Neutrino masses and mixing







We would like to investigate if this reaction, a neutrino flavor transformation during the propagation, is possible, both theoretically and experimentally.

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Neutrino oscillations: well established !



10 8. (1997) μ→μ (Δm², θ₂₃)



 $\mu \rightarrow \mu (\Delta m^2, \theta_{23})$



e →e (Δm²,θ₁₃)



μ→e (Δm²,θ₁₃,θ₂₃)



μ→τ (Δm², θ₂₃)

Data from various types of neutrino experiments: (a) solar, (b) long-baseline reactor, (c) atmospheric, (d) long-baseline accelerator, (e) short-baseline reactor, (f,g) long baseline accelerator (and, in part, atmospheric).

(a) KamLAND [plot]; (b) Borexino [plot], Homestake, Super-K, SAGE, GALLEX/GNO, SNO; (c) Super-K atmosph. [plot], MACRO, MINOS etc.; (d) T2K (plot), MINOS, K2K;
 (e) Daya Bay [plot], RENO, Double Chooz; (f) T2K [plot], MINOS; (g) OPERA [plot], Super-K atmospheric.



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а.

NB: u->v ; d->l Flavor and the Standard Model

The Standard Model is based on the gauge group $SU(3)_{c}xSU(2)_{L}xU(1)_{Y}$, with two interaction terms for fermions :

$$\mathcal{L}_W = -\sqrt{rac{1}{2}g\overline{u_{Li}^I}}\gamma^\mu \mathbf{1}_{ij}d_{Lj}^IW^+_\mu + ext{h.c.}.$$

$$\mathcal{L}_M = -\sqrt{rac{1}{2}} v G_{ij} \overline{d^I_{Li}} d^I_{Rj} - \sqrt{rac{1}{2}} v F_{ij} \overline{u^I_{Li}} u^I_{Rj} + \mathrm{h.c.},$$

Interaction q_L-W (charged current)

Interactions q-Higgs, mass terms

If we consider the mass eigenstates, the charged current is not diagonal in the flavor space $\mathcal{L}_W = -\sqrt{\frac{1}{2}}g\overline{u_{Li}}\gamma^{\mu}V_{ij}d_{Lj}W^+_{\mu} + \text{h.c.}$

$$V_{dL}M_dV_{dR}^\dagger = M_d^{
m diag}, ~~ V_{uL}M_uV_{uR}^\dagger = M_u^{
m diag},$$

With $M_u \propto vG$, $M_d \propto vF$ and $V=V_{uL}V_{dL}^{\dagger}$



The unitary matrix V (CKM for quarks, PMNS for leptons) depends on 3 angles et one (or more) phase

Principle of neutrino oscillations



Notice that the flavor eigenstates differ from the mass eigenstates. Therefore the source produces a linear superposition of the mass eigenstates

$$\mathbf{v}_{\alpha} = \sum_{i}^{N} U_{\alpha i} \mathbf{v}_{i}$$

Greek indices for flavor Latin for mass eigenstates

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Principle of neutrino oscillations

$$\begin{split} |v_{\alpha}\rangle &= \sum_{i}^{N} U_{\alpha i} |v_{i}\rangle & \text{At t=0 a flavor eigenstate} \\ |v(\vec{x},t)\rangle &= \sum_{i}^{N} U_{\alpha i} e^{-ip_{i}x} |v_{i}\rangle & \text{At time t it has evolved} \\ p_{i}x &= E_{i}t - pL = p(t-L) + \frac{M_{i}^{2}L}{2p} \\ E_{i} &= \sqrt{p^{2} + M_{i}^{2}} \approx p + \frac{M_{i}^{2}}{2p} & \text{Assume our v are relativistic M}$$

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Phase shift $(p_1-p_2)x \sim \Delta m^2 L/4p =>$ Interference

"Neutrino oscillation" is due to the phase shift between the lighter states (in advance with respect to phase) versus the heavier. Out of phase linear superposition means the other flavor eigenstates (not present at t=0) appear during the propagation.



