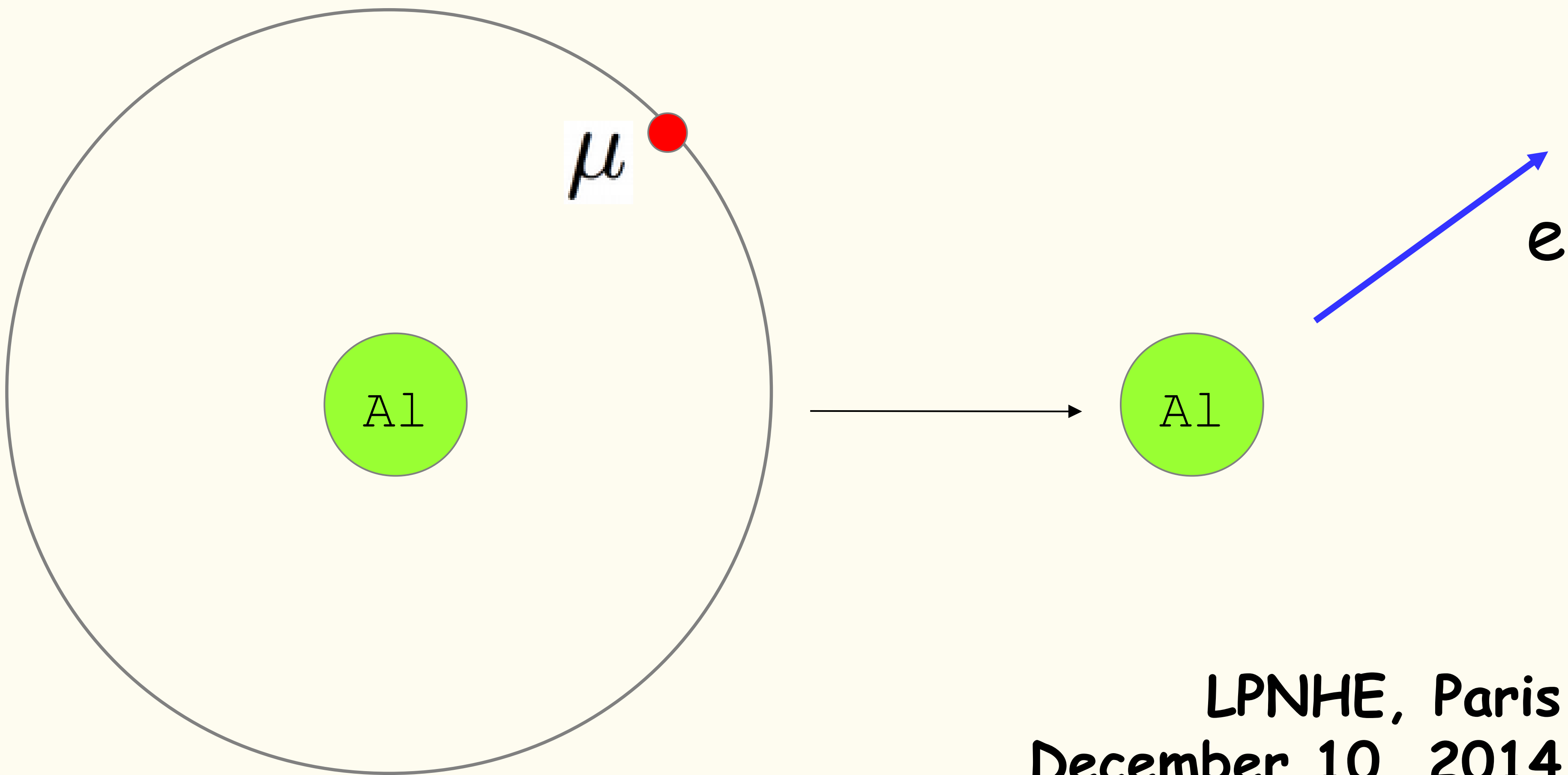
Muon g-2 and Lepton Flavor Violation



LPNHE, Paris
December 10, 2014

Andrzej Czarnecki University of Alberta

Outline

Three promising directions in low-energy searches:

* anomalous magnetic moments (muon vs electron)

