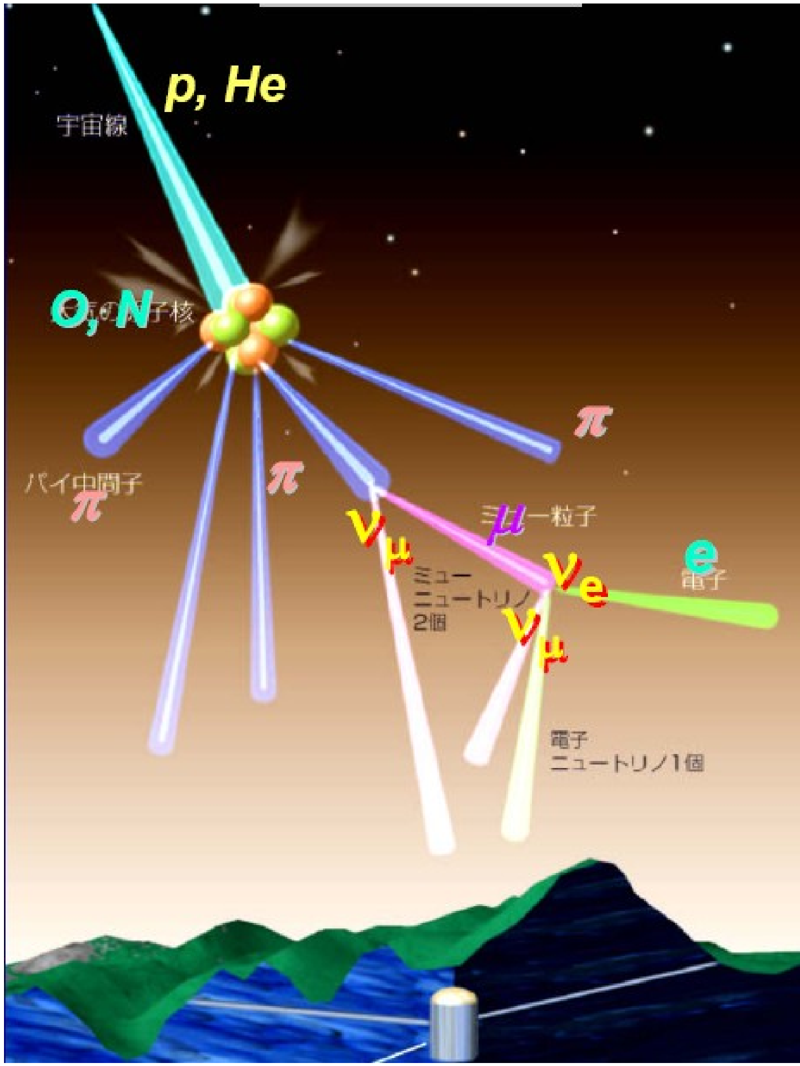
The neutrino mixing matrix and recent experimental progress

Marco Zito

- Atmospheric neutrinos
- The PMNS matrix
- Theta_13
- Measuring CP violation

Fact 2: the atmospheric neutrino anomaly



The source: hadronic showers produced by cosmic rays in the atmosphere

Pion decays: two ν_μ per ν_e

$$p \text{ (He)} + A \rightarrow \pi^{+(-)} + A'$$
$$\pi^{+(-)} \rightarrow \mu^{+(-)} + \nu_\mu (\bar{\nu}_\mu)$$
$$\mu^{+(-)} \rightarrow e^{+(-)} + \nu_e (\bar{\nu}_e) + \bar{\nu}_\mu (\nu_\mu)$$

R (μ/e) = 0.60 ± 0.07 (stat.) ± 0.05(syst.)
(μ/e) Data/ **(μ/e)** MC: R expected to be 1
Kamiokande, Phys. Lett. B280 (1992) 146

$R = [N(\mu)_{obs}/N(\mu)_{MC}] / [N(e)_{obs}/N(e)_{MC}]$

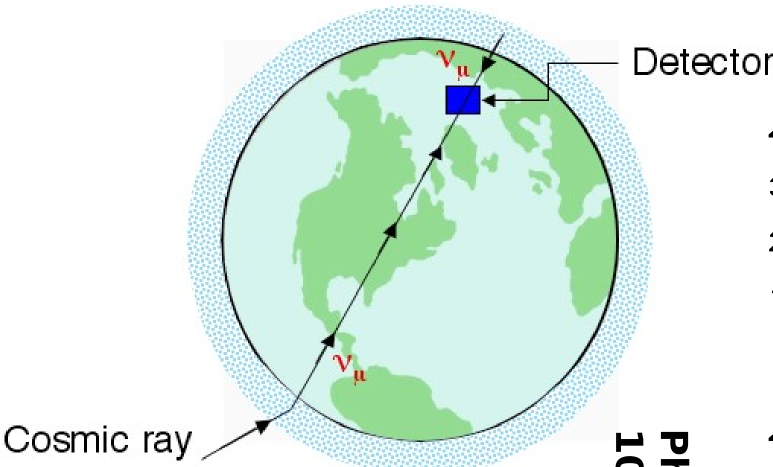
Less ν_μ than ν_e than expected

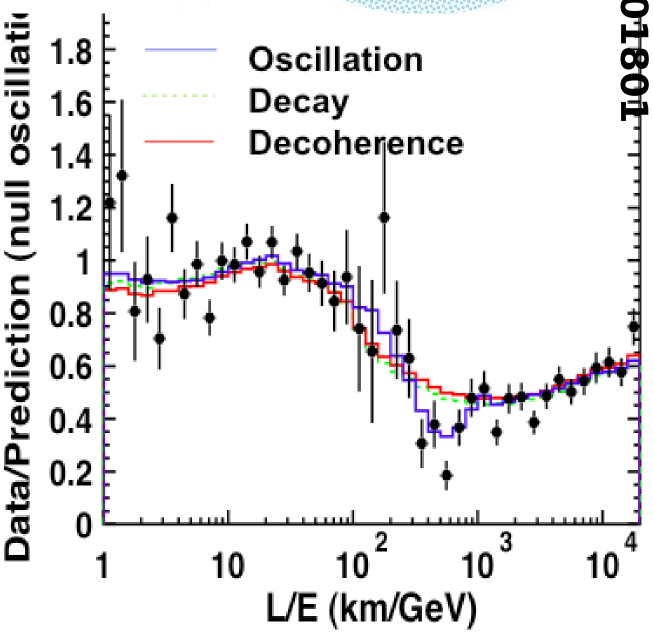
Anomaly observed also by IMB and Soudan

Evidence for atmospheric oscillation : SK

$1-1/2\sin^2 2\theta$

- Dependence on zenith angle (oscillation length)

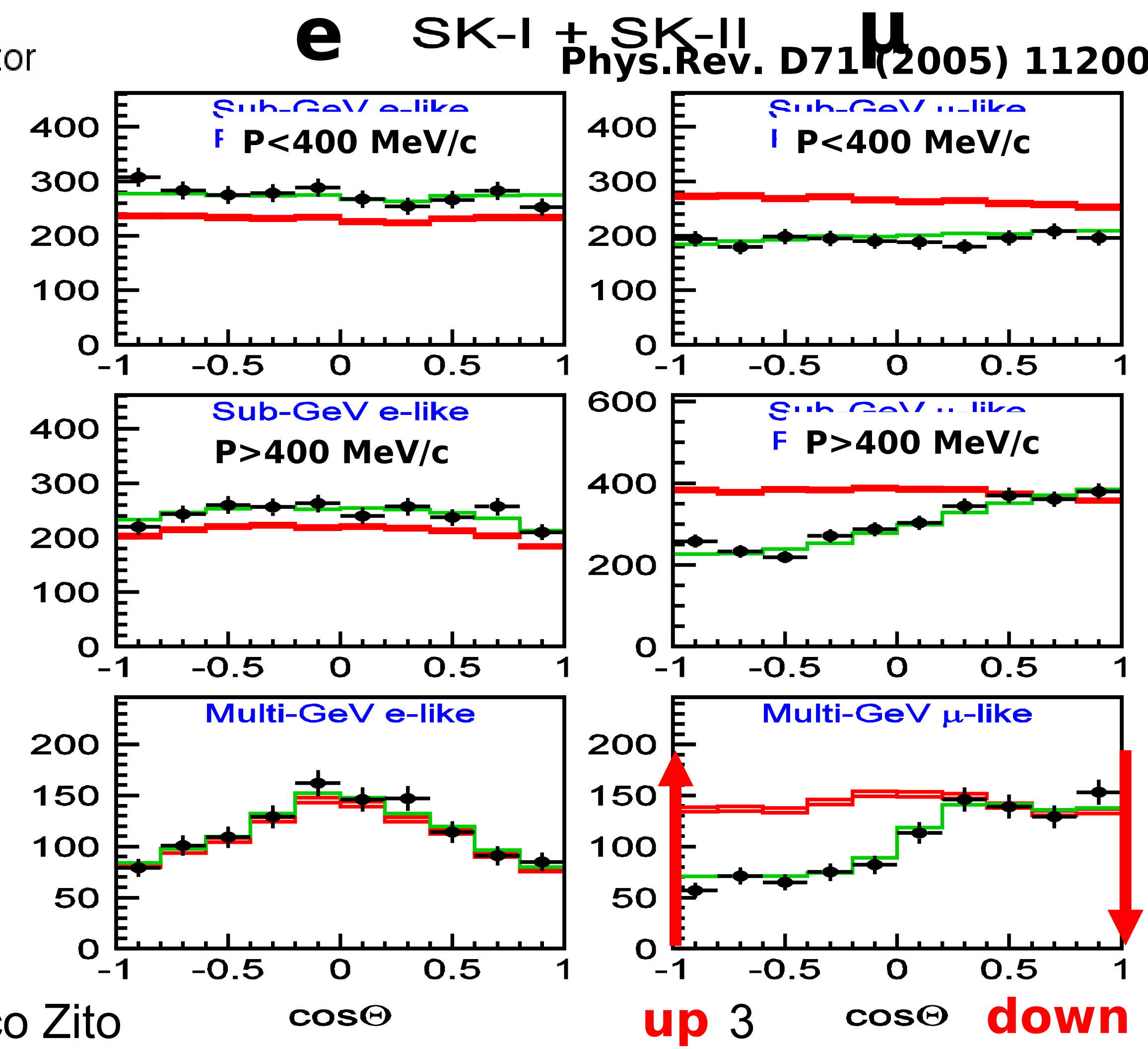




Phys.Rev.Lett. 93 (2004) 101801

SK-I + SK-II

Phys.Rev. D71 (2005) 11200



Impact of neutrino oscillation

- Neutrino oscillation implies that :
  - Neutrinos have non-zero mixing angles
  - Neutrinos have tiny but non-zero masses
  - Lepton flavor is not conserved
  - The Standard Model is incomplete

The neutrino mass and mixings (until 2011)

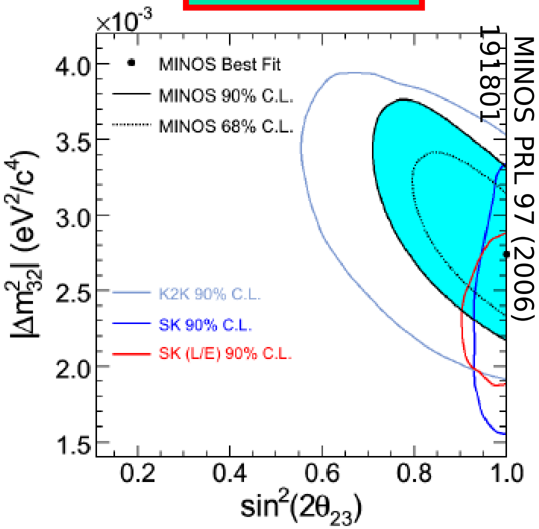
"Atmospheric" "Solar"

$$U=\begin{pmatrix}U_{e1} & U_{e2} & U_{e3}\\ U_{\mu1} & U_{\mu2} & U_{\mu3}\\ U_{\tau1} & U_{\tau2} & U_{\tau3}\end{pmatrix}=\begin{pmatrix}1 & 0 & 0\\ 0 & c_{23} & s_{23}\\ 0 & -s_{23} & c_{23}\end{pmatrix}\begin{pmatrix}c_{13} & 0 & s_{13}e^{-i\delta}\\ 0 & 1 & 0\\ -s_{13}e^{i\delta} & 0 & c_{13}\end{pmatrix}\begin{pmatrix}c_{12} & s_{12} & 0\\ -s_{12} & c_{12} & 0\\ 0 & 0 & 1\end{pmatrix}$$

3 mixing angles (θ_{12}, θ_{23}, θ_{13})
1 CPV phase (δ)
2 (indep.) mass differences ($\Delta m^2_{ij}=m^2_i-m^2_j$)
Additional phases if Majorana neutrinos

Situation until 2011

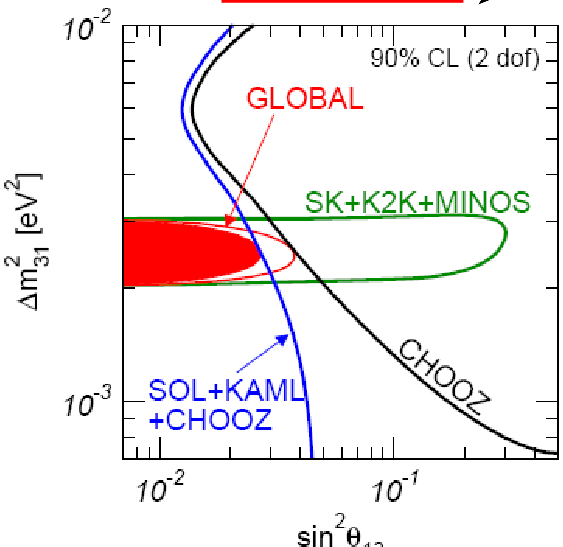
θ_{23}, Δm^2_{32}



MINOS PRL 97 (2006) 191801

$|\Delta m^2_{32}| = 2.74^{+0.44}_{-0.26}(\text{stat}+\text{syst})\times10^{-3}\,\text{eV}^2$
$\sin^2 2\theta_{23} = 1.00_{-0.13}(\text{stat}+\text{syst})$

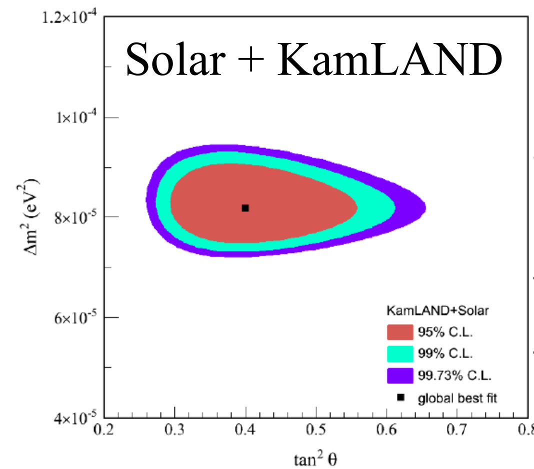
θ_{13}, Δm^2_{31}



T.Schwetz, hep-ph/0606060

Only upper limit on θ_{13}

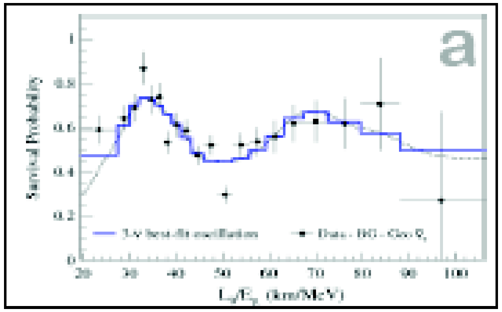
θ_{12}, Δm^2_{21}



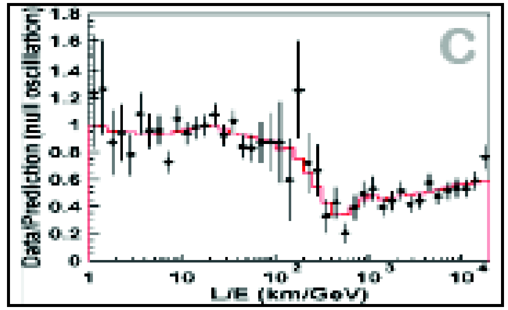
KAMLAND, PRL 94 (2005) 081801

PMNS today

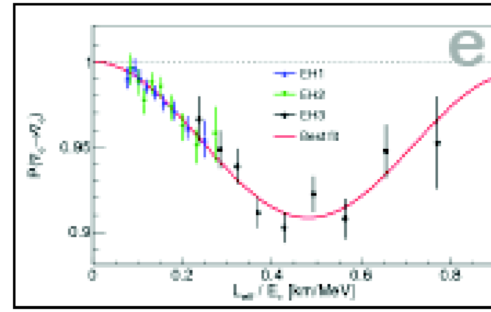
$e \to e\ (\delta m^2, \theta_{12})$



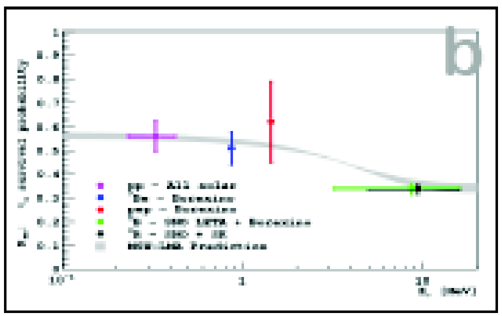
$\mu \to \mu\ (\Delta m^2, \theta_{23})$



