Pion couplings to the scalar B meson

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In collaboration with B. Blossier and N. Garron

Based on [arXiv:1410.3409]

LPT Orsay

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Motivations

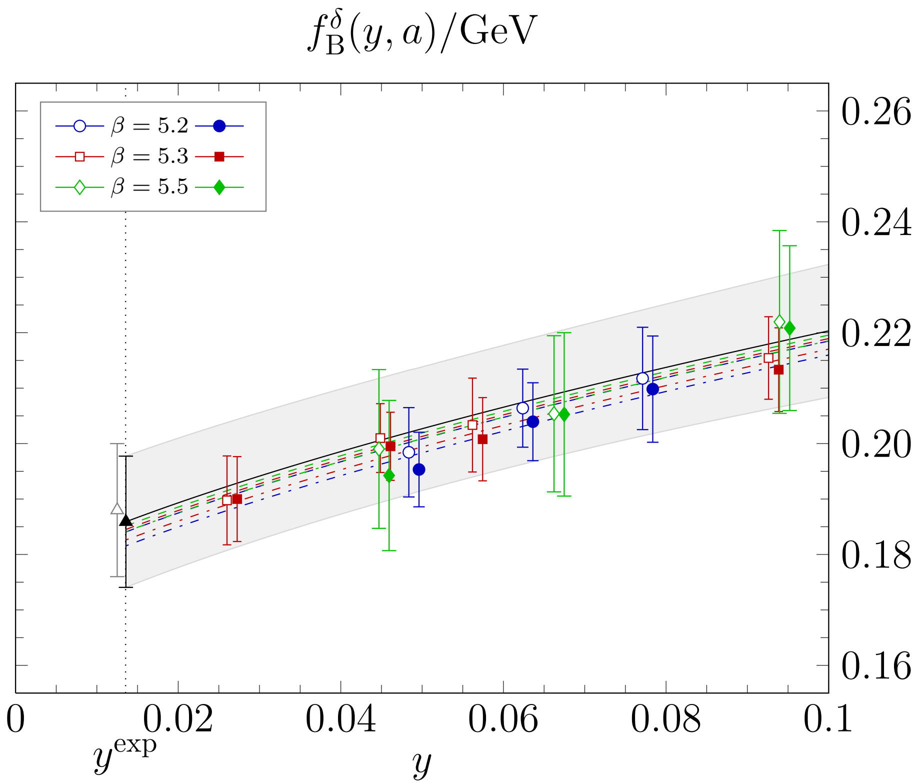
- Lattice simulations are often done at unphysical light quark masses ($\Rightarrow m_\pi > m_\pi^{\text{phys}}$)

  $\rightarrow$ Computing the quark propagator at small pion mass is numerically difficult

  $\rightarrow$ solution : use different quark masses and extrapolate to the physical pion mass

Motivations

- Example: chiral extrapolation of the B mesons decay constant [ALPHA Collaboration] :



$$y = \frac{m_\pi^2}{8\pi f_\pi^2}$$

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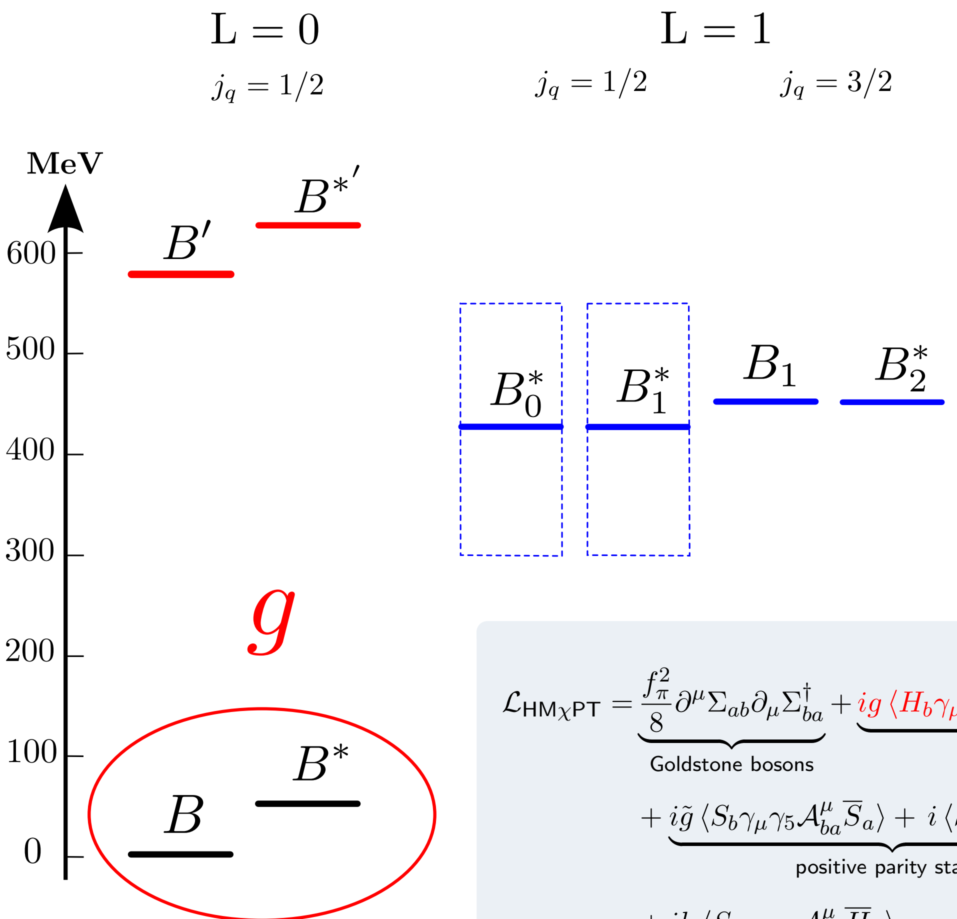
- Heavy-light mesons:

