QCD and jets
Lecture 2: Applications

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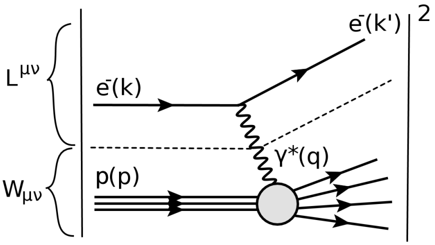
DIS in detail

Deep inelastic scattering (DIS) process



$$Q^2 = -q^2$$
$$x = \frac{Q^2}{2\,p\cdot q} \quad \text{(Bjorken } x\text{)}$$
$$y = \frac{p\cdot q}{k\cdot p}$$

The cross section can be split into leptonic and hadronic piece

$$\frac{d\sigma}{dxdQ^2} = \frac{2\pi\alpha_{\text{em}}^2}{x^2 s^2 Q^2} L^{\mu\nu} W_{\mu\nu}\,.$$

The leptonic part is completely determined from QED

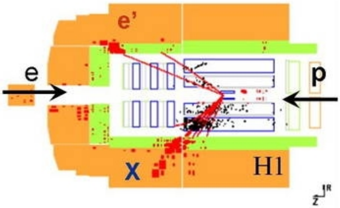
$$L^{\mu\nu} = 2\left(k^\mu k'^\nu + k^\nu k'^\mu - g^{\mu\nu}\, k\cdot k'\right).$$

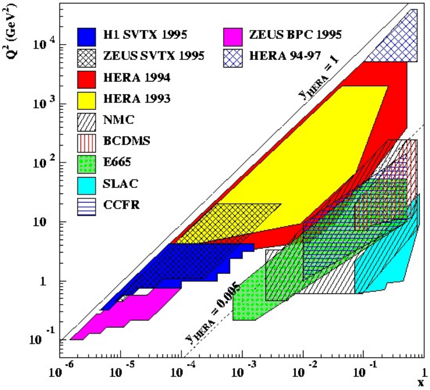
DIS experiments

- A lot of insight into DIS comes from HERA

$$e^+/e^-(30\,\text{GeV}) \quad \longrightarrow \qquad\qquad \longleftarrow \quad p^+(920\,\text{GeV})$$

- But also the fixed-tagged experiments





DIS in detail

Hadronic tensor describes the $\gamma^* p$ interaction

$$W_{\mu\nu} = \frac{1}{4\pi} \sum_{n \in \text{final states}} \langle p|J_\nu(0)|n\rangle\langle n|J_\mu(0)|p\rangle \,(2\pi)^4\delta^4(q + p - p_n)\,,$$

where J_μ is the electromagnetic current.

Using conservation of J_μ and the parity conservation, one can derive the most general form of the hadronic tensor

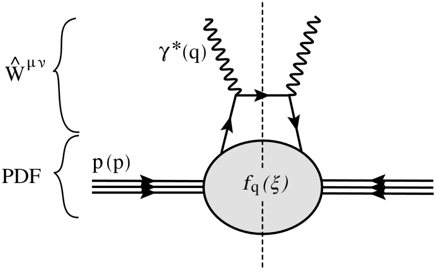
$$W^{\mu\nu} = \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) F_1(x, Q^2) + \left(p^\mu + \tfrac{1}{2x}q^\mu\right)\left(p^\nu + \tfrac{1}{2x}q^\nu\right)\tfrac{1}{p\cdot q} F_2(x, Q^2)\,.$$

That leads to the following expression for the DIS cross section

$$\frac{d^2\sigma}{dxdQ} = \frac{4\pi\alpha_{\text{em}}^2}{Q^4}\left\{\left[1 + (1-y)^2\right] F_1(x, Q^2) + \tfrac{1-y}{x}\left[F_2(x, Q^2) - 2xF_1(x, Q^2)\right]\right\}\,,$$

where F_1 and F_2 are the structure functions.

Parton distribution functions

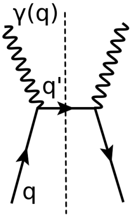


$$W^{\mu\nu}(p,q) = \sum_{i=q,\bar{q},g} \int_0^1 \frac{d\xi}{\xi} f_i(\xi)\, \hat{W}^{\mu\nu}(\xi p, q)$$

- Partonic tensor $\hat{W}^{\mu\nu}(\xi p, q)$ can be computed perturbatively.
- Parton distribution functions $f_i(\xi)$ are genuinely non-perturbative objects which contain information about long-range dynamics.

DIS at leading order

$$F_2(x)\Big|_{\rm LO} = x\sum_{i=q,\bar q}\int_x^1 d\xi\, f_i(\xi)\; g_{\mu\nu}\hat W^{\mu\nu}_{\rm LO}(\xi,x)$$



and

$$\hat W^{\mu\nu}_{\rm LO}(\xi,x) = \frac{1}{4\pi}\int d\Phi_1 \overline{\sum}|\mathcal{M}_{\gamma^* q\to q'}|^2 = g^{\mu\nu}\frac{e_q^2}{4}\,\delta(x-\xi)$$

where $\delta(x-\xi)$ follows from mass-shell constraint for the outgoing quark

$$p_{q'}^2 = (p_q+q)^2 = q^2 + 2p_q\cdot q = -2p\cdot q(x-\xi) = 0\,.$$

Hence, at LO

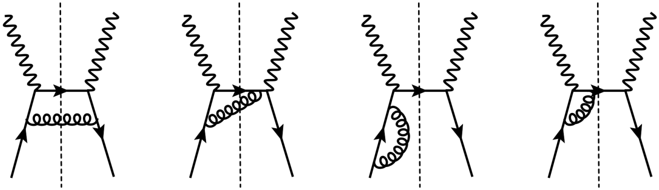
$$F_2(x) = 2\,x\,F_1(x) = x\sum_{i=q,\bar q} e_i^2 f_i(x)$$

- Callan-Gross relation holds exactly for LO DIS.
- Momentum fraction carried by the quark equals exactly to Bjorken x.

