

Octet baryon masses in covariant baryon chiral perturbation theory

Xiu-Lei Ren

School of Physics and Nuclear Energy Engineering, Beihang University
Institut de Physique Nucléaire, IN2P3-CNRS and Université Paris-Sud

Collaborators:

Li-Sheng Geng (BUAA), Jie Meng (PKU & BUAA),
J. Martin-Camalich (U.C. San Diego), and H. Toki (RCNP)

OUTLINE

- Introduction
- Theoretical Framework
- Numerical Details
- Results and Discussion
- Summary

Origin of masses

□ Current quark masses – Explained

- Standard Model → Higgs Mechanism
- LHC @ CERN → Higgs particle

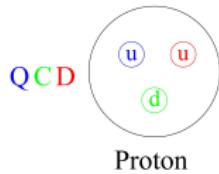
ATLAS Collaboration, PLB716(2012)1

CMS Collaboration, PLB716(2012)30

Nobel Prize 2013



□ Light hadron masses – Complicated



$M_u \sim 3 \text{ MeV}$
 $M_d \sim 6 \text{ MeV}$
 $M_p = 938 \text{ MeV}$
(Strong force)

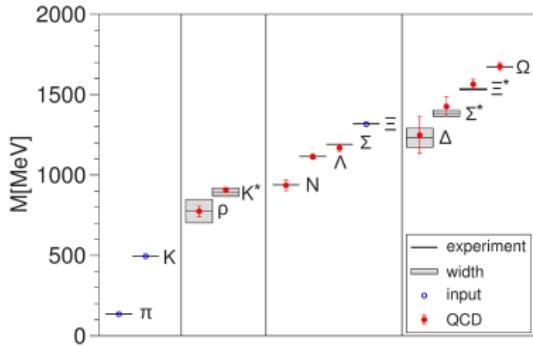
$$M_p \text{ (938MeV)} \gg m_u + m_u + m_d \text{ (12MeV)}$$

- Current quark masses (1-3%)
- Non-perturbative strong interaction (>95%)
 - Lattice QCD
 - Chiral Perturbation Theory
 - Other Models

Octet baryon masses in LQCD

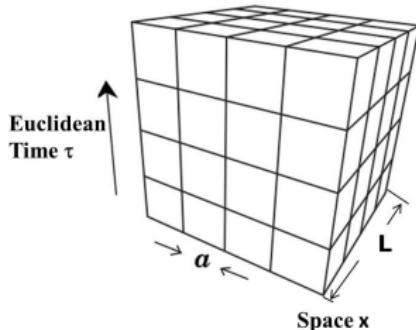
■ $N_f = 2 + 1$ lattice calculation

- BMW, S. Dürr et al., *Science* 322 (2008) 1224
- PACS-CS, S. Aoki et al., *PRD* 79 (2009) 034503
- LHPC, A. Walker-Loud et al., *PRD* 79 (2009) 054502
- HSC, H.-W. Lin et al., *PRD* 79 (2009) 034502
- UKQCD, W. Bietenholz et al., *PRD* 84 (2011) 054509
- NPLQCD, S. Beane et al., *PRD* 84 (2011) 014507



■ Test the consistency – crucial

- Lattice simulations:
 - different fermion/gauge actions
 - different quark masses
 - different lattice volumes ($V = L^4$)
 - different lattice spacings (a)
- In continuum:
should lead to the same theory – QCD



LQCD supplemented BChPT

Cost of LQCD

$$\text{Cost} \propto \left(\frac{L}{a}\right)^4 \frac{1}{a} \frac{1}{m_{u/d} a}.$$

Limitation of LQCD

Input of LQCD	Simulation	Physical World
Light quark masses $m_{u/d}$	~ 10 MeV	3 – 5 MeV
Lattice box size L	2 – 5 fm	Infinite space time
Lattice spacing a	$a \sim 0.1$ fm	Continuum

In order to obtain the physical values

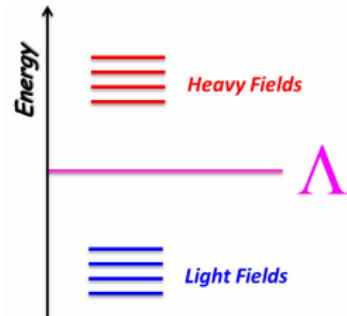
$$\begin{array}{ccc} m_{u/d}^{\text{Lat.}} & \xrightarrow{\text{Chiral extrapolation}} & m_{u/d}^{\text{Phys.}} \\ L & \xrightarrow{\text{Finite-volume corrections}} & \infty \\ a & \xrightarrow{\text{Continuum extrapolation}} & 0 \end{array}$$

Baryon Chiral Perturbation Theory (BChPT) is a powerful tool to perform **the multi-extrapolation** for LQCD simulations.

Baryon Chiral Perturbation Theory

□ Effective Field Theory of low-energy QCD

- Degrees of freedom
 - ✓ Pseudoscalar mesons, ✓ Baryons (Octet and Decuplet)
- Chiral symmetry $SU(3)_L \times SU(3)_R$
- Explicit and spontaneous symmetry breaking



□ Solving the Power Counting Breaking problem

Non-Relativistic	Relativistic
Heavy-Baryon ChPT <i>E.E. Jenkins et al., PLB(1991)</i>	Infrared BCChPT <i>T. Becher et al., EPJC(1999)</i>
Baryon as static source	$H = I + R$
Strict power-counting	$\int_0^1 \dots = \int_0^\infty \dots - \int_1^\infty \dots$

- Several successful applications of **EOMS BCChPT** (in the $SU(3)$ sector)
 - ✓ Baryon magnetic moments, ✓ Baryon axial couplings, ✓ Hyperon vector coupling $f_1(0)$

PRL101,222002,PLB676,63, PRD80,034027

PRD78,014011, PRD90,054502

PRD79,094022, PRD89,113007

Octet baryon masses in BCChPT

□ Up to NNLO

- Heavy Baryon ChPT
 - ☞ failed to describe the lattice data *PACS-CS, PRD(2009), LHPC, PRD(2009)*
- EOMS BCChPT
 - ☞ Improved description of the PACS-CS and LHPC data *J. Martin-Camalich et al., PRD(2010)*
 - ☞ Finite-volume effects in LQCD simulations are very important *L.S. Geng et al., PRD(2011)*
- Finite-range regularization + HBChPT
 - ☞ nice description of the PACS-CS and LHPC data *R.D. Young et al., PRD(2010)*

