Towards Constraining Composite Higgs Models via the Bootstrap

Marco Serone, SISSA, Trieste

in collaboration with Francesco Caracciolo, Alejandro Castedo, Benedict von Harling, 1406.7845

Lyon, November 2014

Plan

Brief Review of Composite Higgs Models (CHM)

Review of CFT and basics of the Bootstrap

How the Bootstrap Can Constrain CHM

Preliminary results

Conclusions

Brief Review of CHM

There are various strongly coupled related scenarios one can consider (technicolor, little Higgs, composite Higgs)

Technicolor, already in trouble with LEP electroweak bounds, is essentially ruled out by a 125 GeV Higgs (techni-dilaton too heavy and different couplings)

Little Higgs are also Composite Higgs Models (CHM)

Little Higgs: thanks to an ingenious symmetry breaking mechanism, the Higgs mass is radiatively generated, while the quartic is not

Composite Higgs: the entire Higgs potential is radiatively generated

In principle little-Higgs models are better, because allow for a separation of scales between the Higgs VEV and the compositeness scale

In practice they are not, because the above ingenious mechanism becomes very cumbersome when fermions are included

Brief Review of CHM

There are various strongly coupled related scenarios one can consider (technicolor, little Higgs, composite Higgs)

Technicolor, already in trouble with LEP electroweak bounds, is essentially ruled out by a 125 GeV Higgs (techni-dilaton too heavy and different couplings)

Little Higgs are also Composite Higgs Models (CHM)

Little Higgs: thanks to an ingenious symmetry breaking mechanism, the Higgs mass is radiatively generated, while the quartic is not

Composite Higgs: the entire Higgs potential is radiatively generated

In principle little-Higgs models are better, because allow for a separation of scales between the Higgs VEV and the compositeness scale

In practice they are not, because the above ingenious mechanism becomes very cumbersome when fermions are included

These are the models I will consider

In absence of a better name I will generically call them CHM

The Higgs field might or might not be a pseudo Nambu-Goldstone boson (pNGB) of a spontaneously broken global symmetry.
Models where the **Higgs is a pNGB are** the most promising

The spontaneously broken global symmetry has also to be explicitly broken (by SM gauge and Yukawa couplings), otherwise the Higgs remains massless

Whole Higgs potential is radiatively generated

The SM gauge group arises as a weak gauging of $H_f \supset G_{SM}$

The SM gauge fields are the analogue of the photon.
The Higgs field is the analogue of the pions

Important difference: fermion fields must now be added (no QCD analogue) to account for the SM fermion sector

One can also impose generalized Weinberg sum rules in CHM to make the Higgs potential calculable [Marzocca, MS, Shu; Pomarol, Riva]

Minimal choice of coset, leading to an Higgs doublet with no additional pNGB's, and custodial symmetry is

$$G_f = SO(5) \times U(1)_X \times SU(3)_c$$
$$H_f = SO(4) \times U(1)_X \times SU(3)_c$$

Implementations in concrete models hard (calculability, flavour problems)

Breakthrough: the composite Higgs paradigm is **holographically** related to theories in extra dimensions!

