Electroweak Symmetry Breaking

Electroweak Effective Theory

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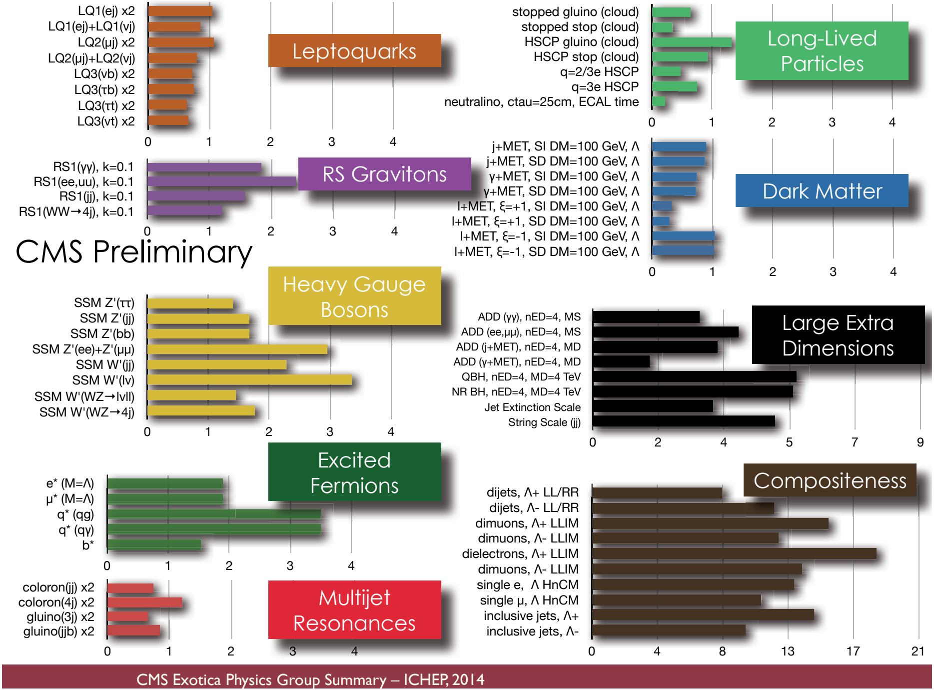
LAPP, Annecy-le-Vieux, 12-13 November 2014

Don Quixote and the Windmills


CMS Preliminary
Leptoquarks
LQ1(ej) x2
LQ1(ej)+LQ1(νj)
LQ2(μj) x2
LQ2(μj)+LQ2(νj)
LQ3(νb) x2
LQ3(τb) x2
LQ3(τt) x2
LQ3(νt) x2
RS Gravitons
RS1(γγ), k=0.1
RS1(ee,μμ), k=0.1
RS1(jj), k=0.1
RS1(WW→4j), k=0.1
Heavy Gauge Bosons
SSM Z'(ττ)
SSM Z'(jj)
SSM Z'(bb)
SSM Z'(ee)+Z'(μμ)
SSM W'(jj)
SSM W'(lν)
SSM W'(WZ→lνll)
SSM W'(WZ→4j)
Excited Fermions
e* (M=Λ)
μ* (M=Λ)
q* (qg)
q* (qγ)
b*
Multijet Resonances
coloron(jj) x2
coloron(4j) x2
gluino(3j) x2
gluino(jjb) x2
Long-Lived Particles
stopped gluino (cloud)
stopped stop (cloud)
HSCP gluino (cloud)
HSCP stop (cloud)
q=2/3e HSCP
q=3e HSCP
neutralino, ctau=25cm, ECAL time
Dark Matter
j+MET, SI DM=100 GeV, Λ
j+MET, SD DM=100 GeV, Λ
γ+MET, SI DM=100 GeV, Λ
γ+MET, SD DM=100 GeV, Λ
l+MET, ξ=+1, SI DM=100 GeV, Λ
l+MET, ξ=+1, SD DM=100 GeV, Λ
l+MET, ξ=-1, SI DM=100 GeV, Λ
l+MET, ξ=-1, SD DM=100 GeV, Λ
Large Extra Dimensions
ADD (γγ), nED=4, MS
ADD (ee,μμ), nED=4, MS
ADD (j+MET), nED=4, MD
ADD (γ+MET), nED=4, MD
QBH, nED=4, MD=4 TeV
NR BH, nED=4, MD=4 TeV
Jet Extinction Scale
String Scale (jj)
Compositeness
dijets, Λ+ LL/RR
dijets, Λ- LL/RR
dimuons, Λ+ LLIM
dimuons, Λ- LLIM
dielectrons, Λ+ LLIM
dimuons, Λ- LLIM
single e, Λ HnCM
single μ, Λ HnCM
inclusive jets, Λ+
inclusive jets, Λ-
CMS Exotica Physics Group Summary – ICHEP, 2014

Energy Scale	Fields	Effective Theory
$\Lambda_{NP} \sim$ TeV	S_n, P_n, V_n, A_n, F_n H, W, Z, γ, g τ, μ, e, ν_i t, b, c, s, d, u	Underlying Dynamics
······ Energy Gap ·····	↓	································
M_W	H, W, Z, γ, g τ, μ, e, ν_i t, b, c, s, d, u	Standard Model

Effective Field Theory

$$\mathcal{L}_{\rm eff} \; = \; \mathcal{L}^{(4)} \, + \, \sum_{D>4} \sum_i \, \frac{c_i^{(D)}}{\Lambda^{D-4}} \, \mathcal{O}_i^{(D)}$$

- **Most general Lagrangian with the SM gauge symmetries**
- **Light ($\mathbf{m \ll \Lambda_{NP}}$) fields only**
- **The SM Lagrangian corresponds to $\mathbf{D = 4}$**
- **$\mathbf{c_i^{(D)}}$ contain information on the underlying dynamics:**

$$\mathcal{L}_{\rm NP} \doteq g_x \, (\bar q_L \gamma^\mu q_L) \, X_\mu \qquad \longrightarrow \qquad \frac{g_x^2}{M_X^2} \, (\bar q_L \gamma^\mu q_L) \, (\bar q_L \gamma_\mu q_L)$$

- **Options for H(126):**
  - **$\mathbf{SU(2)_L}$ doublet (SM)**
  - **Scalar singlet**
  - **Additional light scalars**

Higgs Mechanism:

Gauge invariance

Massless $W^{\pm}$, Z (spin 1)

3×2 polarizations $= 6$

Higgs Mechanism: 3 additional degrees of freedom $\varphi_i(x)$

Gauge invariance

Massless $W^\pm$, Z (spin 1)

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$+$

3 Goldstones $\varphi_i(x)$

SSB ↓

Massive $W^\pm$, Z

3×3 polarizations $= 9$

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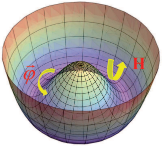
Spontaneous Symmetry Breaking

$$\mathcal{L}_\Phi = (D_\mu\Phi)^\dagger D^\mu\Phi - \mu^2\Phi^\dagger\Phi - \lambda\,(\Phi^\dagger\Phi)^2$$



$\mu^2 < 0$

$$\Phi(x) = \exp\left\{i\,\frac{\vec{\sigma}}{2}\cdot\vec{\varphi}(x)\right\}\,\frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ v + H(x) \end{bmatrix}$$

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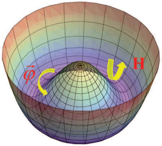

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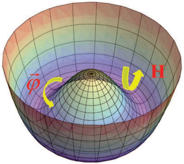
$$D_\mu\Phi = (\partial_\mu + \tfrac{i}{2}\,g\,\vec{\sigma}\cdot\vec{W}_\mu + \tfrac{i}{2}\,g'\,B_\mu)\,\Phi \quad ; \quad v^2 = -\mu^2/\lambda$$

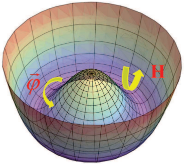
$$(D_\mu\Phi)^\dagger D^\mu\Phi \;\to\; M_W^2\,W_\mu^\dagger W^\mu + \frac{M_Z^2}{2}\,Z_\mu Z^\mu$$

$$\mathbf{M_W = M_Z\cos\theta_W = \frac{1}{2}\,g\,v}$$



$$\mathcal{L}_\Phi \;=\; (D_\mu\Phi)^\dagger D^\mu\Phi - \lambda\left(|\Phi|^2 - \frac{v^2}{2}\right)^2$$

$$\Sigma \equiv (\Phi^c, \Phi) = \begin{pmatrix} \Phi^{0*} & \Phi^+ \\ -\Phi^- & \Phi^0 \end{pmatrix}$$



$$\mathcal{L}_\Phi = (D_\mu\Phi)^\dagger D^\mu\Phi - \lambda\left(|\Phi|^2 - \frac{v^2}{2}\right)^2$$

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Custodial Symmetry

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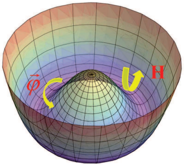

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$SU(2)_L \otimes SU(2)_R \to SU(2)_{L+R}$ **Symmetry:** $\Sigma \to g_L\,\Sigma\,g_R^\dagger$

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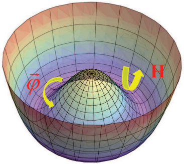
$$\Sigma \equiv (\Phi^c, \Phi) = \begin{pmatrix} \Phi^{0*} & \Phi^+ \\ -\Phi^- & \Phi^0 \end{pmatrix} \equiv \frac{1}{\sqrt{2}} (v + H)\; U(\vec{\varphi}\,)$$

$$U(\vec{\varphi}\,) \equiv \exp\left\{ i\,\vec{\sigma}\cdot\frac{\vec{\varphi}}{v} \right\}$$



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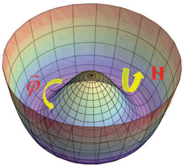

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Same Goldstone Lagrangian as QCD pions:

$$f_\pi \to v \qquad , \qquad \vec{\pi} \to \vec{\varphi} \to W_L^\pm, Z_L$$

EFFECTIVE LAGRANGIAN: $\mathcal{L}(U) = \sum_n \mathcal{L}_{2n}$

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$\langle 0|\, \bar{q}^j_L q^i_R \,|0\rangle$ **(QCD)**, Φ **(SM)** ➡ $U_{ij}(\phi) = \{\exp\left(i\vec{\sigma}\cdot\vec{\varphi}/f\right)\}_{ij}$

