# Renormalization Group Optimized Perturbation: some applications at zero and finite temperature

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1. Introduction, Motivations

2. (Variationally) Optimized Perturbation (OPT)

3. Renormalization Group improvement of OPT (RGOPT)

4. $F_\pi / \Lambda^{QCD}_{\overline{\text{MS}}}$ and $\alpha_S$

5. chiral quark condensate $\langle \bar{q} q \rangle$ (preliminary!)

6. $\lambda \phi^4 (T \neq 0)$: Pressure at two-loops (preliminary!)

Summary, Outlook

## 1. Introduction/Motivations

General goal: get approximations (of reasonable accuracy?) to 'intrinsically nonperturbative' chiral sym. breaking order parameters from unconventional resummation of perturbative expansions

Very general: relevant both at $T=0$ or $T\neq 0$ (also finite density)

$\rightarrow$ address well-known problem of unstable thermal perturbation theory:

(here illustrate for $\lambda\Phi^4$, next goal: real QCD for Quark Gluon Plasma: thermodynamic quantities, comparison with Lattice results).

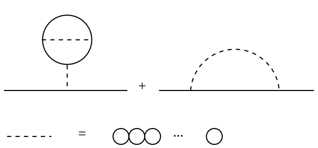
# Chiral Symmetry Breaking ($\chi$SB ) Order parameters

Usually considered hopeless from standard perturbation:

1. $\langle\bar{q}q\rangle^{1/3}$, $F_\pi$,.. $\sim \mathcal{O}(\Lambda_{QCD}) \simeq$ 100–300 MeV
$\to \alpha_S$ (a priori) large $\to$ invalidates pert. expansion

2. $\langle\bar{q}q\rangle$, $F_\pi$,.. perturbative series $\sim (m_q)^d \sum_{n,p} \alpha_s^n \ln^p(m_q)$
vanish for $m_q \to 0$ at any pert. order (trivial chiral limit)

3. More sophisticated arguments e.g. (infrared) renormalons (factorially divergent pert. coeff. at large orders)



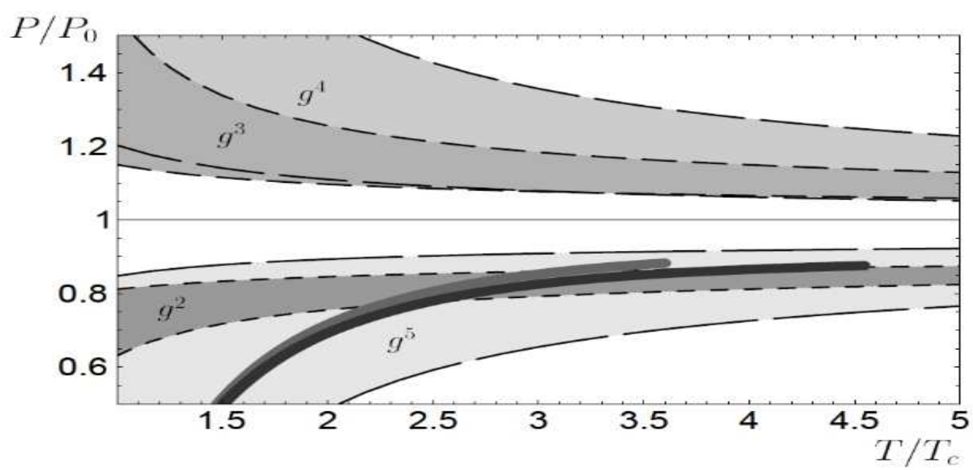
$$\Rightarrow \simeq \int dp^2 \sum_n (\ln \frac{p^2}{\mu^2})^n \sim n!$$

All seems to tell that $\chi$SB parameters are intrinsically NP

•Optimized pert. (OPT): appear to circumvent at least 1., 2., and may give more clues to pert./NP bridge

# $T \neq 0$: perturbative Pressure (QCD or $\lambda\phi^4$)

Know long-standing Pb: poorly convergent and very scale-dependent (ordinary) perturbative expansion




QCD (pure glue) pressure at successive pert. orders

bands=scale-dependence $\mu = \pi T - 4\pi T$

## 2. (Variationally) Optimized Perturbation (OPT)

$$\mathcal{L}_{QCD}(g, m_q) \to \mathcal{L}_{QCD}(\delta\, g, m(1-\delta)) \quad (\alpha_S \equiv g/(4\pi))$$

$0 < \delta < 1$ interpolates between $\mathcal{L}_{free}$ and *massless* $\mathcal{L}_{int}$;
(quark) mass $m_q \to m$: arbitrary trial parameter

- Take any standard (renormalized) QCD pert. series, expand in $\delta$ *after:*

$$m_q \to m\,(1-\delta); \quad \alpha_S \to \delta\,\alpha_S$$

then take $\delta \to 1$ (to recover original massless theory):

BUT a $m$-dependence remains at any finite $\delta^k$-order:
fixed typically by optimization (OPT):

$$\frac{\partial}{\partial m}(\text{physical quantity}) = 0 \text{ for } m = m_{opt}(\alpha_S) \neq 0$$

Manifestation of *dimensional transmutation*!
Expect *flatter* $m$-dependence at increasing $\delta$ orders...

But does this 'cheap trick' always work? and why?

