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EIPS 2014

Particle Accelerators Course

Introduction

ACCELERATORS = PHYSICS

Particle accelerators are extremely sophisticated devices. This implies the application of different concepts and experimental techniques. Accelerator Physics is beteween the most pluridisciplinary physics fields ?

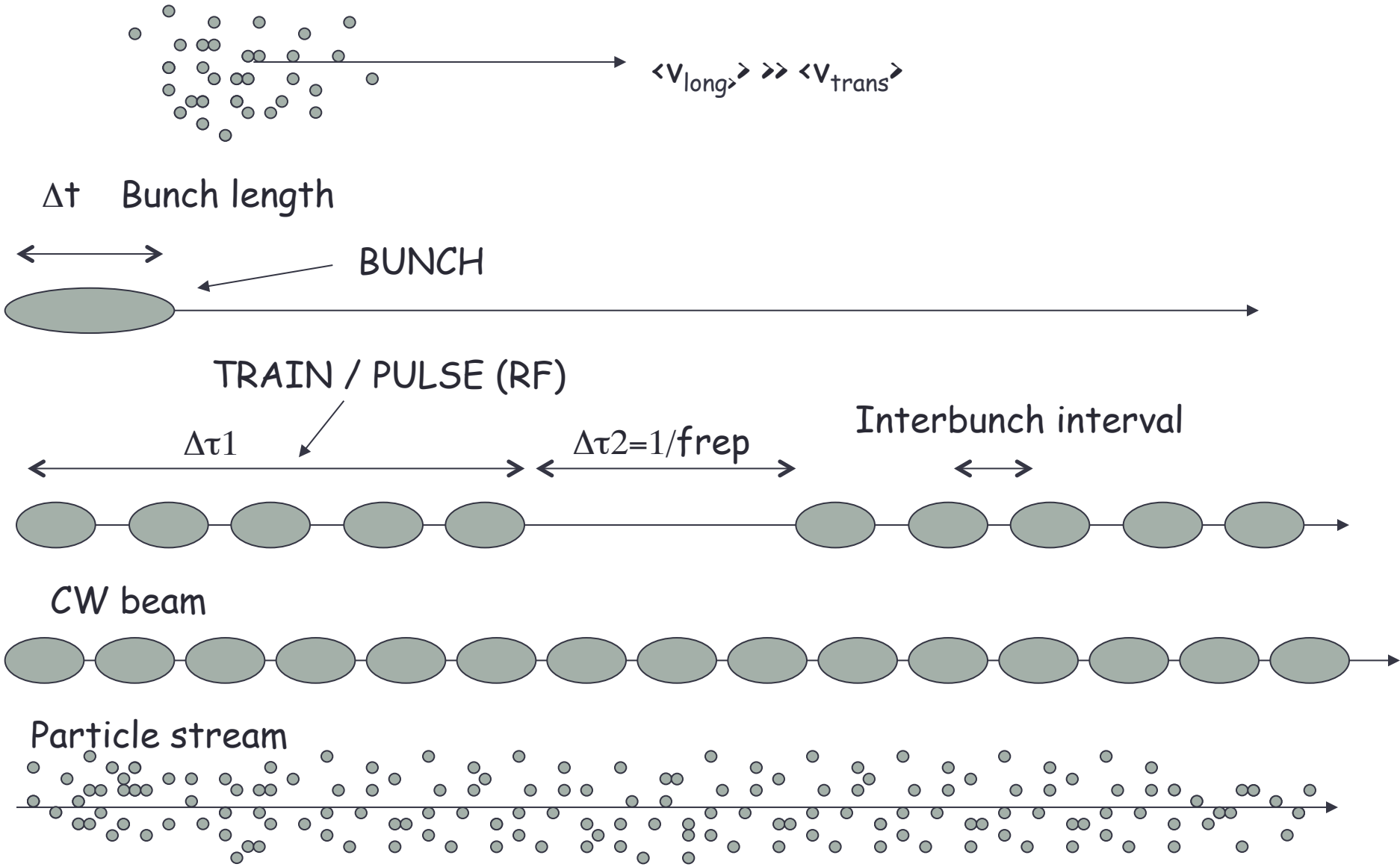
- Classical physics
- Classical mechanics
- Electromagnetism
- Relativity
- Quantum mechanics
- High energy physics
- Statistics
- Non-neutral plasma
- Instrumentation……

Unfortunately we will have a short overview and we will treat only few simple arguments….

What is a particle Accelerator?

- Seems trivial: whatever device that accelerate particles and particle beams. Nevertheless, in this definition, an amazing variety of different type of devices, technologies, ideas and applications are included.
- Remember: often the particle physicists ask to the accelerator physicists bemas with X amperes (with X-> infinite) colliding in Y meters (with Y->zero), without noise and in a reliable machine that never stops. ☺
- BUT 1st, WHAT IS A BEAM?

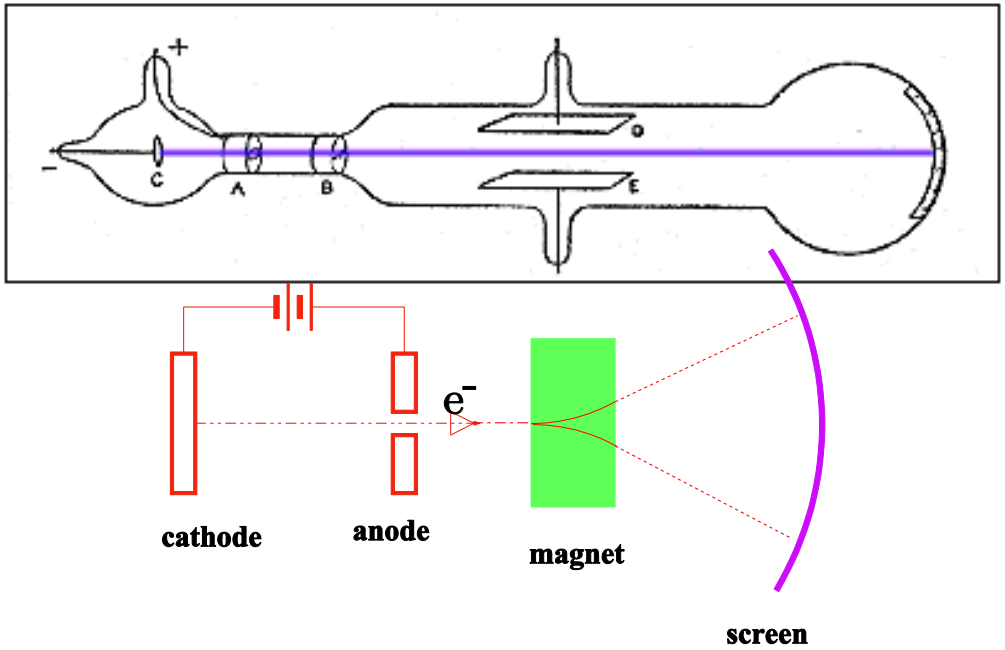
Definition : Particle beams -> $\langle v_{long} \rangle \gg \langle v_{trans} \rangle$



- Particle Accelerators : History

1st Accelerator: Cathode Ray Tube

- Electric voltage between two metallic plates
- Heat the cathode --- something emitted
- Proved the existence of electron in 1897 J.J. Thompson
- TV monitor (until some years ago)



Cock-Croft Electro-Static Accelerator

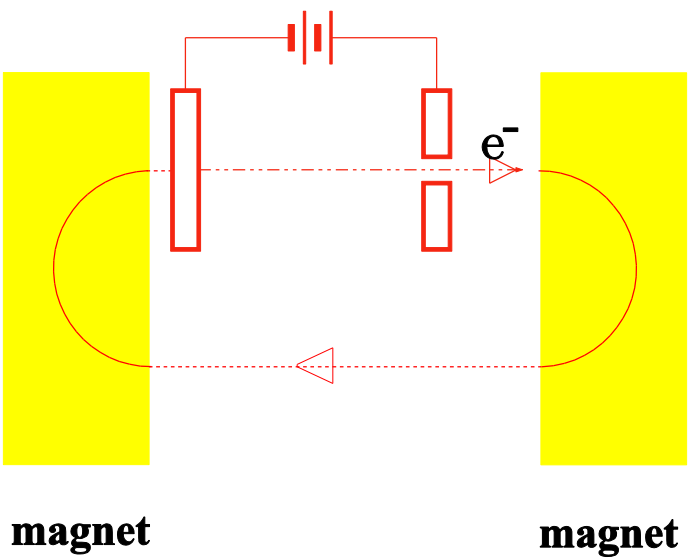
- High voltage by static electricity
- First nuclear transformation by accelerator
  $H + Li \rightarrow 2\ He$
- Cavendish institute in UK, 1932
- 800keV
- Breakdown limit

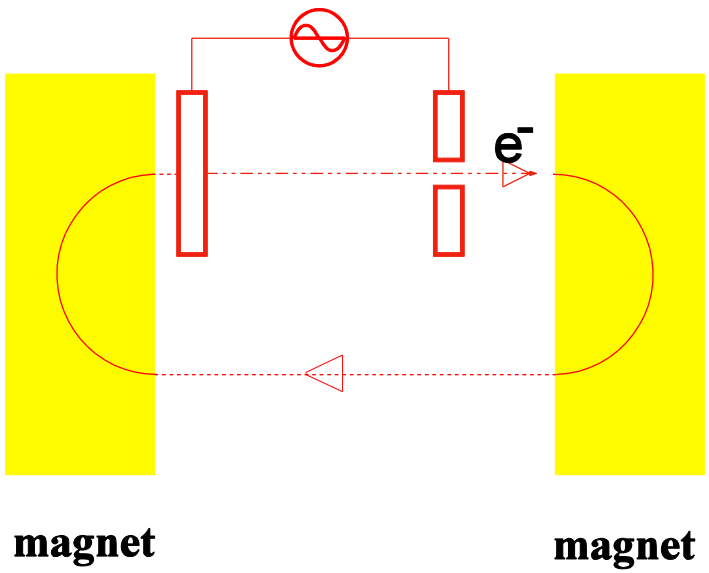
KEK 750keV Cockcroft-Walton

Repeated question: How can we go to higher energies?

- reuse possible?

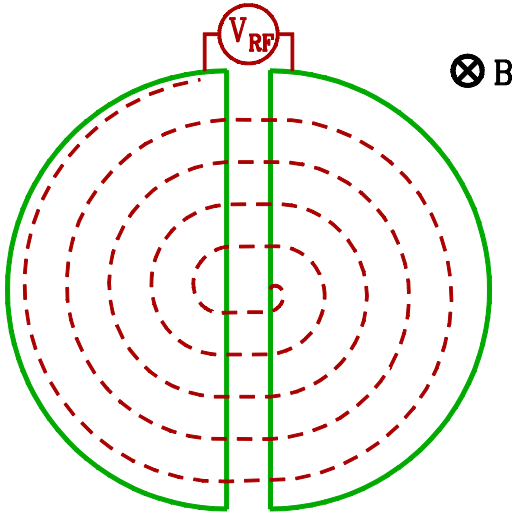
- use of alternating voltage
- high frequency needed

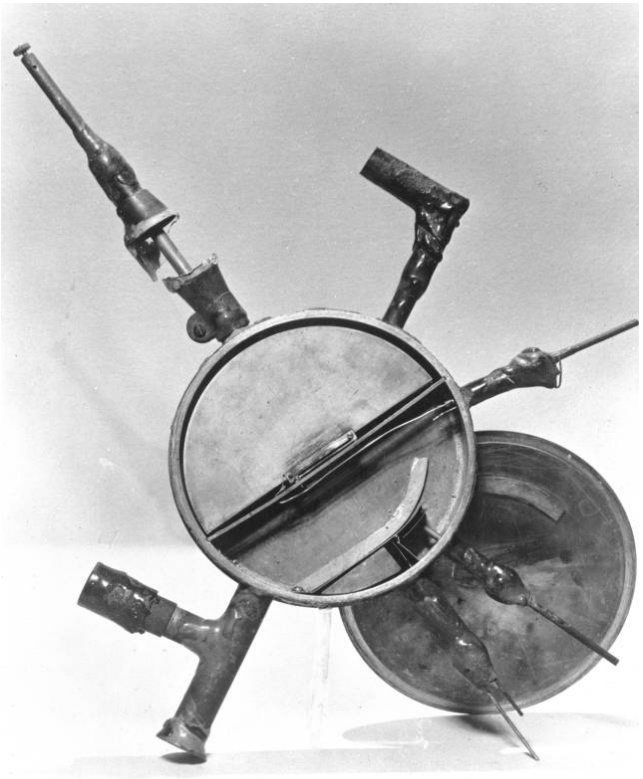


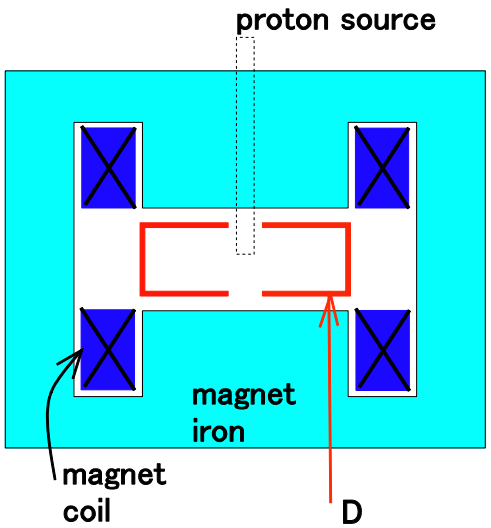


Cyclotron

- E.O.Lorence, 1931
  Berkeley, California
- Revolution period independent of energy



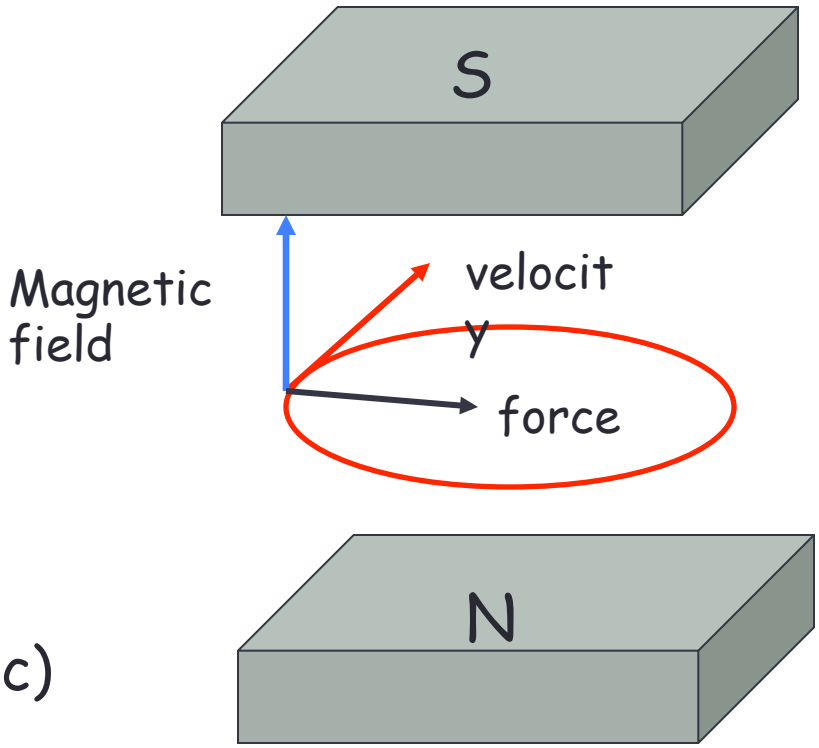




Relation : radius – magnetic field – beam energy – revolution time
Lorentz Force= Centrifugal Force

- Radius

$$\rho[\mathrm{m}] = \frac{p[\mathrm{GeV/c}]}{0.3B[\mathrm{T}]}$$



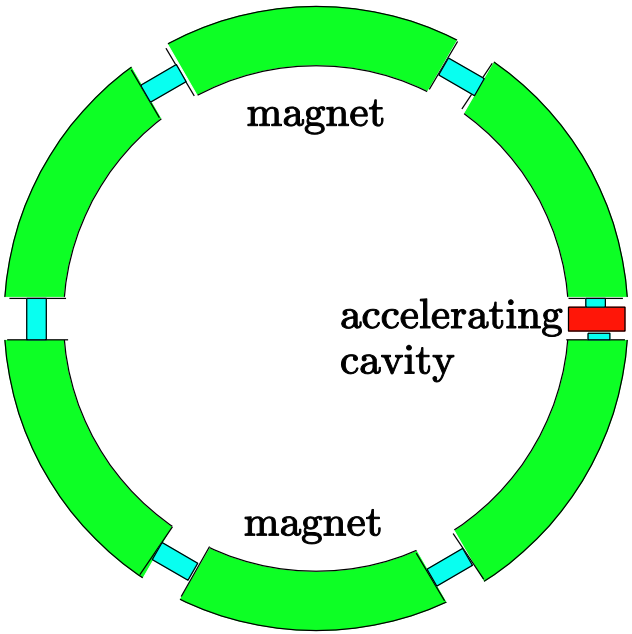
- Revolution period (non-relativistic)

$$T = \frac{2\pi\rho}{v} = 2\pi\frac{m}{eB} = \text{constant}$$

RINGS

Synchrotron

- Make orbit radius independent of energy
  - Raise magnetic field as acceleration
  - Save volume of magnets
  - Area of field is proportional to p (momentum), not p^2
- Gradient magnet needed for focusing. Dipoles poles modified

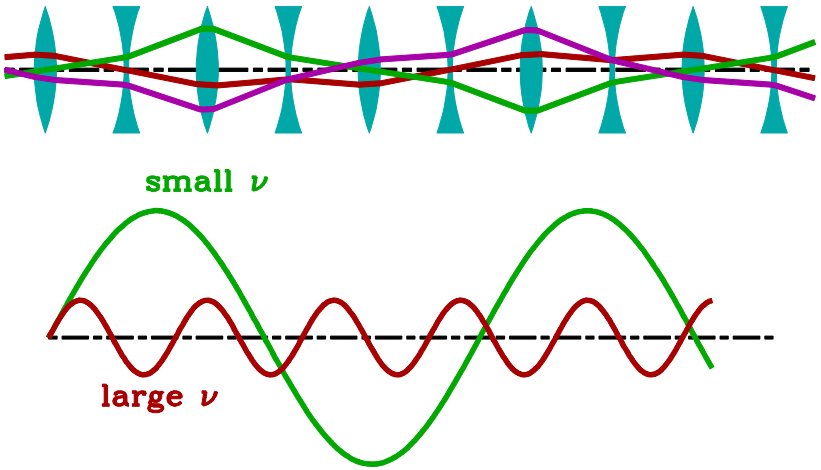


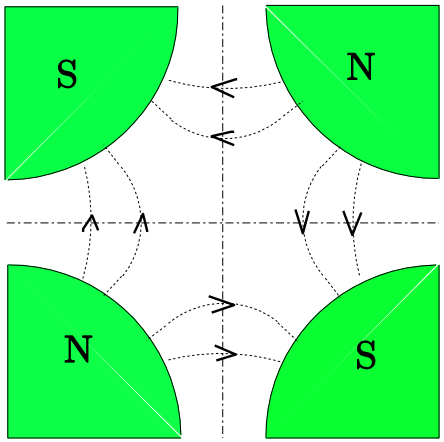
1950's

- A few GeV proton synchrotrons
  - Cosmotron (BNL) 3GeV
  - Bevatron (LBL) 6.2GeV
  - Magnet size became an issue even for synchotron of a few GeV scale
- Many new particles
  - anti-proton, anti-neutron
  - Λ, Σ, Ξ, Ω,....
  - Systematic description introducing "Quarks" by Gell-Mann in 1964

Strong Focusing

- Combination of F-type magnet and D-type can reduce the beam size
- Around 1957
- Quadrupole magnets can also be used
- New issue: accuracy of field and alignment

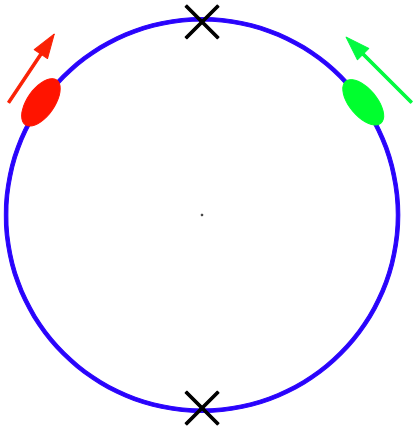




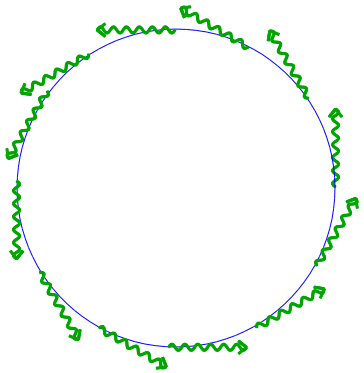
At present => Rings

- After acceleration, for different goals, a beam can be stored in a ring.
- In a ring the particles are maintained on a orbit by magnetic elements
- DIPOLES guide the beam on a circular orbit, QUADRUPOLES provide focalization and MULTIPOLES are used to correct aberrations
- In the lepton case the energy losses given by the synchrotron radiation are recovered by accelerating RF CAVITIES
- In all the rings (leptons and hadrons) anyway the cavities are required to maintain the beam longitudinal bunching
- Synchronization and stabilization of the orbit are provided by electronic feedbacks.
- An important aspects in rings are provided by INSTABILITIES. These are due to the interaction of a bunch (by means of its associated e.m field) with the environment and with the other bunches. Also in this case feedbacks are used to damp the instabilities.

Storage Ring



- Synchrotron can be used to store beams for seconds to days
- Usage
  - Collider
  - Synchrotron light source
- Principle same as synchrotron but
  - no need of rapid acceleration (even no acceleration)
  - longer beam life (e.g., better vacuum)
  - insertion structure (IP, insertions, etc)



The beam quality – cooling & damping rings

- For quite all the applications a good beam must be able to provide a lot of particles in a little space with a little divergence and as monochromatic as possible.
- A parameter, the EMITTANCE, describe the degree of quality of the beam
- Usually the emittance is a motion invariant (the normalized one… we will see what it means), but under special condition it can vary. Usually it increase but some mechanism provides also its reduction. The most important is the synchrotron radiation cooling. Under strong radiation emission the beam reduces its emittance . It is for this reason that in special case (see linear colliders) special rings, called damping rings, are studied to reduce the emittance of the beam.
- The damping rings are special rings where the beam is injected from a linac. Once stored it is forced to strongly emit synchrotron radiation by dipoles and insertions (WIGGLERS, UNDULATORS). In this way the beam emittance it is reduced and the beam can be extracted and send to the interaction point

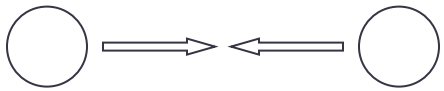
Light sources – SR rings

- A part form the high energy physics (damping rings) the synchrotron radiation is much more used in the light sources.
- Special rings are studied to emit high brillance photon fluxes by synchrotron radiation. The particular characteristics of spectrum and intensity are extremely attractive for a wide spectrum of physics research fields like the material science, medical science, biology, cristallography etc etc
- Also in this case the use of insertion devices is extremely attractive due to their characteristics in flux and spectrum
- To produce and preserve the light beam also the electron beam characteristics must be performing. Again a little emittance and a strong current is required.