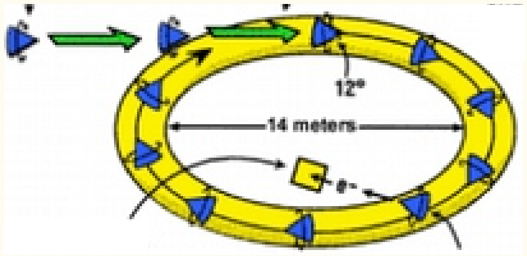
* electric dipole moments (electron)

* lepton flavor violation (muon --> electron)

Anomalous magnetic dipole moments

The puzzle of the muon magnetic moment

The 3.6 sigma discrepancy persists,



$$a_\mu^{\rm exp} - a_\mu^{\rm SM} = 287(80) \times 10^{-11}$$

PRD 86, 095009 (2012)

Talks by Marc Knecht, Maurice Benayoun, and Valentina de Romeri

This is rather large when compared with other bounds on New Physics:

Muon MDM $\quad d_\mu \sim \frac{e}{2m_\mu} a_\mu^{\rm NP} \sim 3 \cdot 10^{-22}\, e \cdot {\rm cm}$

Muon-electron transition moment $\quad |d_{\mu\to e}| < 4 \cdot 10^{-27}\, e \cdot {\rm cm}$ MEG 2013

Talk by Yoshi Kuno

Electron EDM $\quad |d_e| < 8.7 \cdot 10^{-29}\, e \cdot {\rm cm}$ ACME 2014

The puzzle of the muon magnetic moment

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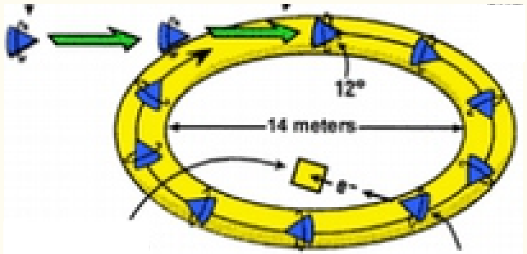

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These moments are expected to scale with the New Physics mass like

$$d_f \sim \frac{m_f}{\Lambda^2}$$

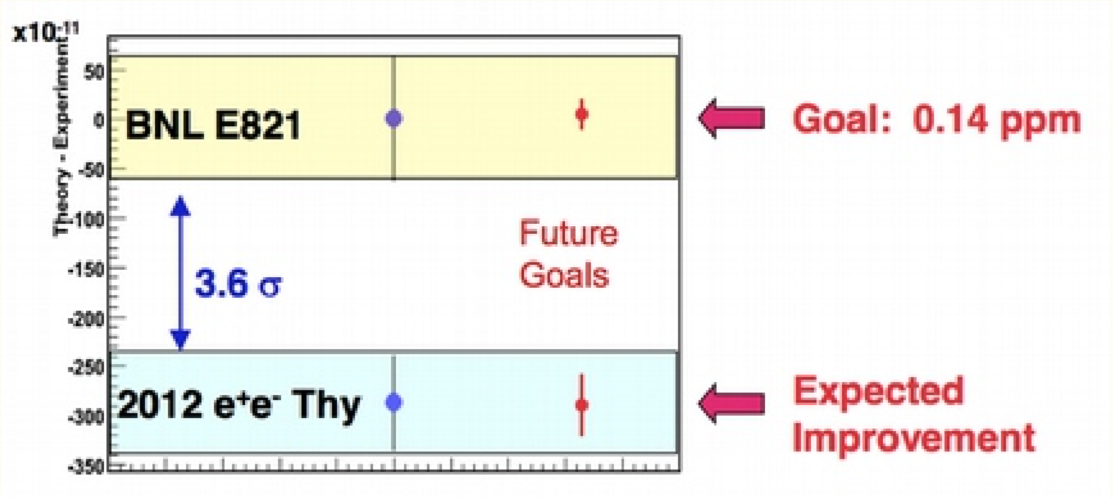
The transition moment probes the highest mass scales,

$$\frac{\Lambda_{\mu\to e}}{\Lambda_{\rm eEDM}} \sim \sqrt{\frac{m_\mu}{46 m_e}} \simeq 2$$

How can g_μ-2 be checked?

