Magnetic Wilson loops in the classical field of high-energy heavy-ion collisions

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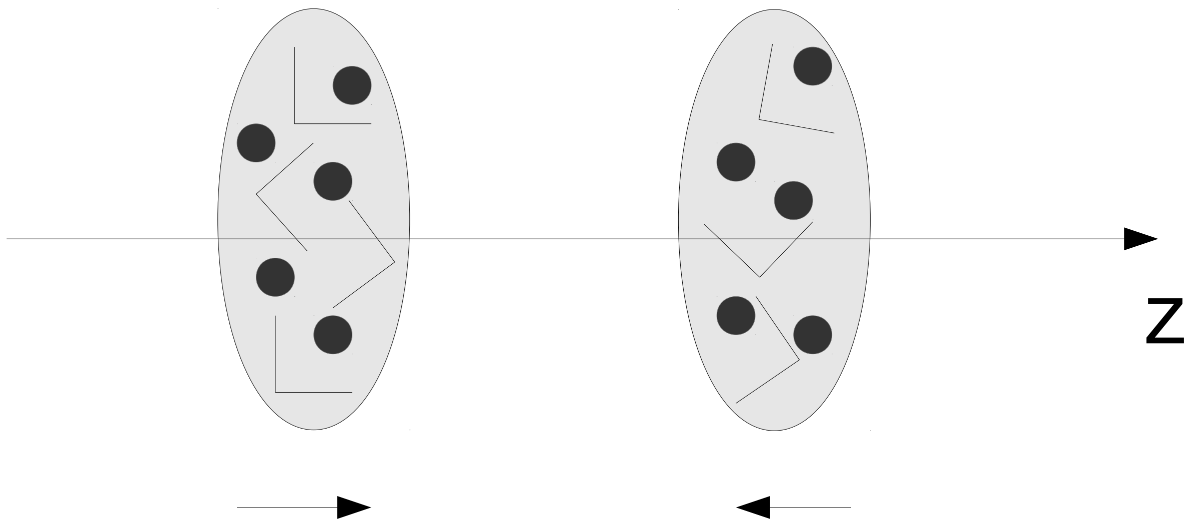
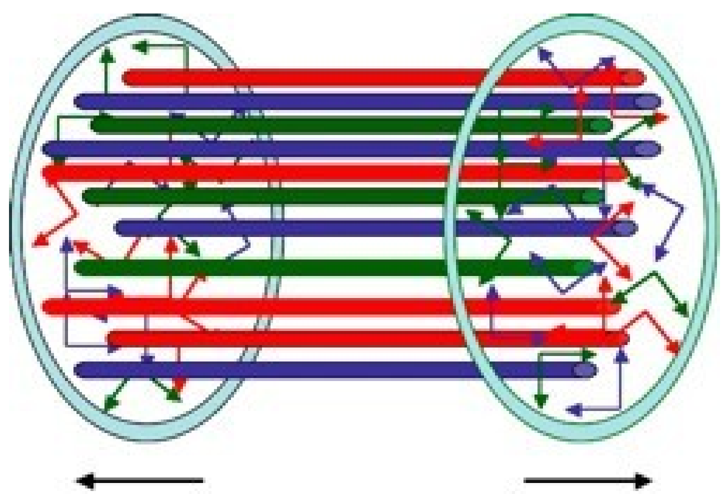
Initial conditions

Before the collision



Projectile Target

After the collision



Target Projectile

Talk by Bin Wu

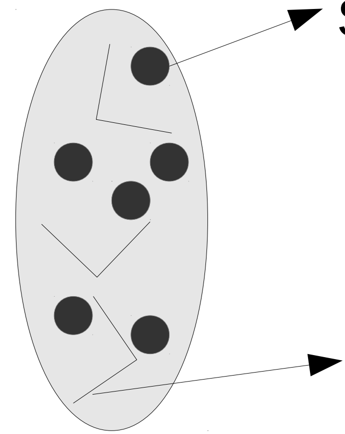
McLerran – Venugopalan model

L. D. McLerran and R. Venugopalan, (1994)

- Large nucleon number

$A^{1/3} \to \infty$

- Small Bjorken -*x*



- High occupation number => Classical treatment of the fields; $\hbar = \frac{1}{k}$
- Weak coupling

Color charge squared per unit transverse area $\mu^2 = \frac{g^2 A}{\pi R^2} \sim A^{1/3}$

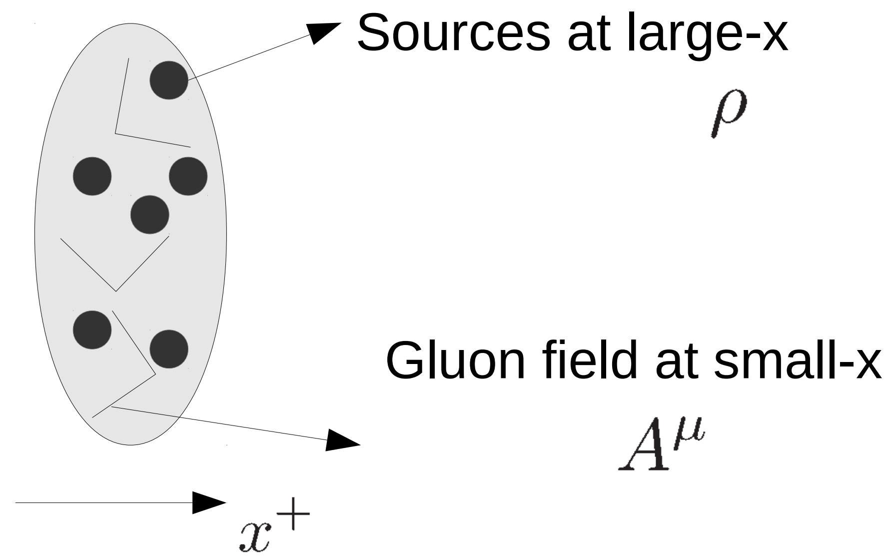
$\mu^2 \gg \Lambda^2_{QCD}$ $\alpha_s \equiv \alpha_s(\mu^2) \ll 1$

Saturation scale $Q_s \sim g^2 \mu \sim A^{1/3}$

Field of a single nucleus

Solve classical Yang-Mills equations of motion

$$[D_\mu, F^{\mu\nu}] = J^\nu$$



$$J^\nu = \delta^{\nu +} J^+$$

$$J^+(x^+ = x_0^+) = g\rho(x^-, x_\perp) \sim g\rho(x_\perp)\delta(x^-)$$

The solution in light-cone gauge is a pure gauge

$$\alpha_m^i = \frac{i}{g} U_m \, \partial^i U_m^\dagger \quad , \quad \partial^i \alpha_m^i = g\rho_m \; .$$

Classical filed after the collision

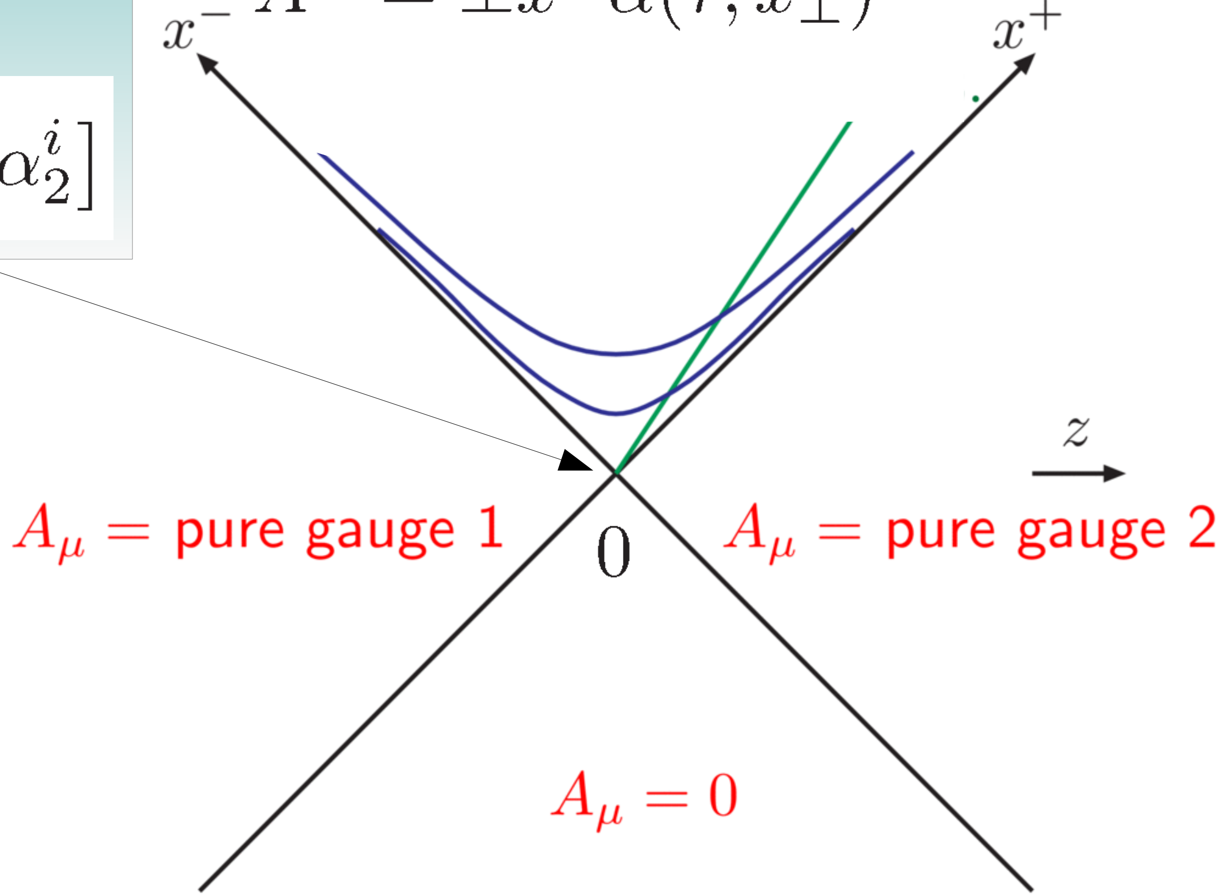
Kovner, McLerran, Weigert (1995)