□ Up to $N^3\text{LO}$ – Few calculations

Many low-energy constants (LECs) need to be fixed

- Partial summation BCChPT
 - ☞ nice description of the BMW, PACS, LHPC and UKQCD data *A. Smeke et al., PRD(2012), M.F.M. Lutz et al., PRD(2013)*
- Infrared BCChPT
 - ☞ nice description of UKQCD data *P.C. Bruns et al., PRD(2013)*

In this work

Calculate the octet baryon masses in the EOMS BChPT up to $N^3\text{LO}$ to systematically study the LQCD data

- ☒ Take into account finite volume corrections (FVCs) self-consistently
- ☒ Perform a simultaneous fit of all the $N_f = 2 + 1$ lattice results
 - ☞ Fix LECs and perform chiral extrapolation
 - ☞ Test the consistency of different LQCD data
- ☒ Perform the continuum extrapolation of LQCD up to $\mathcal{O}(a^2)$
 - ☞ Evaluate the finite lattice spacing discretization effects
- ☒ Accurately predict the sigma terms of octet baryon

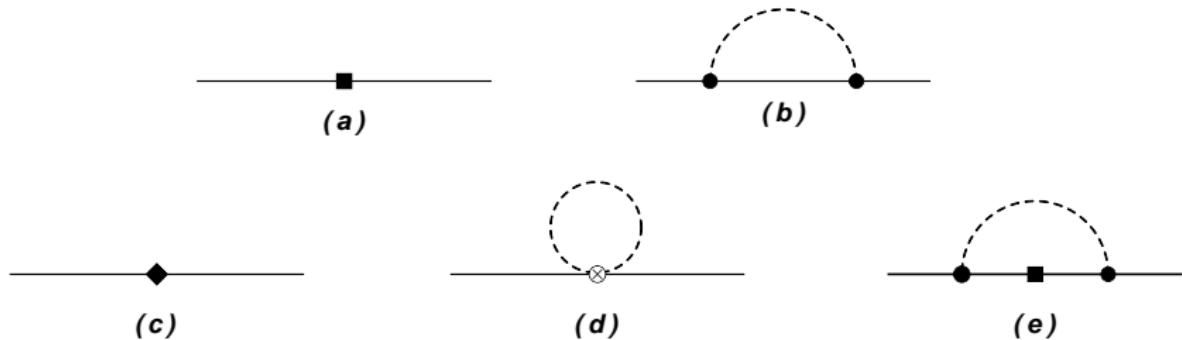
Theoretical Framework

□ Effective Lagrangians up to N³LO

$$\begin{aligned}\mathcal{L}_{\text{eff}} &= \mathcal{L}_{\phi}^{(2)} + \mathcal{L}_{\phi}^{(4)} + \mathcal{L}_{\phi B}^{(1)} + \mathcal{L}_{\phi B}^{(2)} + \mathcal{L}_{\phi B}^{(3)} + \mathcal{L}_{\phi B}^{(4)} \\ &= \frac{\mathbf{F}_{\phi}^2}{4} \langle D_{\mu} U (D_{\mu} U)^{\dagger} \rangle + \frac{\mathbf{F}_{\phi}^2}{4} \langle \chi U^{\dagger} + U \chi^{\dagger} \rangle + \sum_{i=4}^8 \mathbf{L}_i \hat{\mathcal{O}}_{\phi}^{(4)} \\ &\quad + \langle \bar{B} (i \not{D} - \mathbf{M}_0) B \rangle + \frac{\mathbf{D}/\mathbf{F}}{2} \langle \bar{B} \gamma^{\mu} \gamma_5 [u_{\mu}, B]_{\pm} \rangle \\ &\quad + \mathbf{b}_0 \langle \chi_{+} \rangle \langle B \bar{B} \rangle + \mathbf{b}_{D/F} \langle \bar{B} [\chi_{+}, B]_{\pm} \rangle + \sum_{j=1}^8 \mathbf{b}_j \hat{\mathcal{O}}_{\phi B}^{(2)} + \sum_{k=1}^7 \mathbf{d}_k \hat{\mathcal{O}}_{\phi B}^{(4)}.\end{aligned}$$

- The meson Lagrangians. *Gasser,NPB(1985)*
 - LECs from $\mathcal{L}_{\phi}^{(2)}$, $\mathcal{L}_{\phi}^{(4)}$: \mathbf{F}_{ϕ} , \mathbf{L}_i , $i \in (4, 5, 6, 7, 8)$
- The meson-baryon Lagrangians. *Borasoy,A.P(1996)*, *Oller,JHEP(2006)*
 - LECs from $\mathcal{L}_{\phi B}^{(1)}$: \mathbf{m}_0 , \mathbf{D} , \mathbf{F}
 - LECs from $\mathcal{L}_{\phi B}^{(2)}$: \mathbf{b}_0 , \mathbf{b}_D , \mathbf{b}_F , \mathbf{b}_j , $j \in (1, \dots, 8)$
 - LECs from $\mathcal{L}_{\phi B}^{(4)}$: \mathbf{d}_k , $k \in (1, \dots, 7)$

Feynman diagrams up to N³LO



Fields: Solid lines – octet-baryons, Dashed lines – Pseudoscalar mesons

Vertex: Boxes – $\mathcal{L}_{\phi B}^{(2)}$, Diamonds – $\mathcal{L}_{\phi B}^{(4)}$, Solid dot – $\mathcal{L}_{\phi B}^{(1)}$, circle-cross – $\mathcal{L}_{\phi B}^{(2)}$

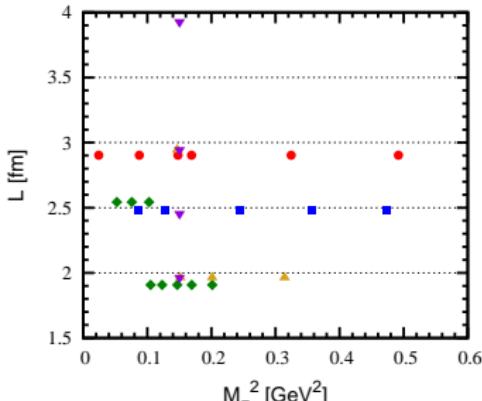
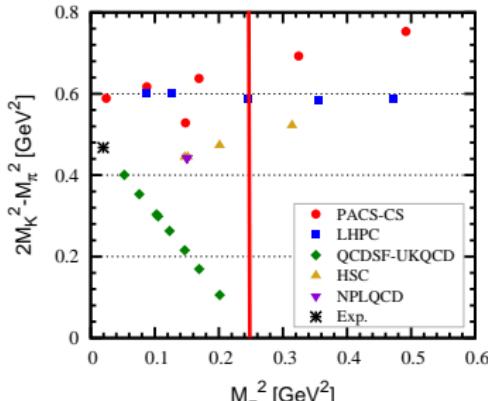
- ① Calculating the baryon self-energy in covariant BChPT
- ② Subtracting the PCB terms with EOMS scheme
- ③ Taking into account the FVC