New experiment at Fermilab

Sam Henry



New experimental concept at J-PARC

Tsutomu Mibe, tomorrow

Can we use g_e-2?

Marc Knecht

New approach to g_μ-2 at J-PARC

Slower muons 300 MeV (instead of the "magic" 3.1 GeV)

Ultracold muons; no electric focusing!

Smaller ring r = 33 cm (instead of 7 m)

$$r\,[\text{in meters}] \simeq \frac{\gamma}{3B\,[\text{in Tesla}]}$$

Strong, very precisely controlled magnetic field.

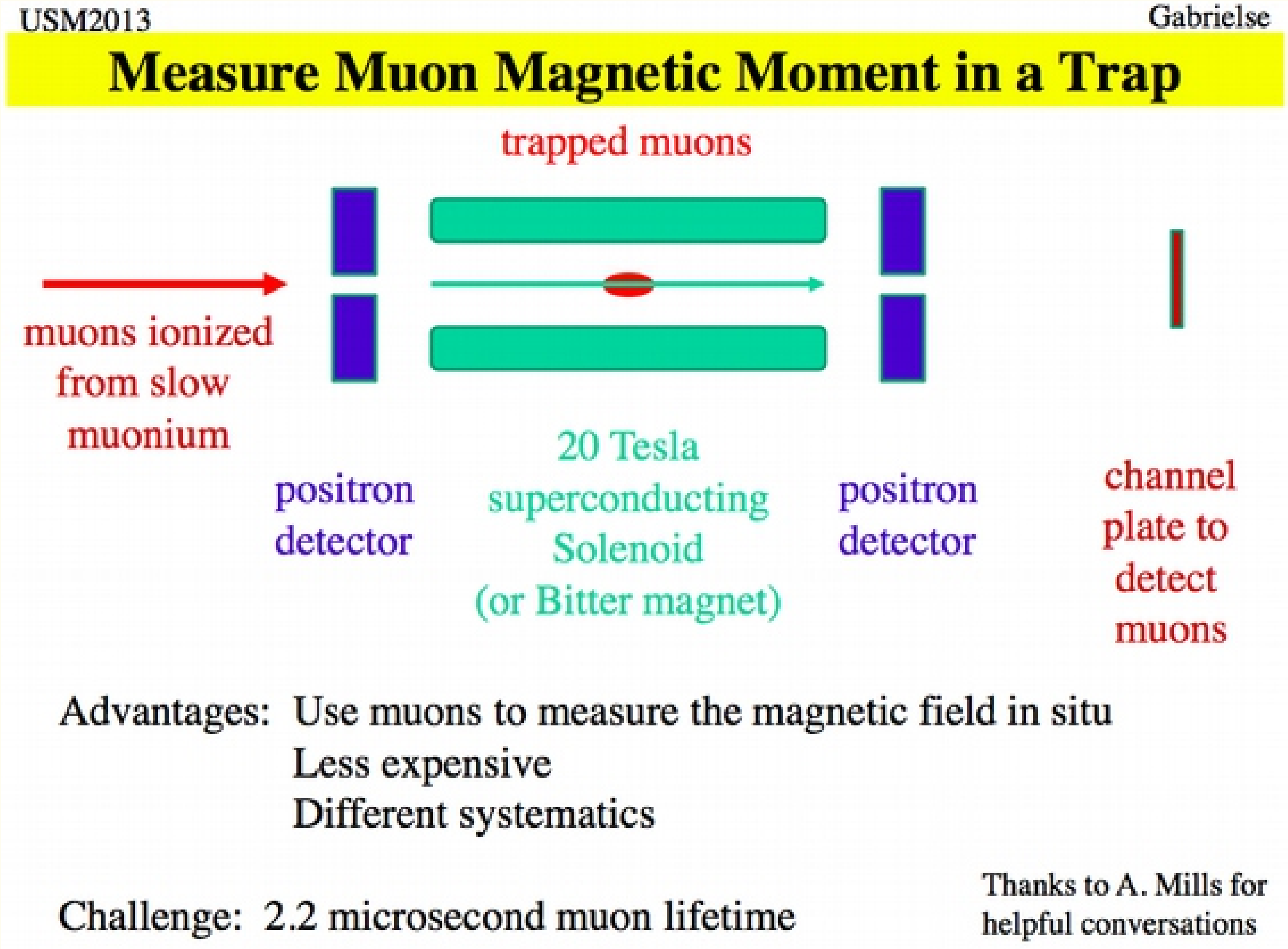
~ 10 times more muons than at Fermilab (compensates shorter lifetime).