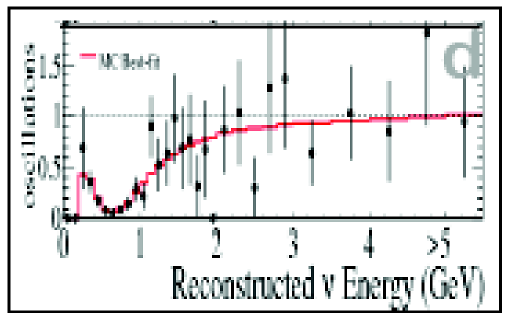
$e \to e\ (\Delta m^2, \theta_{13})$



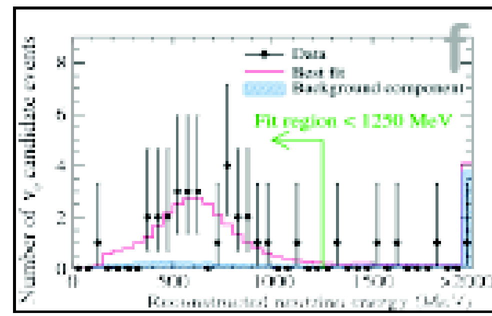
$e \to e\ (\delta m^2, \theta_{12})$



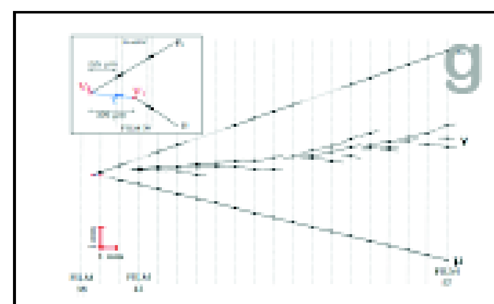
$\mu \to \mu\ (\Delta m^2, \theta_{23})$



$\mu \to e\ (\Delta m^2, \theta_{13}, \theta_{23})$



$\mu \to \tau\ (\Delta m^2, \theta_{23})$

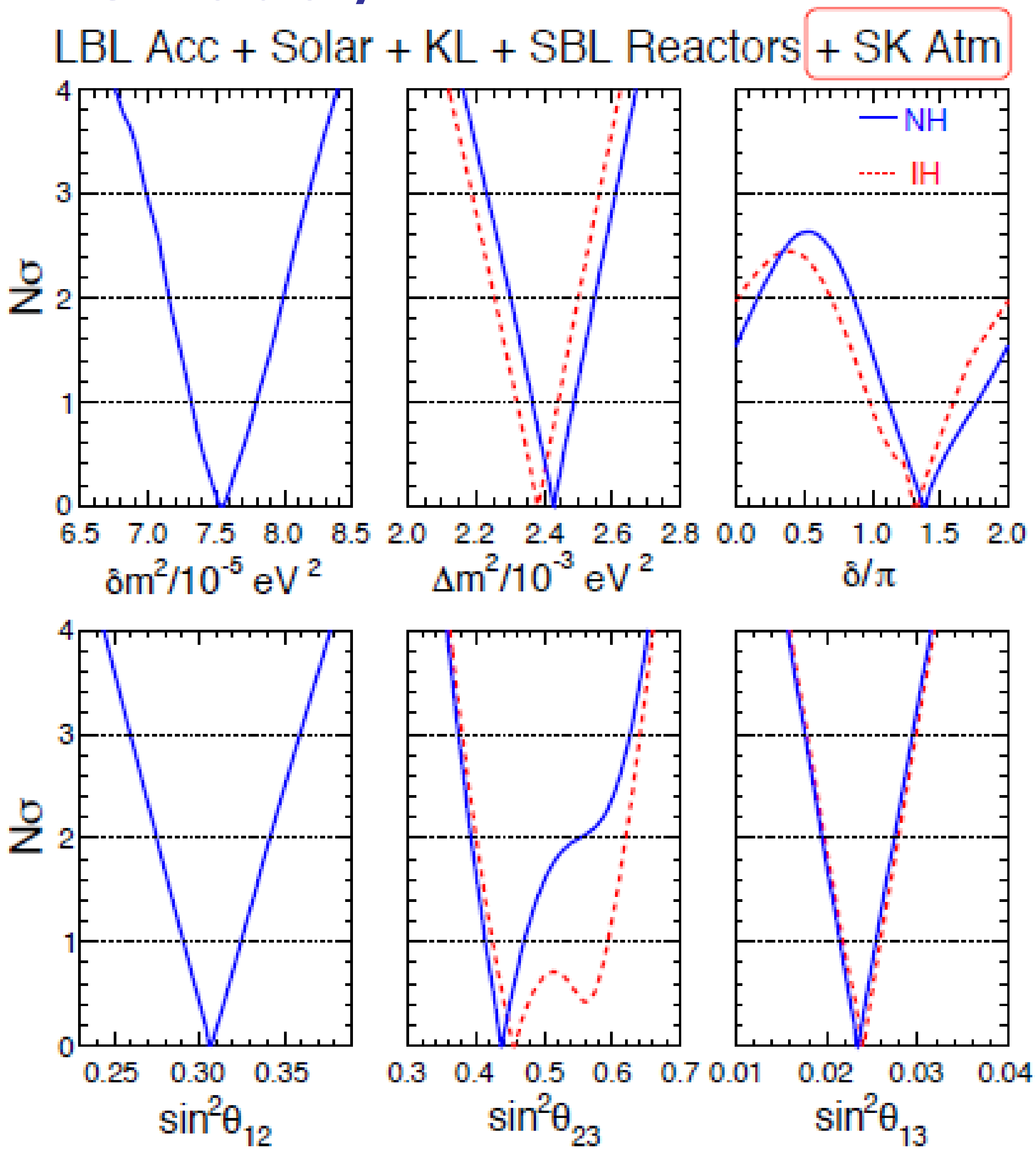


Data from various types of neutrino experiments: (a) solar, (b) long-baseline reactor, (c) atmospheric, (d) long-baseline accelerator, (e) short-baseline reactor, (f,g) long baseline accelerator (and, in part, atmospheric).

(a) KamLAND [plot]; (b) Borexino [plot], Homestake, Super-K, SAGE, GALLEX/GNO, SNO; (c) Super-K atmosph. [plot], MACRO, MINOS etc.; (d) T2K (plot), MINOS, K2K; (e) Daya Bay [plot], RENO, Double Chooz; (f) T2K [plot], MINOS; (g) OPERA [plot], Super-K atmospheric.

See also other talks at this Meeting.

PMNS today


LBL Acc + Solar + KL + SBL Reactors + SK Atm
Nσ
NH
IH
δm²/10⁻⁵ eV²
Δm²/10⁻³ eV²
δ/π
sin²θ12
sin²θ23
sin²θ13

Current precision on neutrino parameters

Nσ ranges for single parameters (pre-Neutrino'14):

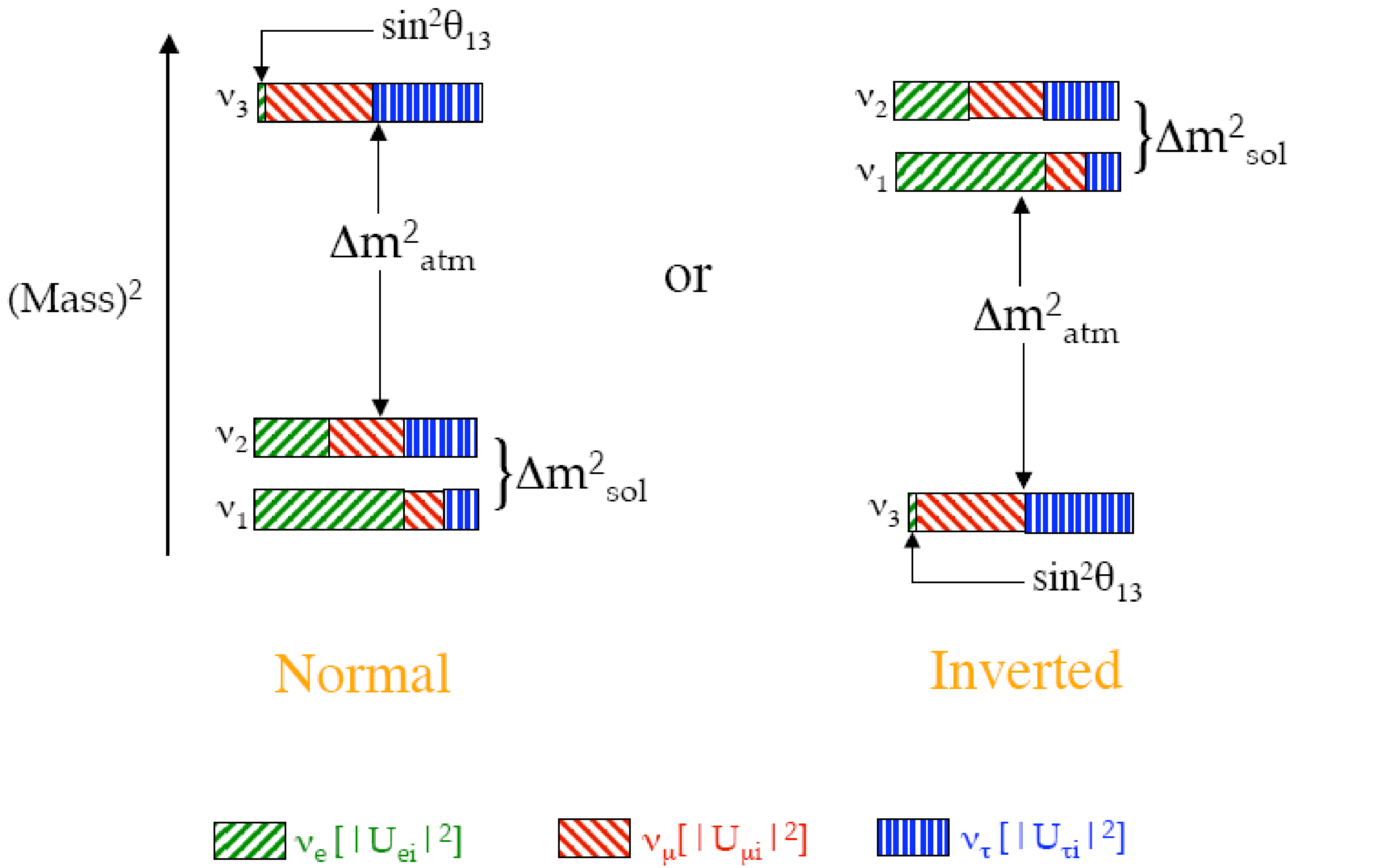
TABLE I: Results of the global 3ν oscillation analysis, in terms of best-fit values and allowed 1, 2 and 3σ ranges for the 3ν mass-mixing parameters. See also Fig. 3 for a graphical representation of the results. We remind that Δm^2 is defined herein as $m_3^2 - (m_1^2 + m_2^2)/2$, with $+\Delta m^2$ for NH and $-\Delta m^2$ for IH. The CP violating phase is taken in the (cyclic) interval $\delta/\pi \in [0, 2]$. The overall χ^2 difference between IH and NH is insignificant ($\Delta\chi^2_{I-N} = -0.3$).

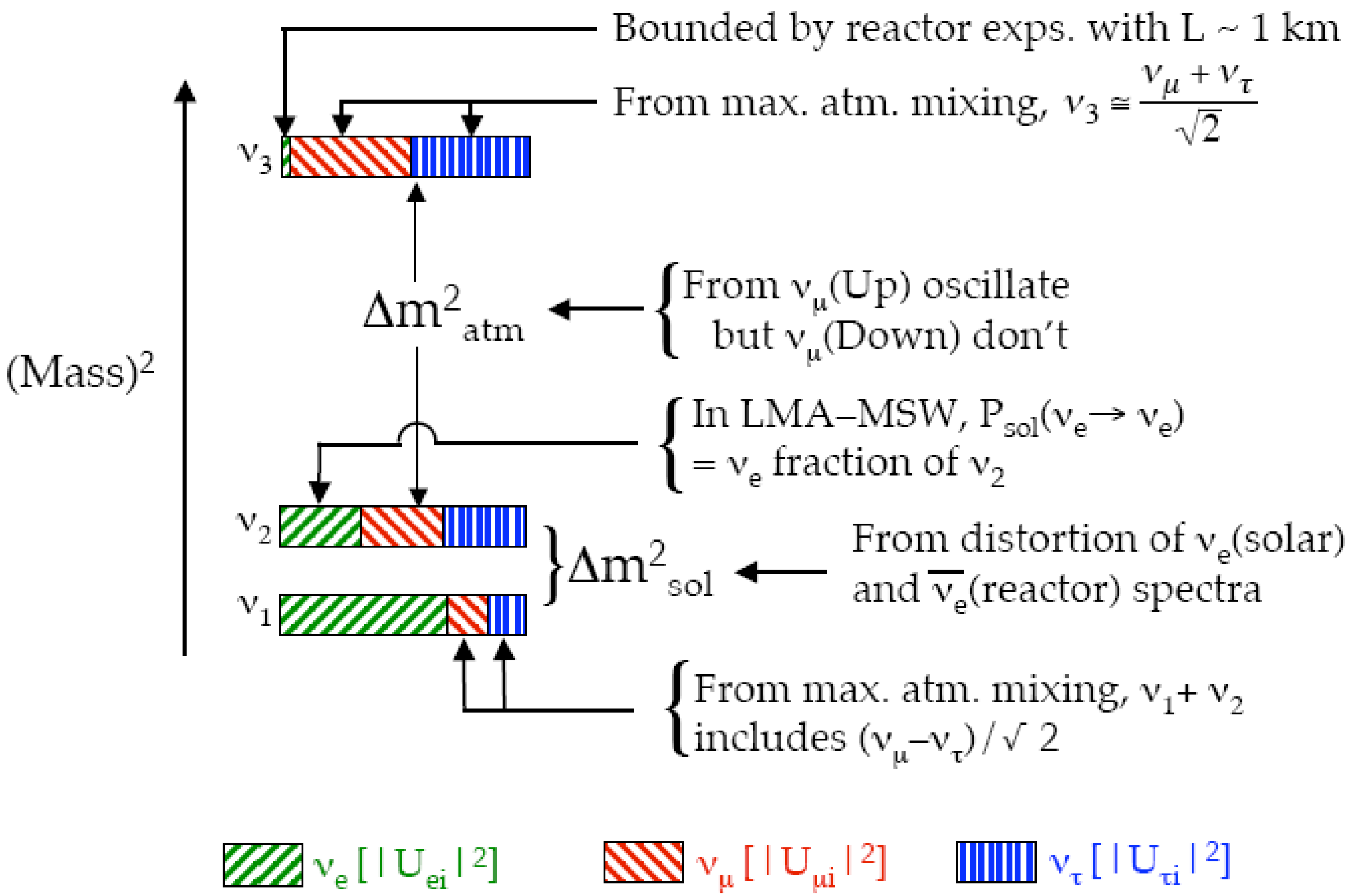
Parameter	Best fit	1σ range	2σ range	3σ range
$\delta m^2/10^{-5}$ eV^2 (NH or IH)	7.54	7.32 – 7.80	7.15 – 8.00	6.99 – 8.18
$\sin^2\theta_{12}/10^{-1}$ (NH or IH)	3.08	2.91 – 3.25	2.75 – 3.42	2.59 – 3.59
$\Delta m^2/10^{-3}$ eV^2 (NH)	2.43	2.37 – 2.49	2.30 – 2.55	2.23 – 2.61
$\Delta m^2/10^{-3}$ eV^2 (IH)	2.38	2.32 – 2.44	2.25 – 2.50	2.19 – 2.56
$\sin^2\theta_{13}/10^{-2}$ (NH)	2.34	2.15 – 2.54	1.95 – 2.74	1.76 – 2.95
$\sin^2\theta_{13}/10^{-2}$ (IH)	2.40	2.18 – 2.59	1.98 – 2.79	1.78 – 2.98
$\sin^2\theta_{23}/10^{-1}$ (NH)	4.37	4.14 – 4.70	3.93 – 5.52	3.74 – 6.26
$\sin^2\theta_{23}/10^{-1}$ (IH)	4.55	4.24 – 5.94	4.00 – 6.20	3.80 – 6.41
δ/π (NH)	1.39	1.12 – 1.77	0.00 – 0.16 ⊕ 0.86 – 2.00	—
δ/π (IH)	1.31	0.98 – 1.60	0.00 – 0.02 ⊕ 0.70 – 2.00	—