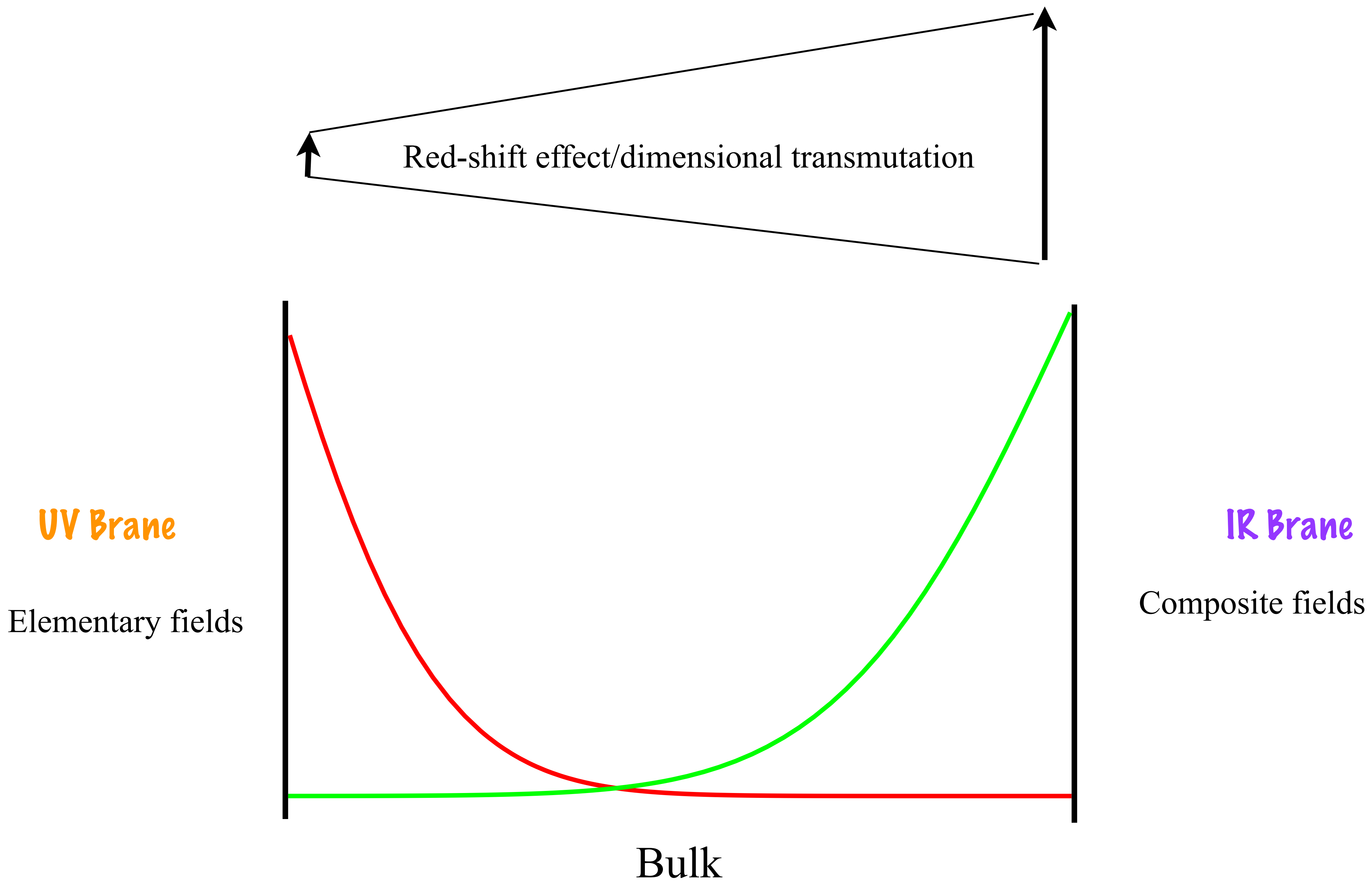
Extra-dimensional models have allowed a tremendous progress

The Higgs becomes the fifth component of a gauge field, leading to

Gauge-Higgs-Unification (GHU) models also known as Holographic Composite Higgs models

Not only relatively weakly coupled description of CHM, Higgs potential fully calculable, but the key points of how to go in model building have been established in higher dimensions

Connection particularly clear in Randall-Sundrum warped models thanks to the celebrated AdS/CFT duality


Red-shift effect/dimensional transmutation
UV Brane
Elementary fields
IR Brane
Composite fields
Bulk

Main lesson learned from holographic 5D models

[Agashe, Contino, Pomarol, ...]

$$\mathcal{L}_{tot} = \mathcal{L}_{el} + \mathcal{L}_{comp} + \mathcal{L}_{mix}$$

Elementary sector: SM particles but Higgs (and possibly top quark)

Composite sector: unspecified strongly coupled theory with unbroken global symmetry $G \supset G_{SM}$

Mixing sector: **mass mixing** between SM fermion and gauge fields and spin 1 or 1/2 bound states of the composite sector

Crucial ingredient in such constructions is the notion of

Partial Compositeness

SM fields get mass by mixing with composite fields: the more they mix the heavier they are (4D counterpart of 5D wave function overlap)

Light generations are automatically screened by new physics effects

Despite purely 4D constructions are possible, Holographic Composite Higgs models still remain among the most interesting ones

Main reason is the dynamical explanation of the smallness of the mixing parameters in the partial compositeness scenario

Theoretically speaking, the weakest point of CHM is the lack of a satisfactory UV completion, despite recent progress in this direction

[Caracciolo, Parolini, MS, 1211,7290; Ferretti, Karateev, 1312.5330; Barnard, Gherghetta, Ray, 1401.8291; Cacciapaglia, Sannino, 1402.0233; Ferretti, 1404.7137]

Holographic Composite Higgs models also require a UV completion, of course, being non-renormalizable 5D theories

Actually, we expect this to involve gravity and hence most likely such a UV completion has to be found in a full-fledged string theory: hard task!

As a matter of fact, we do not even know if such UV completions exist at all!

From a dual 4D point of view, the composite sector in these models should correspond to some unknown CFT with a global symmetry

The mixing sector corresponds to marginal or relevant deformations of the CFT

Maybe we can address the problem of a possible UV completion within 4D CFT's

Let us see some properties that a 4D CFT should have to be a valid hidden sector for a composite Higgs theory

1) A global symmetry $G \supseteq G_{\rm SM} = {\rm SU}(3)_c \times {\rm SU}(2)_L \times {\rm U}(1)_Y$

2) No scalar operator with dimension $\Delta < 4$ which is neutral under G

3) No Landau poles below $\Lambda_{\rm UV}$ for the SM gauge couplings

4) One fermion operator with proper quantum numbers and $\Delta \simeq 5/2$ and others with $\Delta > 5/2$

Of course, there are only necessary, but not sufficient, conditions to get a viable CHM