Colliders

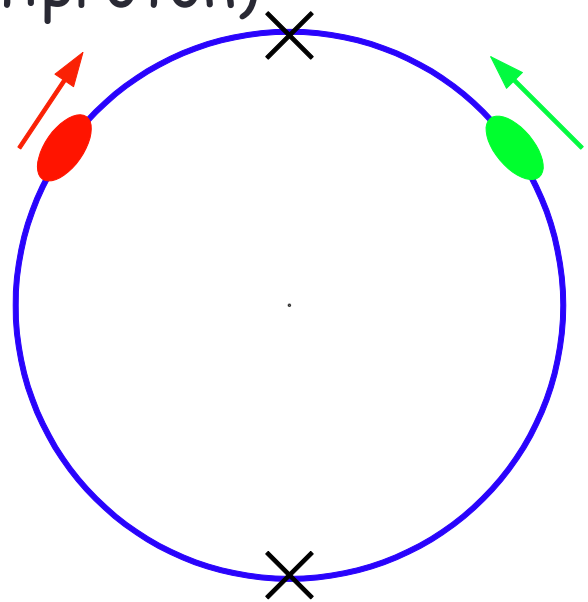
- For HEP. Two beams (particle – antiparticle or particle-particle … see LHC)
- Counter-propagating in two separate rings or in the same ring.
- Same energy or different energy (b factories)
- In a certain region they cross an Interaction Point => Physics
- Usually IP are in the detectors (that usually have solenoids…). Coupling of x/y coordinate ask for special magnets to compensate for the coupling. This is provided by rotated quadrupoles called : skew quadrupoles.
- Also in this case, to increase the luminosity, we need high current and small beam sizes, so little emittances.

Collider



$$E_{CM} \approx 2E$$

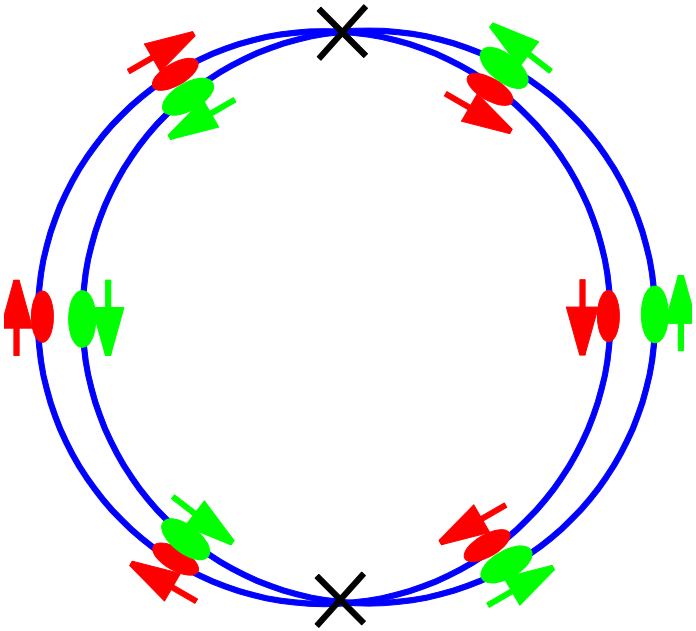
- Can be done in one ring for same energy beams and opposite charge (e.g., e+e-, proton-antiproton)



.... PETRA, TRISTAN, LEP,

..... Spps, Tevatron

- More freedom with two rings

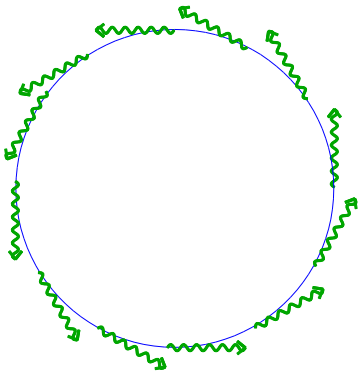


PEPII, KEKB, LHC, ...

Limitation : Synchrotron Radiation

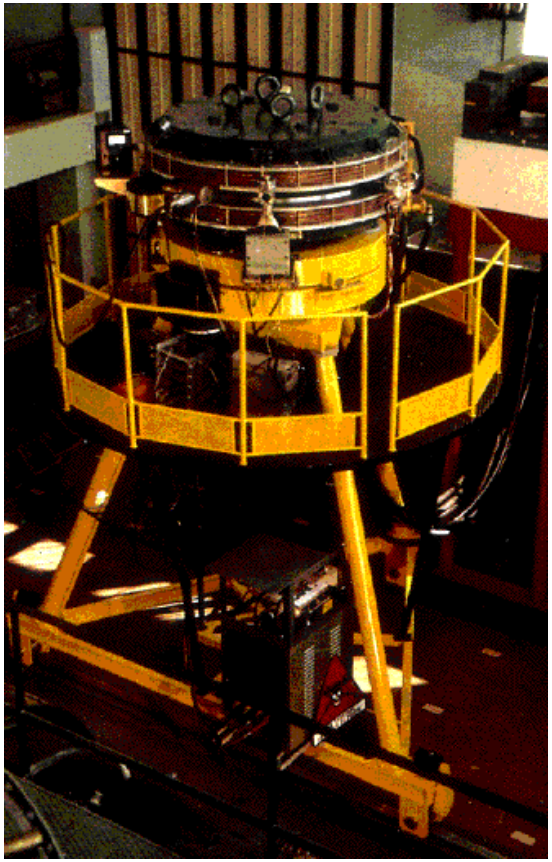
- Charged particles lose energy by synchrotron radiation
- proportional to $1/m^4$
- Loss per turn (electron)

$$U = 0.088 \frac{E^4[\mathrm{GeV}]}{\rho[\mathrm{m}]} \quad [\mathrm{MeV}]$$



The First Electron-Positron Collider: AdA

- First beam in 1961 in Italy
- Moved to Orsay, France
- The first beam collision in 1964
- Orbit radius 65cm, collision energy 0.5GeV



Energy of Collider Ring

- Proton/antiproton
  - Ring size
  - Magnetic field
  - Synchrotron radiation (future)
- Electron/positron
  - Ring size
  - Synchrotron radiation
    - Electric power consumption

Era of Huge Ring Colliders:

SPS: Super Proton Synchrotron

- Large proton synchrotron at CERN
- Operation start in 1976
- Reached 500GeV
- Remodeled into the first proton-antiproton collider

TEVATRON

- FNAL
- Proton-antiproton
- circumference 6.3km
- up to ~1TeV
- Completed in 1983
- Superconducting magnet 4.2Tesla
- 1995 Top Quark
- 2009 shutdown

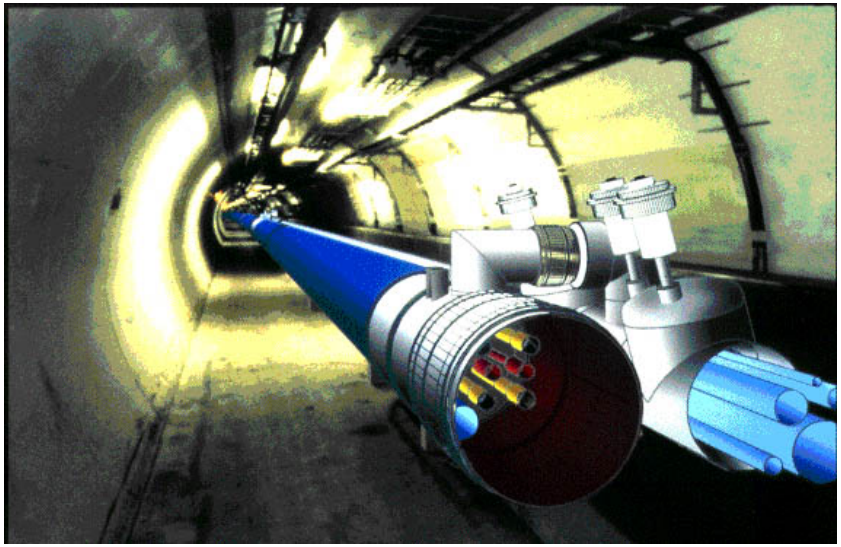
Main Injector in front and Tevatron hehind

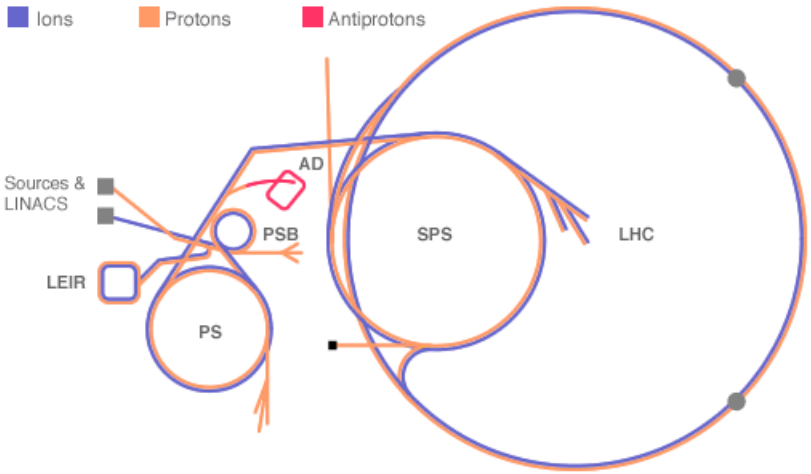
LEP

- LEP (Large Electron-Positron Collider)
  - CERN
  - Construction started in 1983, operation in 1989
- circumference 27km
  - First target Z^0 at 92GeV
  - Final beam energy 104.5GeV
  - end in 2000

LHC

- Latest step to higher energies
- Reuse of LEP tunnel
  - Circumference 27km
- 14TeV proton-proton
  - magnetic field 8.33 Tesla
  - Higgs



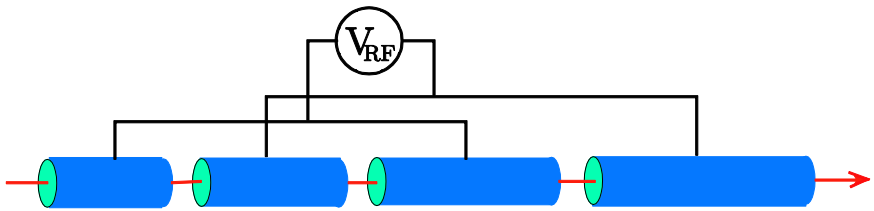


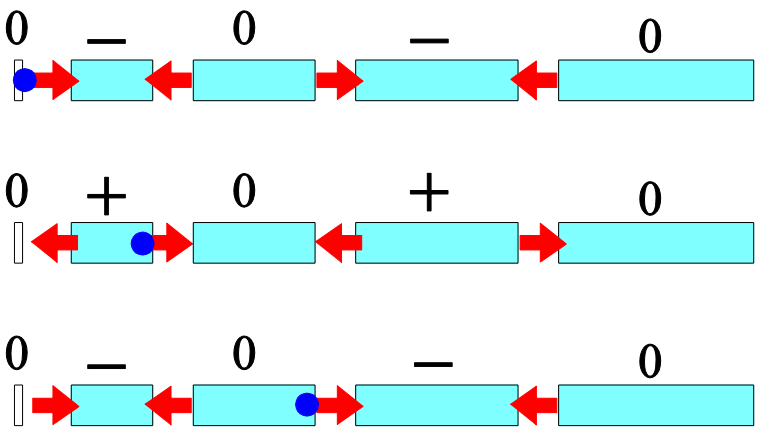
http://athome.web.cern.ch/athome/LHC/lhc.html

LINACS

Linear Accelerator (Linac)

- Drift tube type

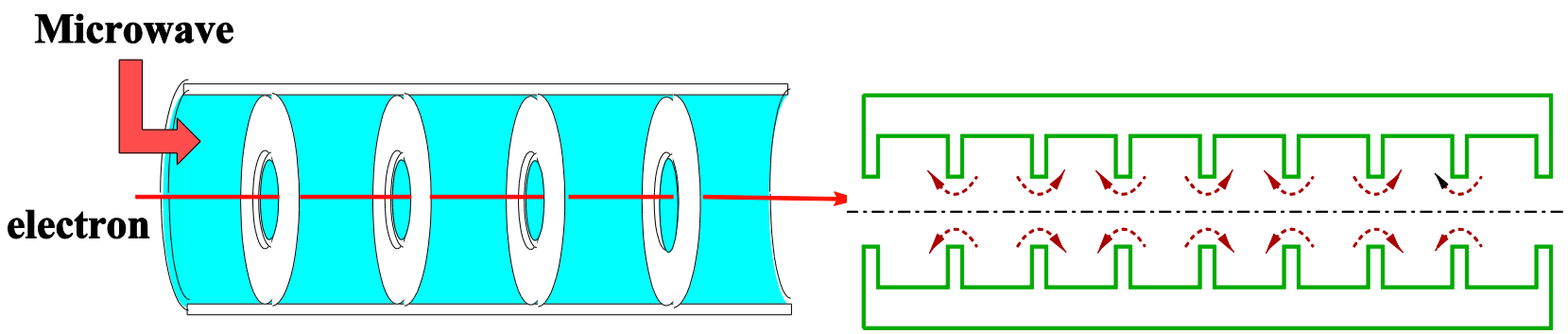




- The progress of microwave technology during World War II
- Application to accelerator after WW II

Electron Linac -> riding the wave

- Velocity is almost constant above MeV
- No need of changing tube length
- Resonator type
- $V_{ph}=V_{part}$ by insertion of iris (if not $v_{ph}>c$...)



Acceleration - LINACS

- Particle are accelerated by linear structures (LINACS) . Two main technologies

a) Warm (usually @ 3 GHz): high gradient are possible ~ 100 MeV/m (high power in short trains, high frequency - > 10 GHz). This only at very reduced duty cycle. It can work in CW with very low gradients (~ 1 MeV/m).

Special devices are able to compress the source energy to reduce the pulse length but increase the gradient. These are called the SLED cavities

b) Cold (SC usually @ 1.3 GHz to take into account wake fields): It can accelerate very important average currents with strong gradients (~ 15 MeV/m CW, ~30 MeV/m pulsed). Gradients are limited by SC state and by critical magnetic field

SLED →

SC example TTF

Linac	TESLA	TTF Linac design/achieved
Accel. grad. [MV/m] with beam	23.4	15 / 14, 19, 22
Unloaded quality factor [10^{10}]	1.0	0.3 / > 1.0
No. of cryomodules	2628	4 / 3 + 2
Energy spread, single bunch rms	5×10^{-4}	$\approx 10^{-3}/10^{-3}$
Energy variation, bunch to bunch	5×10^{-4}	$\approx 2\times10^{-3}/2\times10^{-3}$
Bunch length, rms [μm]	300	1000 / 400
Beam current [mA]	9.5	8 / 7
Beam macro pulse length [μs]	950	800 / 800

Table 1: TESLA-500 – TTF Linac parameters comparison.

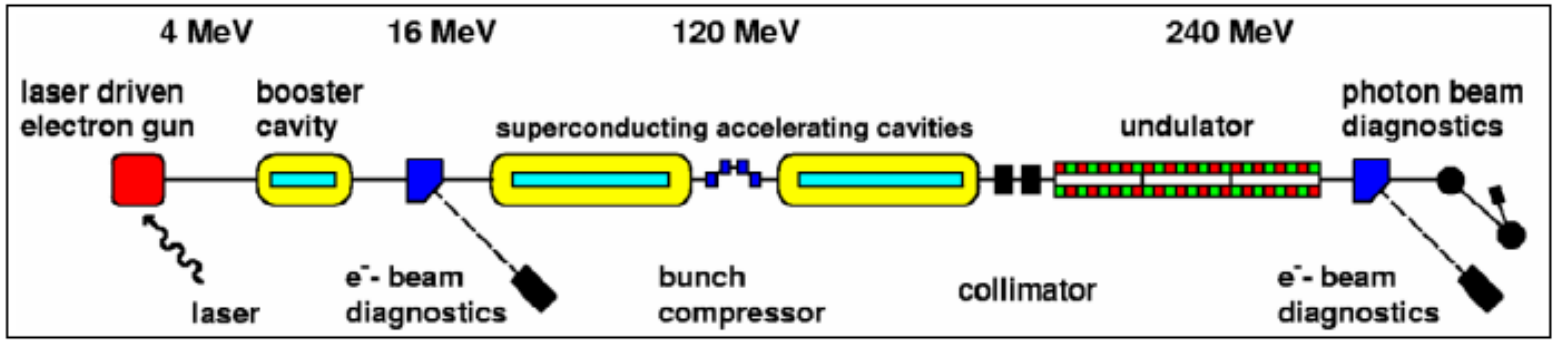
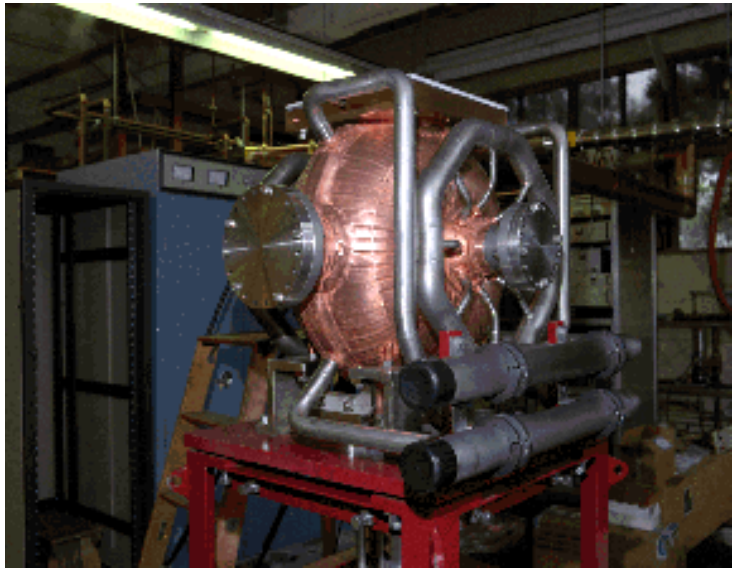
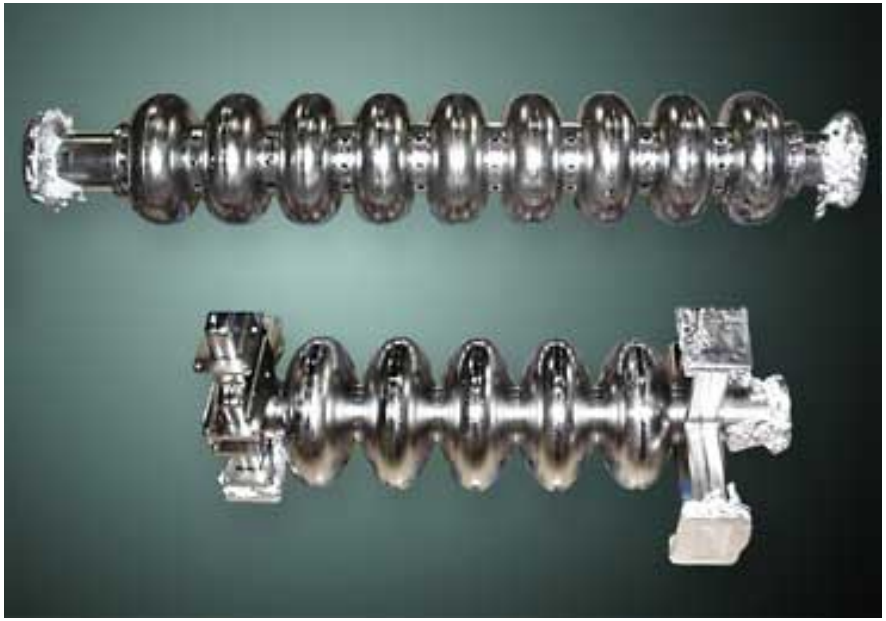


Figure 1: Schematic layout of the TESLA Test Facility Linac (TTFL). The total length is about 120 m.



Normal Conductive

Super Conductive

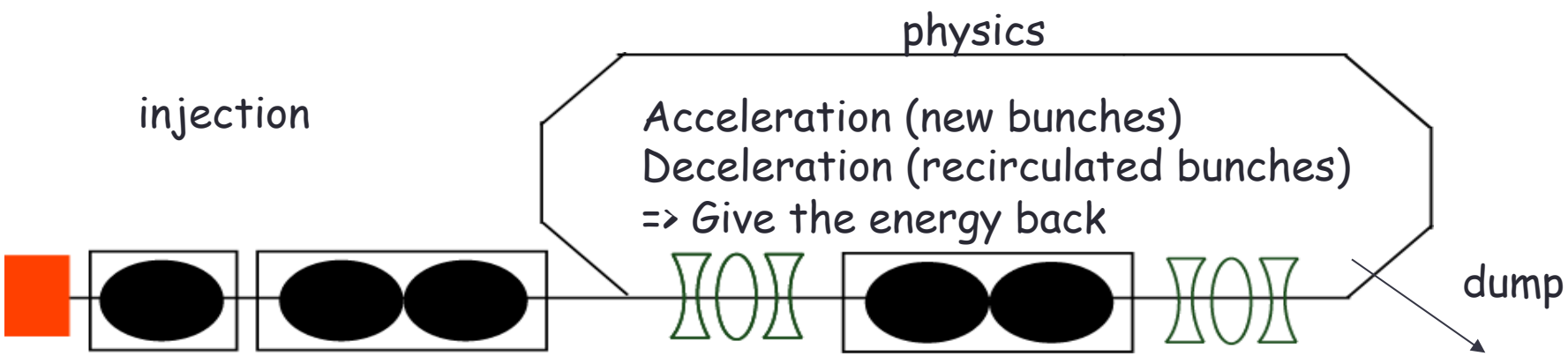


RF BANDS

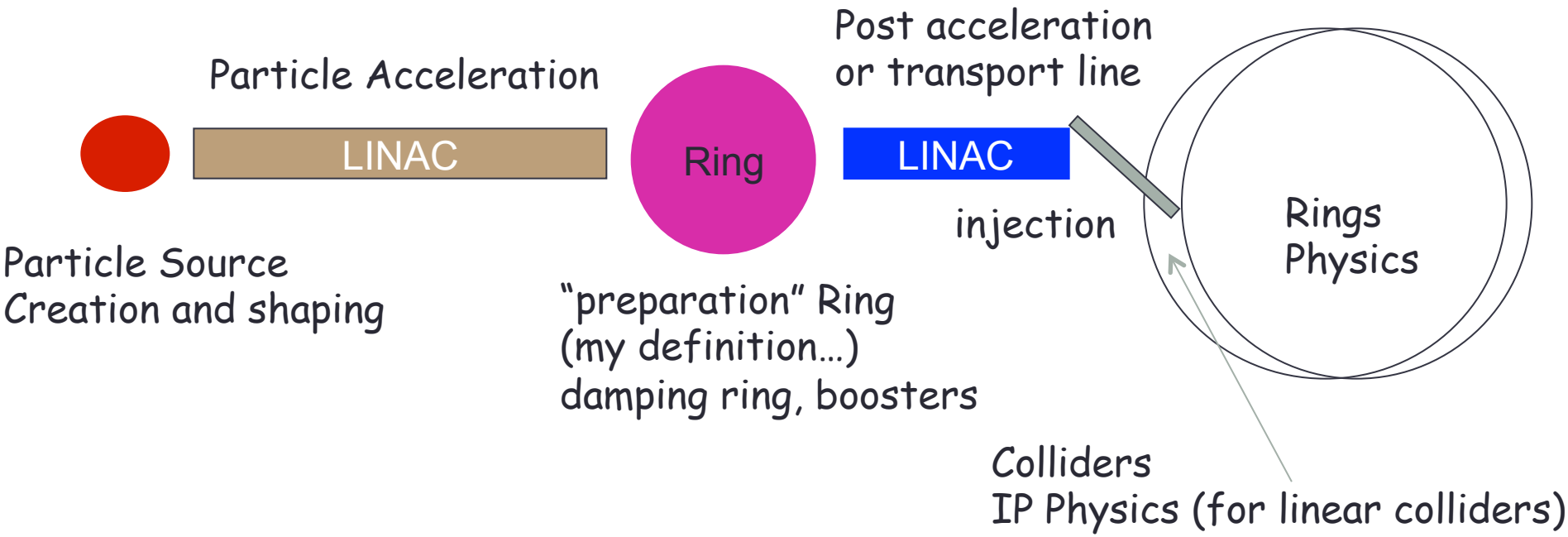
Band	Frequency range	Origin of name
HF band	3 to 30 MHz	High Frequency
VHF band	30 to 300 MHz	Very High Frequency
UHF band	300 to 1000 MHz	Ultra High Frequency
L band	1 to 2 GHz	Long wave (SC accelerators)
S band	2 to 4 GHz	Short wave (NC accelerators)
C band	4 to 8 GHz	Compromise between S and X
X band	8 to 12 GHz	(CLIC II, NLC)
Ku band	12 to 18 GHz	
K band	18 to 27 GHz	(CLIC I)
Ka band	27 to 40 GHz	
V band	40 to 75 GHz	
W band	75 to 110 GHz	W follows V in the alphabet
mm band	110 to 300 GHz	

Summarizing : Particle Accelerators

- Linacs : Linear Accelerators – Can be pulsed or CW (not in HEP… too much power to dump). Particle are accelerated in straight lines and delivered to a) physics b) rings
- Circular, Rings (colliders, boosters, damping rings, synchrotrons, cyclotrons…). Accelerated particles are injected. They stay on a nominal orbit defined by the ring MAGNETS. The energy lost by synchrotron radiation is recuperated by means of e.m cavities.
- ERL : new concept to allow to run ~quasi linear accelerators in cw regime. This needs (only one ecception) the SuperConductive (SC) technology



Accelerator complex: general scheme



Accelerator physics basics

What's the accelerator behavior?

