Recent developments in formal high energy physics

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RPP, January 2015

Quantum field theory

- Feynman integrals and Goncharov polylogarithms
- Effective action in Gaiotto theories
- Truncated conformal space approach

Feynman integrands

- ⋆ On-shell recursion relations [Britto, Cachazo, Feng, Witten]
- ⋆ Generalised unitarity [Bern, Dixon, Dunbar, Kosower]
- ⋆ Duality color/kinematics [Bern, Carrasco, Johansson]

  $\hookrightarrow$ $\mathcal{N} = 4$ superYang–Mills : 5-loop and 6-loop planar
  $\hookrightarrow$ $\mathcal{N} = 8$ supergravity : 4-loop

Feynman integrals as polylogarithms

Example of the 1-loop massless box diagram

$$\frac{d^4\ell(p_1+p_2)^2(p_1+p_3)^2}{\ell^2(\ell+p_1)^2(\ell+p_1+p_2)^2(\ell-p_4)^2}$$
$$= \quad d\ln\left(\frac{\ell^2}{(\ell-\ell^*)^2}\right)d\ln\left(\frac{(\ell+p_1)^2}{(\ell-\ell^*)^2}\right)d\ln\left(\frac{(\ell+p_1+p_2)^2}{(\ell-\ell^*)^2}\right)d\ln\left(\frac{(\ell-p_4)^2}{(\ell-\ell^*)^2}\right)$$

More generally : Goncharov polylogarithms
[Goncharov, Spradlin, Vergu, Volovich]

$$G_{a_1,a_2,\dots a_n}(z)=\int_0^z\frac{dt}{t-a_1}G_{a_2,\dots a_n}(t)=\int_0^z\frac{dt_1}{t_1-a_1}\int_0^{t_1}\frac{dt_2}{t_2-a_2}\dots\int_0^{t_{n-1}}\frac{dt_n}{t_n-a_n}$$

For example $\mathrm{Li}_k(t)=-G_{0^{k-1},1}(t)=\frac{1}{\Gamma(k)}\int_0^\infty dx\frac{x^{k-1}}{e^x t^{-1}-1}$.
Symbol permits to determine the leading transcendental part, modulo identities like the Abel identity

$$\mathrm{Li}_2\left(\frac{x}{1-y}\right)+\mathrm{Li}_2\left(\frac{y}{1-x}\right)-\mathrm{Li}_2\left(\frac{xy}{(1-x)(1-y)}\right)=\mathrm{Li}_2(x)+\mathrm{Li}_2(y)+\ln(1-x)\ln(1-y)$$

Feynman integrals as polylogarithms

Example : two-loop six point MHV amplitude

[Goncharov, Spradlin, Vergu, Volovich]

$$A_6^{\mathrm{MHV}} = A_6^{\mathrm{BDS}}(\epsilon, s_{ij}) \exp\Big(\sum_{i=1}^{3}\big(\mathrm{L}_4(x_i^+, x_i^-) - \tfrac{1}{2}\mathrm{Li}_4(1-1/u_i)\big) - \frac{1}{8}\Big(\sum_{i=1}^{3}\mathrm{Li}_2(1-1/u_i)\Big)^2 + \frac{1}{24}J^4 + \frac{\pi^2}{12}J^2 + \frac{\pi^4}{72}\Big)$$

up to 4-loop [Dixon, Drummond,Duhr, von Hippel, Pennington]

Differential equations for master integrals [Henn, Smirnov]

$$\partial_x \vec{f}(x;\epsilon) = \epsilon \sum_k \frac{A_k}{x - x_k}\vec{f}(x;\epsilon)$$

In the most general case: also eliptic integrals, eliptic polylogarithms.