Fractional errors (defined as 1/6 of ±3σ ranges):

δm^2	2.6 %
Δm^2	2.6 %
$\sin^2\theta_{12}$	5.4 %
$\sin^2\theta_{13}$	8.5 %
$\sin^2\theta_{23}$	~ 10 %

The spectrum, showing its approximate flavor content, is




Bounded by reactor exps. with L ~ 1 km
From max. atm. mixing, $\nu_3 \cong \frac{\nu_\mu + \nu_\tau}{\sqrt{2}}$
ν_3
Δm^2_{atm}
From ν_μ(Up) oscillate but ν_μ(Down) don't
(Mass)^2
In LMA–MSW, $P_{sol}(\nu_e \rightarrow \nu_e)$ = ν_e fraction of ν_2
ν_2
Δm^2_{sol}
From distortion of ν_e(solar) and $\overline{\nu_e}$(reactor) spectra
ν_1
From max. atm. mixing, $\nu_1 + \nu_2$ includes $(\nu_\mu - \nu_\tau)/\sqrt{2}$
ν_e [$|U_{ei}|^2$]
ν_μ [$|U_{\mu i}|^2$]
ν_τ [$|U_{\tau i}|^2$]

Leptons versus quarks: 3-1

$$V_{PMNS} = \begin{pmatrix} 0.8 & 0.5 & 0.2 \\ 0.4 & 0.6 & 0.7 \\ 0.4 & 0.6 & 0.7 \end{pmatrix}$$

The neutrino mixing matrix shows 3 large (and unexpected) angles. This is very different from the CKM matrix.

$$V_{CKM} = \begin{pmatrix} 1 & 0.2 & 0.001 \\ 0.2 & 1 & 0.01 \\ 0.001 & 0.01 & 1 \end{pmatrix}$$

VCKM $\approx \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$

A Numerical coincidence ?

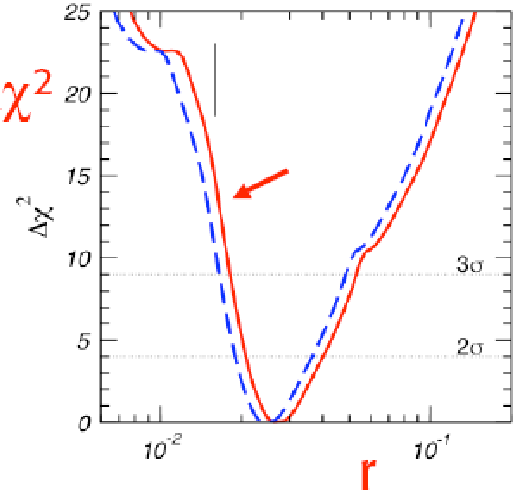
$\theta^{l}_{12} + \theta^{q}_{12} \sim \pi/4$

$12° + 34°$

General remarks

• After KamLAND, SNO and WMAP.... not too much hierarchy is needed for ν masses:

$r \sim \Delta m^2_{sol}/\Delta m^2_{atm} \sim 1/30$



Only a few years ago could be as small as 10^{-8}!

Precisely at 2σ: $0.025 < r < 0.049$

or

$m_{heaviest} < 0.2 - 0.7$ eV
$m_{next} > \sim 8\ 10^{-3}$ eV

For a hierarchical spectrum: $\frac{m_2}{m_3} \approx \sqrt{r} \approx 0.2$

Comparable to $\lambda_C = \sin\theta_C$: $\lambda_C \approx 0.22$ or $\sqrt{\frac{m_\mu}{m_\tau}} \approx 0.24$

Suggests the same "hierarchy" parameters for q, l, ν (small powers of λ_C) ⟶ e.g. θ_{13} not too small!

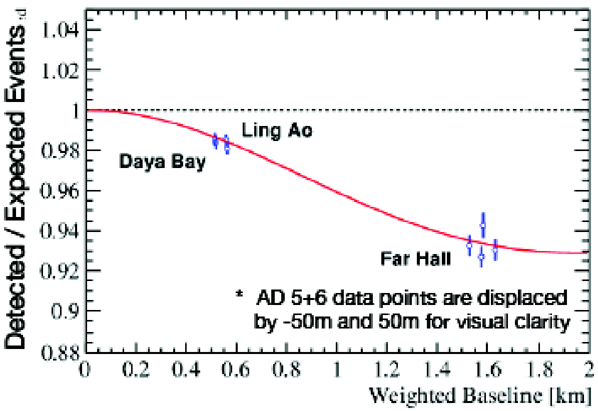
The measurement of theta_13

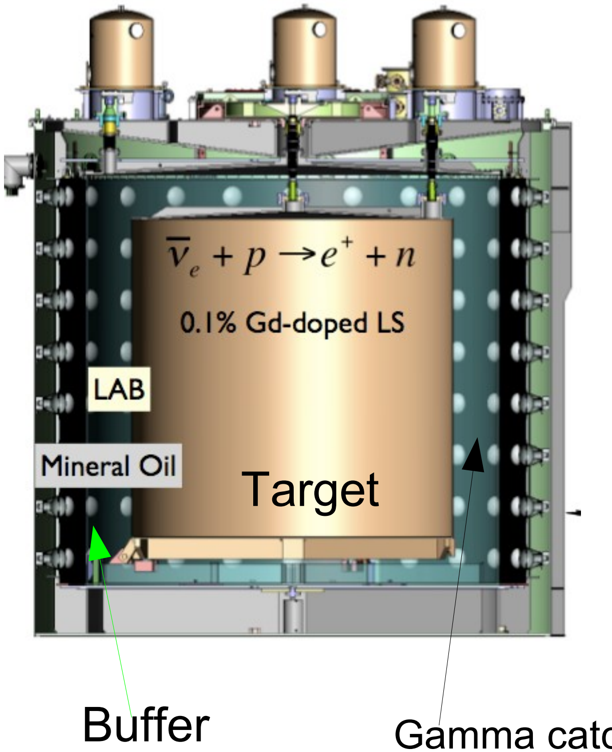
$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

CP Violation effects are proportional to $\sin\theta_{23} \sin\theta_{13} \sin\theta_{12}$

Reactor neutrino experiment: schematic

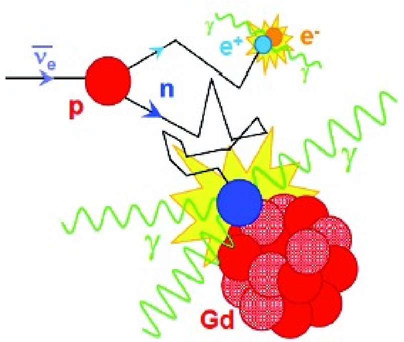
$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \approx 1 - \sin^2 2\theta_{13} \sin^2 \frac{\Delta m^2_{31} L}{4E}$$





Nuclear reactor: >10^{20} ν/s

$$\nu_e\, p \rightarrow e^+\, n$$

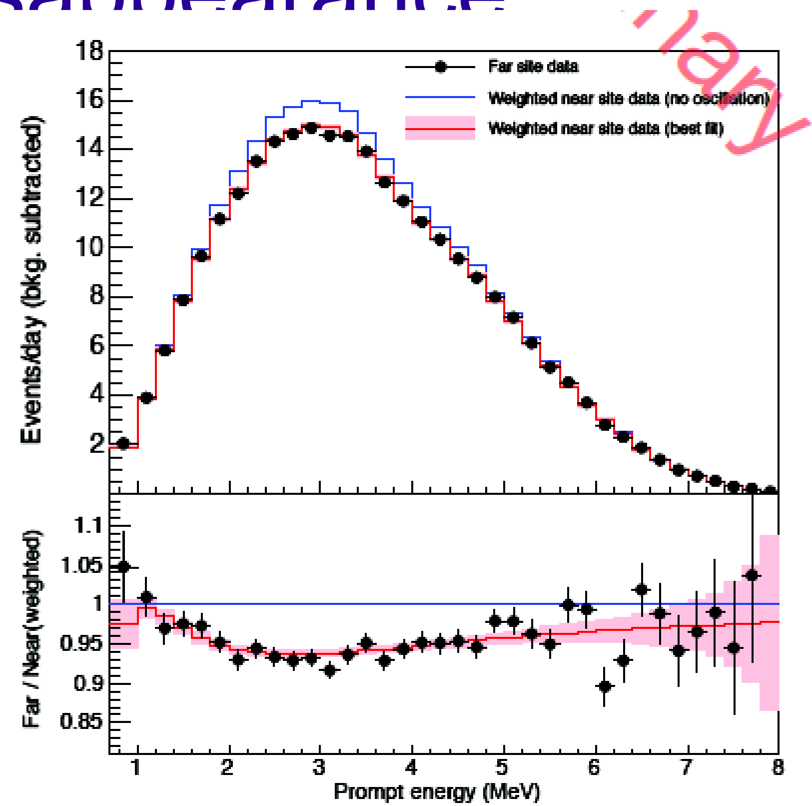


The detection technique is based on the Inverse Beta Decay reaction followed by a neutron capture on Gadolinium (delayed coincidence).
The reactor flux is measured by near detector(s).
Control of the backgrounds and of the systematic uncertainties is crucial.

$\bar{\nu}_e \rightarrow \bar{\nu}_e$

Daya Bay $\bar{\nu}_e$ disappearance

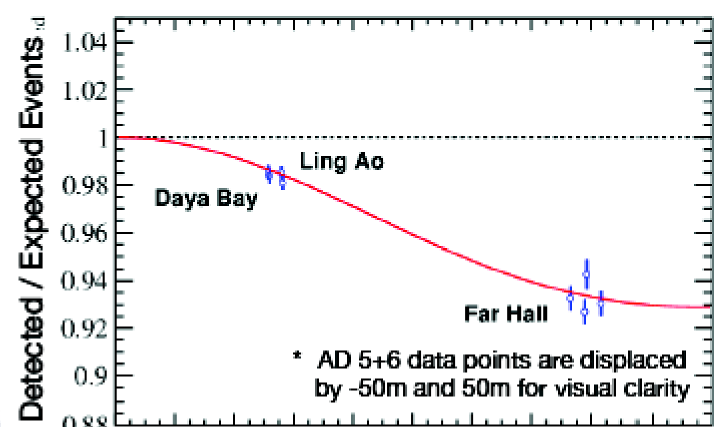
- Four times more statistics (621 days) the previously published result
- Over 1 million antineutrinos detected (150k in the far detectors)
- Most precise measurement of $\sin^2(2\theta$ (6%)
- Shape distortion agrees with oscillatio prediction



$$\sin^2 2\theta_{13} = 0.084^{+0.005}_{-0.005}$$

$$|\Delta m^2_{ee}| = 2.44^{+0.10}_{-0.11} \times 10^{-3}\text{eV}^2$$

$$\chi^2/NDF = 134.7/146$$

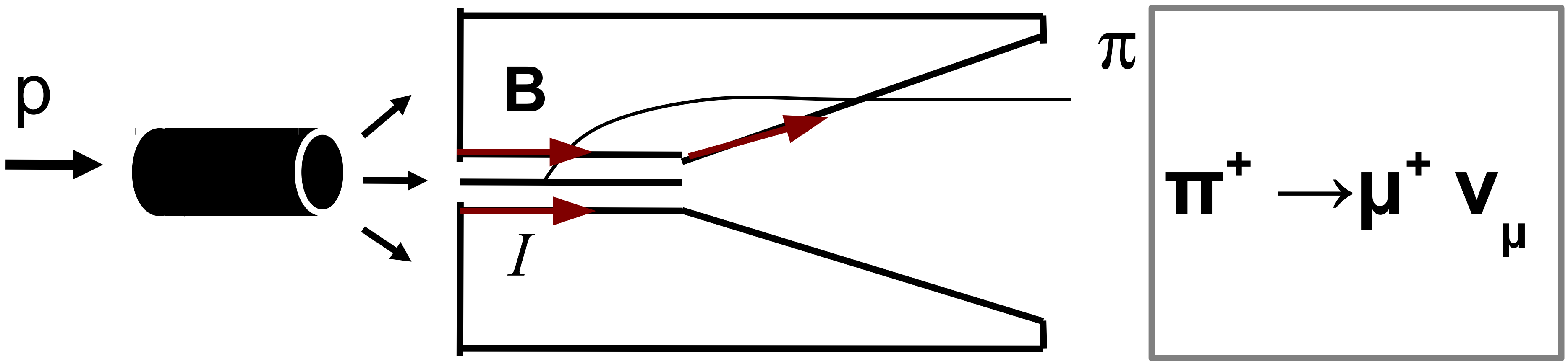


RENO $\sin^2 2\theta_{13} = 0.101 \pm 0.013$

Neutrino oscillation with a beam

- Producing the neutrino beam
- Controlling the flux
- Large neutrino detectors

Producing a neutrino beam



Primary proton beam on target

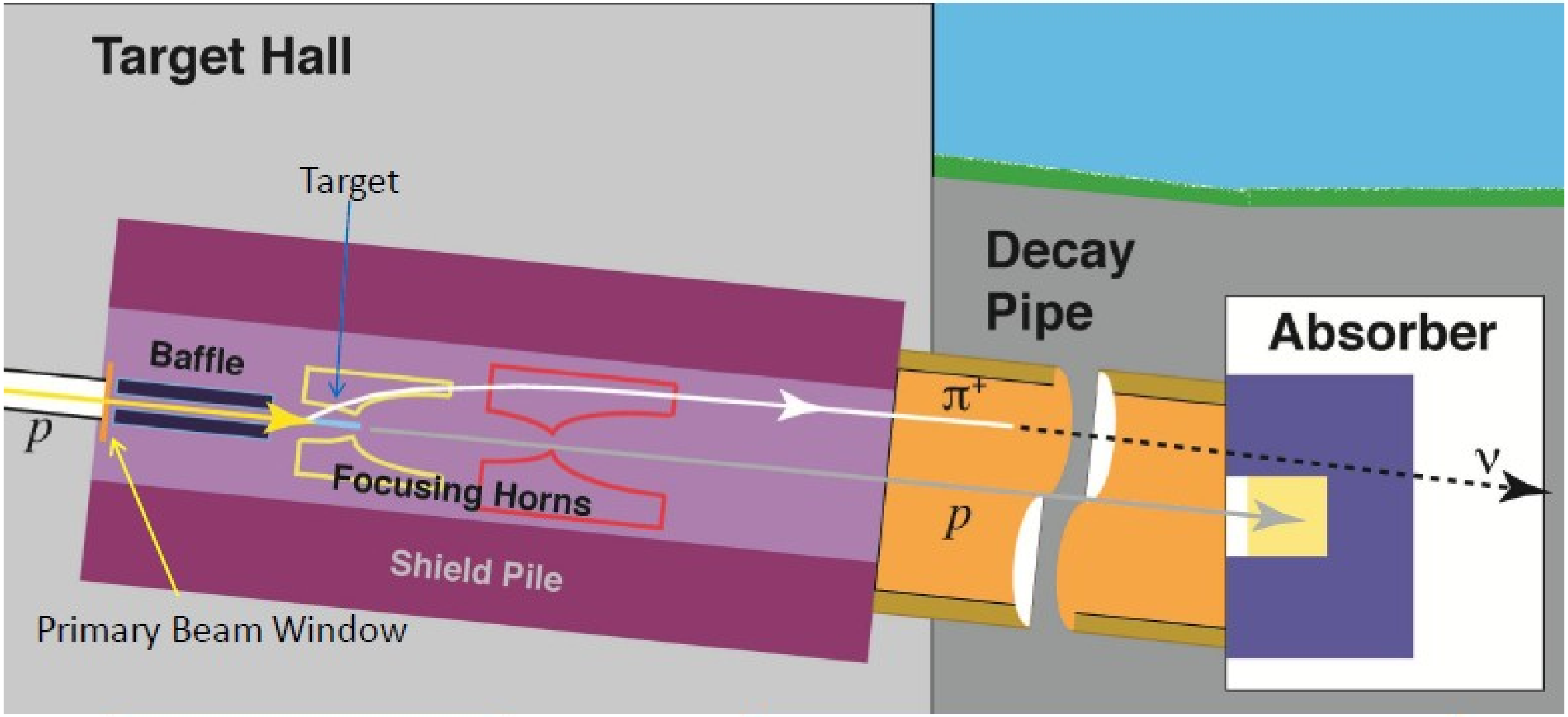
Focusing the pions with a magnetic device (horn). The current allows to focus either pi+ (numu) or pi-(numubar)

Decay volume. Need to stop the mu before they decay (otherwise nue)

Not a novel device !