We focus on Landau poles for SM gauge couplings because

- They have been shown to be the major obstacle for a UV completions of CHM with SUSY [Caracciolo, Parolini, MS, 1211.7290]
- We can estimate their value in the CFT by looking at current-current correlators

Review of CFT and basics of the Bootstrap

Old idea of late 70's recently revived by Rattazzi, Rychkov, Tonni and Vichi, 0807.0004

Basic idea is to make some assumption about the structure of some CFT and check if it is consistent with fundamental principles such as unitarity and crossing symmetry. If it is not, that CFT is ruled out

Starting point is typically a four-point correlation function. For 4 identical scalars we have

$$\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle = \frac{g(u,v)}{x_{12}^{2d}x_{34}^{2d}}$$

$$x_{ij}^2 = (x_i - x_j)_\mu (x_i - x_j)^\mu \qquad u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}\,, \quad v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$$

Using the OPE to pairs of operators, the function g(u,v) can be expressed as a sum over so called conformal blocks

$$g(u,v) = 1 + \sum_{\Delta,l} |\lambda_{\phi\phi\mathcal{O}}|^2 g_{\Delta,l}(u,v)$$

Sum over all possible symmetric traceless operators that can appear in the OPE of two scalars.

$\lambda_{\phi\phi\mathcal{O}}$ is the coefficient of the $\langle\phi\phi\mathcal{O}\rangle$ three-point function

Demanding that the OPE in two different pairings (s and t channels) give the same result, we get a crossing symmetry constraint

$$\sum_{\Delta,l} |\lambda_{\mathcal{O}}|^2 F_{d,\Delta,l}(z,\bar{z}) = 1$$

$$u = z\bar{z}\,, \quad v = (1-z)(1-\bar{z})$$

$$F_{d,\Delta,l}(z,\bar{z}) \equiv \frac{v^d g_{\Delta,l}(u,v) - u^d g_{\Delta,l}(v,u)}{u^d - v^d}$$

Explicit form of $g_{\Delta,l}(u,v)$ known [Dolan,Osborn, hep-th/0011040, hep-th/0309180]

In the original paper the bootstrap equation has been used to put an upper bound on the lowest scalar operator dimension appearing in the OPE

Motivation was conformal technicolor, which has indeed severely constrained by bootstrap analysis

Other possible application: set bounds on OPE coefficients $\lambda_{\phi\phi\mathcal{O}}$

[Caracciolo,Rychkov, 0905.2211]

Suppose we want to set bounds on a specific coefficient $\lambda_{\phi\phi\mathcal{O}_0}$
Look for a linear functional α such that

$$\alpha(F_{d,\Delta_0,l_0}) = 1\,, \qquad \alpha(F_{d,\Delta,l}) \geq 0 \quad \forall(\Delta,l) \neq (\Delta_0,l_0)$$

Applying to bootstrap equation gives

$$|\lambda_{\mathcal{O}_0}|^2 = \alpha(1) - \sum_{(\Delta,l)\neq(\Delta_0,l_0)} |\lambda_{\mathcal{O}}|^2 \alpha(F_{d,\Delta,l}) \leq \alpha(1)$$

In practice

$$\alpha(f(z,\bar z)) = \sum_{m+n\leq 2k} a_{mn} \partial_z^m \partial_{\bar z}^n f(z,\bar z)|_{z=\bar z=1/2}\,,$$

Since then, various generalizations and results have been obtained

Relevant for this talk is the generalization to CFT's with global symmetry, where the scalar field transforms in some representation of the group. In this case the bootstrap equation turns into a system of P+Q equations

$$\sum_i \eta^p_{F,i} \sum_{\mathcal{O}\in r_i} |\lambda_{\mathcal{O}_i}|^2 F_{d,\Delta,l}(z,\bar{z}) = \omega^p_F\,, \quad p = 1,\ldots,P\,,$$

$$\sum_i \eta^q_{H,i} \sum_{\mathcal{O}\in r_i} |\lambda_{\mathcal{O}_i}|^2 H_{d,\Delta,l}(z,\bar{z}) = \omega^q_H\,, \quad q = P+1,\ldots,P+Q\,.$$

i runs over irreducible representations that can appear in the s- and t-channel

$\eta^p_{F,i}$ and $\eta^q_{H,i}$ numerical factors that depend on G

$\omega^p_F = 1$, $\omega^q_H = -1$ if singlet representation appears, otherwise $\omega^p_F = \omega^q_H = 0$.

