On the $\Delta I = 1/2$ rule

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Introduction: the $\Delta I = 1/2$ rule

- In $K \to \pi\pi$ decays, the final state can have isospin 0 or 2
- Experimentally we observe that

$$\mathbb{P}[K \to (\pi\pi)_{\mathrm{I=0}}] \sim 450 \times \mathbb{P}[K \to (\pi\pi)_{\mathrm{I=2}}]$$

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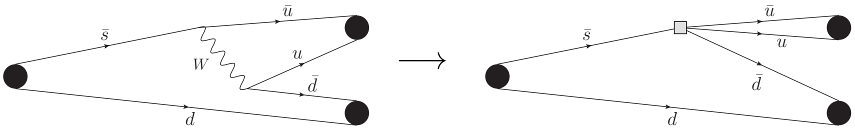
- Is the remaining contribution coming from non-perturbative QCD ? $\longrightarrow$ task for lattice QCD
- We have already computed A_2, we have a pilot computation of A_0

  $\Rightarrow$ Can we extract an explanation for this phenomena ?

Computation of $K \to \pi\pi$ amplitudes

Overview of the computation

Operator Product expansion



Describe $K \to (\pi\pi)_{\mathrm{I}=0,2}$ with an effective Hamiltonian

$$H^{\Delta s=1} = \frac{G_F}{\sqrt{2}} \left\{ \sum_{i=1}^{10} \left(V_{ud} V_{us}^* z_i(\mu) - V_{td} V_{ts}^* y_i(\mu) \right) Q_i(\mu) \right\}$$

Short distance effects factorized in the Wilson coefficients y_i, z_i

Long distance effects factorized in the matrix elements

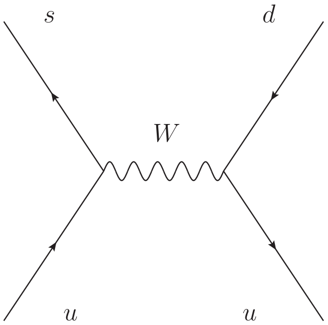
$$\langle \pi\pi | Q_i | K \rangle \longrightarrow \text{ Lattice}$$

See eg [Norman Christ @ Kaon'09] for an overview of different strategies.

and [Lellouch @ Les Houches'09] for an review

4-quark operators

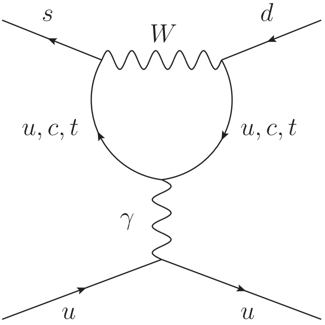
Current diagrams



$Q_1 = (\bar{s}d)_{\mathrm{V-A}}(\bar{u}u)_{\mathrm{V-A}}$ $\qquad$ $Q_2 =$ color mixed

4-quark operators

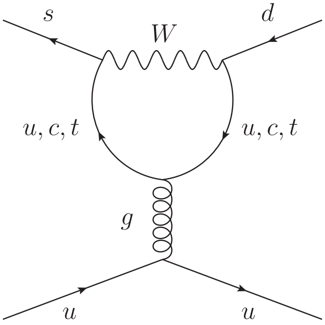
Electroweak penguins



$$Q_7 = \frac{3}{2}(\bar{s}d)_{\mathrm{V-A}} \sum_{q=u,d,s} e_q(\bar{q}q)_{\mathrm{V+A}} \qquad Q_8 = \text{color mixed}$$

$$Q_9 = \frac{3}{2}(\bar{s}d)_{\mathrm{V-A}} \sum_{q=u,d,s} e_q(\bar{q}q)_{\mathrm{V-A}} \qquad Q_{10} = \text{color mixed}$$

QCD penguins



$$Q_3 = (\bar{s}d)_{\mathrm{V-A}} \sum_{q=u,d,s} (\bar{q}q)_{\mathrm{V-A}} \qquad Q_4 = \text{color mixed}$$

$$Q_5 = (\bar{s}d)_{\mathrm{V-A}} \sum_{q=u,d,s} (\bar{q}q)_{\mathrm{V+A}} \qquad Q_6 = \text{color mixed}$$