Simon van der Meer, CERN-61-07 1961

The LBNE target/horn configuration

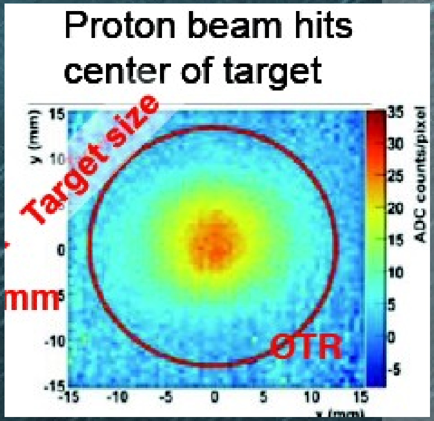


NuMI design Horns.

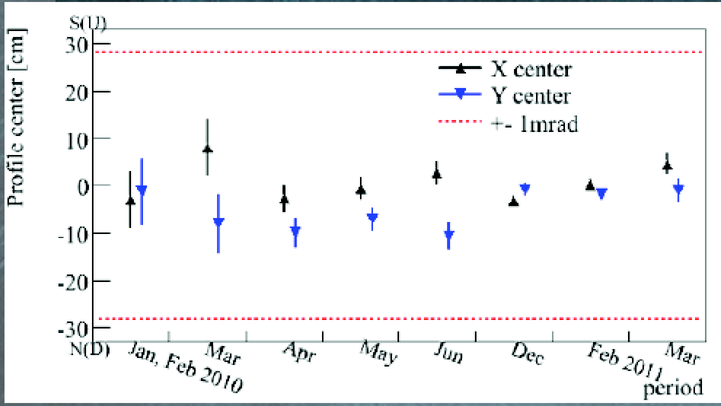
NuMI-like low energy target for 708 kW operation.

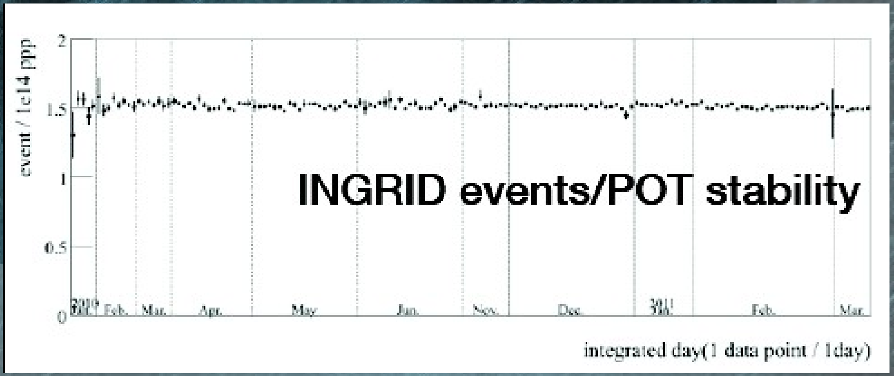
Target inserted into Horn 1.
Upstream end of target at -35 cm relative to the upstream face of Horn 1.
Tunable neutrino energy spectrum.

T2K Beam monitoring



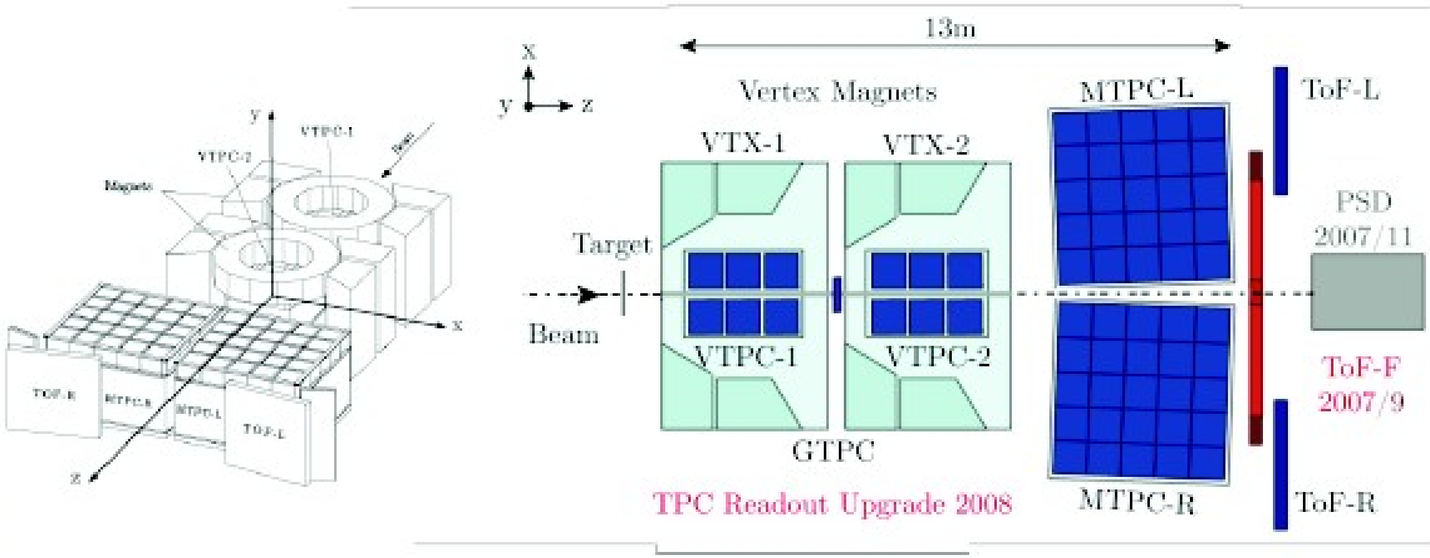
- Beam position on target within 1 mm
- Muon monitor : beam direction within 1 mrad, intensity stable (<1%)
- Neutrino beam (INGRID)





CERN NA61 measurements

Evaluation of Particle Yields in 30 GeV p+C Inelastic Interactions and in the T2K replica target



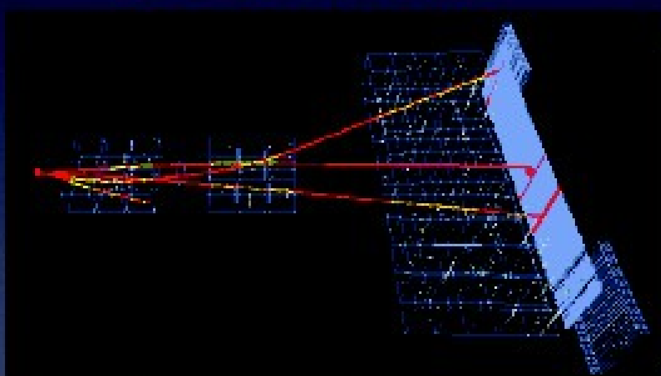
Large acceptance spectrometer:

5 TPCs
2 dipole magnets
$\sigma(p)/p^2 \approx 10^{-4}\,(\mathrm{GeV}/c)^{-1}$
$\sigma(dE/dx)/\langle dE/dx\rangle \approx 0.04$

3 ToFs
$\sigma(\text{ToF-F}) \approx 120$ ps
$\sigma(\text{ToF-L/R}) \approx 60$ ps

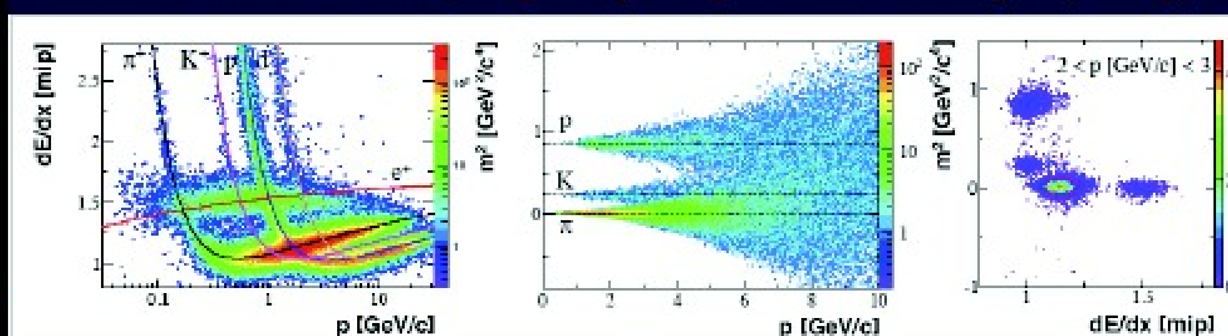
Full Coverage of T2K phase space

p+C @ 31 GeV/c



Particle ID methods used:

1) dE/dx (p < 1 GeV/c, p > 4 GeV/c)
2) Combined dE/dx + ToF (1 < p [GeV/c] < 4)
3) Negatively charged hadron h- analysis (π- only)

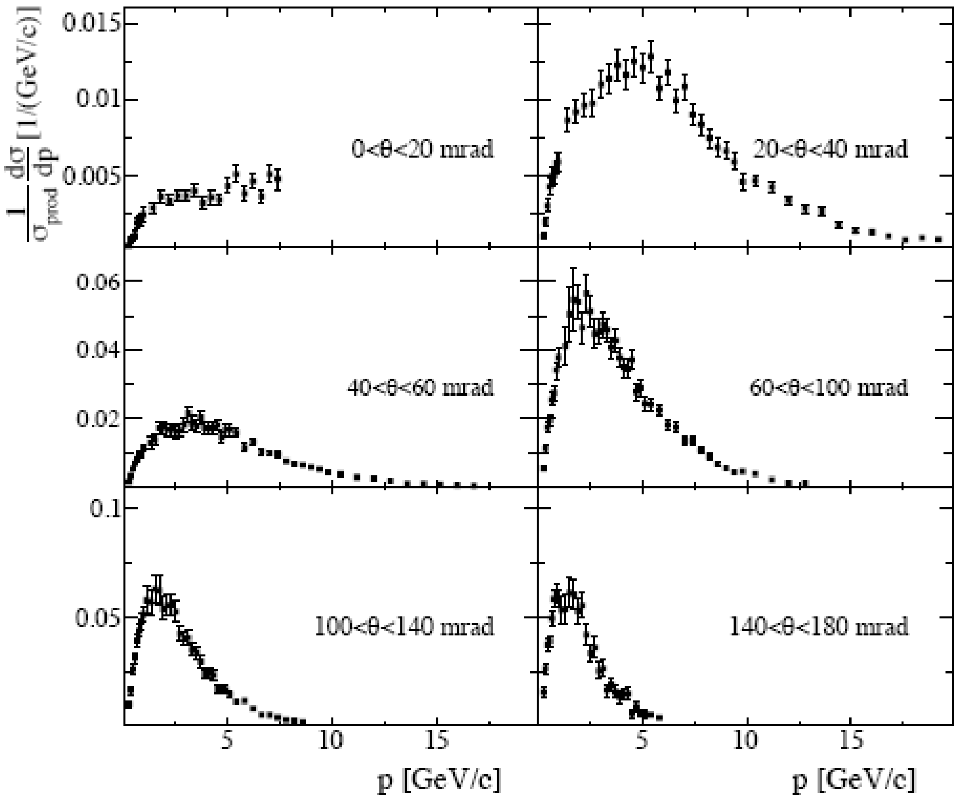


Results of pion production from thin target (2007 data)

N.Abgrall et al., arXiv:1102.0983 [hep-ex]
submitted to Phys.Rev.C (2011)

Differential cross section for π^+ production in 30GeV p+C

Error bars = stat. + syst. in quadrature
no normalization error is shown



Systematic uncertainty was evaluated in each (p,θ) bin

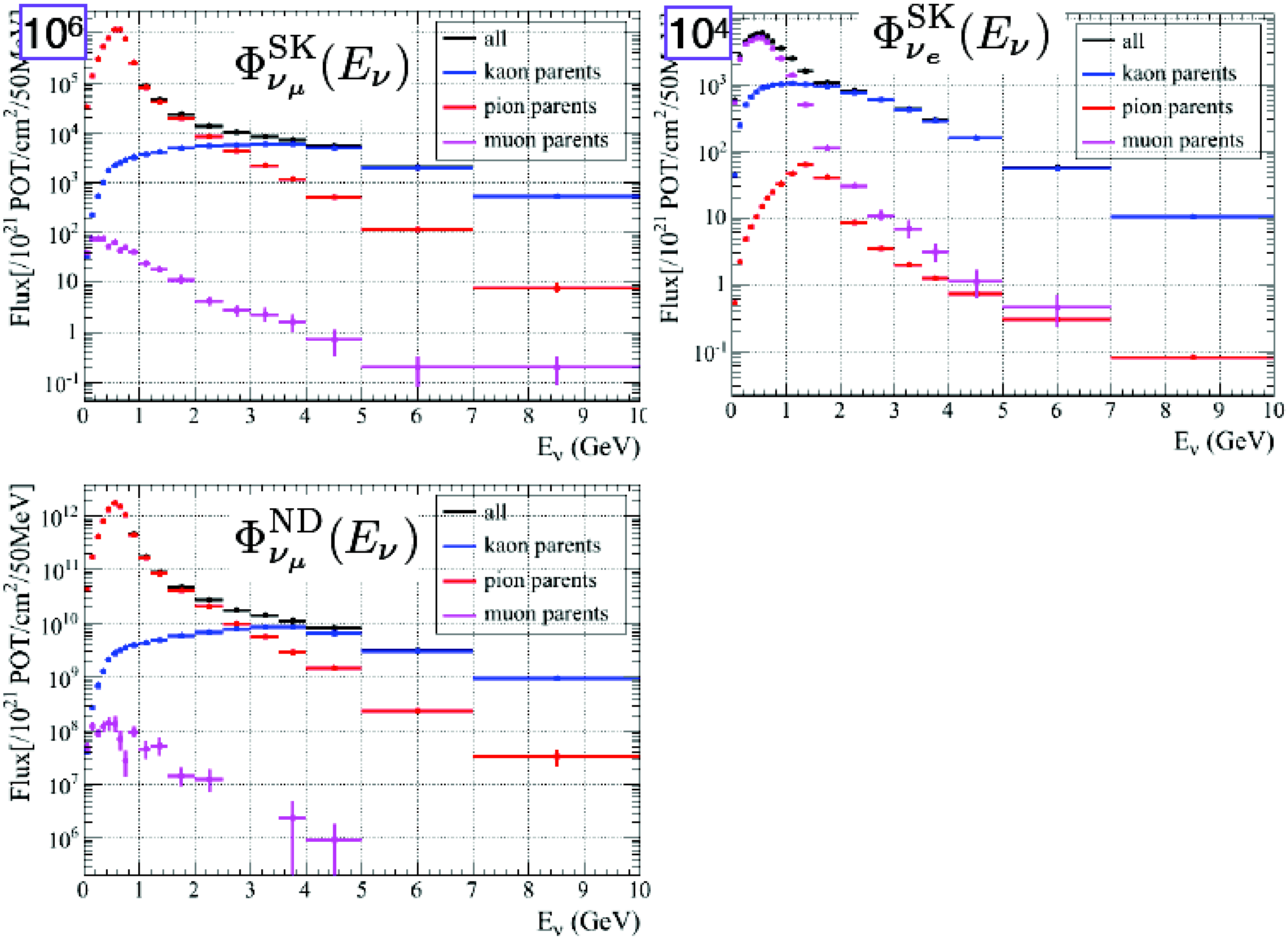
typically 5-10%

The normalization uncertainty is 2.3% on the overall (p,θ)