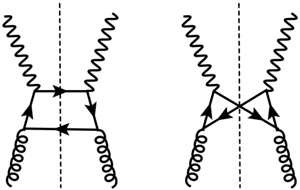
DIS at next-to-leading order

Two types of contributions:

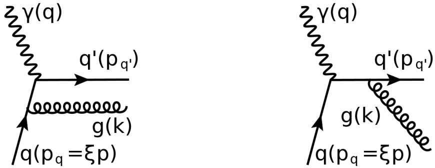
$$\gamma^* q \to qg$$



$$\gamma^* g \to q\bar{q}$$



DIS at next-to-leading order ($\gamma^* q \to qg$)



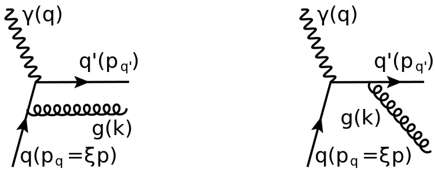
$$\sum |\mathcal{M}_{\gamma^* q \to q' g}|^2 \propto \frac{k \cdot p_{q'}}{k \cdot p_q} + \frac{k \cdot p_q}{k \cdot p_{q'}} + \frac{Q^2 (p_q \cdot p_{q'})}{(k \cdot p_q)(k \cdot p_{q'})}$$

Structure of singularities familiar from e^+e^-

$$|\mathcal{M}_{\gamma^* g \to q' g}|^2 \to \infty \quad \text{if} \quad \begin{cases} (k \cdot p_q)(k \cdot p_{q'}) \to 0 & \text{infrared (soft) limit} \\ k \cdot p_q \to 0 & \text{collinear limit} \\ k \cdot p_{q'} \to 0 & \text{collinear limit} \end{cases}$$

$$\hat{s} = (\xi p + q)^2 = \xi\, 2p \cdot q - Q^2 = Q^2 \left(\frac{\xi}{x} - 1 \right) \equiv Q^2 \left(\frac{1}{z} - 1 \right)$$

DIS at next-to-leading order ($\gamma^* q \to qg$)



Hence, we introduce a variable

$$z \equiv \frac{x}{\xi} = \frac{Q^2}{Q^2 + \hat{s}^2}$$

- $z \in [0, 1]$
- $z \to 1 \Leftrightarrow \xi \to x$
  - In this limit, the fraction of proton momentum is equal to the Bjorken variable, x, just like at the leading order.
  - That implies that as $z \to 1$, there is less and less energy for emitting a gluon. This is the soft limit.

DIS at next-to-leading order ($\gamma^* q \to qg$)

After some work we get the partonic structure function $\hat{F}_2$ in the form

$$\begin{aligned}
\hat{F}_2(x, Q^2; \kappa^2) \quad &\propto \quad \int d\Phi_1 |\mathcal{M}_{\gamma^* g \to q' g}|^2 \\
&\propto \quad \int_{\kappa^2}^{Q^2 \frac{(1-z)}{4z}} \frac{dk_T^2}{k_T^2} \left(\frac{1+z^2}{1-z} \right) + R(z) + \mathcal{O}\left(\kappa^2/Q^2\right)
\end{aligned}$$

where

- k_T is the transverse momentum of emitted gluon
- κ^2 is a cutoff introduced to regularize collinear divergence of $|\mathcal{M}|^2$
- soft singularity, $z \to 1$, is still present in the first term
- $R(z)$ is finite in the collinear $\kappa^2 \to 0$

DIS at next-to-leading order

Adding virtual correction removes the soft singularity

$$\hat{F}_2(x, Q^2; \kappa^2) \quad \propto \quad \int_{\kappa^2}^{Q^2 \frac{(1-z)}{4z}} \frac{dk_T^2}{k_T^2} \left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z) \right] + R(z)$$

$$\propto \quad P_{qq}^{(0)}(z) \ln \frac{Q^2}{\kappa^2} + D_q(z)\,,$$

where we introduced the plus prescription:

$$F(z)_+ = F(z) - \delta(1-z) \int_0^1 dy\, F(y)\,,$$

which has the following property

$$\int_0^1 dz \frac{f(z)}{(1-z)_+} \equiv \int_0^1 dz \frac{f(z) - f(1)}{1-z}\,,$$

and the leading order splitting function

$$P_{qq}^{(0)}(z) = C_F \left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z) \right]\,,$$

together with the coefficient function $D_q(z)$.

DIS at next-to-leading order

Let us now convolute the partonic $\hat{F}_2$ with PDFs to get the full hadronic structure function

$$\frac{F_2(x,Q^2;\kappa^2)}{xe_q^2} = \int_x^1 \frac{dz}{z} f_q\left(\frac{x}{z}\right)\left\{\delta(1-z) + \frac{\alpha_s}{2\pi}\left[P_{qq}^{(0)}(z)\ln\frac{Q^2}{\kappa^2} + D_q(z)\right]\right\},$$

where we have also added the Born contribution $\sim \delta(1-z)$.

