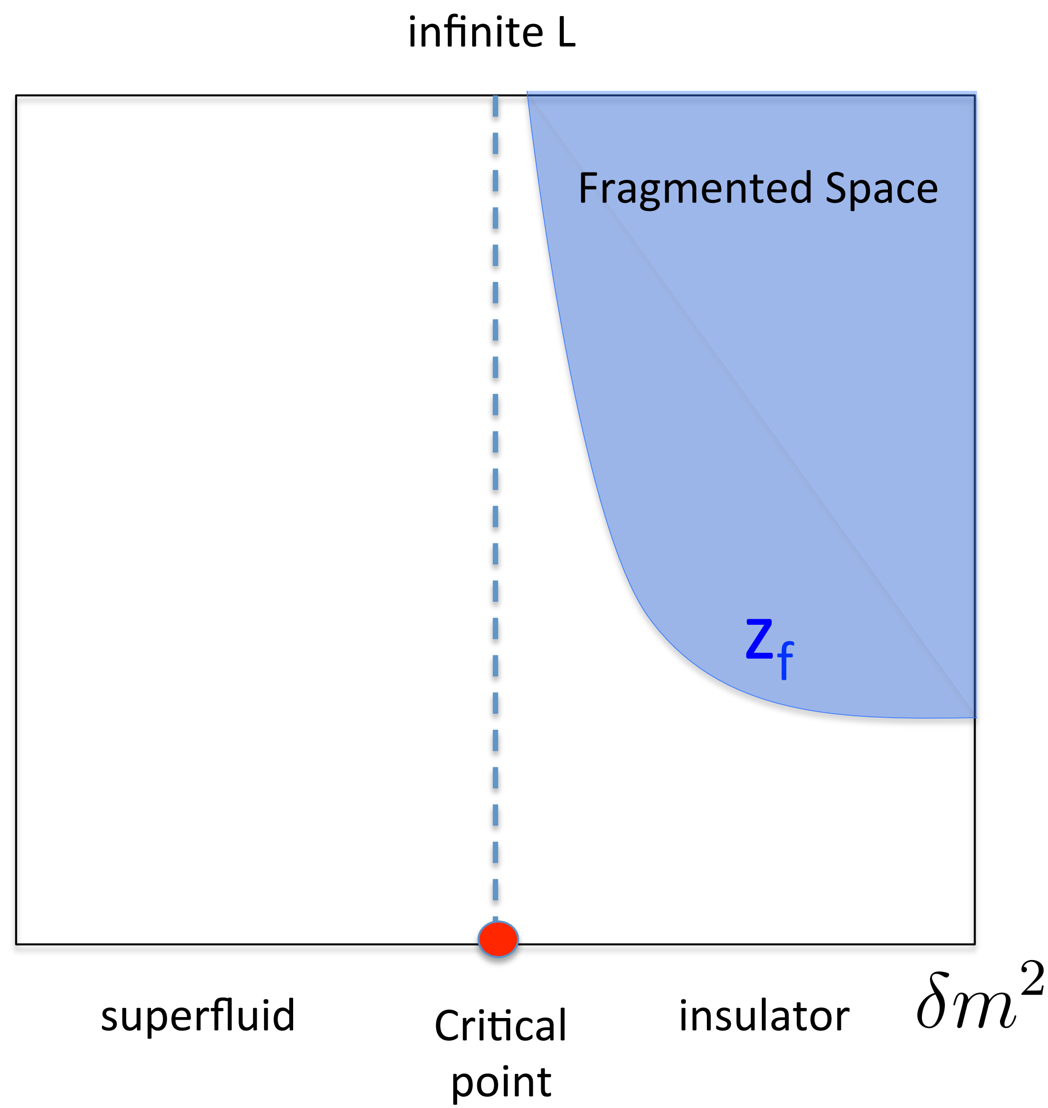


Fragmentation in superfluid phase is a finite size effect

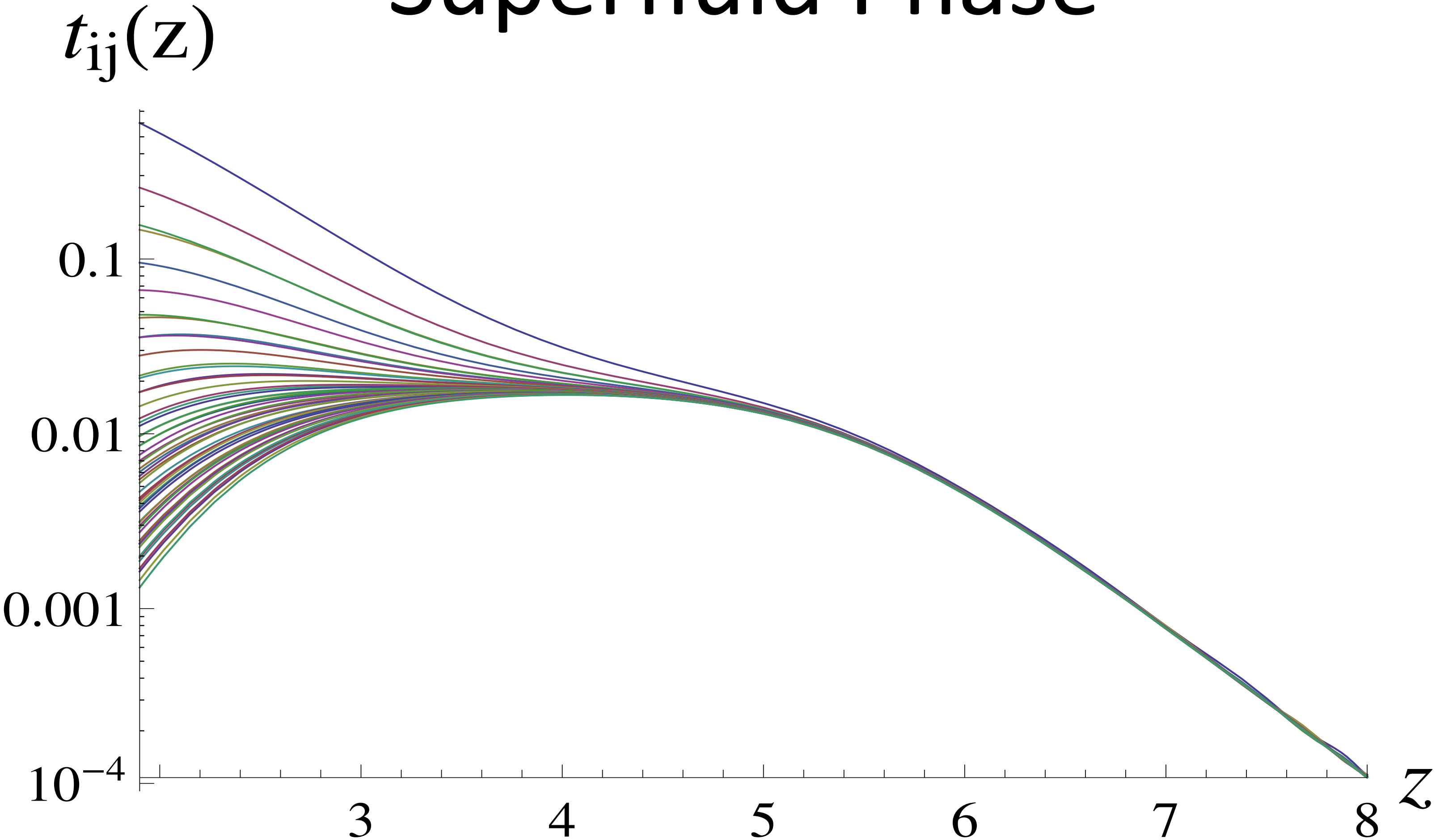


- In the insulating phase, z_F is independent of system size
- In the superfluid phase and at the critical point, z_F diverges in the thermodynamic limit