 $L_{osc} = 2\pi \frac{2p}{\Delta m^2}$

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Two neutrinos oscillation



$$P(v_e \to v_\mu) = \sin^2 2\theta \sin^2 \frac{\left[\Delta m^2 L\right]}{4p}$$
$$P(v_e \to v_e) = 1 - P(v_e \to v_\mu) = 1 - \sin^2 2\theta \sin^2 \frac{\left[\Delta m^2 L\right]}{4p}$$

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Typical Lengths-Δm²

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L_osc ~ 2.5 km E (GeV)/ Δm^2 (eV²)

Source	Energy	Distance	Δm ²
Reactor	4 MeV	100m	0.1 eV ²
Accelerator	1 GeV	1 km	2.5 eV ²
Atmospheric	400 MeV	10 km	0.1 eV ²
		10 000 km	10 ⁻⁴ eV ²
Sun	1 MeV	500s	10 ⁻¹¹ eV ²
	1 GeV	1000 km	2.5 10 ⁻³ eV ²
	1 MeV	25 km	8 10 ⁻⁵ eV ²
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Principle of neutrino oscillations

$$|v(L)\rangle = \sum_{\beta} \left[\sum_{i}^{N} U_{ai} e^{-i\frac{M_{i}^{2}L}{2p}} U_{\beta i^{i}} \right] |v_{\beta}\rangle$$

$$A(\alpha \rightarrow \beta) = \langle v_{\beta} | v(L) \rangle = \left[\sum_{i}^{N} U_{ai} e^{-i\frac{M_{i}^{2}L}{2p}} U_{\beta i^{i}} \right]$$

$$Prob \ (\alpha \rightarrow \beta) = |\langle v_{\beta} | v(L) \rangle|^{2} = \sum_{ij} J^{\alpha\beta}{}^{ij} (1 - 2\sin^{2}\phi_{ij} + i\sin 2\phi_{ij})$$

$$i \delta_{\alpha\beta} - \sum_{i < j} 4 \operatorname{Re}(J^{\alpha\beta}{}^{ij}) \sin^{2}\phi_{ij} - \sum_{i < j} 2 \operatorname{Im}(J^{\alpha\beta}{}^{ij}) \sin \phi_{ij}$$

$$e^{2i\phi} = 1 - 2\sin^{2}\phi + i\sin 2\phi$$

$$J^{\alpha\beta}{}^{ij} \equiv U_{ai} U_{\beta i^{i}} U_{\alpha j^{i}} U_{\beta j} \phi_{ij} \equiv \phi_{i} - \phi_{j} = \frac{\Delta M_{ij}^{2}L}{2p} \Delta M_{ij}^{2} = M_{i}^{2} - M_{j}^{2}$$

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Principle of neutrino oscillations

$$\operatorname{Prob}\left(\alpha \to \beta\right) = \delta_{\alpha\beta} - \sum_{i < j} 4\operatorname{Re}(J^{\alpha\beta}{}_{ij})\sin^2\varphi_{ij} - \sum_{i < j} 2\operatorname{Im}(J^{\alpha\beta}{}_{ij})\sin\varphi_{ij}$$

$$J^{\alpha\beta}{}_{ij} \equiv U_{\alpha i} U_{\beta i}{}^{*} U_{\alpha j}{}^{*} U_{\beta j} \quad \varphi_{ij} \equiv \varphi_{ij} - \varphi_{ij} = \frac{\Delta M_{ij}^{2} L}{2p} \quad \Delta M_{ij}^{2} = M_{i}^{2} - M_{j}^{2}$$

- If all the masses are equal, Prob=I
- Oscillation are sensitive only to difference of mass squared (NOT absolute mass)
- Total flux is conserved

$$\frac{\Delta m^2 L}{4p} = 1.27 \Delta m^2 (eV^2) \frac{L(km)}{p(GeV)}$$

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CP violation

We want to study if $P(\alpha - > \beta)$ is equal to $P(\alpha - > \beta)$

$$P(\overrightarrow{v}_{\alpha} \rightarrow \overrightarrow{v}_{\beta}) = P(v_{\beta} \rightarrow v_{\alpha}) = P(v_{\alpha} \rightarrow v_{\beta}, U \rightarrow U^{*})$$

$$Prob(\alpha \rightarrow \beta) = \delta_{\alpha\beta} - 4\sum_{i < j} \operatorname{Re}(J_{ij}^{\alpha\beta}) \sin^{2} \varphi_{ij} - 2\sum_{i < j} \operatorname{Im}(J_{ij}^{\alpha\beta}) \sin 2\varphi_{ij} \text{ changes sign}$$

$$q_{ij} = \varphi_{i} - \varphi_{j} = \frac{\Delta m^{2}_{ij} x}{4p} \quad \Delta m^{2}_{ij} = m^{2}_{i} - m^{2}_{j} \quad J_{ij}^{\alpha\beta} = U^{*}_{\alpha\beta} U^{*}_{\betai} U_{\alpha} U_{\betaj}$$

If U is complex, CP violation effect can be observed in oscillation

For the N=3, case, Im(J) is proportional to the Jarlskog invariant (area of the unitarity triangle, cf CKM matrix for the quark mixing)