Octet baryon masses in finite volume box

$$\begin{aligned} m_B(M_\phi) &= m_0 + m_B^{(2)}(M_\phi) + m_B^{(3)}(M_\phi) + m_B^{(4)}(M_\phi) \\ &= m_0 + \sum_{\phi=\pi,K} \xi_{B,\phi}^{(a)} M_\phi^2 + \sum_{\phi_1,\phi_2=\pi,K,\eta} \xi_{B,\phi_1,\phi_2}^{(c)} M_{\phi_1}^2 M_{\phi_2}^2 \\ &\quad + \sum_{\phi=\pi,K,\eta} \xi_{B,\phi}^{(b)} \left[H_{\text{loop}}^{(b)} - H_{\text{pcb}}^{(b)} - \Delta H_{\text{FVC}}^{(b)} \right] \\ &\quad + \sum_{\phi=\pi,K,\eta} \xi_{B,\phi}^{(d)} \left[H_{\text{loop}}^{(d)} - \Delta H_{\text{FVC}}^{(d)} \right] \\ &\quad + \sum_{\phi=\pi,K,\eta} \xi_{B,\phi}^{(e)} \left[H_{\text{loop}}^{(e)} - H_{\text{pcb}}^{(e)} - \Delta H_{\text{FVC}}^{(e)} \right]. \end{aligned}$$

Numerical Details

- Fitting data: LQCD results (11-sets) + Exp. values
 - PACS-CS, LHPC, QCDSF-UKQCD, HSC, NPLQCD
 - Lattice data with $M_\pi < 500$ MeV
 - ☞ reduce the higher order contributions of chiral expansions
 - Lattice data with $M_\phi L > 4$
 - ☞ minimize finite-volume effects of LQCD
 - **Fitting points:** 44(LQCD) + 4(Exp.) = 48



Results and Discussion

Octet baryon masses in EOMS BChPT

Assuming: virtual decuplet effects can be absorbed by LECs.

- Fitting methods

	Fitting formula	Free parameters	
NLO	$m_0 + m_B^{(2)}$	m_0, b_0, b_D, b_F	4
NNLO	$m_0 + m_B^{(2)} + m_B^{(3)}$	m_0, b_0, b_D, b_F	4
N^3LO	$m_0 + m_B^{(2)} + m_B^{(3)} + m_B^{(4)}$	$m_0, b_0, b_D, b_F, b_i, d_j$	19

- Other parameters

- $L_{4,5,6,7,8}^r$, J. Bijnens et al., NPB(2012), with $\mu = 1 \text{ GeV}$
- $F_0 = 0.0871 \text{ GeV}$, G. Amoros et al., NPB(2001)
- $D = 0.80, F = 0.46$

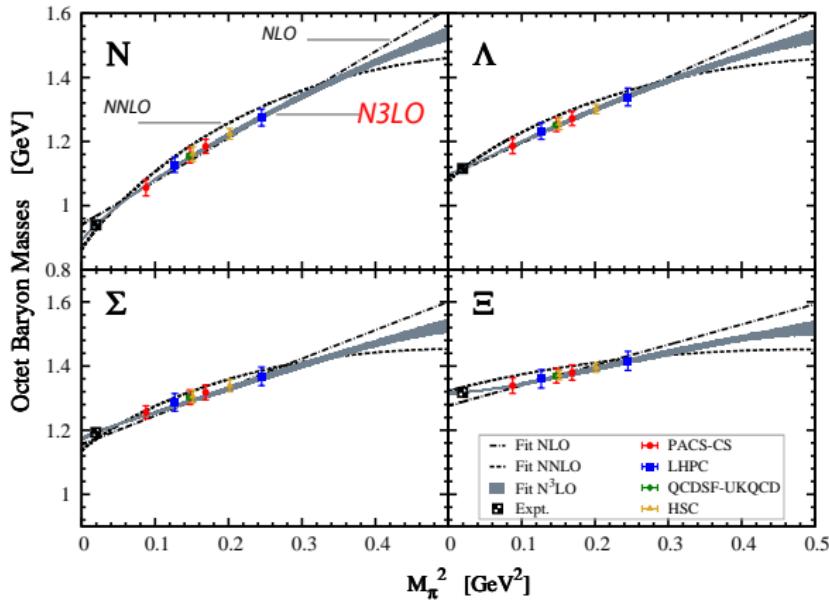
Best fitting results

	NLO	NNLO	$N^3\text{LO}$
m_0 [MeV]	900(6)	767(6)	880(22)
b_0 [GeV $^{-1}$]	-0.273(6)	-0.886(5)	-0.609(19)
b_D [GeV $^{-1}$]	0.0506(17)	0.0482(17)	0.225(34)
b_F [GeV $^{-1}$]	-0.179(1)	-0.514(1)	-0.404(27)
b_1 [GeV $^{-1}$]	-	-	0.550(44)
b_2 [GeV $^{-1}$]	-	-	-0.706(99)
b_3 [GeV $^{-1}$]	-	-	-0.674(115)
b_4 [GeV $^{-1}$]	-	-	-0.843(81)
b_5 [GeV $^{-2}$]	-	-	-0.555(144)
b_6 [GeV $^{-2}$]	-	-	0.160(95)
b_7 [GeV $^{-2}$]	-	-	1.98(18)
b_8 [GeV $^{-2}$]	-	-	0.473(65)
d_1 [GeV $^{-3}$]	-	-	0.0340(143)
d_2 [GeV $^{-3}$]	-	-	0.296(53)
d_3 [GeV $^{-3}$]	-	-	0.0431(304)
d_4 [GeV $^{-3}$]	-	-	0.234(67)
d_5 [GeV $^{-3}$]	-	-	-0.328(60)
d_7 [GeV $^{-3}$]	-	-	-0.0358(269)
d_8 [GeV $^{-3}$]	-	-	-0.107(32)
$\chi^2/\text{d.o.f.}$	11.8	8.6	1.0

- EOMS-BChPT shows a clear improvement order by order
- Different lattice QCD calculations are consistent with each other
- Values of LECs from EOMS- $N^3\text{LO}$ look very natural
- $m_0 = 880$ MeV consistent with the SU(2)-BChPT results.
M. Procura et al., PRD(2003,2006)
L. Alvarez-Ruso et al., PRD(2013)
- Neglecting finite-volume corrections would lead to $\chi^2/\text{d.o.f.} = 1.9$.