Irrep of $SU(3)_L \otimes SU(3)_R$

$$\begin{aligned} \overline{3} \otimes 3 &= 8 + 1 \\ \overline{8} \otimes 8 &= 27 + \overline{10} + 10 + 8 + 8 + 1 \end{aligned}$$

Decomposition of the 4-quark operators gives

$$\begin{aligned} Q_{1,2} &= Q_{1,2}^{(27,1),\Delta I=3/2} + Q_{1,2}^{(27,1),\Delta I=1/2} + Q_{1,2}^{(8,8),\Delta I=1/2} \\ Q_{3,4} &= Q_{3,4}^{(8,1),\Delta I=1/2} \\ Q_{5,6} &= Q_{5,6}^{(8,1),\Delta I=1/2} \\ Q_{7,8} &= Q_{7,8}^{(8,8),\Delta I=3/2} + Q_{7,8}^{(8,8),\Delta I=1/2} \\ Q_{9,10} &= Q_{9,10}^{(27,1),\Delta I=3/2} + Q_{9,10}^{(27,1),\Delta I=1/2} + Q_{9,10}^{(8,8),\Delta I=1/2} \end{aligned}$$

see eg [Claude Bernard @ TASI'89] and [RBC'01]

$SU(3)_L \otimes SU(3)_R$ and isospin decomposition

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Only 7 are independent: one $(27, 1)$ four $(8, 1)$, and two $(8, 8)$, $\Rightarrow$ we called them Q'

$$\begin{aligned}
(27,1) \quad Q'_1 &= Q'^{(27,1),\Delta I=3/2}_1 + Q'^{(27,1),\Delta I=1/2}_1 \\
(8,1) \quad Q'_2 &= Q'^{(8,1),\Delta I=1/2}_2 \\
Q'_3 &= Q'^{(8,1),\Delta I=1/2}_3 \\
Q'_5 &= Q'^{(8,1),\Delta I=1/2}_5 \\
Q'_6 &= Q'^{(8,1),\Delta I=1/2}_6 \\
(8,8) \quad Q'_7 &= Q'^{(8,8),\Delta I=3/2}_7 + Q'^{(8,8),\Delta I=1/2}_7 \\
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Q'_8 &= Q_8'^{(8,8),\Delta I=3/2} + Q_8'^{(8,8),\Delta I=1/2}
\end{aligned}$$

A challenge !

Many obstacles:

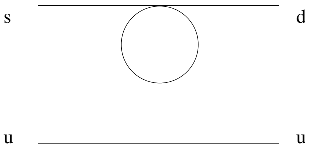
- Final state with two pions
- Many operators that mix under renormalisation
- Require the evaluation of disconnected graphs

Moreover, using a chiral disctretisation of the Dirac operator is probably unavoidable.

Plus the usual difficulties: light dynamical quarks, large volume, . . .

Isospin channels

- Only 3 of these operators contribute to the $\Delta I = 3/2$ channel
  - A tree-level operator
  - 2 electroweak penguins
- No disconnect graphs contribute to the $\Delta I = 3/2$ channel



$\Rightarrow$ A_2 is much simpler than A_0

Still highly non-trivial, but perfect challenge for lattice QCD with chiral fermions

Lattice computation of A_2 by RBC-UKQCD

Overview of the computation

- Lellouch-Lüscher method Lellouch Lüscher '00 to obtain the physical matrix element from the finite-volume Euclidiean amplitude and the derivative of the phase shift

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- Combine
  - Wigner-Eckart theorem (Exact up to isospin symmetry breaking)

$$\langle\pi^+(p_1)\pi^0(p_2)|O^{\Delta I=3/2}_{\Delta I_Z=1/2}|K^+\rangle = 3/2\langle\pi^+(p_1)\pi^+(p_2)|O^{\Delta I=3/2}_{\Delta I_Z=3/2}|K^+\rangle$$

  and then compute the unphysical process $K^+ \to \pi^+\pi^+$

  - Use Anti-periodic B.C. to eliminate the unwanted (wrong-kinematic) state Sachrajda & Villadoro '05

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- Renormalise at low energy $\mu_0 \sim 1.1 \text{ GeV}$ on and run non-perturbatively using finer lattices to $\mu = 3 \text{ GeV}$ and match to $\overline{\text{MS}}$ Arthur, Boyle '10, Arthur, Boyle, N.G. , Kelly, Lytle '11

$$\lim_{a_1\to 0} \underbrace{\left[Z(\mu_1,a_1)Z^{-1}(\mu_0,a_1)\right]}_{\text{fine lattice}} \times \underbrace{Z(\mu_0,a_0)}_{\text{coarse lattice}} = Z(\mu_1,a_0)$$