Fragmentation in the insulating phase

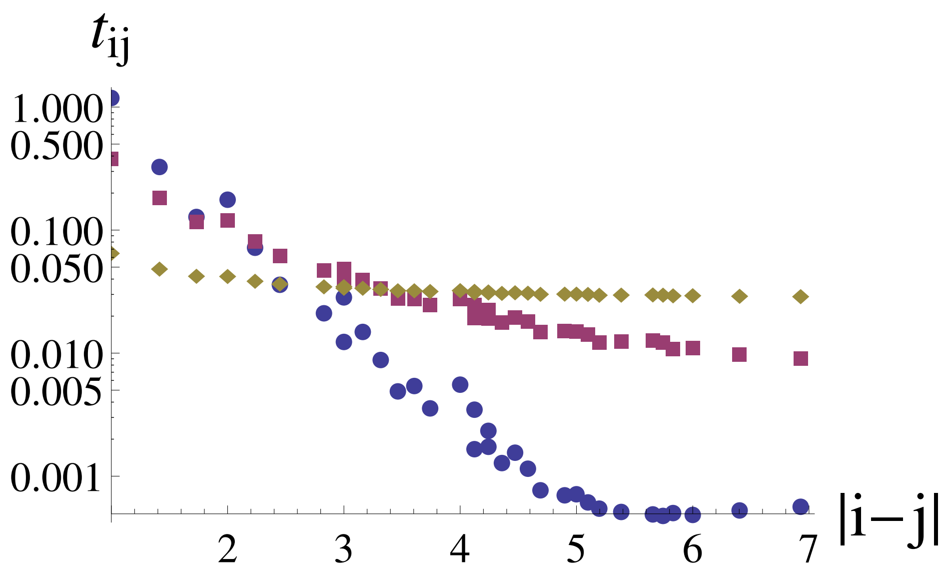


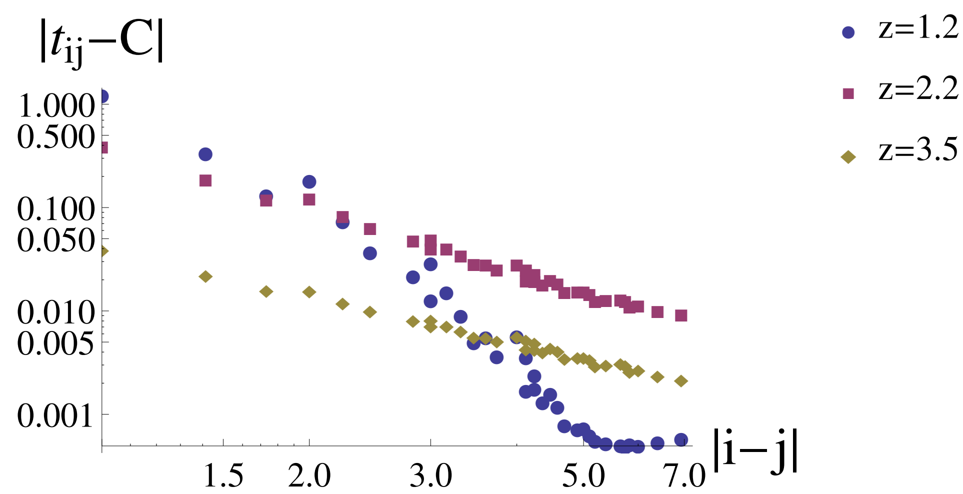
Superfluid Phase



- In the supefluid phase, locality is lost in the bulk

Superfluid Phase





$Z < Z_H$

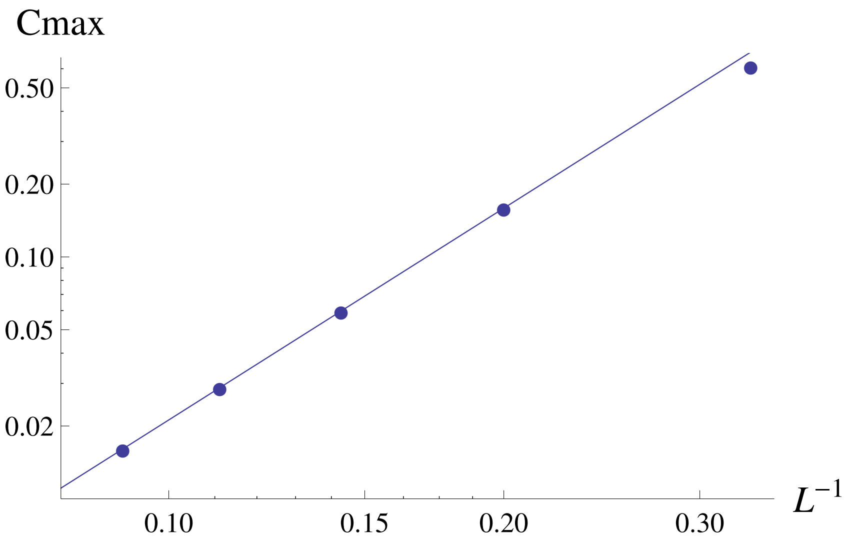
$$t_{ij}(z) \sim e^{-\psi(z)|i-j|}$$

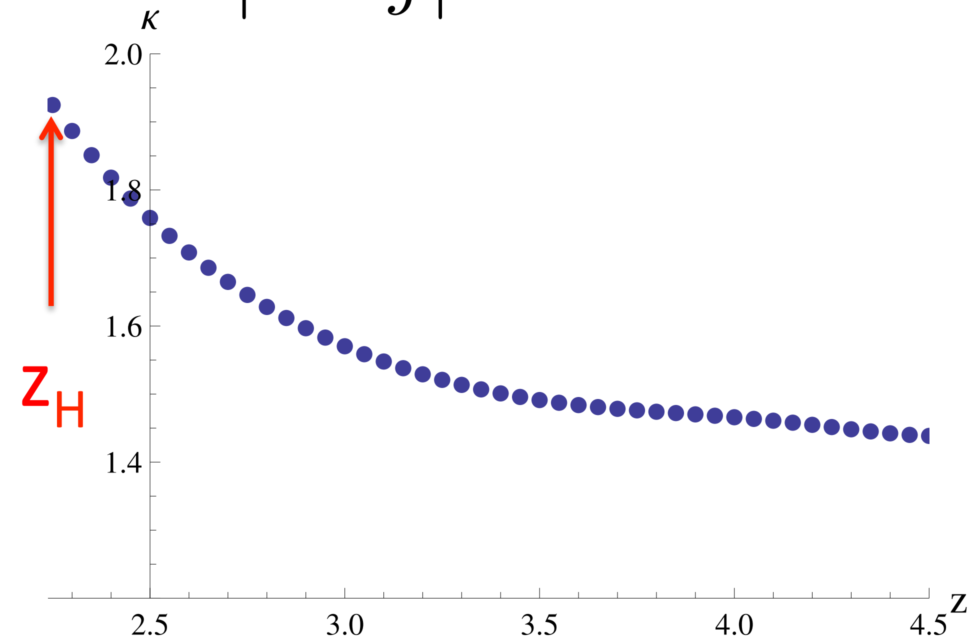