Table: Values of the LECs.

Chiral extrapolation



- **NLO fitting** linear and can not describe the experimental value
- **NNLO fitting** more curved and can not well describe lattice data
- **N 3 LO fitting** can give a good description of LQCD and Exp. data, confirm the linear dependence of the lattice data on M_π^2

Up to now...

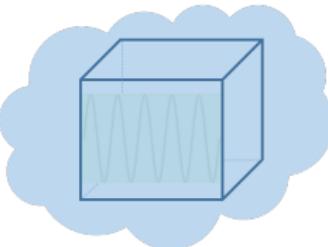
Multi-extrapolation of LQCD

$m_{u/d}^{\text{Lat.}}$	<u>Chiral extrapolation</u>	$m_{u/d}^{\text{Phys.}}$	✓
L	<u>Finite-volume corrections</u>	∞	✓
a	<u>Continuum extrapolation</u>	0	

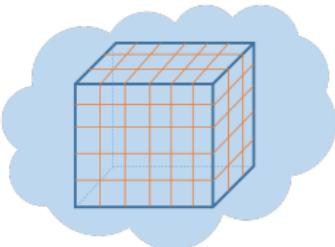
Infinite time-space



Finite hypercube



Finite lattice spacing



Discretization effects of LQCD

- ✓ Formulate the discretization effects $m_B^{(a)}$ up to $\mathcal{O}(a^2)$ for the Wilson action

- Use Symanzik effective action [K.Symanzik, NPB\(1983\)](#)

$$S_{\text{eff}} = S_0^{\text{QCD}} + aS_1 + a^2S_2 + \dots$$

- Construct effective Lagrangians up to $\mathcal{O}(p^4)$ ($p^2 \sim \frac{m_q}{\Lambda_{\text{QCD}}} \sim a\Lambda_{\text{QCD}}$)

$$\mathcal{L}_a^{\text{eff}} = \mathcal{L}^{\mathcal{O}(a)} + \mathcal{L}^{\mathcal{O}(am_q)} + \mathcal{L}^{\mathcal{O}(a^2)}.$$

- Calculate discretization effects of LQCD with the Wilson fermion

$$m_B^{(a)} = m_B^{\mathcal{O}(a)} + m_B^{\mathcal{O}(am_q)} + m_B^{\mathcal{O}(a^2)}.$$

- ✓ Self-consistently including FVCs and discretization effects in the octet baryon masses

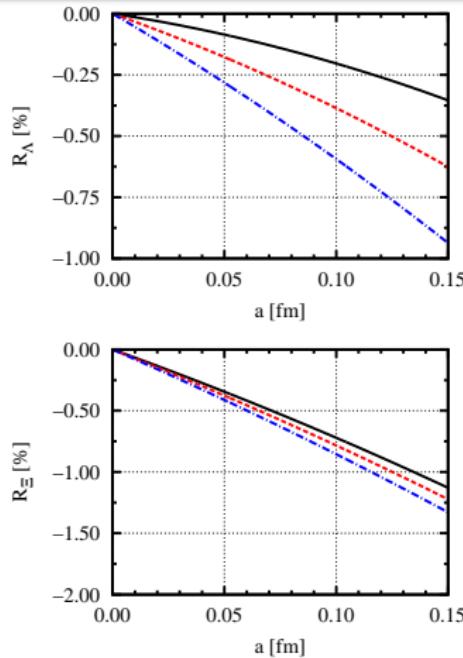
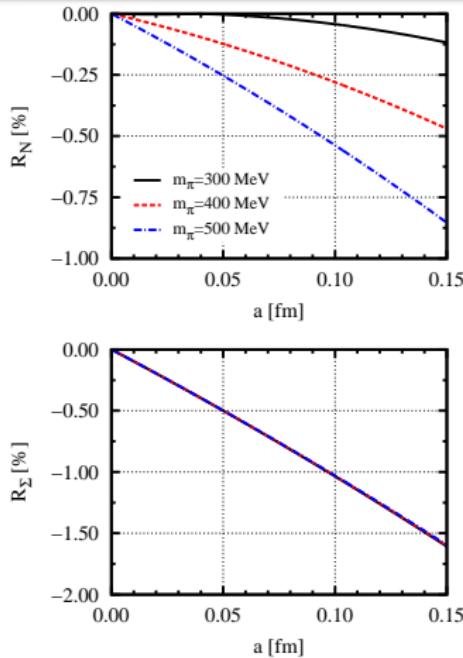
$$m_B = m_0 + m_B^{(2)} + m_B^{(3)} + m_B^{(4)} + \textcolor{red}{m}_B^{(a)}.$$

- 19 LECs + 4 new LECs (related to lattice spacing)

- ✓ Study the LQCD results obtained with $\mathcal{O}(a)$ -improved Wilson actions

- 10 sets: PACS-CS, QCDSF-UKQCD, HSC, NPLQCD

Evolution of discretization effects with a and m_π



$$R_B = \frac{m_B^{(a)}}{m_B}$$

- Pion mass m_π fixed:

$$a \uparrow \sim m_B^{(a)} \uparrow$$

- Lattice spacing a fixed:

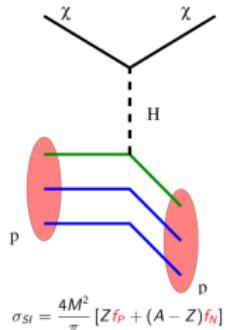
$$m_\pi \uparrow \sim m_B^{(a)} \uparrow$$

- Discretization effects on baryon masses **do not exceed 2% for $a = 0.15$ fm**
- Consistent** with early LQCD studies [S. Durr et al., Phys. Rev. D79, \(2009\) 014501](#).
- Up to $\mathcal{O}(a^2)$, discretization effects are **small** and can be safely **ignored**