A_2 from RBC-UKQCD

- Very challenging both theoretically and numerically
- Computation performed with state-of-the-art algorithm and large-scale computer resources
- Possible because of various ingenious methods

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- $2+1$ chiral fermions (Domain-Wall on IDSDR $a \sim 0.14\ \mathrm{fm}$)
- lightest unitary pion mass $\sim 170\ \mathrm{MeV}$ (partially quenched $140\ \mathrm{MeV}$)
- Non-perturbative-renormalization through RI-SMOM schemes

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- Find $\mathrm{Re}A_2 = 1.381(46)_{\mathrm{stat}}(258)_{\mathrm{syst}} 10^{-8}$ GeV, experimental value is $1.479(4)\,10^{-8}$ GeV
- And $\mathrm{Im}A_2 = -6.54(46)_{\mathrm{stat}}(120)_{\mathrm{syst}}$ GeV

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- And $\mathrm{Im}A_2 = -6.54(46)_{\mathrm{stat}}(120)_{\mathrm{syst}}$ GeV
- Important computation in the field: first realistic computation of a hadronic decay
- 2012 Ken Wilson lattice award

Toward a full computation of $K \to (\pi\pi)$

and an understanding of the $\Delta I = 1/2$ rule ?

"Pilot" computation of the full process

T. Blum, Boyle, Christ, N.G., Goode, Izubuchi, Lehner, Liu, Mawhinney, Sachrajda, Soni, Sturm, Yin, Zhou, PRD'11.

Unphysical:

- "Heavy" pions (lightest $\sim m_\pi \sim 300\ \mathrm{MeV}$), small volume
- Non-physical kinematics: pions at rest

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- Two-pion state
- All the contractions of the 7 fourk-operators are computed
- Renormalisation done non-perturbatively

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obtain

$$\begin{aligned} \mathrm{Re}\, A_0 &= 3.80(82) \times 10^{-7}\mathrm{GeV} \\ \mathrm{Im}\, A_0 &= -2.5(2.2) \times 10^{-11}\mathrm{GeV} \end{aligned}$$

We combine our physical computation of $\Delta I = 3/2$ part with our non-physical computation of the $\Delta I = 1/2$

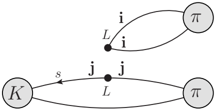
	$1/a$ [GeV]	m_π [MeV]	m_K [MeV]	$\mathrm{Re}A_2$ [10^{-8}GeV]	$\mathrm{Re}A_0$ [10^{-8} GeV]	$\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2}$	kinematics
16^3 **IW**	1.73(3)	422(7)	878(15)	4.911(31)	45(10)	9.1(2.1)	threshold
24^3 **IW**	1.73(3)	329(6)	662(11)	2.668(14)	32.1(4.6)	12.0(1.7)	threshold
32^3 **ID**	1.36(1)	142.9(1.1)	511.3(3.9)	1.38(5)(26)	-	-	physical
Exp	–	135 - 140	494 - 498	1.479(4)	33.2(2)	22.45(6)	

Pattern which could explain the $\Delta I = 1/2$ enhancement

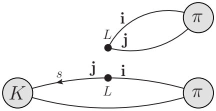
Boyle, Christ, N.G., Goode, Izubuchi, Janowski, Lehner, Liu, Lytle, Sachrajda, Soni, Zhang, PRL'13

Toward an quantitative understanding of the $\Delta I = 1/2$ rule

Two kinds of contraction for each $\Delta I = 3/2$ operator



Contraction ①



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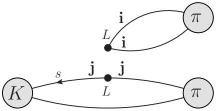

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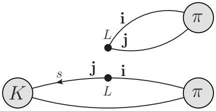

Contraction ②

- ReA_2 is dominated by the tree level operator (EWP $\sim 1\%$)

$$ReA_2 \sim ① + ②$$

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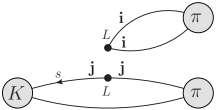

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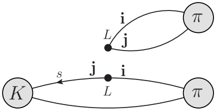