[Brown, Levin, Bloch, Vanhove, Remiddi, Tancredi, Adams, Bogner, Weinzierl]

The $(2,0)$ theory and Gaiotto theories

Effective theories for M5 branes in M-theory
D3-brane decoupling limit $\Leftrightarrow$ large N $\mathcal{N}=4$ Yang–Mills
M5-brane decoupling limit $\Leftrightarrow$ $(2,0)$ theory in $d=6$
No action principle, selfdual tensor degrees of freedom

$$3\partial_{[\mu}B_{\nu\sigma]} = \frac{1}{2}\varepsilon_{\mu\nu\sigma}{}^{\rho\kappa\lambda}\partial_\rho B_{\kappa\lambda}$$

A mother theory for superconformal field theories in four dimensions [Witten Gaiotto]

$(2,0)$ theory on $S^4 \times \Sigma$
$\hookrightarrow$ $\mathcal{N}=2$ superconformal YM on S^4 $\hookrightarrow$ Toda QFT on Σ
$\hookrightarrow$ Nekrasov–Pestun partition function

Truncated conformal space approach

Local operator satisfy to operator product expansion in exactly solved CFT's

$$\mathcal{O}_i(x)\mathcal{O}_j(0) = \sum_k \frac{f_{ijk}}{|x|^{\Delta_i+\Delta_j-\Delta_k}}(\mathcal{O}_k(0)+\ldots)$$

with on a cylinder $\mathbb{R}\times S_R^{d-1}$

$$\langle i|H_0|j\rangle = R^{-1}\Delta_j\delta_{ij}\ .$$

For $H = H_0 + V$,

$$\langle i|V|j\rangle \propto R^{-1}R^{d-\Delta_\mathcal{V}} f_{\mathcal{O}_i^\dagger\mathcal{V}\mathcal{O}_j}\ .$$

Solve numerically on a truncated set of operator, *e.g.* bound on the dimension of the operators. [Hogervorst, Rychkov, van Rees]
So far works for super-renormalizable theories.

Superstring and supergravity

- Amplitudes
  - Subleading infrared behaviour
  - 3-loop superstring amplitude
  - Off-shell Green functions
- Black holes
  - Entropy from localisation
  - Mathur conjecture and superstrata

Subleading infrared behaviour

Weinberg soft theorem for gravitational amplitudes

$$\mathcal{M}_{n+1}(p_1, p_2, \dots p_n, q) = \Big(\sum_{a=1}^{n} \frac{e_{\mu\nu}(q) p_a^\mu p_a^\nu}{q p_a}\Big) \mathcal{M}_n(p_1, p_2, \dots p_n) + \mathcal{O}(q^0)$$

ensuring that loop infrared divergences are compensated in any cross section including integration over soft momenta. One has moreover [Cachazo, Strominger]

$$\mathcal{M}_{n+1}(p_1, p_2, \dots p_n, q) = \Big(\sum_{a=1}^{n} \frac{e_{\mu\nu}(q)\big(p_a^\mu p_a^\nu - i q_\sigma J_a^{\sigma\mu} p_a^\nu - \frac{1}{2} q_\sigma J_a^{\sigma\mu} q_\rho J_a^{\rho\nu}\big)}{q p_a}\Big) \mathcal{M}_n(p_1, p_2, \dots p_n) + \mathcal{O}(q^2)$$

$\mathcal{O}(q^{-1})$ and $\mathcal{O}(q^0)$ terms are exact, and $\mathcal{O}(q)$ gets an exact 1-loop correction. [Broedel, De Leeuw, Plefka, Rosso]
Interpretation through Virasoro extended BMS (Bondi, Burg, Metzner, Sachs) symmetry [Barnich, Troessaert]

3-loop superstring amplitude

2-loop [D'Hoker, Phong]
3-loop [Mafra, Gomez]

Using Berkovits pure spinor formalism

$$\lambda^\alpha \Gamma^\mu{}_{\alpha\beta} \lambda^\beta = 0$$

Reproduce the $SL(2,\mathbb{Z})$-duality invariant low energy effective action [Green, Vanhove]

$$S = \cdots + \int d^{10}x \sqrt{-g} \nabla^6 R^4 \left(4{\zeta_3}^2 e^{-2\phi} + 8\zeta_2\zeta_3 + \frac{48}{5}{\zeta_2}^2 e^{2\phi} + \frac{8}{9}\zeta_6 e^{4\phi}\right)$$

up to a factor of 3.