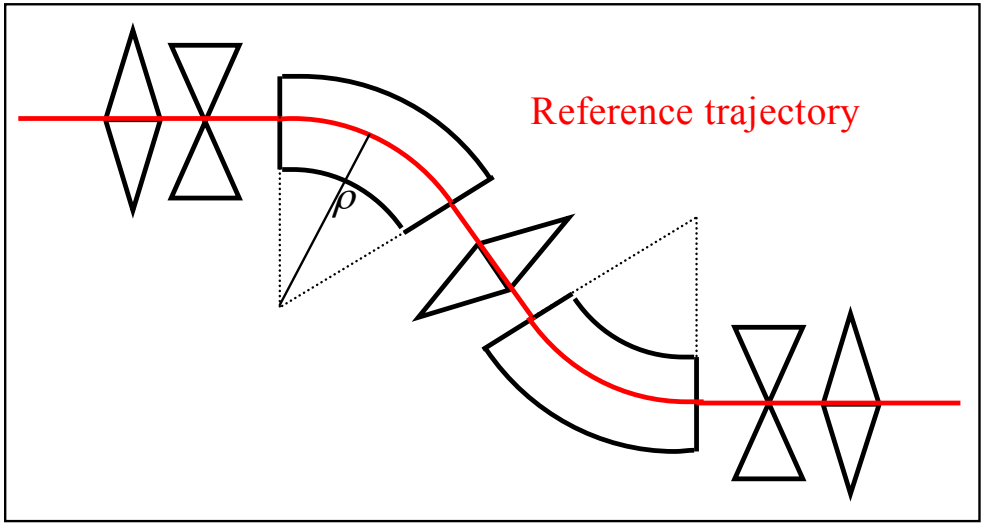
A huge and very particular Harmonic Oscillator

Particle transverse motion

- Let's start by tracking one particle
- 1st => chose the good reference system (fundamental)
- For accelerator can be very complex (due to the design geometry) but
- We can define a design trajectory
- We obtain a reference trajectory.
- The goal is to keep all the particle 'confined' in respect to the reference trajectory remember the beam definition..) -> Recall Force

An accelerator is designed around a reference trajectory (design orbit in circular accelerators), which is :

1. Straight line in drift and focusing element (no field on the axe)
2. Arc of circle in dipole magnet, horizontal or vertical (Transfer lines)
3. r is the local radius of curvature



On this trajectory, a particle is represented by a curved abscissa : *s*

Reference system -> riding the reference particle

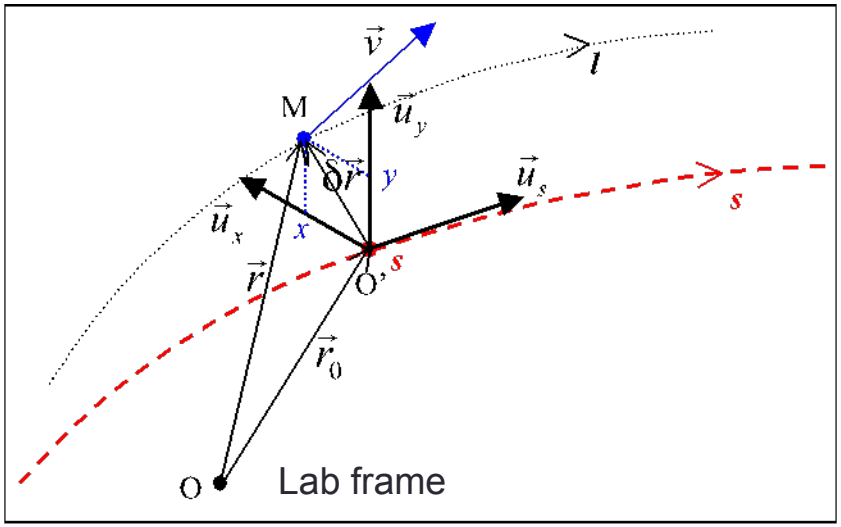
So we will describe the particle motion as a little deviation form the reference particle, moving on the reference trajectory in s coordinate: travelling reference system! => Frenet-Serret System

$$\left(\vec{u}_x, \vec{u}_y, \vec{u}_s\right)$$

$\vec{u}_s$: tangent to the reference trajectory

$\vec{u}_y$: vertical

$\vec{u}_x$: in horizontal plan



Recall forces

MAGNETS

- Why magnets ?:
- 1) Not needed to be integrated under vacuum
- 2) Efficiency given by the technological limits:

In theory the magnetic force is efficient at relativistic velocity : $F_E = \beta F_B$

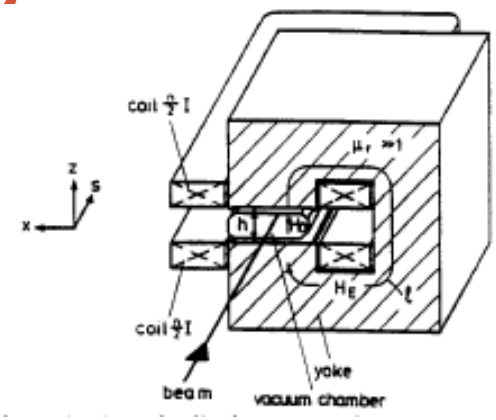
But let's take into account the technological limits. Iron saturation is 2T, Electric field breakdown threshold in vacuum ~ 10 MV/m. So also taking into account a β of 0.1 we have:

$$F_E / F_B = 10\ MV/m / 2T * 3\ 10^8 = 0.1 \Rightarrow F_E / F_B = 10\ MV/m / 2T * 0.1 * 3\ 10^8 = 1/6$$

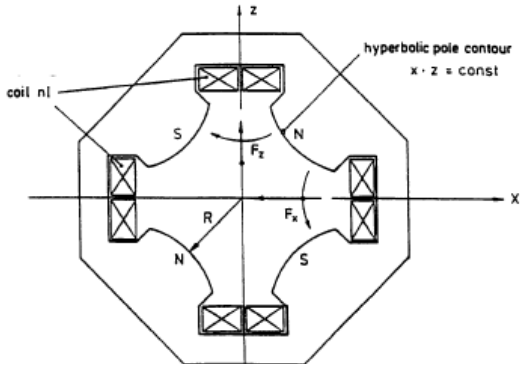
So already at non relativistic energy B field win!!!

Main type of magnets

- To bend : Dipoles



- To focalise : Quadrupoles



- Chromatic corrections :sextupoles, octupoles

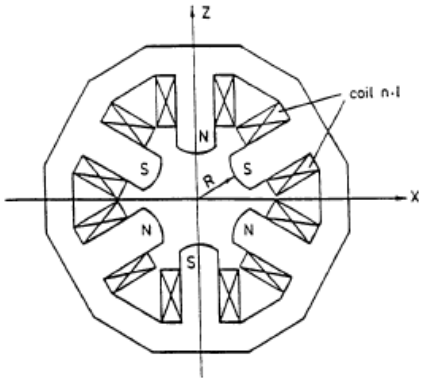
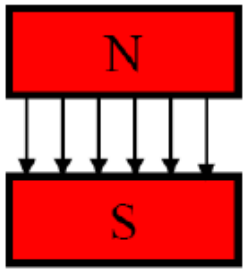


Figure 12: Sextupole magnet

Multipoles

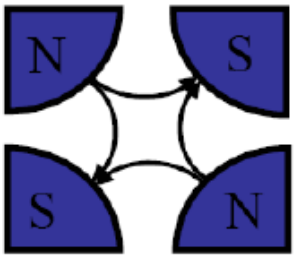
The general equation for B allows us to write the field for any n-pole magnet. Examples of upright magnets:

n=1: Dipole



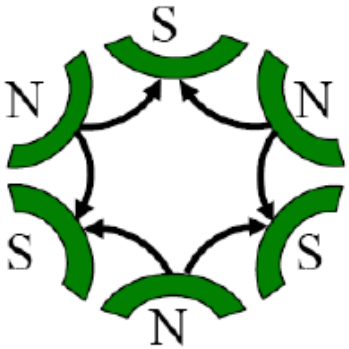
180° between poles

n=2: Quadrupole



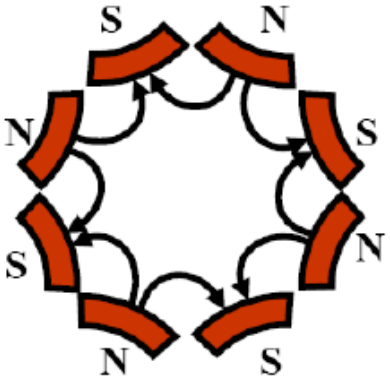
90° between poles

n=3: Sextupole



60° between poles

n=4: Octupole



45° between poles

- In general, poles are 360°/2n apart.
- The skew version of the magnet is obtained by rotating the upright magnet by 180°/2n.

Dipole

Quadrupole

Sextupole

Equation of motion

- To get a final motion equation we have to transform r_p and v_p in the lab system, and d/dt in d/ds (reference)

- At the end we can plug the components in the equation given by the Lorentz force:

$$r = q/m\,(r \times B)$$

This gives the system :

Linearization

$$x''\dot{s}^2 + x'\ddot{s} - \frac{\dot{s}^2}{\rho}\left(1+\frac{x}{\rho}\right) = \frac{q}{m}\left(\dot{s}\left(1+\frac{x}{\rho}\right)B_y - y'B_s\right)$$

$$y''\dot{s}^2 + y'\ddot{s} - \frac{\dot{s}^2}{\rho}\left(1+\frac{x}{\rho}\right) = \frac{q}{m}\left(-\dot{s}\left(1+\frac{x}{\rho}\right)B_x + \dot{s}\,x'B_s\right)$$

We are looking for linear motion : we need to linearize.

Our Approximations :

1) Velocity effect in the magnet negligible (vs >> vtrans) - > $\ddot{s} = 0$

2) $\vec{B} = (B_x, B_y, 0)$ (almost true)

3) $\frac{\mathrm{v}}{\dot{s}} = \sqrt{\left(1+\frac{x}{\rho}\right)^2 + y'^2 + x'^2} \approx \left(1+\frac{x}{\rho}\right)$ (little deviations in transverse plane)

4) $\frac{\mathrm{p}_0}{\mathrm{p}} \cong \left(1-\frac{\Delta p}{p}\right)$ little momentum deviations

5) $\frac{\mathrm{q}}{\mathrm{p}}B_y \approx kx - \frac{1}{\rho}$ and $\frac{\mathrm{q}}{\mathrm{p}}B_x \approx ky$ Espansions of the B field up to the linear term (Qpoles). Little deviations

HILL's equation

- After the substitution (ref) in the equations we get to the linear equations of motion

$$x'' - (k - 1/\rho^2)x = 1/\rho \, \Delta p/p$$

$$y'' + ky = 0$$

- Harmonic oscillators but k and r are k(s) and r(s) !!!!!!!!
- In general the Hill's equation will be

$$x'' + k(s)x = a(s)$$

Homogeneous solutions

- Being solution of a 2nd order differential equation the solution will be oscillatory…..we call it Sin Like and Cos like: S(s), C(s)
- They obviously satisfy:

$$S_H''(s)+k(s)S_H(s)=0$$

$$C_H''(s)+k(s)C_H(s)=0$$

For every transport element…so knowing k(s) we can find S and C…but we also know that from the general solution:

$$x(s)=A\,C(s)+B\,S(s)$$

$$x'(s)=A\,C'(s)+B\,S'(s)$$

Imposing * C(0)=1, C'(0)=0, S(0)=0,S'(0)=1

$$x(s)=C(s)\,x_0+S(s)\,x_0'$$

$$x'(s)=C'(s)\,x_0+S'(s)\,x_0'$$

So in matrix form : $\begin{pmatrix} x \\ x' \end{pmatrix}=\begin{pmatrix} C & S \\ C' & S' \end{pmatrix}\begin{pmatrix} x_0 \\ x_0' \end{pmatrix}$

Like in the general case…!!!

So : $x(s)=x_0\,C(s)+x_0'\,S(s)$+NH solution

Non Homogeneous Solutions

$x(s)=x_H(s)+x_{NH}(s)$:

$$x_{NH}''(s)+k(s)x_{NH}(s)=1/\rho(s)\,\Delta p/p_0$$

In a beam (particle ~ at the same momentum) $\Delta p/p_0$ is supposed to be a constant. So we can normalize the non homogeneous solution. The normalized solution will be the Dispersion function D(s):

$$D(s)=x_{NH}(s)/\Delta p/p_0$$

So the general solution will be : $x(s)=x_0\,C(s)+x_0'\,S(s)+D(s)\Delta p/p_0$

Green=Homogeneous / Bleu=Non Homogeneous

Physical meaning : The dispersion function give the deviation from the reference orbit due to a difference in the reference momentum. Since it is a deviation its dimensions are [m]

Summarizing

- Tracking (for sake of simplicity we took only the horizontal component, the plane of motion…in y usually D=0)

$$\begin{pmatrix} x \\ x' \\ \dfrac{\Delta p}{p_0} \end{pmatrix}_s = \begin{pmatrix} C(s) & S(s) & D(s) \\ C'(s) & S'(s) & D'(s) \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_0 \\ x'_0 \\ \dfrac{\Delta p}{p_{0\ @0}} \end{pmatrix}$$

C, S, D defined by the Hill's equations solutions in each element.
After that we have the matrix with each element we can multiply them to define a line or a ring for LINEAR particle tracking

Back to accelerators and particles

- x,y transverse coordinates in respect the reference particle
- x′, y′ can be angles defining the TRACE SPACE (x,x′) or momenta defining the PHASE SPACE (x,P_x)
- The vector (x,x′) represent the dynamical state of the particle
- In complex transport system, in linear approximation, we can solve the equations for each part of the trajectory obtaining M_1, $M_2 \ldots M_n$.
- The final particle state will be represented by $r_f = M_n \cdot M_{n-1} \cdot \ldots M_1 \cdot r_0$ with r=(x,x′)
- M coefficients COS AND SIN like -> H oscillator

NOW:

1) Let's see the application to accelerators (coordinates transformation, eq of motion, recall forces…)

2) Also after this the problem is not solved : N particles (10^{10} in a bunch), N turns, N elements/turn, collective effects….

….impossible to design with tracking…we need parametrization

Periodic focusing (PF) : solutions

$$u'' + K(s)\cdot u = 0$$

Hill equation

With : $K(s+S) = K(s)$

S : focusing period

ANSATZ COS like Solution : Given by the Floquet theorem

$$u(s) = \sqrt{\varepsilon \cdot \beta(s)} \cdot \cos(\phi(s))$$

$$\beta(s+S) = \beta(s)$$

The **beta function** at position s

$$\mu(s/s_0) = \phi(s) - \phi(s_0) = \int_{s_0}^{s} \frac{ds}{\beta(s)}$$

The ***phase advance*** between s_0 and s
For one turn = Q = TUNE

(ε = EMITTANCE and invariant given by particle initial conditions)

The motion is then a pseudo-harmonic oscillation with <u>varying amplitude and frequency</u>. This transverse motion is called the ***betatron oscillation***.

PF : Courant-Snyder parameters

Let's define :

$$\alpha(s) = -\frac{\beta'(s)}{2} \qquad \gamma(s) = \frac{1+\alpha(s)^2}{\beta(s)}$$

α, β and γ are the Courant-Snyder parameters of the motion

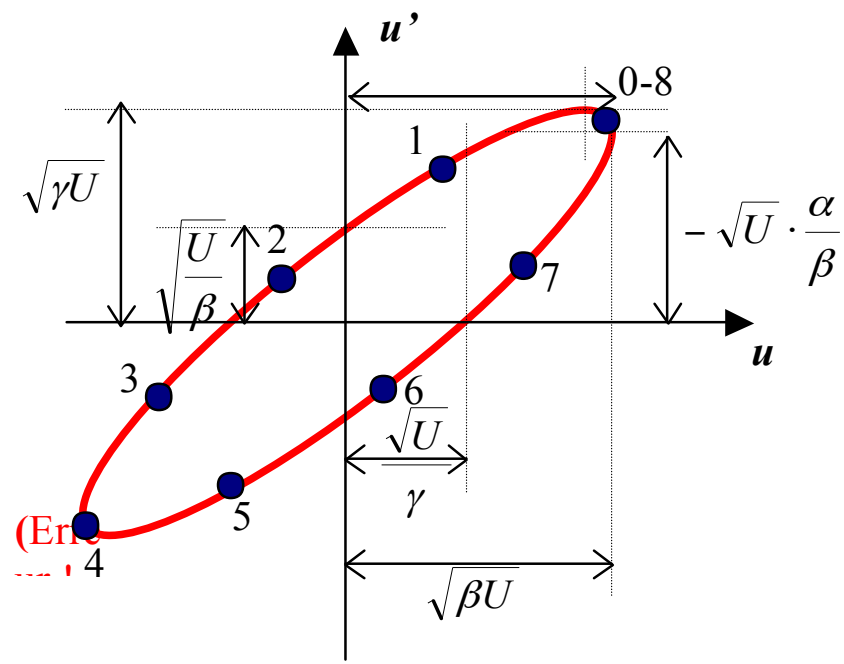
It is easy to show that :

$$\gamma \cdot u^2 + 2 \cdot \alpha \cdot u \cdot u' + \beta \cdot u'^2 = U$$

This is an ellipse equation.

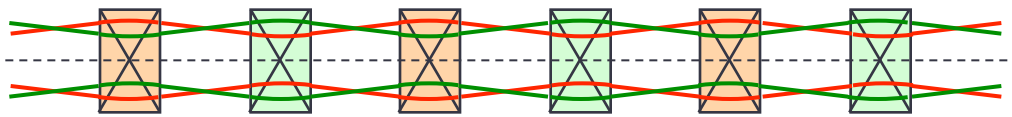
Particle is moving on an ellipse whose shape is given by Courant-Syder parameters.

$U=\varepsilon$ is the courant-Snyder invariant linked to a particle

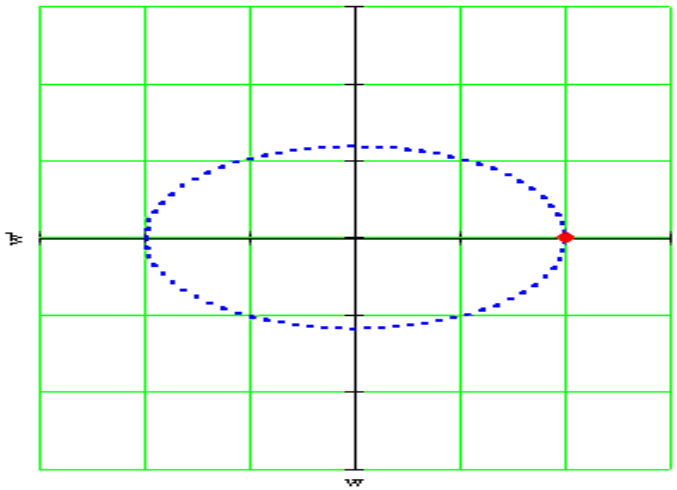


● Particle

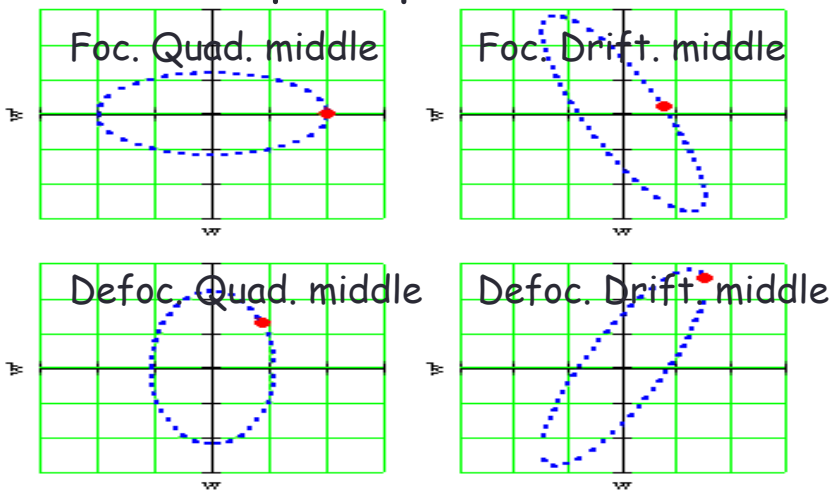
····· Particle ellipse

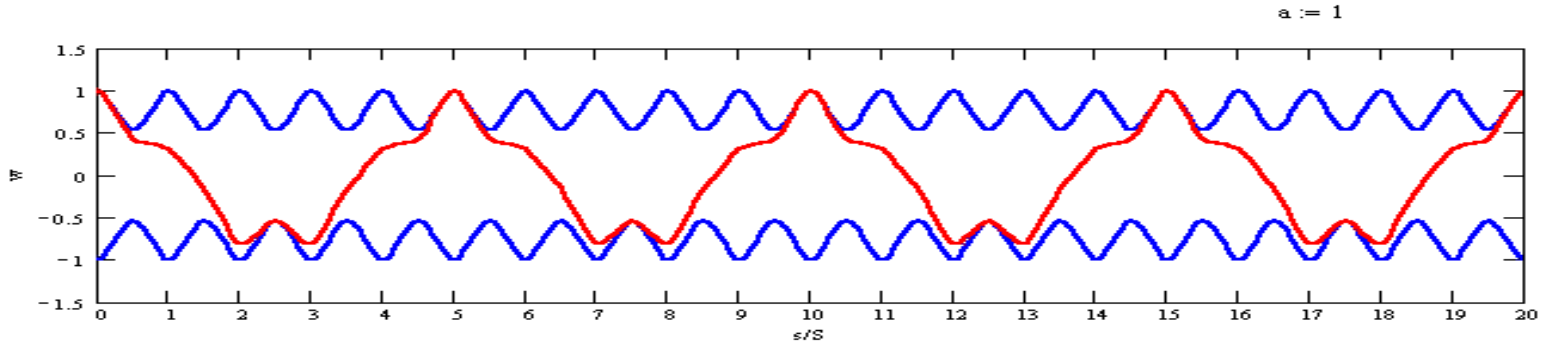


Phase-space trajectory



Phase-space periodic looks





Particle trajectory — Particle ellipse maximum size

Let's calculate the transfer matrix $M(s/s_0)$ from s_0 to s.

$$u(s) = \sqrt{\frac{\beta}{\beta_0}} \cdot (\cos\mu + \alpha_0 \cdot \sin\mu) \cdot u(s_0) + \sqrt{\beta\beta_0} \cdot \sin\mu \cdot u'(s_0)$$

$$u'(s) = -\left(\frac{\alpha - \alpha_0}{\sqrt{\beta\beta_0}} \cdot \cos\mu + \frac{1 + \alpha\alpha_0}{\sqrt{\beta\beta_0}} \cdot \sin\mu \right) \cdot u(s_0) + \sqrt{\frac{\beta_0}{\beta}} \cdot (\cos\mu - \alpha \cdot \sin\mu) \cdot u'(s_0)$$

m is the phase advance from s_0 to s.

$$M(s/s_0) = \begin{pmatrix} \sqrt{\frac{\beta}{\beta_0}} \cdot (\cos\mu + \alpha_0 \cdot \sin\mu) & \sqrt{\beta\beta_0} \cdot \sin\mu \\ \frac{\alpha_0 - \alpha}{\sqrt{\beta\beta_0}} \cdot \cos\mu - \frac{1 + \alpha\alpha_0}{\sqrt{\beta\beta_0}} \cdot \sin\mu & \sqrt{\frac{\beta_0}{\beta}} \cdot (\cos\mu - \alpha \cdot \sin\mu) \end{pmatrix}$$

Transfer matrix of 1 period

$$M(s_0 + S / s_0) = \begin{pmatrix} \cos\mu + \alpha_0 \cdot \sin\mu & \beta_0 \cdot \sin\mu \\ -\gamma_0 \cdot \sin\mu & \cos\mu - \alpha_0 \cdot \sin\mu \end{pmatrix}$$

If M is the matrix of one period (or lattice) from s_0 to s_0+S :

$$\boxed{\begin{cases} \cos\mu = \dfrac{1}{2} \cdot (M_{11} + M_{22}) \\ \beta_0 = \dfrac{M_{12}}{\sin\mu} \\ \gamma_0 = -\dfrac{M_{21}}{\sin\mu} \\ \alpha_0 = \dfrac{M_{11} - M_{22}}{2 \cdot \sin\mu} \end{cases}}$$

Errors

- Impulsional D errors in s_0 can be expressed in the form of the Green functions:

$$X_{co}(s)=G(s,s_0)\theta(s_0)$$

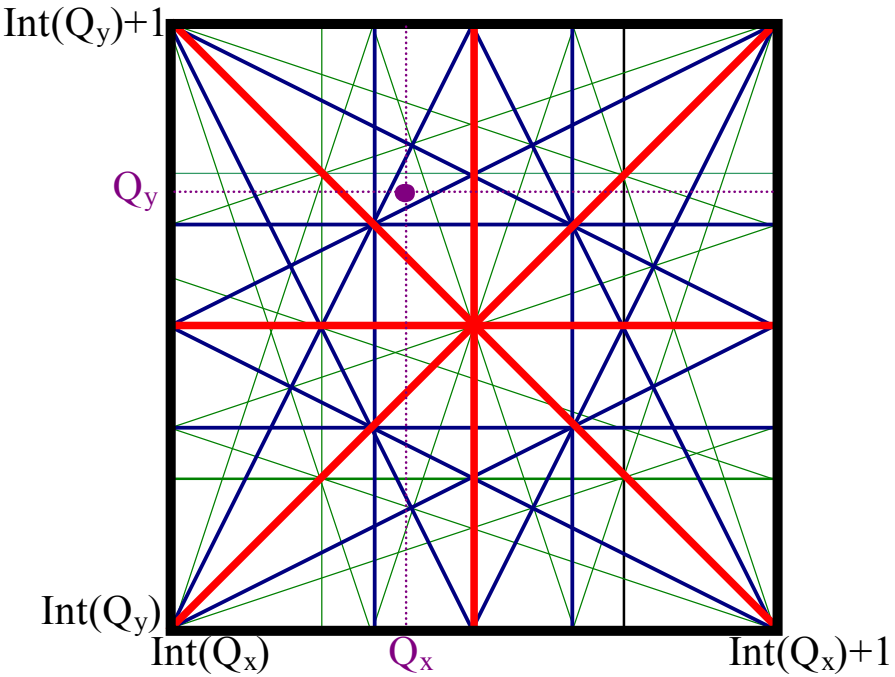
With $G(s,s_0)=\sqrt{\beta\beta_0}/2\sin\pi Q\cos(\pi Q-|\varphi(s)-\varphi(s_0)|)$
(believe me…)

So the orbit response is the product of the Green function by the kick itself. Q at denominator!!!!!!!!!!!!!!