  $\rightarrow$ Light quark dynamics is well described by chiral perturbation theory (χPT)
  $\rightarrow$ Heavy quarks are described by the Heavy Quark Effective Theory (HQET)
  } HMχPT = HQET + χPT

  $\hookrightarrow$ Use HMχPT (Heavy Meson Chiral Perturbation Theory) to extrapolate lattice data

  $\hookrightarrow$ HMχPT is parametrized by three couplings g, h and $\tilde{g}$ at leading order

Motivations

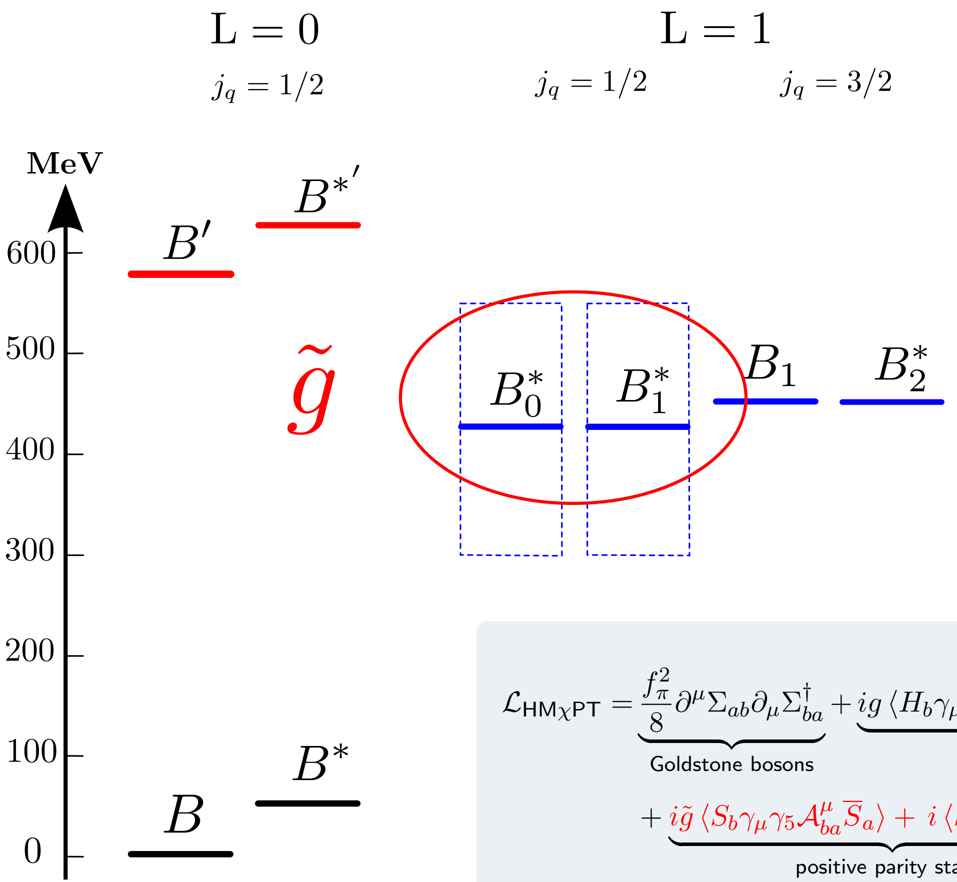


$$\mathcal{L}_{\mathsf{HM}\chi\mathsf{PT}} = \underbrace{\frac{f_\pi^2}{8}\partial^\mu \Sigma_{ab}\partial_\mu \Sigma^\dagger_{ba}}_{\text{Goldstone bosons}} + \underbrace{ig\left\langle H_b \gamma_\mu \gamma_5 \mathcal{A}^\mu_{ba}\overline{H}_a\right\rangle + i\left\langle H_b v^\mu \mathcal{D}_{\mu ba}\overline{H}_a\right\rangle}_{\text{negative parity states}}$$

$$+ \underbrace{i\tilde{g}\left\langle S_b \gamma_\mu \gamma_5 \mathcal{A}^\mu_{ba}\overline{S}_a\right\rangle + i\left\langle S_b v^\mu \mathcal{D}_{\mu ba}\overline{S}_a\right\rangle}_{\text{positive parity states}}$$

$$+ ih\left\langle S_b \gamma_\mu \gamma_5 \mathcal{A}^\mu_{ba}\overline{H}_a\right\rangle$$

Motivations



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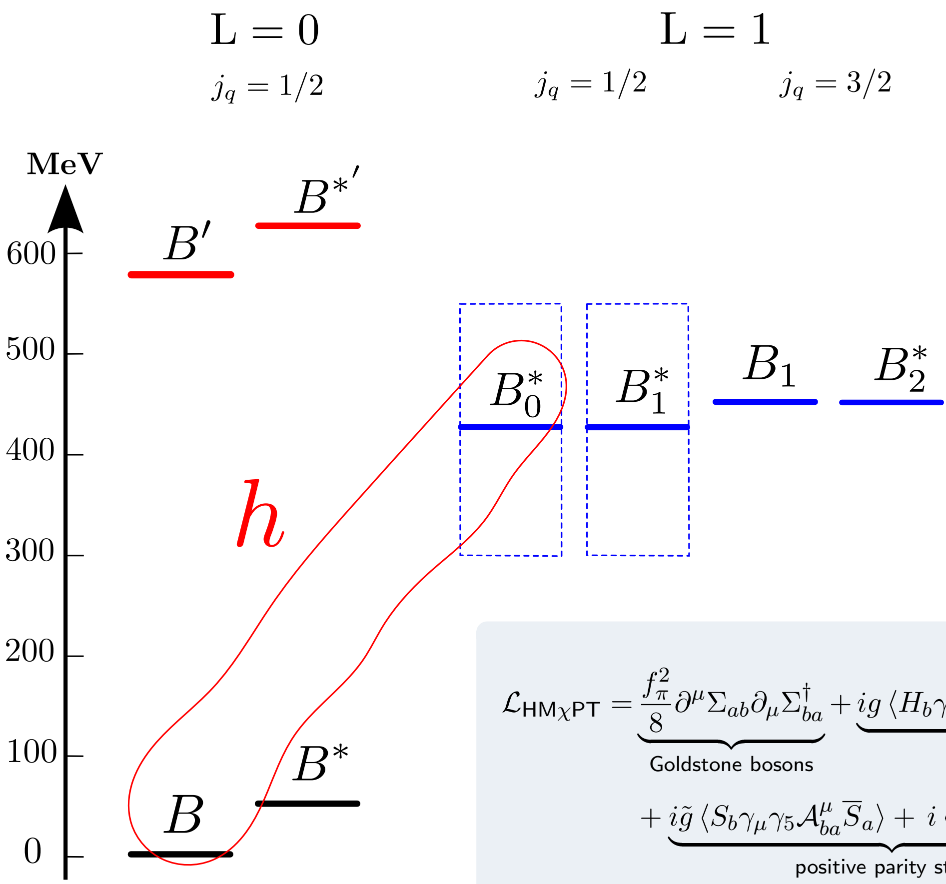

