From OPE to Soft-Wall

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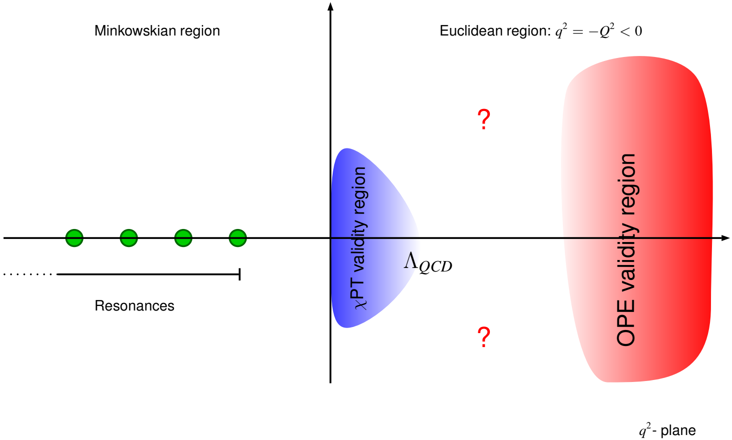
- QCD has well described different regimes: low energy (*e.g.* χPT), high energy (OPE), Minkowskian sector (resonances)
- Unfortunately the intermediate sector is unknown...
- Because of the existence of **Non-Perturbative** effects.



From now we assume Large-N_c limit

Several ways to address this issue

- Treat the 2 points Green functions as Padé approximants.
  Minimal Hadronic Approximation and related models
  S. Peris and E. de Rafael, JHEP 9805 (1998)
  M. Golterman and S. Peris, JHEP 0101 (2001)
- Resummation of Large-N_c resonances properties.
  O. Cata, M. Golterman and S. Peris, JHEP 0508, 076 (2005)
  E. de Rafael, Indian Academy of Sciences, Vol. 78, N6 June 2012
- Non-Analytic reconstruction method.
  D. G. and S. Peris, Phys.Rev. D82 (2010)
- AdS/CFT correspondence.

J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998)

S. S. Gubser, I. R. Klebanov, and A. M. Polyakov, Phys. Lett. B **428**, 105 (1998)

A. Karch, E. Katz, D. T. Son and M. A. Stephanov, Phys. Rev. D **74**, 015005 (2006)

The Maldacena conjecture

4 - dimensional		5 - dimensional
Non perturbative CFT Large N_c	$\longleftrightarrow$	Perturbative AdS Classical Theory
Green functions		Green functions

Practically...

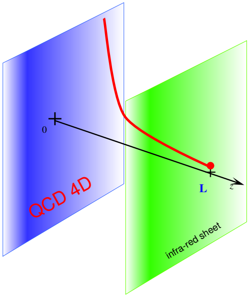
4 - dimensional		5 - dimensional
Sources coupled to currents v, a, s, p	$\xleftarrow[z\to 0]{}$	Fields $\Phi(x, z)$ on a gravitationnal background

Two different models: Hard - Wall and Soft-Wall

Let consider a conformally flat metric in 5D,

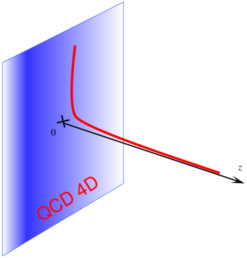
$$g_{MN}\,\mathrm{d}x^M\,\mathrm{d}x^N = w(z)^2\left(\eta_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu - \mathrm{d}z^2\right)$$

Hard - Wall



$$0 \leqslant z \leqslant L\,,\quad w(z)^2 = \frac{1}{z^2}$$

Soft - Wall



$$0 \leqslant z \leqslant \infty\,,\quad w(z)^2 = \frac{1}{z^2}\mathrm{e}^{-\Phi(z)}$$

Recovered Large N_c QCD Properties

J. Hirn and V. Sanz, JHEP **0512**, 030 (2005)
L. Da Rold and A. Pomarol, Nucl. Phys. B **721**, 79 (2005)
A. Karch, E. Katz, D. T. Son and M. A. Stephanov, Phys. Rev. D **74** (2006)
H. J. Kwee and R. F. Lebed, JHEP **0801**, 027 (2008)

	Hard - Wall	**Soft - Wall** $\Phi(z) = \kappa^2 z^2$
Parton Log	**YES**	**YES**
OPE	**NO**	**NO**
Chiral symmetry breaking	**YES** / **NO**	**NO**
Regge-like Spectrum of resonances	**NO**	**YES**

Purpose of this work

We want to modified the dilaton field $\Phi(z)$ such that we obtain:

- The right OPE.
- A chiral symmetry breaking mechanism: axial field and a pion pole (*i.e.* $F_\pi \neq 0$)

Lagrangian

$$S_5 = -\frac{1}{4g_5^2}\int d^4x \int_0^\infty dz \sqrt{g}\, e^{-\Phi(z)}\, g^{MN} g^{RS}\, \mathrm{Tr}\left[\mathbb{F}_{MR}\,\mathbb{F}_{NS}\right] \;,$$

with $g = |\det g_{MN}|$, the field strength $\mathbb{F}_{MN} = \partial_M \mathbb{V}_M - \partial_N \mathbb{V}_M - i\,[\mathbb{V}_M, \mathbb{V}_N]$ and in order to reproduce the parton log

$$g_5^2 = \frac{12\pi}{N_c} \;.$$

The AdS/CFT correspondence prescribes that the boundary value of the 5D gauge field $\mathbb{V}_M$ as to be identified with the classical source v_μ coupled to the the 4-dimensional vectorial current $J^a_{V\ \mu} =: \bar{q}\,\gamma_\mu\, t^a q :$,

$$\lim_{z\to 0} \mathbb{V}^a_{\mu,z}(x,z) = \mathrm{v}^a_\mu(x) \;.$$

The 4D-Fourier transform f_V of $\mathbb{V}_M$ satisfies the equation of motion

$$\partial_z^2 f_V + \partial_z\left[\ln w_0(z)\right]\partial_z f_V + q^2 f_V = 0 \;,$$

with $w_0(z) \doteq \frac{e^{-\Phi(z)}}{z^2} = \frac{e^{-\kappa^2 z^2}}{z^2}$ and the boundary condition $f_V(-q^2, 0) = 1$

Vectorial correlator properties

OPE

$$\Pi_V(Q^2) \underset{Q^2\to\infty}{\sim} \frac{F_V^2}{M^2}\ln\left(\frac{\Lambda_V^2}{Q^2}\right) + \langle\mathcal{O}_2\rangle\frac{1}{Q^2} + \langle\mathcal{O}_4\rangle\frac{1}{Q^4} + \langle\mathcal{O}_6\rangle_V\frac{1}{Q^6}$$

where in the large-N$_c$ limit the coefficients of the OPE are given by

$$\begin{cases} \langle\mathcal{O}_2\rangle = 0 \\ \langle\mathcal{O}_4\rangle = \frac{1}{12\pi}\ \alpha_s\ \langle G^2\rangle \\ \langle\mathcal{O}_6\rangle_V = -\frac{28\pi}{9}\ \alpha_s\ \langle\bar{\psi}\psi\rangle^2 \end{cases}$$

– Second Weinberg's sum rule –

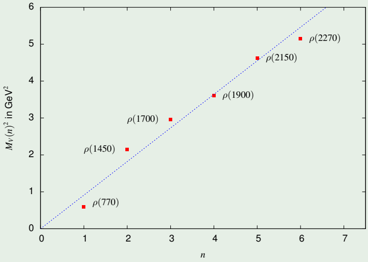
Regge Resonances spectrum

$$\Pi_V(Q^2) = \sum_{n=0}^{\infty}\frac{F_V(n)^2}{Q^2 + M_V(n)^2}$$

with

$$M_V(n)^2 \underset{n\to\infty}{\sim} \sigma n\ ,$$

σ is related to the confining string tension.



Connection between the two point Green function and the 5D model

$$Q^2\Pi_V(Q^2) = \frac{1}{g_5^2}\lim_{z\to 0} w_0(z)\, f_V(Q^2,z)\, \partial_z f_V(Q^2,z)$$

Dilaton profile prescription

In order to recover the right OPE of the vectorial correlator, we take in $w(z)$

$$\Phi(z) \longmapsto \Phi(z) + B(z) \quad \text{where} \quad B(z) = \sum_{k=1}^{3} \frac{b_{2k}}{2k} z^{2k} \ ,$$

We prove that : $b_{2k} \longleftrightarrow \langle \mathcal{O}_{2k} \rangle$

Regge Resonances spectrum

$$Q^2\Pi_V(Q^2) = \sum_{k,n} \mathcal{P}_{k,n}\left(\frac{Q^2}{4\kappa^2}\right)\ \psi^{(k)}\left(\frac{Q^2}{4\kappa^2}\right)\ ,$$