	Brookhaven	Fermilab	J-PARC
Muon momentum	3.09 GeV/c		0.3 GeV/c
gamma	29.3		3
Storage field	B=1.45 T		3.0 T
Focusing field	Electric quad		None
# of detected μ+ decays	5.0E9	1.8E11	1.5E12
# of detected μ- decays	3.6E9	-	-
Precision (stat)	0.46 ppm	0.1 ppm	0.1 ppm

gamma (29.3) $\simeq \sqrt{\frac{2\pi}{\alpha}}$

From N. Saito

(Futuristic?) approach to g_μ-2



Magnetic moment of the electron

$$a_e = \frac{g_e - 2}{2}$$

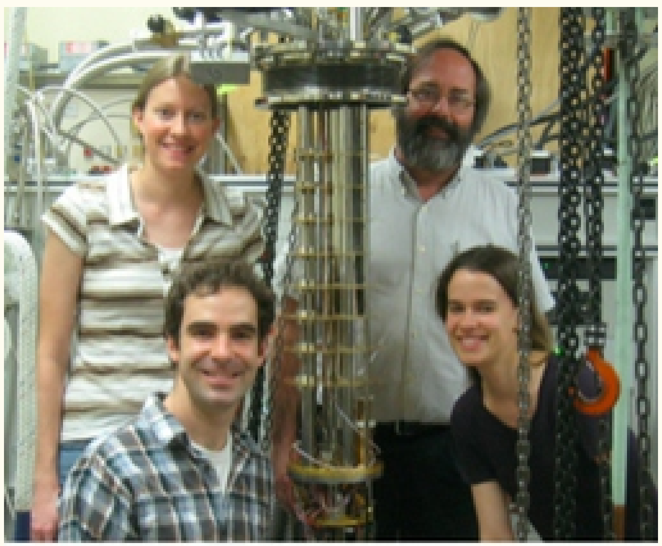
Measured with relative error $25 \cdot 10^{-11}$

Phys. Rev. Lett. 100, 120801 (2008)

Provides the fine structure constant with the same precision,

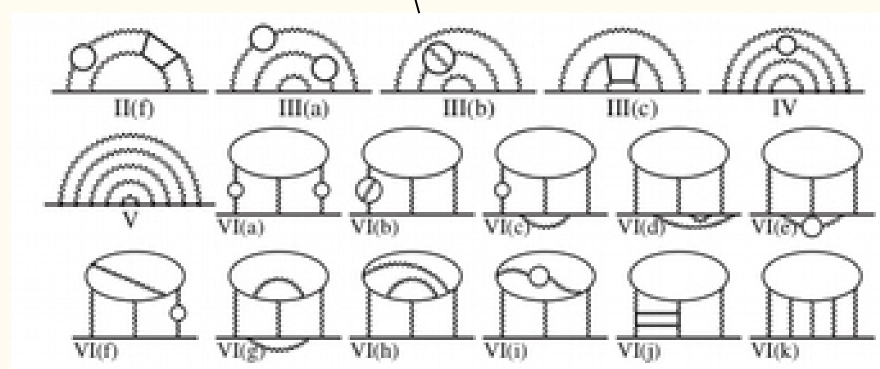
$$\alpha^{-1}(a_e) = 137.035\,999\,1736\,(331)\,(86)$$

Phys. Rev. Lett. 109, 111807 (2012)



Experimental error dominates (for now)

Numerical errors from 4- and 5-loop diagrams



Kinoshita

How to use g_e-2 to check g_μ-2?