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- only g is actually considered in chiral extrapolations

  $\longrightarrow$ $\Delta = m_{B_0^*} - E_{B\pi}$ is usually not $\gg m_\pi$ on the lattice

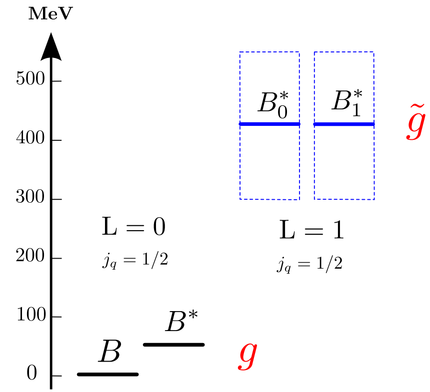
  $\longrightarrow$ the coupling h is large

  $\longrightarrow$ h and $\tilde{g}$ may have important contributions

Determination of $\widetilde{g}$

The soft coupling $\tilde{g}$

$$\tilde{g}\,\epsilon_i = \langle B_0^*(\vec{0})\,|\,\overline{\psi}_l\gamma_i\gamma_5\psi_l\,|\,B_1^*(\epsilon_i,\vec{0})\rangle$$



→ Three-point correlation function: $A_\mu = \overline{\psi}_l(x)\gamma_\mu\gamma_5\psi_l(x)$ (axial current)

$$C^{(3)}(t,t_1) = Z_A\,\frac{1}{V^3}\sum_{\vec{x},\vec{y},\vec{z}}\frac{1}{3}\langle\,\mathcal{A}_k(\vec{z},t)\;A_k(\vec{y},t_1)\;\mathcal{S}^\dagger(\vec{x},0)\,\rangle$$

$$\mathcal{A}_k(x) = \overline{\psi}_h(x)\Gamma_k^A\psi_l(x) \qquad \mathcal{S}(x) = \overline{\psi}_h(x)\Gamma^S\psi_l(x)$$

Z_A was determined non-perturbatively by the ALPHA Collaboration [Nucl.Phys. B865 (2012) 397-429]

→ Two-point correlation functions

$$C_{\mathcal{S}}^{(2)}(t) = \langle\,\sum_{\vec{x},\vec{y}}\mathcal{S}(\vec{x},t)\mathcal{S}^\dagger(\vec{y},0)\,\rangle \quad , \quad C_{\mathcal{A}}^{(2)}(t) = \langle\,\sum_{\vec{x},\vec{y}}\mathcal{A}_k(\vec{x},t)\mathcal{A}_k^\dagger(\vec{y},0)\,\rangle$$

→ Finally, the following ratio converge to the coupling $\tilde{g}$:

$$R(t) = \frac{C^{(3)}(t,t_1)}{\sqrt{C_{\mathcal{S}}^{(2)}(t)C_{\mathcal{A}}^{(2)}(t)}} \quad \xrightarrow[t_0=t-a]{t\gg 1} \quad \tilde{g} \;+\; \text{excited states}$$

Excited states : "summed" GEVP

- Use $N = 3$ interpolating operators with different overlaps with excited states $\rightarrow$ matrix of correlators
- Solve the Generalized Eigenvalue Problem (GEVP): [Michael, '85; Lüscher and Wolff, '90]

$$C_{\mathcal{S}}^{(2)}(t) v_n(t,t_0) = \lambda_n(t,t_0) C_{\mathcal{S}}^{(2)}(t_0) v_n(t,t_0)$$

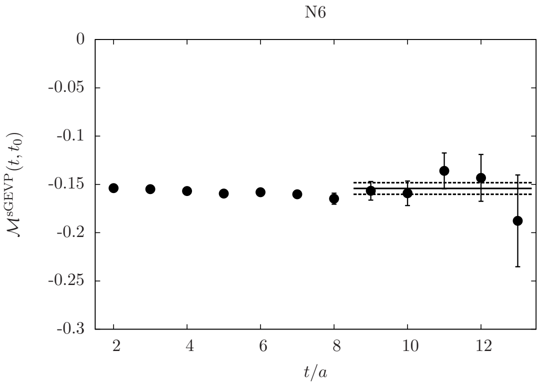
- Eigenvectors (v_n) and eigenvalues (λ_n) can be used to construct the summed ratio $\mathcal{M}_{nn}^{\mathrm{sGEVP}}(t,t_0)$:

$$\mathcal{M}_{nn}^{\mathrm{sGEVP}}(t,t_0) = -\partial_t \left(\frac{\left(v_n(t,t_0), \left[K(t,t_0)/\lambda_n(t,t_0) - K(t_0,t_0)\right] v_n(t,t_0)\right)}{\left(v_n(t,t_0), C_{\mathcal{S}}^{(2)}(t_0) v_n(t,t_0)\right)^{1/2} \left(v_n(t,t_0), C_{\mathcal{A}}^{(2)}(t_0) v_n(t,t_0)\right)^{1/2}} \right)$$