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Derivative Coupling

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Derivative Coupling

Goldstones become free at zero momenta

Electroweak Symmetry Breaking

$$\mathcal{L}_2 = \frac{v^2}{4}\,\mathrm{Tr}\left(D_\mu U^\dagger D^\mu U\right) \quad \xrightarrow{U=1} \quad \mathcal{L}_2 = M_W^2\, W_\mu^\dagger W^\mu + \frac{1}{2} M_Z^2\, Z_\mu Z^\mu$$

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$$\hat{W}^{\mu\nu} = \partial^\mu \hat{W}^\nu - \partial^\nu \hat{W}^\mu - i\,[\hat{W}^\mu, \hat{W}^\nu] \quad , \quad \hat{B}^{\mu\nu} = \partial^\mu \hat{B}^\nu - \partial^\nu \hat{B}^\mu - i\,[\hat{B}^\mu, \hat{B}^\nu]$$

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(explicit symmetry breaking)

Electroweak Symmetry Breaking

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- **EW Goldstones are responsible for $M_{W,Z}$ (not the Higgs!)**
- **QCD pions also generate small W, Z masses:** $\delta_\pi M_W = \frac{1}{2}\, g\, f_\pi$

Goldstone interactions are determined by the underlying symmetry

$$\frac{v^2}{4}\langle\partial_\mu U^\dagger\partial^\mu U\rangle = \partial_\mu\varphi^-\partial^\mu\varphi^+ + \frac{1}{2}\,\partial_\mu\varphi^0\partial^\mu\varphi^0$$

$$+ \frac{1}{6v^2}\left\{\left(\varphi^+\overleftrightarrow{\partial}_\mu\,\varphi^-\right)\left(\varphi^+\overleftrightarrow{\partial}^\mu\varphi^-\right) + 2\left(\varphi^0\overleftrightarrow{\partial}_\mu\,\varphi^+\right)\left(\varphi^-\overleftrightarrow{\partial}^\mu\varphi^0\right)\right\}$$

$$+ O\left(\varphi^6/v^4\right)$$

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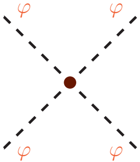

$$T\left(\varphi^+\varphi^- \to \varphi^+\varphi^-\right) \;=\; \frac{s+t}{v^2}$$

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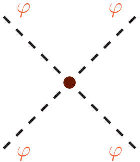
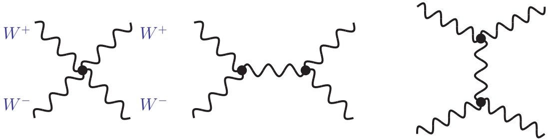


$$T\left(\varphi^+\varphi^-\to\varphi^+\varphi^-\right) = \frac{s+t}{v^2}$$

Non-Linear Lagrangian: $2\varphi\to 2\varphi,\ 4\varphi\ \cdots$ related

Equivalence Theorem



Cornwall–Levin–Tiktopoulos

Vayonakis

Lee–Quigg–Thacker

$$T(W_L^+W_L^- \to W_L^+W_L^-) = \frac{s+t}{v^2} + O\left(\frac{M_W}{\sqrt{s}}\right)$$
$$= T(\varphi^+\varphi^- \to \varphi^+\varphi^-) + O\left(\frac{M_W}{\sqrt{s}}\right)$$

The scattering amplitude grows with energy

Goldstone dynamics ⟷ **derivative interactions**

Tree-level violation of unitarity

Longitudinal Polarizations

$$k^\mu = \left(k^0, 0, 0, |\vec{k}|\right) \quad \longrightarrow \quad \epsilon_L^\mu(\vec{k}) = \frac{1}{M_W}\,\left(|\vec{k}|, 0, 0, k^0\right) \;=\; \frac{k^\mu}{M_W} + O\left(\frac{M_W}{|\vec{k}|}\right)$$

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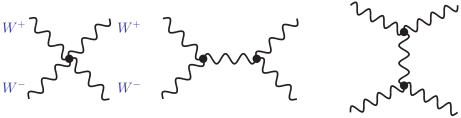
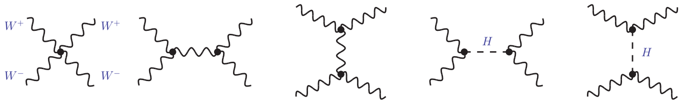


Gauge Cancelation

$$T(W_L^+ W_L^- \to W_L^+ W_L^-) = \frac{s+t}{v^2} + O\left(\frac{M_W}{\sqrt{s}}\right)$$

$$= T(\varphi^+ \varphi^- \to \varphi^+ \varphi^-) + O\left(\frac{M_W}{\sqrt{s}}\right)$$

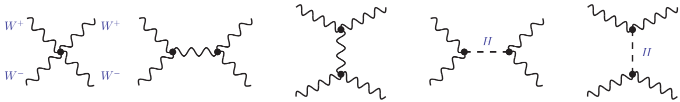
$W_L^+ W_L^- \to W_L^+ W_L^-$:



$$T_{\rm SM} = \frac{1}{v^2}\left\{s+t-\frac{s^2}{s-M_H^2}-\frac{t^2}{t-M_H^2}\right\} = -\frac{M_H^2}{v^2}\left\{\frac{s}{s-M_H^2}+\frac{t}{t-M_H^2}\right\}$$

Higgs-exchange exactly cancels the $O(s,t)$ terms in the SM

$W_L^+ W_L^- \to W_L^+ W_L^-$:



$$T_{\rm SM} = \frac{1}{v^2}\left\{ s + t - \frac{s^2}{s - M_H^2} - \frac{t^2}{t - M_H^2}\right\} = -\frac{M_H^2}{v^2}\left\{\frac{s}{s - M_H^2} + \frac{t}{t - M_H^2}\right\}$$

Higgs-exchange exactly cancels the $O(s,t)$ terms in the SM

When $s \gg M_H^2$, $\quad T_{\rm SM} \approx -\frac{2M_H^2}{v^2} \quad , \quad a_0 \equiv \frac{1}{32\pi}\int_{-1}^{1} d\cos\theta \; T_{\rm SM} \approx -\frac{M_H^2}{8\pi v^2}$

Unitarity: Lee–Quigg–Thacker

$$|a_0| \le 1 \quad \longrightarrow \quad \mathbf{M_H} < \sqrt{8\pi}\,\mathbf{v} \underbrace{\sqrt{2/3}}_{W^+W^-,\,ZZ,\,HH} \approx \mathbf{1\ TeV}$$

What happens in QCD?

- **QCD satisfies unitarity** **(it is a renormalizable theory)**
- **Pion scattering unitarized by exchanges of resonances (composite objects):**
  - **P-wave $(J = 1)$ unitarized by ρ exchange**
  - **S-wave $(J = 0)$ unitarized by σ exchange**
- **The σ meson is the QCD equivalent of the SM Higgs**
- **BUT, the σ is an 'effective' object generated through π rescattering (summation of pion loops)**

Does not seem to work this way in the EW case, but . . .

Higher-Order Goldstone Interactions

$$\mathcal{L}_{\mathrm{EW}}^{(4)}\Big|_{\mathrm{CP-even}} = \sum_{i=0}^{14} a_i\, \mathcal{O}_i$$

(Appelquist, Longhitano)

$$\mathcal{O}_0 = v^2\, \langle T_L V_\mu\rangle^2$$

$$\mathcal{O}_1 = \langle U\, \hat{B}_{\mu\nu}\, U^\dagger\, \hat{W}^{\mu\nu}\rangle \qquad \mathcal{O}_2 = i\, \langle U\, \hat{B}_{\mu\nu}\, U^\dagger\, [V^\mu, V^\nu]\rangle$$

$$\mathcal{O}_3 = i\, \langle \hat{W}_{\mu\nu}\, [V^\mu, V^\nu]\rangle \qquad \mathcal{O}_4 = \langle V_\mu V_\nu\rangle\, \langle V^\mu V^\nu\rangle$$

$$\mathcal{O}_5 = \langle V_\mu V^\mu\rangle^2 \qquad \mathcal{O}_6 = 4\, \langle V_\mu V_\nu\rangle\, \langle T_L V^\mu\rangle\, \langle T_L V^\nu\rangle$$

$$\mathcal{O}_7 = 4\, \langle V_\mu V^\mu\rangle\, \langle T_L V_\nu\rangle^2 \qquad \mathcal{O}_8 = \langle T_L \hat{W}_{\mu\nu}\rangle^2$$

$$\mathcal{O}_9 = -2\, \langle T_L \hat{W}_{\mu\nu}\rangle\, \langle T_L\, [V^\mu, V^\nu]\rangle \qquad \mathcal{O}_{10} = 16\, \{\langle T_L V_\mu\rangle\, \langle T_L V_\nu\rangle\}^2$$

$$\mathcal{O}_{11} = \langle (D_\mu V^\mu)^2\rangle \qquad \mathcal{O}_{12} = 4\, \langle T_L D_\mu D_\nu V^\nu\rangle\, \langle T_L V^\mu\rangle$$

$$\mathcal{O}_{13} = 2\, \langle T_L D_\mu V_\nu\rangle^2 \qquad \mathcal{O}_{14} = -2i\, \varepsilon^{\mu\nu\rho\sigma}\, \langle \hat{W}_{\mu\nu} V_\rho\rangle\, \langle T_L V_\sigma\rangle$$

$$V_\mu \equiv D_\mu U\, U^\dagger \quad , \quad D_\mu V_\nu \equiv \partial_\mu V_\nu - i\, [\hat{W}_\mu, V_\nu] \quad , \quad (V_\mu, D_\mu V_\nu, T_L) \to g_L\, (V_\mu, D_\mu V_\nu, T_L)\, g_L^\dagger$$

Symmetry breaking: $\quad T_L \equiv U\, \frac{\sigma_3}{2}\, U^\dagger \quad , \quad \hat{B}_{\mu\nu} \equiv -g'\, \frac{\sigma_3}{2}\, B_{\mu\nu}$

NLO Predictions

- $\mathcal{L}_{\mathrm{EW}}^{(2)}$ **at one loop:** **Unitarity**

  Non-local (logarithmic) dependences unambiguously predicted

- $\mathcal{L}_{\mathrm{EW}}^{(4)}$ **at tree level:** **Local (polynomic) amplitude**

  Short-distance information encoded in the a_i couplings

  Loop divergences reabsorbed through renormalized a_i

$$a_i \;=\; a_i^r(\mu) + \frac{\gamma_i}{16\pi^2}\,\left[\frac{2\,\mu^{D-4}}{4-D} + \log\left(4\pi\right) - \gamma_E\right]$$