If the muon anomaly is due to New Physics, the expected effect for the electron is likely smaller by $\frac{m_e^2}{m_\mu^2} \sim \frac{1}{43000}$

$$\Delta a_\mu \sim 287 \cdot 10^{-11} \rightarrow \Delta a_e \sim 7 \cdot 10^{-14}$$

This means relative uncertainty $\frac{\Delta a_e}{a_e} \sim 7 \cdot 10^{-11}$

and requires a factor of 4 improvement of the latest measurement.

In addition, an independent determination of the fine structure constant is needed, with matching precision.

AC, Nature 442 (2006) 516
Giudice, Paradisi, Passera, JHEP 11 (2012) 113
Terranova & Tino, PRA 89 (2014) 052118

How to use g_e-2 to check g_μ-2?

Nature 442, 516 (2006)
PRA 89, 052118 (2014)

The second best determination of alpha:
from atomic spectroscopy

$$R_\infty = \frac{m_e c \alpha^2}{2h}$$

Needed precision:

$14 \cdot 10^{-11}$

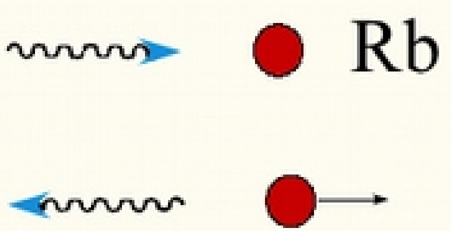
$$\alpha^2 = \frac{R_\infty}{2} \cdot \frac{u}{m_e} \cdot \frac{M_X}{u} \cdot \frac{h}{M_X}$$

R_∞: $7 \cdot 10^{-12}$ (but is it for sure?)

u/m_e: $8 \cdot 10^{-11}$ NEW Nature 2014 Sturm et al

M_X/u: $12 \cdot 10^{-11}$ for Rb (better for He)

h/M_X: $124 \cdot 10^{-11}$ improvement needed by factor ~10

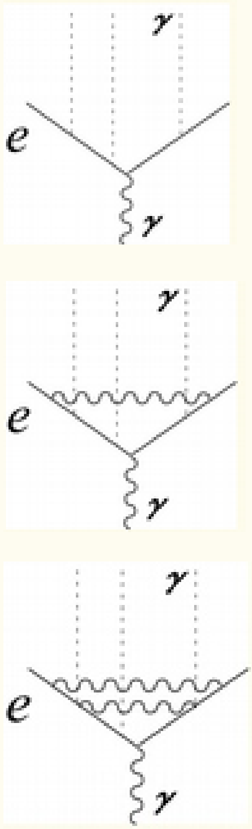


gives h/m

$$\alpha(\mathrm{Rb}) = 1/137.035\,999\,049(90) \quad [66 \cdot 10^{-11}]$$

PRL 106, 080801 (2011)

Bound-electron g-2: theory needed for u/m_e



$$g = 2 - \frac{2\left(Z\alpha\right)^2}{3} - \frac{\left(Z\alpha\right)^4}{6} + \ldots$$

$$+\frac{\alpha}{\pi}\left[1 + \frac{\left(Z\alpha\right)^2}{6} + \left(Z\alpha\right)^4\left(a_{41}\ln Z\alpha + a_{40}\right) + \ldots\right]$$

$$+\left(\frac{\alpha}{\pi}\right)^2\left[-0.65..\left(1 + \frac{\left(Z\alpha\right)^2}{6}\right) + \left(Z\alpha\right)^4\left(b_{41}\ln Z\alpha + b_{40}\right) + \ldots\right]$$