- The result still shows the logarithmic divergence coming from the collinear singularity.
- The collinear region $\kappa^2 \to 0$ is a domain of non-perturbative physics and should be contained in the PDFs.

We introduce a new scale μ_F, called factorization scale, which separates the domains of short-distance (perturbative) and long-distance (non-perturbative) physics.
That allows us to write

$$\ln\frac{Q^2}{\kappa^2} = \ln\frac{\mu_F^2}{\kappa^2} + \ln\frac{Q^2}{\mu_F^2}$$

and push the singular $\ln\frac{\mu_F^2}{\kappa^2}$ into PDFs.

Collinear factorization

With this we obtain

$$\frac{F_2(x,Q^2)}{xe_q^2} = f_q^F(x,\mu_F^2) + \int_x^1 \frac{dz}{z} f_q^F\left(\frac{x}{z},\mu_F^2\right)\frac{\alpha_s}{2\pi}\left[P_{qq}^{(0)}(z)\ln\frac{Q^2}{\mu_F^2} + D_q(z) - D_q^F(z)\right]$$

with the physical parton distribution function

$$f_q^F(x,\mu_F^2) = f_q(x) + \int_x^1 \frac{dz}{z} f_q\left(\frac{x}{z}\right)\frac{\alpha_s}{2\pi}\left[P_{qq}^{(0)}(z)\ln\frac{\mu_F^2}{\kappa^2} + D_q^F(z)\right]$$

- The "bare" PDF $f_q(x)$ contains logarithmic divergence in κ^2 that compensates $\ln \mu_F^2/\kappa^2$ above, giving the finite result for $f_q^F(x,\mu_F^2)$.
- There is a freedom to move an arbitrary finite piece $D_q^F(z)$ together with the singular term $\ln \mu_F^2/\kappa^2$ into PDFs. This defines the factorization scheme.
- $F_2(x,Q^2)$ is independent of arbitrary factorization scale μ_F and arbitrary factorization scheme defined by $D_q^F(z)$ up to order $\mathcal{O}(\alpha_s)$.

Collinear factorization

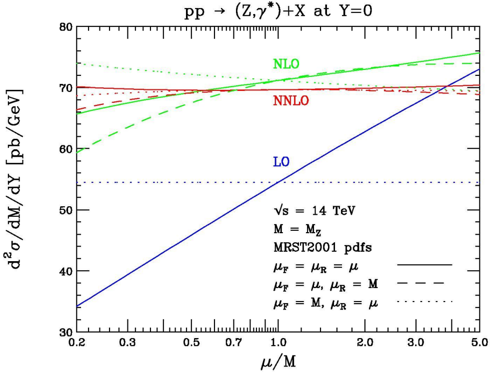
Further comments:

- Two most popular factorization schemes:
  - $\overline{\text{MS}}$ scheme: $D_q^F(z) = 0$ (only singular term is absorbed into PDFs)
  - DIS scheme: $D_q^F(z) = D_q(z)$ (all finite terms are absorbed into PDFs)
- "Physical" PDF $f_q^F(x, \mu_F^2)$ is not an observable. It is always defined within certain factorization scheme.
  - However, when convoluted with a coefficient function in the same scheme F, it results in a scheme-independent quantity.
  - This is true for any process, like lepton pair production or dijet production (details later).
- Even though the choice of μ_F is in principle arbitrary, in practice, taking $\mu_F^2 = Q^2$ removes the $P_{qq}^{(0)}(z) \ln \frac{Q^2}{\mu_F^2}$ piece and results in a simpler expression

$$\frac{F_2(x, Q^2)}{x e_q^2} = f_q^F(x, \mu_F^2) + \int_x^1 \frac{dz}{z} f_q^F\left(\frac{x}{z}, \mu_F^2\right) \frac{\alpha_s}{2\pi} \left[D_q(z) - D_q^F(z)\right]$$

Residual μ_F and μ_R dependence

- Variation of μ_F can be used to study the residual dependence on the factorization scale and that (together with μ_R variation) can be used to estimate the uncertainty of the prediction at a given order.
- Even though this is being commonly done, that procedure is really rough and comes with no warranty! (more in lecture 3)



[Anastasiou, Dixon, Melnikov & Petriello '03]

DIS at next-to-leading order: dimensional regularization

Instead of regularizing by the cut-off κ^2, we can use dimensional regularization. That results in the following form of the structure function

$$\frac{F_2^{\epsilon}(x,Q^2)}{xe_q^2} = \int_x^1 \frac{dz}{z} f_q\left(\frac{x}{z}\right)\frac{\alpha_s}{2\pi}\left\{-P_{qq}^{(0)}(z)\left[\frac{1}{\epsilon} - \gamma_E + \ln(4\pi) - \ln\frac{Q^2}{\mu^2}\right] + D_q^{\overline{\text{MS}}}(z)\right\}$$

Absorbing $\frac{1}{\epsilon} - \gamma_E + \ln(4\pi)$ into PDF defines $\overline{\text{MS}}$ factorization scheme.