→ Propagate the systematic uncertainty in each (p,θ) bin into the expected number of events in T2K

→ Input to T2K neutrino beam simulation

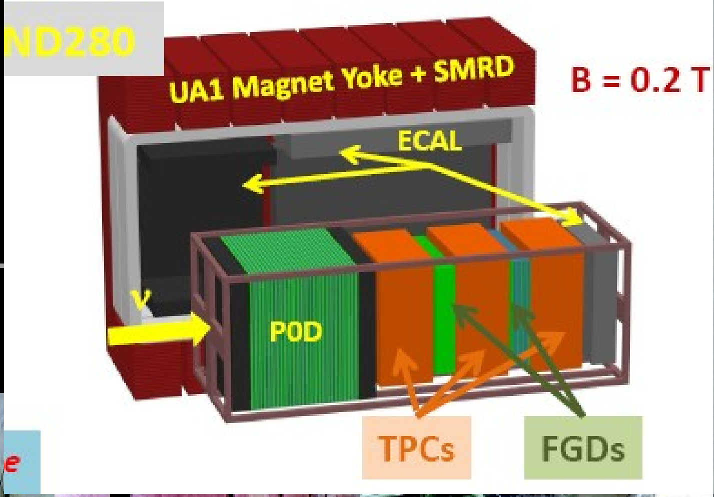
Predicted neutrino flux (center value)

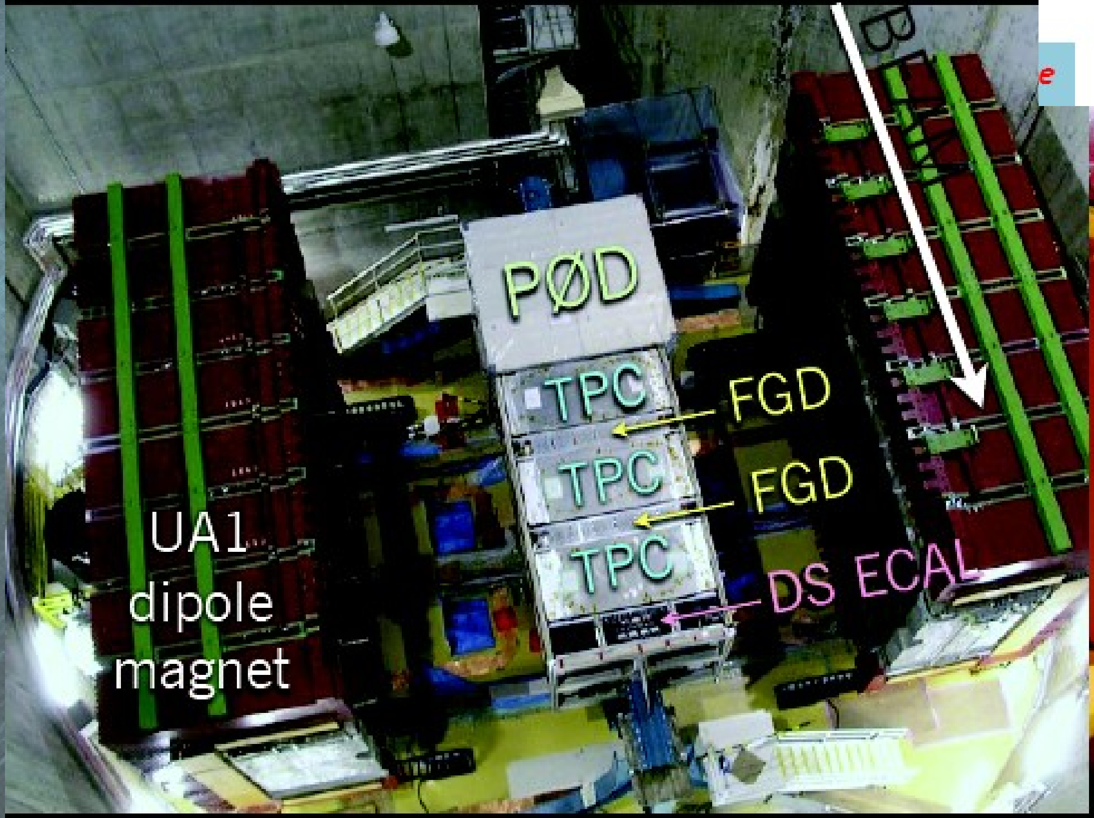


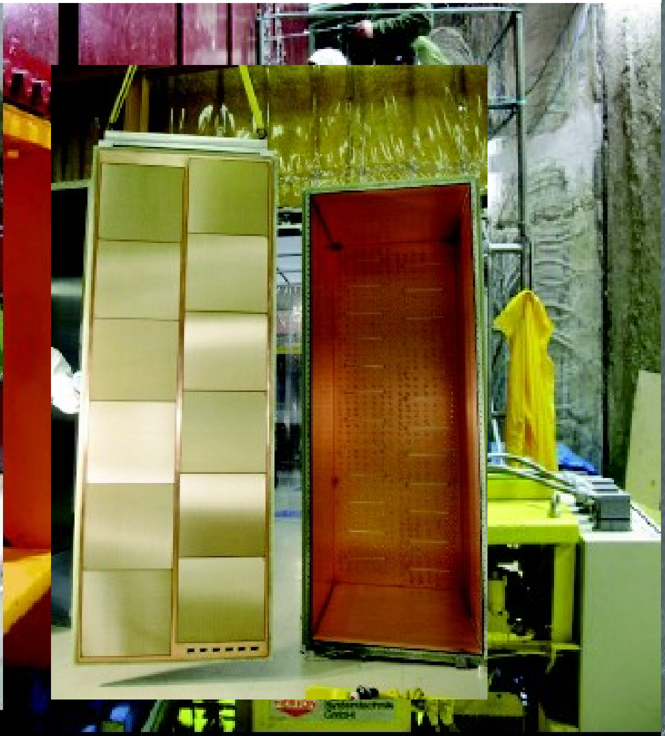
Controlling the neutrino flux with near detectors

- Two strategies:
- 1) use a near detector (almost) identical to far detectors (eg reactor experiments)
- 2) use the most sophisticated near detector possible (T2K)

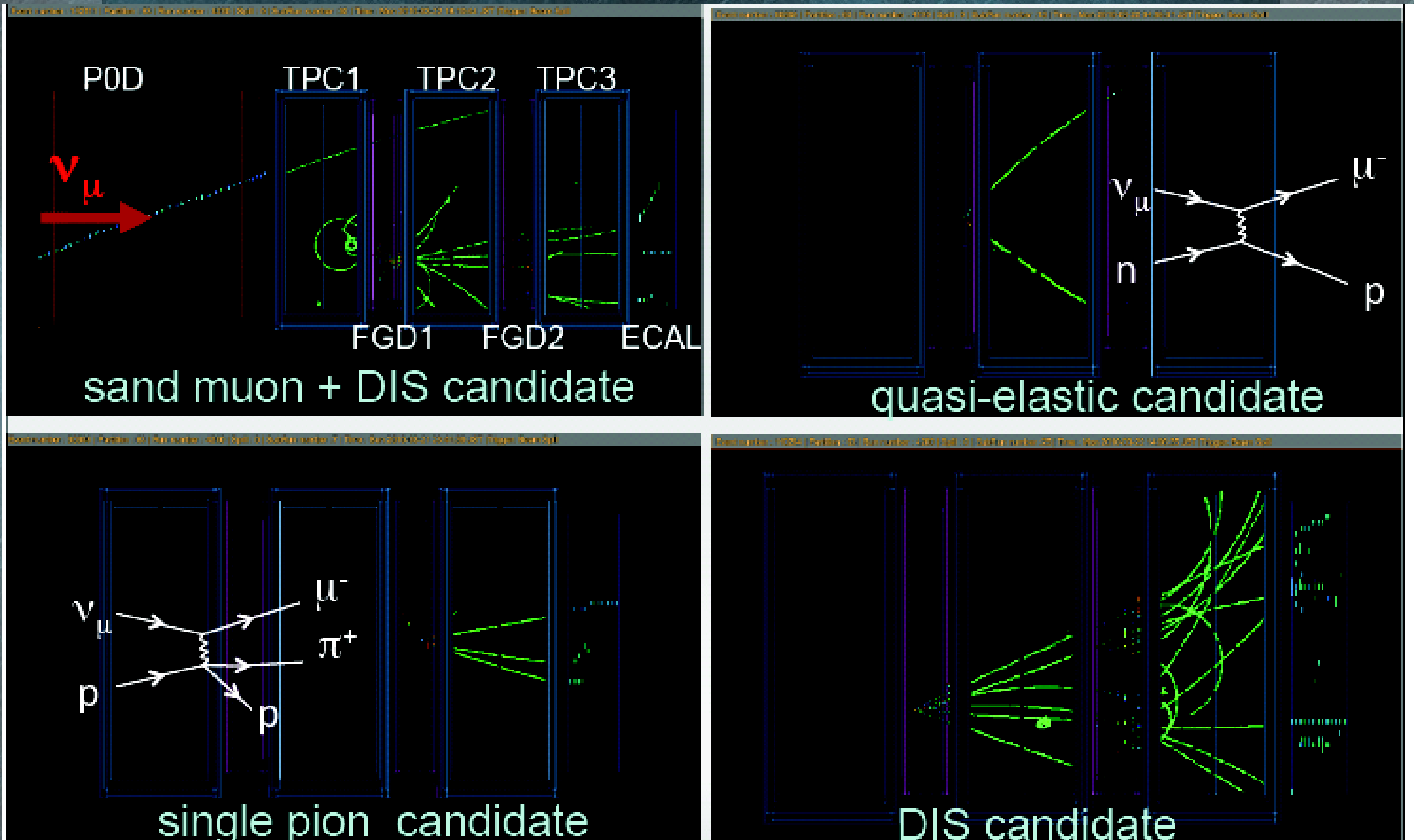
Off-axis Near Detector



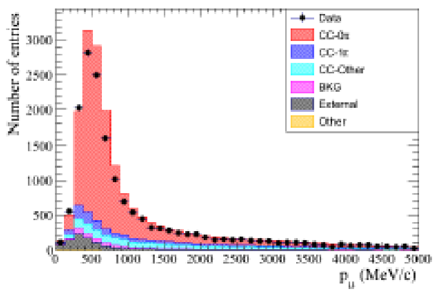


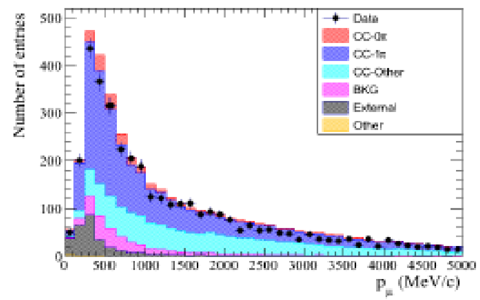


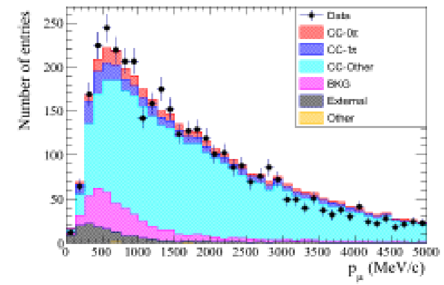
Neutrino interactions in ND280



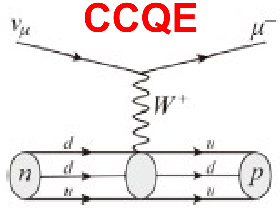
T2K Near detector constraint



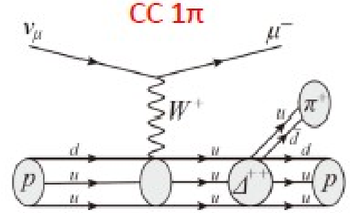




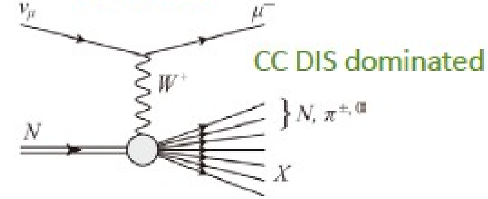
CCQE



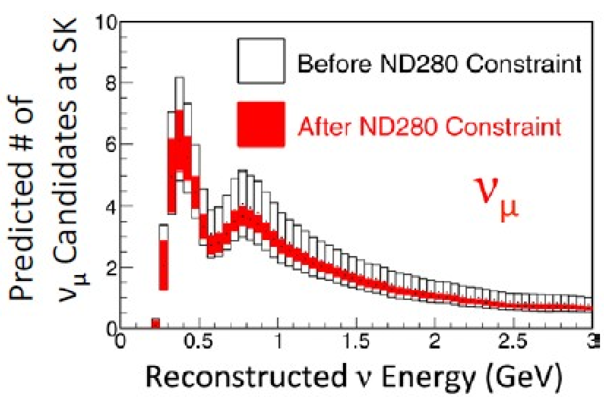
CC 1π

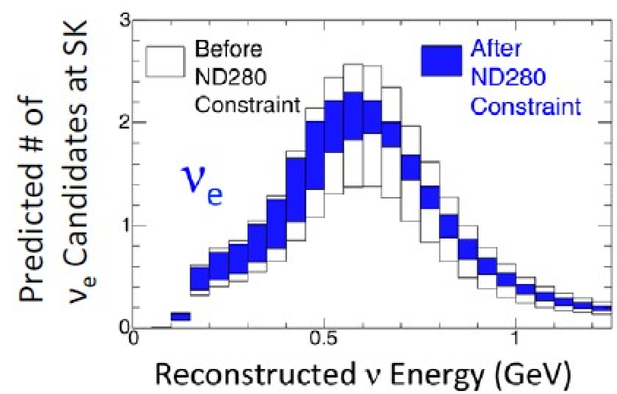


CC Other



Flux and cross-section systematic uncertainty on N_{SK} significantly reduced to ~7%





Neutrino detectors

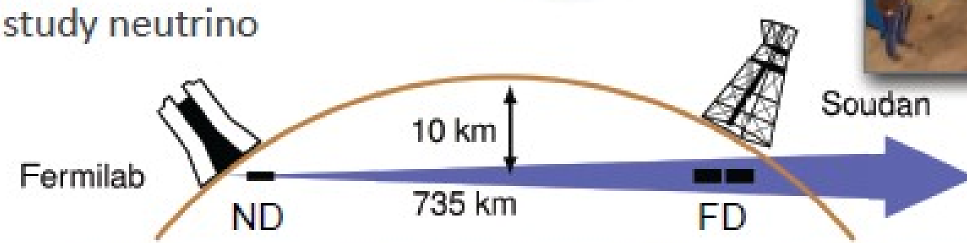
- Need a large instrumented mass to overcome the small cross-section
- A calorimeter fulfills the requirements, especially if it is totally active
- Main options
  - Water Cherenkov (SuperKamiokande, Antares, IceCube) (up to 1 Mt envisaged)
  - Liquid Scintillator (Kamland, JUNO)
  - Liquid Argon TPC (Icarus, LBNE, LBNO)
  - Magnetised iron, scintillator (MINOS)