A new function appears

$$H_{d,\Delta,l}(z,\bar{z}) \equiv \frac{v^d g_{\Delta,l}(u,v) + u^d g_{\Delta,l}(v,u)}{u^d + v^d}$$

Bound on $\lambda_{\phi\phi\mathcal{O}}$ generalizes to

$$|\lambda_{\mathcal{O}_0}|^2 \leq \sum_{p=1}^{P} \alpha_p(\omega_p^F) + \sum_{q=P+1}^{P+Q} \alpha_q(\omega_q^H)$$

We will be interested in the OPE coefficient associated with a conserved vector current J_μ of a global symmetry, $\Delta_0 = 3$ and $l_0 = 1$. At leading order, this coefficient λ_J governs the CFT contribution to the β-function of the corresponding gauge coupling

$$\mathcal{L}_{\text{gauged}} = \mathcal{L}_{\text{CFT}} + g J^\mu_A A^A_\mu - \frac{1}{4} F^A_{\mu\nu} F^{\mu\nu}_A$$

$$e^{-\Gamma(A)} = \int \mathcal{D}\Phi_{\text{CFT}}\; e^{-\int d^4x\, \mathcal{L}_{\text{gauged}}}$$

$$\Gamma(A) \supset -\frac{1}{4} \int d^4x\, Z F^A_{\mu\nu} F^{\mu\nu}_A$$

$$Z = (1 + \delta Z_{\rm CFT}) \qquad \beta_{\rm CFT} = g\mu \frac{d}{d\mu}\sqrt{Z} = \frac{1}{2} g\mu \frac{d}{d\mu} \delta Z_{\rm CFT}$$

Take two derivatives with respect to $A_\mu^A(p)$

$$\delta_{AB}\delta Z_{\rm CFT}(\delta_{\mu\nu}p^2 - p_\mu p_\nu) = -g^2 \langle J_\mu^A(-p) J_\nu^B(p)\rangle_{g=0}$$

λ_J is actually fixed by Ward identites,

but two -point function has unknown coefficient

$$\langle J_\mu^A(x) J_\nu^B(0)\rangle_{g=0} = \frac{3\kappa\delta^{AB}}{4\pi^4}\left(\delta_{\mu\nu} - 2\frac{x_\mu x_\nu}{x^2}\right)\frac{1}{x^6}$$

κ measures how many charged degrees of freedom are present in the CFT,

analogue of central charge c for energy momentum tensor $\quad \lambda_J^2 \propto \frac{1}{\kappa}$

$$\langle J_\mu^A(-p) J_\nu^B(p)\rangle_{g=0} = (\delta_{\mu\nu}p^2 - p_\mu p_\nu)\frac{\kappa}{16\pi^2}\delta^{AB}\log\left(\frac{p^2}{\mu^2}\right)$$

$$\Rightarrow \qquad \beta_{\rm CFT} = g^3 \frac{\kappa}{16\pi^2}$$

Estimate how severe Landau pole problem can be in CHM with SO(5)→SO(4)

Consider $\mathrm{SU}(3)_c$ coupling α_c

$$\Lambda_L \simeq \mu \exp\left(\frac{2\pi}{(\kappa-7)\alpha_c(\mu)}\right)$$

for $\kappa > 7$, $\mu \sim \mathcal{O}(\mathrm{TeV})$ is scale where CFT breaks spontaneously.

Estimate: CHM with fermions in fundamental of $SO(5)$. Free theory value is

$$\kappa_{\text{free}} = \frac{2}{3} \times 30 = 20$$

corresponding to $\Lambda_L \sim 200\ \ \mathrm{TeV}$

Important to set lower bounds on κ in a generic CFT.

Let us recall properties that a 4D CFT should have to be a valid hidden sector for composite Higgs theory

1) A global symmetry $G \supseteq G_{\rm SM} = {\rm SU}(3)_c \times {\rm SU}(2)_L \times {\rm U}(1)_Y$

2) No scalar operator with dimension $\Delta < 4$ which is neutral under G

3) No Landau poles below $\Lambda_{\rm UV}$ for the SM gauge couplings

4) One fermion operator with proper quantum numbers and $\Delta \simeq 5/2$ and others with $\Delta > 5/2$

Ideal framework: analyze 4-point functions of fermions with $\Delta \simeq 5/2$ with above assumptions and see if solution exists. In particular look for the upper bound on λ_J arising in the OPE of the four fermions

Unfortunately this is technically challenging

Drastic approximation: replace fermions with scalars

Preliminary results

We have studied lower bounds on κ coming from CFT's with global symmetry

SO(N), SU(N), SO(N)$\times$ SO(M), SO(N)$\times$ SU(M)