When we add also the $\gamma g \to q\bar{q}$ channel, we get the complete answer

$$\frac{F_2(x,Q^2)}{x} = \int_x^1 \frac{dz}{z}\sum_{i=q,\bar{q}} e_q^2 \left\{ f_q^{\overline{\text{MS}}}(x,\mu_F^2)\left[\delta(1-z) + \frac{\alpha_s}{2\pi}\left[P_{qq}^{(0)}(z)\ln\frac{Q^2}{\mu_F^2} + D_q^{\overline{\text{MS}}}(z)\right]\right.\right.$$
$$\left.\left. + f_g^{\overline{\text{MS}}}(x,\mu_F^2)\frac{\alpha_s}{2\pi}\left[P_{qq}^{(0)}(z)\ln\frac{Q^2}{\mu_F^2} + D_q^{\overline{\text{MS}}}(z)\right]\right\}\right.$$

DIS at next-to-leading order: dimensional regularization

Where the quark and the gluon $\overline{\text{MS}}$ PDFs are given by

$$f_q^{\overline{\text{MS}}}(x,\mu_F^2) = \sum_{i=q,g} \int_x^1 \frac{dz}{z} f_i\left(\frac{x}{z},\epsilon\right)\left[\delta(1-z)\delta_{qi} - \frac{\alpha_s}{2\pi} P_{qi}^{(0)}(z)\left(S_\epsilon - \ln\frac{\mu_F^2}{\mu^2}\right)\right]$$

$$f_g^{\overline{\text{MS}}}(x,\mu_F^2) = \sum_{i=q,\bar{q},g} \int_x^1 \frac{dz}{z} f_i\left(\frac{x}{z},\epsilon\right)\left[\delta(1-z)\delta_{gi} - \frac{\alpha_s}{2\pi} P_{gi}^{(0)}(z)\left(S_\epsilon - \ln\frac{\mu_F^2}{\mu^2}\right)\right]$$

where $S_\epsilon = \frac{1}{\epsilon} - \gamma_E + \ln(4\pi)$.

The corresponding coefficient functions read

$$D_q^{\overline{\text{MS}}}(z) = \frac{C_F}{2}\left[\frac{1+z^2}{1-z}\left(\ln\frac{1-z}{z} - \frac{3}{4}\right) + \frac{9+5z}{4}\right]_+$$

$$D_g^{\overline{\text{MS}}}(z) = T_R\left[\left((1-z)^2+z^2\right)\ln\frac{1-z}{z} - 8z^2 + 8z - 1\right]$$

DGLAP equation

Recall the simplified formula for the structure function (with $\mu_F = \mu$)

$$\frac{F_2(x, Q^2)}{x e_q^2} = f_q(x, \mu^2) + \int_x^1 \frac{dz}{z} f_q\left(\frac{x}{z}, \mu^2\right) \frac{\alpha_s}{2\pi} \left[P_{qq}^{(0)}(z) \ln \frac{Q^2}{\mu^2} + \tilde{D}_q(z)\right]$$

- $f_q(x, \mu^2)$ is defined in a particular factorization scheme.
- L.h.s. is μ_F-independent up to $\mathcal{O}(\alpha_s)$: $\partial_{\mu^2} F_2(x, Q^2) = 0 + \mathcal{O}\left(\alpha_s^2\right)$.

Differentiating both sides w.r.t. μ^2 gives:

$$\mu^2 \frac{\partial f_q(x, \mu^2)}{\partial \mu^2} = \int_x^1 \frac{dz}{z} f_q\left(\frac{x}{z}, \mu^2\right) \frac{\alpha_s}{2\pi} P_{qq}^{(0)}(z)$$

This is the DGLAP equation [Dokshitzer-Gribov-Lipatov-Altarelli-Parisi '77].

- $f_q(x, \mu^2)$ is a non-perturbative object and we do not know how to calculate it.
- What we know, however, is how $f_q(x, \mu^2)$ evolves with μ^2 – that is given by the DGLAP equation.

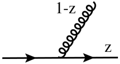
DGLAP equation

The most general form of DGLAP equation is a $(2n_f+1)$ matrix equation

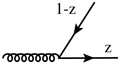
$$t\frac{\partial}{\partial t}\begin{bmatrix} q_i(x,t) \\ g(x,t)\end{bmatrix} = \frac{\alpha_s(t)}{2\pi}\sum_{q_i,\bar{q}_i}\int_x^1 \frac{dz}{z}\begin{bmatrix} P_{q_iq_j}(\frac{x}{z},\alpha_s) & P_{q_ig}(\frac{x}{z},\alpha_s) \\ P_{gq_j}(\frac{x}{z},\alpha_s) & P_{gg}(\frac{x}{z},\alpha_s)\end{bmatrix}\begin{bmatrix} q_i(x,t) \\ g(x,t)\end{bmatrix}$$

where we used the notation $t=\mu^2$.