The MINOS+ Concept

- Long-baseline neutrino oscillation experiment

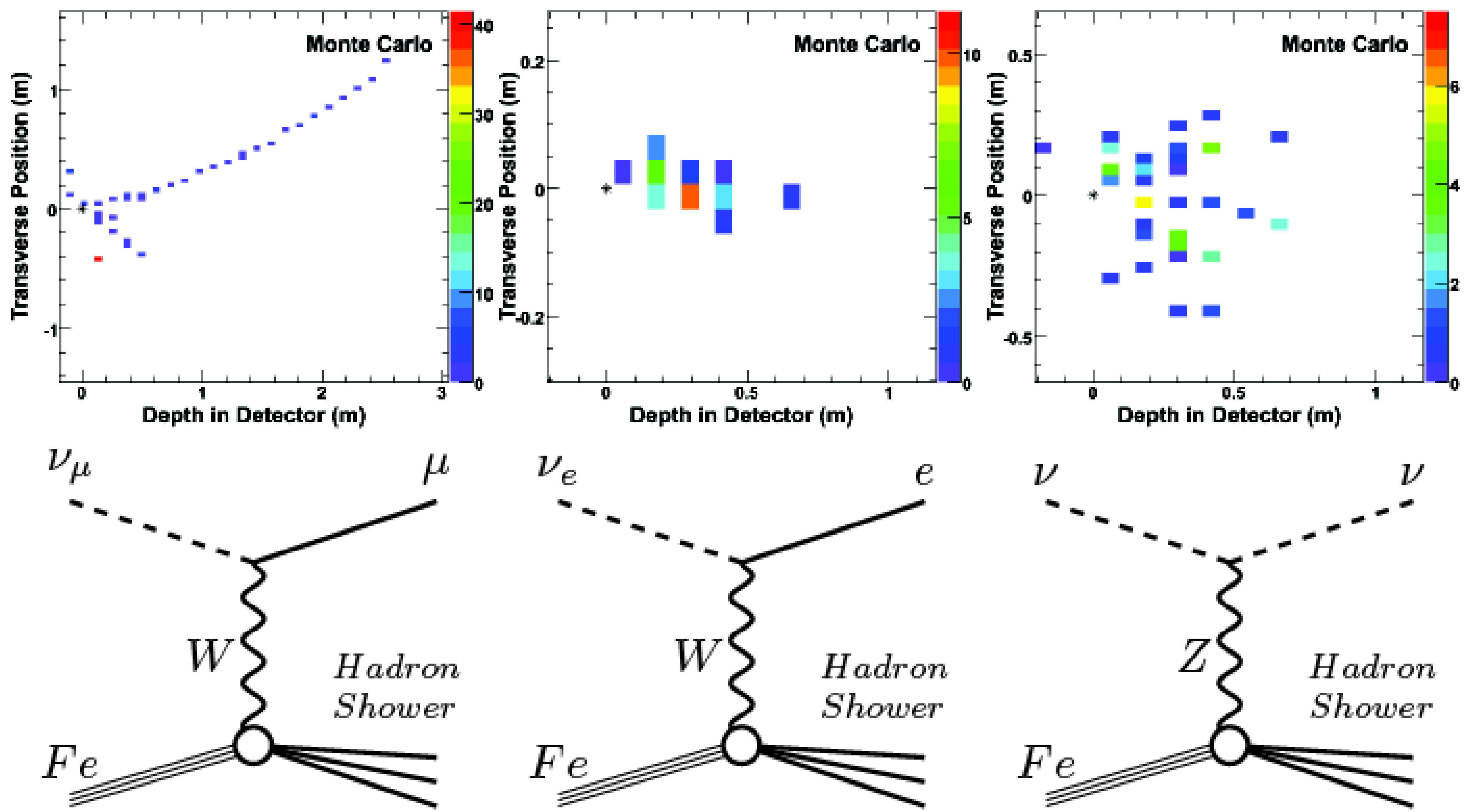
- Measure NuMI Neutrino beam energy and flavor composition with two detectors over 735 km
  - L/E ~ 500 km/GeV

- Near Detector at Fermilab
- Far Detector at Soudan Underground Lab, MN
- Compare Near and Far measurements to study neutrino mixing



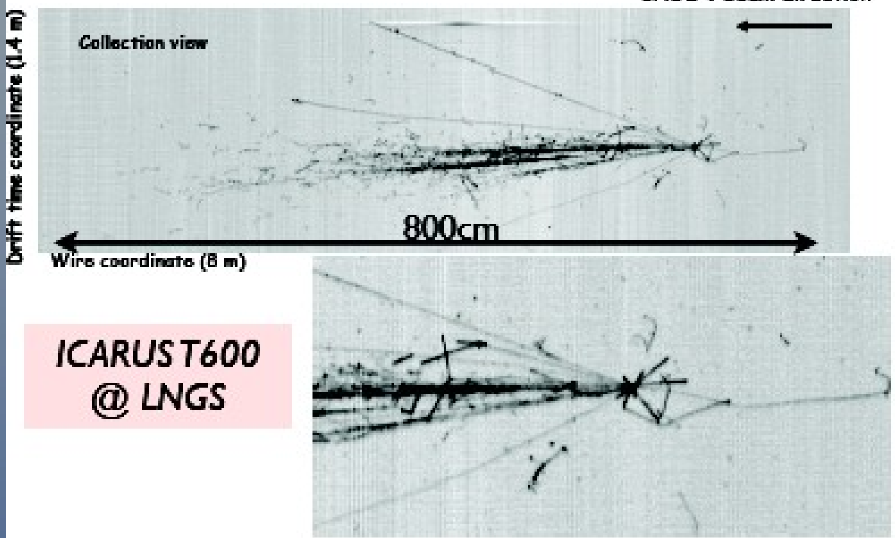
MINOS event topology

ν_μ-CC event | ν_e-CC event | NC event

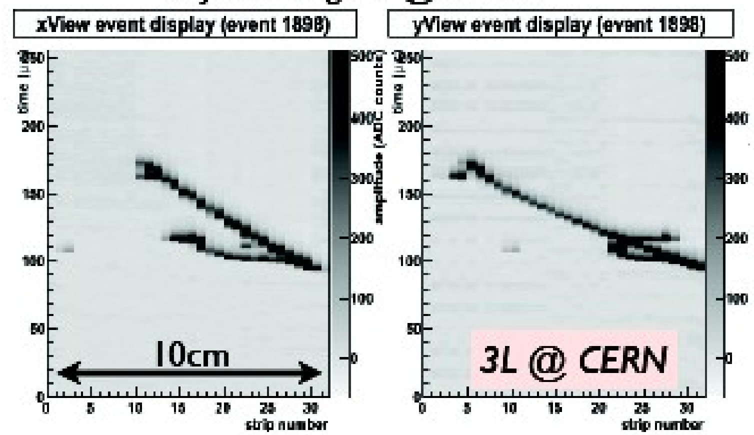


Liquid Argon Detectors (LArTPC)

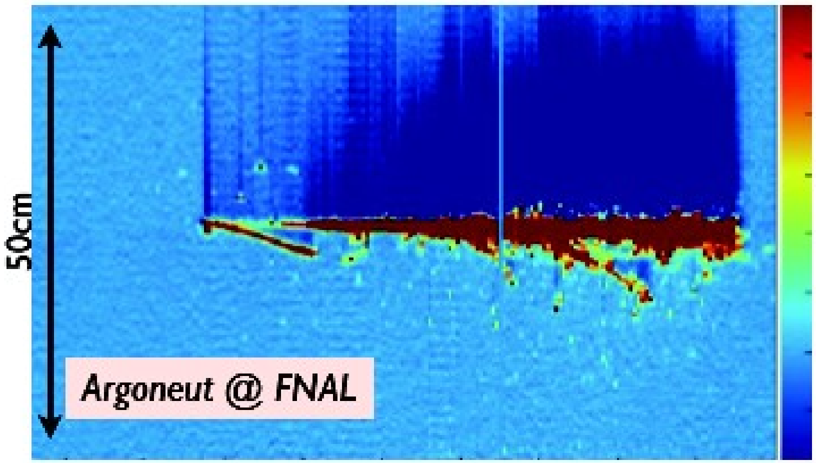
"electronic bubble chambers"



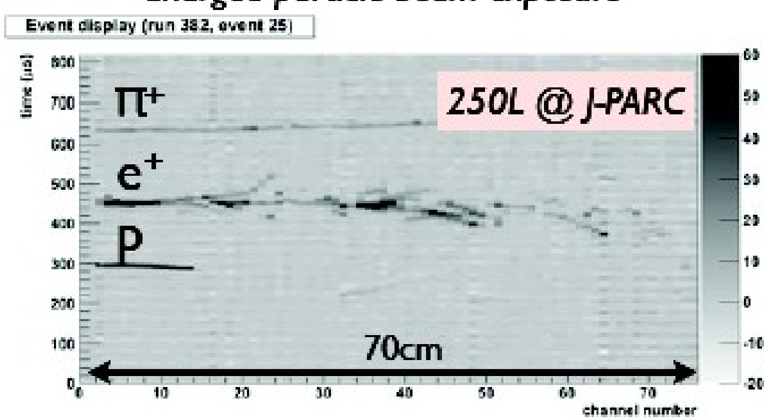
Cosmic track in double phase 3L LAr-LEM TPC with adjustable gain @ CERN



Much improved S/N (>100) compared to single-phase LAr operation (≈15)



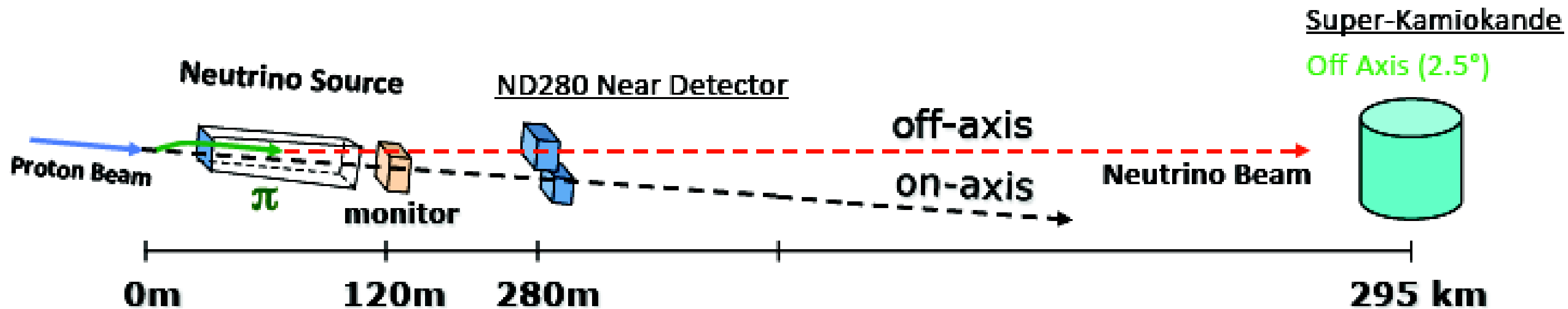
Charged particle beam exposure

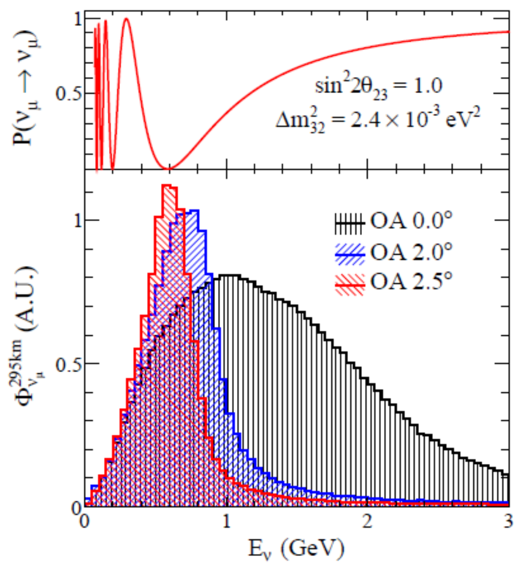


The Tokai to Kamioka (T2K) experiment

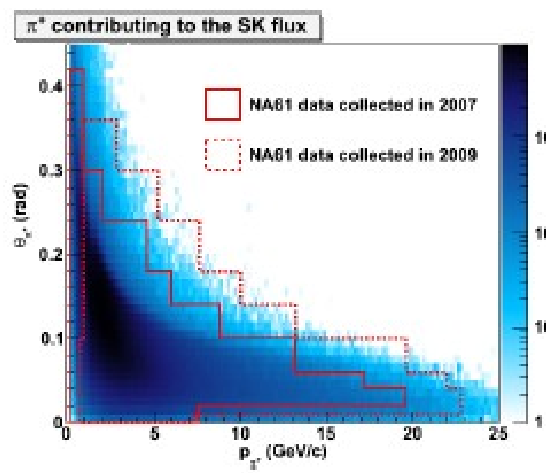
- Long-baseline (295 km) neutrino experiment in Japan between J-PARC (Tokai) and Super-Kamiokande (SK).
- Primary proton beam: 30 GeV/c, 235 kW (RUN4) 6.57 10^{20} Proton On Target (8% of the final design exposure)
- SK: 22.5 kt fiducial mass. ~100% livetime

T2K: Main Experimental Features

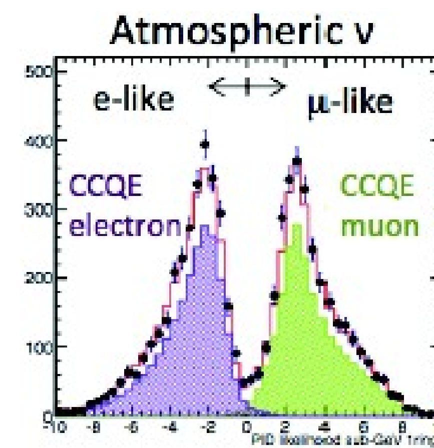




Off-axis beam.
Flux has a narrow peak tuned for the first oscillation maximum



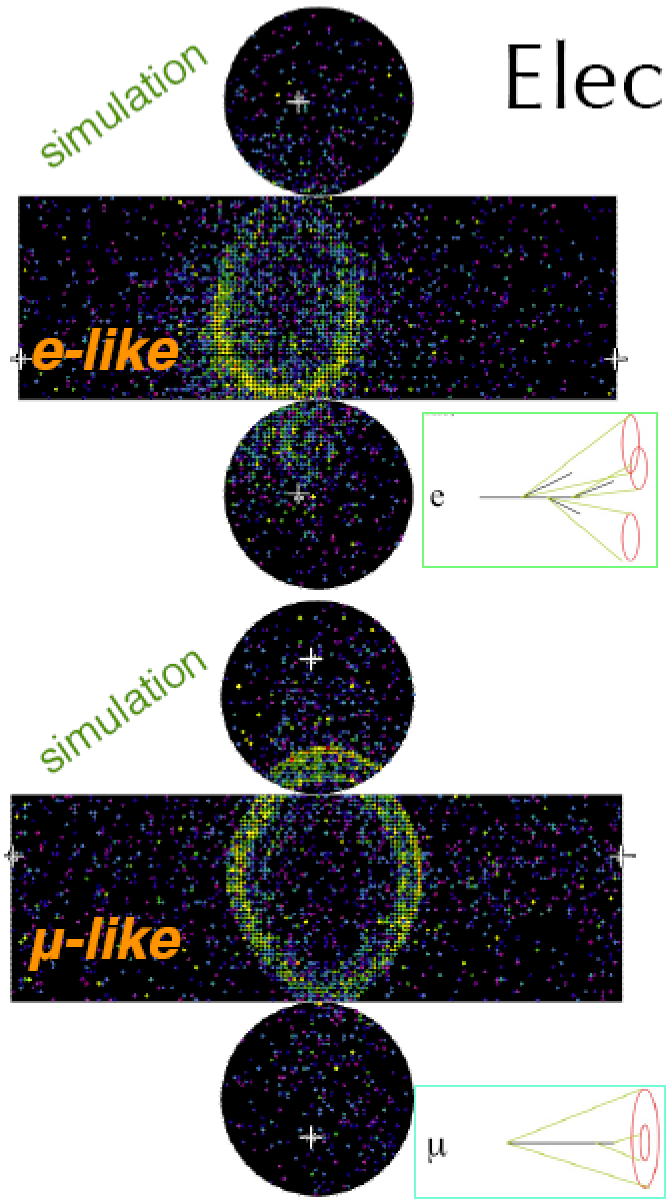
Pion and kaon production measured by the NA61 exp. at CERN