$$A^i = \alpha_3^i(\tau, x_\perp)$$

$$A^\pm = \pm x^\pm \alpha(\tau, x_\perp)$$

$$A^i = \alpha_1^i + \alpha_2^i$$

$$\alpha = -\frac{ig}{2}\left[\alpha_{1i}, \alpha_2^i\right]$$



Solution at proper time $\tau = \sqrt{t^2 - z^2} = 0$

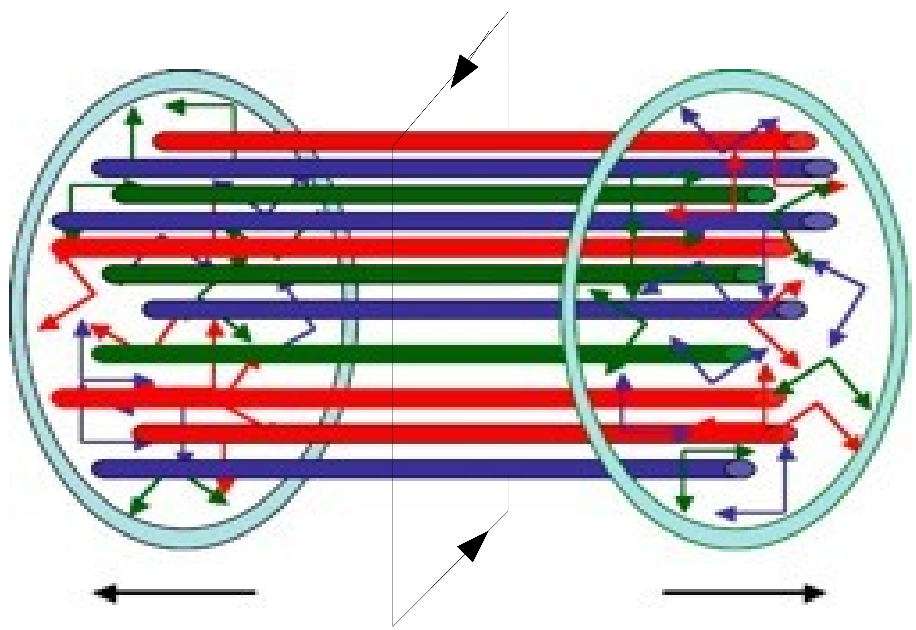
Longitudinal chromo-electric and chromo-magnetic fields at

$$\tau = \sqrt{t^2 - z^2} = 0$$

Kharzeev, Krasnitz, Venugopalan, 2002
R. Fries, J. Kapusta, Y. Li, 2006
Lappi, McLerran, 2006

$$E_z = ig[\alpha_1^i, \alpha_2^i] \ ,$$

$$B_z = ig\epsilon^{ij}[\alpha_1^i, \alpha_2^j] \quad , \quad (i, j = 1, 2)$$



Non-Abelian Wilson loop in the classical field of a collision

$$M(R) = \mathcal{P}\exp\left(ig\oint dx^i A^i\right) = \mathcal{P}\exp\left[ig\oint dx^i\,(\alpha_1^i + \alpha_2^i)\right]$$

Calculate loop's expectation value:

$$W_M(R) = \frac{1}{N_c}\langle\ \mathrm{tr}\ M(R)\rangle$$

Gaussian distribution of sources:

$$W[\rho] = \exp\left[-\int d^2x_\perp \frac{\delta^{ab}\rho^a(x_\perp)\rho^b(x_\perp)}{2\mu^2}\right]$$

Abelian case, Wilson loop measures magnetic flux:

$$\oint d\vec{l}\;\vec{A} = \int d\vec{a}\;\vec{B} \equiv \Phi \qquad exp\left[ig\oint d\vec{l}\;\vec{A}\right] = exp\left[ig\Phi\right]$$

Single valued *A* field, flux quantization: $g\oint d\vec{l}\;\vec{A} = 2\pi n$

Non-Abelian case, gauge fields transform as a representation of $SU(N)/Z_N$

$$Z_N = \left\{e^{2\pi i n/N}\mathbb{1}, n = 0\ldots N-1\right\}$$

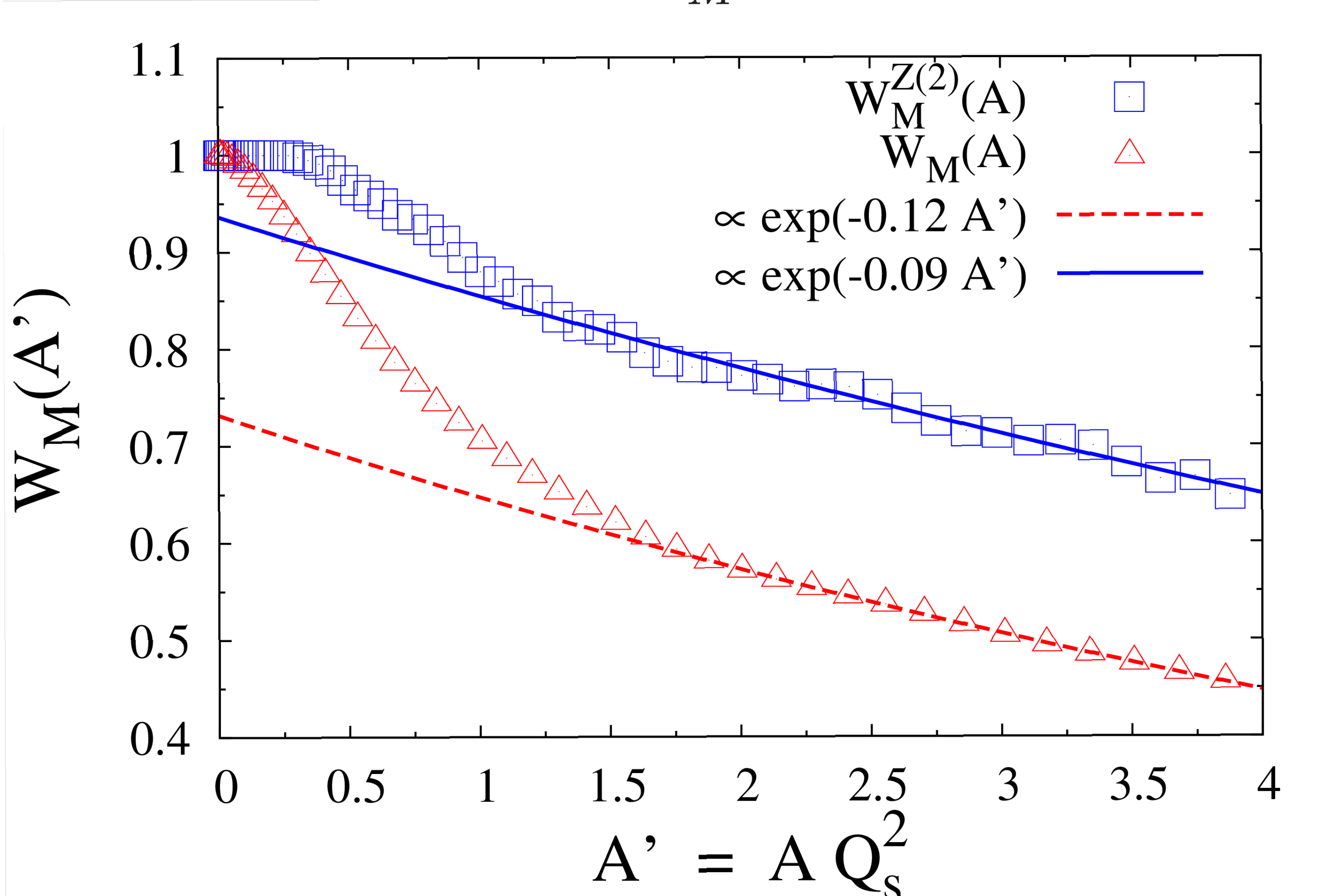
$$\mathcal{P}exp\left[ig\oint d\vec{l}\;\vec{A}\right] = e^{2\pi i n/N}\mathbb{1} \qquad e^{2\pi i(n_1+n_2+n_3)/N}\mathbb{1}$$

For completely uncorrelated magnetic vortices => area law behavior.

$$\langle W\rangle \sim \exp[-\mathrm{Area}]$$

Numerical results for W_M

A. Dumitru, Y. Nara, E. P., 2013



Area law behavior: $W_M(R) \sim \exp\left(-\sigma_M A\right)$ $\sigma_M \simeq 0.12 Q_s^2$