Pion- and strangeness-octet baryon sigma terms

□ Nucleon-sigma term

- Related to chiral quark condensate $\langle \bar{q}q \rangle$
- Understand the **composition of the nucleon**
- Key input for direct **dark matter** searches
- **Strangeness-nucleon sigma term: $0 \sim 300$ MeV**



□ Feynman-Hellmann Theorem

$$\begin{aligned}\sigma_{\pi B} &= m_l \langle B(p) | \bar{u}u + \bar{d}d | B(p) \rangle \equiv m_l \frac{\partial \mathbf{m}_B}{\partial m_l}, \\ \sigma_{s B} &= m_s \langle B(p) | \bar{s}s | B(p) \rangle \equiv m_s \frac{\partial \mathbf{m}_B}{\partial m_s}.\end{aligned}$$

□ Three key factors to accurately predict baryon sigma terms

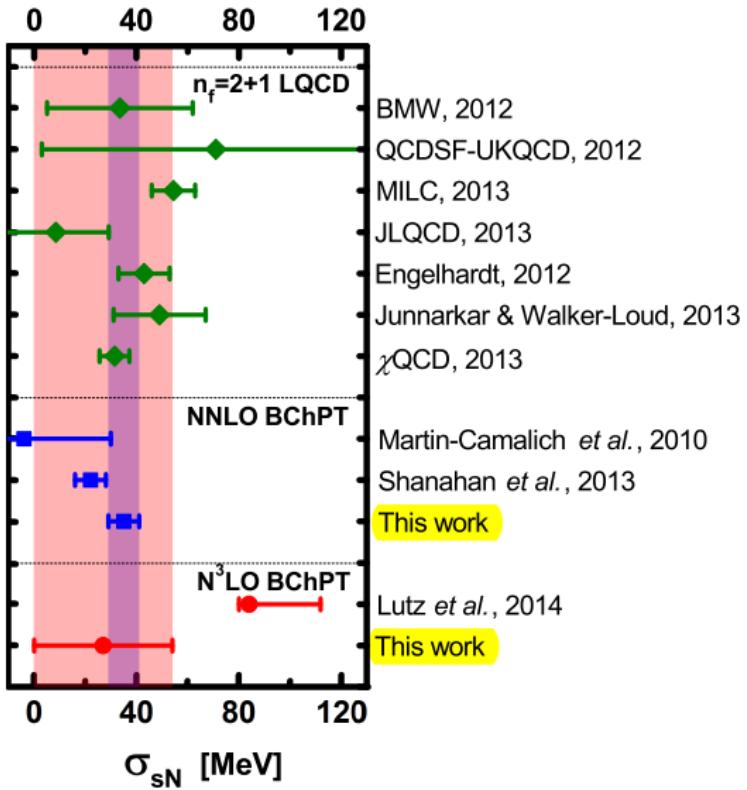
- **Effects of lattice scale setting:** mass independent vs. mass dependent
- **Strong isospin breaking effects:** better constrain the values of LECs
- **Chiral expansion truncations:** systematic uncertainties of sigma terms

Octet baryon sigma terms from N³LO BChPT

	MIS		MDS
	a fixed	a free	
$\sigma_{\pi N}$	55(1)(4)	54(1)	51(2)
$\sigma_{\pi \Lambda}$	32(1)(2)	32(1)	30(2)
$\sigma_{\pi \Sigma}$	34(1)(3)	33(1)	37(2)
$\sigma_{\pi \Xi}$	16(1)(2)	18(2)	15(3)
$\sigma_{s N}$	27(27)(4)	23(19)	26(21)
$\sigma_{s \Lambda}$	185(24)(17)	192(15)	168(14)
$\sigma_{s \Sigma}$	210(26)(42)	216(16)	252(15)
$\sigma_{s \Xi}$	333(25)(13)	346(15)	340(13)

- Three scale setting methods yield **similar baryon sigma terms**.
- **The scale setting effects** on the sigma terms are very small.
 - MIS: mass independent scale setting
 - a -fixed: lattice spacing from LQCD paper
 - a -free: EOMS BChPT consistent to fix lattice spacing
 - MDS: mass dependent scale setting

σ_{sN} : comparison with earlier studies



- Our result **consistent** with the latest LQCD and NNLO BChPT results
- N^3 LO σ_{sN} result has **a larger uncertainty** compared to the NNLO
- Call for more LQCD data of octet baryon masses, especially with **the different strange quark masses**

Summary

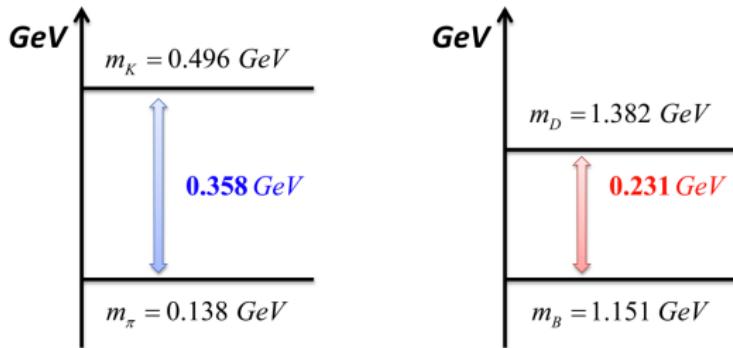
- We have systematically studied the LQCD octet baryon masses with **the EOMS BChPT up to N^3LO**
- **Finite-volume and discretization effects** on the lattice data are taken into account self-consistently
- Through simultaneously fitting "all" the current LQCD data:
 - ☞ Covariant BChPT shows **a clear improvement order by order**
 - ☞ LQCD results are **consistent with each other**, though their setups are quite different
 - ☞ Up to $\mathcal{O}(a^2)$, the discretization effects on the LQCD baryon masses are shown to be **small** and can be safely **ignored**
- **An accurate determination** of the octet baryon sigma terms via the Feynman-Hellmann theorem.

Thank you!