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- $\text{Re}A_2$ is dominated by the tree level operator (EWP $\sim 1\%$)

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- Naive factorisation approach: ② $\sim 1/3$①

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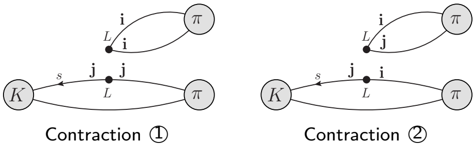

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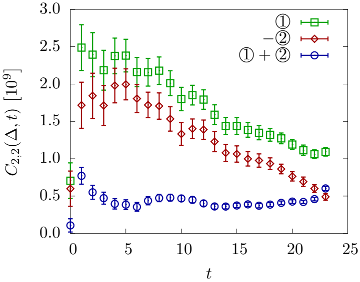
$$ReA_2 \sim ① + ②$$

- Naive factorisation approach: $② \sim 1/3①$
- Our computation: $② \sim -0.7①$

$\Rightarrow$ large cancellation in $\mathrm{Re}A_2$



ReA_0 is also dominated by the tree level operators

i	Q_i^{lat} [GeV]	$Q_i^{\overline{\text{MS}}\text{-NDR}}$ [GeV]
1	8.1(4.6) 10^{-8}	6.6(3.1) 10^{-8}
2	2.5(0.6) 10^{-7}	2.6(0.5) 10^{-7}
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4	–	2.3(2.1) 10^{-9}
5	-1.2(0.5) 10^{-9}	4.0(2.6) 10^{-10}
6	4.7(1.7) 10^{-9}	-7.0(2.4) 10^{-9}
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Toward an quantitative understanding of the $\Delta I = 1/2$ rule

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$$\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2} \sim \frac{2② - ①}{① + ②}$$

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5	-1.2(0.5) 10^{-9}	4.0(2.6) 10^{-10}
6	4.7(1.7) 10^{-9}	-7.0(2.4) 10^{-9}
7	1.5(0.1) 10^{-10}	6.3(0.5) 10^{-11}
8	-4.7(0.2) 10^{-10}	-3.9(0.1) 10^{-10}
9	–	2.0(0.6) 10^{-14}
10	–	1.6(0.5) 10^{-11}
ReA_0	3.2(0.5) 10^{-7}	3.2(0.5) 10^{-7}

Dominant contribution to $Q_2^{\rm lat}$ is $\propto$ (2② − ①) $\Rightarrow$ Enhancement in ReA_0

$$\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2} \sim \frac{2②-①}{①+②}$$

With this unphysical computation (kinematics, masses) we find

$$\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2} = 9.1(2.1) \text{ for } m_K = 878 \text{ MeV } m_\pi = 422 \text{ MeV}$$

$$\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2} = 12.0(1.7) \text{ for } m_K = 662 \text{ MeV } m_\pi = 329 \text{ MeV}$$

Toward an quantitative understanding of the $\Delta I = 1/2$ rule

- This sign implies both a cancellation in A_2 and an enhancement in A_0
- Need to be confirmed with physical quark masses and physical kinematics
- Analytic work in that direction Pich & de Rafael '96, Buras
- See also discussion in Lellouch @ Les Houches '09]

Going further: 2014-2015 update

Lattice 2014 update

- $\Delta I = 3/2$

  Main limitation on the previous computation : only one coarse lattice spacing

  IDSDR $32^3 \times 64$, with $a^{-1} \sim 1.37$ GeV $\Rightarrow a \sim 0.14$ fm, $L \sim 4.6$ fm

  Current computation:

  two lattice spacing, $n_f = 2 + 1$, large volume at the physical point

  New discretisation of the Domain-Wall fermion forumlation: Möbius Brower, Neff, Orginos '12

  - $48^3 \times 96$, with $a^{-1} \sim 1.729$ GeV $\Rightarrow a \sim 0.11$ fm, $L \sim 5.5$ fm
  - $64^3 \times 128$ with $a^{-1} \sim 2.358$ GeV $\Rightarrow a \sim 0.084$ fm, $L \sim 5.4$ fm
  - $am_{res} \sim 10^{-4}$

  Status: Computation finished, draft in final stage

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  Status: Computation finished, draft in final stage