$\Longrightarrow$ Uncorrelated magnetic flux vortices with radius:

$$R_{\rm vtx} \sim 1/Q_s$$

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Numerical results for W_M

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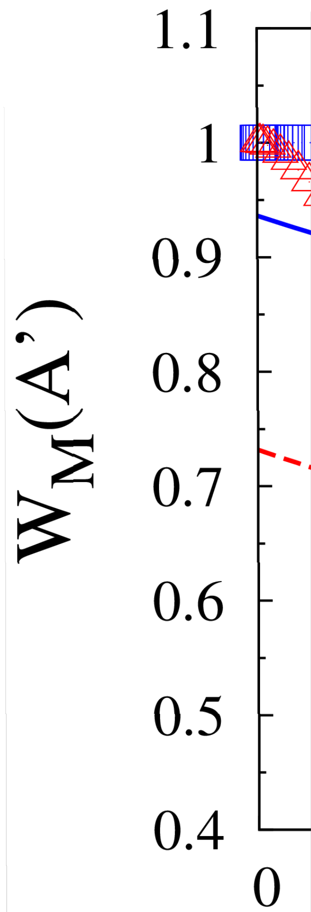
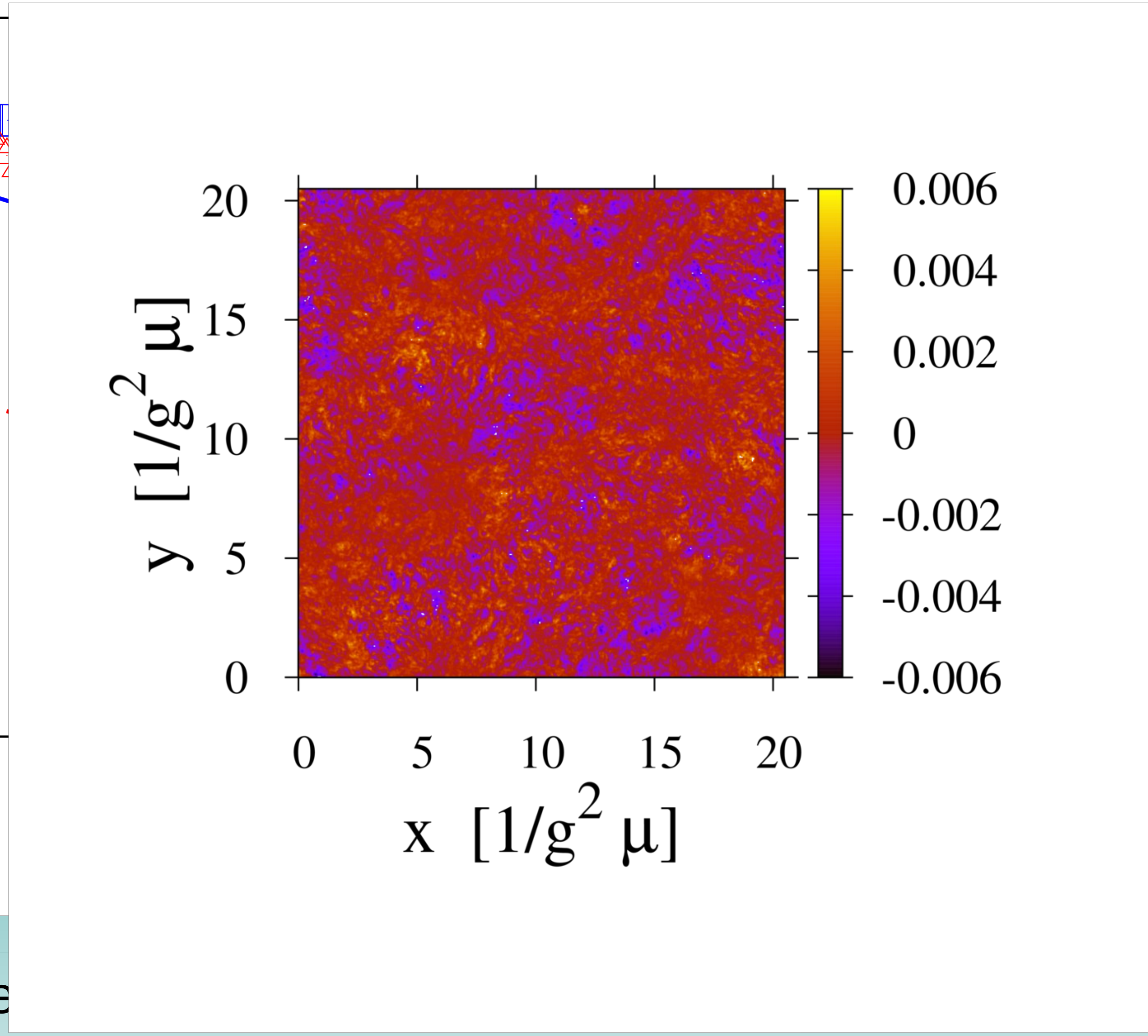




Area law be ... Q_s^2

$\Longrightarrow$ Uncorrelated magnetic flux vortices with radius:

$$R_{\rm vtx} \sim 1/Q_s$$

Perturbative calculation

Single nucleus: $M(R) = \mathcal{P} \exp\left(ig \oint dx^i \alpha_m^i \right) = \mathbb{1}$

In a collision: $M(R) = \mathcal{P} \exp\left[ig \oint dx^i \left(\alpha_1^i + \alpha_2^i \right) \right] = \mathcal{P} \exp\left[X_1 + X_2 \right]$

$$X_m = ig \oint dx^i \alpha_m^{ai} t^a$$

$$\exp\ X \exp\ Y = \exp\left\{ X + Y + \frac{1}{2}[X, Y] + \dots \right\}$$

$$W_M(R) \simeq \frac{1}{N_c} \left\langle \mathrm{tr} \exp\left(-\frac{1}{2}[X_1, X_2] \right) \right\rangle \simeq 1 - \frac{1}{2N_c} \left\langle g^2 h^2 \right\rangle$$

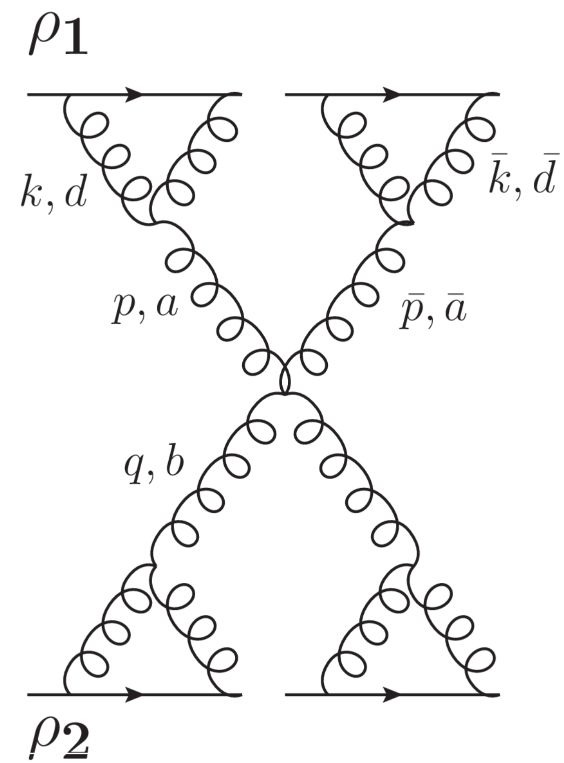
$$g^2 h^2 = \frac{1}{16} f^{abc} f^{\bar{a}\bar{b}c} X_1^a X_1^{\bar{a}} X_2^b X_2^{\bar{b}}$$

Perturbative calculation

Diagram representation of:

$$h^2 = \frac{1}{16g^2} f^{abc} f^{\bar{a}\bar{b}c} X_1^a X_1^{\bar{a}} X_2^b X_2^{\bar{b}}$$

$$dx^i \alpha_m^i \Big) = \mathbb{1}$$



$$\exp\ X \exp\ Y = \exp\left\{X + Y + \frac{1}{2}[X\right.$$

$$W_M(R) \simeq \frac{1}{N_c}\left\langle \mathrm{tr}\exp\left(-\frac{1}{2}[X_1\right.\right.$$

$$g^2 h^2 = \frac{1}{16} f^{abc} f^{\bar{a}\bar{b}c} X_1^a X_1^{\bar{a}} X_2^b X_2^{\bar{b}}$$