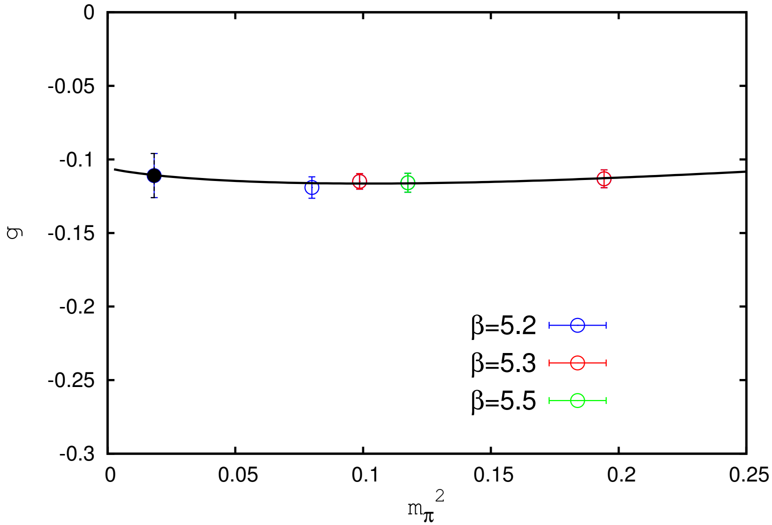
with : $K_{ij}(t,t_0) = \sum_{t_1} C_{ij}^{(3)}(t,t_1)$ *"summed GEVP"* [JHEP 1201 (2012) 140]

$$\mathcal{M}_{11}^{\mathrm{sGEVP}}(t) \xrightarrow[t_0=t-1]{t\gg 1} \widetilde{g} + \mathcal{O}\left(t e^{-\Delta_{N+1,n} t}\right)$$

$\longrightarrow$ $\Delta_{N+1,n} = E_{N+1} - E_n$
(excited states contribution is reduced)



Extrapolation to the physical point



- small dependence on the lattice spacing
- small dependence on the pion mass

$$\widetilde{g} = -0.122(8)(6)$$

$$\widetilde{g} = \alpha \left[1 - \frac{2 + 4\tilde{g}^2}{(4\pi f_\pi)^2} m_\pi^2 \log(m_\pi^2) + \frac{h^2}{(4\pi f_\pi)^2} \frac{m_\pi^2}{8\Delta^2} \left(3 + \frac{g}{\tilde{g}} \right) m_\pi^2 \log(m_\pi^2) \right] + C m_\pi^2 \qquad (HM\chi PT)$$

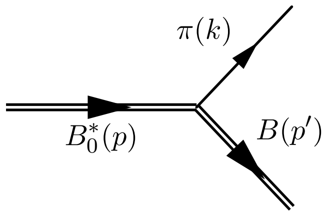
Determination of h

Lattice computation of h

- Continuum

$$g_{B_0^* B\pi} = \langle \pi(k) B(p') | B_0^*(p) \rangle = \sqrt{m_B m_{B_0^*}} \frac{m_{B_0^*}^2 - m_B^2}{m_{B_0^*} f_\pi} \times h$$

(static limit)



The decay rate Γ is given by:

$$\Gamma(B_0^* \to B\pi) = \frac{|\vec{k}|}{8\pi m_{B_0^*}^2} g_{B_0^* B\pi}^2$$

$$|\vec{k}| = \frac{\sqrt{\left(m_{B_0^*}^2 - (m_B + m_\pi)^2\right)\left(m_{B_0^*}^2 - (m_B - m_\pi)^2\right)}}{2 m_{B_0^*}}$$

- Lattice: Fermi Golden rule [Phys.Rev. D63 (2001)] (McNeile et al.)

$$\Gamma\left(B_0^* \to B\pi\right) = (2\pi)\, x^2\, \rho \quad , \quad x = \langle B_0^* | B\pi \rangle$$

ρ is the density of final states on the lattice:

$$\rho = \frac{L^3 k E_\pi}{2\pi^2}$$

$$\frac{\Gamma\left(B_0^* \to B\pi\right)}{k} = \frac{1}{\pi}\left(\frac{L}{a}\right)^3 (aE_\pi) \times (ax)^2$$

- Conclusion: one can access to h through x, computed on the lattice

Lattice computation: $x = \langle B_0^* | B\pi \rangle$

- Two-point correlation function: $x = \langle B_0^* | B\pi \rangle$

$$C_{B_0^*-B\pi}(t) = \langle \mathcal{O}^{B\pi}(t)\mathcal{O}^{B_0^*}(0)^\dagger \rangle = \sum_{t_1} \underbrace{\langle 0|\hat{\mathcal{O}}^{B_0^*}|B_0^*\rangle e^{-m_{B_0^*}t_1}} \times x \times \underbrace{\langle B\pi|\hat{\mathcal{O}}^{B\pi}|0\rangle \, e^{-E_{B\pi}(t-t_1)}}$$

- Near threshold ($m_{B_0^*} \approx E_{B\pi}$)

$$C_{B_0^*-B\pi}(t) \approx \langle 0|\hat{\mathcal{O}}^{B_0^*}|B_0^*\rangle \, x \, \langle B\pi|\hat{\mathcal{O}}^{B\pi}|0\rangle \, t \, e^{-Et}$$

- Consider the following ratio

$$R(t) = \frac{C^{(2)}_{B_0^*-B\pi}(t)}{\left(C^{(2)}_{B_0^*-B_0^*}(t)C^{(2)}_{B\pi-B\pi}(t)\right)^{1/2}} \approx \; xt \; + \; \text{excited states}$$