$\mathcal{P}_{k,n}$ a polynomial and $\psi^{(k)}$ is the k^{th} derivative of the Digamma ψ function with poles

$$\frac{Q^2}{4\kappa^2} = -n$$

(Expected) Axial correlator properties

OPE

$$\Pi_A(Q^2) \underset{Q^2\to\infty}{\sim} \frac{F_A^2}{M^2} \ln\left(\frac{\Lambda_A^2}{Q^2}\right) + \langle \mathcal{O}_2 \rangle \frac{1}{Q^2} + \langle \mathcal{O}_4 \rangle \frac{1}{Q^4} + \langle \mathcal{O}_6 \rangle_A \frac{1}{Q^6}$$

where in the large-N_c limit the coefficients of the OPE are given by

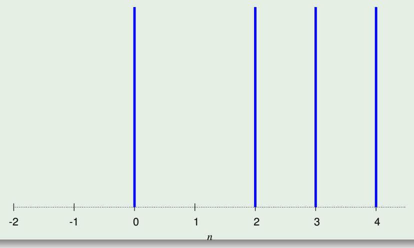
$$\begin{cases} \langle \mathcal{O}_2 \rangle = 0 \\ \langle \mathcal{O}_4 \rangle = \frac{1}{12\pi} \alpha_s \left\langle G^2 \right\rangle \\ \langle \mathcal{O}_6 \rangle_A = -\frac{11}{7} \langle \mathcal{O}_6 \rangle_V \end{cases}$$

Axial spectrum

$$\Pi_A(Q^2) = -\frac{F_\pi^2}{Q^2} + \sum_{n=0}^{\infty} \frac{F_A(n)^2}{Q^2 + M_A(n)^2}$$

with

$$M_A(n)^2 \underset{n\to\infty}{\sim} \sigma n$$



H. J. Kwee and R. F. Lebed, JHEP **0801**, 027 (2008)

Problems

- Since axial field obeys to the same dilaton field then it is impossible to have a different OPE from the vectorial correlator.
- We need a mechanism to explain the pion pole and the difference between the axial and the vectorial spectrum.

Solution: extra-scalar field on the bulk $\mathbb{X}$

$$S_5 = \int d^4x \int_0^\infty dz \sqrt{g}\, e^{-\Phi(z)}\, \mathrm{Tr}\Big\{ g^{MN} \left(D_M\mathbb{X}\right)^\dagger \left(D_N\mathbb{X}\right) - m^2\mathbb{X}^2 - \frac{1}{4g_5^2} g_{MN} g_{RS} \left(\mathbb{F}_V^{MR}\mathbb{F}_V^{NS} + \mathbb{F}_A^{MR}\mathbb{F}_A^{NS}\right)\Big\}$$

where we use

$$D^M\mathbb{X} = \partial^M\mathbb{X} - i[\mathbb{V}^M,\mathbb{X}] - i\{\mathbb{A}^M,\mathbb{X}\}$$
$$\mathbb{F}_V^{MN} = \partial^M\mathbb{V}^N - \partial^N\mathbb{V}^M - i[\mathbb{V}^M,\mathbb{V}^N] - i[\mathbb{A}^M,\mathbb{A}^N]$$
$$\mathbb{F}_A^{MN} = \partial^M\mathbb{A}^N - \partial^N\mathbb{A}^M - i[\mathbb{V}^M,\mathbb{A}^N] - i[\mathbb{A}^M,\mathbb{V}^N]$$

H. J. Kwee and R. F. Lebed, JHEP **0801**, 027 (2008)

Solution: extra-scalar field on the bulk $\mathbb{X}$

If $v(z)$ is the vacuum expectation value for the scalar field $\mathbb{X}$,

$$\mathrm{Tr}|\mathbb{X}|^2 = 2v(z)^2$$

The equation of motion for the Fourier transform over the 4D space of the axial field $\mathbb{A}$, $f_A(-q^2, z)$, becomes

$$\partial_z^2 f_A + \partial_z \left[\ln w(z)\right] \partial_z f_A - Q^2 f_A = \left(\frac{v(z)}{z}\right)^2 f_A$$

where $w(z) = \frac{1}{z} \mathrm{e}^{-\Phi(z) - B(z)}$.

Results

We prove that is possible to constrain $v(z)$ such that :

- We recover the right axial OPE *i.e.* $\langle \mathcal{O}_6 \rangle_A = -\frac{11}{7} \langle \mathcal{O}_6 \rangle_V$.
- The axial spectrum is recovered and we have analytically that $F_\pi \neq 0$.

Conclusion

- We have shown that it is possible to recover the right OPE for the vectorial field from a modified dilaton profile in the SW model
- We have shown that naturally the Regge behaviour of the spectrum is keep.
- We have shown that by the use of an extra scalar field it is possible to have axial field in SW model with the right OPE and the axial spectrum.
- What about the intermediate region? This method provides an unique way to do the analytic continuation from the OPE to the χPT and Minkowskian regions.