and a 4-point function of scalars in (bi)-fundamental of the global group

assuming (or not) lowest scalar operator has $\Delta \geq 4$

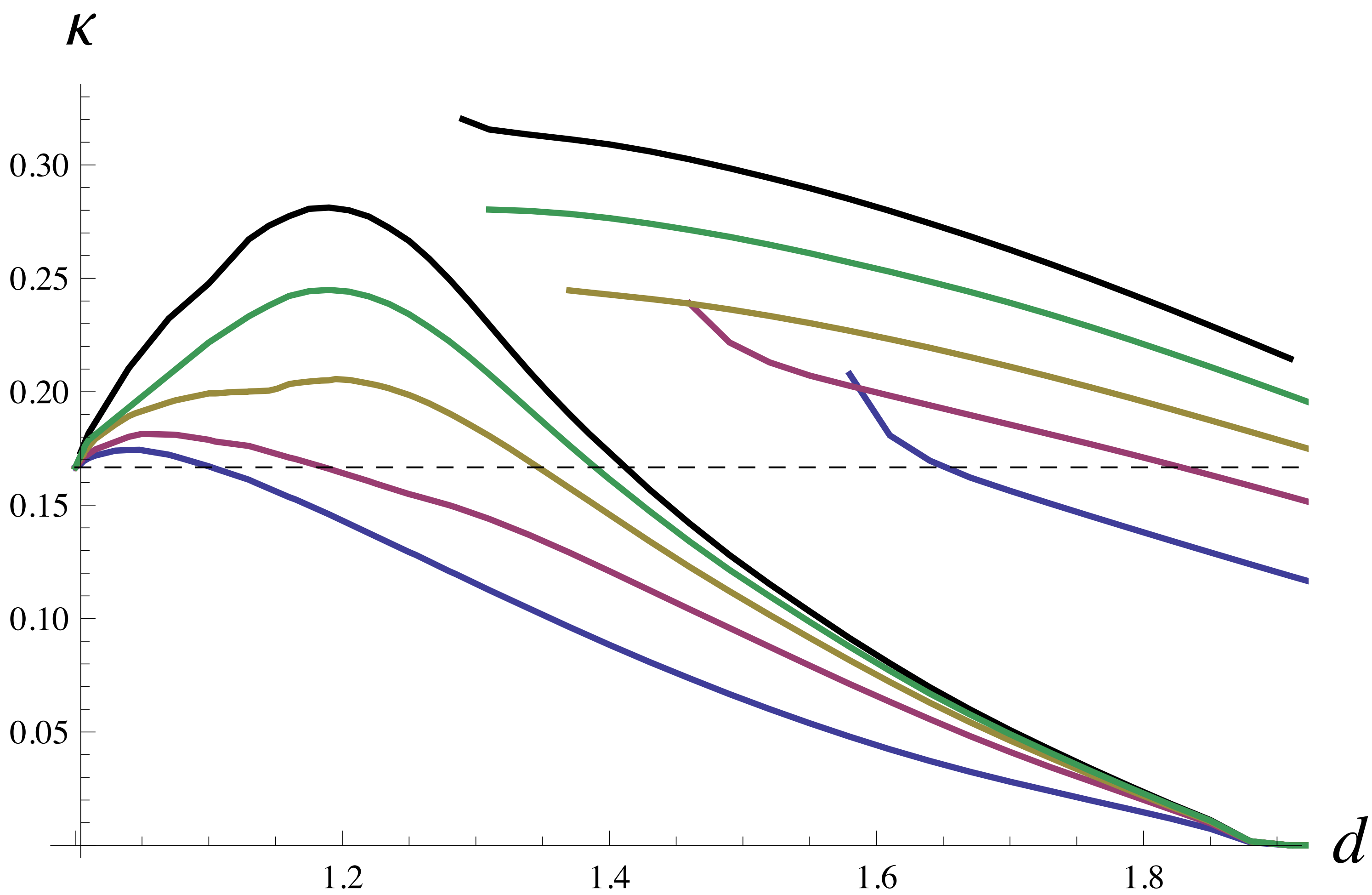
Actually, SU(N) and SO(2N) give identical bounds

Similarly SO(N)$\times$ SU(M) and SO(N)$\times$ SO(2M)

Enough to report results for SO(N) and SO(N)$\times$ SO(M)

Technically speaking, we use the semi-definite numerical algorithm as developed by

[Poland, Simmons-Duffin 1009.2087; Poland, Simmons-Duffin, Vichi, 1109.5176]



Lower bounds κ between two SO(N) or SU($N/2$) currents as obtained from a four-point function of scalars in the fundamental with dimension d at $k = 10$. From below, the lines which start at $d = 1$ correspond to $N = 2$ (blue), $N = 6$ (red), $N = 10$ (brown), $N = 14$ (green), $N = 18$ (black), with no assumption on the spectrum. In the same order and using the same color code, the bound assuming no scalar operator in the singlet channel with $\Delta_S < 4$. The free-theory value $\kappa_{\text{free}} = 1/6$ is a black dashed line.

Lower bounds on κ with no assumption on the spectrum agree with

[Poland, Simmons-Duffin, Vichi, 1109.5176] where similar bounds were obtained

Assumption on the scalar spectrum makes bounds quite stronger

In this way we can put bounds using one one field in fundamental

A free CFT with N scalars in fundamental of SO(M) or SU(M) will have

$$\kappa_{\rm free} = \frac{N}{6}$$

The larger N, the more constraining the lower bounds.