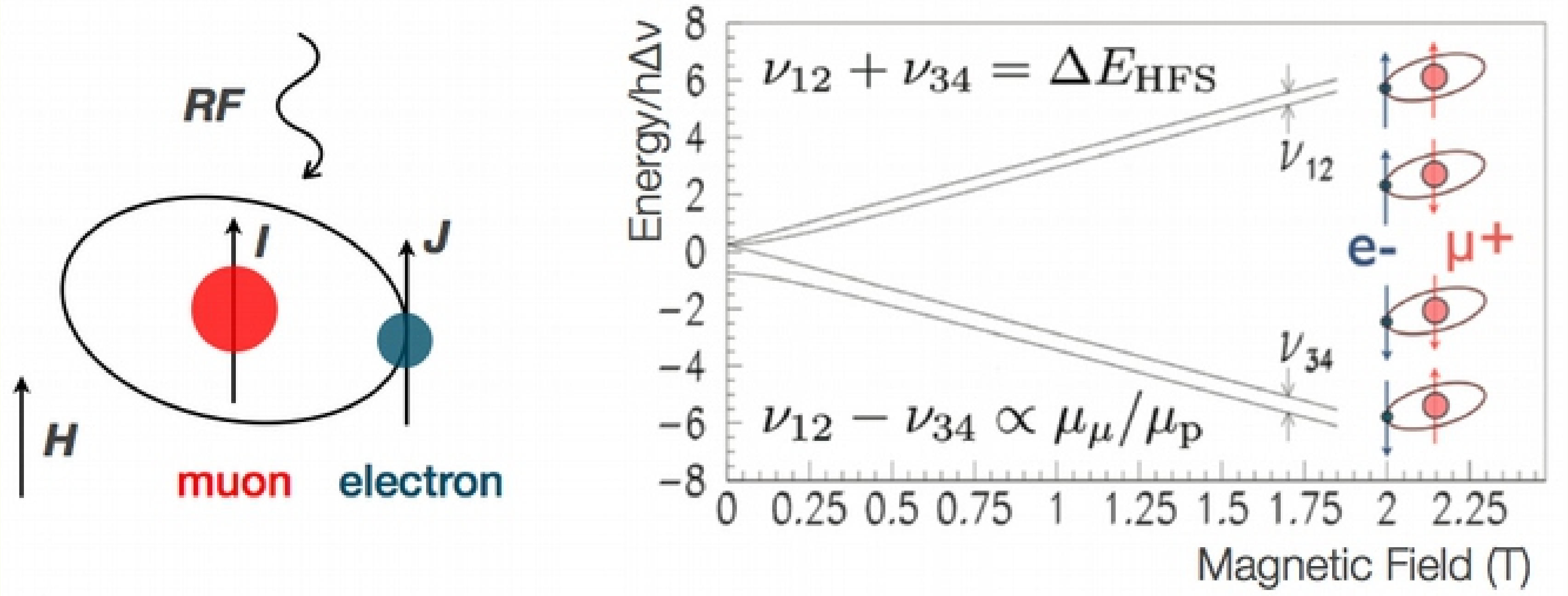
two-loop corrections

$$b_{41} = \frac{28}{9}$$

$$b_{40} = -16.4$$

Pachucki, AC Jentschura, Yerokhin

Similar binding corrections needed for the muonium: input to g-2

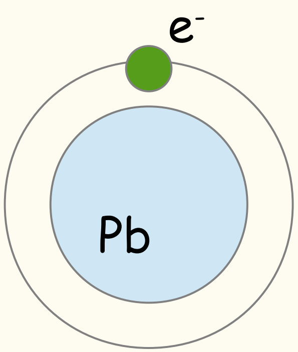


From Sohtaro Kanda

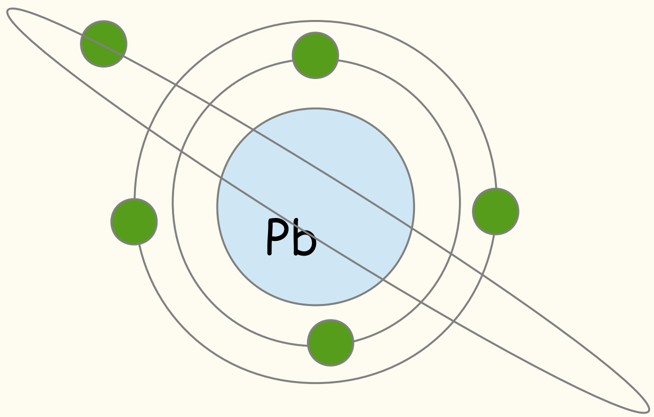
Another source of alpha: highly-charged ions

$$g \simeq 2 - \frac{2(Z\alpha)^2}{3} \longrightarrow \frac{\delta\alpha}{\alpha} \sim \frac{1}{(\alpha Z)^2}\sqrt{(\delta g_{\text{exp}})^2 + (\delta g_{\text{th}})^2}$$