# Simpler model's support + properties

•Convergence proof of this procedure for $D=1\ \lambda\phi^4$ oscillator (cancels large pert. order factorial divergences!) Guida et al '95

particular case of 'order-dependent mapping' Seznec+Zinn-Justin '79 (exponentially fast convergence for ground state energy $E_0 = const.\lambda^{1/3}$; good to % level at second $\delta$-order)

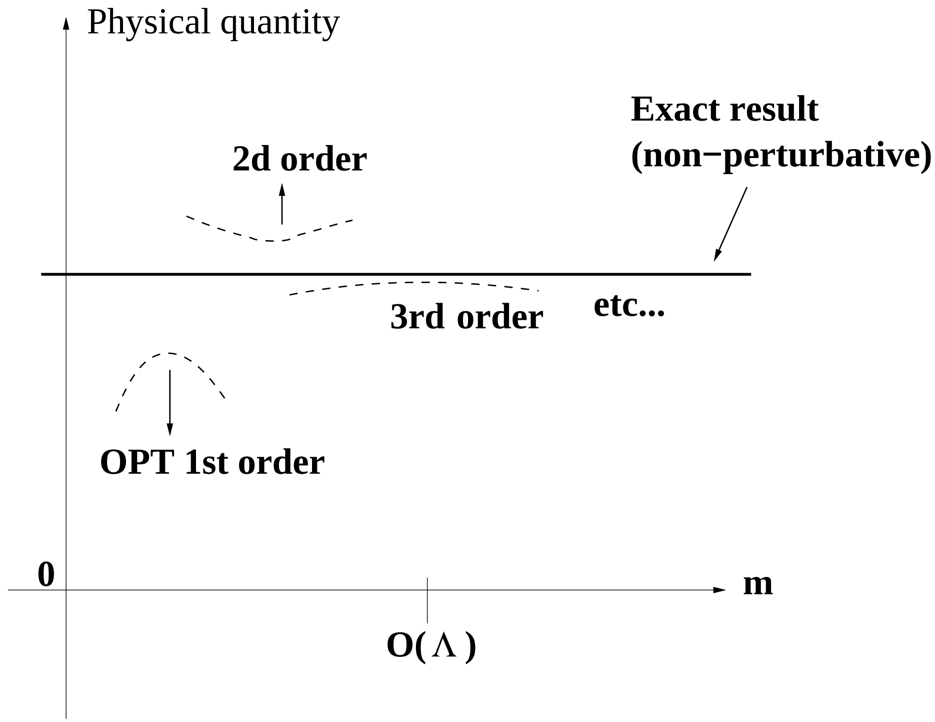
•In renormalizable QFT, first order consistent with Hartree-Fock (or large $N$) approximation

•Also produces factorial damping at large pert. orders ('delay' infrared renormalon behaviour to higher orders)( JLK, Reynaud '2002 )

•Flexible, Renormalization-compatible, gauge-invariant: applications also at finite temperature (phase transitions beyond mean field approx. in 2D, 3D models, QCD...)

(many variants, many works)

# Expected behaviour (Ideally...)




But not quite what happens.. (except in simple oscillator)

Most elaborated calculations (e.g $T \neq 0$) (very) difficult beyond first order: $\rightarrow$ what about convergence?

Main pb at higher order: OPT: $\partial_m(\ldots) = 0$ has multi-solutions (some complex!), how to choose right one??

# 3. RG improved OPT (RGOPT)

Our main new ingredient (JLK, A. Neveu 2010):

Consider a physical quantity (perturbatively RG invariant), e.g. pole mass M:
in addition to OPT Eq: $\frac{\partial}{\partial m} M^{(k)}(m, g, \delta = 1)|_{m \equiv \tilde{m}} \equiv 0$

Require ($\delta$-modified!) series at order $\delta^k$ to satisfy a standard perturbative Renormalization Group (RG) equation:

$$\mathrm{RG}\left(M^{(k)}(m, g, \delta = 1)\right) = 0$$

with standard RG operator:

$$\mathrm{RG} \equiv \mu \frac{d}{d\,\mu} = \mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g} - \gamma_m(g)\, m \frac{\partial}{\partial m}$$

$[\beta(g) \equiv -2b_0 g^2 - 2b_1 g^3 + \cdots\,, \;\; \gamma_m(g) \equiv \gamma_0 g + \gamma_1 g^2 + \cdots\,]$

$\rightarrow$ Combined with OPT, RG Eq. takes a reduced form:

$$\left[\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g}\right] M^{(k)}(m, g, \delta = 1) = 0$$

Note: OPT+RG completely fix $m \equiv \tilde{m}$ and $g \equiv \tilde{g}$ (two constraints for two parameters).

• Now $\Lambda_{\overline{\text{MS}}}(g)$ satisfies by def. $\left[\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g}\right] \Lambda_{\overline{\text{MS}}} \equiv 0$ consistently at a given pert. order for $\beta(g)$.

Thus equivalent to:

$$\frac{\partial}{\partial\, m}\left(\frac{M^k(m, g, \delta = 1)}{\Lambda_{\overline{\text{MS}}}(g)}\right) = 0\ ; \quad \frac{\partial}{\partial\, g}\left(\frac{M^k(m, g, \delta = 1)}{\Lambda_{\overline{\text{MS}}}(g)}\right) = 0$$

# OPT + RG main features

•OPT: (too) much freedom in the interpolating Lagrangian?:

$$m \to m\,(1-\delta)^{a}$$

in most previous works: linear case $a = 1$ for 'simplicity'...