$Z \geq Z_H$

$$t_{ij}(z) = C(z) + \frac{B(z)}{|i-j|^{\kappa}}$$

Non-local geometry beyond horizon

$$t_{ij}(z) = C(z) + \frac{B(z)}{|i-j|^{\kappa}}$$

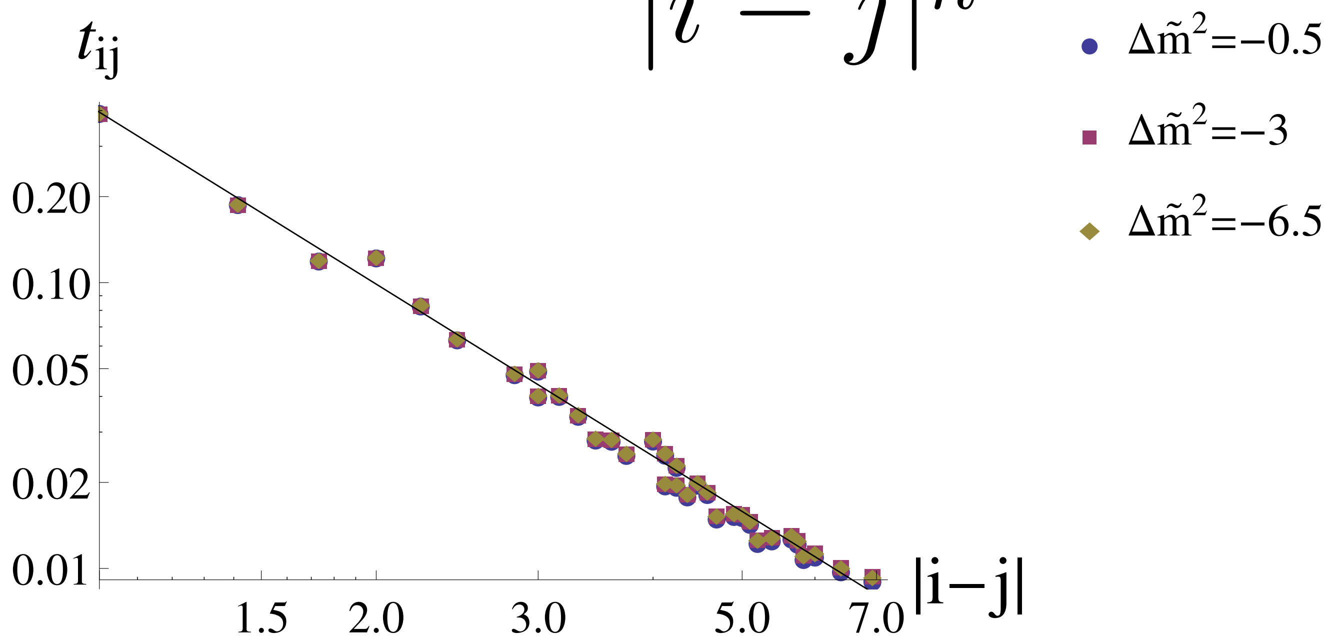




- C(z) vanishes in the thermodynamic limit
- What distiniguish the horizon and the region inside the horizon is the exponent