Tune diagram

Resonance : $$n_x \cdot Q_x + n_y \cdot Q_y = n$$

Resonance's order : $$|n_x| + |n_y|$$



<u>Avoid resonances</u> :
find the best working point in tune diagram

Emittance and Twiss parameters

The best ellipse fitting the particle distribution is :

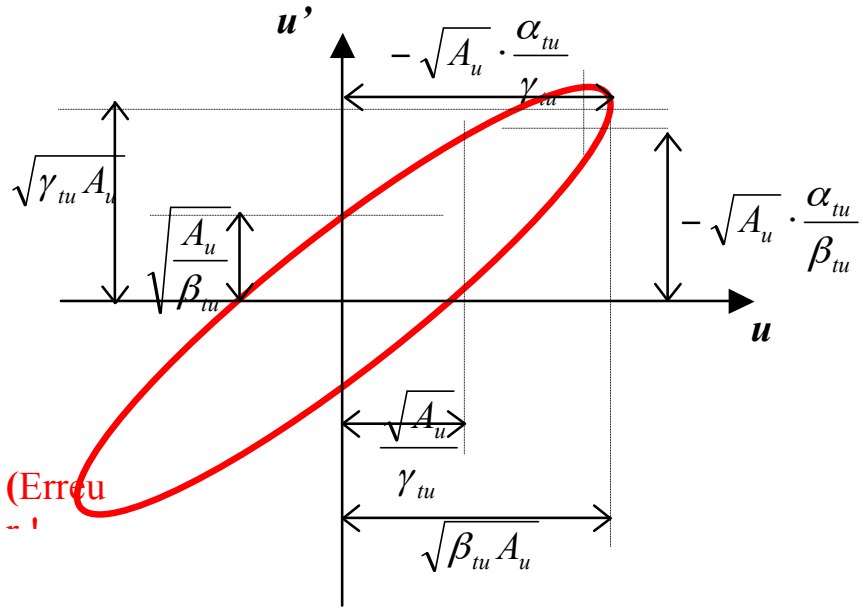
$$\gamma_{tu} \cdot u^2 + 2 \cdot \alpha_{tu} \cdot u \cdot u' + \beta_{tu} \cdot u'^2 = A_u$$

Such as :

$$\beta_{tu} = \frac{u_{rms}^2}{\varepsilon_{u,rms}} = \frac{\sigma_u}{\varepsilon_{u,rms}}$$

$$\gamma_{tu} = \frac{u_{rms}'^2}{\varepsilon_{u,rms}} = \frac{\sigma_u'}{\varepsilon_{u,rms}}$$

$$\alpha_{tu} = -\frac{uu'_{rms}}{\varepsilon_{u,rms}} = -\frac{\sigma_{uu'}}{\varepsilon_{u,rms}}$$



are the beam ***Twiss Parameters***

Sigma matrix : definition

Let's concentrate on ellipse :

$$\gamma_t \cdot u^2 + 2 \cdot \alpha_t \cdot u \cdot u' + \beta_t \cdot u'^2 = \varepsilon_{u,rms}$$

It can be also written :

$$U^T \cdot [\sigma]^{-1} \cdot U = I$$

$U = \begin{pmatrix} u \\ u' \end{pmatrix}$ is a vector given the particle position in phase-space

$[\sigma] = \begin{pmatrix} \sigma_u & \sigma_{uu'} \\ \sigma_{uu'} & \sigma_{u'} \end{pmatrix} = \begin{pmatrix} \beta_{tu} & -\alpha_{tu} \\ -\alpha_{tu} & \gamma_{tu} \end{pmatrix} \cdot \varepsilon_{u,rms}$ is a beam ***sigma matrix***

- We described the accelerators with parameters : we can describe the general behavior and FIT!!!!!

- α,β,γ,Q,D, (Accelerator)
- ε,σ (Beam)

The statistical emittance : Beam RMS (Root Mean Square) dimensions. Covariance ellipse of the particle distribution. Liouville Theorem

Average of function $A(\vec{r},\vec{r}')$ on the beam :

$$\langle A(\vec{r},\vec{r}')\rangle = \frac{1}{n}\cdot\sum_{i=1}^{n} A(\vec{r}_i,\vec{r}_i') = \frac{1}{N}\cdot\iint f(\vec{r},\vec{r}')\cdot A(\vec{r},\vec{r}')\cdot d\vec{r}\cdot d\vec{r}'$$

<u>Ex</u> : C.o.g. position : $(\langle u\rangle,\langle u'\rangle)$

RMS Size : $u_{rms} = \sqrt{\sigma_u} = \sqrt{\langle (u-\langle u\rangle)^2\rangle}$

RMS Divergence : $u'_{rms} = \sqrt{\sigma_{u'}} = \sqrt{\langle (u'-\langle u'\rangle)^2\rangle}$

Coupling term : $uu'_{rms} = \sigma_{uu'} = \langle (u-\langle u\rangle)\cdot(u'-\langle u'\rangle)\rangle$

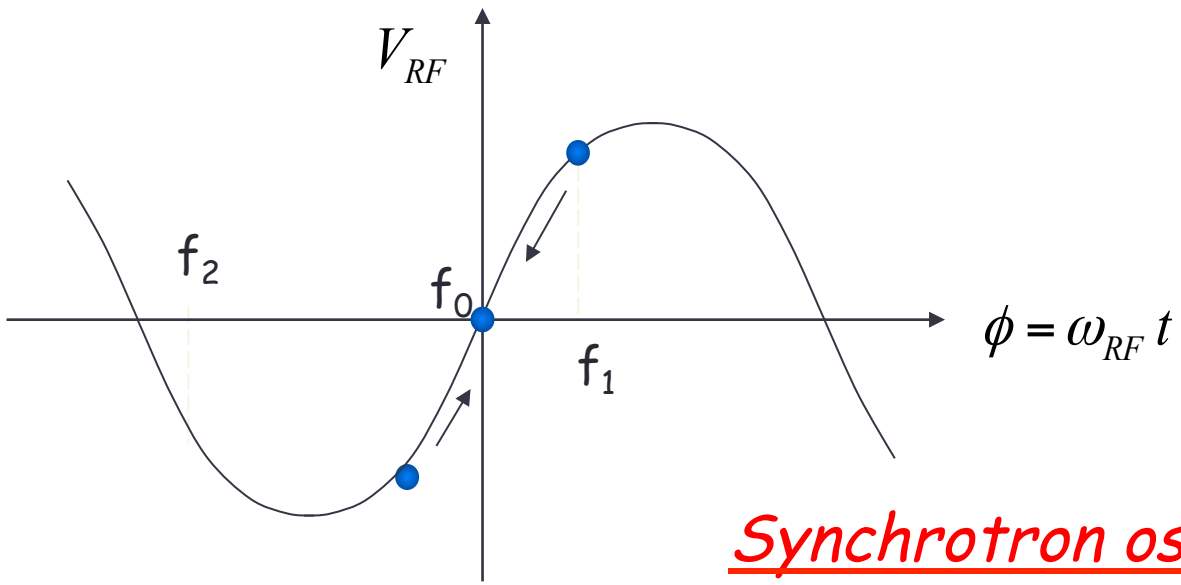
RMS emittance : $\varepsilon_{u,rms} = \sqrt{u_{rms}^2\cdot u'^2_{rms} - (uu'_{rms})^2}$

Longitudinal plane:
Same big H Oscillator but:

1) The recall force is given by a RF Cavity
2) Due to 1) non linearities

f_1 - The particle is accelerated
- The particle arrives earlier – tends toward f_0

The cavity acts as a longitudinal lens



Synchrotron oscillations

f_2 - The particle is decelerated
- The particle arrives later – tends toward f_0

For small phase oscillation in the phase space (phase,energy)
We can get again to a second order differential equation (damped Harmonic oscillator….)

$$\ddot{\varphi} + 2\tau\dot{\varphi} + \Omega^2\varphi = 0,$$

Let's think to a very slow damping so τ=0

with $\varphi = \Psi - \Psi_s$, τ = damping decrement , Ω^2 = *synchrotron* frequency

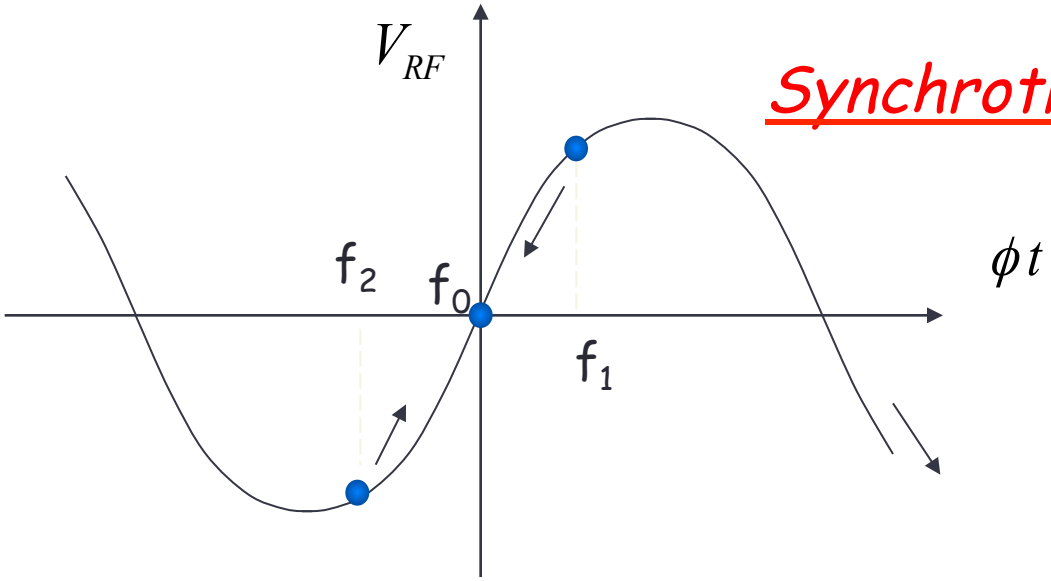
if the accelerating potential is RF (sinusoidal) $\Omega^2 = \dfrac{ck\eta_c}{cT_0 p_0} q\hat{V} Cos\Psi_s$

with T_0 = time of flight = $L_0 / c\beta$, $\hat{V}$ = *ref* potential $V = \hat{V}\sin\Psi$

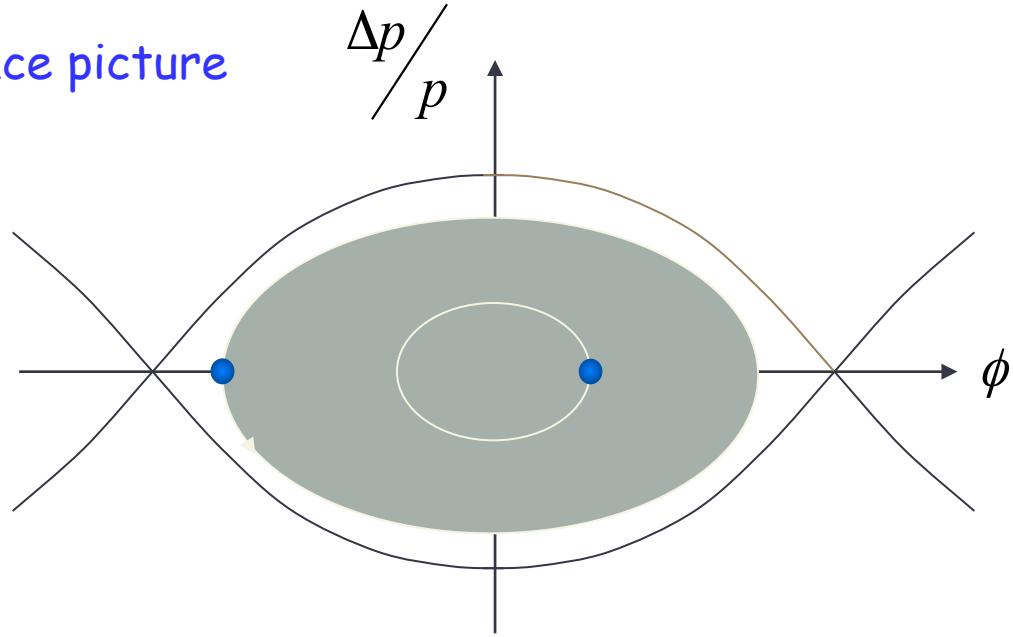
if $f_{rf} = hf_{rev}$ (h = integer) $\quad \Omega^2 = \omega_{rev}^2 \dfrac{h\eta_c}{2\pi\beta c p_0} q\hat{V} Cos\Psi_s$

similar to betatron motion we have a syncrotron tune $\nu_s = \dfrac{\Omega}{\omega_{rev}}$

Synchrotron oscillations



Phase space picture



- So also for the longitudinal coordinates (phase space) the particle follow an oscillatory motion characterized by a frequency (a wavelength for β) and a tune.
- The area of the longitudinal phase space that represent a stable motion is called "bucket"
- Before we neglect the τ. But this is important (cooling). This is possible if the energy loss is dependent from the particle energy itself.

$\tau=-(1/2T_0)(dU/dE)_{E_0}$

Synchroton and Betatron motion are damped by synchrotron radiation emission!!!!!

Energy loss per turn and related parameters for various electron storage rings

	E (GeV)	ρ (m)	L (m)	T_0 (μs)	$U_{0,dip}$* (MeV)
Adone	.51	5	105	.35	.001
DAΦNE	.51	1.4	98	.31	.004
PEP B LE	3.1	30.5	2200	13.6	.27
PEP B HE	9.0	165	2200	13.6	3.5
LEP	100.	3100	$3\ 10^4$	89	2855

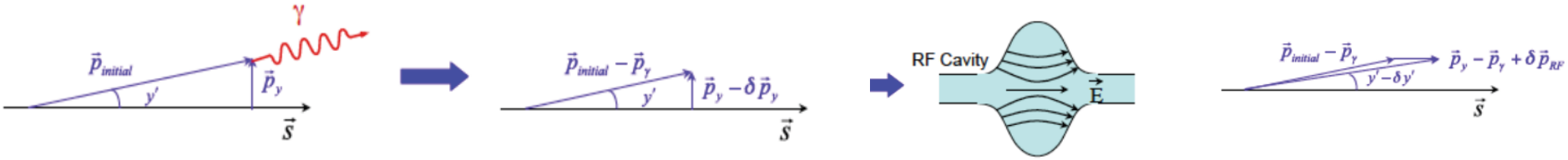
The same quantities for the LHC proton storage ring

	E (GeV)	ρ (m)	L (m)	T_0 (s)	$U_{0,dip}$ (MeV)
LHC	7700	2568	$3\ 10^4$	89	.011

* dip = from dipoles, excluding contributions from wigglers

Synchrotron Cooling (heating?)

- Without energy losses the 6D emittance is an invariant of motion
- But particle loose energy by synchrotron radiation. This energy is restored in the RF cavities by a longitudinal field
- In longitudinal motion higher energy particles lose more energy than the lower energy ones. They recover energy with the same field -> The motion is damped -> cooling
- In transverse space the synchrotron radiation is emitted in a 1/γ cone. Loss of transverse momentum. The energy is recovered longitudinally -> Angle damping, position unchanged -> cooling



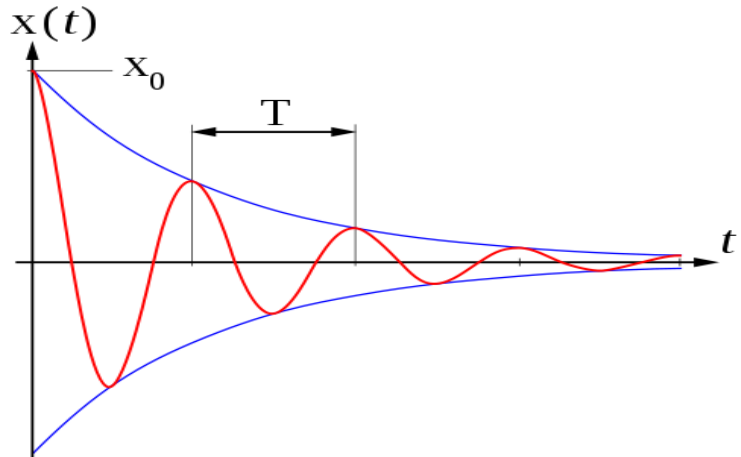
Longitudinal damping

- (longitudinal motion)

$$d^2\tau/dt^2 + 2\alpha_E\, d\tau/dt + \omega_s^2\, \tau = 0$$

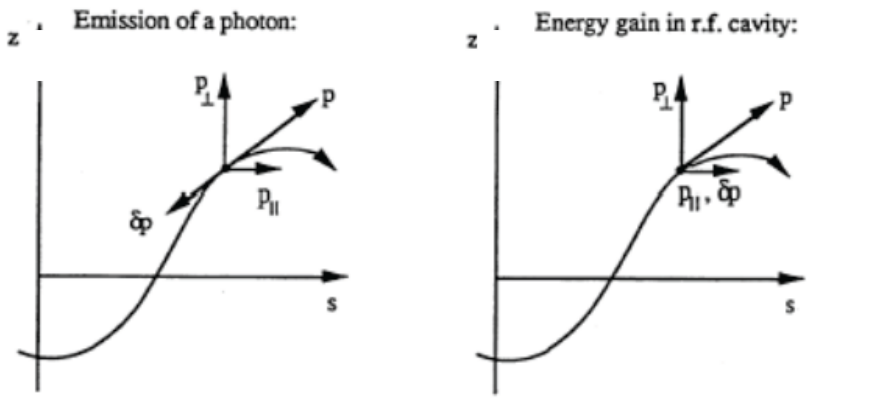
with $\alpha_E = W/2T_0$, $W = dU/dE|_{E=E_0}$, U=synchrotron radiated energy,
$\omega_s = -\alpha_c\, qV/ET_0$, α_c =momentum compaction

Damped solution: $\tau(t) = Ae^{-\alpha_E t}\cos(\omega_s t - \varphi_0)$



Transverse Damping, Vertical motion

Effect of energy loss due to photon emission and energy gain in the RF cavity



In the vertical plane the particle undergoes Betatron oscillations. So the vertical displacement will be $y=A\sqrt{\beta}\cos\theta$, $y' = -A/\sqrt{\beta}\sin\theta$ where the amplitude A is given by the Courant Snyder invariant $A^2 = \gamma y^2 + 2\alpha y y' + \beta y'^2$

For every turn the lost energy (SR with 1/γ angle) is restored by the RF cavity (zero angle). This change the longitudinal momentum. For the zero synchrotron amplitude particle :

$\Delta p/p = \Delta y'/y' = U/E$ with U -> energy lost by SR and E -> nominal energy.

AVERAGE EFFECT: Using the Amplitude definition it is possible to demonstrate that after many kicks, in average,

(averaged on all the betatron phases) the amplitude to the first order will vary as : $\langle \Delta A \rangle / A = -U/2E$

So we will have that $dA/dt = -U/2ET_0 \; A$ with a damped solution Ae^{-t/α_y} with $\alpha_y = U/2ET_0$ -> damping decrement.

Can we go to zero emittance?

- NO
- Why?
- Photons are stochastically emitted in a very short time $\sim\rho/c\gamma$
- This is much shorter than the revolution period. So emissions are instantaneous.. In this Δt the particle makes a discontinuous energy jump. Emissions are independent so the obey the Poisson distribution. These emissions perturb the particle orbit adding a random noise spread, so increasing the average oscillations.
- Equilibrium is attained when the quantum fluctuation rate equals the radiation damping

Quantum excitation

- In the energy domain a particle with ΔE in respect to the nominal energy undergoes to synchrotron oscillations with amplitude A.
- If at t_1 a photon with energy u is emitted ΔE will be

$$\Delta E = A_0 e^{j\omega(t-t_0)} - u e^{j\omega(t-t_0)} = A e^{j\omega(t-t_0)}$$

with $A^2 = A_0^2 + u^2 - 2A_0 u \cos\omega(t-t_0)$

Since the emission is independent t is equally distributed in time and in average the oscillatory term will disappear giving:

$$\delta A^2 = \langle A^2 - A_0^2 \rangle_t = u^2$$

So we will have:

$$\langle dA^2/dt \rangle = d\langle A^2 \rangle/dt = \mathcal{N} u^2 \text{ with}$$

$\mathcal{N}$=emission rate

COLLISIONS

Collisions energy in a collider

The frame of the center of mass for a system of particle is defined as the frame in which the momenta sum is zero

Let's take a head-on collision, the invariance requires:

$$\boxed{\left(\sum_i \frac{E_i}{c}\right)^2 - \sum_i p_i \sum_i p_i = \left(\sum_i \frac{E_i^{CM}}{c}\right)^2}$$

For two particles: $$\frac{\left(E_1^{CM} + E_2^{CM}\right)^2}{c^2} = \frac{\left(E_1 + E_2\right)^2}{c^2} - p_1^2 - p_2^2 - 2p_1 p_2$$

But if relaticistic p~mc=E/c and taking => $p_1=-p_2$

Substituting we obtain : $$E_1^{CM} + E_2^{CM} = 2\sqrt{E_1 E_2}$$

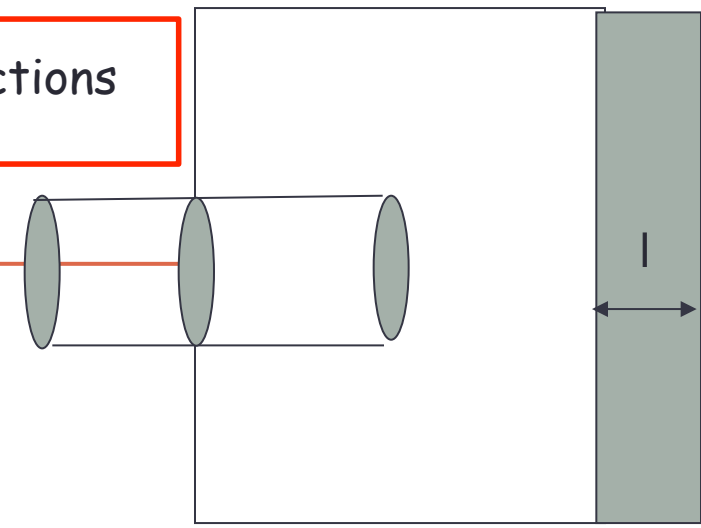
Cross section

Let's first take into account a Fixed target of length l and infinite size.
The target has density n.