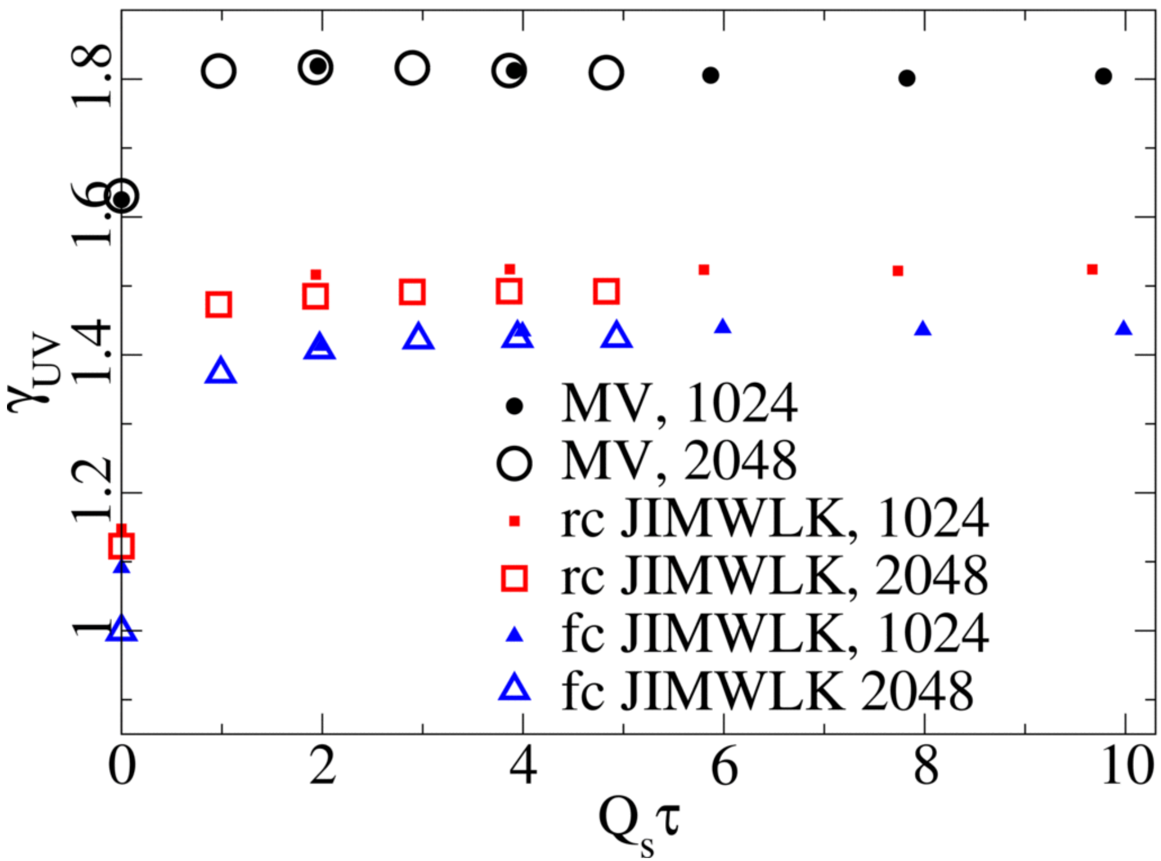
Perturbative result for the expectation value of the magnetic Wilson loop:

$$W_M(R) \simeq 1 - \frac{\pi^2 N_c^6}{64(N_c^2-1)^3} \frac{Q_{s1}^4 Q_{s2}^4}{\Lambda^4} A^2$$

Lattice calculations for small loops

$$e^{-3.5} < AQ_s^2 < e^{-0.5}$$

A. Dumitru, T. Lappi, Y. Nara, (2014)

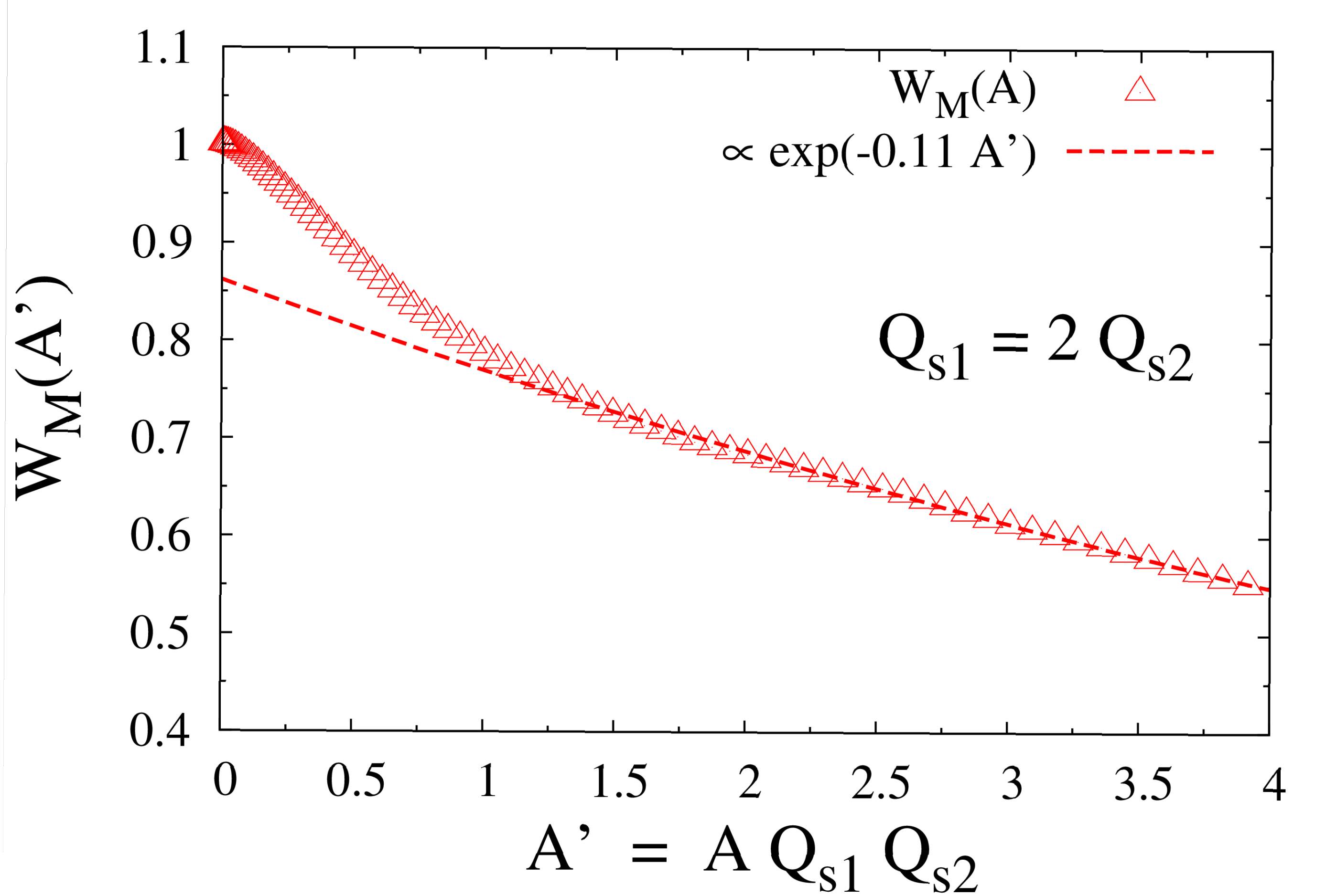


$$W(A) = \exp\left[-(\sigma A)^{\gamma}\right]$$

A term proportional to the area of the loop: $\sim A\, Q_{s1} Q_{s2}$

Gaussian contractions can give only powers of $Q_{s1}^2\ ,\ Q_{s2}^2$

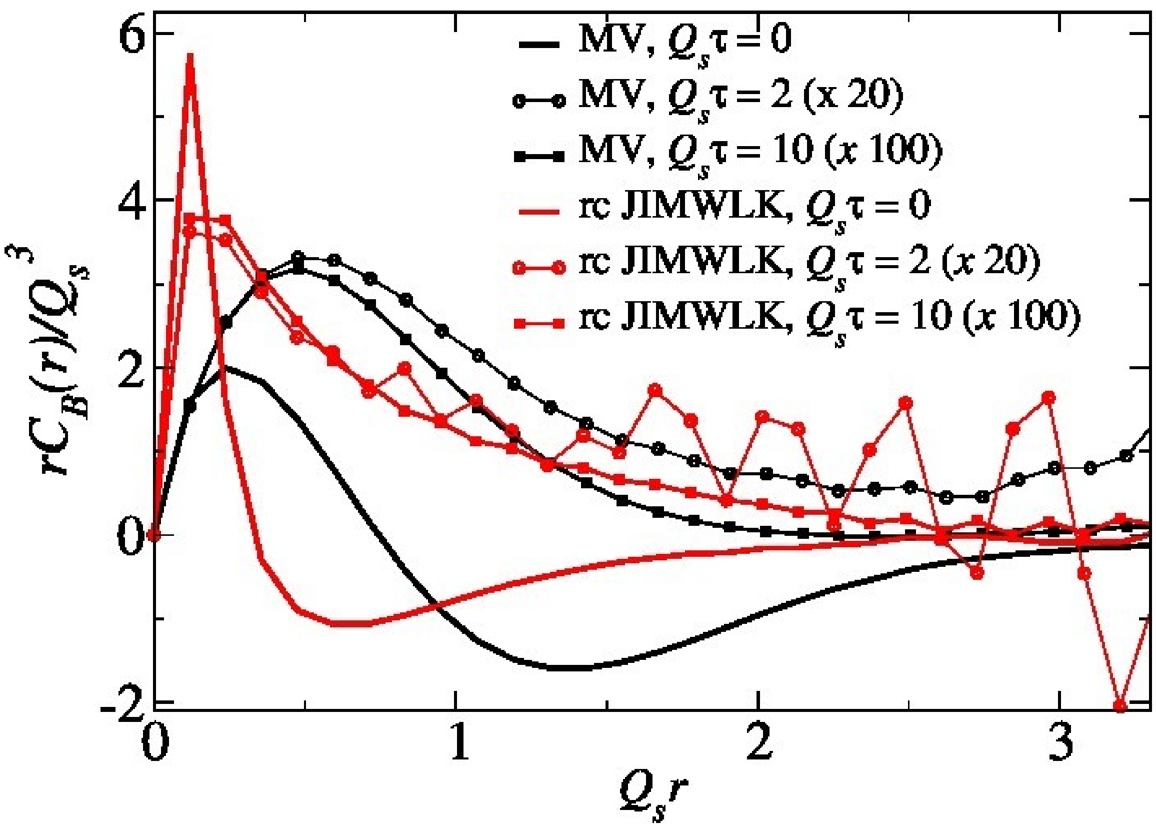
$$\langle \rho_m^a(\mathbf{x})\, \rho_m^b(\mathbf{y}) \rangle = \mu_m^2 \delta^{ab} \delta(\mathbf{x}-\mathbf{y}) \sim Q_{s_m}^2$$