[exceptions: Bose-Einstein Condensate $T_c$ shift, calculated from $O(2)\lambda\phi^4$, *requires* $a \neq 1$:

gives real solutions +related to critical exponents (Kleinert,Kastening; JLK,Neveu,Pinto '04)

•OPT, RG Eqs. are polynomial in $(L \equiv \ln\frac{m}{\mu}, g = 4\pi\alpha_S)$:

serious drawback: polynomial Eqs of order $k \to$ (too) many solutions, and often complex, at increasing $\delta$-orders

•Our *compelling* way out: require solutions to match standard perturbation (i.e. Asympt. Freedom for QCD):

$\alpha_S \to 0, |L| \to \infty$: $\alpha_S \sim -\frac{1}{2b_0 L} + \cdots$

$\to$ at arbitrary RG order, AF-compatible RG + OPT branches *only* appear for a specific universal $a$ value:

$$m \to m\,(1-\delta)^{\frac{\gamma_0}{2b_0}};$$ (e.g. $\frac{\gamma_0}{2b_0}^{QCD}(n_f = 3) = \frac{4}{9}$)

+ Removes spurious solutions incompatible with AF!

# Pre-QCD guidance: Gross-Neveu model

•$D = 2\ O(2N)$ GN model shares many properties with QCD (asymptotic freedom, (discrete) chiral sym., mass gap,..)

$$\mathcal{L}_{GN} = \bar{\Psi} i \not{\partial} \Psi + \frac{g_0}{2N} (\sum_1^N \bar{\Psi}\Psi)^2 \text{ (}\textit{massless}\text{)}$$

Standard mass-gap (massless, large $N$ approx.):

consider $V_{eff}(\sigma)$, $\sigma \sim \bar{\Psi}\Psi$;

$\sigma \equiv M = \mu e^{-\frac{2\pi}{g}} \equiv \Lambda_{\overline{\mathrm{MS}}}$

•Mass gap known exactly for any $N$:

$$\frac{M_{exact}(N)}{\Lambda_{\overline{\mathrm{MS}}}} = \frac{(4e)^{\frac{1}{2N-2}}}{\Gamma[1 - \frac{1}{2N-2}]}$$

(From $D = 2$ integrability: Bethe Ansatz) Forgacs et al '91

# Massive GN model

Now consider *massive* case (still large $N$):

$M(m,g) \equiv m(1+g\ln\frac{M}{\mu})^{-1}$: Resummed mass ($g/(2\pi) \to g$)

$= m(1-g\ln\frac{m}{\mu}+g^2(\ln\frac{m}{\mu}+\ln^2\frac{m}{\mu})+\cdots)$ (pert. re-expanded)

• Only fully summed $M(m,g)$ gives right result, upon:

-identify $\Lambda \equiv \mu e^{-1/g}$; $\to M(m,g) = \frac{m}{g\ln\frac{M}{\Lambda}} \equiv \frac{\hat{m}}{\ln\frac{M}{\Lambda}}$;

-take *reciprocal*: $\hat{m}(F \equiv \ln\frac{M}{\Lambda}) = F\, e^F\, \Lambda \sim F$ for $\hat{m} \to 0$;

$\to M(\hat{m} \to 0) \sim \frac{\hat{m}}{\hat{m}/\Lambda + \mathcal{O}(\hat{m}^2)} = \Lambda_{\overline{\text{MS}}}$

never seen in standard perturbation: $M_{pert}(m \to 0) \to 0$!

•But (RG)OPT gives $M = \Lambda_{\overline{\text{MS}}}$ at *first* (and any) $\delta$-order!

(at any order, OPT sol.: $\ln\frac{m}{\mu} = -\frac{1}{g}$,  RG sol.: $g = 1$ )

•At $\delta^2$-order (2-loop), RGOPT $\sim 1-2\%$ from $M_{exact}$(any $N$)

# 4. QCD Application: Pion decay constant $F_\pi$

Consider $SU(n_f)_L \times SU(n_f)_R \to SU(n_f)_{L+R}$ for $n_f$ massless quarks. ( $n_f = 2$, $n_f = 3$)

Define/calculate pion decay constant $F_\pi$ from

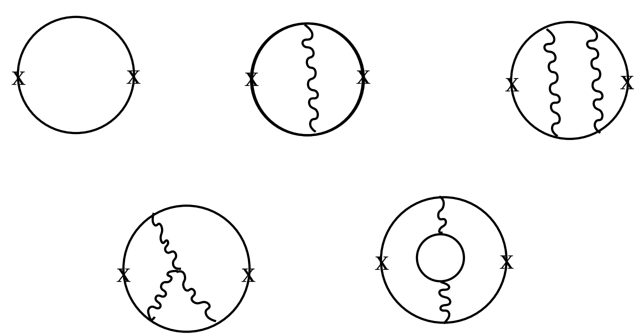
$$i\langle 0|T A^i_\mu(p) A^j_\nu(0)|0\rangle \equiv \delta^{ij} g_{\mu\nu} F^2_\pi + \mathcal{O}(p_\mu p_\nu)$$

where quark axial current: $A^i_\mu \equiv \bar{q}\gamma_\mu\gamma_5 \frac{\tau_i}{2}\, q$

$F_\pi \neq 0$: Chiral symmetry breaking order parameter

Advantage: Perturbative expression known to 3,4 loops

(3-loop Chetyrkin et al ’95; 4-loop Maier et al ’08 ’09, +Maier, Marquard private comm.)