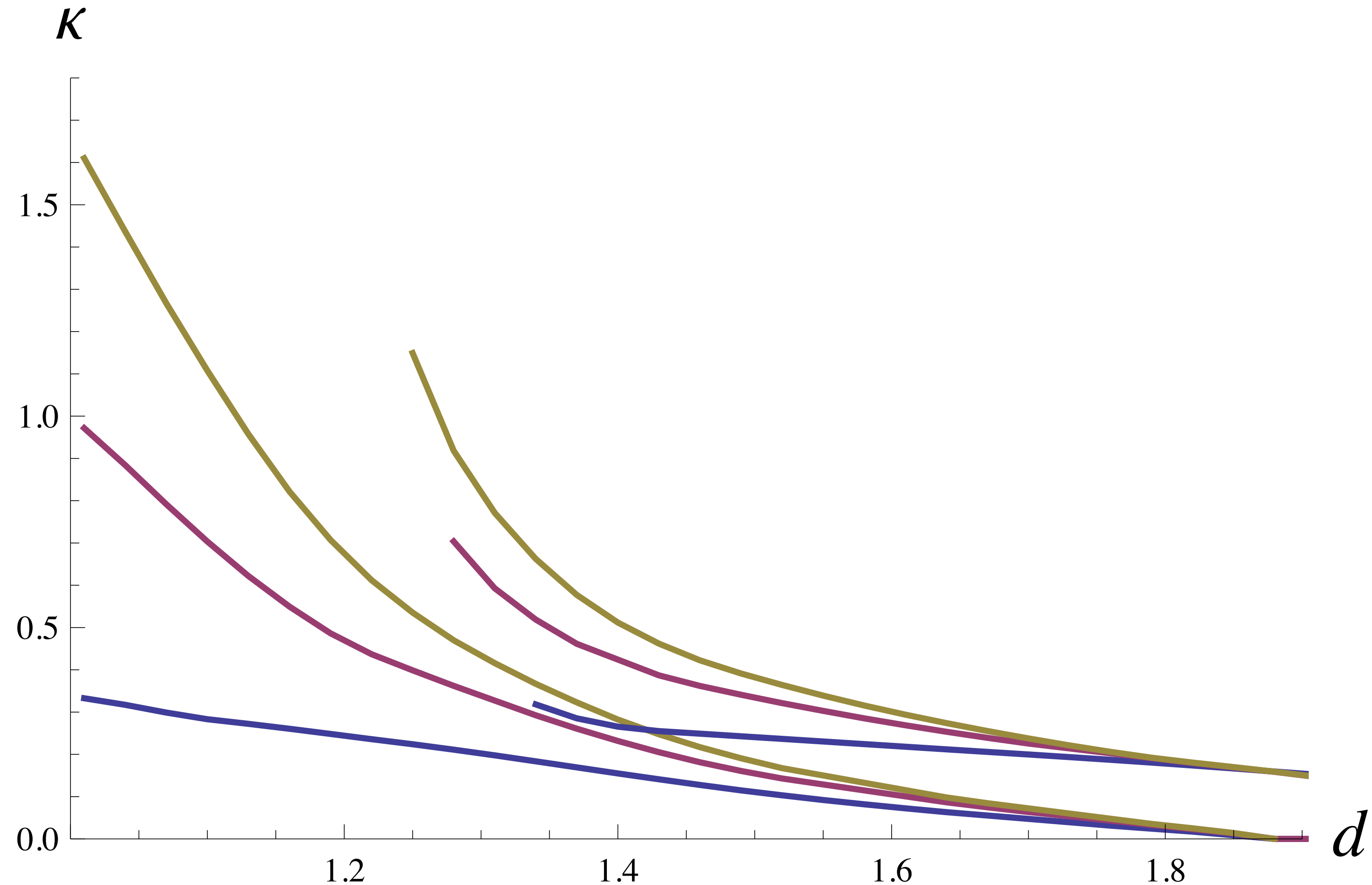
Bootstrap with non identical scalars more involved.
Mimic multiplicity by taking fields in bi-fundamental of two groups.