A. Dumitru, T. Lappi, Y. Nara, 2014

Magnetic field correlator

$$C_B \equiv 2g^2 \mathrm{Tr} \left\langle B_z(x_\perp) U_{x_\perp \to y_\perp} B_z(y_\perp) U^\dagger_{x_\perp \to y_\perp} \right\rangle$$

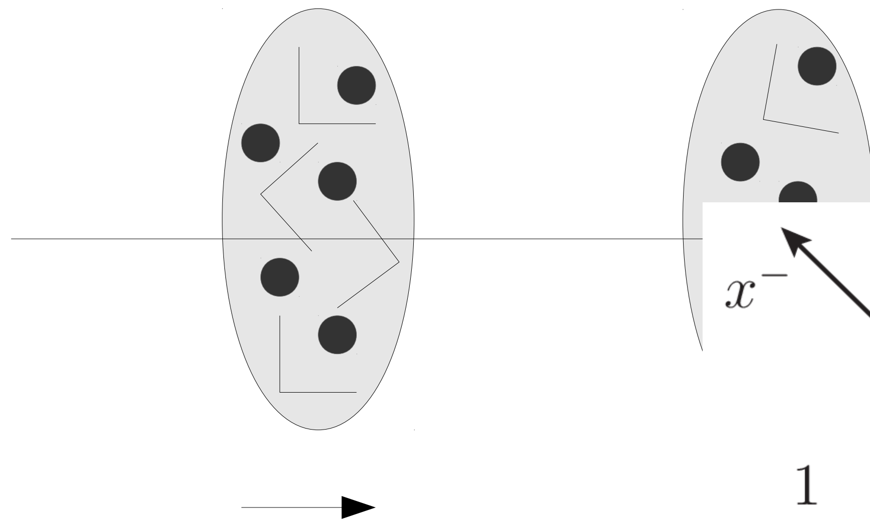


Thank you

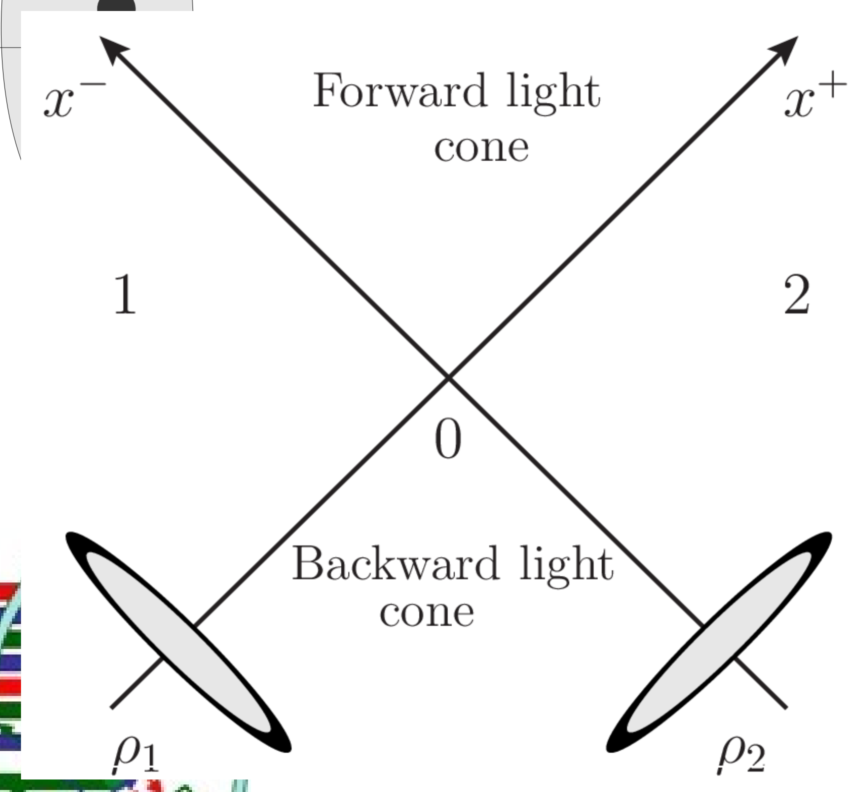
Back up

Initial conditions

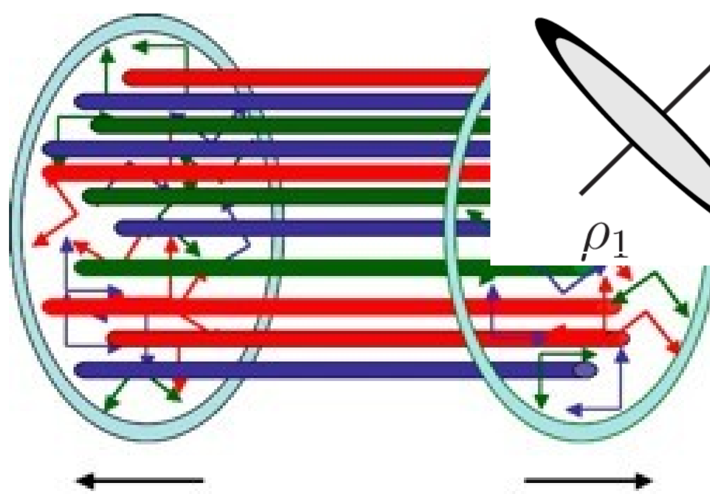
Before the collision



Projectile



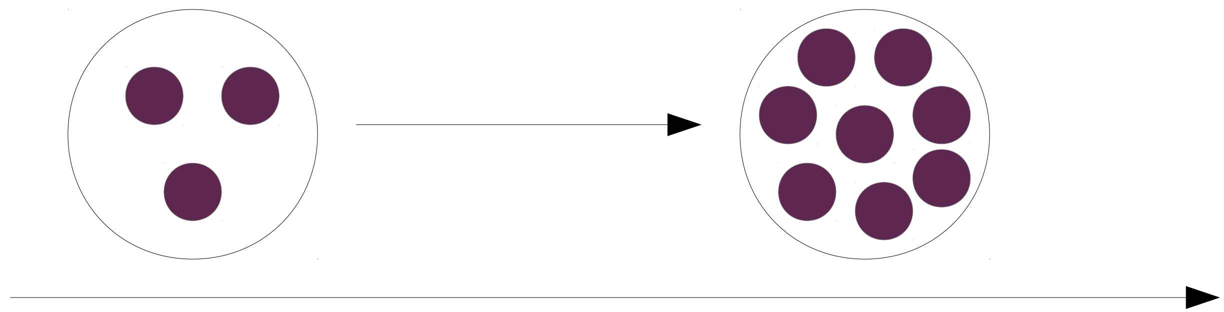
After the collision



Target

Projectile

Energy evolution



$$\ln\frac{1}{x}$$

$$x = \frac{k_z}{P_z}$$ Longitudinal momentum fraction

Linear BFKL equation ----> Fast growth of the gluon densities with decreasing x:

Balitsky, Fadin, Kuraev, Lipatov 1975-1978

$$\phi(x, k_\perp^2) \sim \left(\frac{1}{x}\right)^\lambda \qquad \lambda = \frac{4\alpha_s N_c}{\pi}\ln 2$$

Violates Froissart bound

Small-x limit

Saturation of parton distributions

$$Q_s^2 \sim A^{\frac{1}{3}} \left(\frac{1}{x}\right)^{\lambda}$$

Saturation scale

Gribov, Levin, Ryskin (1983)

The Wilson loop measures magnetic flux:

$$\int d\vec{l}\ \vec{A} = \int d\vec{a}\ \vec{B} \equiv \Phi$$

Area law indicates uncorrelated domains.