large Z favorable



Hydrogen-like lead



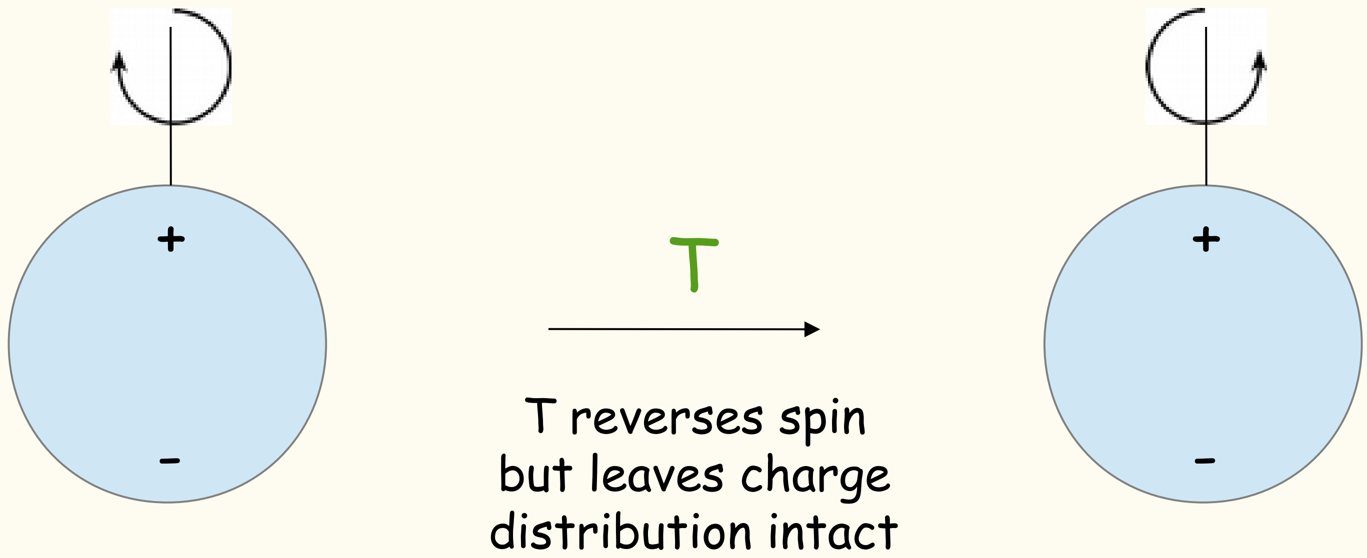
Boron-like lead

There is a combination of *g*-factors in both ions where the sensitivity to the nuclear structure largely cancels, but the sensitivity to alpha remains.