Obvious price to be paid: scalar dimensions of fields all equal

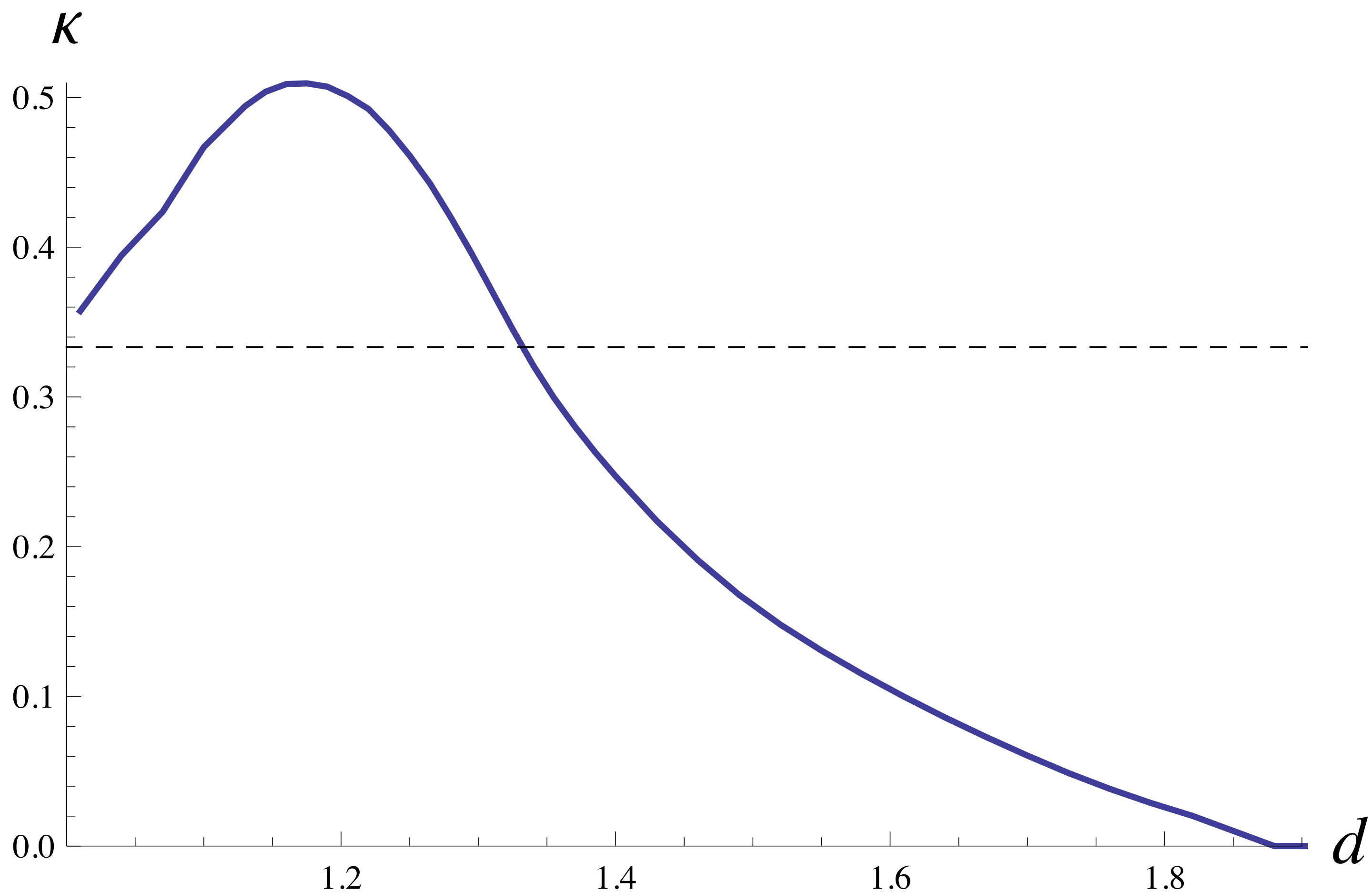
This is main motivation to consider global symmetries $SO(N) \times SO(M)$:

a way to obtain lower bounds on $\kappa_{SO(N)}$

in presence of more fields charged under $SO(N)$.



Lower bounds on κ between two conserved SO(N) or SU($N/2$) currents as obtained from a four-point function of scalar operators with dimension d in the bi-fundamental of SO(N)$\times$SO(M), at $k = 9$. We take $N = 6$. From below, the lines which start at $d = 1$ correspond to $M = 2$ (blue), $M = 6$ (red), $M = 10$ (brown), with no assumption on the spectrum. In the same order and using the same color code, the bound assuming no scalar operator in the singlet channel has dimension $\Delta_S < 4$.



Lower bounds on κ between two conserved SO(30) or SU(15) currents as obtained from a four-point function of scalar operators with dimension d in the bi-fundamental of SO(30)$\times$SO(2), at $k = 9$. No assumption on the spectrum is made. The free-theory value $\kappa_{\rm free} = 1/3$ is a black dashed line.

Notice change of behaviour of the bounds on $\kappa_{SO(N)}$ depending on $SO(M)$:

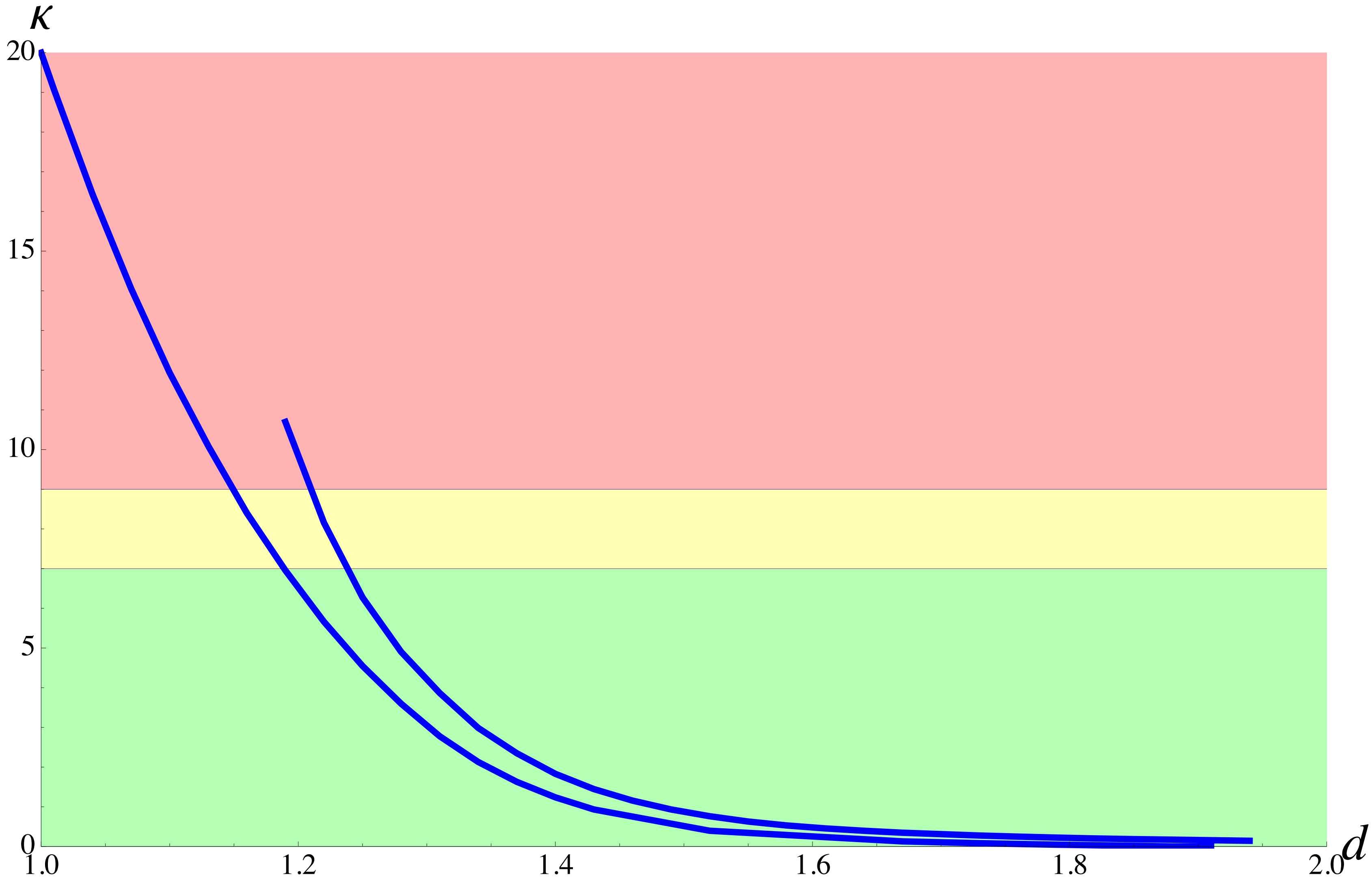
For $N \gg M$, clear maximum and then decrease (like in single SO(N) case)

For $N \leq M$, bound goes down as d increases

It would be interesting to understand why this different behaviours

Let us see lower bounds on κ for the group SO(6)$\times$SO(120)

SO(120) because 120 free complex scalar triplets to the $\mathrm{SU}(3)_c \subset \mathrm{SO}(6)$ current-current two-point function gives $\kappa = 20$. This is the same κ of fermion triplets needed to give mass to SM quarks in the SO(5)$\rightarrow$SO(4) CHM



Lower bounds on κ between two conserved $\mathrm{SO}(6) \supset \mathrm{SU}(3)_c$ currents as obtained from a four-point function of scalar operators with dimension d in the bi-fundamental of $\mathrm{SO}(6){\times}\mathrm{SO}(120)$, at $k = 9$. In the green region α_c remains asymptotically free, while in the orange and red regions α_c develops trans-Planckian and sub-Planckian Landau poles, respectively.

Conclusions

Composite Higgs Models with partial compositeness are a promising idea to solve the SM gauge hierarchy problem.

The most pressing problem from a theoretical point of view is to understand whether all the necessary ingredients to get a phenomenologically viable model can fit into a UV complete theory

For holographic models, this turns into the problem of looking for a 4D CFT with the desired properties.

The bootstrap approach can be useful in constraining the parameter space

More specifically, useful information is expected to arise by imposing crossing symmetry in 4-point functions involving fermions with scaling dimension 5/2 (top partners after conformal symmetry breaking)

Bootstrap equations for 4D fermion 4-point functions are not yet available

As drastic simplification, we have studied the bound on the current - current correlator coming from scalar 4-point functions to detect possible Landau poles for the SM gauge couplings.

Optimistic point of view: constraints are weak, CHM are ok

Pessimistic point of view: bootstrap approach not very constraining

Results too preliminary to have a phenomenological impact

Of course, the next natural step is to study the 4-fermion correlator.

This requires preliminary CFT results not yet available. Work is in progress towards this ultimate goal

Of course, the next natural step is to study the 4-fermion correlator.

This requires preliminary CFT results not yet available. Work is in progress towards this ultimate goal

Thank You