$\hat{a}_i \equiv a_i/(16\pi)^2$ for different limits of the SM

	$M_H \to \infty$	$M_{t',b'} \to \infty$	$M_t \to \infty$
$\hat{a}_0$	$-\frac{3}{4}g'^2\left[\log(M_H/\mu) - \frac{5}{12}\right]$	0	$\frac{3}{2}\frac{M_t^2}{v^2}$
$\hat{a}_1$	$-\frac{1}{6}\log(M_H/\mu) + \frac{5}{72}$	$-\frac{1}{2}$	$\frac{1}{3}\log(M_t/\mu) - \frac{1}{4}$
$\hat{a}_2$	$-\frac{1}{12}\log(M_H/\mu) + \frac{17}{144}$	$-\frac{1}{2}$	$\frac{1}{3}\log(M_t/\mu) - \frac{3}{4}$
$\hat{a}_3$	$\frac{1}{12}\log(M_H/\mu) - \frac{17}{144}$	$\frac{1}{2}$	$\frac{3}{8}$
$\hat{a}_4$	$\frac{1}{6}\log(M_H/\mu) - \frac{17}{72}$	$\frac{1}{4}$	$\log(M_t/\mu) - \frac{5}{6}$
$\hat{a}_5$	$\frac{2\pi^2 v^2}{M_H^2} + \frac{1}{12}\log(M_H/\mu) - \frac{79}{72} + \frac{9\pi}{16\sqrt{3}}$	$-\frac{1}{8}$	$-\log(M_t/\mu) + \frac{23}{24}$
$\hat{a}_6$	0	0	$-\log(M_t/\mu) + \frac{23}{24}$
$\hat{a}_7$	0	0	$\log(M_t/\mu) - \frac{23}{24}$
$\hat{a}_8$	0	0	$\log(M_t/\mu) - \frac{7}{12}$
$\hat{a}_9$	0	0	$\log(M_t/\mu) - \frac{23}{24}$
$\hat{a}_{10}$	0	0	$-\frac{1}{64}$
$\hat{a}_{11}$	—	$-\frac{1}{2}$	$-\frac{1}{2}$
$\hat{a}_{12}$	—	0	$-\frac{1}{8}$
$\hat{a}_{13}$	—	0	$-\frac{1}{4}$
$\hat{a}_{14}$	0	0	$\frac{3}{8}$

Unitary Gauge: $\mathbf{U} = \mathbf{1}$

All invariants reduce to polynomials of gauge fields

- **Bilinear terms:** $\mathcal{O}_0,\, \mathcal{O}_1,\, \mathcal{O}_8,\, \mathcal{O}_{11},\, \mathcal{O}_{12},\, \mathcal{O}_{13}$

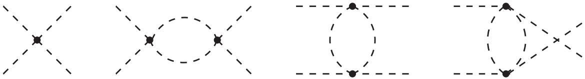
  ➡ **Oblique corrections** $(\Delta r,\, \Delta\rho,\, \Delta k$ ⟷ $S,\, T,\, U)$

- **Trilinear terms:** $\mathcal{O}_2,\, \mathcal{O}_3,\, \mathcal{O}_9,\, \mathcal{O}_{14}$

- **Quartic terms:** $\mathcal{O}_4,\, \mathcal{O}_5,\, \mathcal{O}_6,\, \mathcal{O}_7,\, \mathcal{O}_{10}$

- $\mathcal{O}_{11} \sim m_t^2\, (\bar\psi\psi)(\bar\psi\psi)$**:** $Z\bar{b}b,\, B^0\text{–}\bar{B}^0,\, \varepsilon_K \ldots$

$\varphi^a \varphi^b \to \varphi^c \varphi^d$:

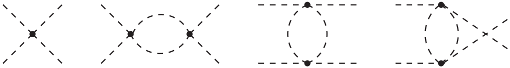


$$\mathcal{A}(\varphi^a\varphi^b \to \varphi^c\varphi^d) \;=\; A(s,t,u)\,\delta_{ab}\,\delta_{cd} + A(t,s,u)\,\delta_{ac}\,\delta_{bd} + A(u,t,s)\,\delta_{ad}\,\delta_{bc}$$

$$A(s,t,u) \;=\; \frac{s}{v^2} + \frac{4}{v^2}\left[a_4^r(\mu)\,(t^2+u^2) + 2\,a_5^r(\mu)\,s^2\right]$$
$$+ \frac{1}{16\pi^2 v^2}\left\{\frac{5}{9}\,s^2 + \frac{13}{18}\,(t^2+u^2) + \frac{1}{12}\,(s^2-3t^2-u^2)\,\log\left(\frac{-t}{\mu^2}\right)\right.$$
$$\left. + \frac{1}{12}\,(s^2-t^2-3u^2)\,\log\left(\frac{-u}{\mu^2}\right) - \frac{1}{2}\,s^2\,\log\left(\frac{-s}{\mu^2}\right)\right\}$$

$$a_i \;=\; a_i^r(\mu) + \frac{\gamma_i}{16\pi^2}\left[\frac{2\,\mu^{D-4}}{4-D} + \log(4\pi) - \gamma_E\right] \quad , \quad \gamma_4 = -\frac{1}{12} \quad , \quad \gamma_5 = -\frac{1}{24}$$

$$\varphi^a \varphi^b \to \varphi^c \varphi^d$$



+ Higgs (tree + 1-loop) contributions

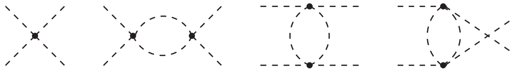
$$\mathcal{L} = \frac{v^2}{4} \langle D^\mu U^\dagger D_\mu U \rangle \left[1 + 2\, a \frac{H}{v} + b \frac{H^2}{v^2} \right]$$

Espriu–Mescia–Yencho, Delgado–Dobado–Llanes-Estrada

$$\begin{aligned} A(s,t,u) &= \frac{s}{v^2}\,(1-a^2) + \frac{4}{v^2}\left[a_4^r(\mu)\,(t^2+u^2) + 2\, a_5^r(\mu)\, s^2 \right] \\ &+ \frac{1}{16\pi^2 v^2} \left\{ \frac{1}{9}\,(14\, a^4 - 10\, a^2 - 18\, a^2 b + 9\, b^2 + 5)\, s^2 + \frac{13}{18}\,(1-a^2)^2\,(t^2+u^2) \right. \\ &\qquad - \frac{1}{2}\,(2\, a^4 - 2\, a^2 - 2\, a^2 b + b^2 + 1)\, s^2 \log\left(\frac{-s}{\mu^2}\right) \\ &\qquad \left. + \frac{1}{12}\,(1-a^2)^2 \left[(s^2 - 3t^2 - u^2) \log\left(\frac{-t}{\mu^2}\right) + (s^2 - t^2 - 3u^2) \log\left(\frac{-u}{\mu^2}\right) \right] \right\} \end{aligned}$$

$$\gamma_4 = -\tfrac{1}{12}\,(1-a^2)^2 \quad , \quad \gamma_5 = -\tfrac{1}{48}\,(2 + 5\, a^4 - 4\, a^2 - 6\, a^2 b + 3\, b^2)$$

$$\varphi^a \varphi^b \to \varphi^c \varphi^d$$



+ Higgs (tree + 1-loop) contributions

$$\mathcal{L} \;=\; \frac{v^2}{4}\,\langle D^\mu U^\dagger D_\mu U\rangle\;\left[1 + 2\,a\,\frac{H}{v} + b\,\frac{H^2}{v^2}\right]$$

Espriu–Mescia–Yencho, Delgado–Dobado–Llanes-Estrada

$$\begin{aligned}
A(s,t,u) \;=\; & \frac{s}{v^2}\,(1-a^2) \;+\; \frac{4}{v^2}\,\left[a_4^r(\mu)\,(t^2+u^2) + 2\,a_5^r(\mu)\,s^2\right] \\
& + \frac{1}{16\pi^2 v^2}\,\left\{\frac{1}{9}\,(14\,a^4 - 10\,a^2 - 18\,a^2 b + 9\,b^2 + 5)\,s^2 + \frac{13}{18}\,(1-a^2)^2\,(t^2+u^2)\right. \\
& \qquad - \frac{1}{2}\,(2\,a^4 - 2\,a^2 - 2\,a^2 b + b^2 + 1)\,s^2\,\log\left(\frac{-s}{\mu^2}\right) \\
& \qquad \left. + \frac{1}{12}\,(1-a^2)^2\,\left[(s^2 - 3t^2 - u^2)\,\log\left(\frac{-t}{\mu^2}\right) + (s^2 - t^2 - 3u^2)\,\log\left(\frac{-u}{\mu^2}\right)\right]\right\}
\end{aligned}$$

$$\gamma_4 \;=\; -\tfrac{1}{12}\,(1-a^2)^2 \qquad , \qquad \gamma_5 \;=\; -\tfrac{1}{48}\,(2 + 5\,a^4 - 4\,a^2 - 6\,a^2 b + 3\,b^2)$$

SM: $a = b = 1$, $a_4 = a_5 = 0$ ➡ $A(s,t,u) \sim \mathcal{O}(M_H^2/v^2)$

Yukawa Couplings

$$\mathcal{L}_Y = -v \left\{ \bar{Q}_L\, U(\varphi) \left[\hat{\mathbf{Y}}_{\mathbf{u}}\, \mathcal{P}_+ + \hat{\mathbf{Y}}_{\mathbf{d}}\, \mathcal{P}_- \right] Q_R + \bar{L}_L\, U(\varphi)\, \hat{\mathbf{Y}}_{\ell}\, \mathcal{P}_+\, L_R + \text{h.c.} \right\}$$

$$Q = \begin{pmatrix} u \\ d \end{pmatrix} \quad , \quad L = \begin{pmatrix} \nu_\ell \\ \ell \end{pmatrix}$$

$$U(\varphi) \to g_L\, U(\varphi)\, g_R^\dagger \quad , \quad Q_L \to g_L\, Q_L \quad , \quad Q_R \to g_R\, Q_R \quad , \quad \mathcal{P}_\pm \to g_R\, \mathcal{P}_\pm\, g_R^\dagger$$