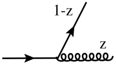
The splitting functions are calculable as a perturbative series in α_s:
$P_{ab}(z) = P_{ab}^{(0)}(z) + \frac{\alpha_s}{2\pi}P_{ab}^{(1)}(z) + \cdots$ and, at the leading order, they read



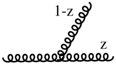
$$P_{qq}^{(0)}(z) = C_F\left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right]$$



$$P_{qg}^{(0)}(z) = T_R\left[z^2+(1-z)^2\right]$$



$$P_{gq}^{(0)}(z) = C_F\left[\frac{1+(1-z)^2}{z}\right]$$



$$P_{gg}^{(0)}(z) = 2C_A\left[\frac{z}{(1-z)_+} + \frac{1-z}{z} + z(1-z)\right] + \delta(1-z)\frac{11C_A - 4n_fT_R}{6}$$

Splitting functions

- Leading order splitting functions satisfy the following sum rules
  - quark (baryon) number conservation

$$\int_0^1 dz\, P_{qq}^{(0)}(z) = 0$$

  - momentum conservation for quarks

$$\int_0^1 dz\, z \left[P_{qq}^{(0)}(z) + P_{gq}^{(0)}(z) \right] = 0$$

  - momentum conservation for gluons

$$\int_0^1 dz\, z \left[2n_f\, P_{qg}^{(0)}(z) + P_{gg}^{(0)}(z) \right] = 0$$

- Leading order splitting functions $P_{ab}^{(0)}(z)$ have the interpretation of probabilities of finding a parton a in a parton b with a fraction z of its longitudinal momentum.
- We also know the splitting functions at $\mathcal{O}(\alpha_s)$ [Curci, Furmanski and Petronzio '80] and at $\mathcal{O}(\alpha_s^2)$ [Moch, Vermaseren and Vogt '04]

PDF fits to DIS data

- In practice, DGLAP equation is solved numerically in (x, Q^2) space.
- The initial condition is chosen at some reference scale Q_0^2 by parametrizing the PDFs in a form of the type:

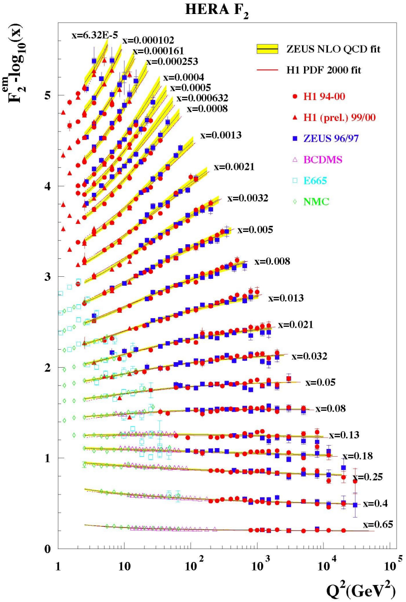
$$q(x, Q_0^2) = A\, x^\alpha (1 + c\sqrt{x} + dx)(1 - x)^b \,.$$

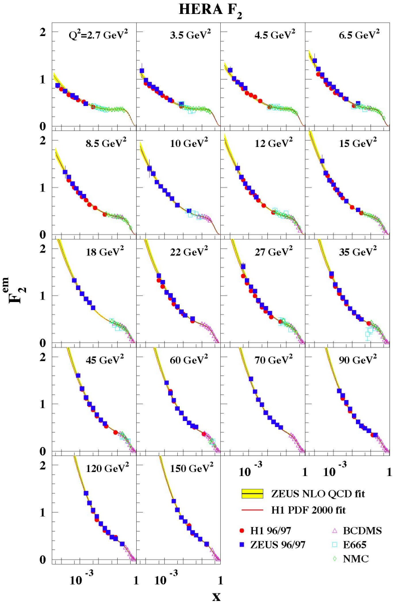
- The parameters are fitted such that the evolved distributions give the values of the structure functions $F_i(x, Q^2)$ that agree with those measured in the DIS process.
- Λ_{QCD}, or equivalently $\alpha_s(m_Z^2)$, is also fitted together with PDFs.
- $\overline{\mathrm{MS}}$ factorization scheme is the standard to use.
- Current fits use up to NNLO DGLAP equation.

There is a number of groups that work on PDF fits. Each of them has slightly different methodologies and hence produces different sets of parton distribution functions.

The most common sets are: MSTW, CTEQ, NNPDF, HERAPDF.

PDF fits to DIS data





PDF fits

Typically we use all available data to constrain the PDFs: fixed target and HERA DIS; W/Z/jets at Tevatron; W/Z/jets at the LHC.

- valence quarks

  $u_v = u - u_{\text{sea}}$

  $d_v = d - d_{\text{sea}}$

- sea quarks

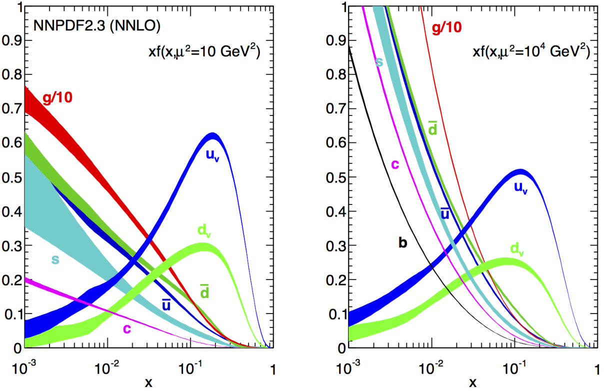
  $u_{\text{sea}} = \bar{u}$

  $d_{\text{sea}} = \bar{d}$

  $s = \bar{s}$

  $c = \bar{c}$

  $b = \bar{b}$

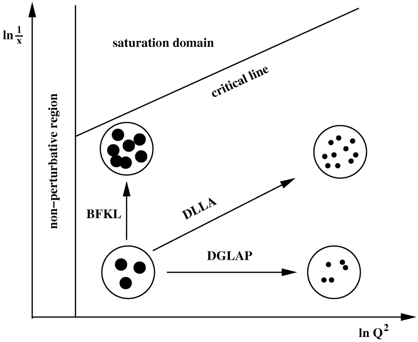


As we go from $\mu^2 = 10\,\text{GeV}^2 \to 10^4\,\text{GeV}^2$:

- u_v and d_v valence quarks decrease at large x ← This is the effect of
- gluon and the sea quarks increase at small x ← DGLAP evolution!