Appendix

Decuplet resonances in BChPT

□ Baryon Spectrum in SU(3)-BChPT



□ Perturbative parameters

$$\frac{m_K}{\Lambda_{\text{ChPT}}} > \frac{\delta}{\Lambda_{\text{ChPT}}}$$

Effects of virtual decuplet baryons should be carefully studied

Baryon masses in octet + decuplet EOMS BChPT

☞ Virtual decuplet baryons are explicitly included in BChPT

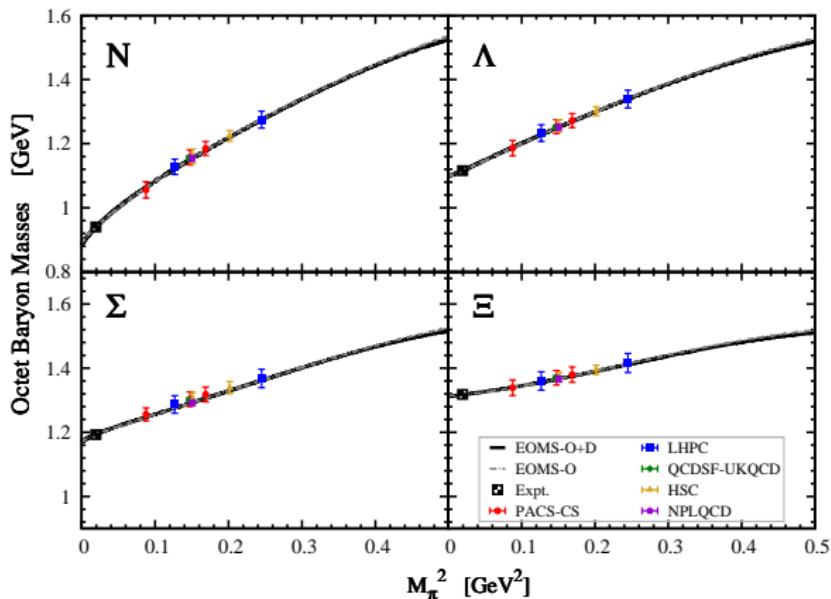
- Octet baryon masses: $m_0 + m_B^{(2)} + m_B^{(3)} + m_B^{(4)} + m_B^{(D)}$

☞ Fit the same lattice data as previous

☞ There is no new unknown LECs

- Octet-decuplet mass splitting: $\delta = 0.231$ GeV
- Meson-octet-decuplet coupling constant: $C = 0.85$ *J. M. Alarcon et al., 1209.2870*
- Fixed from the experimental decuplet masses *J. Martin-Camalich et al., PRD(2010)*
 - $m_D = m_0 + \delta = m_0 + 0.231$ GeV
 - $t_0 = (m_0 + 0.231 - 1.215)/0.507$ GeV $^{-1}$
 - $t_D = -0.326$ GeV $^{-1}$

Virtual decuplet effects on the chiral extrapolation

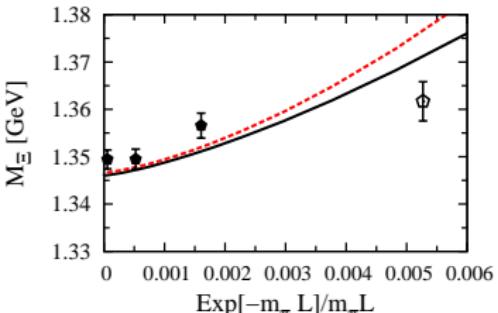
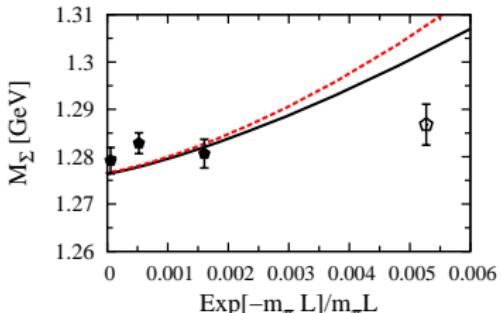
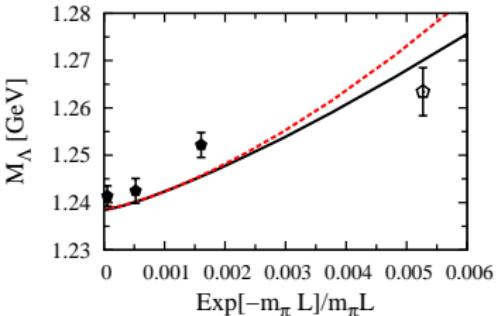
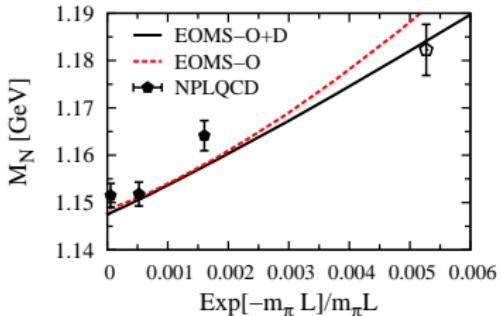


Fit the 11 LQCD data sets with and without decuplet

- Decuplet effects on the chiral extrapolation are **small**
- Previous assumption is confirmed:** virtual decuplet contributions can be absorbed by 19 LECs of octet only version

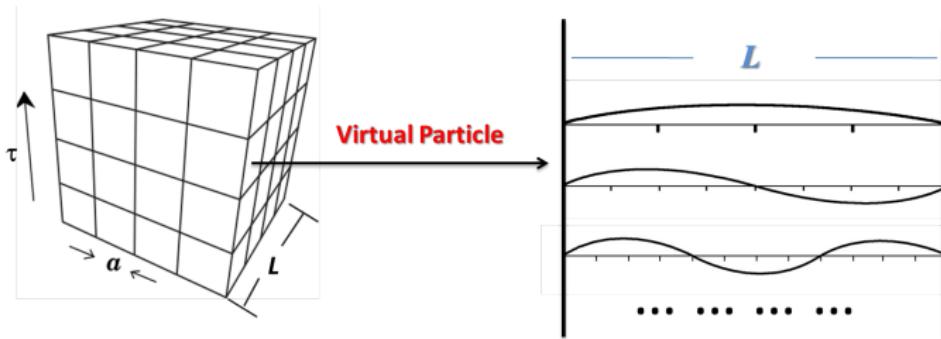
Virtual decuplet effects on finite-volume corrections

- Use the previous best fit results to describe the NPLQCD lattice data
- Virtual decuplet contributions can give a better description of the FVCs at small volume region