Lattice computation: $x = \langle B_0^* | B\pi \rangle$

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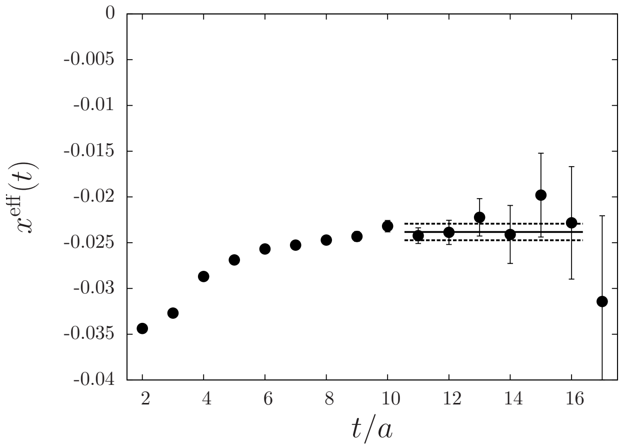
- Or the *GEVP ratio* to reduce the excited states contribution

$$R^{\mathrm{GEVP}}(t, t_0) = \frac{\left(v_{B_0^*}(t,t_0), C_{B_0^*-B\pi}(t) \, v_{B\pi}(t,t_0) \right)}{\sqrt{\left(v_{B_0^*}(t,t_0), C_{B_0^*-B_0^*}(t) \, v_{B_0^*}(t,t_0) \right) \times \left(v_{B\pi}(t,t_0), C_{B\pi-B\pi}(t) \, v_{B\pi}(t,t_0) \right)}} \approx \; xt + \text{excited states}$$

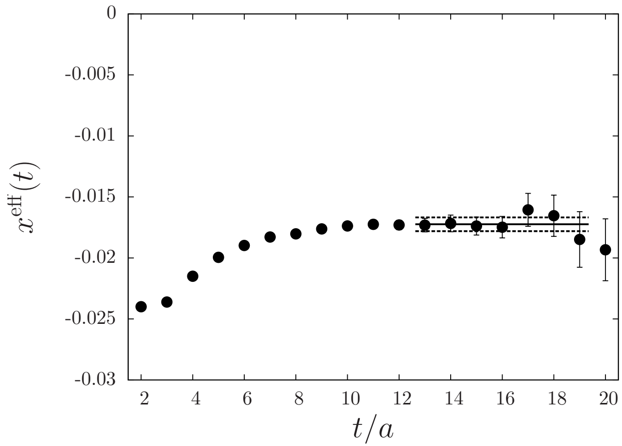
$$x^{\mathrm{eff}}(t) = \partial_t R^{\mathrm{GEVP}}(t) \xrightarrow[t_0 = t-a]{t \gg 1} \quad x$$

Lattice computation: $x = \langle B_0^* | B\pi \rangle$

- We have considered the corrections coming from $\Delta = m_{B_0^*} - E_{B\pi} \neq 0$
- We have considered the contribution from excited states



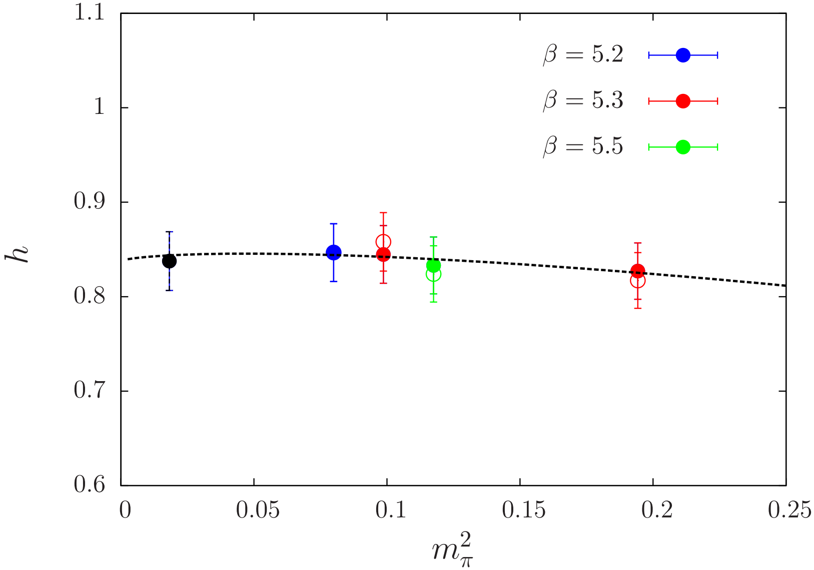
$a = 0.065$ fm, $m_\pi = 440$ MeV



$a = 0.048$ fm, $m_\pi = 340$ MeV

m_π	280 MeV	310 MeV	340 MeV	435 MeV
ax	$-0.0156(4)$	$-0.0159(3)$	$-0.0174(6)$	$-0.0241(10)$
h	0.86(4)	0.86(3)	0.85(4)	0.84(5)

h: chiral and continuum extrapolations



→ small dependence on the lattice spacing

→ small dependence on the quark mass

$$h = 0.86(4)_{\rm stat}(2)_\chi$$

NLO HMχPT :
$$h = h_0 \left[1 - \frac{3}{4} \frac{3\hat{g}^2 + 3\tilde{g}^2 + 2\hat{g}\tilde{g}}{(4\pi f_\pi)^2} \left(m_\pi^2 \log(m_\pi^2) - (m_\pi^{\rm exp})^2 \log((m_\pi^{\rm exp})^2) \right) \right] + C \left(m_\pi^2 - (m_\pi^{\rm exp})^2 \right)$$

Conclusion

- We have computed the soft pion couplings h and $\tilde{g}$ with $N_f = 2$ dynamical quarks

$$h = 0.84(3)(2) \quad , \quad \widetilde{g} = -0.122(8)(6)$$

- Good control under/over systematics:

  $\rightarrow$ in both case, the continuum and chiral extrapolations are performed

  $\rightarrow$ we used the GEVP to reduce the contamination from excited states

- These couplings can be used in the chiral extrapolations of relevant heavy-light meson properties