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Pontecorvo, Maki, Nakagawa, Sakata

The PMNS matrix

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

NxN unitary matrix, N(N-1)/2 angles, N(N+1)/2 phases (2N**2 real parameters, Unitarity: N conditions on the diagonal+

 $\frac{1}{2}N(N-1) Im(V1k V2k^*=0) + \frac{1}{2}N(N-1) Re(V1k V2k^*=0) =>N^{**2}$ real parameters)

N phases can be rotated away by redefining the charged lepton fields

N-1 phases can be rotated away by redefining the neutrino fields

However this is not possible if Majorana (not invariant under U(1))

How many physical phases? (N-1)(N-2)/2 phases in general $U = U' \cdot$ (N-1) additional phases if Majorana

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$$\begin{array}{c}
1 \\
e^{i\phi_2} \\
\vdots \\
e^{i\phi_N} \\
e^{i\phi_N}
\end{array}$$



What happens if we measure VERY precisely the momenta of π and μ so that we are selecting (by energy momentum conservation) the neutrino mass eigenstate that is propagating to the detector ? Then oscillation should be destroyed but why and how ?

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Quantum effects-1

 This is because precise momentum measurement implies that the information about position is lost according to the uncertainty principle

 $M_v^2 = E_v^2 - p_v^2$ Where E and p comes from measurement of pi and mu

$$\sigma(M_{v}^{2}) = \sqrt{(2E_{v})^{2}\sigma^{2}(E_{v}) + (2p_{v})^{2}\sigma^{2}(p_{v})}$$

$$(2p_{v})\sigma(p_{v}) < \left|M_{1}^{2} - M_{2}^{2}\right| \text{ To select one mass eigenstate}$$

$$\Delta x > \frac{2p_{v}}{\left|M_{1}^{2} - M_{2}^{2}\right|} = \frac{L_{osc}}{2\pi}$$

The uncertainty on the position of the source is much larger the oscillation length, this uncertainty smears out the oscillations NB in practice for p=100 MeV, to distinguish $\Delta m^2 = 1 \text{ eV}^2$, we need $\sigma(p)/p = 10^{-16}!$

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Oscillation in matter

- Up to know we have derived oscillations in vacuum
- Matter effect are relevant for solar neutrino, atmospheric neutrino and also long baseline experiments
- To do this we need to:
 - I Write the Hamiltonian in the flavor basis
 - 2 Find the interaction potential
 - 3 Solve the equation: eigenvalues, effective mixing angle

$$\begin{pmatrix} v_e \\ v_\mu \end{pmatrix} = U \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

$$H = U \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} U^{-1} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

$$= \begin{pmatrix} \cos^2\theta E_1 + \sin^2\theta E_2 & -\sin\theta\cos\theta E_1 + \sin\theta\cos\theta E_2 \\ -\sin\theta\cos\theta E_1 + \sin\theta\cos\theta E_2 & \cos^2\theta E_2 + \sin^2\theta E_1 \end{pmatrix}$$

$$H = \frac{1}{2} (E_1 + E_2) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{1}{2} (E_2 - E_1) \begin{pmatrix} -\cos2\theta & \sin2\theta \\ \sin2\theta & \cos2\theta \end{pmatrix}$$

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2 Oscillation in matter (diagrams and potential)

Neutrino oscillations in matter

When neutrinos propagate in matter (Earth, Sun...), the amplitude for this propagation is modified due to coherent forward scattering on electrons and nucleons:

Wolfenstein 1978



 $\mathcal{H}_{CC} = \sqrt{2}G_F \left[\bar{\epsilon} \gamma_{\mu} P_L \nu_{\epsilon} \right] \left[\bar{\nu}_{\epsilon} \gamma^{\mu} P_L \epsilon \right] = \sqrt{2}G_F \left[\bar{\epsilon} \gamma_{\mu} P_L \epsilon \right] \left[\bar{\nu}_{\epsilon} \gamma^{\mu} P_L \nu_{\epsilon} \right]$

 $\langle \bar{e} \gamma_{\mu} P_L e \rangle_{unpol.medium} = \delta_{\mu 0} N_e$

For v_e, V=sqrt(2) $G_F N_e + C$

The term C is the same for all neutrino flavors We are interested in phase differences =>we can neglect C