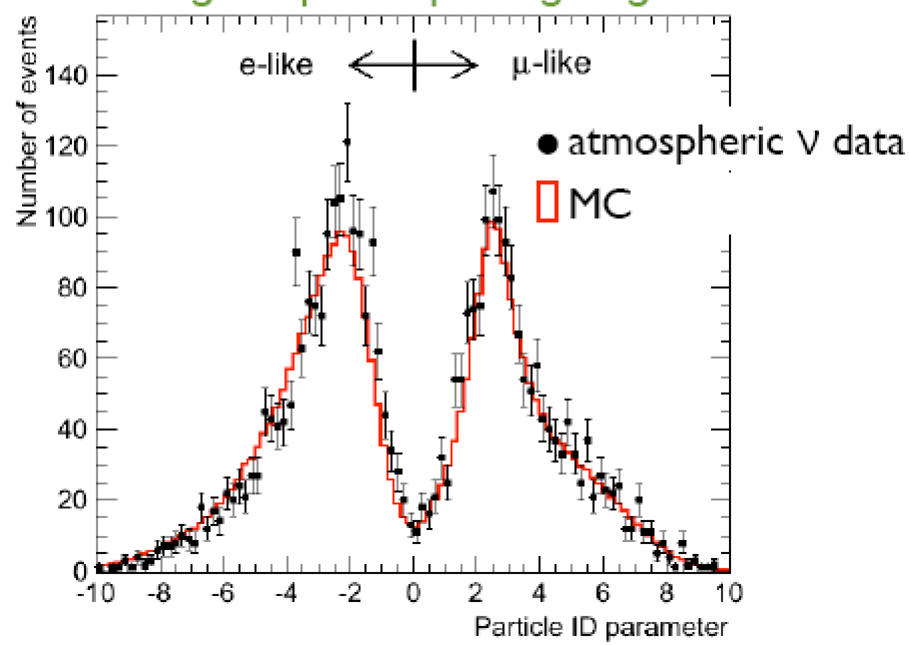
Excellent particle identification capabilities in SK (misid <1%)

Super Kamiokande: electron muon

Electron-like and muon-like event at SK



Particle identification using ring shape & opening angle

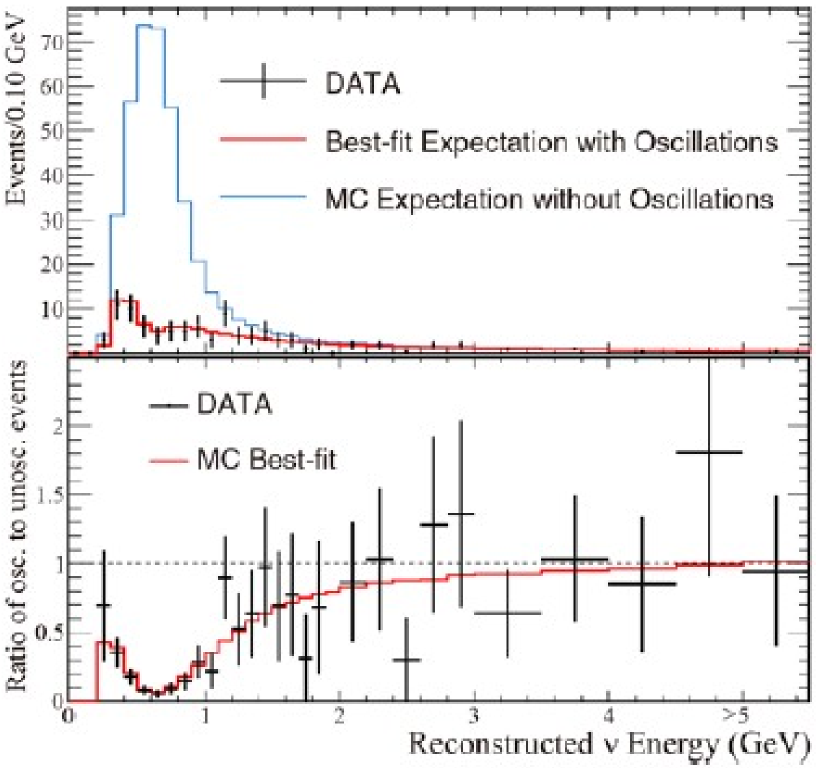


Probability that μ is mis-identified as electron is ~1%

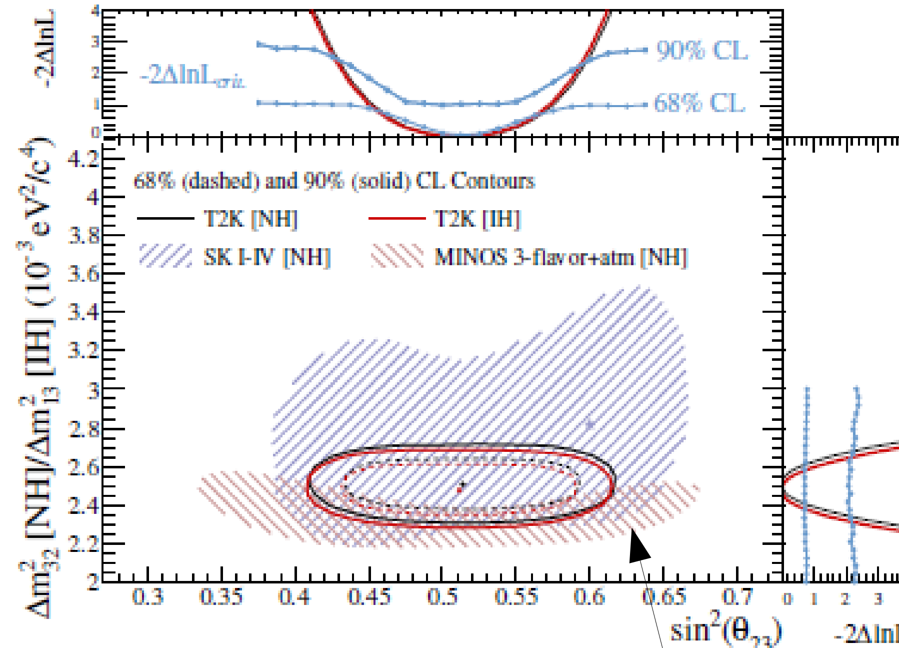
$\nu_\mu \rightarrow \nu_\mu$

Phys. Rev. Lett. 112, 181801 (2014)

T2K ν_μ disappearance measurement



Talk by A. Himmel



Notice recent MINOS 3-flavor result

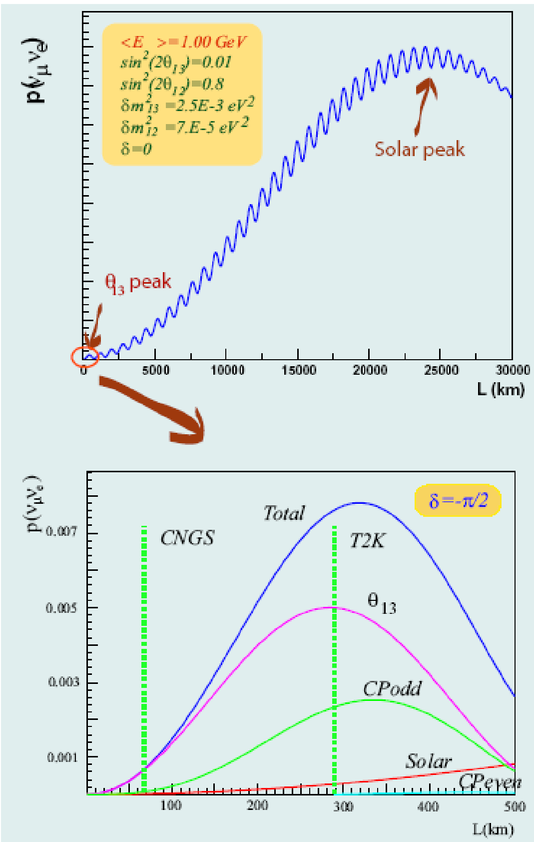
Oscillation parameters		Best-fit value
NH	$\sin^2(\theta_{23})$	$0.514^{+0.055}_{-0.056}$
	Δm^2_{32} (10^{-3} eV^2)	2.51 ± 0.10
IH	$\sin^2(\theta_{23})$	0.511 ± 0.055
	Δm^2_{13} (10^{-3} eV^2)	2.48 ± 0.10

N_{obs} = 120

N_{exp} (no osc) = 446±23(syst.)

Data favor maximum disappearance.

Sub leading $\nu_\mu - \nu_e$ oscillations



$$p(\nu_\mu \to \nu_e) = 4c_{13}^2 s_{13}^2 s_{23}^2 \sin^2 \frac{\Delta m_{13}^2 L}{4E} \quad \theta_{13} \text{ driven}$$

$$+ 8c_{13}^2 s_{12}s_{13}s_{23}(c_{12}c_{23}cos\delta - s_{12}s_{13}s_{23}) \cos \frac{\Delta m_{23}^2 L}{4E} \sin \frac{\Delta m_{13}^2 L}{4E} \sin \frac{\Delta m_{12}^2 L}{4E} \quad \text{CPeven}$$

$$- 8c_{13}^2 c_{12}c_{23}s_{12}s_{13}s_{23}\sin\delta \sin \frac{\Delta m_{23}^2 L}{4E} \sin \frac{\Delta m_{13}^2 L}{4E} \sin \frac{\Delta m_{12}^2 L}{4E} \quad \text{CPodd}$$

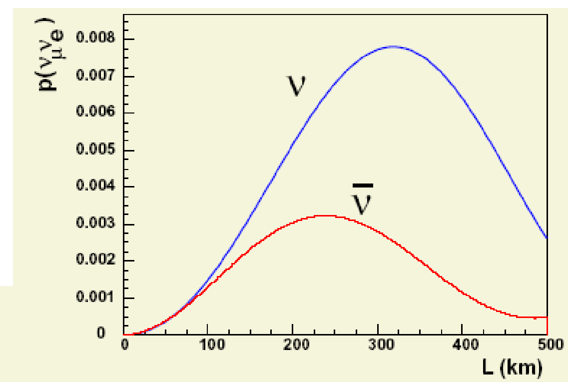
$$+ 4s_{12}^2 c_{13}^2 \{c_{13}^2 c_{23}^2 + s_{12}^2 s_{23}^2 s_{13}^2 - 2c_{12}c_{23}s_{12}s_{23}s_{13}cos\delta\} \sin \frac{\Delta m_{12}^2 L}{4E} \quad \text{solar driven}$$

$$- 8c_{12}^2 s_{13}^2 s_{23}^2 \cos \frac{\Delta m_{23}^2 L}{4E} \sin \frac{\Delta m_{13}^2 L}{4E} \frac{aL}{4E}(1 - 2s_{13}^2) \quad \text{matter effect (CP odd)}$$

θ_{13} discovery requires total probability greater than solar driven probability

Leptonic CP discovery requires

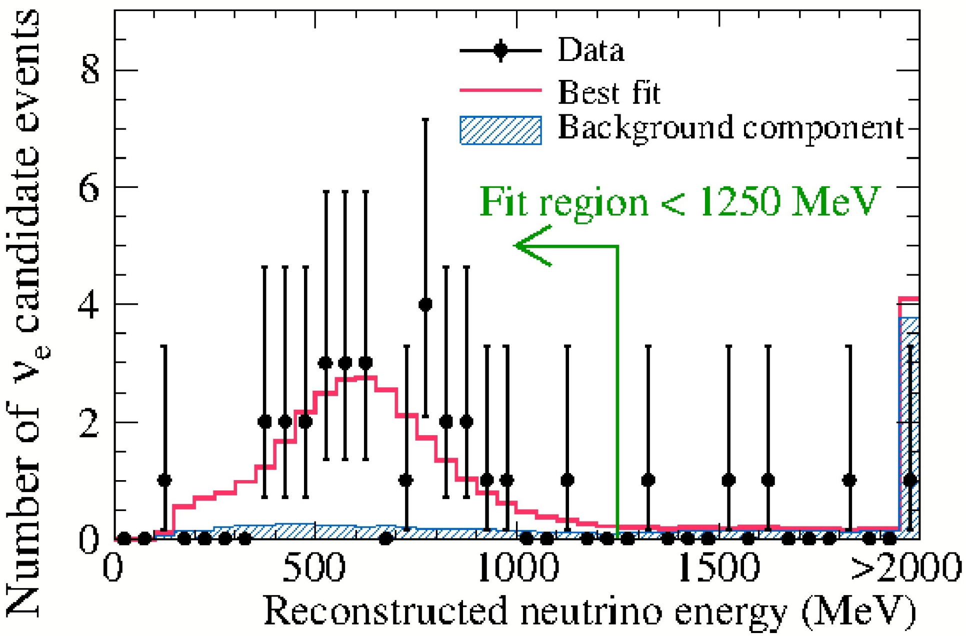
$$A_{CP} = \frac{P(\nu_\mu \to \nu_e) - P(\bar{\nu}_\mu \to \bar{\nu}_e)}{P(\nu_\mu \to \nu_e) + P(\bar{\nu}_\mu \to \bar{\nu}_e)} \neq 0$$

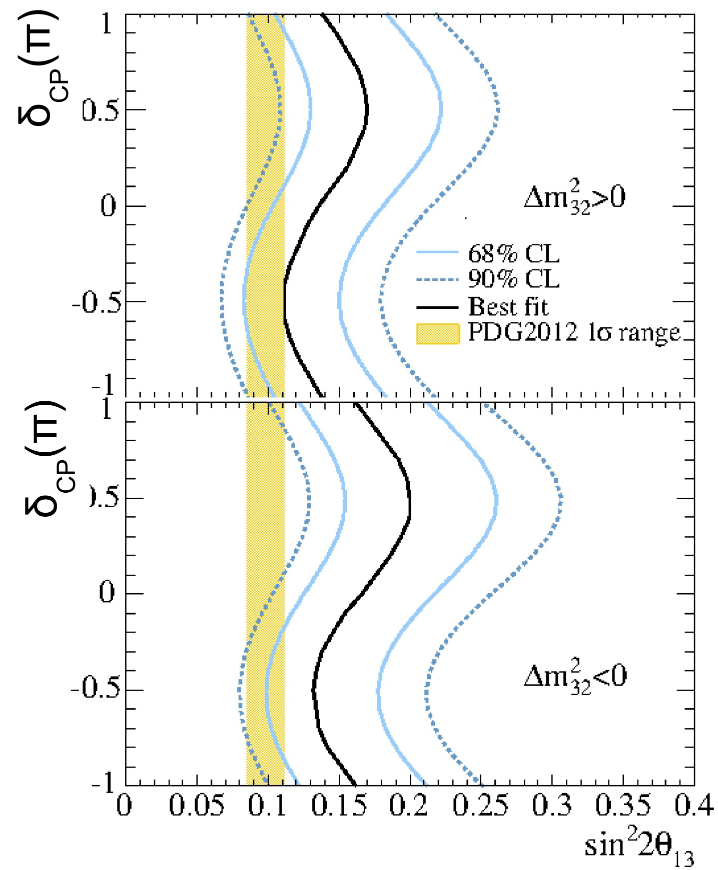


$\nu_\mu \rightarrow \nu_e$

Phys. Rev. Lett. 112, 061802 (2014)

T2K observation of $\nu_\mu \rightarrow \nu_e$ appearance





N_{obs} = 28

N_{exp} (bck. only) = 4.9±0.6(syst.)