For $N_c = 2$ $\qquad W_M^{Z(2)}(R) = \langle \mathrm{sgn}\ \mathrm{tr}\ M(R) \rangle$

$$M(R) = \exp\left[\frac{2\pi i n}{N_c}\right] \mathbb{1}$$

$\Rightarrow$

$$R_{\mathrm{vtx}} \sim 0.8/Q_s$$

Uncorrelated magnetic flux vortices with radius:

Gaussian distribution of sources

Calculation of observables $$\langle O[\rho]\rangle \equiv \frac{\int \mathcal{D}\rho\, W[\rho] O[\rho]}{\int \mathcal{D}\rho\, W[\rho]}$$

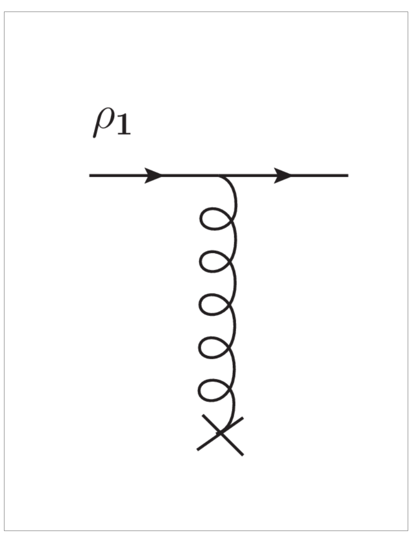
$$\boxed{W[\rho] = \exp\left[-\int d^2 x_\perp \frac{\delta^{ab}\rho^a(x_\perp)\rho^b(x_\perp)}{2\mu^2}\right]}$$

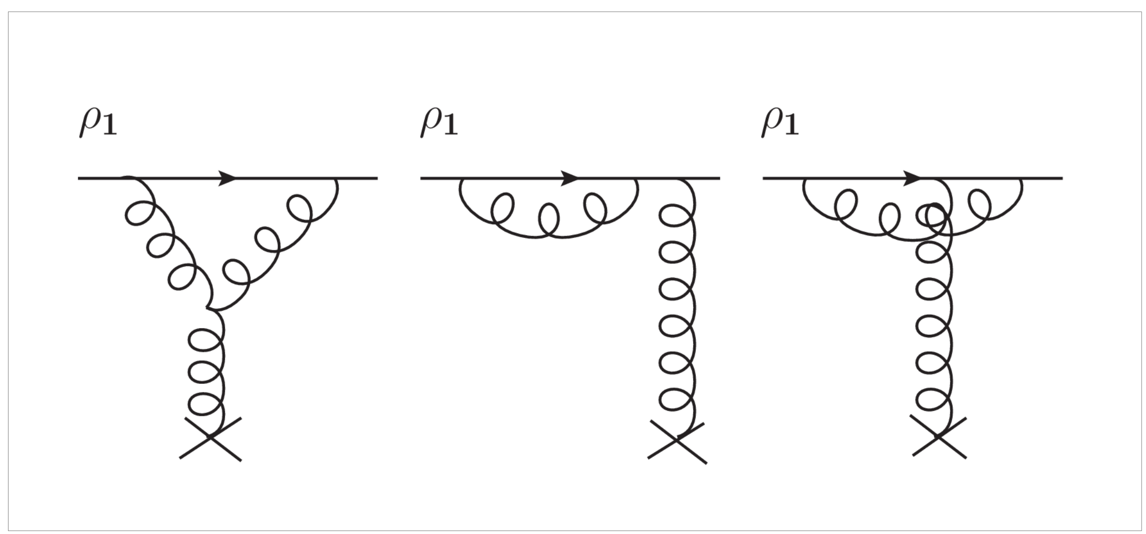
$$\mu^2 = \frac{g^2 A}{\pi R^2}$$ Color charge squared per unit transverse area

Valid for a large nucleus: $A^{1/3} \to \infty$

Diagram representation:

$$\alpha_m^i = -\partial^i\Phi_m + \frac{ig}{2}\left(\delta^{ij} - \partial^i \frac{1}{\nabla_\perp^2}\partial^j\right)\left[\Phi_m, \partial^j\Phi_m\right] + \mathcal{O}\left(\Phi_m^3\right)$$





$$\alpha^i_m = -\partial^i \Phi_m + \frac{ig}{2}\left(\delta^{ij} - \partial^i \frac{1}{\nabla^2_\perp}\partial^j\right)\left[\Phi_m, \partial^j \Phi_m\right] + \mathcal{O}\left(\Phi^3_m\right)$$

Find X_m

$$X_m = ig \oint dx^i \alpha^{ai}_m t^a$$

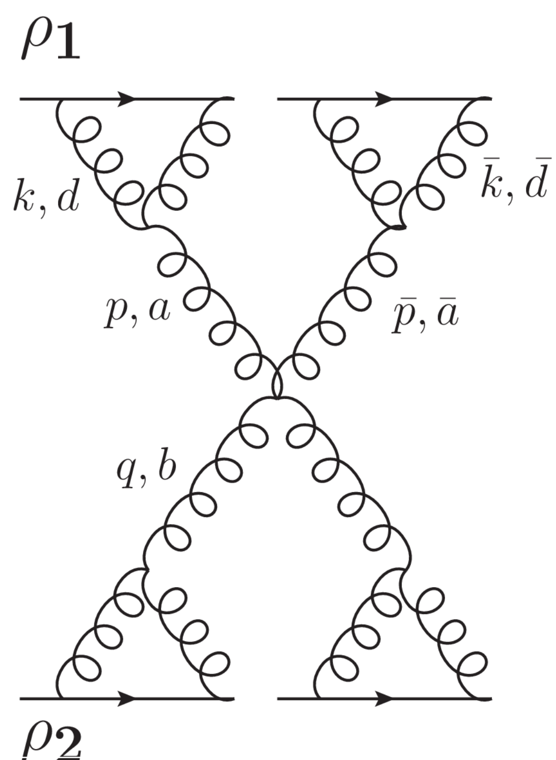
The lowest order does not contribute. Use the term $\sim \Phi^2$

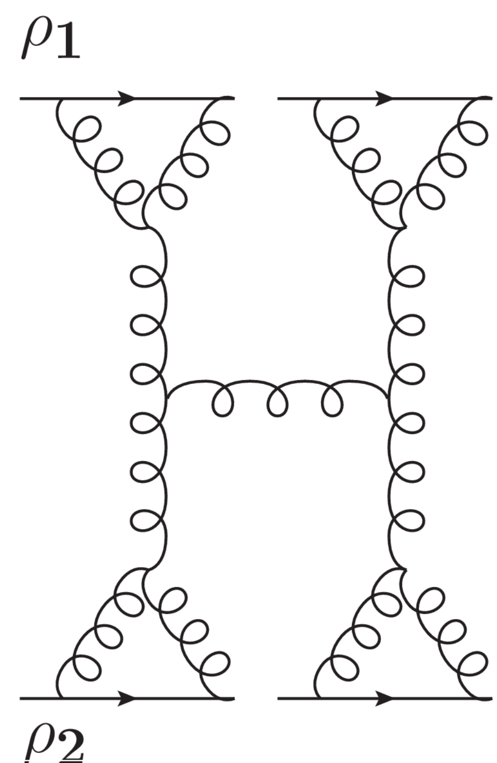
$$\boxed{X^a_m = -\frac{g^2}{2} f^{ade} \oint dx^i \; \Phi^d_m \partial^i \Phi^e_m}$$

Diagram representation of:

$$h^2 = \frac{1}{16g^2} f^{abc} f^{\bar{a}\bar{b}c} X_1^a X_1^{\bar{a}} X_2^b X_2^{\bar{b}}$$

$$X_m^a = -\frac{ig^2}{2} f^{ade} \oint dx^i \; \Phi_m^d \partial^i \Phi_m^e$$





Correction diagram

Calculate $\left\langle \Phi^d_m(\mathbf{k})\ \Phi^e_m(\mathbf{p}-\mathbf{k})\ \Phi^{\bar{d}}_m(\bar{\mathbf{k}})\ \Phi^{\bar{e}}_m(\bar{\mathbf{p}}-\bar{\mathbf{k}})\right\rangle_{\rho_m}$

$$\Phi^a(\mathbf{k}) = -\frac{g}{k^2}\rho^a(\mathbf{k}) \qquad \langle\rho^a(\mathbf{k})\,\rho^b(\mathbf{p})\rangle = \mu^2\delta^{ab}(2\pi)^2\delta(\mathbf{k}+\mathbf{p})$$

Extended action:

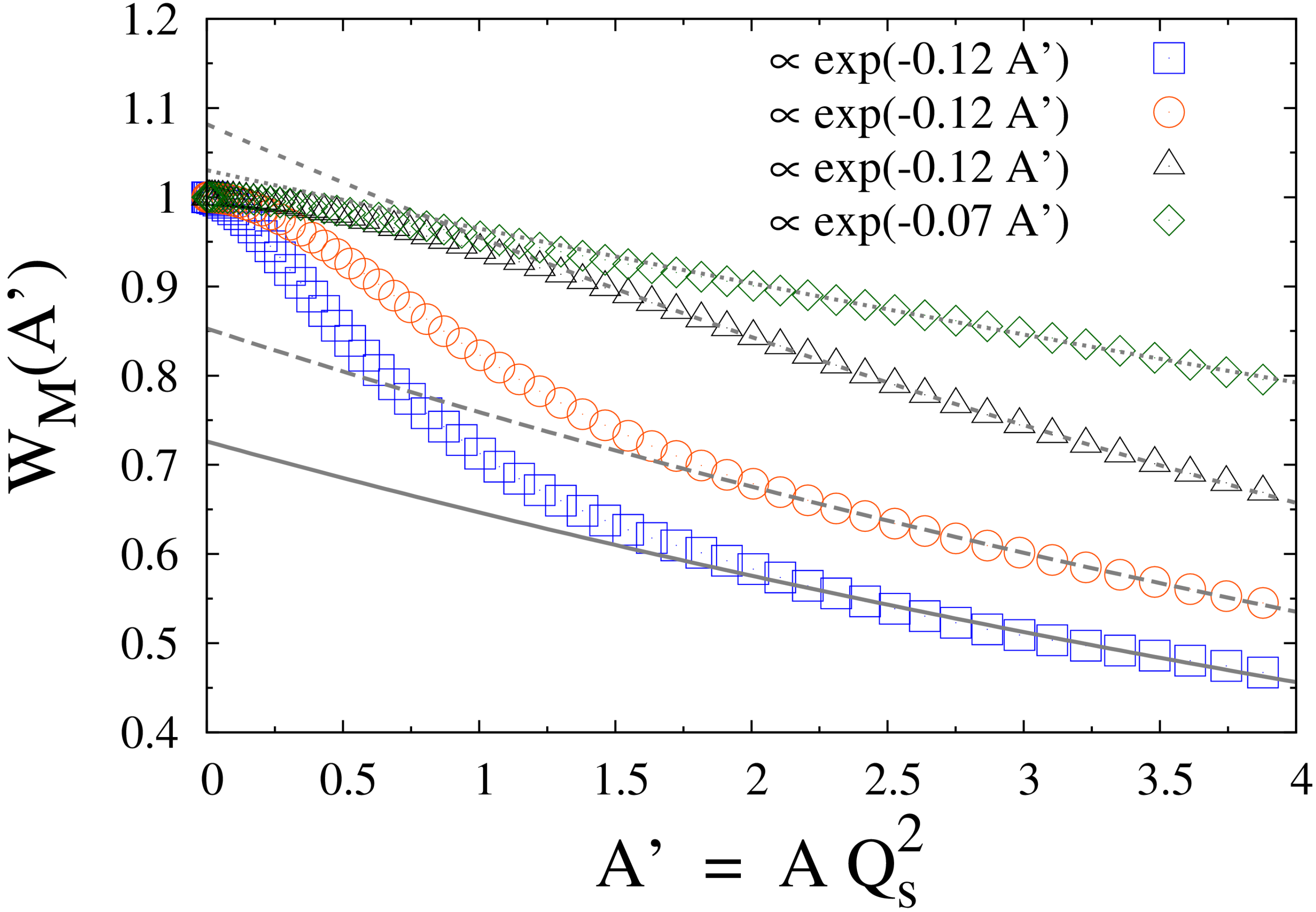
$$S[\rho(x)] \simeq \int d^2x\left[\frac{\delta^{ab}\,\rho^a\rho^b}{2\mu^2} - \frac{d^{abc}\,\rho^a\rho^b\rho^c}{\kappa_3} + \frac{\delta^{ab}\delta^{cd}+\delta^{ac}\delta^{bd}+\delta^{ad}\delta^{bc}}{\kappa_4}\rho^a\rho^b\rho^c\rho^d\right]$$

Cubic term does not contribute.

Quartic term correction to the four-point function:

$$f^{ade}f^{\bar{a}\bar{d}\bar{e}}\left(\delta^{de}\delta^{\bar{d}\bar{e}}+\delta^{d\bar{d}}\delta^{e\bar{e}}+\delta^{d\bar{e}}\delta^{e\bar{d}}\right)=0$$

Time evolution



$$\left\langle g^2 h^2 \right\rangle = \frac{1}{16} f^{abc} f^{\bar{a}\bar{b}c} \left\langle X_1^a X_1^{\bar{a}} \right\rangle_{\rho_1} \left\langle X_2^b X_2^{\bar{b}} \right\rangle_{\rho_2}$$

$$X_m^a = -\frac{ig^2}{2} f^{ade} \oint dx^i \; \Phi_m^d \partial^i \Phi_m^e$$

$$X_m^a = -\frac{ig^2}{2(2\pi)^3} f^{ade} R \int d^2\mathbf{k} d^2\mathbf{p} |\mathbf{k}| J_1(R|\mathbf{p}|) \; \sin(\alpha - \theta) \Phi_m^d(\mathbf{k}) \Phi_m^e(\mathbf{p} - \mathbf{k})$$

$$\left\langle X_m^a X_m^{\bar{a}} \right\rangle_{\rho_m} = -\frac{g^4}{4(2\pi)^6} f^{ade} f^{\bar{a}\bar{d}\bar{e}} R^2 \times$$

$$\int d^2\mathbf{k} \; d^2\mathbf{p} \; d^2\bar{\mathbf{k}} \; d^2\bar{\mathbf{p}} \; |\mathbf{k}||\bar{\mathbf{k}}| \; J_1(R|\mathbf{p}|) J_1(R|\bar{\mathbf{p}}|) \times$$

$$\sin(\alpha - \theta) \sin(\bar{\alpha} - \bar{\theta}) \left\langle \Phi_m^d(\mathbf{k}) \; \Phi_m^e(\mathbf{p} - \mathbf{k}) \; \Phi_m^{\bar{d}}(\bar{\mathbf{k}}) \; \Phi_m^{\bar{e}}(\bar{\mathbf{p}} - \bar{\mathbf{k}}) \right\rangle_{\rho_m}$$

With Gaussian action:

$$\left\langle X_m^a X_m^{\bar{a}} \right\rangle_{\rho_m} = -\frac{g^8 \mu_m^4}{16\pi^2} f^{ade} f^{\bar{a}de} R^2 \times$$

$$\int d^2\mathbf{k}\; d^2\mathbf{p}\; d^2\bar{\mathbf{k}}\; d^2\bar{\mathbf{p}}\; \frac{J_1(R|\mathbf{p}|)J_1(R|\bar{\mathbf{p}}|)}{|\mathbf{k}||\bar{\mathbf{k}}|(\mathbf{p}-\mathbf{k})^2(\bar{\mathbf{p}}-\bar{\mathbf{k}})^2} \times$$

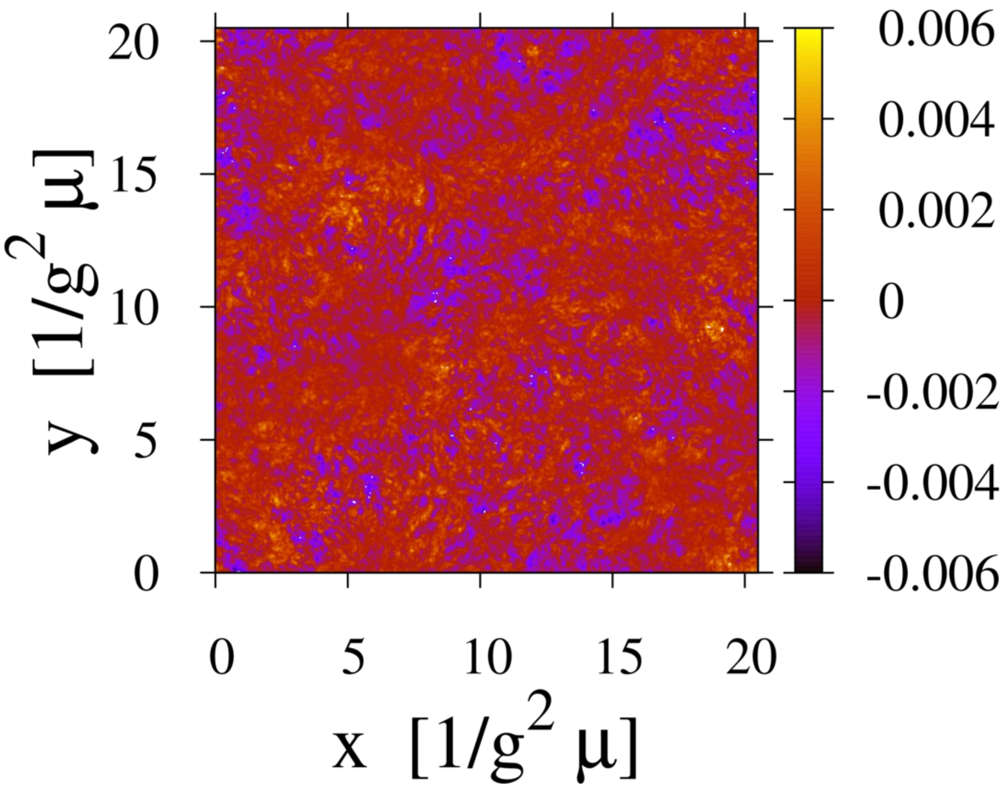
$$\sin(\alpha-\theta)\sin(\bar{\alpha}-\bar{\theta})\times$$

$$\left[\delta(\mathbf{k}+\bar{\mathbf{k}})\delta(\mathbf{p}-\mathbf{k}+\bar{\mathbf{p}}-\bar{\mathbf{k}}) - \delta(\mathbf{k}+\bar{\mathbf{p}}-\bar{\mathbf{k}})\delta(\mathbf{p}-\mathbf{k}+\bar{\mathbf{k}})\right]$$