## (Standard) perturbative available information

$$F_\pi^2(pert)_{\overline{\text{MS}}} = N_c \frac{m^2}{2\pi^2}\left[-L + \frac{\alpha_S}{4\pi}(8L^2 + \frac{4}{3}L + \frac{1}{6})\right.$$
$$\left.+(\frac{\alpha_S}{4\pi})^2[f_{30}(n_f)L^3 + f_{31}(n_f)L + f_{32}(n_f)L + f_{33}(n_f)] + \mathcal{O}(\alpha_S^3)\right]$$

$L \equiv \ln\frac{m}{\mu}$, $n_f = 2(3)$

Note: finite part (after mass + coupling renormalization) not separately RG-inv: (i.e. $F_\pi^2$, as defined, mixes with $m^2\,1$ operator)

$\rightarrow$ (extra) renormalization by subtraction of the form:
$S(m,\alpha_S) = m^2(s_0/\alpha_S + s_1 + s_2\alpha_S + \ldots)$ where $s_i$ fixed requiring RG-inv order by order: $s_0 = \frac{3}{16\pi^3(b_0-\gamma_0)}$, $s_1 = \ldots$

Same feature for $\langle\bar{q}q\rangle$, related to vacuum energy, needs an extra (additive) renormalization in $\overline{\text{MS}}$-scheme to be RG consistent.

# Warm-up calculation: pure RG approximation

neglect non-RG (non-logarithmic) terms:

$$F_\pi^2(\text{RG-1}, \mathcal{O}(g)) = 3\frac{m^2}{2\pi^2}\left[-L + \frac{\alpha_S}{4\pi}(8L^2 + \frac{4}{3}L) - (\frac{1}{8\pi(b_0-\gamma_0)\,\alpha_S} - \frac{5}{12})\right]$$

$$\to F_\pi^2(m \to m(1-\delta)^{\gamma_0/(2b_0)}, \alpha_S \to \delta\alpha_S, \mathcal{O}(\delta))|_{\delta\to 1} =$$

$$3\frac{m^2}{2\pi^2}\left[-\frac{102\pi}{841\,\alpha_S} + \frac{169}{348} - \frac{5}{29}L + \frac{\alpha_S}{4\pi}(8L^2 + \frac{4}{3}L)\right]$$

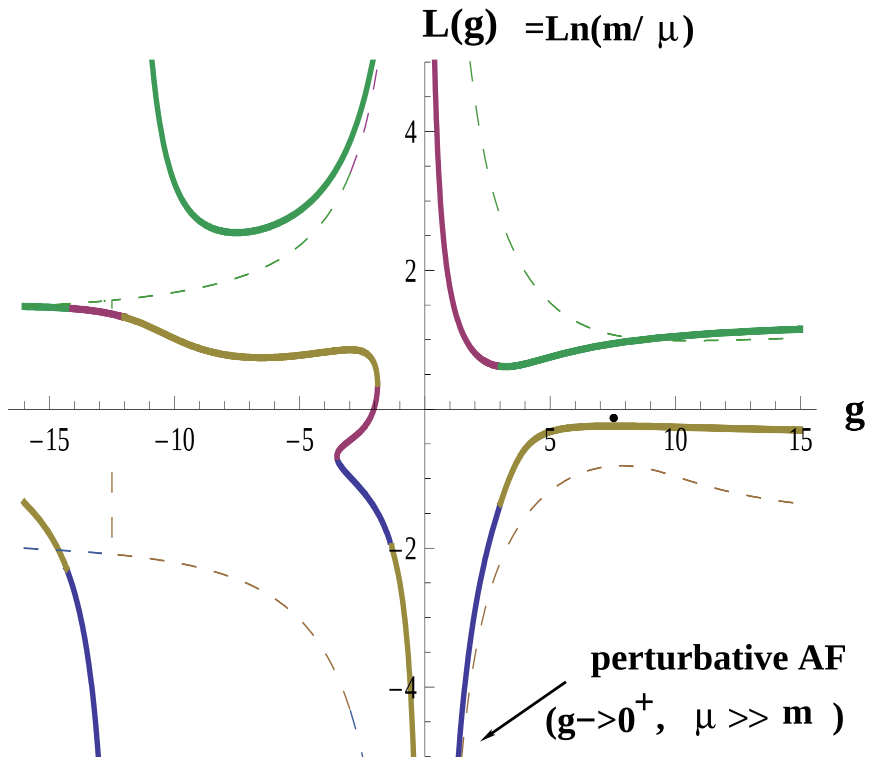
OPT+RG: $\partial_m(F_\pi^2/\Lambda^2_{\overline{\text{MS}}}), \partial_{\alpha_S}(F_\pi^2/\Lambda^2_{\overline{\text{MS}}}) \equiv 0$: have a unique AF-compatible real solution: $\tilde{L} \equiv \ln\frac{\tilde{m}}{\mu} = -\frac{\gamma_0}{2b_0}$ ; $\tilde{\alpha}_S = \frac{\pi}{2}$

$$\to F_\pi(\tilde{m}, \tilde{\alpha}_S) = (\frac{5}{8\pi^2})^{1/2}\tilde{m} \simeq 0.25\Lambda_{\overline{\text{MS}}}$$

•Includes higher orders +non-RG terms: $\tilde{m}_{opt}$ remains $\mathcal{O}(\Lambda_{\overline{\text{MS}}})$ (rather than $m \sim 0$): RG-consistent 'mass gap',

And OPT *stabilizes* $\alpha_S^{opt} \simeq .5$: more perturbative values

# Exact $F_\pi$ RG+OPT solutions at 4-loops ($\overline{\rm MS}$)




All branches of RG (thick) and OPT(dashed) solutions $Re[L \equiv \ln \frac{m}{\mu}(g)]$ to the $\delta$-modified 3rd order (4-loop) perturbation ($g = 4\pi\alpha_S$). Unique AF compatible sol.: black dot

- However beyond lowest order, AF-compatibility and reality of solutions appear mutually exclusive...