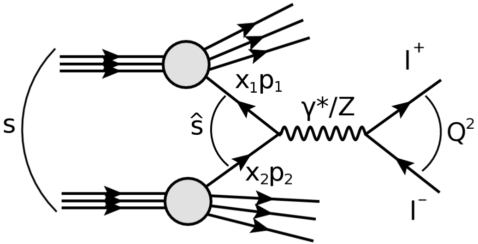
Evolution of parton densities: summary

- For scales $Q^2 \gg \Lambda^2$, we can use pQCD to derive evolution equations for PDFs.
- DGLAP equation allows us to study evolution with Q^2.
- Photon of virtuality Q^2 can only resolve objects with transverse size $\sim 1/Q$.
- Increasing Q^2 can be seen as improving the resolution which leads to seeing more partons.



- On the other hand, decreasing x for fixed Q^2 means increasing energy.
- More energy means more gluon emissions, hence the growth of gluon density at low x. That is what we see in DGLAP fits.
- Evolution in the Bjorken x variable is described by the linear Balitsky-Fadin-Kurayev-Lipatov (BFKL) equation.
- At very low x, the density of gluons is so high that they start to recombine: this is the saturation regime. The region around the critical line is described by nonlinear Balitsky-Kovchegov (BK) equation.

Drell-Yan process



Production of a lepton pair with $m^2_{l^+l^-} = Q^2 \gg 1\text{GeV}^2$

- large scale allows for perturbative treatment!

One of the most important/interesting process at hadron colliders

- finite state is colorless: easier to handle theoretically, cleaner to measure in the experiment
- leading order contribution sensitive to $\bar{q}(x, Q^2)$ distribution: very useful at constraining the anti-quark PDFs
- direct relation to DIS and $e^+e^- \to q\bar{q}$ via crossing

Drell-Yan process: leading order

In analogy to DIS the DY cross section at LO is given by the convolution

$$\frac{d\sigma^{(0)}_{p_1 p_2 \to l^+ l^-}}{dQ^2} = \int_0^1 dx_1 \int_0^1 dx_2 \sum_q \left[q_{p_1}(x_1)\bar{q}_{p_2}(x_2) + \bar{q}_{p_1}(x_1) q_{p_2}(x_2) \right] \frac{d\hat{\sigma}^{(0)}_{q\bar{q} \to l^+ l^-}}{dQ^2}$$

The partonic LO cross section reads

$$\frac{d\hat{\sigma}^{(0)}_{q\bar{q} \to l^+ l^-}}{dQ^2} = \frac{\sigma^{(0)}_{\mathrm{DY}}}{Q^2} e_q^2\, \delta(1-\hat{\tau}) \qquad \text{where} \qquad \sigma^{(0)}_{\mathrm{DY}} = \frac{4\pi\alpha_{\mathrm{em}}}{3N_c Q^2},$$

and we have introduced the variables

$$\hat{\tau} = \frac{Q^2}{\hat{s}} = \frac{\tau}{x_1 x_2} \qquad \text{and} \qquad \tau = \frac{Q^2}{s}.$$

Notice the general relation (for massless protons)

$$\hat{s} = (x_1 p_1 + x_2 p_2)^2 = x_1 x_2\, 2p_1 p_2 = x_1 x_2 s\,.$$

Drell-Yan process: leading order

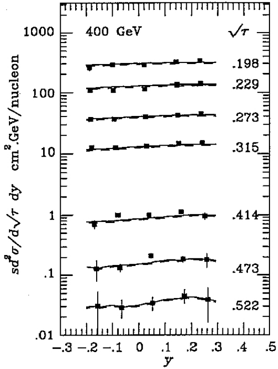
Substituting the LO partonic cross section into factorization formula gives

$$\frac{d\sigma^{(0)}_{p_1 p_2 \to l^+ l^-}}{dQ^2} = \frac{\sigma^{(0)}_{\mathrm{DY}}}{Q^2}\,\tau \int_\tau^1 dx \sum_q e_q^2 \left[q_{p_1}(x)\bar{q}_{p_2}\left(\frac{\tau}{x}\right) + \bar{q}_{p_1}(x) q_{p_2}\left(\frac{x}{\tau}\right) \right]$$

$$\propto \frac{\tau F(\tau)}{Q^4}$$

DY cross section at LO $\times$ Q^4 exhibits scaling in variable $\tau = Q^2/s$

- this is analogous to the Bjorken scaling of $F_{1,2}(x)$ in DIS
- indeed seen in the data