The cross section will measure the number of interactions per particle and unit density & interaction length

$$N = \sigma \times n \times l$$

$$\sigma = N/(n \times l)$$



Cross Section [cm^2]

So if we have 1 particle crossing a target of 1 cm with a density of 1 x cm^{-3} we will obtain σ interactions.

σ DEPEND ON THE CONSIDERED EVENTS!!!

Unit = **barn** (symbol b) Système international, 10^{-28} m^2.

Luminosity

- Let's take into account a beam with Np particles per second impinging on our target. The Rate will be:
- $R = dN/dt = \sigma\, n\, Np\, l = \sigma\, L$ (with $L = n\, Np\, l$)

Luminosity [$cm^{-2}s^{-1}$]:
Luminosity is the event rate per unit cross section

Usually
Istantaneous L =Inverse Barn s^{-1}
Integrated L=Inverse Barn

So if we have cross section = 1 we will count L events per second
Independent from physics…depends on the machine

Example

- Single Bunch
- Head-on Collision
- Counter-rotating Beams with Longitudinal Speed v
- Revolution Frequency f_R

$$L = 2 v f_R \iiiint dx\,dy\,dz\,dt\, n_+(x,y,z,t)\, n_-(x,y,z,t)$$

MAXIMIZE THE EXPLOITABLE LUMINOSITY IS THE GOAL OF WHATEVER COLLIDER....

Gaussian beams, crossing angle, no hourglass

- $L = f_{rep} N_1 N_2 / 2\pi \, 1/\sqrt{\sigma_{y1}^2 + \sigma_{y2}^2} \, 1/\sqrt{(\sigma_{x1}^2 + \sigma_{x2}^2)\cos\vartheta + (\sigma_{z1}^2 + \sigma_{z2}^2)\sin\vartheta}$

If beam sizes are the same and the crossing angle is little

$$L = f_{rep} N_1 N_2 / 4\pi \, 1/\sigma_y \, 1/\sqrt{\sigma_x^2 + (\sigma_z \vartheta)^2} = f_{rep} N_1 N_2 / 4\pi \, 1/\sigma_x \sigma_y \, 1/\sqrt{1 + (\sigma_z / \sigma_x \vartheta)^2} =$$

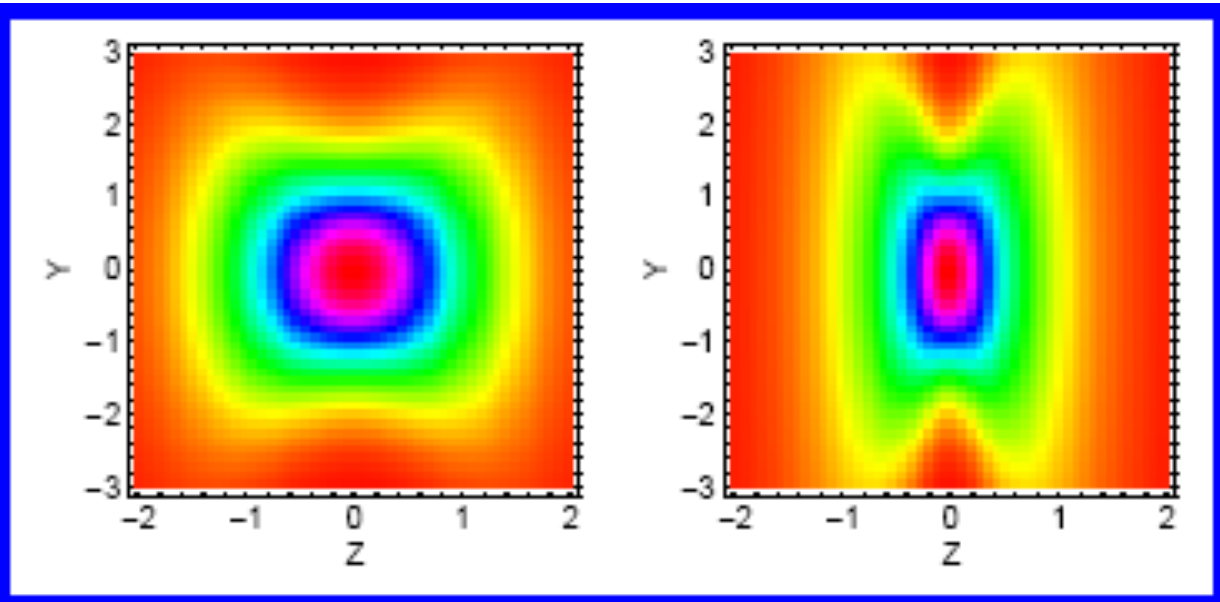
$$\boxed{L = cost/Area \, 1/\sqrt{1+\varphi^2} \Rightarrow \varphi = \sigma_z / \sigma_x \vartheta = PIWINSKI\ ANGLE}$$

What are the luminosity limits?

Geometrical effects

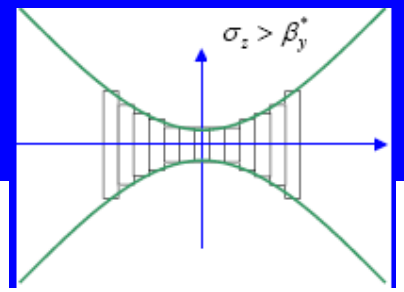
Hourglass Effect:

Too small beta reduce the "good" interaction point longitudinal region…



β = "depth of focus"

reasonable lower limit for β is bunch length σ_z



In laser science= RAYLEIGH LENGTH

Final focus and Hourglass

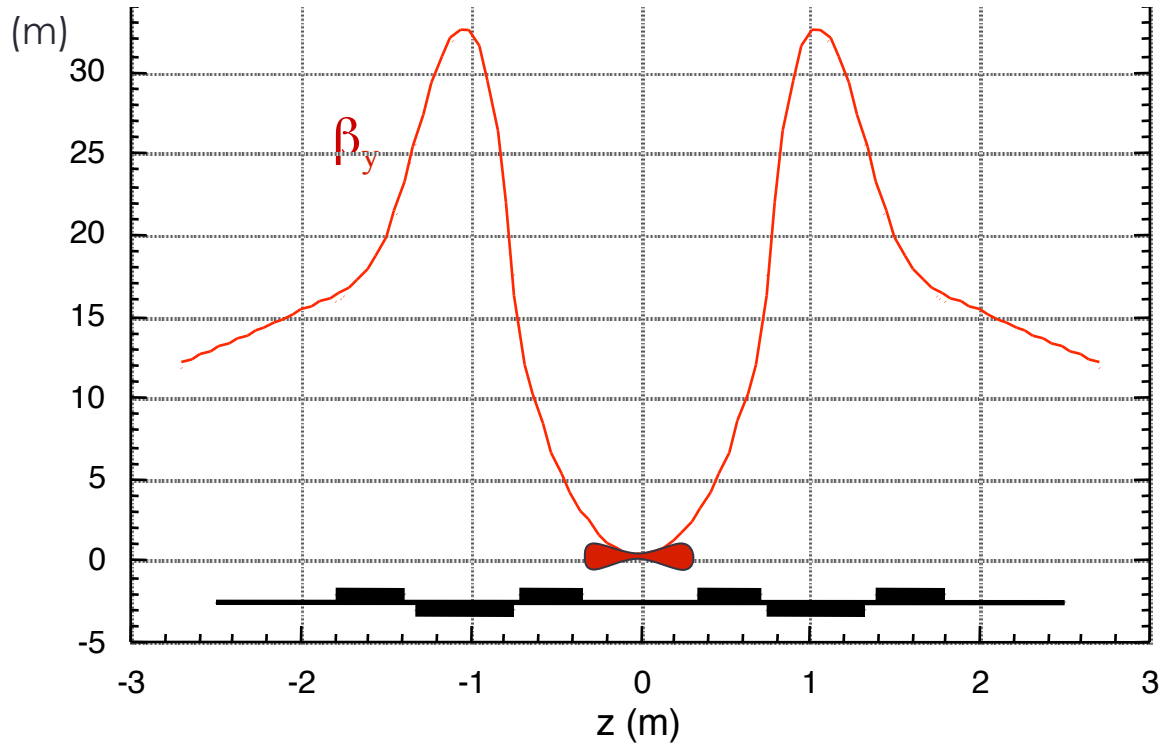


Photo in time
Math file
luminosity

$$\mathcal{L} \propto \frac{N}{\sigma_x \sigma_y} P_{beam} =$$

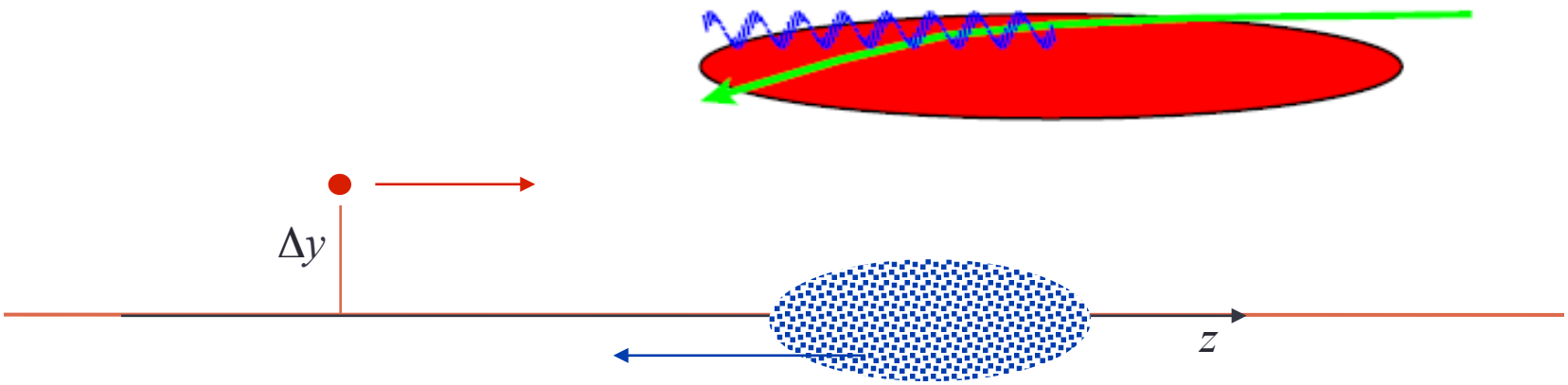
The image cannot be displayed. Your computer may not have enough memory to open the image, or the image may have been corrupted. Restart your computer, and then open the file again. If the red x still appears, you may have to delete the image and then insert it again.

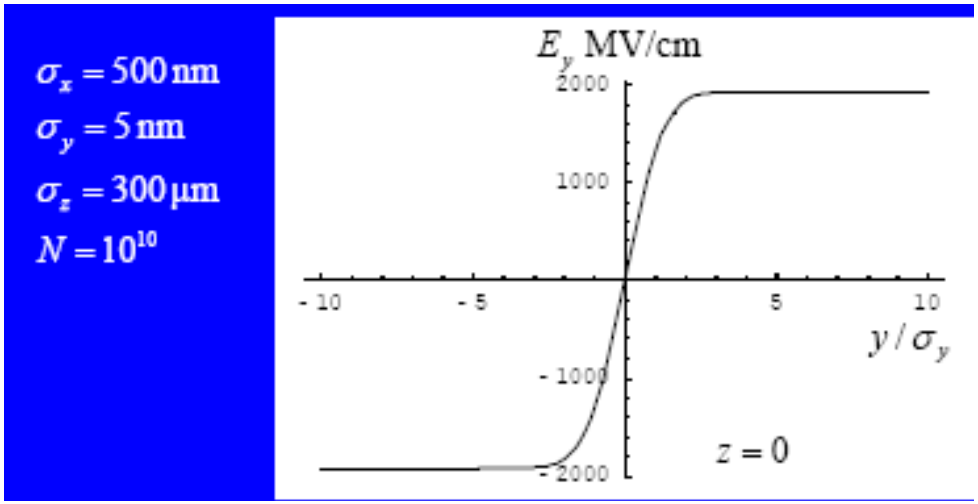
Question: Why this ?

The image cannot be displayed. Your computer may not have enough memory to open the image, or the image may have been corrupted. Restart your computer, and then open the file again. If the red x still appears, you may have to delete the image and then insert it again.

Physical effects

Beam-beam effects: the beam





Beam-beam effects

- Different effects:
- 1)Increase luminosity (focusing (pinch) effect)
- 2)Disruption (no re-using)
- 3)Tune shift (spread….), luminosity decrease

Beam-Beam Deflection (off-center particles)

$$\Delta y << \sigma_y^* \qquad \Delta x << \sigma_x^*$$

$$\Delta y' \cong -\frac{1}{f_y}\Delta y \qquad f_y^{-1} = \frac{2Nr_e}{\gamma\sigma_y^*\left(\sigma_x^* + \sigma_y^*\right)}$$

$$\Delta x' \cong -\frac{1}{f_x}\Delta x \qquad f_x^{-1} = \frac{2Nr_e}{\gamma\sigma_x^*\left(\sigma_x^* + \sigma_y^*\right)}$$

Focusing Quadrupole (thin lens):

$$\mathbf{Q}_F^k \equiv \begin{pmatrix} 1 & 0 \\ -1/f_k & 1 \end{pmatrix} \qquad k = x, y$$

Linear Beam-Beam Tune Shift

$$\xi_y^+ = \frac{N_- r_e \beta_y^{*+}}{2\pi\gamma\,\sigma_y^-\left(\sigma_y^- + \sigma_x^-\right)} = \Delta Q_y \qquad \xi_x^+ = \frac{N_- r_e \beta_x^{*+}}{2\pi\gamma\,\sigma_x^-\left(\sigma_y^- + \sigma_x^-\right)} = \Delta Q_x$$

Pinch Enhancement

- During collision, the bunches focus each other (f-focusing or pinching) leading to an increase in luminosity
- Luminosity enhancement factor : $H_D = \frac{L}{L_0} = \frac{\sigma_{x0}\sigma_{y0}}{\sigma_x \sigma_y}$
- Very few analytical results exist on this parameter.
- Empirical fit to beam-beam simulation results gives

$$H_{D_{x,y}} = 1 + D_{x,y}^{1/4}\left(\frac{D_{x,y}^3}{1+D_{x,y}^3}\right)\left(\ln\left(\sqrt{D_{x,y}}+1\right) + 2\ln\left(\frac{0.8\beta_{x,y}}{\sigma_z}\right)\right)$$

Only a function of disruption parameter $D_{x,y}$

Hour glass effect

Disruption

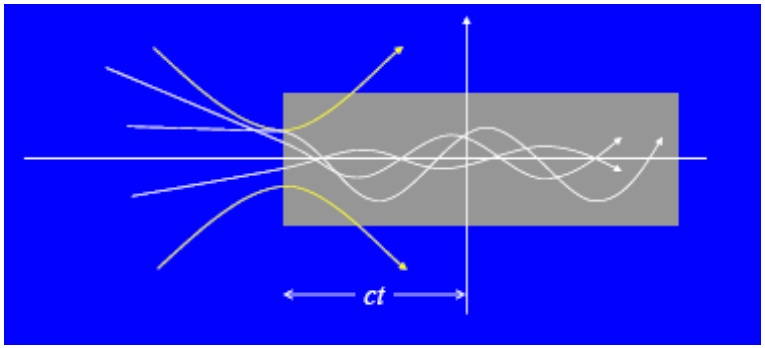
Let's take a test particle crossing the opposite beam:

- Opposite bunch => Gaussian
-Test particle $\Delta y << \sigma_y$ (particle near the axis)
-Initial $P_T=0$

-We have seen the deflection angles

-We introduce $D_{x,y}=\sigma_z/f_{x,y}$

-D<<1 =>the beam is a thin lens
-D>>1 long focalizing lens. The particle trajectory follows an oscillatory path inside the bunch.



BeamStrahlung

Particles travel on curved trajectories
Synchrotron radiation -> in beam-beam interaction called beamtrahlung
Quantum recoil -> Energy losses and spread
Particles collides at different energies than the reference one
Physics cross section are affected with threshold scans
Particles with large energy loss cannot circulate around the ring (momentum band-width)=>Affects the beam life time

- The critical energy is characterized by the upsilon parameter

$$\Upsilon \equiv \frac{2}{3}\frac{\hbar\omega_c}{E} = \frac{\lambda_e\gamma^2}{\rho} = \gamma\frac{2B}{B_c} = \frac{e}{m^3}\sqrt{\left|(F_{\mu\nu}p^\nu)^2\right|}$$

$$B_c = m^2/e \approx 4.4\text{GTeslas}$$

$$\Upsilon_{average} = \frac{5}{6}\frac{Nr_e^2\gamma}{\alpha\sigma_z(\sigma_x+\sigma_y)}$$

- ~0.1 for 500GeV collider

Luminosity and Beamstrahlung, why flat beams?

- Beamstrahlung causes a spread in the center of mass energy of $e^- e^+$. This effect is characterized by the parameter δ_{BS}.
- Limiting the beamstrahlung emission is of great concern for the design of the interaction region.

Low energy regime, $\Upsilon \rightarrow 0$

$$\delta_{BS} \approx 0.86 \frac{e r_e^3}{2 m_0 c^2} \left[\frac{E_{CM}}{\sigma_z} \right] \frac{N^2}{(\sigma_x + \sigma_y)^2}$$

Luminosity : $$\mathcal{L} = \frac{n_b N^2 f_{rep}}{4\pi \sigma_x \sigma_y} H_D$$

we would like to make $\sigma_x \sigma_y$ small to maximise Luminosity.

BUT keep $(\sigma_x + \sigma_y)$ large to reduce δ_{SB}.

Using flat beams $\sigma_x >> \sigma_y$; $$\delta_{BS} \propto \left[\frac{E_{CM}}{\sigma_z} \right] \frac{N^2}{\sigma_x^{\,2}}$$

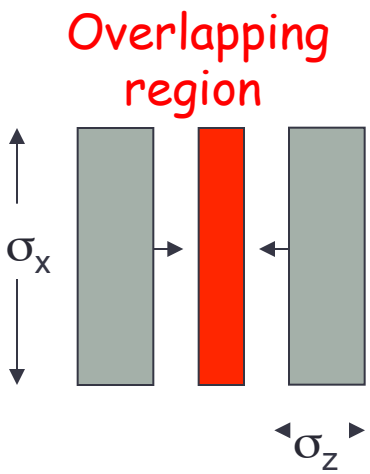
Set σ_x to fix δ_{SB}, and make σ_y as small as possible to achieve high luminosity.

The last idea on the market How to reach 10^{36}?

- The crab waist

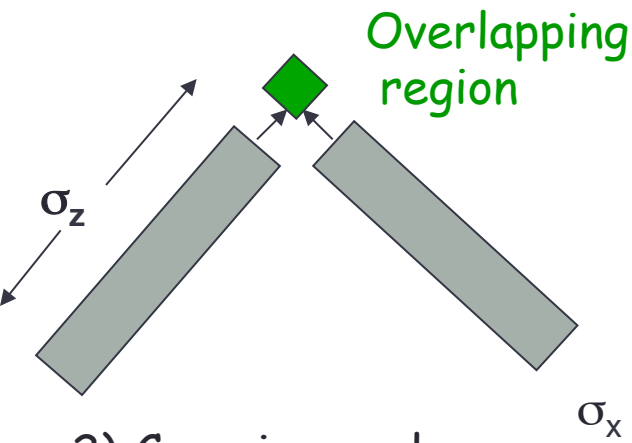
Crossing angle concepts

1) Standard short bunches

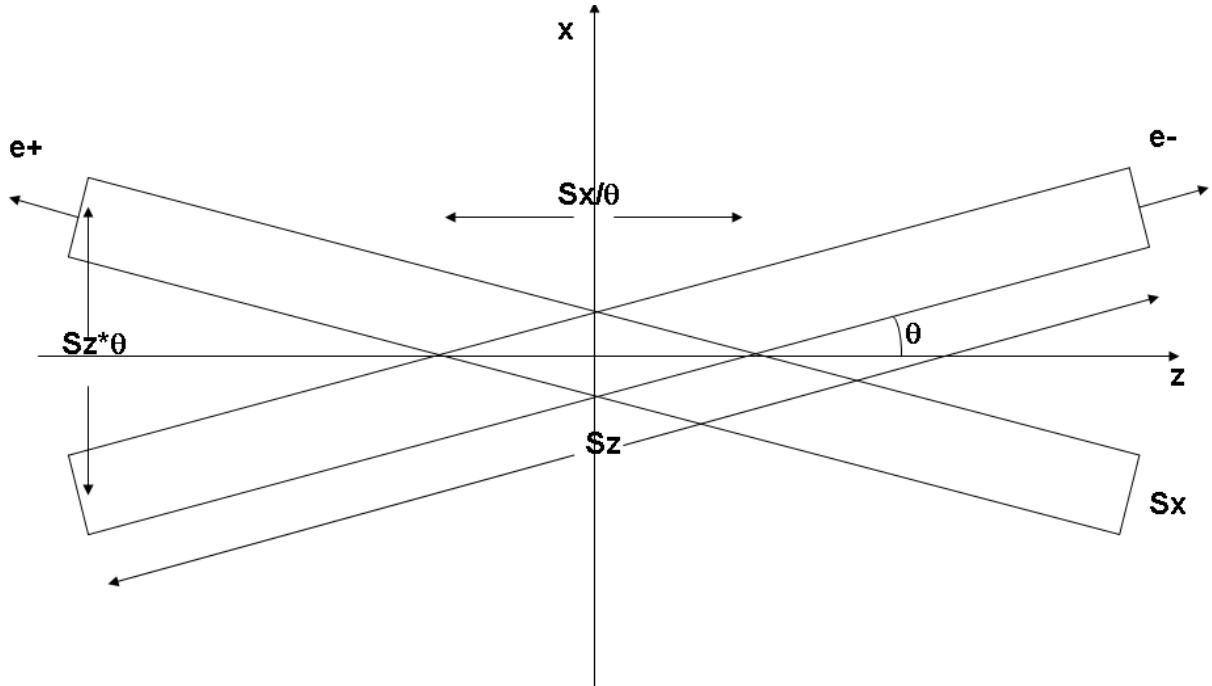


Both cases have the same luminosity,
(2) has longer bunch and smaller σ_x

In a ring is not difficult to have little emittances. Sigma x is usually big due to the tune shift. With large crossing angle X and Z requirements are swapped:



2) Crossing angle



But not only crossing angle.....

Large Piwinski angle $\Phi = \frac{\sigma_z}{\sigma_x} tg(\frac{\vartheta}{2})$ => (so long bunches and little H emittances).