But, complex solutions are artefacts of solving *exactly* the RG and OPT (polynomial in $L$) Eqs...

# Recovering real AF-compatible solutions

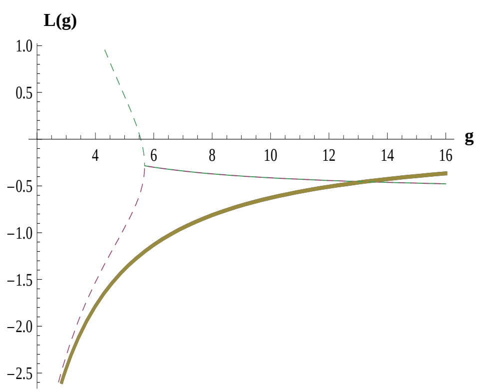
Are there *perturbative* 'deformations' consistent with RG?:

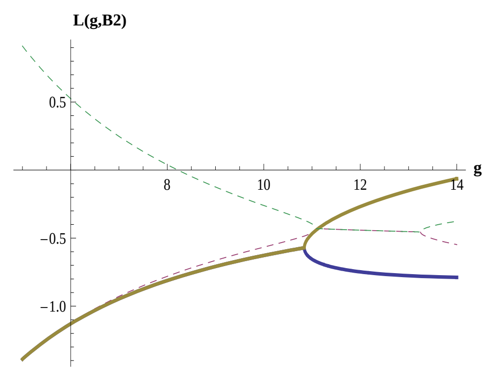
Evidently: Renormalization scheme changes (RSC)!

$m \to m'(1 + B_1 g' + B_2 g'^2 + \cdots)$, $g \to g'(1 + A_1 g' + A_2 g'^2 + \cdots)$

Require *contact* solution (thus closest to $\overline{\text{MS}}$):

$$\frac{\partial}{\partial g}\text{RG}(g, L, B_i)\frac{\partial}{\partial L}\text{OPT}(g, L, B_i) - \frac{\partial}{\partial L}\text{RG}\,\frac{\partial}{\partial g}\text{OPT} \equiv 0$$

$\mathcal{O}(\delta)$, $\overline{\text{MS}}$: 


$\to$ 


RSC affects pert. coefficients, but with property:

$$F_\pi^{\overline{MS}}(\overline{m}, g; \overline{f}_{ij}) = F'_\pi(m', g'; f'_{ij}(B_i)) + g^{k+1}\text{remnant}(B_i)$$

$\to$ differences *should* decrease with perturbative order!

# Results, with theoretical uncertainties JLK, Neveu 1305.6910 PRD

Beside recovering real solution, RSC offer natural, reasonably convincing uncertainty estimates: non-unique RSC
$\to$ we take differences between those as th. uncertainties

Table 1: Main optimized results at successive orders ($n_f = 3$)

| $\delta^k$ order | nearest-to-$\overline{\text{MS}}$ RSC $\tilde{B}_i$ | $\tilde{L}'$ | $\tilde{\alpha}_S$ | $\frac{F_0}{\overline{\Lambda}_{4l}}$ (RSC uncertainties) |
|---|---|---|---|---|
| $\delta$, RG-2l | $\tilde{B}_2 = 2.38\,10^{-4}$ | $-0.523$ | $0.757$ | $0.27 - 0.34$ |
| $\delta^2$, RG-3l | $\tilde{B}_3 = 3.39\,10^{-5}$ | $-1.368$ | $0.507$ | $0.236 - 0.255$ |
| $\delta^3$, RG-4l | $\tilde{B}_4 = 1.51\,10^{-5}$ | $-1.760$ | $0.374$ | $0.2409 - 0.2546$ |

$n_f = 2$: $\frac{F}{\overline{\Lambda}}(\delta^2) = 0.213 - 0.269$ ($\tilde{\alpha}_S = 0.46 - 0.64$)
$\frac{F}{\overline{\Lambda}}(\delta^3) = 0.2224 - 0.2495$ ($\tilde{\alpha}_S = 0.35 - 0.42$)

- Empirical stability/convergence exhibited, with $-2b_0\tilde{g}\tilde{L} \simeq 1$ i.e. $\tilde{m}_{opt} \simeq \mu\, e^{-1/(2b_0\tilde{g})}$ (like pure 1rst RG order)

# More realistic: explicit symmetry breaking

•Need to "subtract" effect from explicit chiral symmetry breaking from genuine quark masses $m_u, m_d, m_s \neq 0$: Unfortunately relies at this stage on other (mainly Lattice) results:

$\frac{F_\pi}{F} \sim 1.073 \pm 0.015$ [robust, $n_f = 2$ ChPT + Lattice]

$\frac{F_\pi}{F_0} \sim 1.172(3)(43)$ (Lattice MILC collaboration '10 using NNLO ChPT fits)

But quite different values by other collaborations

+ hint of slower convergence of $n_f = 3$ ChPT, e.g. Bernard, Descotes-Genon, Toucan '10

Alternative: implement explicit sym. break. within OPT (to be fully independent of ChPT+lattice results):

$m \to m^{true}_{u,d,s} + m(1-\delta)^{\gamma_0/(2b_0)}$: promising but rather involved RG+OPT Eqs. (no longer polynomial), work in progress...)