N_{exp} ($\sin^2 2\theta_{13}$=0.1) = 21.6

7.3 σ evidence of non-zero θ_{13}

First direct observation of a new flavor appearance

Opens the way to the determination of the CP violation parameter δ

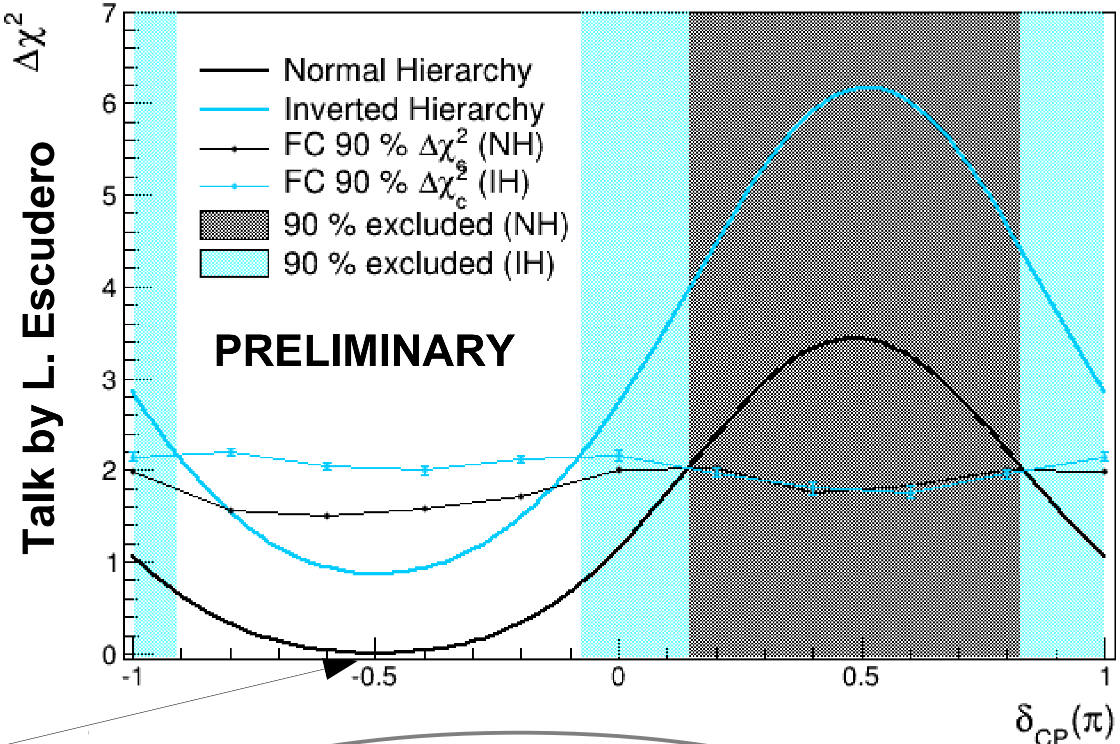
T2K combined fit to appearance and disappearance data

Combined fit to the ν_μ and ν_e samples

Using PDG 2013 θ_{13} T2K obtains an indication favoring $\delta = -\pi/2$

Similar results from an independent analysis using Markov chain MC producing Bayesian credible intervals

If nature has chosen this happy spot: a) a generous help to experiments b) a solution that satisfies the leptogenesis bound with no additional CP violation



Leptogenesis bound

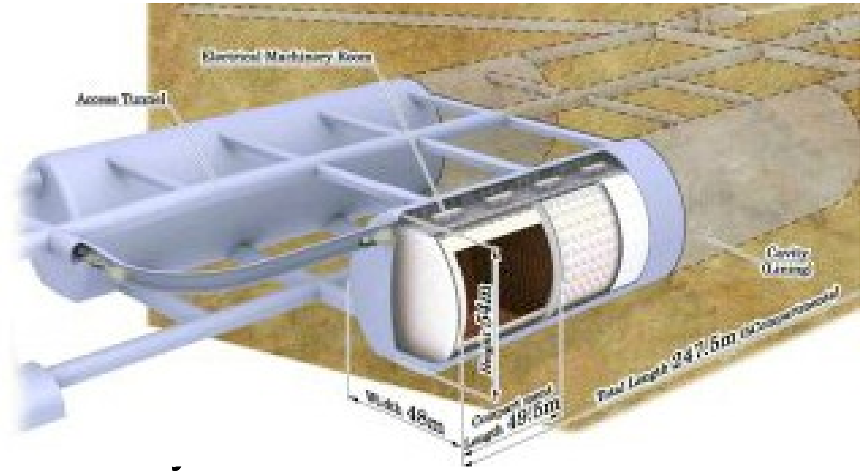
$$|\sin\theta_{13}\sin\delta| \geq 0.11$$

Pascoli Petcov Riotto 2007

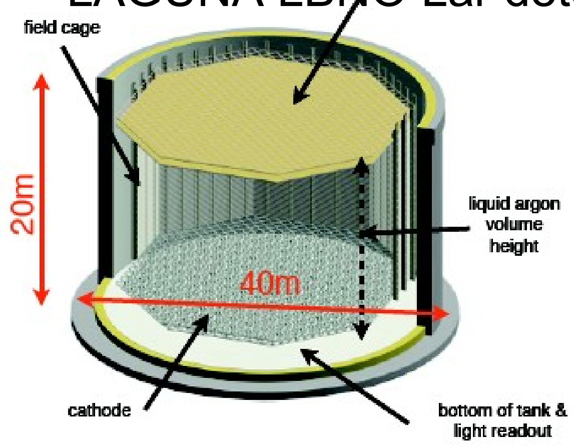
Future long baseline projects

- Aim at measuring CP violation effects in the neutrino sector
- Technique is the same as the current operating long baseline accelerator experiments but higher beam power and larger detector mass
- Two strategies:
  - Short baseline (~100-300 km), lower energy (<1 GeV), narrow beam, large Water Cherenkov (~500 kT). Concentrates on $\nu/\bar{\nu}$ asymmetry, “counting” experiment.
  - Longer baseline (>1000 km), higher energy (>1 GeV), wide beam, Liquid Argon TPC. All final states accessible, E/L oscillation pattern and second maximum

HK: 20 times SK: 0.5 Mt fiducial



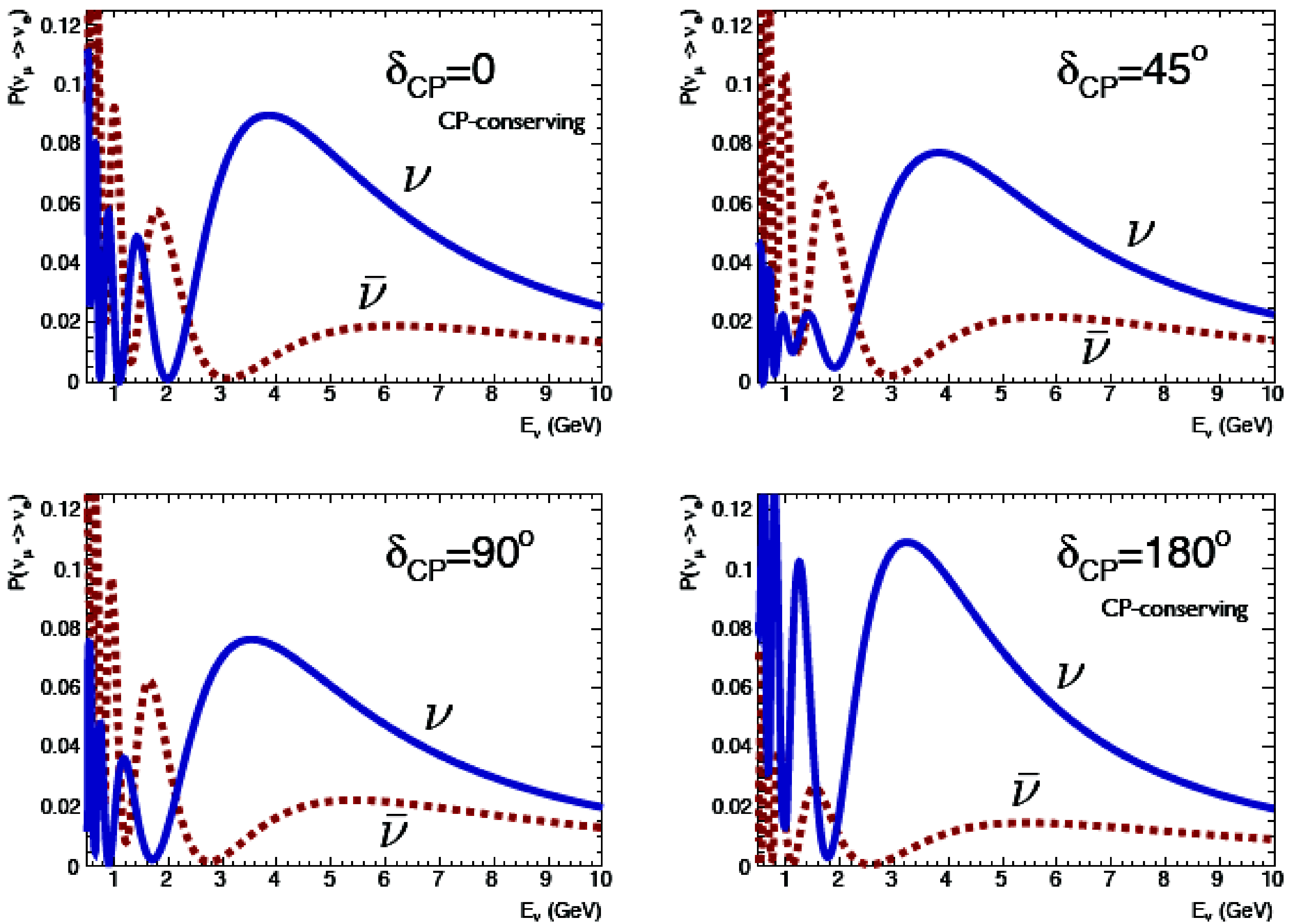
LAGUNA LBNO Lar detector



CERN-Pyhäsalmi: oscillations

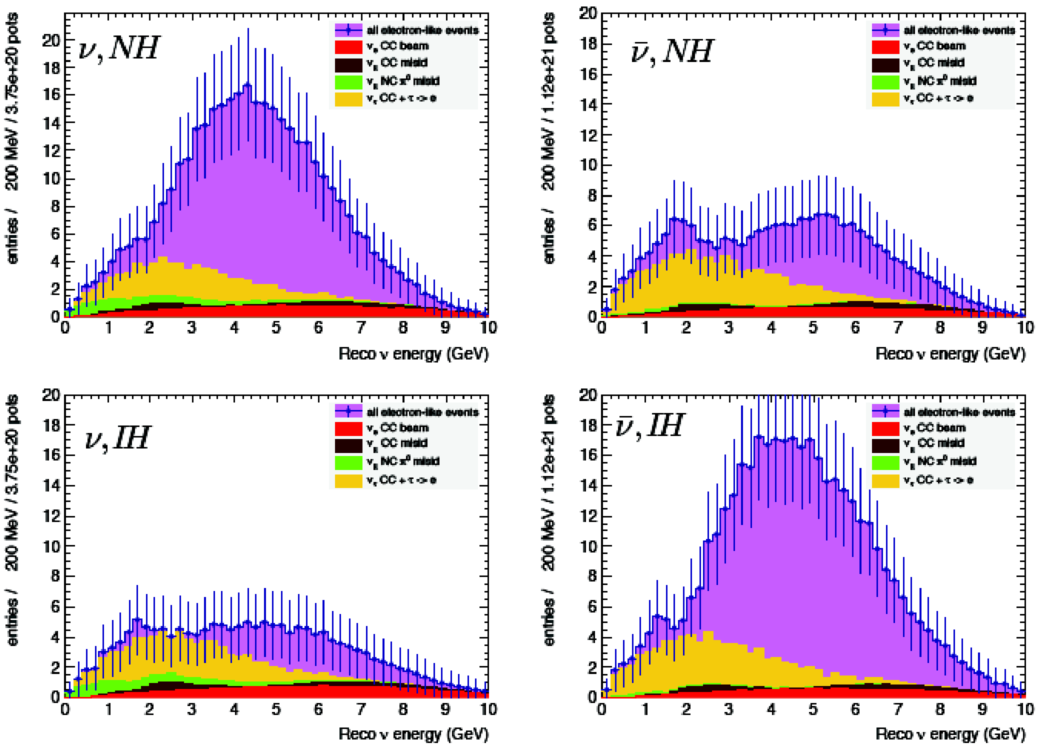
★Normal mass hierarchy L=2300 km

$\sin^2(2\theta_{13}) = 0.09$

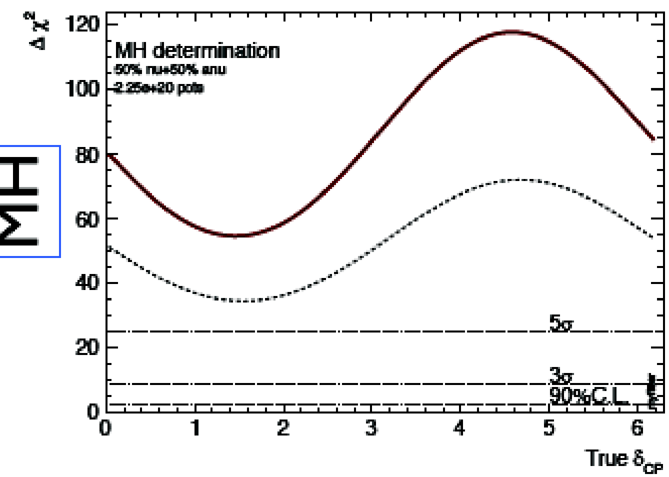


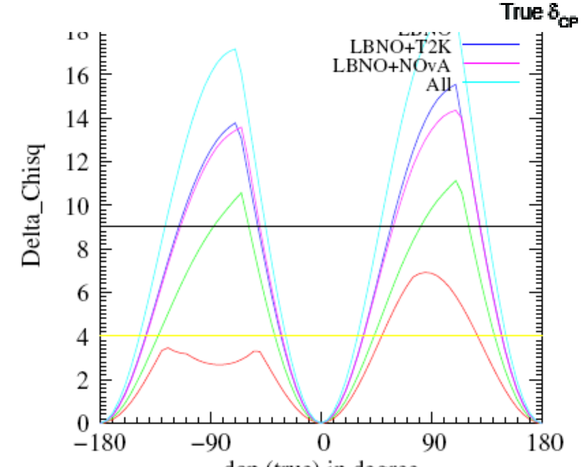
Projected performances

- Mass hierarchy : 100 % coverage at 5 σ in a few years
- CPV: 71 (44) % coverage at 90% (3σ)in 10 years



MH





Conclusions

- Neutrino oscillations established
- Increased precision measurement on the angles θ_{13} and θ_{23}
- Next step will be the measurement of the CP violation (relation to leptogenesis)

Tri-bimaximal Mixing

Harrison, Perkins, Scott

A simple mixing matrix compatible with all present data

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

In the basis of diagonal ch. leptons:

$m_\nu = U \mathrm{diag}(m_1, m_2, m_3) U^T$

$$m_\nu = \frac{m_3}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} + \frac{m_2}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} + \frac{m_1}{6}\begin{bmatrix} 4 & -2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{bmatrix}$$

Eigenvectors: $m_3 \to \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$ $\quad m_2 \to \frac{1}{\sqrt{3}}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $\quad m_1 \to \frac{1}{\sqrt{6}}\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$

Note: mixing angles independent of mass eigenvalues

CPT and CP in neutrino oscillations

- Prob($\nu_\alpha \to \nu_\beta$) → CPT → Prob($\bar{\nu}_\beta \to \bar{\nu}_\alpha$)
- then Prob($\nu_\alpha \to \nu_\alpha$) → CPT → Prob($\bar{\nu}_\alpha \to \bar{\nu}_\alpha$)
- We cannot test CP violation using the disappearance probability
- We need to have a flavor in the final state that is different from the initial state to probe CP violation -> search for CP violation in appearance probability

Testing CP violation in ν sector

Changes sign under CP!

$$\text{Prob}\left(\alpha \to \beta\right) = \delta_{\alpha\beta} - \sum_{i<j} 4\,\text{Re}\left(J^{\alpha\beta}{}_{ij}\right)\sin^2\phi_{ij} - \sum_{i<j} 2\,\text{Im}\left(J^{\alpha\beta}{}_{ij}\right)\sin\phi_{ij}$$

$$J^{\alpha\beta}{}_{ij} \equiv U_{\alpha i} U_{\beta i}^{*} U_{\alpha j}^{*} U_{\beta j} \quad \phi_{ij} \equiv \phi_i - \phi_j = \frac{\Delta M_{ij}^2 L}{2p} \quad \Delta M_{ij}^2 = M_i^2 - M_j^2$$

$$A_{\alpha\beta}^{CP} \equiv \frac{P(\nu_\alpha \to \nu_\beta) - P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)}{P(\nu_\alpha \to \nu_\beta) + P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)} = \frac{2\sin\delta c_{13}\sin 2\theta_{13} \overbrace{\sin 2\theta_{12}\frac{\Delta m_{12}^2 L}{4E_\nu}}^{\text{solar}} \overbrace{\sin 2\theta_{23}\sin^2\frac{\Delta m_{13}^2 L}{4E_\nu}}^{atmos}}{P_{\nu_\alpha\nu_\beta}^{\text{CP-even}}}$$

Need to take into account large “fake CP” effects due to propagation of the neutrinos in the matter