Off-shell Green functions

String amplitudes computed on-shell

$$M^n_{a_1,\dots a_n}(k_1,\dots k_n) = \sum_{h=0}^{\infty} e^{(2h-2)\phi} \int_{\mathcal{M}_{h,n}} \left\langle \prod_i V_{a_i}(k_i) \right\rangle_{\mathrm{CFT}}$$

Superconformal invariance $\hookrightarrow$ ${k_i}^2 = {m_i}^2$

In quantum field theory (BPHZ)

$$M^n_{a_1,\dots a_n}(k_1,\dots k_n) = \lim_{{k_i}^2 \to {m_i}^2} G^n_{a_1,\dots a_n}(k_1,\dots k_n) \prod_i \left(Z_i^{-\frac{1}{2}} ({k_i}^2 + {m_i}^2) \right)$$

Off-shell computation [Ashoke Sen]

⋆ Renormalised mass and S-matrix do not depend on the local coordinates and unitarity is satisfied

⋆ Wave function renormalisation and unphysical states mass do

Applies as well for spontaneously generated scalar field potential.

Entropy from localisation

Supersymmetric black holes with $AdS_2 \times S^2$ horizon, have a classical entropy (in microcanonical ensemble)

$$d(q,p) = e^{S(q,p)} \sim e^{\pi\sqrt{\Delta(q,p)}} .$$

Matching microscopic counting [Maldacena, Moore, Strominger]

$$d(q,p) = \#\text{D-brane} = \{8, 12, 39, 56, 152, 208, 513, \ldots\}$$

with Sen supergravity definition

$$d(q,p) = \left\langle \exp\left(-iq_I \int A^I\right) \right\rangle_{\mathrm{AdS}_2}$$

Using localisation, perturbatively [Dabholkar, Gomes, Murthy]

$$d(q,p) = \{7.97, 12.20, 38.99, 55.72, 152.04, 208.46, 512.96, \ldots\}$$

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Using localisation, exact [Dabholkar, Gomes, Murthy]

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Mathur conjecture

Usual belief that quantum gravity effects are negligible at the horizon.

$$\Psi[g,\phi] \approx \delta[g-g_{\rm BH}]\Psi[\phi]_{g_{\rm BH}}$$

Mathur argument on information paradox

$\hookrightarrow$ $\mathcal{O}(1)$ corrections at the horizon

Mathur conjecture in string theory

$$\Psi[g,\phi] \approx \sum_{g_s} \delta[g-g_s]\Psi[\phi]_{g_s}$$

where g_s are smooth globally hyperbolic space-times in 10 dimensions, that asymptotically reproduce the $M_{\rm BH}\times T^6$ geometry.

$$M_{ADM} = \int_\Sigma \Big(\iota_{\partial_t} G_{I\, D-p} \wedge F_p^I + \iota_{\partial_t} F_p^I \wedge G_{I\, D-p} \Big)$$

Superstrata

For supersymmetric black holes in five dimensions

$$S_{\mathrm{BH}} = 2\pi\sqrt{Q_1 Q_2 Q_3}$$

one has solutions in six dimensions with fundamental S^3 cycles,

$$\frac{1}{2\pi^2}\int_{S^3_A} dB^I = p^I \ ,$$

which shape is defined by functions of two angle variables [Bena, Shigemori, Warner] such that the number of solutions (after semi-classical quantization)

$$\#\text{superstrata} \sim \exp\left(2\pi\sqrt{\tfrac{1}{6} Q_1 Q_2 Q_3}\right)$$