Oscillation in matter

$$\begin{split} H_{\nu}|v_{1}\rangle &= E_{1}|v_{1}\rangle \\ H_{\nu}|v_{2}\rangle &= E_{2}|v_{2}\rangle \\ V|v_{e}\rangle &= (C + \sqrt{2}G_{F}N_{e})|v_{e}\rangle \\ V|v_{\mu}\rangle &= C|v_{\mu}\rangle \\ H_{\nu} &= \begin{pmatrix} E_{1}\cos^{2}\vartheta_{\nu} + E_{2}\sin^{2}\vartheta_{\nu} & (E_{2} - E_{1})\cos\vartheta_{\nu}\sin\vartheta_{\nu} \\ (E_{2} - E_{1})\cos\vartheta_{\nu}\sin\vartheta_{\nu} & E_{2}\cos^{2}\vartheta_{\nu} + E_{1}\sin^{2}\vartheta_{\nu} \end{pmatrix} \\ H &= \begin{pmatrix} E_{1}\cos^{2}\vartheta_{\nu} + E_{2}\sin^{2}\vartheta_{\nu} + C + \sqrt{2}G_{F}N_{e} & (E_{2} - E_{1})\cos\vartheta_{\nu}\sin\vartheta_{\nu} \\ (E_{2} - E_{1})\cos\vartheta_{\nu}\sin\vartheta_{\nu} & E_{2}\cos^{2}\vartheta_{\nu} + E_{1}\sin^{2}\vartheta_{\nu} + C \end{pmatrix} \\ H' &= \begin{pmatrix} -\frac{1}{2}(E_{2} - E_{1})\cos2\vartheta_{\nu} + \sqrt{2}G_{F}N_{e} & \frac{1}{2}(E_{2} - E_{1})\sin2\vartheta_{\nu} \\ \frac{1}{2}(E_{2} - E_{1})\sin2\vartheta_{\nu} & \frac{1}{2}(E_{2} - E_{1})\cos2\vartheta_{\nu} \end{pmatrix} \\ H' &= \frac{\Delta m^{2}}{4E} \begin{pmatrix} -\cos2\vartheta_{\nu} & \sin2\vartheta_{\nu} \\ \sin2\vartheta_{\nu} & \cos2\vartheta_{\nu} \end{pmatrix} + \sqrt{2}G_{F}N_{e} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \end{split}$$

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Oscillation in matter

$$H = \begin{pmatrix} a(r) & x \\ x & b \end{pmatrix}$$

a(r) because in the sun the density N e is a function of the radius r For E=8MeV 5 $\sqrt{2}G_F N_e = \frac{\Delta m^2}{2E} \cos 2\theta = \frac{\Delta m^2}{2E} (\cos^2 \theta - \sin^2 \theta)$ 0.75 10⁻⁵ eV**2/MeV $\tan 2\vartheta_M = \frac{2x}{a-b}$ If a=b, resonance for θ_m : MSW resonant effect (Mikheyev-Smirnov-Wolfenstein)

$$E_{\pm} = \frac{a+b}{2} \pm \frac{1}{2}\sqrt{(a-b)^2 + 4x^2}$$

Far away from the resonance, $(a-b)^{**2} > 4x^{**2}$ the eigenvalues correspond to the unmixed case, a and b. However, for non-zero mixing, the quantity under the square root never goes to 0, the energy level never cross!

Mixing in matter

$$\diamondsuit \quad \sin^2 2\theta_m = \frac{\sin^2 2\theta \cdot (\frac{\Delta m^2}{2E})^2}{\left[\frac{\Delta m^2}{2E}\cos 2\theta - \sqrt{2}G_F N_e\right]^2 + (\frac{\Delta m^2}{2E})^2 \sin^2 2\theta}$$

Mikheyev - Smirnov - Wolfenstein (MSW) resonance:

$$0.75 \ 10-5 \qquad \qquad \sqrt{2}G_F N_e = \frac{\Delta m^2}{2E} \cos 2\theta$$

At the resonance: $\theta_m = 45^\circ (\sin^2 2\theta_m = 1) - \text{maximal mixing}$

$$|\nu_e\rangle = \cos\theta_m |\nu_{1m}\rangle + \sin\theta_m |\nu_{2m}\rangle \qquad N_e \gg (N_e)_{\rm res}: \quad \theta_m \approx 90^\circ |\nu_\mu\rangle = -\sin\theta_m |\nu_{1m}\rangle + \cos\theta_m |\nu_{2m}\rangle \qquad N_e = (N_e)_{\rm res}: \quad \theta_m = 45^\circ N_e \ll (N_e)_{\rm res}: \quad \theta_m \approx \theta$$

 $|\nu_{1m}\rangle$, $|\nu_{2m}\rangle$ – eigenstates of *H* in matter (matter eigenstates)