Yukawa Couplings

$$\mathcal{L}_Y = -v \left\{ \bar{Q}_L \, U(\varphi) \left[\hat{\mathbf{Y}}_{\mathbf{u}} \, \mathcal{P}_+ + \hat{\mathbf{Y}}_{\mathbf{d}} \, \mathcal{P}_- \right] Q_R + \bar{L}_L \, U(\varphi) \, \hat{\mathbf{Y}}_{\ell} \, \mathcal{P}_+ \, L_R + \text{h.c.} \right\}$$

$$Q = \begin{pmatrix} u \\ d \end{pmatrix} \quad , \quad L = \begin{pmatrix} \nu_\ell \\ \ell \end{pmatrix}$$

$$U(\varphi) \to g_L \, U(\varphi) \, g_R^\dagger \quad , \quad Q_L \to g_L \, Q_L \quad , \quad Q_R \to g_R \, Q_R \quad , \quad \mathcal{P}_\pm \to g_R \, \mathcal{P}_\pm \, g_R^\dagger$$

Symmetry Breaking: $\quad \mathcal{P}_\pm = \frac{1}{2} \left(\mathrm{I}_2 \pm \sigma_3 \right)$

Yukawa Couplings

$$\mathcal{L}_Y = -v \left\{ \bar{Q}_L \, U(\varphi) \left[\hat{\mathbf{Y}}_{\mathbf{u}} \, \mathcal{P}_+ + \hat{\mathbf{Y}}_{\mathbf{d}} \, \mathcal{P}_- \right] Q_R + \bar{L}_L \, U(\varphi) \, \hat{\mathbf{Y}}_{\boldsymbol{\ell}} \, \mathcal{P}_+ \, L_R + \mathrm{h.c.} \right\}$$

$$Q = \begin{pmatrix} u \\ d \end{pmatrix} \quad , \quad L = \begin{pmatrix} \nu_\ell \\ \ell \end{pmatrix}$$

$$U(\varphi) \to g_L \, U(\varphi) \, g_R^\dagger \quad , \quad Q_L \to g_L \, Q_L \quad , \quad Q_R \to g_R \, Q_R \quad , \quad \mathcal{P}_\pm \to g_R \, \mathcal{P}_\pm \, g_R^\dagger$$

Symmetry Breaking: $\mathcal{P}_\pm = \frac{1}{2} \left(\mathrm{I}_2 \pm \sigma_3 \right)$

Flavour Structure: $\hat{\mathbf{Y}}_{\mathbf{u},\mathbf{d},\boldsymbol{\ell}}$ **3×3 matrices in flavour space**

NLO Operators

Buchalla–Catá

$$\mathcal{O}_{\psi V1} = i\,\bar{Q}_L\gamma^\mu Q_L\,\langle V_\mu T_L\rangle \qquad \mathcal{O}_{\psi V2} = i\,\bar{Q}_L\gamma^\mu T_L Q_L\,\langle V_\mu T_L\rangle \qquad \mathcal{O}_{\psi V3} = i\,\bar{Q}_L\gamma^\mu \tilde{P}_{12} Q_L\,\langle V_\mu \tilde{P}_{21}\rangle$$

$$\mathcal{O}_{\psi V4} = i\,\bar{u}_R\gamma^\mu u_R\,\langle V_\mu T_L\rangle \qquad \mathcal{O}_{\psi V5} = i\,\bar{d}_R\gamma^\mu d_R\,\langle V_\mu T_L\rangle \qquad \mathcal{O}_{\psi V6} = i\,\bar{u}_R\gamma^\mu d_R\,\langle V_\mu \tilde{P}_{21}\rangle$$

$$\mathcal{O}_{\psi V7} = i\,\bar{L}_L\gamma^\mu L_L\,\langle V_\mu T_L\rangle \qquad \mathcal{O}_{\psi V8} = i\,\bar{L}_L\gamma^\mu T_L L_L\,\langle V_\mu T_L\rangle \qquad \mathcal{O}_{\psi V9} = \bar{L}_L\gamma^\mu \tilde{P}_{12} L_L\,\langle V_\mu \tilde{P}_{21}\rangle$$

$$\mathcal{O}_{\psi V10} = \bar{\ell}_R\gamma^\mu \ell_R\,\langle V_\mu T_L\rangle \qquad \mathcal{O}^\dagger_{\psi V3} \qquad \mathcal{O}^\dagger_{\psi V6}$$

$$\mathcal{O}_{\psi S1,2} = \bar{Q}_L\tilde{P}_\pm U Q_R\,\langle D_\mu U^\dagger D^\mu U\rangle \qquad \mathcal{O}_{\psi S3,4} = \bar{Q}_L\tilde{P}_\pm U Q_R\,\langle V_\mu T_L\rangle^2$$

$$\mathcal{O}_{\psi S5} = \bar{Q}_L\tilde{P}_{12} U Q_R\,\langle V_\mu \tilde{P}_{21}\rangle\,\langle V^\mu T_L\rangle \qquad \mathcal{O}_{\psi S6} = \bar{Q}_L\tilde{P}_{12} Q_R\,\langle V_\mu \tilde{P}_{12}\rangle\,\langle V^\mu T_L\rangle$$

$$\mathcal{O}_{\psi S7} = \bar{L}_L\tilde{P}_- U L_R\,\langle D_\mu U^\dagger D^\mu U\rangle \qquad \mathcal{O}_{\psi S8} = \bar{L}_L\tilde{P}_- U L_R\,\langle V^\mu T_L\rangle^2$$

$$\mathcal{O}_{\psi S9} = \bar{L}_L\tilde{P}_{12} U L_R\,\langle V_\mu \tilde{P}_{12}\rangle\,\langle V^\mu T_L\rangle$$

$$\mathcal{O}_{\psi T1} = \bar{Q}_L\sigma^{\mu\nu}\tilde{P}_{12} U Q_R\,\langle V_\mu \tilde{P}_{21}\rangle\,\langle V^\nu T_L\rangle \qquad \mathcal{O}_{\psi T2} = \bar{Q}_L\sigma^{\mu\nu}\tilde{P}_{21} U Q_R\,\langle V_\mu \tilde{P}_{12}\rangle\,\langle V^\nu T_L\rangle$$

$$\mathcal{O}_{\psi T3,4} = \bar{Q}_L\sigma^{\mu\nu}\tilde{P}_\pm U Q_R\,\langle V_\mu \tilde{P}_{12}\rangle\,\langle V_\nu \tilde{P}_{21}\rangle \qquad \mathcal{O}_{\psi T5} = \bar{L}_L\sigma^{\mu\nu}\tilde{P}_{12} U L_R\,\langle V_\mu \tilde{P}_{21}\rangle\,\langle V^\nu T_L\rangle$$

$$\mathcal{O}_{\psi T6} = \bar{L}_L\sigma^{\mu\nu}\tilde{P}_- U L_R\,\langle V_\mu \tilde{P}_{12}\rangle\,\langle V_\nu \tilde{P}_{21}\rangle$$

$$V_\mu = D_\mu U\, U^\dagger \quad , \quad T_L = U\,\frac{\sigma_3}{2}\,U^\dagger \quad , \quad \tilde{P}_{12} = U\,\frac{\sigma_{1+i2}}{2}\,U^\dagger \quad , \quad \tilde{P}_{21} = U\,\frac{\sigma_{1-i2}}{2}\,U^\dagger \quad , \quad \tilde{P}_\pm = U\,P_\pm\,U^\dagger$$

NLO Operators (cont.)

Buchalla–Catá

$\mathcal{O}_{LL6} = \bar{Q}_L\gamma^\mu T_L Q_L\ \bar{Q}_L\gamma_\mu T_L Q_L$ $\quad \mathcal{O}_{LL7} = \bar{Q}_L\gamma^\mu T_L Q_L\ \bar{Q}_L\gamma_\mu Q_L$ $\quad \mathcal{O}_{LL8} = \bar{q}_{L\alpha}\gamma^\mu T_L Q_{L\beta}\ \bar{Q}_{L\beta}\gamma_\mu T_L Q_{L\alpha}$

$\mathcal{O}_{LL10} = \bar{Q}_L\gamma^\mu T_L Q_L\ \bar{L}_L\gamma_\mu T_L L_L$ $\quad \mathcal{O}_{LL11} = \bar{Q}_L\gamma^\mu T_L Q_L\ \bar{L}_L\gamma_\mu L_L$ $\quad \mathcal{O}_{LL9} = \bar{Q}_{L\alpha}\gamma^\mu T_L Q_{L\beta}\ \bar{Q}_{L\beta}\gamma_\mu Q_{L\alpha}$

$\mathcal{O}_{LL12} = \bar{Q}_L\gamma^\mu Q_L\ \bar{L}_L\gamma_\mu T_L L_L$ $\quad \mathcal{O}_{LL13} = \bar{Q}_L\gamma^\mu T_L L_L\ \bar{L}_L\gamma_\mu T_L Q_L$ $\quad \mathcal{O}_{LL14} = \bar{Q}_L\gamma^\mu T_L L_L\ \bar{L}_L\gamma_\mu Q_L$

$\mathcal{O}_{LL15} = \bar{L}_L\gamma^\mu T_L L_L\ \bar{L}_L\gamma_\mu T_L L_L$ $\quad \mathcal{O}_{LL16} = \bar{L}_L\gamma^\mu T_L L_L\ \bar{L}_L\gamma_\mu L_L$

$\mathcal{O}_{LR10} = \bar{Q}_L\gamma^\mu T_L Q_L\ \bar{u}_R\gamma_\mu u_R$ $\quad \mathcal{O}_{LR12} = \bar{Q}_L\gamma^\mu T_L Q_L\ \bar{d}_R\gamma_\mu d_R$ $\quad \mathcal{O}_{LR11} = \bar{Q}_L\gamma^\mu t^a T_L Q_L\ \bar{u}_R\gamma_\mu t^a u_R$

$\mathcal{O}_{LR14} = \bar{u}_R\gamma^\mu u_R\ \bar{L}_L\gamma_\mu T_L L_L$ $\quad \mathcal{O}_{LR15} = \bar{d}_R\gamma^\mu d_R\ \bar{L}_L\gamma_\mu T_L L_L$ $\quad \mathcal{O}_{LR13} = \bar{Q}_L\gamma^\mu t^a T_L Q_L\ \bar{d}_R\gamma_\mu t^a d_R$

$\mathcal{O}_{LR16} = \bar{Q}_L\gamma^\mu T_L Q_L\ \bar{\ell}_R\gamma_\mu \ell_R$ $\quad \mathcal{O}_{LR17} = \bar{L}_L\gamma^\mu T_L L_L\ \bar{\ell}_R\gamma_\mu \ell_R$ $\quad \mathcal{O}_{LR18} = \bar{Q}_L\gamma^\mu T_L L_L\ \bar{\ell}_R\gamma_\mu d_R$