## Combined results with theoretical uncertainties:

Average different RSC +average $\delta^2$ and $\delta^3$ results:

$\overline{\Lambda}^{n_f=2}_{4-loop} \simeq 359^{+38}_{-26} \pm 5$ MeV

$\overline{\Lambda}^{n_f=3}_{4-loop} \simeq 317^{+14}_{-7} \pm 13$ MeV

To be compared with some recent lattice results, e.g.:

•'Schrödinger functional scheme' (ALPHA coll. Della Morte et al '12):

$\Lambda_{\overline{\text{MS}}}(n_f = 2) = 310 \pm 30$ MeV

•Wilson fermions (Göckeler et al '05)

$\Lambda_{\overline{\text{MS}}}(n_f = 2) = 261 \pm 17(\text{stat}) \pm 26(\text{syst})$ MeV

•Twisted fermions (+NP power corrections) (Blossier et al '10):

$\Lambda_{\overline{\text{MS}}}(n_f = 2) = 330 \pm 23 \pm 22_{-33}$ MeV

•static potential (Jansen et al '12): $\Lambda_{\overline{\text{MS}}}(n_f = 2) = 315 \pm 30$ MeV

## Extrapolation to $\alpha_S$ at high (perturbative) $q^2$

Use only $\Lambda_{\overline{\mathsf{MS}}}^{n_f=3}$ (more perturbative trustable threshold crossings)

•In $\overline{\mathsf{MS}}$-scheme, no *explicit* decoupling of large masses:

$m_{u,d} \ll m_s \ll \Lambda_{\overline{\mathsf{MS}}} \ll m_{charm} \ll m_{bottom} \cdots$

•need non-trivial decoupling/matching: $\Lambda_{\overline{\mathsf{MS}}}(n_f)$ and 'jumps': standard perturbative extrapolation (3,4-loop with $m_c$, $m_b$ threshold etc):

$$\alpha_S^{n_f+1}(\mu) = \alpha_S^{n_f}(\mu)\left(1 - \frac{11}{72}\left(\frac{\alpha_S}{\pi}\right)^2 + (-0.972057 + .0846515 n_f)\left(\frac{\alpha_S}{\pi}\right)^3\right)$$

$$\rightarrow \overline{\alpha}_S(m_Z) = 0.1174^{+.0010}_{-.0005} \pm .0010 \pm .0005_{evol}$$

$$\overline{\alpha}_S^{n_f=3}(m_\tau) = 0.308^{+.007}_{-.004} \pm .007 \pm .002_{evol}$$

Compare to 2013 world average: $\alpha_S(m_Z) = .1185 \pm .0007$

# 5. Chiral quark condensate $\langle \bar{q}q \rangle$

In a nutshell:

$$\langle \bar{q}q \rangle \equiv -2m \int_0^\infty d\lambda \frac{\rho(\lambda)}{\lambda^2 + m^2}$$

$\rho(\lambda)$ is the spectral density of the (euclidean) Dirac operator.

Banks-Casher relation:

$$lim_{m \to 0} \langle \bar{q}q \rangle = -\pi \rho(0)$$

Again an intrinsical nonperturbative quantity, vanishing to all orders of ordinary perturbation.

Conversely: $\rho(\lambda) = \frac{1}{2\pi} \left( \langle \bar{q}q \rangle (i\lambda - \epsilon) - \langle \bar{q}q \rangle (i\lambda + \epsilon) \right) |_{\epsilon \to 0}$

i.e. $\rho(\lambda)$ determined by dicontinuities of $\langle \bar{q}q \rangle (m)$ across imaginary axis.

Pert. expansion known to 3-loops (Chetyrkin et al) $\to$ $\ln(m \to i\lambda)$ discontinuities.

# RGOPT 3-loop for $\langle \bar{q} q \rangle$ ($n_f = 2, 3$) (preliminary!)

Real solutions:

$n_f = 2$: $\tilde{\alpha}_S \simeq 0.43 - 0.48$; $\ln \frac{\tilde{m}}{\mu} \simeq -(0.69 - 0.70)$

$-\frac{\langle \bar{q} q \rangle^{1/3}}{\Lambda_{\overline{\text{MS}}}}(n_f = 2)(\tilde{\mu} \simeq 1 GeV) \simeq 0.79 - 0.80$

$n_f = 3$: $\tilde{\alpha}_S \simeq 0.44 - 0.47$; $\ln \frac{\tilde{m}}{\mu} \simeq -(0.69 - 0.79)$

$-\frac{\langle \bar{q} q \rangle^{1/3}}{\Lambda_{\overline{\text{MS}}}}(n_f = 3)(\tilde{\mu} \simeq 1 GeV) \simeq 0.78 - 0.79$

$\rightarrow$ Appears to have very mild dependence on $n_f$.