Luminosity & IP parameters for a circular collider (beam re-used)

+ crossing angle & beam-beam

$$L \propto D \frac{f_r N^2}{(\sigma_x \sigma_y)} \frac{1}{\sqrt{1+\Phi^2}}$$

$$\Phi = \frac{\sigma_z}{\sigma_x} tg(\frac{\vartheta}{2})$$

Working with large Piwinski angle decreases the L but: WE CAN WORK WITH VERY SMALL BETA !!!

$$\textit{from}\ \text{Hourglass effect} \Rightarrow \beta_y \geq \sigma_z$$

- So the first assumption for CW scheme is to collide with large Piwinski angle

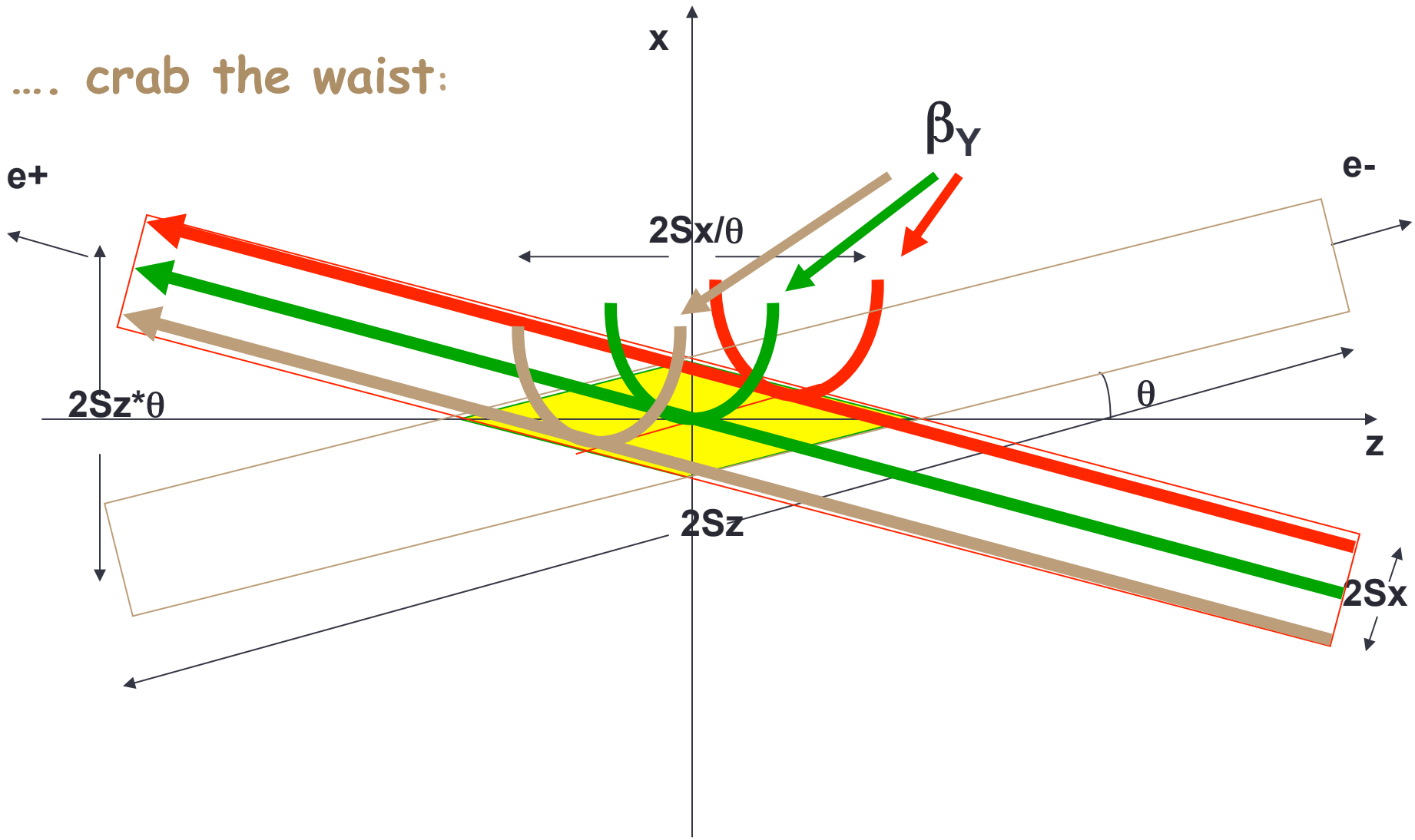
1) Large Piwinski angle - high σ_z and collision angle. (slight L decrease)
⇒allows to work with small beta & decreases the disruption due to the effective z overlap & minimises parasitic collision. The ring stability profits from long bunches (CSR, HOM…) but
Introduces B-B and S-B resonances (strong coordinates coupling).

2)Extremely short β^*_y - so little σ^*_y (high L gain…)

3)Small horizontal emittance (horizontal tune compensated by large Piwinski angle)

4) Small disruption (or same D increasing N) without luminosity loss. The beam can be re-utilised in an accumulation ring => High repetition frequency and charge per bunch (High L gain)

…. crab the waist:

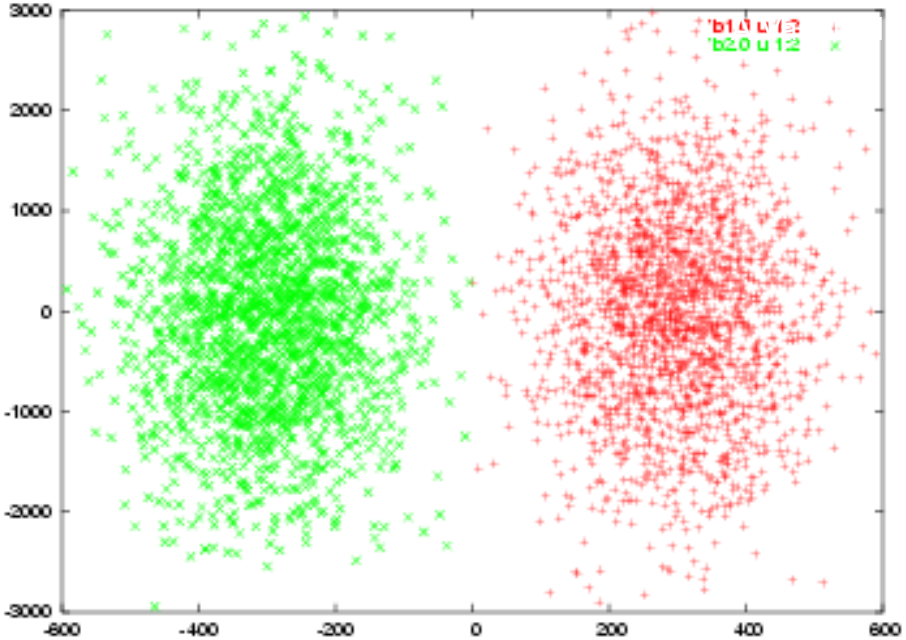


Why? Crabbed waist removes betratron coupling resonances introduced by the crossing angle (betatron phase and amplitude modulation)

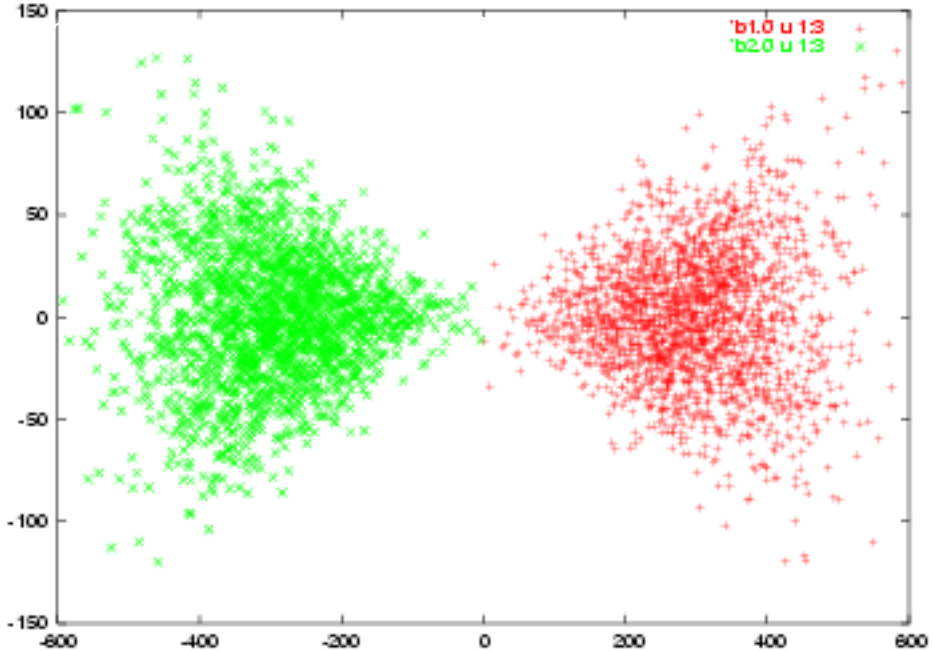
Summarizing => Large angle and waist is crabbed in the IP (slight gain L, very good for S-B B-B resonances suppression)

Resonances:

- The large collision angle and the short β^* introduce strong amplitude and phase modulation of the V beam kick as a function of the H coordinate. The crab waist strongly suppresses these resonances.

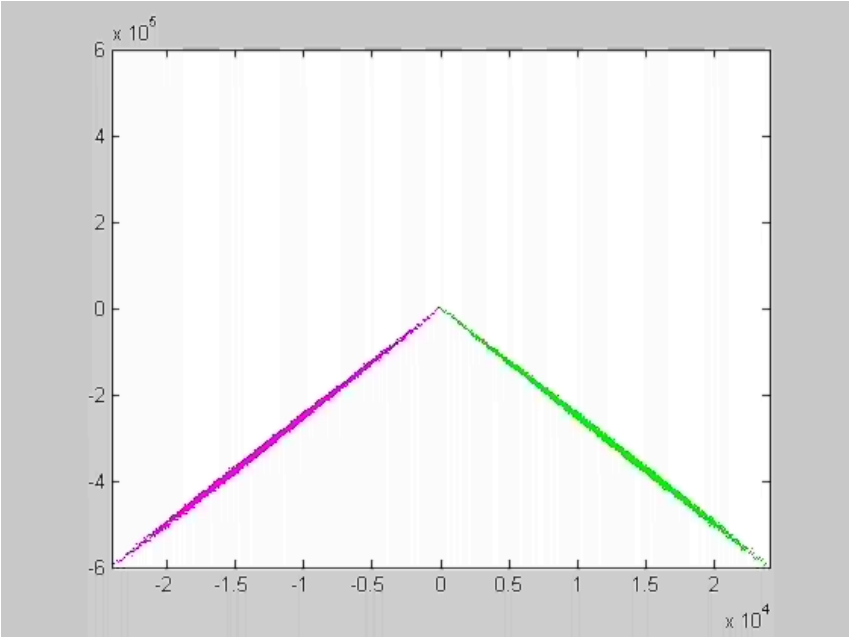


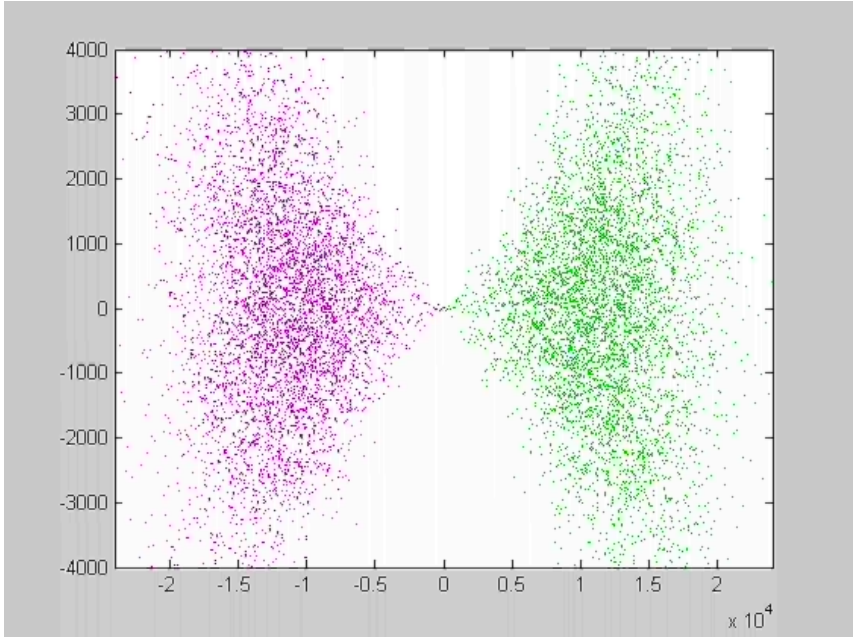
Horizontal Collision



Vertical collision

No crossing angle,
$\varepsilon_{yout}/\varepsilon_{yin}$= 200y/10x





Horizontal Plane **Vertical Plane**

Another example: Collisions with uncompressed beams
Crossing angle = 2*25mrad.
Relative Emittance growth per collision about $2.5*10^{-3}$
$\varepsilon_{yout}/\varepsilon_{yin}=1.0025$

What's next?

- Hadron Colliders
- Lepton Colliders
  - e+e-
    - Linear
    - Ring
  - $\mu+\mu-$
  - $\gamma\gamma$
  - New acceleration mechanism

Hadron Collider

- Hadron (proton/antiproton) is easier to accelerate to high energies owing to the absence of synchrotron radiation
- Already 14TeV will be reached in a few years (LHC)
- Events are complicated because proton is not an elementary particle
  - p = uud
  - Very high event rate: most of them are unnecessary
- Higher energies are possible only by
  - Higher magnetic field
  - or larger ring

  - Increasing the energy also SR losses start to play a role

HELHC: Higher Energy LHC

- proposed after the luminosity upgrade to HL-LHC
- Upgrade the magnets of LHC
- 8.33 Tesla → 20 Tesla ?
- E_{CM} 33TeV
- According to the present price of magnet (if possible), 80km ring is cheaper

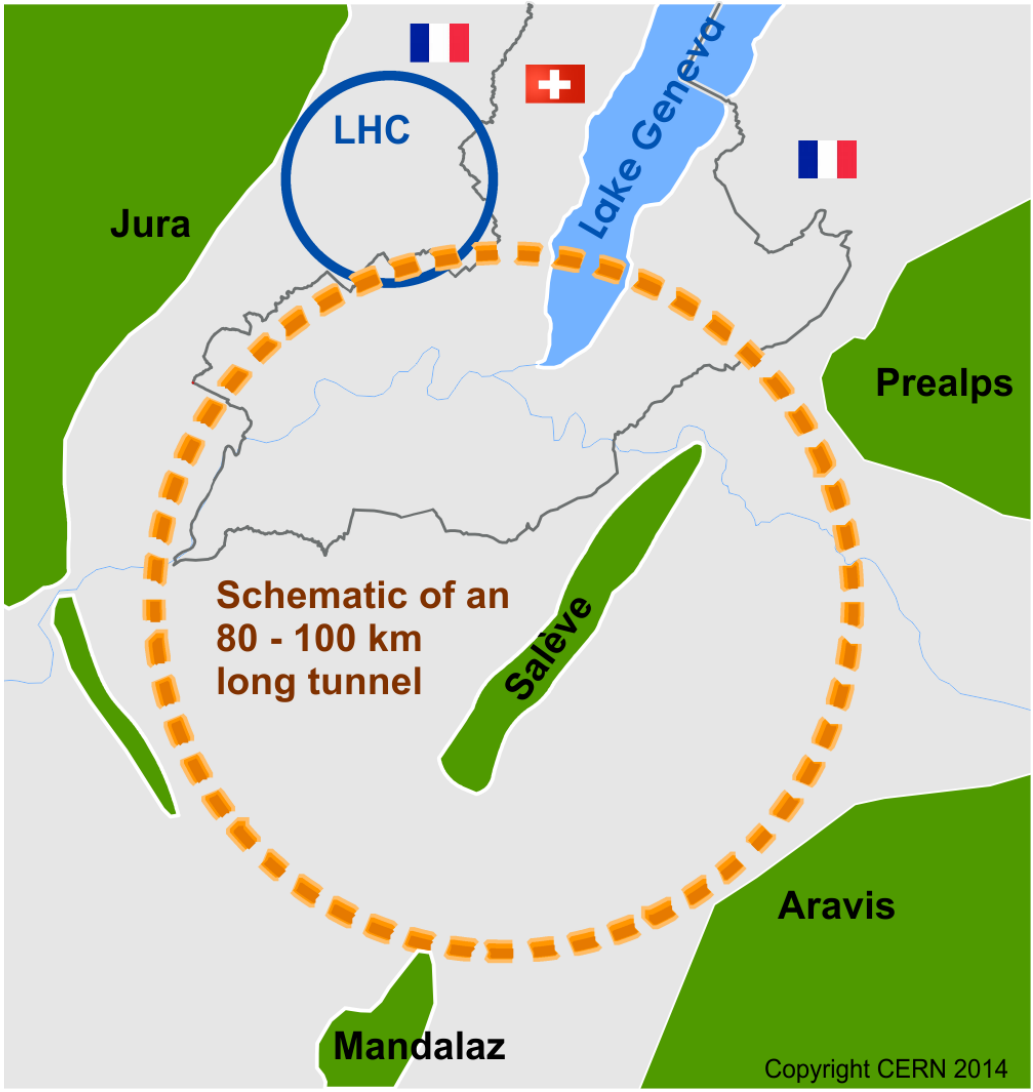
Future Circular Collider Study - SCOPE
CDR and cost review for the next ESU (2018)

international collaboration to study:

- ***pp*-collider (*FCC-hh*)** → defining infrastructure requirements

~16 T ⇒ 100 TeV *pp* in 100 km
~20 T ⇒ 100 TeV *pp* in 80 km

- ***e⁺e⁻* collider (*FCC-ee*)** as potential intermediate step
- ***p-e* (*FCC-he*) option**
- **80-100 km infrastructure** in Geneva area



Physics Parameters

	LHC	HL-LHC	HE-LHC	FCC-hh
Cms energy [TeV]	14		33	100
Luminosity [$10^{34}cm^{-2}s^{-1}$]	1	5	5	5
Bunch distance [ns]	25			25 (5)
Background events/bx	27	135	147	170 (34)
Bunch length [cm]	7.5	7.5	7.5	8

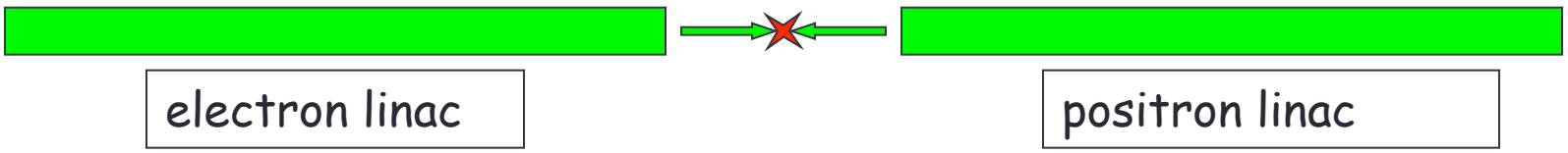
- Two main experiments sharing the beam-beam tuneshift
  - Two reserve experimental areas not contributing to tuneshift

- Currently assume 25ns as baseline
  - May be able to reduce bunch spacing and background
- Might be able to increase bunch length
  - Will explore this if experiments find it useful

- 80% of circumference filled with bunches

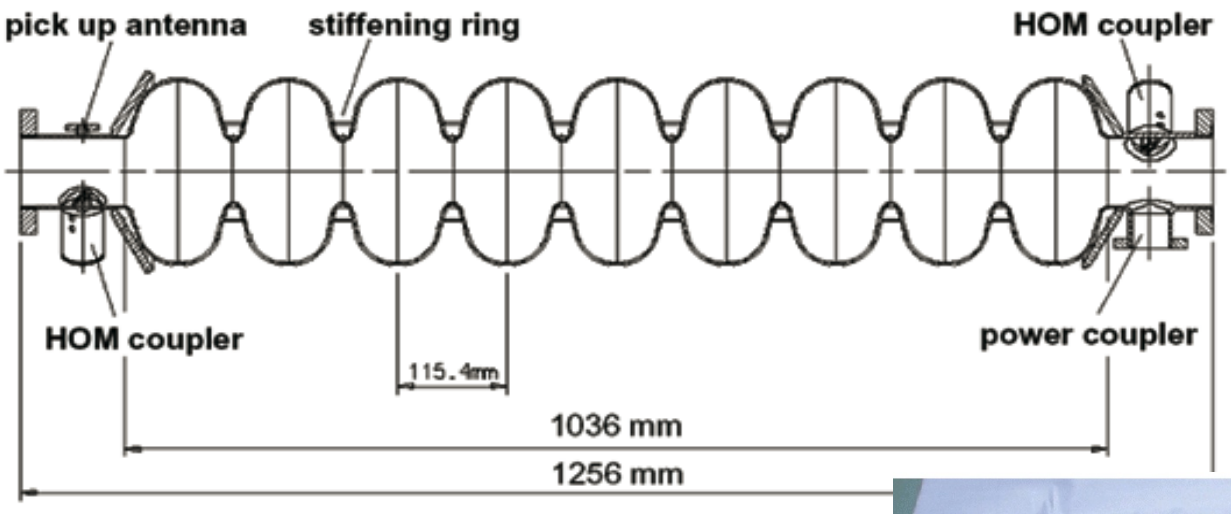
Electron-Positron Collider

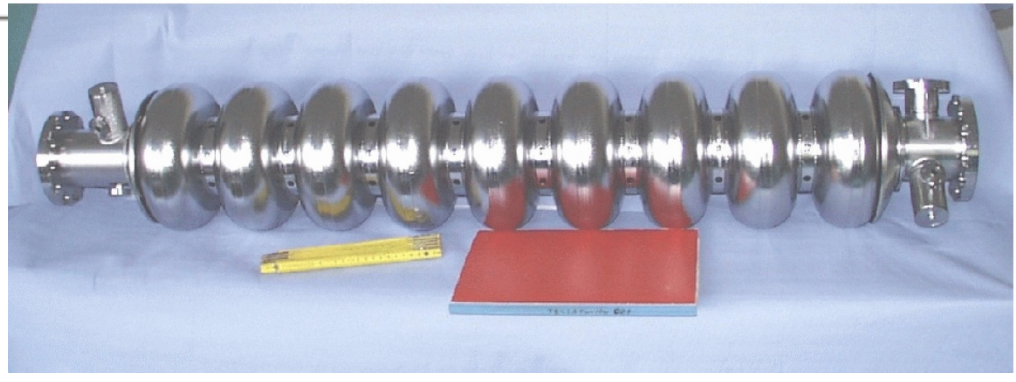
- Ring collider is limited due to synchtrotron radiation (→ later slides)
  - LEP ended at E_{cm}=209GeV
- Beyond the radiation limit, the only possibility is linear collider
- First key issues of linear collider are
  - Acceleration gradient
  - Luminosity
    because of single-pass