$\mathcal{O}_{ST5} = \bar{Q}_L\tilde{P}_+ U Q_R\ \bar{Q}_L\tilde{P}_- U Q_R$ $\quad \mathcal{O}_{ST6} = \bar{Q}_L\tilde{P}_{21} U Q_R\ \bar{Q}_L\tilde{P}_{12} U Q_R$ $\quad \mathcal{O}_{ST7} = \bar{Q}_L t^a\tilde{P}_+ U Q_R\ \bar{Q}_L t^a\tilde{P}_- U Q_R$

$\mathcal{O}_{ST9} = \bar{Q}_L\tilde{P}_+ U Q_R\ \bar{L}_L\tilde{P}_- U L_R$ $\quad \mathcal{O}_{ST10} = \bar{Q}_L\tilde{P}_{21} U Q_R\ \bar{L}_L\tilde{P}_{12} U L_R$ $\quad \mathcal{O}_{ST8} = \bar{Q}_L t^a\tilde{P}_{21} U Q_R\ \bar{Q}_L t^a\tilde{P}_{12} U Q_R$

$\mathcal{O}_{ST11} = \bar{Q}_L\sigma^{\mu\nu}\tilde{P}_+ U Q_R\ \bar{L}_L\sigma_{\mu\nu}\tilde{P}_- U L_R$ $\quad \mathcal{O}_{ST12} = \bar{Q}_L\sigma^{\mu\nu}\tilde{P}_{21} U Q_R\ \bar{L}_L\sigma_{\mu\nu}\tilde{P}_{12} U L_R$

$\mathcal{O}_{FY4} = \bar{Q}_L t^a\tilde{P}_- U Q_R\ \bar{Q}_L t^a\tilde{P}_- U Q_R$ $\quad \mathcal{O}_{FY8} = \bar{Q}_L\sigma^{\mu\nu}\tilde{P}_- U Q_R\ \bar{L}_L\sigma_{\mu\nu}\tilde{P}_- U L_R$

$\mathcal{O}_{FY1} = \bar{Q}_L\tilde{P}_+ U Q_R\ \bar{Q}_L\tilde{P}_+ U Q_R$ $\quad \mathcal{O}_{FY3} = \bar{Q}_L\tilde{P}_- U Q_R\ \bar{Q}_L\tilde{P}_- U Q_R$ $\quad \mathcal{O}_{FY2} = \bar{Q}_L t^a\tilde{P}_+ U Q_R\ \bar{Q}_L t^a\tilde{P}_+ U Q_R$

$\mathcal{O}_{FY5} = \bar{Q}_L\tilde{P}_- U Q_R\ \bar{Q}_R U^\dagger\tilde{P}_+ Q_L$ $\quad \mathcal{O}_{FY7} = \bar{Q}_L\tilde{P}_- U Q_R\ \bar{L}_L\tilde{P}_- U L_R$ $\quad \mathcal{O}_{FY6} = \bar{Q}_L t^a\tilde{P}_- U Q_R\ \bar{Q}_R t^a U^\dagger\tilde{P}_+ Q_L$

$\mathcal{O}_{FY9} = \bar{L}_L\tilde{P}_- U L_R\ \bar{Q}_R U^\dagger\tilde{P}_+ Q_R$ $\quad \mathcal{O}_{FY10} = \bar{L}_L\tilde{P}_- U L_R\ \bar{L}_L\tilde{P}_- U L_R$ $\quad \mathcal{O}_{FY11} = \bar{L}_L\tilde{P}_- U Q_R\ \bar{Q}_R U^\dagger\tilde{P}_+ L_L$

Including a Light (singlet) Higgs in the EWET

Let us assume that h(126) is an $SU(2)_{L+R}$ scalar singlet

All Higgsless operators can be multiplied by an arbitrary function of **h**:

$$\mathcal{O}_X \quad \longrightarrow \quad \tilde{\mathcal{O}}_X \equiv \mathcal{F}_X(h)\, \mathcal{O}_X$$

$$\mathcal{F}_X(h) = \sum_{n=0} c_X^{(n)} \left(\frac{h}{v}\right)^n$$

In addition, the LO Lagrangian should include the **scalar potential**:

$$V(h) = v^4 \sum_{n=2} c_V^{(n)} \left(\frac{h}{v}\right)^n$$

Plus operators with derivatives $(\partial_\mu h)$: $F_X \equiv F_X(h)$

$$\mathcal{O}_{D7} = -\langle V_\mu V^\mu \rangle \frac{\partial_\nu h\, \partial^\nu h}{v^2} F_{D7} \qquad \mathcal{O}_{D8} = -\langle V_\mu V_\nu \rangle \frac{\partial^\mu h\, \partial^\nu h}{v^2} F_{D8} \qquad \mathcal{O}_{D11} = \frac{(\partial_\mu h\, \partial^\mu h)^2}{v^4} F_{D11}$$

$$\mathcal{O}_{D6} = -\langle T_L V_\mu V_\nu \rangle \langle T_L V^\mu \rangle \frac{\partial^\nu h}{v} F_{D6} \qquad \mathcal{O}_{D9} = -\langle T_L V_\mu \rangle \langle T_L V^\mu \rangle \frac{\partial_\nu h\, \partial^\nu h}{v^2} F_{D9}$$

$$\mathcal{O}_{D10} = -\langle T_L V_\mu \rangle \langle T_L V_\nu \rangle \frac{\partial^\mu h\, \partial^\nu h}{v^2} F_{D10} \qquad \mathcal{O}_{\psi S18} = \bar{L}_L \tilde{P}_- U L_R \frac{\partial_\mu h\, \partial^\mu h}{v^2} F_{\psi S18}$$

$$\mathcal{O}_{\psi S10} = -i\, \bar{Q}_L \tilde{P}_+ U Q_R \langle T_L V_\mu \rangle \frac{\partial^\mu h}{v} F_{\psi S10} \qquad \mathcal{O}_{\psi S11} = -i\, \bar{Q}_L \tilde{P}_- U Q_R \langle T_L V_\mu \rangle \frac{\partial^\mu h}{v} F_{\psi S11}$$

$$\mathcal{O}_{\psi S12} = -i\, \bar{Q}_L \tilde{P}_{12} U Q_R \langle \tilde{P}_{21} V_\mu \rangle \frac{\partial^\mu h}{v} F_{\psi S12} \qquad \mathcal{O}_{\psi S13} = -i\, \bar{Q}_L \tilde{P}_{21} U Q_R \langle \tilde{P}_{12} V_\mu \rangle \frac{\partial^\mu h}{v} F_{\psi S13}$$

$$\mathcal{O}_{\psi S14} = \bar{Q}_L \tilde{P}_+ U Q_R \frac{\partial_\mu h\, \partial^\mu h}{v^2} F_{\psi S14} \qquad \mathcal{O}_{\psi S15} = \bar{Q}_L \tilde{P}_- U Q_R \frac{\partial_\mu h\, \partial^\mu h}{v^2} F_{\psi S15}$$

$$\mathcal{O}_{\psi S16} = -i\, \bar{L}_L \tilde{P}_- U L_R \langle T_L V_\mu \rangle \frac{\partial^\mu h}{v} F_{\psi S16} \qquad \mathcal{O}_{\psi S17} = -i\, \bar{L}_L \tilde{P}_{12} U L_R \langle \tilde{P}_{21} V_\mu \rangle \frac{\partial^\mu h}{v} F_{\psi S17}$$

$$\mathcal{O}_{\psi T7} = -i\, \bar{Q}_L \sigma_{\mu\nu} \tilde{P}_+ U Q_R \langle T_L V^\mu \rangle \frac{\partial^\nu h}{v} F_{\psi T7} \qquad \mathcal{O}_{\psi T8} = -i\, \bar{Q}_L \sigma_{\mu\nu} \tilde{P}_- U Q_R \langle T_L V^\mu \rangle \frac{\partial^\nu h}{v} F_{\psi T8}$$

$$\mathcal{O}_{\psi T9} = -i\, \bar{Q}_L \sigma_{\mu\nu} \tilde{P}_{21} U Q_R \langle \tilde{P}_{12} V^\mu \rangle \frac{\partial^\nu h}{v} F_{\psi T9} \qquad \mathcal{O}_{\psi T10} = -i\, \bar{Q}_L \sigma_{\mu\nu} \tilde{P}_{12} U Q_R \langle \tilde{P}_{21} V^\mu \rangle \frac{\partial^\nu h}{v} F_{\psi T10}$$

$$\mathcal{O}_{\psi T11} = -i\, \bar{L}_L \sigma_{\mu\nu} \tilde{P}_- U L_R \langle T_L V^\mu \rangle \frac{\partial^\nu h}{v} F_{\psi T11} \qquad \mathcal{O}_{\psi T12} = -i\, \bar{L}_L \sigma_{\mu\nu} \tilde{P}_{12} U L_R \langle \tilde{P}_{21} V^\mu \rangle \frac{\partial^\nu h}{v} F_{\psi T12}$$

Buchalla–Catà–Krause

Linear Realization: $SU(2)_L \otimes U(1)_Y$

Assumes that H(126) and $\vec{\varphi}$ combine into an $SU(2)_L$ doublet:

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{2}\,(v+H)\; U(\vec{\varphi}) \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

The SM Lagrangian is the low-energy effective theory with $D = 4$

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm SM} + \sum_{D>4} \sum_i \frac{c_i^{(D)}}{\Lambda^{D-4}}\, \mathcal{O}_i^{(D)}$$

- **1 operator with D = 5:** $\mathcal{O}^{(5)} = \bar{L}_L \tilde{\Phi} \tilde{\Phi}^T L_L^c$ **(violates L by 2 units)**
  Weinberg
- **59 independent $\mathcal{O}_i^{(6)}$ preserving B and L (for 1 generation)**
  Buchmuller–Wyler, Grzadkowski–Iskrzynski–Misiak–Rosiek
- **5 independent $\mathcal{O}_i^{(6)}$ violating B and L (for 1 generation)**
  Weinberg, Wilczek–Zee, Abbott–Wise,

$D = 6$ Operators (other than 4-fermion ones)