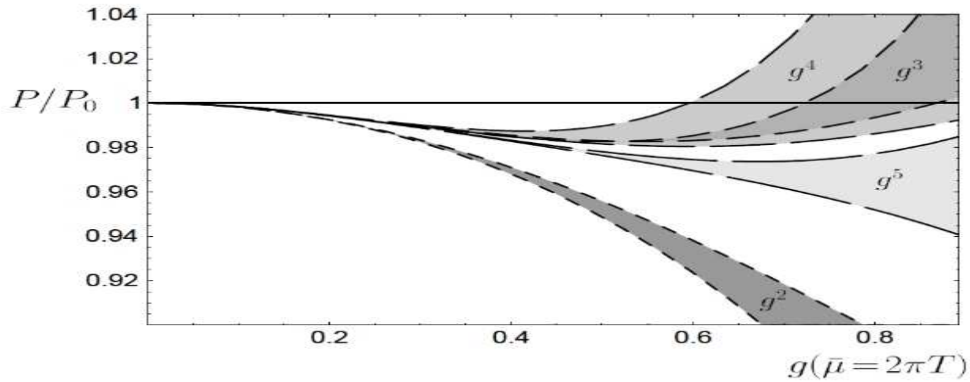
However, (see previous) $\Lambda_{\overline{\text{MS}}}(n_f = 2) > \Lambda_{\overline{\text{MS}}}(n_f = 3)$ (with larger $\Lambda_{\overline{\text{MS}}}(n_f = 2)$ uncertainty)

$\rightarrow -\langle \bar{q} q \rangle^{1/3}(n_f = 2, \mu = 1 GeV) \simeq 284^{+30}_{-20} MeV$

$\rightarrow -\langle \bar{q} q \rangle^{1/3}(n_f = 3, \mu = 1 GeV) \simeq 247^{+20}_{-13} MeV$

with uncertainties mostly from $\Lambda_{\overline{\text{MS}}}$ ones

# 6. RGOPT $\lambda\phi^4$ Pressure (JLK +M. Pinto, preliminary!)




$\lambda\phi^4$ pressure at successive ordinary pert. orders

bands=scale-dependence $\mu = \pi T - 4\pi T$

Many efforts to improve this, motivated by QGP (review e.g. Blaizot et al '03) Screened Pert. (Karsh et al '97, $\sim$ Hard Thermal loop (HTL) resummation (Andersen, Braaten, Strickland) -Functional RG, 2-particle irreducible (2PI) formalism (Blaizot, Iancu, Rebhan '01)

Culprit (in a nutshell): mix up of *hard* $p \sim T$ and *soft* $p \sim \lambda T$ modes.
Yet thermal 'Debye' screening mass $m_D^2 \lambda T^2$ generated gives IR cutoff,
BUT $\rightarrow$ Perturbative expansion in $\sqrt{\lambda}$ ($\sqrt{\alpha}_S$ in QCD) $\rightarrow$ slower convergence
Yet most of interesting physics happens at moderate $\lambda$ values..
But large scale-dependence (increasing with order) remains very odd, specially for HTL
RGOPT cures this, essentially by treating thermal mass 'RG consistently'

(NB some qualitative links with Blaizot, Wschebor 1409.4795)

# RGOPT($\lambda\phi^4$)

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{m^2}{2}(1-\delta)^{2\frac{\gamma_0}{b_0}}\phi^2 - \delta\frac{\lambda}{4!}\phi^4$$

NB $2\gamma_0/b_0 = 1/3$.

2-loop Vacuum energy ($\overline{\text{MS}}$ scheme):

$$(4\pi)^2\mathcal{F}_0 = \mathcal{E}_0 - \tfrac{1}{8}m^4(3 + 2\ln\tfrac{\mu^2}{m^2}) - \tfrac{1}{2}J_0(\tfrac{m}{T})T^4$$
$$+\tfrac{1}{8}\tfrac{\lambda}{16\pi^2}\left[(\ln\tfrac{\mu^2}{m^2}+1)m^2 - J_1(\tfrac{m}{T})T^2\right]^2$$

T-dependent part: $J_0(\frac{m}{T}) =\sim \int_0^\infty dp \frac{1}{\sqrt{p^2+m^2}}\,\frac{1}{E^{\sqrt{p^2+m^2}}-1}$

$\mathcal{E}_0$: *finite* vacuum energy terms: $\mathcal{E}_0(\lambda, m) = -\frac{m^4}{\lambda}\sum_{k\geq 0} s_k\lambda^k$

$$s_0 = \frac{1}{2(b_0-4\gamma_0)} = 8\pi^2; \quad s_1 = \frac{(b_1-4\gamma_1)}{8\gamma_0\,(b_0-4\gamma_0)} = -1$$

(NB $T$-independent, determined consistently by requiring RG invariance!)

NB: non-trivial OPT solution $\tilde{m}(\lambda, T)$ already at one-loop (not the case for HTLpt).