- $h \gg \hat{g}$: B meson orbital excitations cannot be neglected in chiral loops

- [Becirevic et al. (2012)]: $h = 0.66(10)(6)$

  $\rightarrow$ They used three-point correlation functions to obtain the form factor $A_+(\Delta^2)$

  $\rightarrow$ They compute the radial density to obtain the form factor in the correct limit $A_+(\Delta^2) \rightarrow A_+(0) \sim g_{B_0^* B\pi}$

- PDG: $\Gamma_{D_0^*} = 267(40)$ MeV, $m_{D_0^*} = 2318(29)$ MeV $\Rightarrow$ $h = 0.74(16)$

  $\rightarrow$ $1/m_c$ corrections are expected to be sizable for D meson

Lattice setup

Lattice discretization

- $N_f = 2$ $O(a)$ improved Wilson-Clover Fermions
- HYP1-2 discretization for the static quark action

Discretization effects

- 3 lattice spacings a :
  (0.048, 0.065, 0.075) < 0.1 fm

Light quark mass chiral extrapolations

- different pion masses in the range [280 MeV, 440 MeV]

CLS based

$\Rightarrow$ **total of 4 ensembles**

Correlation functions

- quark-antiquark interpolating operators

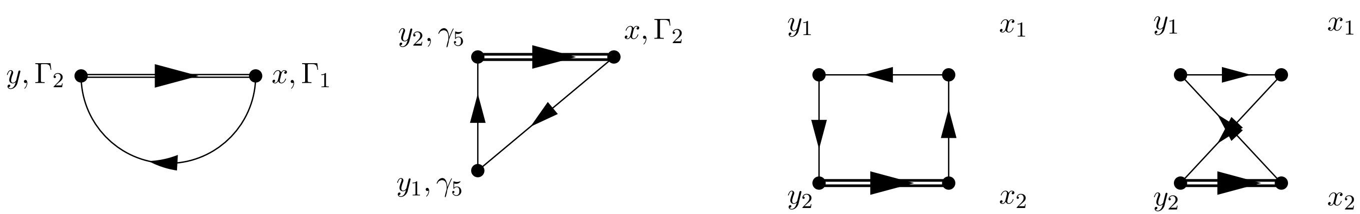
$$\mathcal{O}^{B}_{\Gamma,n}(t) = \frac{1}{V}\sum_{\vec{x}}\left[\overline{d}^{(n)}(x)\Gamma b(x)\right]$$

- meson-meson interpolating operators

$$\longrightarrow \quad \sqrt{\frac{2}{3}}\pi^{+}(0)B^{-}(0) - \sqrt{\frac{1}{3}}\pi^{0}(0)\overline{B}^{0}(0)$$

$$\mathcal{O}^{B\pi}_{\Gamma,n} = \frac{1}{V^2}\sum_{\vec{x}_i}\sqrt{\frac{2}{3}}\left[\overline{d}(x_1)\Gamma u(x_1)\right]\left[\overline{u}^{(n)}(x_2)\Gamma b(x_2)\right] - \sqrt{\frac{1}{6}}\left[\overline{u}(x_1)\Gamma u(x_1) - \overline{d}(x_1)\Gamma d(x_1)\right] \times \left[\overline{d}^{(n)}(x_2)\Gamma b(x_2)\right]$$

- local ($\Gamma = \gamma_0,\ \gamma_5$) and derivative ($\Gamma = \gamma_i\gamma_0\gamma_5\nabla_i,\ \gamma_i\nabla_i$) interpolating operators
- 4 levels of gaussian smearing



Systematic errors on h: corrections from $\Delta = m_{B_0^*} - E_{B\pi} \neq 0$

Neglecting excited states, at threshold we have:

$$C_{B_0^*-B\pi}(t) = \sum_{t_1} \langle 0|\hat{\mathcal{O}}^{B_0^*}|B_0^*\rangle \, x \, \langle B\pi|\hat{\mathcal{O}}^{B\pi}|0\rangle \, e^{-m_{B_0^*} t_1} \, e^{-E_{B\pi}(t-t_1)} \approx \langle 0|\hat{\mathcal{O}}^{B_0^*}|B_0^*\rangle \, x \, \langle B\pi|\hat{\mathcal{O}}^{B\pi}|0\rangle \, t \, e^{-Et}$$

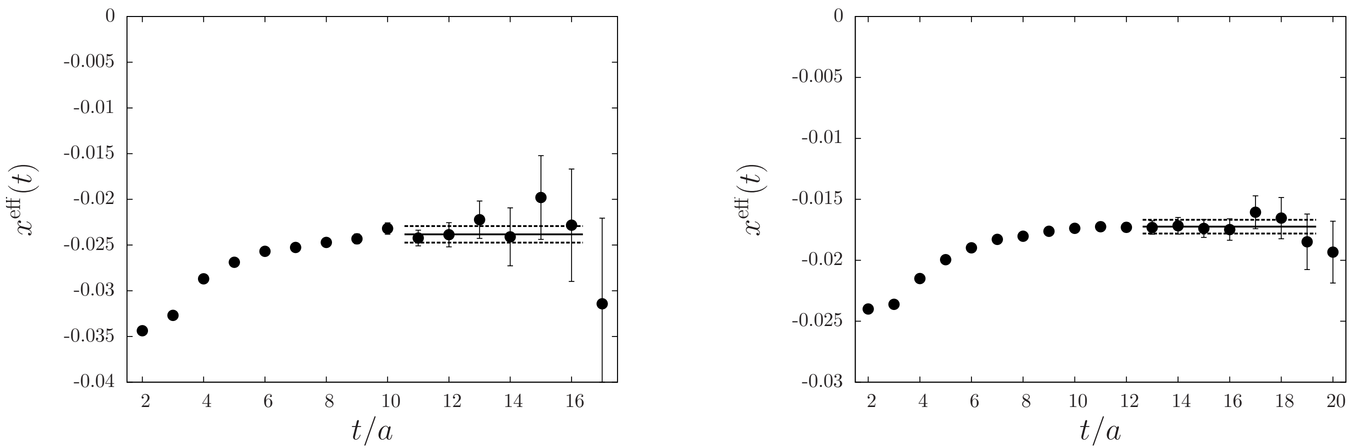
when the threshold condition is only approximately fulfilled, the linear time dependance becomes:

$$t \longrightarrow \frac{2}{\Delta}\sinh\left(\frac{\Delta}{2}t\right) = t + \frac{\Delta^2 t^3}{24} + \mathcal{O}(\Delta^4)$$

$$\left(\frac{3t^2\Delta^2}{24} \ll 1 \quad \text{for} \quad t \in [0-20]\right)$$

m_π	280 MeV	310 MeV	340 MeV	435 MeV
$a\Delta$	0.036(4)	0.026(8)	0.010(3)	−0.012(6)

Table: $\Delta = m_{B_0^*} - E_{B\pi}$



$a = 0.065$ fm, $m_\pi = 440$ MeV $\qquad$ $a = 0.048$ fm, $m_\pi = 340$ MeV

Systematic errors on h: Excited states contribution

$$C_{B_0^*-B\pi}(t) = \langle \mathcal{O}^{B_0^*}(t)\mathcal{O}^{B\pi}(0)^\dagger\rangle = \sum_{t_1}\langle 0|\hat{\mathcal{O}}^{B_0^*}|B_0^*\rangle\, x\,\langle B\pi|\hat{\mathcal{O}}^{B\pi}|0\rangle\, e^{-m_{B_0^*}t_1}e^{-E_{B\pi}(t-t_1)}$$

When excited states are taken into account ($X_1 = B_0^*$, $X_2 = B\pi$) :

$$x_{nm} = \langle X_n|X_m\rangle$$

$$C_{B_0^*-B\pi}(t) = \sum_{nm}\sum_{t_1}\langle 0|\hat{\mathcal{O}}^{B_0^*}|X_n\rangle x_{nm}\langle X_m|\hat{\mathcal{O}}^{B\pi}|0\rangle e^{-E_n t_1}e^{-E_m(t-t_1)}$$

Assumptions:

- I consider only the contribution from the first excited state X_3 : $m_{B_0^*} \approx E_{B\pi} < E_{X_3} < E_{X_4} < \cdots$
- X_3 has a non-negligible overlap with $\mathcal{O}^{B_0^*} \to \langle 0|\hat{\mathcal{O}}^{B_0^*}|X_3\rangle \neq 0$ and $\langle 0|\hat{\mathcal{O}}^{B\pi}|X_3\rangle \approx 0$

  $\to$ the symmetric case (a non-negligible overlap with $\mathcal{O}^{B\pi}$) is similar

$$\sum_{t_1}\langle 0|\hat{\mathcal{O}}^{B_0^*}|X_3\rangle x_{32}\langle B\pi|\hat{\mathcal{O}}^{B\pi}|0\rangle e^{-E_3 t_1}e^{-E(t-t_1)}$$

$$= \underbrace{t\langle 0|\hat{\mathcal{O}}^{B_0^*}|B_0^*\rangle x\langle B\pi|\hat{\mathcal{O}}^{B\pi}|0\rangle\, e^{-Et}}_{\text{ground state contribution}} \times \frac{1}{t}\,\frac{\langle 0|\hat{\mathcal{O}}^{B_0^*}|X_3\rangle}{\langle 0|\hat{\mathcal{O}}^{B_0^*}|B_0^*\rangle}\,\frac{x_{32}}{x}\sum_{t_1}e^{(E_3-E)t_1}$$

$\to$ Excited states contributions are suppressed by a factor t

$\to$ To be compared with the usual exponential suppression

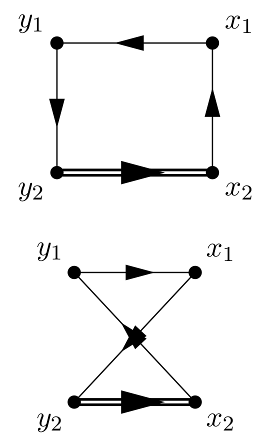
$$R^{\mathrm{GEVP}}(t) \approx A + xt$$

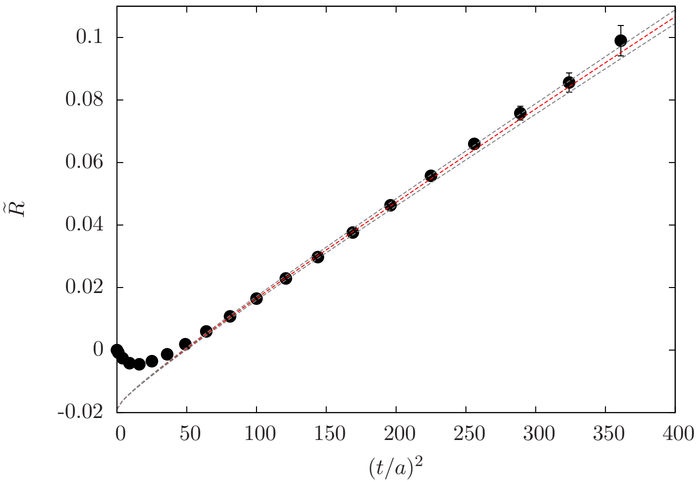
Cross-check: box and cross diagrams

[Phys Lett B556 (2004)] (McNeile et al.)

$$\widetilde{R}(t) = \frac{(v_{B\pi}(t,t_0), C_{\text{connected}}(t)\, v_{B\pi}(t,t_0))}{(v_{B\pi}(t,t_0), C_{B\pi-B\pi}(t)\, v_{B\pi}(t,t_0))} = B + \frac{1}{2}x^2t^2 + \mathcal{O}(t)$$

$$C_{\text{connected}}(t) = -\frac{3}{2}C_{\text{box}}(t) + \frac{1}{2}C_{\text{cross}}(t)\,.$$