Grzadkowski–Iskrzynski–Misiak–Rosiek

X^3		Φ^6 and $\Phi^4 D^2$		$\psi^2\Phi^3$	
$\mathcal{O}_G$	$f^{ABC}\, G_\mu^{A\nu}\, G_\nu^{B\rho}\, G_\rho^{C\mu}$	$\mathcal{O}_\Phi$	$(\Phi^\dagger\Phi)^3$	$\mathcal{O}_{e\Phi}$	$(\Phi^\dagger\Phi)\,(\bar{l}_p e_r \Phi)$
$\mathcal{O}_{\widetilde{G}}$	$f^{ABC}\, \widetilde{G}_\mu^{A\nu}\, G_\nu^{B\rho}\, G_\rho^{C\mu}$	$\mathcal{O}_{\Phi\Box}$	$(\Phi^\dagger\Phi)\,\Box\,(\Phi^\dagger\Phi)$	$\mathcal{O}_{u\Phi}$	$(\Phi^\dagger\Phi)\,(\bar{q}_p u_r \widetilde{\Phi})$
$\mathcal{O}_W$	$\varepsilon^{IJK}\, W_\mu^{I\nu}\, W_\nu^{J\rho}\, W_\rho^{K\mu}$	$\mathcal{O}_{\Phi D}$	$\left(\Phi^\dagger D^\mu \Phi\right)^\star\, \left(\Phi^\dagger D_\mu \Phi\right)$	$\mathcal{O}_{d\Phi}$	$(\Phi^\dagger\Phi)\,(\bar{q}_p d_r \Phi)$
$\mathcal{O}_{\widetilde{W}}$	$\varepsilon^{IJK}\, \widetilde{W}_\mu^{I\nu}\, W_\nu^{J\rho}\, W_\rho^{K\mu}$				

$X^2\Phi^2$		$\psi^2 X \Phi$		$\psi^2\Phi^2 D$	
$\mathcal{O}_{\Phi G}$	$\Phi^\dagger\Phi\; G_{\mu\nu}^A\, G^{A\mu\nu}$	$\mathcal{O}_{eW}$	$(\bar{l}_p \sigma^{\mu\nu} e_r)\; \tau^I \Phi\; W_{\mu\nu}^I$	$\mathcal{O}_{\Phi l}^{(1)}$	$(\Phi^\dagger i \overleftrightarrow{D}_\mu\, \Phi)\,(\bar{l}_p \gamma^\mu l_r)$
$\mathcal{O}_{\Phi\widetilde{G}}$	$\Phi^\dagger\Phi\; \widetilde{G}_{\mu\nu}^A\, G^{A\mu\nu}$	$\mathcal{O}_{eB}$	$(\bar{l}_p \sigma^{\mu\nu} e_r)\; \Phi\; B_{\mu\nu}$	$\mathcal{O}_{\Phi l}^{(3)}$	$(\Phi^\dagger i \overleftrightarrow{D}_\mu^I\, \Phi)\,(\bar{l}_p \tau^I \gamma^\mu l_r)$
$\mathcal{O}_{\Phi W}$	$\Phi^\dagger\Phi\; W_{\mu\nu}^I\, W^{I\mu\nu}$	$\mathcal{O}_{uG}$	$(\bar{q}_p \sigma^{\mu\nu}\, T^A u_r)\; \widetilde{\Phi}\; G_{\mu\nu}^A$	$\mathcal{O}_{\Phi e}$	$(\Phi^\dagger i \overleftrightarrow{D}_\mu\, \Phi)\,(\bar{e}_p \gamma^\mu e_r)$
$\mathcal{O}_{\Phi\widetilde{W}}$	$\Phi^\dagger\Phi\; \widetilde{W}_{\mu\nu}^I\, W^{I\mu\nu}$	$\mathcal{O}_{uW}$	$(\bar{q}_p \sigma^{\mu\nu} u_r)\; \tau^I \widetilde{\Phi}\; W_{\mu\nu}^I$	$\mathcal{O}_{\Phi q}^{(1)}$	$(\Phi^\dagger i \overleftrightarrow{D}_\mu\, \Phi)\,(\bar{q}_p \gamma^\mu q_r)$
$\mathcal{O}_{\Phi B}$	$\Phi^\dagger\Phi\; B_{\mu\nu}\, B^{\mu\nu}$	$\mathcal{O}_{uB}$	$(\bar{q}_p \sigma^{\mu\nu} u_r)\; \widetilde{\Phi}\; B_{\mu\nu}$	$\mathcal{O}_{\Phi q}^{(3)}$	$(\Phi^\dagger i \overleftrightarrow{D}_\mu^I\, \Phi)\,(\bar{q}_p \tau^I \gamma^\mu q_r)$
$\mathcal{O}_{\Phi\widetilde{B}}$	$\Phi^\dagger\Phi\; \widetilde{B}_{\mu\nu}\, B^{\mu\nu}$	$\mathcal{O}_{dG}$	$(\bar{q}_p \sigma^{\mu\nu}\, T^A d_r)\; \Phi\; G_{\mu\nu}^A$	$\mathcal{O}_{\Phi u}$	$(\Phi^\dagger i \overleftrightarrow{D}_\mu\, \Phi)\,(\bar{u}_p \gamma^\mu u_r)$
$\mathcal{O}_{\Phi WB}$	$\Phi^\dagger \tau^I \Phi\; W_{\mu\nu}^I\, B^{\mu\nu}$	$\mathcal{O}_{dW}$	$(\bar{q}_p \sigma^{\mu\nu} d_r)\; \tau^I \Phi\; W_{\mu\nu}^I$	$\mathcal{O}_{\Phi d}$	$(\Phi^\dagger i \overleftrightarrow{D}_\mu\, \Phi)\,(\bar{d}_p \gamma^\mu d_r)$
$\mathcal{O}_{\Phi\widetilde{W}B}$	$\Phi^\dagger \tau^I \Phi\; \widetilde{W}_{\mu\nu}^I\, B^{\mu\nu}$	$\mathcal{O}_{dB}$	$(\bar{q}_p \sigma^{\mu\nu} d_r)\; \Phi\; B_{\mu\nu}$	$\mathcal{O}_{\Phi ud}$	$i(\widetilde{\Phi}^\dagger D_\mu \Phi)\,(\bar{u}_p \gamma^\mu d_r)$

$q = q_L$, $l = l_L$, $u = u_R$, $d = d_R$, $e = e_R$, $p, r =$ generation indices

$D = 6$ Four-Fermion Operators

Grzadkowski–Iskrzynski–Misiak–Rosiek

$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
$\mathcal{O}_{ll}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$	$\mathcal{O}_{ee}$	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$	$\mathcal{O}_{le}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)$
$\mathcal{O}_{qq}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$	$\mathcal{O}_{uu}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$	$\mathcal{O}_{lu}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma^\mu u_t)$
$\mathcal{O}_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu\tau^I q_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	$\mathcal{O}_{dd}$	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$	$\mathcal{O}_{ld}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t)$
$\mathcal{O}_{lq}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$	$\mathcal{O}_{eu}$	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$	$\mathcal{O}_{qe}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)$
$\mathcal{O}_{lq}^{(3)}$	$(\bar{l}_p\gamma_\mu\tau^I l_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	$\mathcal{O}_{ed}$	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$	$\mathcal{O}_{qu}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t)$
		$\mathcal{O}_{ud}^{(1)}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)$	$\mathcal{O}_{qu}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{u}_s\gamma^\mu T^A u_t)$
		$\mathcal{O}_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$	$\mathcal{O}_{qd}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)$
				$\mathcal{O}_{qd}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{d}_s\gamma^\mu T^A d_t)$

$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$		B-violating	
$\mathcal{O}_{ledq}$	$(\bar{l}_p^j e_r)(\bar{d}_s q_t^j)$	$\mathcal{O}_{duq}$	$\varepsilon^{\alpha\beta\gamma}\,\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
$\mathcal{O}_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\,\varepsilon_{jk}\,(\bar{q}_s^k d_t)$	$\mathcal{O}_{qqu}$	$\varepsilon^{\alpha\beta\gamma}\,\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$\mathcal{O}_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\,\varepsilon_{jk}\,(\bar{q}_s^k T^A d_t)$	$\mathcal{O}_{qqq}^{(1)}$	$\varepsilon^{\alpha\beta\gamma}\,\varepsilon_{jk}\,\varepsilon_{mn}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$\mathcal{O}_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\,\varepsilon_{jk}\,(\bar{q}_s^k u_t)$	$\mathcal{O}_{qqq}^{(3)}$	$\varepsilon^{\alpha\beta\gamma}\,(\tau^I\varepsilon)_{jk}\,(\tau^I\varepsilon)_{mn}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$\mathcal{O}_{lequ}^{(3)}$	$(\bar{l}_p^j\sigma_{\mu\nu} e_r)\,\varepsilon_{jk}\,(\bar{q}_s^k\sigma^{\mu\nu} u_t)$	$\mathcal{O}_{duu}$	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$

$q = q_L\,,\; l = l_L\,,\; u = u_R\,,\; d = d_R\,,\; e = e_R$, $p, r, s, t =$ generation indices

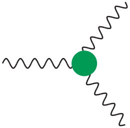
Equivalent Bases of CP-even Operators (EW bosons only)