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Why solar neutrino disappear



$$H' = \frac{\Delta m^2}{4E} \begin{pmatrix} -\cos 2\vartheta_V & \sin 2\vartheta_V \\ \sin 2\vartheta_V & \cos 2\vartheta_V \end{pmatrix} + \sqrt{2}G_F N_e \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

At the center of the Sun, for 8MeV v, N_e term dominates, the electron neutrino is the heavier eigenstate.
For an <u>adiabatic oscillation</u>, this state follows the v_2 trajectory, it exits the sun as the « wrong » eigenstate.
What comes out of the Sun (for this energy) has a probability sinθ of interacting as a v_e

•There is a deficit of solar neutrinos

Survival probability for v_e



Figure 7: Averaged survival probability vs $E/\Delta m^2$ in the case of neutrino oscillations in a matter of monotonically decreasing density. Solid line - small θ_0 , dashed line - large θ_0 .

What we have learned

- Neutrino oscillations need several ingredients:
 - A non trivial mixing matrix
 - Neutrino masses
 - Neutrino mass differences
- Neutrino oscillations are marvellous quantum mechanical effect on macroscopic distances
- Neutrino oscillation experiments can measure mixing angles and neutrino mass differences (not absolute mass scales!)
- We can also measure CP violating effects

Neutrino oscillation experiments

Several sources have been used:

- The sun
- Atmospheric neutrinos
- Nuclear reactors (neutrino discovery!)
- Accelerators (several options)
- Supernova neutrinos
- Ultra high energy neutrinos

The solar neutrinos

The sun is a very bright source of neutrinos. We are confident we understand the basic reaction (nuclear fusion) in its "engine"

 $4p + 2e \rightarrow {}^{4}\text{He} + 2\nu_e \qquad (Q = 26.7 \,\text{MeV}).$

 And we have precisely measured the power (giving the total neutrino luminosity) from the energy flux
 K_sun=8.53 10*11 MeV cm-2 s-1



The spectrum of solar neutrinos



Experimental methods

- Chlorine v_e + 37Cl -> 37Ar +e- Threshold 0.814 MeV
- Gallium: v_e + 37Cl -> 37Ar +e-. Threshold 0.233
 MeV
- Water Cherenkov $v + e > v + e E \sim 6 MeV$
- Heavy water $v_{\alpha} + d \rightarrow n + p + v_{\alpha}$ (all flavor are contributing)



Homestake Chlorine experiment





The chlorine experiment

Only a few Argom atoms are produced per week in a detector as large as a olympic swiming pool

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SuperKamiokande



• SuperKamiokande (1998): $\nu_e + e^- \rightarrow \nu_e + e^-$

Get information on direction and energy of the u's !



The neutrinos definitely come from the Sun, no spectral distorsion and no significant day-night asymmetry

Sudbury Neutrino Observatory



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v Reactions in SNO

$$\underbrace{\mathbf{ES}}_{\mathbf{v}_x} + \mathbf{e}^- \Rightarrow \mathbf{v}_x + \mathbf{e}^-$$

- Both SK, SNO
- Mainly sensitive to $\nu_{e,},$ less to ν_{μ} and ν_{τ}
- Strong directional sensitivity

$$c v_e + d \Rightarrow p + p + e^{-1}$$

- Good measurement of ν_e energy spectrum - Weak directional sensitivity $\propto 1\text{-}1/3\text{cos}(\theta)$

 $-\nu_e ONLY$

NC
$$v_x + d \Rightarrow p + n + v_x$$

- Measure total ⁸B v flux from the sun.

- Equal cross section for all v types

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Evidence for solar oscillation: SNO

•Charged current reactions measure v_e flux •Neutral current reactions measure $(v_e + v_\mu + v_\tau)$ • $\Phi_{CC}/\Phi_{NC} = \Phi(v_e) / \Phi(v_e + v_\mu + v_\tau) < 1 =>$ neutrino flavor •transformation!





Figure 7: Averaged survival probability vs $E/\Delta m^2$ in the case of neutrino oscillations of monotonically decreasing density. Solid line - small θ_0 , dashed line - large θ_0 .

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E_v [MeV]

pp



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What we have learned

- Neutrino masses imply the existence of physics beyond the SM (either right handed neutrinos, or super-heavy neutrinos)
- We can test neutrino masses using neutrino oscillations (=neutrino interferometry)
- Different phenomena for oscillation in vacuum or in matter
- Solar neutrino experiments show a neutrino deficit in agreement with MSW adiabatic conversion
- This has been confirmed on earth by long distance experiment with reactor neutrinos (Kamland)