Interlude: rapidity

Kinematics of a particle is specified by its 4-momentum (E, p_x, p_y, p_z).
Rapidity is defined as

$$y = \frac{1}{2} \ln \left(\frac{E + p_z}{E - p_z} \right) .$$

For massless particles, it is equals to pseudorapidity, η, and it is directly related to the polar angle with respect to the beam θ

$$\eta = -\ln \tan(\theta/2) .$$

Hence, for massless particles:

$$\begin{aligned} \theta = 0 \quad &\Leftrightarrow \quad y = \infty \quad \text{(forward)} \\ \theta = \frac{\pi}{2} \quad &\Leftrightarrow \quad y = 0 \quad \text{(central)} \\ \theta = \pi \quad &\Leftrightarrow \quad y = -\infty \quad \text{(backward)} \end{aligned}$$

Together with the transverse momentum p_T and the azimuthal angle in the transverse plane ϕ, it gives the following parametrization of 4-momentum

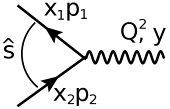
$$p^\mu = (m_T \cosh y, p_T \sin \phi, p_T \cos \phi, m_T \sinh y)$$

where $m_T = \sqrt{p_T^2 + m^2}$.

Interlude: rapidity

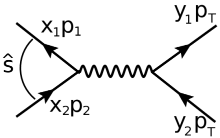
Energy-momentum conservation allows one to relate the rapidities and the transverse momenta of the outgoing particles with the momentum fractions of the incoming partons.

For $2 \to 1$ process:



$$x_{1,2} = \frac{Q}{\sqrt{s}} e^{\pm y}$$

For $2 \to 2$ process:

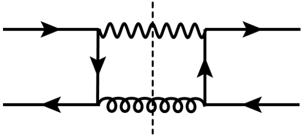


$$x_{1,2} = \frac{2p_T}{\sqrt{s}} \left(e^{\pm y_1} + e^{\pm y_2}\right)$$

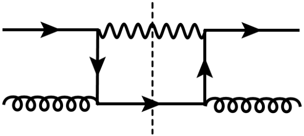
Drell-Yan: next-to-leading order

The partonic cross section $\frac{d\hat{\sigma}^{(0)}_{ab\to l^+l^-}}{dQ^2}$ has of course a perturbative expansion. Three types of contributions:

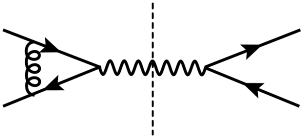
$$|\mathcal{M}_{q\bar{q}\to\gamma g}|^2$$



$$|\mathcal{M}_{qg\to q\gamma}|^2$$



$$\mathcal{M}^{\text{virt}}_{q\bar{q}\to\gamma} \times \left(\mathcal{M}^{\text{Born}}_{q\bar{q}\to\gamma}\right)^\dagger$$



Drell-Yan: next-to-leading order

As in the case of e^+e^- and DIS, real and virtual diagrams have collinear and soft divergencies. Phase space integration can be performed via dimensional regularization and we get

$$\frac{d\hat{\sigma}^{(1)}_{q\bar{q}}}{dQ^2} = \frac{\sigma^{(0)}_{\mathrm{DY}}}{Q^2} e_q^2 \frac{\alpha_s}{2\pi} \left\{ -2P^{(0)}_{qq}(\hat{\tau}) \left[S_\epsilon - \ln \frac{Q^2}{\mu^2} \right] + D_q(\hat{\tau}) \right\}$$

$$\frac{d\hat{\sigma}^{(1)}_{qg}}{dQ^2} = \frac{\sigma^{(0)}_{\mathrm{DY}}}{Q^2} e_q^2 \frac{\alpha_s}{2\pi} \left\{ -2P^{(0)}_{qg}(\hat{\tau}) \left[S_\epsilon - \ln \frac{Q^2}{\mu^2} \right] + D_g(\hat{\tau}) \right\}$$

where $S_\epsilon = \frac{1}{\epsilon} - \gamma_E + \ln(4\pi)$. The D_q and D_g functions are finite.

Overall structure of the above result is the same as in the case of DIS!

- Soft singularities cancelled in the sum of real and virtual corrections.
- Partonic cross sections exhibit collinear divergence.
- The coefficients of the $\frac{1}{\epsilon}$ pole are identical to those in DIS.

In analogy to DIS, we push the $\overline{\mathrm{MS}}$-type singular terms to the PDFs. Since those terms are identical to the ones found in DIS our "physical" PDFs will be exactly those introduced for DIS.

Drell-Yan: next-to-leading order

The complete NLO result for the Drell-Yan process in $\overline{\text{MS}}$ scheme reads