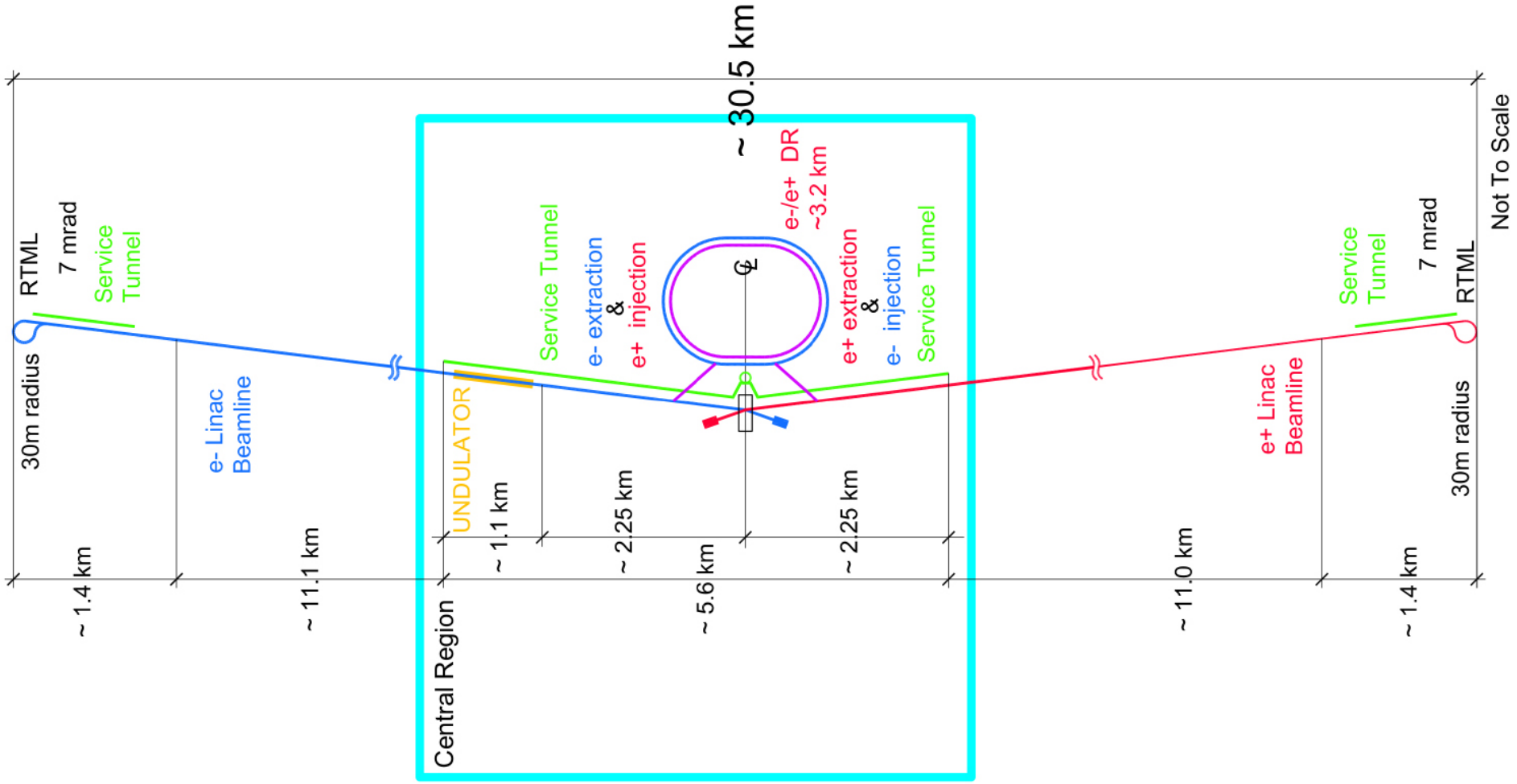
ILC: International Linear Collider

- Key technology: superconducting RF cavities
- Average accelerating gradient 31.5 MV/m



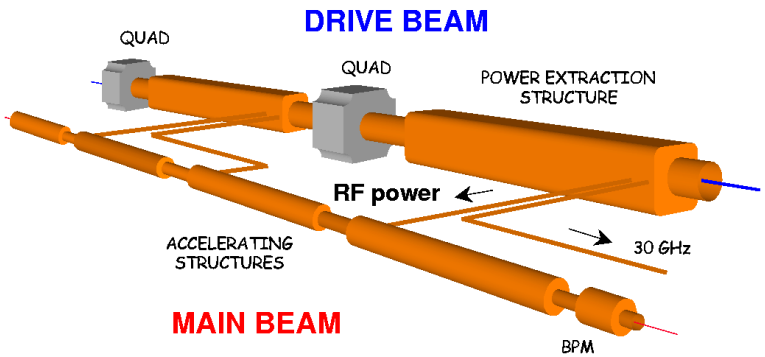


ILC Layout

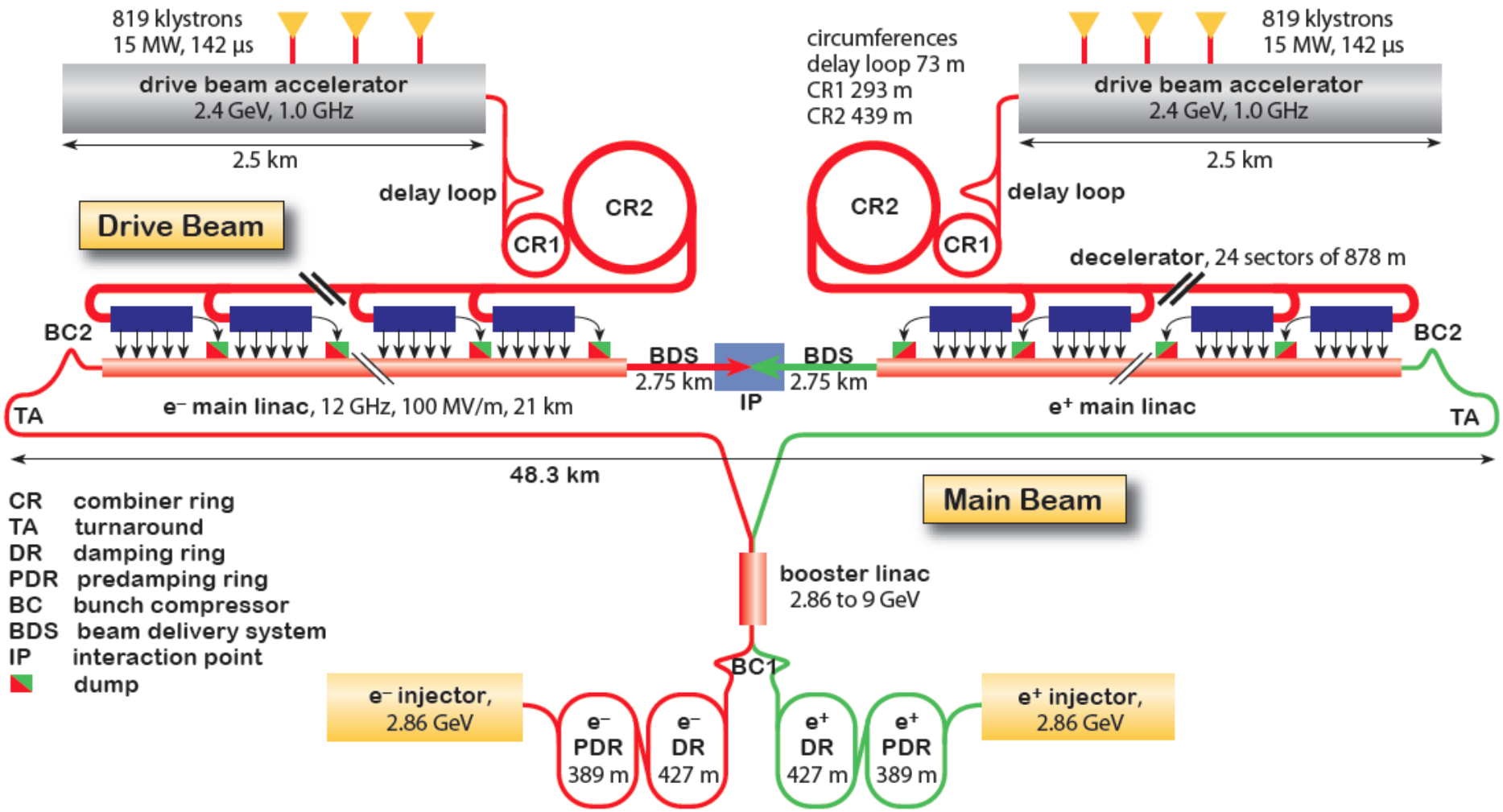

~ 30.5 km
RTML
7 mrad
Service Tunnel
30m radius
e- Linac Beamline
~ 1.4 km
~ 11.1 km
Central Region
UNDULATOR
~ 1.1 km
Service Tunnel
e- extraction & e+ injection
~ 2.25 km
~ 5.6 km
e-/e+ DR ~3.2 km
e+ extraction & e- injection
Service Tunnel
~ 2.25 km
~ 11.0 km
e+ Linac Beamline
Service Tunnel
7 mrad
RTML
30m radius
~ 1.4 km
Not To Scale

CLIC: Compact Linear Collider

- Two-beam scheme
  - Accelerate long train of electron beam to GeV
  - lead it to decelerating structure (PET: Power Extraction Structure)
  - transfer the generated microwave to linac (normal conducting) side by side with PET
  - Huge klystron
  - First proposed at CERN in 1987(?)
  - New scheme proposed by R. Ruth
    - Manipulation of long bunch train
    - Frequency determined by drive bunch interval and PET

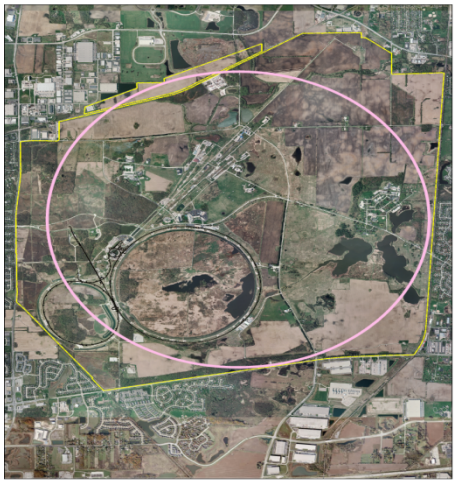


CLIC (CERN Linear Collider)

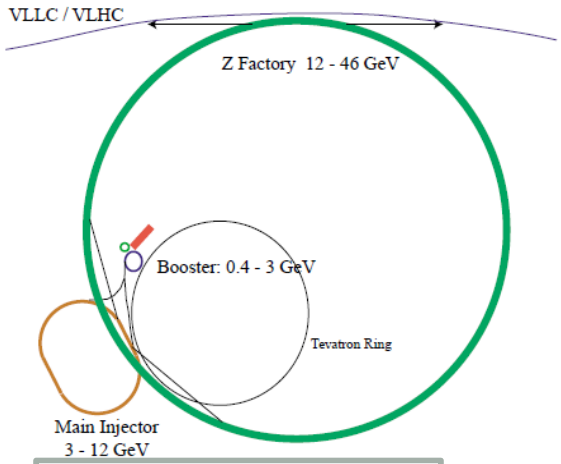


Revival of e+e- Ring Colliders ?

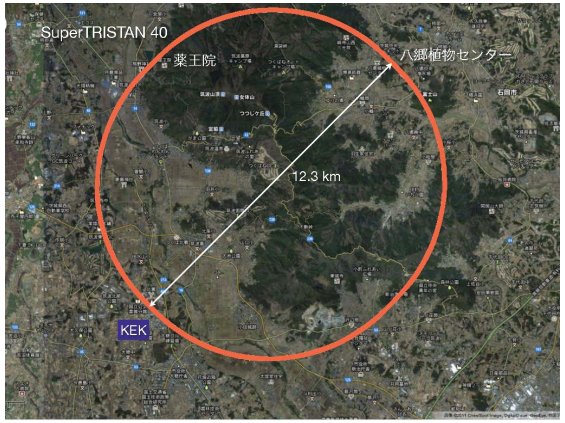
- To create Higgs by e+e- → ZH requires E_{CM}~240GeV
- This is not too high compared with the final energy 209GeV at LEP



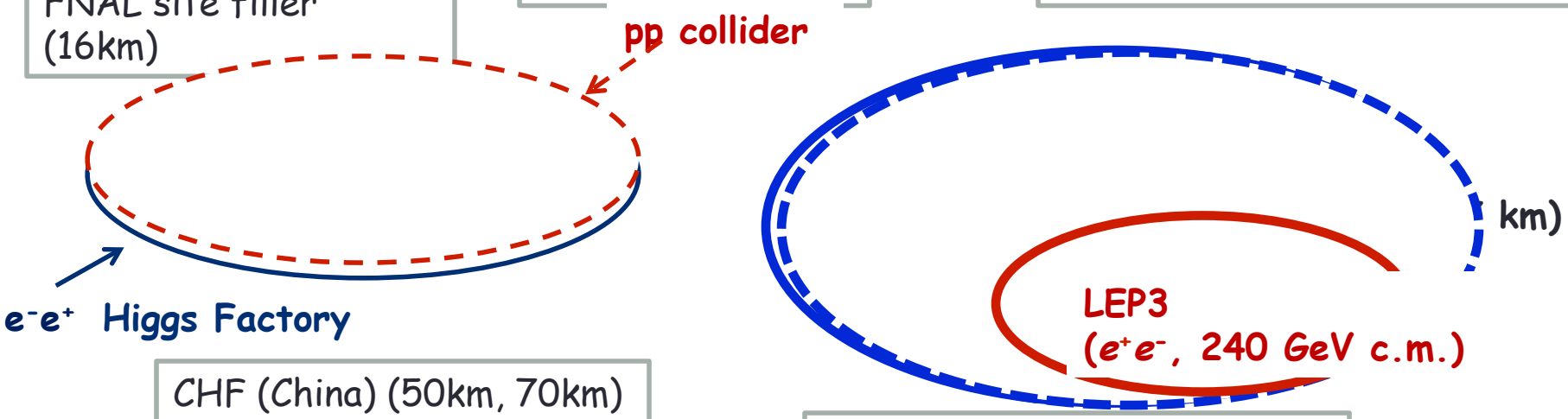
FNAL site filler (16km)



VLCC (233km)



SuperTRISTAN (40km, 60km)

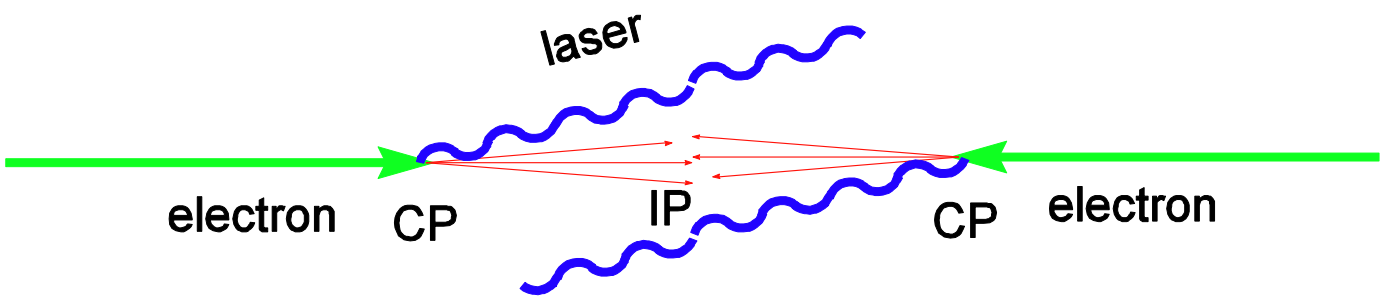


CHF (China) (50km, 70km)

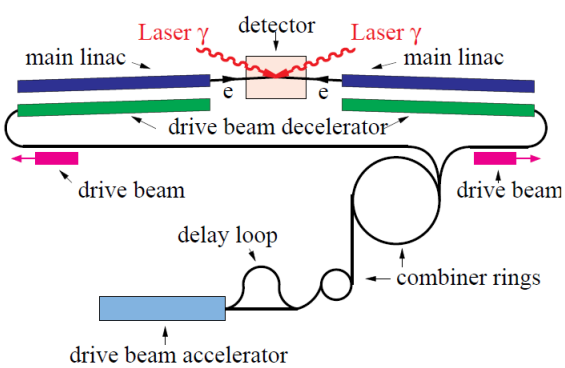
LEP3 (27km), TLEP (80km)

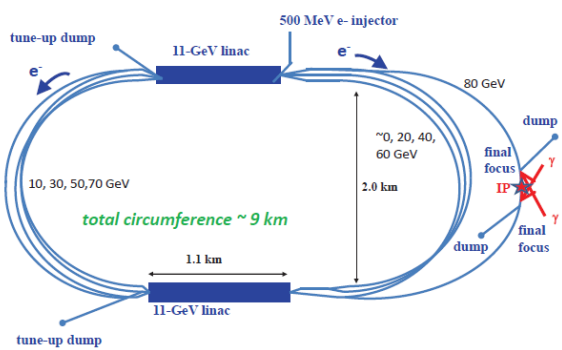
Gamma-Gamma Collider

- electron-electron collider
- irradiate lasers just before ee collision
- create high energy photons, which made to collide
- no need of positrons



CP : conversion point
IP : interaction point



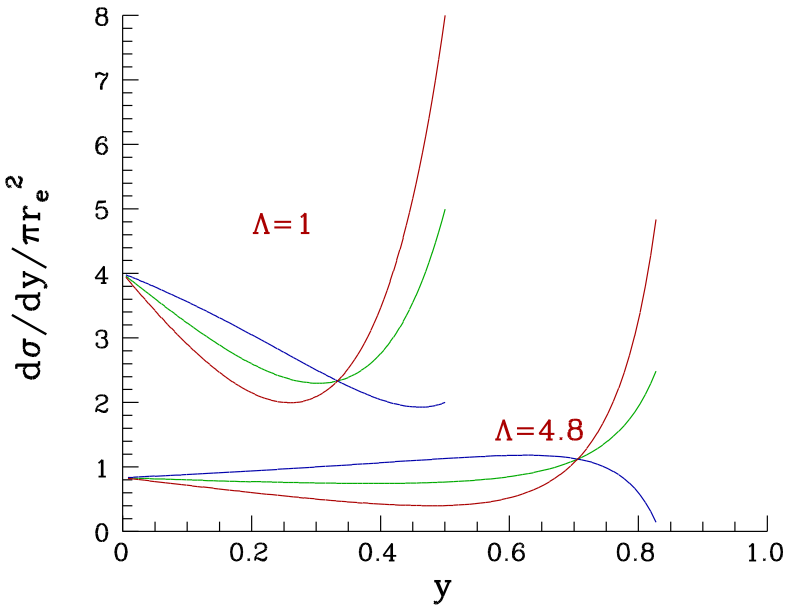


Kinetics of gamma conversion

- maximum photon energy

$$\omega = \frac{x}{1+x+\xi^2}E_e, \qquad x \equiv \frac{4E_e\omega_L}{m^2}$$

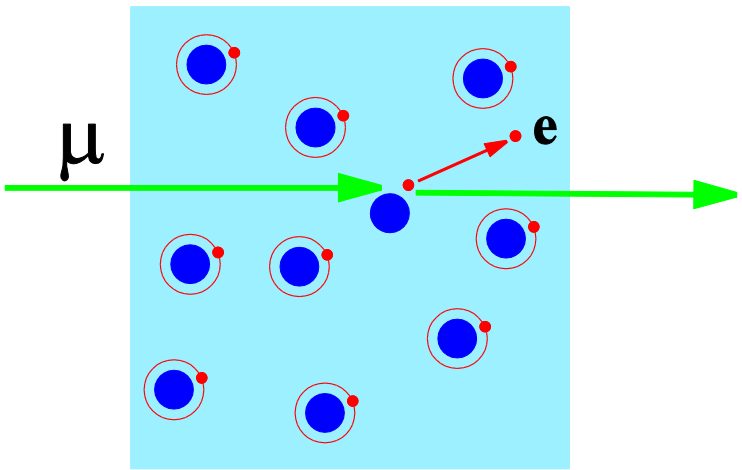
- electron polarization (longitudinal) is essential to create sharp photon energy spectrum



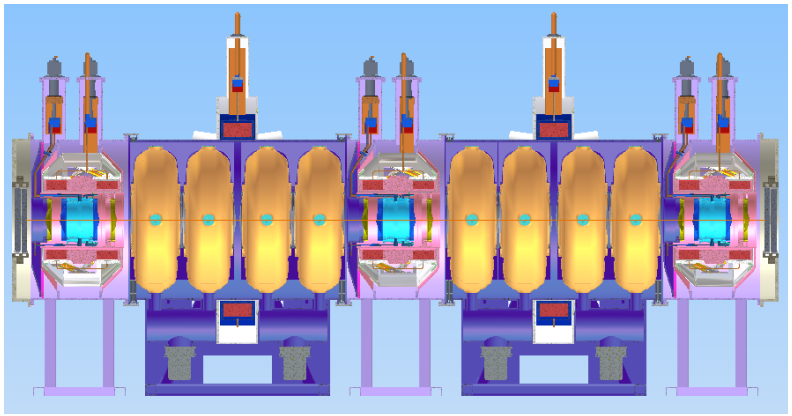
- required laser flush energy to convert most of the electrons is a few (5-10) Joules (weakly depends on electron bunch length)

Muon Collider

- Properties of muons are quite similar to electron/positron
  - What can be done in e+e- can also be done in $\mu^+\mu^-$
- but muon is 200x heavier → can be accelerated to high energies in circular accelerator
- $\mu^+\mu^-$ collider is much cleaner than e+e- (beamstrahlung negligible)
- except the problem of background from muon decay
- But muons do not exist naturally
- need cooling like antiproton
- "Ionization cooling" invented by Skrinsky-Parkhomchuk 1981, Neuffer 1983

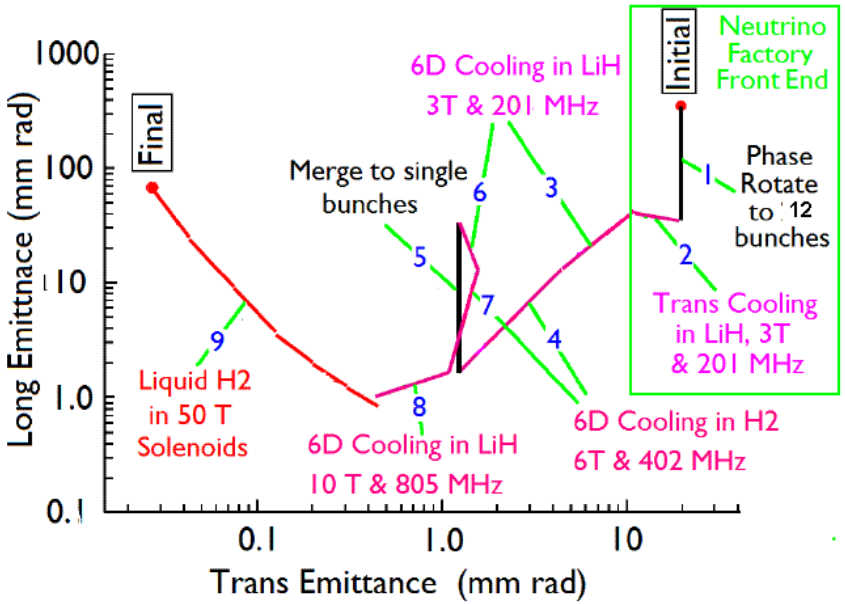


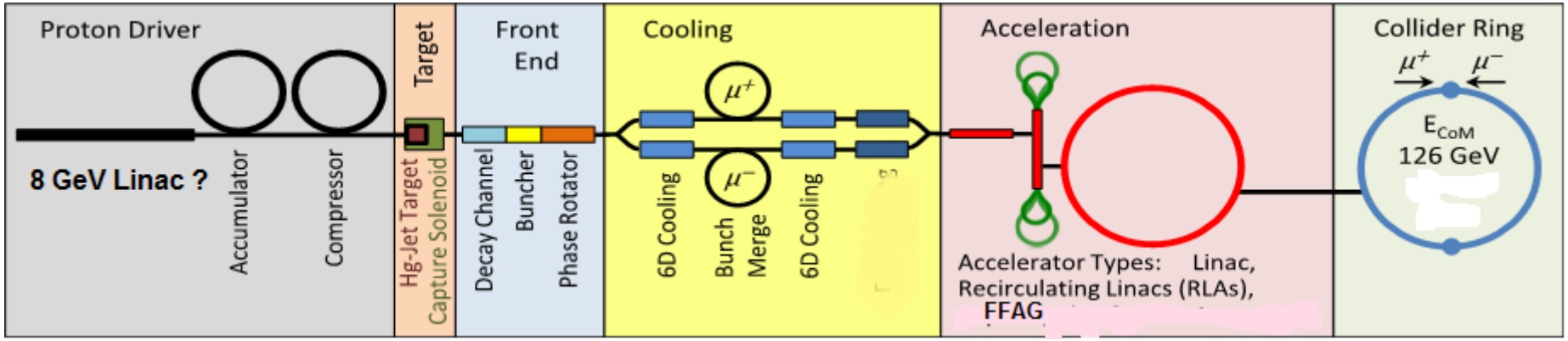
Ionization cooling test at MICE



Create and Cool Muon Beam

- Can be created by hadron collision
- Muons decay within 2μs in the rest frame
  - must be accelerated quickly
- Long way to collider