# RGOPT one-loop ($\mathcal{O}(\delta^0)$)

exact OPT solution: $\tilde{m}^2 = \frac{\lambda}{2}\left[b_0\tilde{m}^2(\ln\frac{\tilde{m}^2}{\mu^2}-1)+T^2 J_1(\frac{\tilde{m}}{T})\right]$

approximate $m/T \lesssim 1$ OPT Eq. form is simple quadratic (sufficient for all purpose):

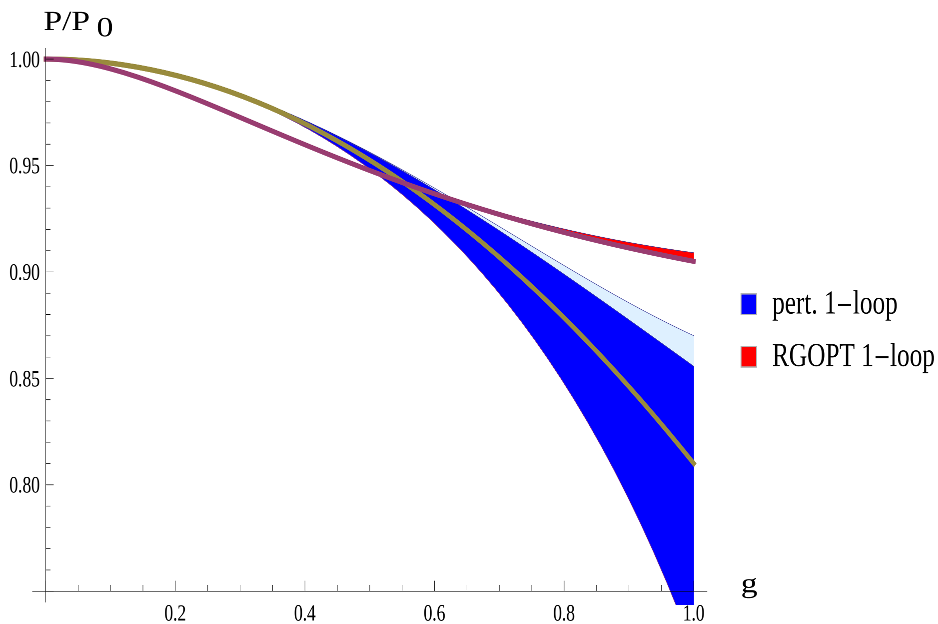
$$(\frac{1}{b_0\,\lambda}+\gamma_E+\ln\frac{\mu}{4\pi T})\,(\frac{m}{T})^2+2\pi\,\frac{m}{T}-2\frac{\pi^2}{3}=0$$

$$\frac{\tilde{m}^{(1)}}{T}=\pi\frac{\sqrt{1+\frac{2}{3}(\frac{1}{b_0\lambda}+L_T)}-1}{\frac{1}{b_0\lambda}+L_T}\sim\frac{1}{2\sqrt{2}}\sqrt{\lambda}-\pi b_0\lambda+\frac{3}{128\pi^2\sqrt{2}}(3-2L_T)\lambda^{3/2}+\cdots$$

$$L_T\equiv\gamma_E+\ln\frac{\mu}{4\pi T}$$

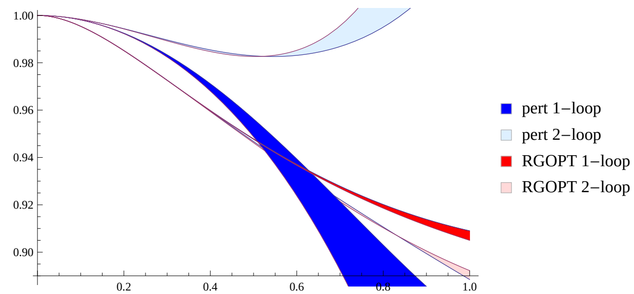
- explicitly exactly scale-invariant!
- reproduces qualitatively more sophisticated 2PI (first order) results!

# RGOPT Pressure: one-loop

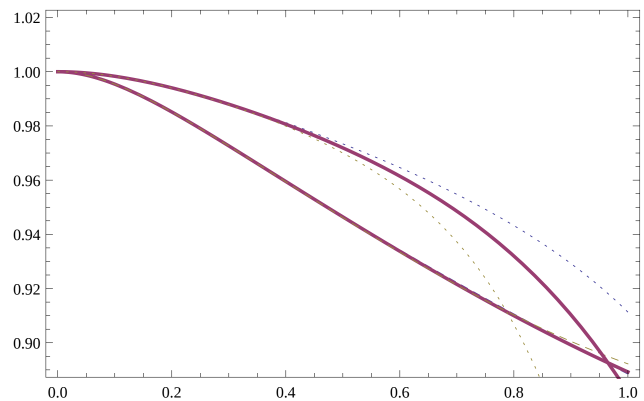



$g(\mu) = (\lambda(\mu)/24)^{1/2}$ with scale-dependence $\mu = \pi T - 4\pi T$

# Two-loops




$g(\mu) = (\lambda(\mu)/24)^{1/2}$ with scale-dependence $\mu = \pi T - 4\pi T$




RGOPT versus standard OPT (HTLPT)

# 5. Summary and Outlook

•OPT gives a simple procedure to resum perturbative expansions, using only perturbative information.

•Our RGOPT version includes 2 major differences w.r.t. most previous OPT approaches:

1) OPT+ RG minimizations fix optimized $\tilde{m}$ *and* $\tilde{g} = 4\pi\tilde{\alpha}_S$
2) Requiring AF-compatible solutions uniquely fixes the basic interpolation $m \to m(1-\delta)^{\gamma_0/(2b_0)}$: discards spurious solutions and accelerates convergence.

($\mathcal{O}(10\%)$ accuracy at 1-2-loops, empirical stability exhibited at 3-loop)

Straightforward to apply for $T \neq 0$: exhibit remarkable stability + scale independence (sharp contrast with 'standard' OPT $\sim$ HTLpt)

•Outlook: (almost) straightforward application to thermal QCD (start with pure gluon pressure)