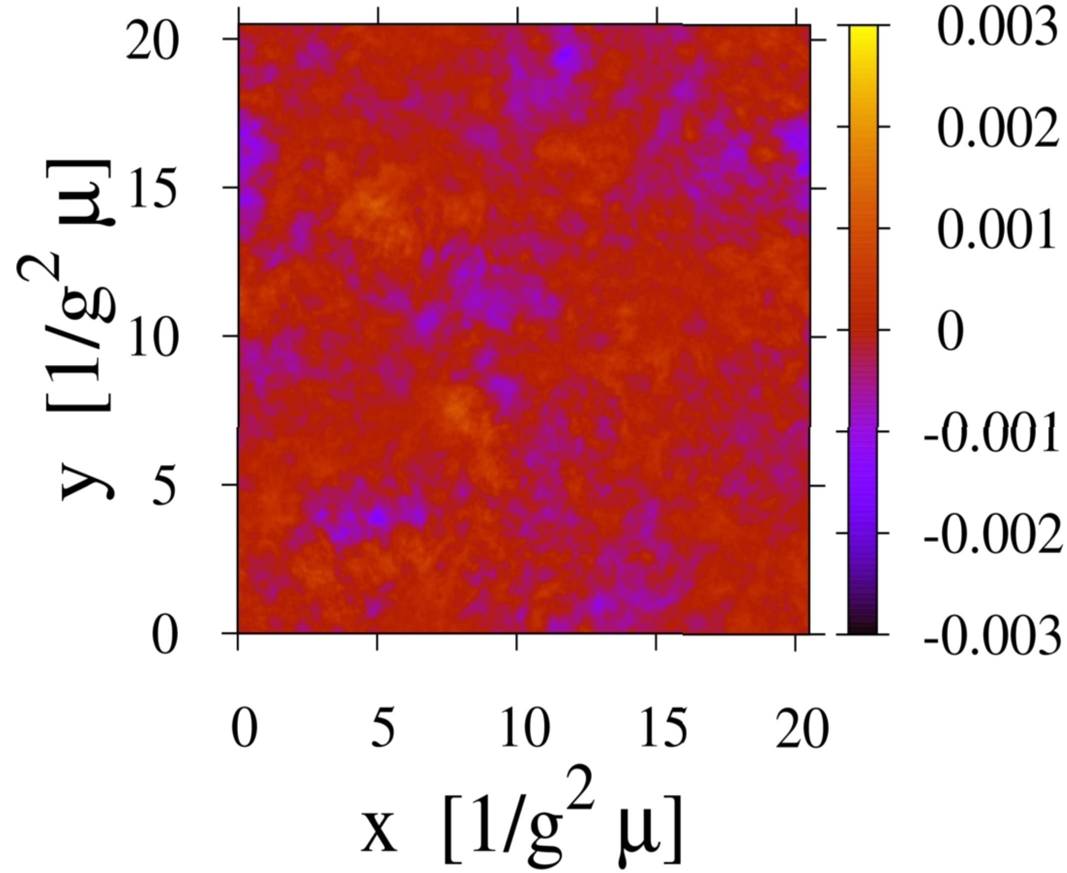
$$\left\langle X_m^a X_m^{\bar{a}} \right\rangle_{\rho_m} = \frac{g^8 \mu_m^4}{8} N_c \delta^{a\bar{a}} R^2 \int \frac{dk}{k^3} \int dp \frac{J_1^2(R|\mathbf{p}|)}{|\mathbf{p}|}$$

$$\int_\Lambda^\infty \frac{dk}{k^3} = \frac{1}{2\Lambda^2}$$

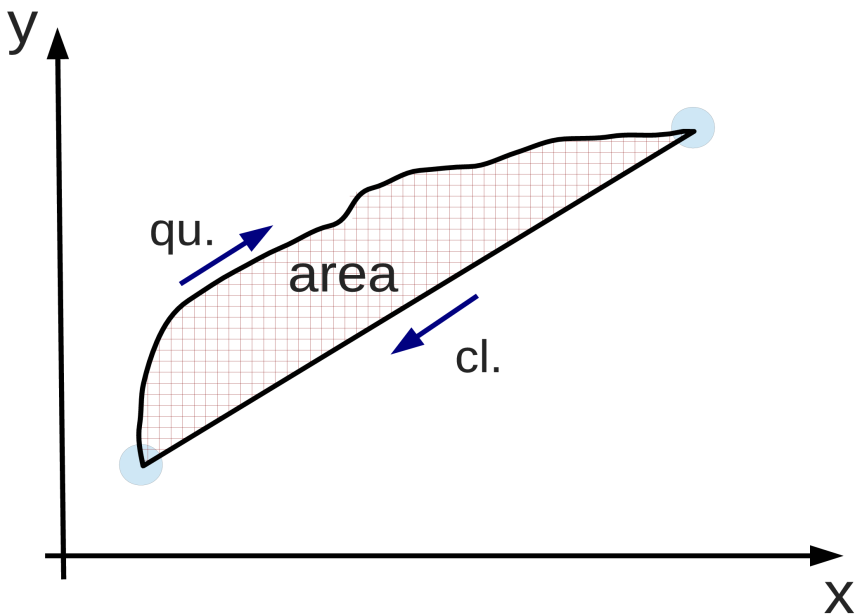
$$\left\langle X_m^a X_m^{\bar{a}} \right\rangle_{\rho_m} = \frac{g^8 \mu_m^4}{32\Lambda^2} N_c \delta^{a\bar{a}} R^2$$



y [1/g^2 μ]
x [1/g^2 μ]
0.006
0.004
0.002
0
-0.002
-0.004
-0.006



y [1/g^2 μ]
x [1/g^2 μ]
0.003
0.002
0.001
0
-0.001
-0.002
-0.003



$$\int_0^\infty ds \int \mathcal{D}x^\mu \left\langle \exp\, i \int\limits_0^s d\tau \left(m\dot{x}^2 + gA_\mu \dot{x}^\mu\right) \right\rangle \sim$$

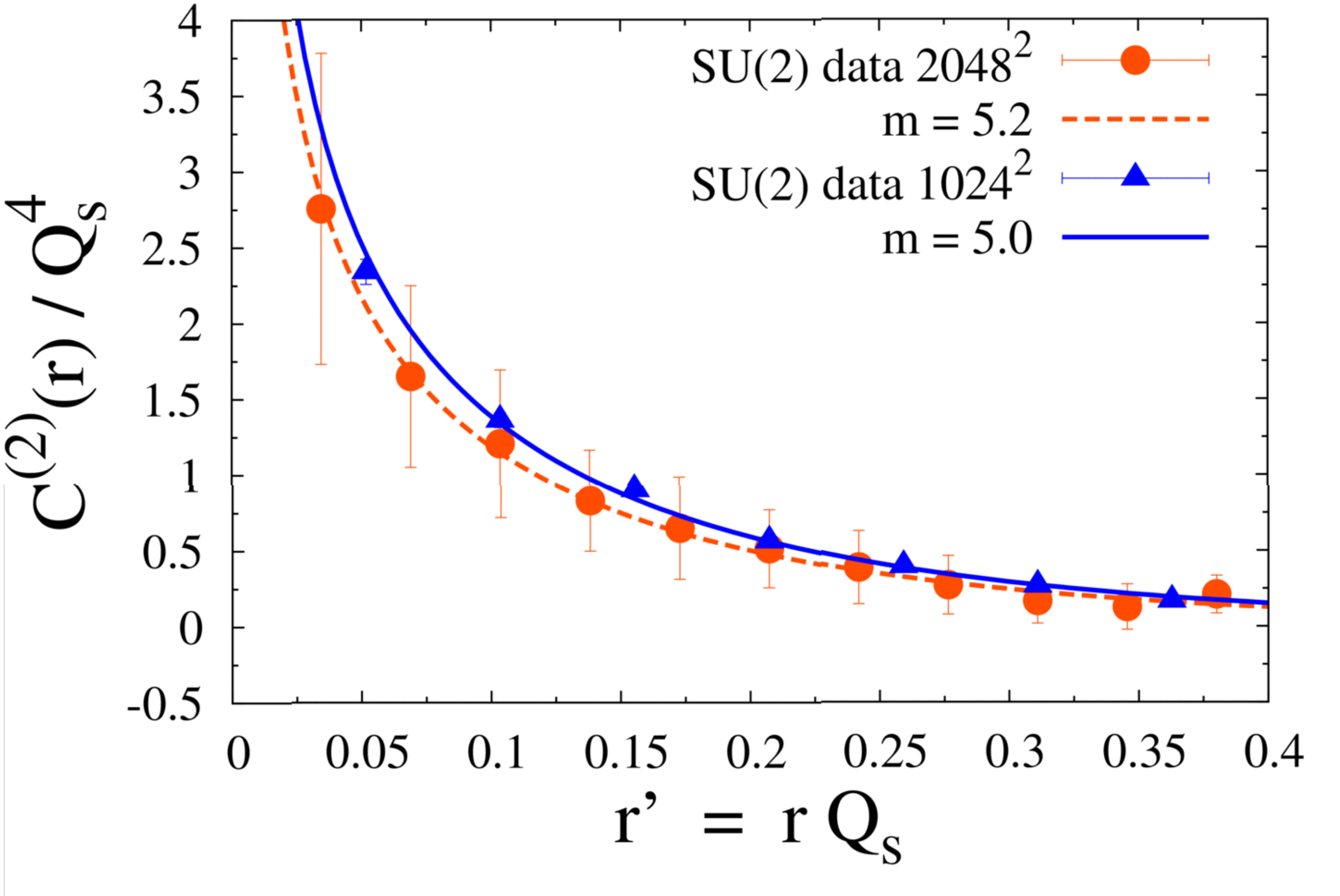
$$\int_0^\infty ds \int \mathcal{D}x^\mu \exp\left(i \int\limits_0^s d\tau\, m\dot{x}^2\right)\, \exp(-\sigma_M A)$$

$$\frac{i}{p^2 + i\sigma_M \frac{m}{p_T}}$$

Area law is due to screening of magnetic fields.

$$G(\mathbf{x}) = g\, U(\mathbf{0} \to \mathbf{x})\, F_{xy}(\mathbf{x})\, U(\mathbf{x} \to \mathbf{0})$$

$$C^{(2)}(r) = \langle \mathrm{tr}\, G(\mathbf{0})\, G(\mathbf{x}) \rangle$$



"Naive" perturbation theory cannot capture the presence of screening corrections.