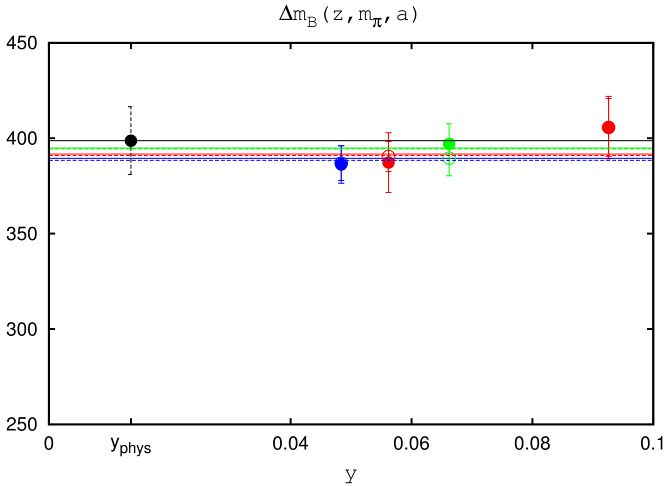
$a = 0.065$ fm, $m_\pi = 440$ MeV

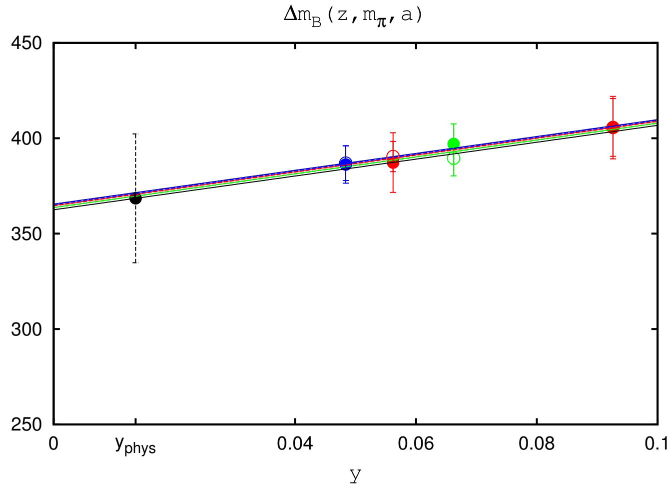
- $\beta(t) = \partial_t \widetilde{R}$
- Previous analysis : $ax = -0.0241(10)$
- Box + Cross diagrams : $ax = -0.0237(8)$

↪ Perfect agreement!

Mass of the scalar B_0^* meson

$$a\Delta m(a, m_\pi) = E^{s}_{\rm stat}(a, m_\pi) - E^{ps}_{\rm stat}(a, m_\pi) \qquad \text{with} \qquad E_n^{\rm eff}(t, t_0) = a^{-1} \log \frac{\lambda_n(t, t_0)}{\lambda_n(t+a, t_0)}$$





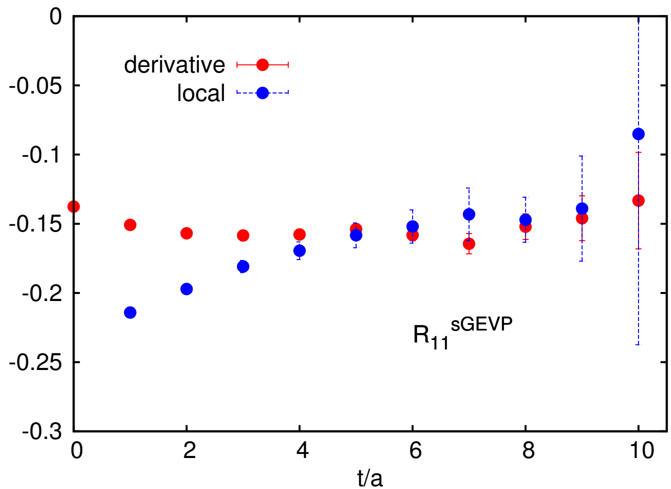
$$\Delta m_{B_0^*} = 385(21)_{\rm stat}(30)_{\rm syst}$$

Local vs Derivative interpolating operator

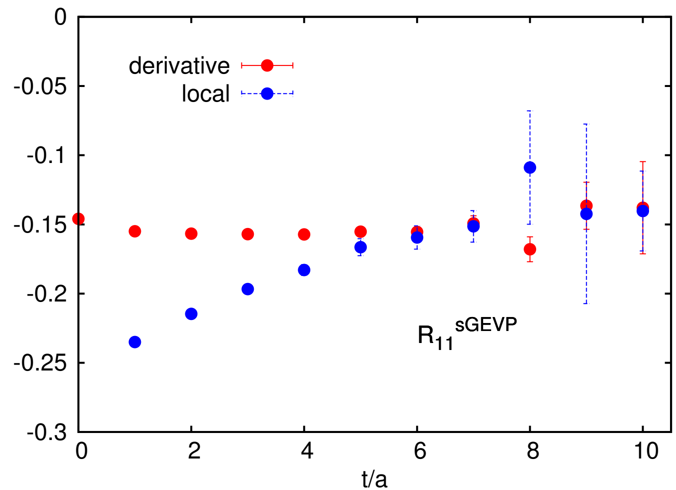
J^P	Local	Derivative
0^+	$\Gamma = \gamma_0$	$\Gamma = \gamma_i \overleftarrow{\nabla}_i$
1^+	$\Gamma = \gamma_5 \gamma_i$	$\Gamma = \gamma_5 \overleftarrow{\nabla}_i$

Table: Interpolating operators

→ Interpolating operators built from covariant derivatives are beneficial to reduce the contamination from higher excited states



E5g : $a = 0.065$ fm, $m_\pi = 440$ MeV



F6 : $a = 0.065$ fm, $m_\pi = 310$ MeV