Lattice Finite-Volume Corrections

□ Physical picture of FVCs



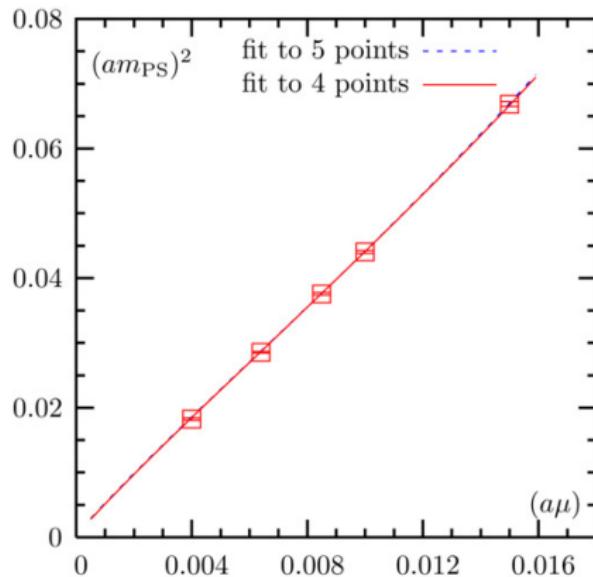
- Momentum of virtual particle discretized

$$k_i = \frac{2\pi}{L} \cdot n_i, \quad (i = 0, 1, 2, 3) \quad \Rightarrow \quad \int_{-\infty}^{\infty} dk \sim \sum_{n=-\infty}^{\infty} \frac{2\pi}{L} \cdot n$$

- Definition of FVCs:

$$\Delta H_{\text{FVC}}^{(b)} = \int \frac{dk_0}{2\pi} \cdot \left(\frac{1}{L^3} \sum_{\vec{k}} \square - \int \frac{d\vec{k}}{(2\pi)^3} \square \right) \quad \text{with } L_{\text{time}} \sim 5L_{\text{space}}.$$

$m_q \propto M_\pi^2$ in LQCD calculation



$$m_\pi^2 \propto m_q$$

ETM collaboration, hep-lat/0701012

HB ChPT cannot describe PACS-CS data

TABLE VI. Fit results with the SU(3) HBChPT for the octet baryon masses. NLO results are obtained with (case 2) and without (case 1) fixing D , F , and \mathcal{C} at the phenomenological estimate.

	LO	NLO	Case 1	Case 2	Phenomenological
m_B	0.410(14)		0.391(39)	-0.15(9)	
α_M	-2.262(62)		-2.62(62)	-15.3(2.0)	
β_M	-1.740(58)		-2.6(1.5)	-21.3(3.0)	
σ_M	-0.53(12)		-0.71(34)	-9.6(1.4)	
D		0.000(16) $\times 10^{-8}$		0.80 fixed	0.80
F		0.000(9) $\times 10^{-8}$		0.47 fixed	0.47
\mathcal{C}		0.36(30)		1.5 fixed	1.5
χ^2/dof	1.10(63)		1.39(77)	153(82)	

Relatively large Strangeness Sigma terms?

□ Lattice-scale setting

- PACS-CS data with mass **independent** scale-setting:
assume that the lattice scale, at constant bare coupling, is independent of the bare quark mass.

$$\sigma_{sN} = 59 \pm 7 \text{ (MeV)}$$

- PACS-CS data with mass **dependent** scale-setting:
The scale for the PACS-CS lattice data was set assuming that the dimensionful Sommer scale r_0 is independent of quark mass.

$$\sigma_{sN} = 21 \pm 6 \text{ (MeV)}$$

P.E. Shanahan, A.W. Thomas and R.D. Young, PRD 87, 074503 (2013)

□ Whether other LQCD data will show the same trend?

$\mathcal{O}(a)$ -improved Wilson action

$$\begin{aligned} S_{\text{quark}} = & \sum_{q=u,d,s} \left[\sum_n \bar{q}_n q_n - \kappa_q c_{\text{SW}} \sum_n \sum_{\mu,\nu} \frac{i}{2} \bar{q}_n \sigma_{\mu\nu} F_{\mu\nu}(n) q_n \right. \\ & - \kappa_q \sum_n \sum_{\mu} \{ \bar{q}_n (1 - \gamma_\mu) U_{n,\mu} q_{n+\hat{\mu}} \right. \\ & \left. \left. + \bar{q}_n (1 + \gamma_\mu) U_{n-\hat{\mu},\mu}^\dagger q_{n-\hat{\mu}} \} \right] \right], \end{aligned} \quad (2)$$

Power-Counting

In ChPT, graphs are analyzed in terms of powers of small external momenta over the large scale: $(Q/\Lambda_\chi)^\nu$, where Q is generic for a momentum (nucleon three-momentum or pion four-momentum) or a pion mass and $\Lambda_\chi \sim 1$ GeV is the chiral symmetry breaking scale.

Determining the power ν has become known as power-counting.
Following the Feynman rules of covariant perturbation theory, a nucleon propagator Q^{-1} , a pion propagator Q^{-2} , each derivative in any interaction is Q , and each four-momentum integration Q^4 . **This is known as naive dimensional analysis.**

Chiral order:

$$\nu = 4L - 2N_M - N_B + \sum_k kv_k.$$

Power-Counting

□ Systematic Power-Counting important for **perturbation theory**

- Due to the calculation with a perturbation theory, we cannot calculate up to infinity order. Therefore, we should perform at a specific order, that's always called **chiral order** in chiral perturbation theory.
- At this order, the corresponding **chiral effective lagrangians** can be written out with chiral symmetry and other symmetries.
- Undering these lagrangians, there exist **infinity feynman diagrams**, obviously, we cannot calculate these infinity feynman diagrams.
- **Then we should have a principle to determine which diagrams are important at the specific order.** Then **this principle is called power-counting**, which can deal with the previous problem to pick out the important feynman diagrams.
- Using this systematic power-counting, we can **test the convergence of chiral expansion** with increase the chiral order.