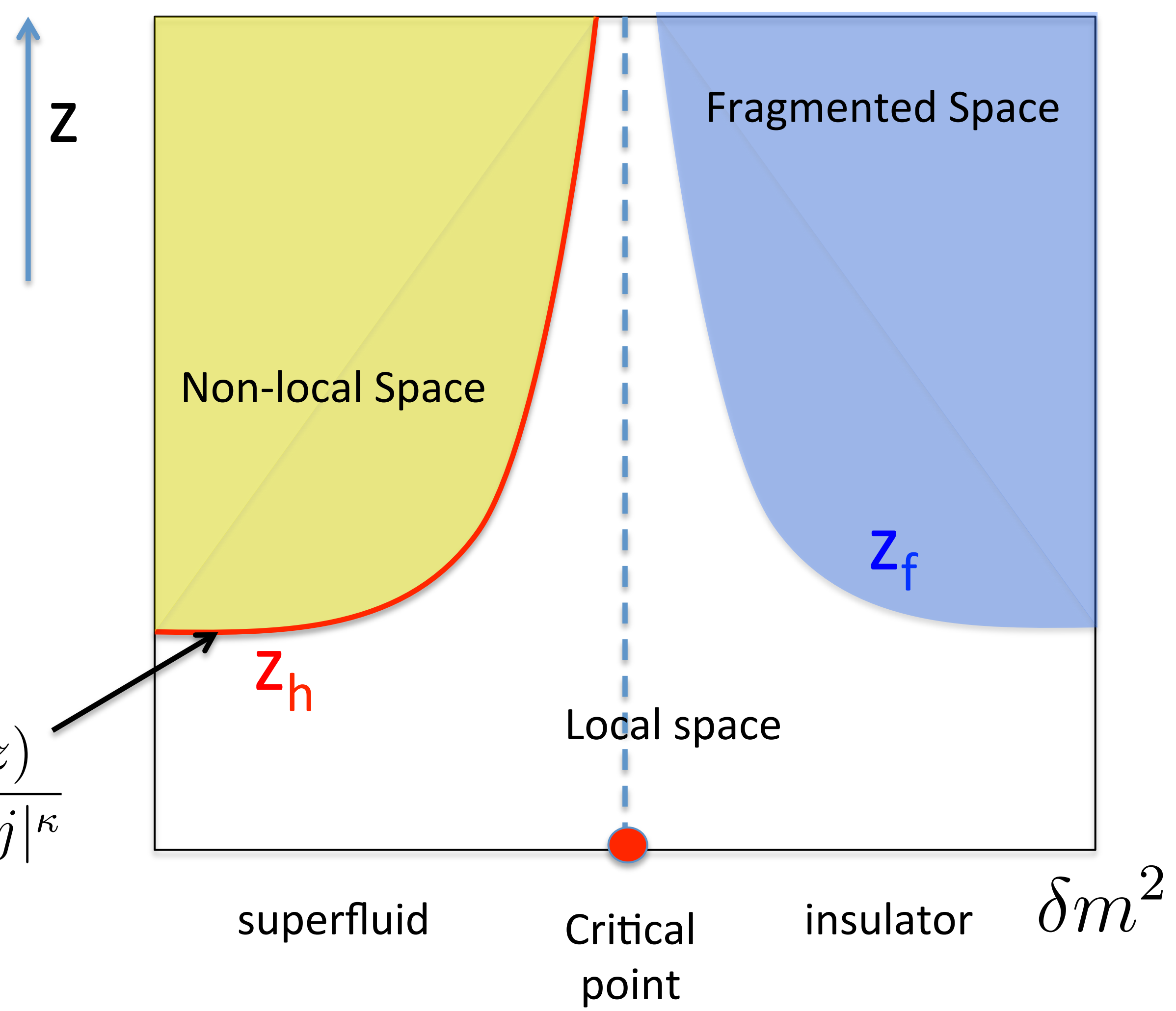
Universal power-law at the horizon

$$t_{ij}(z) = \frac{B(z)}{|i-j|^{\kappa}}$$



- At the horizon, the hopping fields decay with a universal exponent independent of δm^2

Holographic phase diagram



$$t_{ij}(z) = \frac{B(z)}{|i-j|^{\kappa}}$$

Summary

- Quantum RG = Quantum GR
- (Non-)Locality serves as holographic order parameter
  - Insulator : fragmented geometry
  - Superfluid : Non-locality geometry behind horizon
  - Critical point : IR region asymptotically approaches the horizon geometry