- $\Delta I = 1/2$

  Main limitation on the previous computation : non-physical kinematic

  New formulation: G-parity boundary conditions

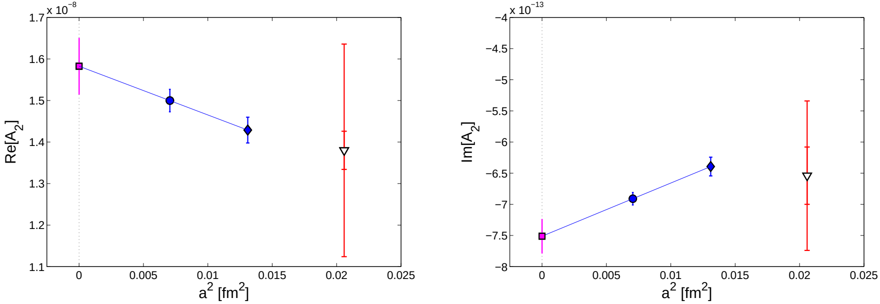
  Status: First computation almost finished

$K \to (\pi\pi)_{I=2}$ Lattice 2014 update

2012 Blum, Boyle, Christ, N.G.,Goode, Izubuchi, Jung, Kelly, Lehner, Lightman, Liu, Lytle, Mawhinney, Sachrajda, Soni, Sturm, *PRL'12, PRD'12*

$\mathrm{Re}\,A_2 = 1.381(46)_{\rm stat}(258)_{\rm syst}\ 10^{-8}\ \mathrm{GeV}$ $\qquad$ $\mathrm{Im}\,A_2 = -6.54(46)_{\rm stat}(120)_{\rm syst}\,10^{-13}\ \mathrm{GeV}$

2014 RBC-UKQCD Work in progress, draft in final stage



Preliminary results, very close to final numbers

see also talk by T.Janowski @ lat'13 Janowski, Sachrajda, Boyle, Christ, Mawhinney, Yin, Zhang, N.G., Lytle

$2+1$ Domain-Wall fermions

Chiral-Flavour symmetry (almost) exact at finite lattice spacing

Finite fith dimension L_s $\rightarrow$ small additive quark mass renormalisation m_{res}

$2 + 1$ Domain-Wall fermions

Chiral-Flavour symmetry (almost) exact at finite lattice spacing

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- 2008: IW $a^{-1} = 1.729(18)$ GeV $\leftrightarrow$ $a \sim 0.1145$ fm, on $24^3 \times 64 \times 16$, ie $L \sim 2.74$ fm

  Unitary pion masses $m_\pi = 331, 419, (557)$ MeV

RBC-UKQCD setup - History- Present

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- 2010: IW $a^{-1} = 2.282(28)$ GeV $\leftrightarrow$ $a \sim 0.0868$ fm, on $32^3 \times 64 \times 16$, ie $L \sim 2.77$ fm

  Unitary pion masses $m_\pi = 290, 345, 394$ MeV

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- 2012: IDSDR $a^{-1} = 1.372(10)$ GeV $\leftrightarrow a \sim 0.144$ fm, on $32^3 \times 64 \times 32$, ie $L \sim 4.62$ fm

  Unitary pion mass $m_\pi = 171$ MeV

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  Unitary pion mass $m_\pi = 171$ MeV

- 2014: Möbius, Unitary pion mass 139 MeV
  - $a^{-1} = 1.730(4)$ GeV $\leftrightarrow$ $a \sim 0.1145$ fm, on $48^3 \times 96 \times 24$ ie $L \sim 4.62$ fm
  - $a^{-1} = 2.359(7)$ GeV $\leftrightarrow$ $a \sim 0.0839$ fm, on $64^3 \times 128 \times 12$ ie $L \sim 5.475$ fm

Conclusions

- Observe a mechanism which contributes to a large enhancement in A_0/A_2
- Is this enhancement enough, or do we need something else ?
- Clearly, a non-perturbative method is required
- New: Continuum limit of $K \to (\pi\pi)_{I=2}$ at the physical point
- First realistic results of $K \to (\pi\pi)_{I=0}$ (with physical kinematics) should be available in a few months(?), thanks to G-parity boundary conditions
- Other kaon pheno applications: kl_3 or BSM matrix elements Boyle, N.G. Hudspith'12

  $\Rightarrow$ Provides important tests of the SM and help to understand/constrain BSM theories