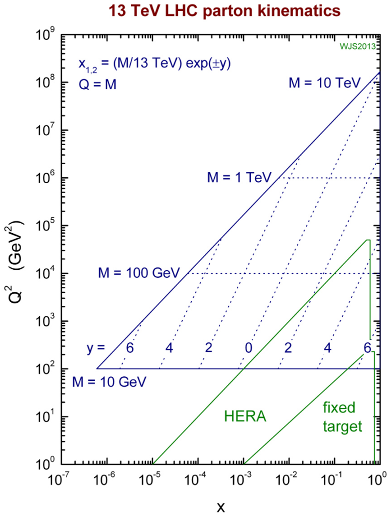
$$
\begin{aligned}
\frac{d\sigma^{\text{NLO}}_{p_1 p_2 \to l^+ l^-}}{dQ^2} &= \frac{\sigma^{(0)}_{\text{DY}}}{Q^2} \int_0^1 dx_1 \int_0^1 dx_2 \\
&\times \Big\{ \sum_q e_q^2 \left[q^{\overline{\text{MS}}}(x_1, \mu_F^2) \bar{q}^{\overline{\text{MS}}}(x_2, \mu_F^2) + \bar{q}^{\overline{\text{MS}}}(x_1, \mu_F^2) q^{\overline{\text{MS}}}(x_2, \mu_F^2) \right] \\
&\qquad \times \left(\delta(1-\hat{\tau}) + \frac{\alpha_s}{2\pi} \left[2P^{(0)}_{qq}(\hat{\tau}) \ln \frac{Q^2}{\mu_F^2} + D_{q\bar{q}}(\hat{\tau}) \right] \right) \\
&\quad + \sum_{f=q,\bar{q}} e_q^2 \left[g^{\overline{\text{MS}}}(x_1, \mu_F^2) f_q^{\overline{\text{MS}}}(x_2, \mu_F^2) + f_q^{\overline{\text{MS}}}(x_1, \mu_F^2) g^{\overline{\text{MS}}}(x_2, \mu_F^2) \right] \\
&\qquad \times \frac{\alpha_s}{2\pi} \left[2P^{(0)}_{qg}(\hat{\tau}) \ln \frac{Q^2}{\mu_F^2} + D_{qg}(\hat{\tau}) \right] \Big\}
\end{aligned}
$$

- Universality: The PDFs are identical to those introduced for DIS, hence, we can plug the PDFs fitted to $F_2(x, Q^2)$ and get the *prediction* for the DY cross section at NLO.
- Alternatively, we can use DY result to improve fits of PDFs.

Complementarity of Drell-Yan and DIS

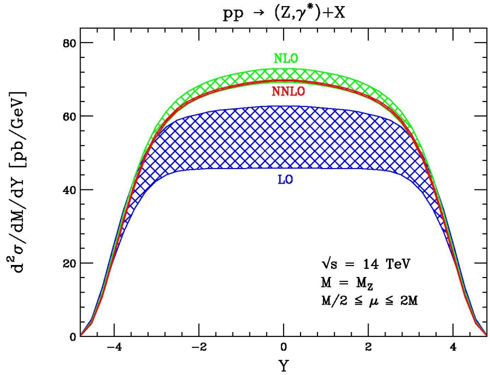
DIS at HERA and DY at LHC cover different regions in (x, Q^2) space!

- Test predictions in the overlap region.
- Improve PDF fits using complementary data from different processes.



Drell-Yan: magnitude of higher order corrections

Rapidity distribution at the LHC



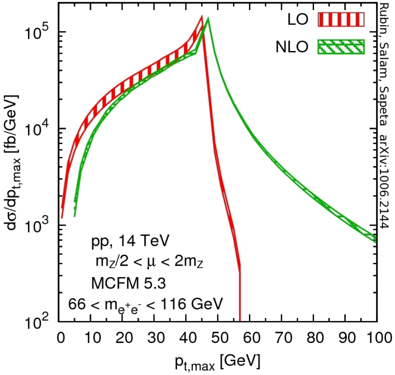
[Anastasiou, Dixon, Melnikov & Petriello '03]

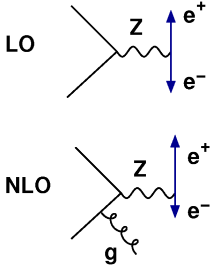
- Significant correction from LO to NLO. Results stabilize at NNLO.

Two fully differential NNLO tools: FEWZ, DYNNLO

Drell-Yan: magnitude of higher order corrections

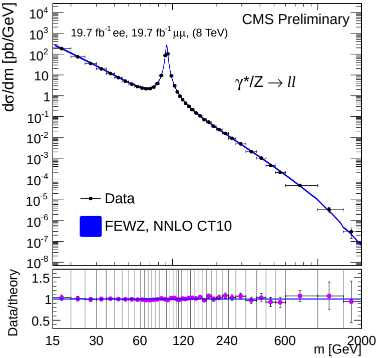
More drastic example:

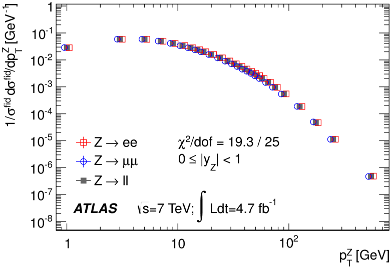




- Higher order corrections may lead to a huge K factor (ratio of NLO to LO cross section)!
- Here, the initial-state gluon causes boost of the e^+e^- pair such that one of the leptons gets significant p_T.

Drell-Yan: distributions in m_Z and p_T





- This is a very strong test of both perturbative QCD and the collinear factorization theorem!

Summary of lecture 2

- We have discussed two of the most important processes of high energy physics: deep inelastic electron-proton scattering (DIS) and lepton pair production in proton-proton collisions (DY).
- Both processes involve short- and long-distance interactions.
- Those two domains can be separated by factorization procedure.
- The short-distance physics is amenable to perturbative treatment with the partonic cross sections calculated order by order in α_s.
- The long-distance physics is described by non-perturbative objects called parton distribution functions (PDFs).
- Parton distribution functions are universal, *i.e.* the same functions can be used across different processes like DIS and DY.
- Evolution of PDFs can be calculated from pQCD and it is described by the DGLAP equation.
- PDFs can be determined by fitting parametrizations of the initial conditions of the DGLAP equation.
- Variation of the renormalization and the factorization scale allows for some estimate of theoretical uncertainties but there are many cases in which this procedure is not working.