Willenbrock–Zhang

BW	HISZ	GGPR
$\mathcal{O}_W = \epsilon^{IJK} W^{I\nu}_{\mu} W^{J\rho}_{\nu} W^{K\mu}_{\rho}$	$\mathcal{O}_{WWW} = \mathrm{Tr}\,[\hat{W}_{\mu\nu} \hat{W}^{\nu\rho} \hat{W}^{\mu}_{\rho}]$	$\mathcal{O}_{3W} = \frac{1}{3!}\, g\, \epsilon_{abc} W^{a\nu}_{\mu} W^{b}_{\nu\rho} W^{c\rho\mu}$
$\mathcal{O}_{\Phi W} = \Phi^\dagger\Phi\; W^I_{\mu\nu} W^{I\mu\nu}$	$\mathcal{O}_{WW} = \Phi^\dagger \hat{W}_{\mu\nu} \hat{W}^{\mu\nu} \Phi$	—
$\mathcal{O}_{\Phi B} = \Phi^\dagger\Phi\; B_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{BB} = \Phi^\dagger \hat{B}_{\mu\nu} \hat{B}^{\mu\nu} \Phi$	$\mathcal{O}_{BB} = g'^2\, \lvert H\rvert^2\, B_{\mu\nu} B^{\mu\nu}$
$\mathcal{O}_{\Phi WB} = \Phi^\dagger \sigma^I \Phi\; W^I_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{BW} = \Phi^\dagger \hat{B}_{\mu\nu} \hat{W}^{\mu\nu} \Phi$	—
—	$\mathcal{O}_W = (D_\mu\Phi)^\dagger \hat{W}^{\mu\nu} (D_\nu\Phi)$	$\mathcal{O}_{HW} = ig\, (D^\mu H)^\dagger \sigma^a (D^\nu H)\, W^a_{\mu\nu}$
—	$\mathcal{O}_B = (D_\mu\Phi)^\dagger \hat{B}^{\mu\nu} (D_\nu\Phi)$	$\mathcal{O}_{HB} = ig' (D^\mu H)^\dagger (D^\nu H) B_{\mu\nu}$
—	$\mathcal{O}_{DW} = \mathrm{Tr}\left([D_\mu, \hat{W}_{\nu\rho}][D^\mu, \hat{W}^{\nu\rho}]\right)$	$\mathcal{O}_{2W} = -\frac{1}{2}\, (D^\mu W^a_{\mu\nu})^2$
—	$\mathcal{O}_{DB} = -\frac{g'^2}{2}\, (\partial_\mu B_{\nu\rho})(\partial^\mu B^{\nu\rho})$	$\mathcal{O}_{2B} = -\frac{1}{2}\, (\partial^\mu B_{\mu\nu})^2$
—	—	$\mathcal{O}_W = \frac{ig}{2}\, (H^\dagger \sigma^a \overleftrightarrow{D}^\mu H)\, D^\nu W^a_{\mu\nu}$
—	—	$\mathcal{O}_B = \frac{ig'}{2}\, (H^\dagger \overleftrightarrow{D}^\mu H)\, \partial^\nu B_{\mu\nu}$
$\mathcal{O}_{\Phi D} = (\Phi^\dagger D^\mu \Phi)^*\, (\Phi^\dagger D_\mu \Phi)$	$\mathcal{O}_{\Phi,1} = (D_\mu\Phi)^\dagger \Phi\Phi^\dagger (D^\mu\Phi)$	$\mathcal{O}_T = \frac{1}{2}\, (H^\dagger \overleftrightarrow{D}^\mu H)^2$
$\mathcal{O}_{\Phi\Box} = (\Phi^\dagger\Phi)\, \Box\, (\Phi^\dagger\Phi)$	$\mathcal{O}_{\Phi,2} = \frac{1}{2}\, \partial_\mu (\Phi^\dagger\Phi)\, \partial^\mu (\Phi^\dagger\Phi)$	$\mathcal{O}_H = \frac{1}{2}\, (\partial^\mu \lvert H\rvert^2)^2$
$\mathcal{O}_\Phi = (\Phi^\dagger\Phi)^3$	$\mathcal{O}_{\Phi,3} = \frac{1}{3}\, (\Phi^\dagger\Phi)^3$	$\mathcal{O}_6 = \lambda\, \lvert H\rvert^6$
—	$\mathcal{O}_{\Phi,4} = (D_\mu\Phi)^\dagger (D^\mu\Phi)\, (\Phi^\dagger\Phi)$	—

BW = Buchmuller–Wyler, Grzadkowski et al. **HISZ** = Hagiwara et al. **GGPR** = Giudice et al.

Anomalous Triple Gauge Couplings



$$\mathcal{L}_{WWV} = -i\, g_{WWV} \Big\{ g_1^V \left(W^+_{\mu\nu} W^{-\mu} V^\nu - W^+_\mu W^{-\mu\nu} V_\nu \right) + \kappa_V\, W^+_\mu W^-_\nu V^{\mu\nu} + \frac{\lambda_V}{M_W^2}\, W^+_{\mu\nu} W^{-\nu\rho} V_\rho^{\;\mu} \Big\}$$

$g_{WW\gamma} = e = g\,\cos\theta_W$, $g_{WWZ} = g\,\cos\theta_W$, $\kappa_V = 1 + \Delta\kappa_V$, $g_1^V = 1 + \Delta g_1^V$

Relevant operators:

$\mathcal{O}_W,\, \mathcal{O}_{\Phi WB},\, \mathcal{O}_{\phi l}^{(3)}$ (BW)

$\mathcal{O}_{WWW},\, \mathcal{O}_W,\, \mathcal{O}_B,\, \mathcal{O}_{BW},\, \mathcal{O}_{DW}$ (HISZ)

$\mathcal{O}_{3W},\, \mathcal{O}_{HW},\, \mathcal{O}_{HB},\, \mathcal{O}_{2W},\, \mathcal{O}_W,\, \mathcal{O}_B$ (GGPR)

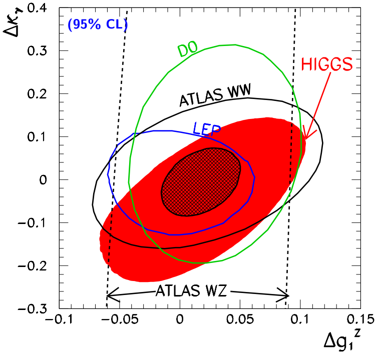
D = 6 Relations: $\lambda_\gamma = \lambda_Z$, $\Delta g_1^Z = \Delta\kappa_Z + \tan^2\theta_W\, \Delta\kappa_\gamma$

Anomalous Triple Gauge Couplings

HISZ basis: $\mathcal{L}_{\rm eff} = \sum_n \frac{f_n}{\Lambda^2}\, \mathcal{O}_n$ Corbett–Eboli–González-Fraile–González-García

$$\lambda_\gamma = \lambda_Z = \frac{3g^2 M_W^2}{2\Lambda^2}\, f_{WWW} \quad , \quad \Delta\kappa_\gamma = \frac{g^2 v^2}{8\Lambda^2}\, (f_W + f_B)$$

$$\Delta\kappa_Z = \frac{g^2 v^2}{8\Lambda^2}\, \left(f_W - \tan^2\theta_W\, f_B\right) \quad , \quad \Delta g_1^Z = \frac{g^2 v^2}{8\cos^2\theta_W \Lambda^2}\, f_W$$



Higgs data: (90% CL)

$-0.047 \le \Delta g_1^Z \le 0.089 \quad , \quad -0.19 \le \Delta\kappa_\gamma \le 0.099$

➡ $-0.019 \le \Delta\kappa_Z \le 0.083$

Combined: (90% CL)

$-0.005 \le \Delta g_1^Z \le 0.040 \quad , \quad -0.058 \le \Delta\kappa_\gamma \le 0.047$

➡ $-0.004 \le \Delta\kappa_Z \le 0.040$

LEP, D0, ATLAS assume $\lambda_\gamma = \lambda_Z = 0$

Global Fit to $\mathcal{L}_{\text{eff}}^{(D=6)}$

Pomarol–Riva

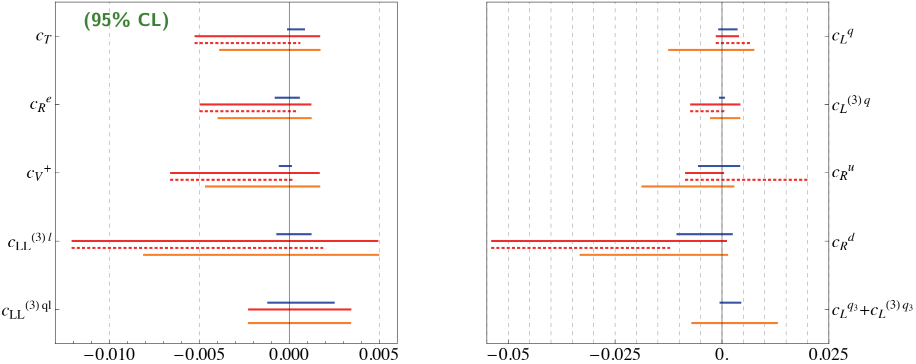
GGPR basis:

$\mathcal{O}_H = \frac{1}{2}\,(\partial^\mu |H|^2)^2$
$\mathcal{O}_T = \frac{1}{2}\,(H^\dagger \overleftrightarrow{D}_\mu H)^2$
$\mathcal{O}_6 = \lambda\,|H|^6$
$\mathcal{O}_W = \frac{ig}{2}\,(H^\dagger\sigma^a \overleftrightarrow{D}_\mu H) D_\nu W^{a\,\mu\nu}$
$\mathcal{O}_B = \frac{ig'}{2}\,(H^\dagger \overleftrightarrow{D}_\mu H)\partial_\nu B^{\mu\nu}$

$\mathcal{O}_{BB} = g'^2\,|H|^2 B_{\mu\nu}B^{\mu\nu}$
$\mathcal{O}_{GG} = g_s^2\,|H|^2 G^A_{\mu\nu}G^{A\mu\nu}$
$\mathcal{O}_{HW} = ig\,(D^\mu H)^\dagger\sigma^a(D^\nu H)W^a_{\mu\nu}$
$\mathcal{O}_{HB} = ig'\,(D^\mu H)^\dagger(D^\nu H)B_{\mu\nu}$
$\mathcal{O}_{3W} = \frac{1}{3!}\,g\,\epsilon_{abc}W^{a\,\nu}_\mu W^b_{\nu\rho}W^{c\,\rho\mu}$

$\mathcal{O}_{y_u} = y_u\,|H|^2\bar{Q}_L\widetilde{H}u_R + \text{h.c.}$	$\mathcal{O}_{y_d} = y_d\,|H|^2\bar{Q}_L H d_R + \text{h.c.}$	$\mathcal{O}_{y_e} = y_e\,|H|^2\bar{L}_L H e_R + \text{h.c.}$
$\mathcal{O}^u_R = i\,(H^\dagger \overleftrightarrow{D}_\mu H)(\bar{u}_R\gamma^\mu u_R)$ $\mathcal{O}^q_L = i\,(H^\dagger \overleftrightarrow{D}_\mu H)(\bar{Q}_L\gamma^\mu Q_L)$ $\mathcal{O}^{(3)\,q}_L = i\,(H^\dagger\sigma^a \overleftrightarrow{D}_\mu H)(\bar{Q}_L\sigma^a\gamma^\mu Q_L)$	$\mathcal{O}^d_R = i\,(H^\dagger \overleftrightarrow{D}_\mu H)(\bar{d}_R\gamma^\mu d_R)$	$\mathcal{O}^e_R = i\,(H^\dagger \overleftrightarrow{D}_\mu H)(\bar{e}_R\gamma^\mu e_R)$
$\mathcal{O}^{(3)\,ql}_{LL} = (\bar{Q}_L\sigma^a\gamma_\mu Q_L)\,(\bar{L}_L\sigma^a\gamma^\mu L_L)$	$\mathcal{O}^{(3)\,l}_{LL} = (\bar{L}_L\sigma^a\gamma^\mu L_L)\,(\bar{L}_L\sigma^a\gamma_\mu L_L)$	