Power-counting breaking

- The nucleon mass does not vanish in the chiral limit.
- The consequence of all this is that **the one-to-one correspondence between the expansion in terms of small external momenta and pion masses (Chiral Expansion) and the expansion in pion loops (Loop Expansion) is destroyed.**

Power-counting in mesonic and baryonic sector

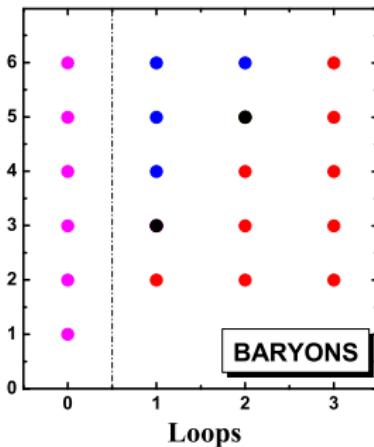
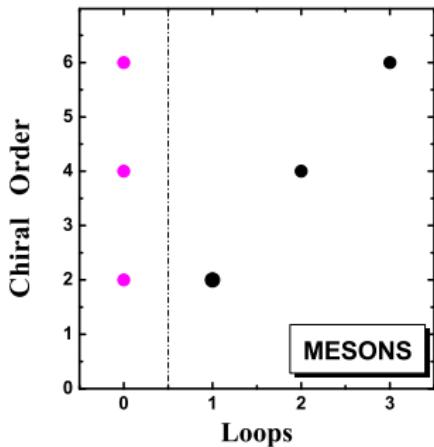
□ Mesonic sector

- ChPT has gained great achievements
- Calculation up to $\mathcal{O}(p^6)$ is standard

□ Baryonic sector – Baryon ChPT

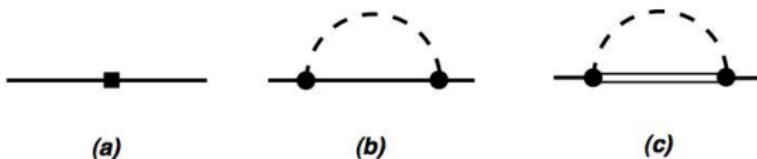
- A systematic power-counting lost
- Because $m_B \neq 0$ in the chiral limit

Chiral Order $4L - 2N_M - N_B + \sum_k kv_k.$



Power-Counting Breaking Problem

- Take the nucleon mass up to $\mathcal{O}(p^3)$ for example



$$m_N = m_0 + bM_\pi^2 + \text{loops}.$$

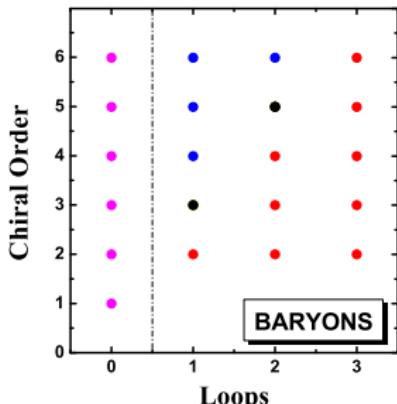
(b or c) Chiral order: $1+1+4-1-2=3$.

If the systematic power counting exists:

$$\text{loop} = cM_\pi^3 + \dots$$

However the truth:

$$\text{loop} = \alpha m_0^3 + \beta m_0 M_\pi^2 + cM_\pi^3 + \dots$$



Solving the PCB problem

Non-Relativistic

Heavy-Baryon ChPT

Jenkins, PLB(1991)

Baryon masses as static sources

Expansion in powers of $1/m_B$

✓ strictly power-counting

✗ breaks analyticity

✗ converges slowly

Relativistic

Infrared BChPT

Becher, EPJC(1999)

$$H = I + R$$

$$\int_0^1 \dots = \int_0^\infty \dots - \int_1^\infty \dots$$

✗ breaks analyticity

✗ converges slowly

EOMS BChPT

Gegelia, PRD(1999), Fuchs, PRD(2003)

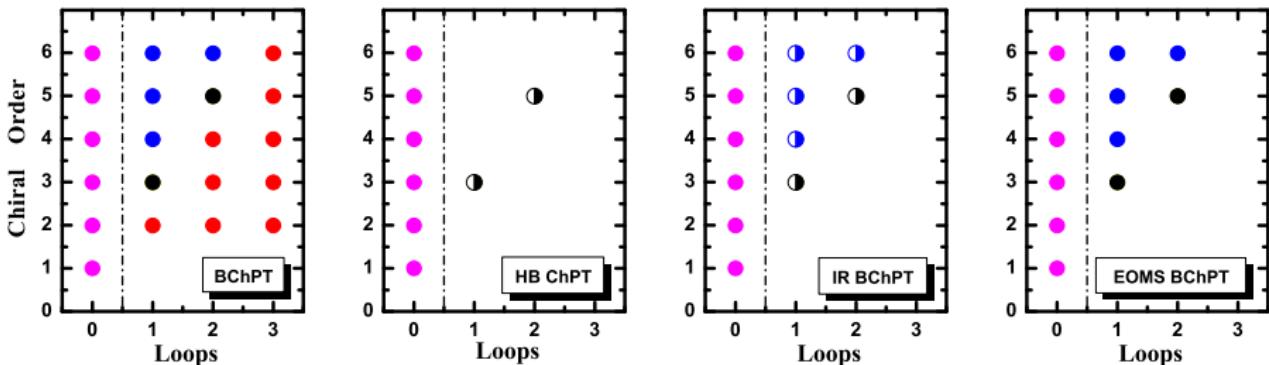
the PCB terms subtracted

Redefinition the LECs

✓ fulfills all symmetry

✓ satisfies analyticity

✓ converges relatively fast



Extended-On-Mass-Shell (EOMS)

$$\begin{aligned}m_N &= m_0 + \text{tree} + \text{loop} \\&= m_0 + b m_\pi^2 + \alpha m_0^3 + \beta m_0 M_\pi^2 + c M_\pi^3 + \dots.\end{aligned}$$

- Directly throw away the power counting breaking terms

$$\alpha = 0, \beta = 0$$

$$m_N = m_0 + b M_\pi^2 + c M_\pi^3 + \dots \quad (\mathcal{O}(p^3)).$$

- Equivalently, redefinition the corresponding LECs

$$m_0^r = m_0(1 + \alpha m_0^2), b^r = b + \beta m_0$$

$$m_N = m_0^r + b^r M_\pi^2 + c M_\pi^3 + \dots \quad (\mathcal{O}(p^3)).$$

lattice spacing power-counting

In LQCD simulations, the following hierarchy of energy scales should be satisfied

$$m_q \ll \Lambda_{\text{QCD}} \ll \frac{1}{a}. \quad (1)$$

If one assumes that the size of the chiral symmetry breaking due to the light-quark masses and the discretization effects are of comparable size, as done in Refs. [Beane:2003xv, Bar:2003mh, Tiburzi:2005vy], one has the following expansion parameters

$$p^2 \sim \frac{m_q}{\Lambda_{\text{QCD}}} \sim a \Lambda_{\text{QCD}}, \quad (2)$$

where p denotes a generic small quantity and $\Lambda_{\text{QCD}} \approx 300$ MeV denotes the typical low energy scale of QCD.