Plasma Accelerator

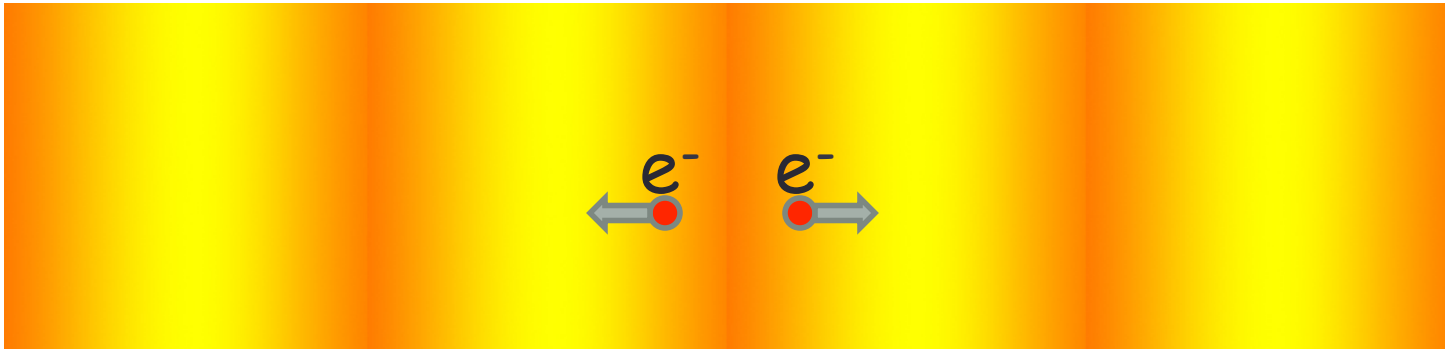
- Linac in the past has been driven by microwave technology
- Plane wave in vacuum cannot accelerate beams: needs material to make boundary condition
- Breakdown at high gradient
  - binding energy of matter: eV/angstrom = 10GeV/m
- Need not worry about breakdown with plasma
  - can reach > 10GeV/m

Plasma Wave

- Plasma is a mixture of free electrons and nucleus (ions), normally neutral
- By perturbation, electrons are easily moved while nuclei are almost sitting, density modulation created.
- The restoring force generates plasma wave
- Charged particles on the density slope are accelerated, like surfing.
- Plasma oscillation frequency and wavelength are given by

$$\omega_p = \sqrt{\frac{e^2}{\epsilon_0 m_e} n_0}, \qquad \lambda_p = \frac{2\pi c}{\omega_p} = \frac{3.3 \times 10^4}{\sqrt{n_e[\mathrm{cm}^{-3}]}} \quad [\mathrm{m}]$$

$$n_e = \text{plasma density}$$



How to Generate Plasma Wave

- PWFA (Plasma Wakefield Accelerator)
  - Use particle (normally electron) beam of short bunch
- LWFA (Laser Wakefield Accelerator)
  - Use ultra-short laser beam
- In both cases the driving beam
  - determines the phase velocity of plasma wave, which must be close to the velocity of light
  - must be shorter than the plasma wavelength required
  - can also ionize neutral gas to create plasma

LWFA

- Laser intensity characterized by the parameter a_0
  - $a_0 < 1$: linear regime
  - $a_0 > 1$: blow-out regime

$$a_0 \approx 8.5 \times 10^{-10} \lambda_L[\mu\mathrm{m}] I^{1/2}[\mathrm{W/cm^2}]$$

- Accelerating field

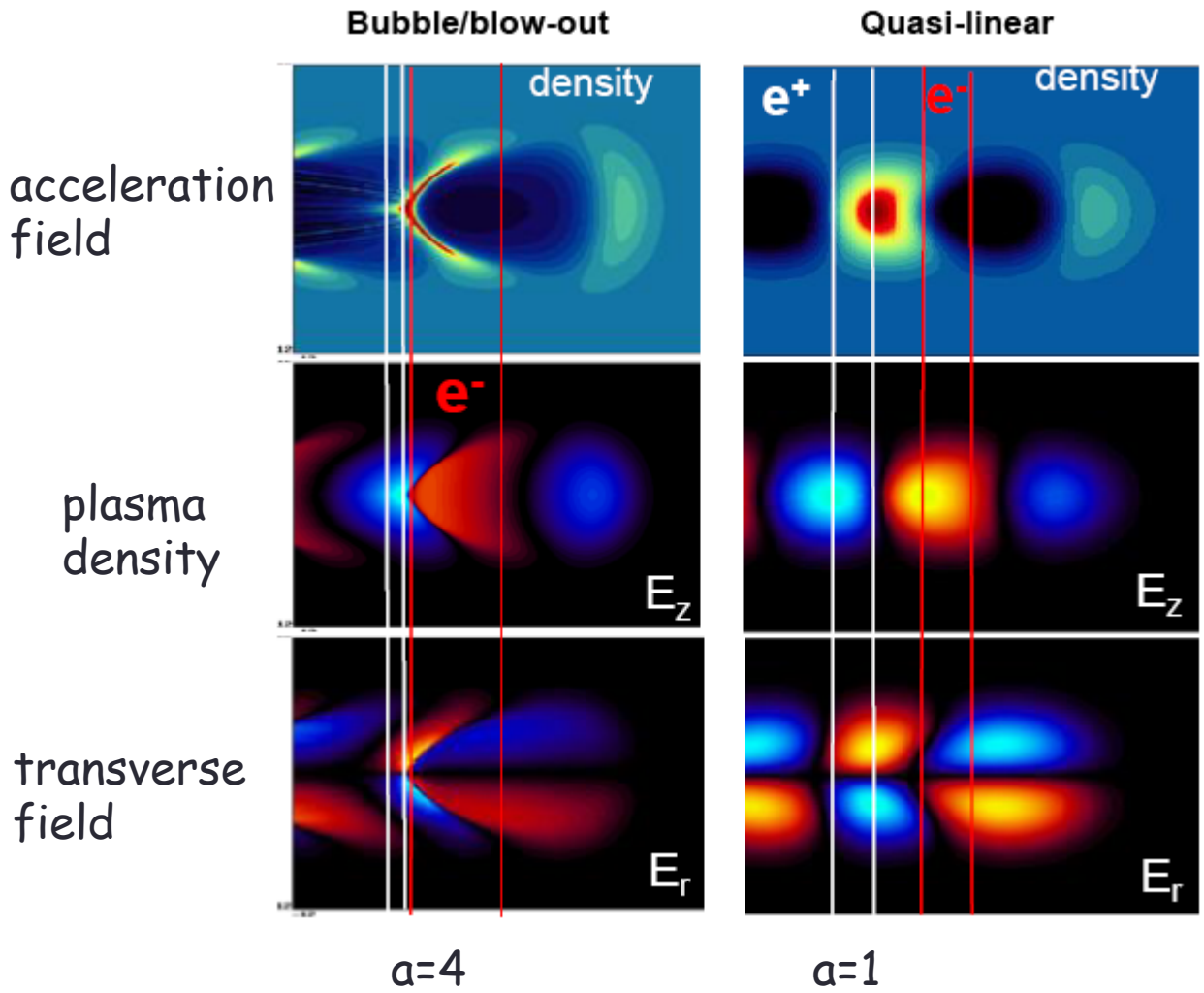
$$E = E_0 \frac{a_0^2/2}{\sqrt{1 + a_0^2/2}}$$

$$E_0 = c m_e \omega_p / e = 96 n_0^{1/2}[\mathrm{cm^{-3}}]$$

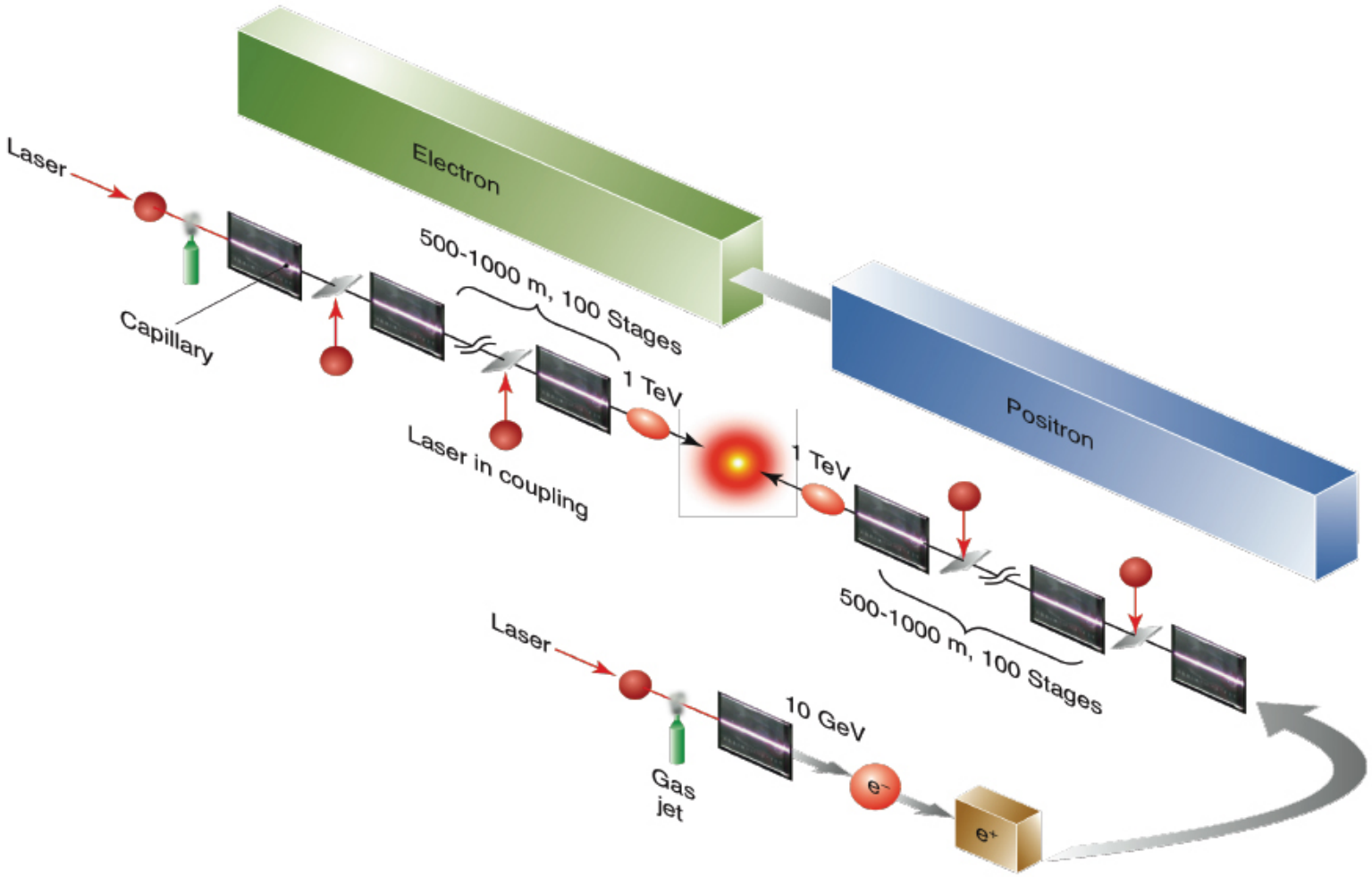
Blowout and Linear Regime

The gradient can be higher in the blowout regime but

- difficult to accelerate positron
- very narrow region of acceleration and focusing



Concept of LWFA Collider. VFF (very far future)



Example Laser Parameters of 1/10TeV Collider

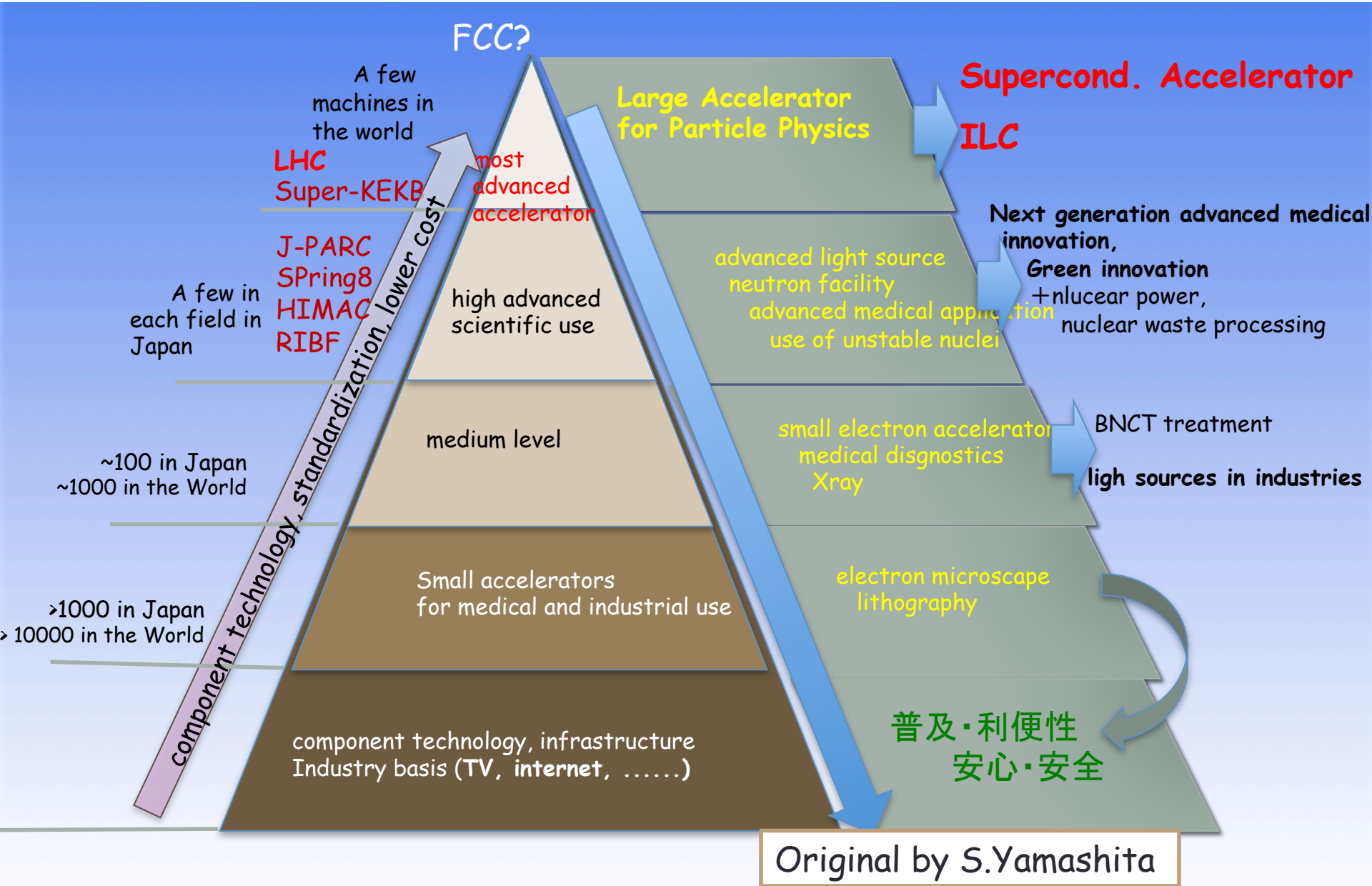
Case: CoM Energy (Plasma density)	1 TeV (10^{17} cm^{-3})	1 TeV (2×10^{15} cm^{-3})	10 TeV (10^{17} cm^{-3})	10 TeV (2×10^{15} cm^{-3})
Wavelength (μm)	1	1	1	1
Pulse energy/stage (kJ)	0.032	11	0.032	11
Pulse length (ps)	0.056	0.4	0.056	0.4
Repetition rate (kHz)	15	0.3	15	0.3
Peak power (PW)	0.24	12	0.24	12
Average laser power/stage (MW)	0.48	3.4	0.48	3.4
Energy gain/stage (GeV)	10	500	10	500
Stage length [LPA + in-coupling] (m)	2	500	2	500
Number of stages (one linac)	50	1	500	10
Total laser power (MW)	48	3.4	480	34
Total wall power (MW)	160	23	960	138
Laser to beam efficiency (%) [laser to wake 50% + wake to beam 40%]	20	20	20	20
Wall plug to laser efficiency (%)	30	30	50	50
Laser spot rms radius (μm)	69	490	69	490
Laser intensity (W/cm^2)	3×10^{18}	3×10^{18}	3×10^{18}	3×10^{18}
Laser strength parameter a_0	1.5	1.5	1.5	1.5
Plasma density (cm^{-3}), with tapering	10^{17}	2×10^{15}	10^{17}	2×10^{15}
Plasma wavelength (mm)	0.1	0.75	0.1	0.75

From ICFA Beamdynamics News Letter 56

What's Needed for Plasma Collider

- High rep rate, high power laser
- Beam quality (fundamental limitations?)
  - Small energy spread << 1%
  - emittance preservation
- High power efficiency from wall-plug to beam
  - Wall-plug → laser
  - Laser → plasma wave
  - plasma wave → beam
- Staging
  - laser phase
  - beam optics matching
- Very high component reliability
- Low cost per GeV
- Colliders need all these, but other applications need only some of these
- Application of plasmas accelerators would start long before these requirements are established

Piramid of Accelerators



Original by S.Yamashita

- END

- THANKS to the school organization for this possibility !

- Thanks also to all the slides 'borrower'…K.Yokoya, N Pichoff, F.Sannibale, M.Biagini….

SR Properties

- INTEGRATING in $d\omega$ => Angular distribution

$$dP/d\Omega = 7\, q^2 / 96\, \pi\, c\, \epsilon_0\ \gamma^2\, \omega_c / (1+X^2)^{5/2}\ \ (1+5X^2/7(1+X^2))$$

- And INTEGRATING on the full angular range => Energy flux

$$I(\omega) = 2q^2\, \gamma / 9\epsilon_0\, c\, S(\omega/\omega_c),\ \ with\ S(x) = 9\sqrt{3}/8\pi\, x \int_x^\infty K_{5/3}(j)dj \qquad \int_0^\infty S(x)dx = 1$$

- The energy flux gives the instantaneous radiated power:

$$P_\gamma = 1/2\pi\rho \int_0^{\exists\infty} I(\omega)d\omega = 4q^2\, \gamma\omega_c / 36\, \pi\, \rho\, \epsilon_0\ \ or\ in\ a\ more\ convenient\ form:$$

$$P_\gamma = c\, C_\gamma / 2\pi\, E^4 / \rho^2\ \ \sim E^2\, B^2\ \ \ with\ C_\gamma = 8.85\ 10^{-5}\ [m/(GeV)^3]$$

The critical photon energy is $\hbar\omega_c = 0.665E^2[GeV]B[T]$

The total energy radiated in one revolution is : $U_0 = E^4\, C_\gamma / 2\pi \oint ds/\rho^2$

and for an isomagnetic ring -> $U_0 = E^4\, C_\gamma / \rho$, average power -> $\langle P_\gamma \rangle = U_0 / T_0 = c\, C_\gamma / 2\pi\, E^4 / R\rho$ ($T_0 = \beta c / 2\pi R$ = revolution period)

Twiss vector transport

The beam can also be represented by a vector made of the Twiss parameters :

$$\begin{pmatrix} \beta_t \\ \alpha_t \\ \gamma_t \end{pmatrix}$$

With a transfer matrix :

$$M(s/s_0) = \begin{pmatrix} C & S \\ C' & S' \end{pmatrix}$$

The Twiss vector is transported with a ***transport matrix*** :

$$\begin{pmatrix} \beta_t(s) \\ \alpha_t(s) \\ \gamma_t(s) \end{pmatrix} = \begin{pmatrix} C^2 & -2SC & S^2 \\ -CC' & S'C + SC' & -SS' \\ C'^2 & -2S'C' & S'^2 \end{pmatrix} \cdot \begin{pmatrix} \beta_t(s_0) \\ \alpha_t(s_0) \\ \gamma_t(s_0) \end{pmatrix}$$

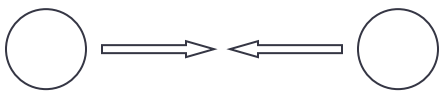
Collider (B.Touscheck)

- What matters in physics is the Center-of-Mass energy

Fixed target

$$E_{CM} \approx \sqrt{2Em}$$

Collider



$$E_{CM} \approx 2E$$

- Energy of each beam can be lower in colliding scheme for given E_{CM}
- Colliding scheme much better in relativistic regime
  - e.g., for electrons, collision of 1GeV electrons is equivalent to 1TeV electron on sitting electron

Beamstrahlung

- Synchrotron radiation during collision due to the field by the on-coming beam
- Causes
  - spread in the collision energy
  - background to the experiment
- The critical energy is characterized by the upsilon parameter

$$\Upsilon \equiv \frac{2}{3}\frac{\hbar\omega_c}{E} = \frac{\lambda_e\gamma^2}{\rho} = \gamma\frac{2B}{B_c} = \frac{e}{m^3}\sqrt{\left|(F_{\mu\nu}p^\nu)^2\right|}$$

$$B_c = m^2/e \approx 4.4\text{GTeslas}$$

Factor 2 in front of B comes from the sum of electric and magnetic fields

- Expressed by the beam parameters

$$\Upsilon_{average} = \frac{5}{6}\frac{N r_e^2 \gamma}{\alpha\sigma_z(\sigma_x + \sigma_y)}$$

- Order of 0.1 in 500GeV collider

Energy loss and number of photons by beamstrahlung

- Average number of photons per electron

$$n_\gamma \approx 1.08 \frac{2Nr_e\alpha}{\sigma_x+\sigma_y} U_0(\Upsilon),$$

$$U_0(\Upsilon) \approx \frac{1}{\sqrt{1+\Upsilon^{2/3}}}$$

- Average energy loss

$$\delta_E = \left\langle -\frac{\Delta E}{E} \right\rangle \approx 0.209 \frac{N^2 r_e^3 \gamma}{\sigma_z} \left(\frac{2}{\sigma_x+\sigma_y} \right)^2 U_1(\Upsilon)$$

$$U_1(\Upsilon) \approx \frac{1}{[1+(1.5\Upsilon)^{2/3}]^2}$$

- Average photon energy

$$\left\langle \frac{\omega}{E} \right\rangle = \begin{cases} 0.462\Upsilon & (\Upsilon \to 0) \\ 16/23 = 0.254 & (\Upsilon \to \infty) \end{cases}$$

2 Aspects of Synchrotron Radiation Loss

- Energy loss by individual particles must be compensated for

$$U = 0.088 \frac{E^4[\mathrm{GeV}]}{\rho[\mathrm{m}]} \quad [\mathrm{MeV}]$$

- This (almost) determines RF voltage per turn
  - ~7GeV in LEP tunnel
  - Still possible owing to the improvement of superconducting cavity technology
- But, to get required electric power, you must multiply the beam current
  - Real limitation comes from the wall-plug power
  - Reduce the beam current
  - Small beam size for high luminosity

Luminosity Scaling of e^+e^- Ring Colliders

V. Telnov, arXiv:1203.6563v, 29 March 2012

- For given Upsilon, the momentum band width must be

$$\eta \equiv [\Delta p/p]_{max} \gtrsim 15\Upsilon$$

- Then, the luminosity at beamstrahlung limit and tune-shift limit is given by

$$\mathcal{L} \propto \frac{\rho P_{SR}}{E^{13/3}} \left(\frac{\xi_y \eta^2}{\varepsilon_{g,y}} \right)^{1/3}$$

P_{SR} : syn.rad.power

ρ : bending radius

ξ_y : tune-shift

$\varepsilon_{g,y}$: geometric emit.

Luminosity vs. Energy

- Key parameters
  - momentum band width
  - vertical emittance
  - beam-beam tune-shift
- Ring Collider can be a choice ?
  if e+e- at >~350GeV is not needed at all

example with

- η=2%
- ξ_y=0.15
- ε_{gy}=0.1nm