LEP-I + SLC + KLOE (CKM univ) + pp → $l\bar{\nu}$ (LHC)



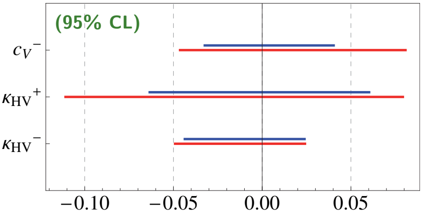
All ; without KLOE ; include $\mathcal{O}_L^t + \mathcal{O}_L^{(3)t}$; All other $\mathcal{O}_i$ set to zero

$$\Delta\mathcal{L} = \frac{c_T}{v^2}\,\mathcal{O}_T + \frac{c_V^+}{M_W^2}\,(\mathcal{O}_W + \mathcal{O}_B) + \frac{c_{LL}^{(3)\,l}}{v^2}\,\mathcal{O}_{LL}^{(3)\,l} + \frac{c_R^e}{v^2}\,\mathcal{O}_R^e$$

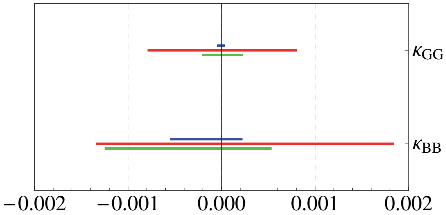
$$+ \frac{c_L^q}{v^2}\,\mathcal{O}_L^q + \frac{c_L^{(3)\,q}}{v^2}\,\mathcal{O}_L^{(3)\,q} + \frac{c_R^u}{v^2}\,\mathcal{O}_R^u + \frac{c_R^d}{v^2}\,\mathcal{O}_R^d + \frac{c_{LL}^{(3)\,ql}}{v^2}\,\mathcal{O}_{LL}^{(3)\,ql}$$

$$c_V^\pm = \tfrac{1}{2}(c_W \pm c_B)$$

LEP-II + $H \to Z\gamma$ (LHC)



$H \to VV, f\bar{f}$ (LHC)



— All ; — All other $\mathcal{O}_i$ set to zero ; — c_H and c_{y_f} effects constrained to be below 50% of SM

$$\Delta\mathcal{L}_{\rm TGC} = \frac{\kappa_{HV}^+}{m_W^2}\left(\mathcal{O}_{HW} + \mathcal{O}_{HB}\right) + \frac{c_V^- + \kappa_{HV}^-}{2m_W^2}\,\mathcal{O}_+ + \frac{\kappa_{3W}}{m_W^2}\,\mathcal{O}_{3W}$$

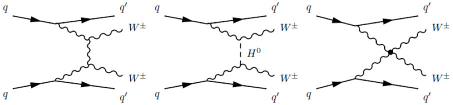
$$\Delta\mathcal{L}_{\rm Higgs} = \frac{c_H}{v^2}\,\mathcal{O}_H + \sum_{f=t,b,\tau}\frac{c_{y_f}}{v^2}\,\mathcal{O}_{y_f} + \frac{c_6}{v^2}\,\mathcal{O}_6 + \frac{\kappa_{BB}}{M_W^2}\,\mathcal{O}_{BB} + \frac{\kappa_{GG}}{M_W^2}\,\mathcal{O}_{GG} + \frac{c_V^- - \kappa_{HV}^-}{2M_W^2}\,\mathcal{O}_-$$

$$\mathcal{O}_\pm \equiv (\mathcal{O}_W - \mathcal{O}_B) \pm (\mathcal{O}_{HW} - \mathcal{O}_{HB}) \quad , \quad \mathcal{O}_{WW} = 4\,\mathcal{O}_- + \mathcal{O}_{BB} \quad , \quad \kappa_{HV}^\pm = \tfrac{1}{2}(\kappa_{HW} \pm \kappa_{HB})$$

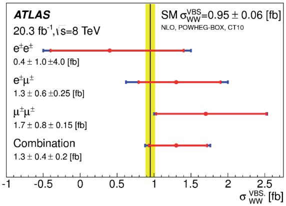
WW Scattering @ LHC

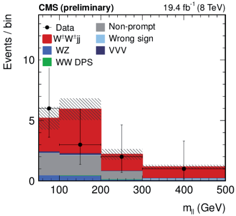
Berryhill

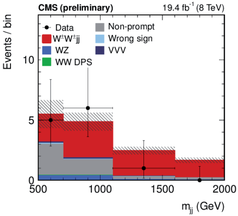
First evidence of $W^\pm W^\pm$ scattering (3.6 σ)

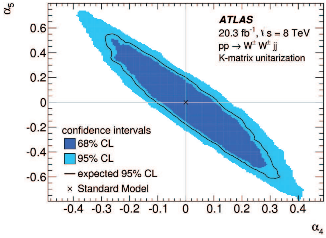


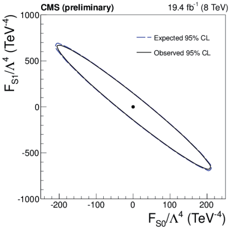
ATLAS, arxiv:1405.6241











CMS-PAS-SMP-13-015

Strongly-Coupled Scenarios

- **Symmetry Breaking:** $SU(2)_L \otimes SU(2)_R \to SU(2)_{L+R}$
- **Goldstone Dynamics** ➨ **Electroweak Effective Theory**
- **Strong Electroweak Dynamics** ➨ **Heavy Resonances**
- **Many possibilities:** **Technicolour, Walking Technicolour, Conformal Technicolour, CFT, 5D . . .**
- **Light Scalar Resonance $S_1(125)$**

  ➨ **Pseudo-Goldstone (composite) Higgs, Dilaton . . .**

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$$\omega = \kappa_W = \kappa_Z = a = \begin{cases} 1 & \text{SM} \\ \sqrt{1 - v^2/F^2} & \text{MCHM4, MCHM5} \\ v^2/F^2 & \text{Dilaton} \end{cases}$$

Resonance Effective Theory

- **Towers of heavy states are usually present in strongly-coupled models of EWSB: Technicolour, Walking TC...**

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This program works in QCD: RχT **(Ecker–Gasser–Leutwyler–Pich–de Rafael)**

Good dynamical understanding at large N_C

Gauge Boson Self-Energies: S, T

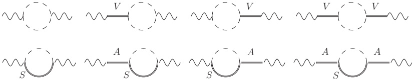
LO: Sensitive to vector and axial states

Peskin-Takeuchi

$$S_{\rm LO} = 4\pi \left(\frac{F_V^2}{M_V^2} - \frac{F_A^2}{M_A^2} \right) = \frac{4\pi v^2}{M_V^2} \left(1 + \frac{M_V^2}{M_A^2} \right) \quad \Rightarrow \quad M_A > M_V > 1.5 \text{ TeV}$$

NLO: Sensitive to the light scalar $S_1(125)$

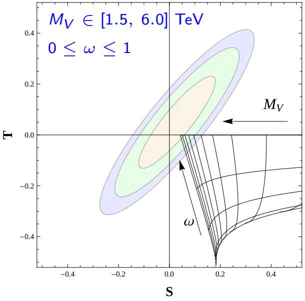
AP, Rosell, Sanz-Cillero, 1212.6769



$$\omega \equiv \frac{g_{SWW}}{g_{HWW}^{\rm SM}} \in [0.94,\, 1] \qquad (95\%\ \text{CL})$$

$$M_A \approx M_V > 4 \text{ TeV}$$

Assumes a good UV behaviour as in asymptotically-free gauge theories



$1^{\rm st} + 2^{\rm nd}$ **WSRs**

EW Effective Theory Summary

- **Effective Field Theory:** powerful low-energy tool
- **Mass Gap:** $E, m_{light} \ll \Lambda_{NP}$
- **Assumption:** relevant symmetries (breakings) & light fields
- Most general $\mathcal{L}_{eff}(\phi_{light})$ allowed by symmetry
- Short-distance dynamics encoded in LECs
- LECs constrained phenomenologically
- **Goal:** get hints on the underlying fundamental dynamics

→ **New Physics**

Learning from QCD experience. EW problem more difficult

Fundamental Underlying Theory unknown

QCD	?
↓	↑
χPT	Standard Model

Additional dynamical input (fresh ideas!) needed

Status & Outlook

- **A new boson discovered**
- **H(125) behaves as the SM Higgs particle**
- **The SM appears to be the right theory at the EW scale**
- **New physics needed to explain many pending questions**
  Flavour, CP, baryogenesis, dark marrer, cosmology . . .

- **How far is the Scale of New Physics Λ_{NP}?**
- **Which symmetry keeps M_H away from Λ_{NP}?**
  Supersymmetry, scale/conformal symmetry . . .
- **Which kind of New Physics?**

Awaiting great discoveries @ LHC

Too many operators/couplings ➡ **Further input needed**

$$\mathcal{F}_X(h) \;=\; \sum_{n=0} c_X^{(n)} \left(\frac{h}{v}\right)^n \;=\; \sum_{n=0} \tilde{c}_X^{(n)} \left(\frac{g_h\, h}{\Lambda_{\mathrm{NP}}}\right)^n$$

- **Weak coupling:** $g_h \ll 1$
- **Strong coupling:** $g_h \sim 4\pi = \Lambda_{\mathrm{NP}}/f$ ➡ $\mathcal{F}_X(h/f)$
- **v ≪ f** ➡ $\xi \equiv \frac{v^2}{f^2}$, $c_X^{(n)} \;=\; \tilde{c}_X^{(n)}\, \xi^{n/2}$

Short-Distance Conditions (UV Behaviour)

- **In asymptotically-free gauge theories:** $\langle VV - AA \rangle \sim s^{-3}$ at $s \to \infty$

  ➔ Weinberg Sum Rules $(F_V^2 - F_A^2 = v^2 \quad , \quad F_V^2 M_V^2 = F_A^2 M_A^2)$

- **Softer requirement valid in theories with non-trivial UV fixed points:**

  $\langle VV - AA \rangle \sim s^{-2}$ ➔ 1^{st} WSR only (assume still $M_V < M_A$)

$$S_{\rm LO} > \frac{4\pi v^2}{M_V^2}$$

$\omega \equiv g_{SWW} / g_{HWW}^{\mathsf{SM}}$ **very different from the SM requires large (unnatural) mass splitings**

AP, Rosell, Sanz-Cillero, 1212.6769