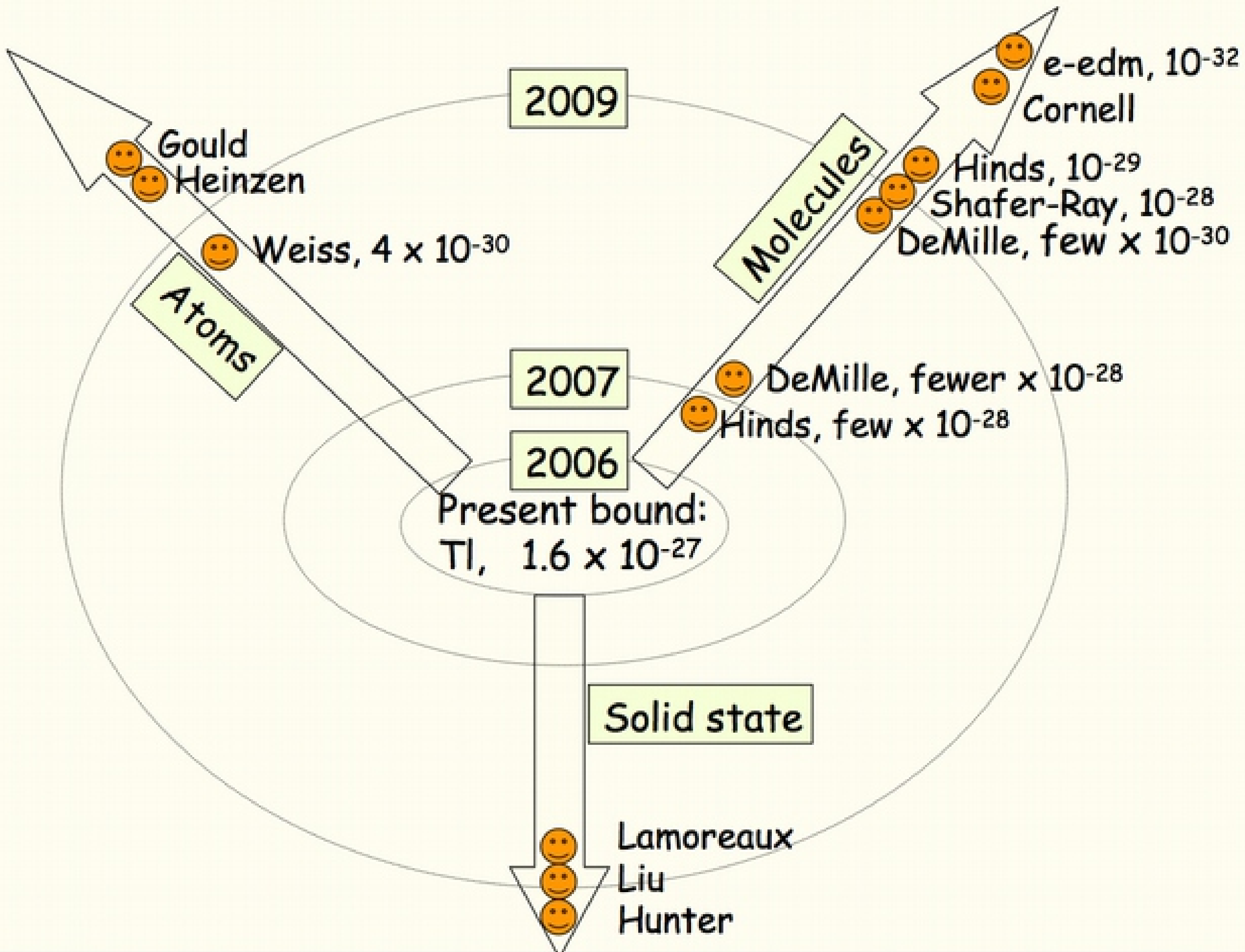
Much interesting theoretical work remains to be done!

Shabaev, Glazov, Oreshkina, Volotka, Plunien, Kluge, Quint

Electron electric dipole moment



Electron EDM: what used to be expected (2006)



In 2010, Maxim Pospelov was suspicious:

Recent great progress

2011: YbF @ Imperial $$|d_e| < 10.5 \times 10^{-28} e\,\mathrm{cm}$$

LETTER

doi:10.1038/nature10104

Improved measurement of the shape of the electron

J. J. Hudson[1], D. M. Kara[1], I. J. Smallman[1], B. E. Sauer[1], M. R. Tarbutt[1] & E. A. Hinds[1]

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2014: ThO by ACME $|d_e| < 8.7 \times 10^{-29}\, e \cdot \text{cm}$

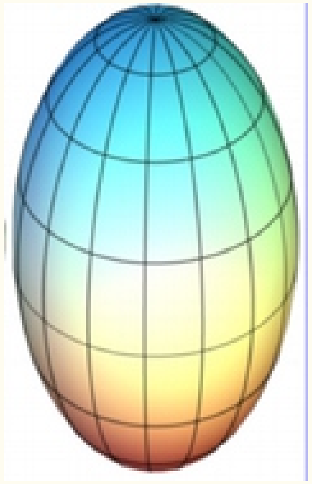
Order of Magnitude Smaller Limit on the Electric Dipole Moment of the Electron

The ACME Collaboration,* J. Baron,[1] W. C. Campbell,[2] D. DeMille,[3]† J. M. Doyle,[1]† G. Gabrielse,[1]† Y. V. Gurevich,[1]‡ P. W. Hess,[1] N. R. Hutzler,[1] E. Kirilov,[3]§ I. Kozyryev,[3]|| B. R. O'Leary,[3] C. D. Panda,[1] M. F. Parsons,[1] E. S. Petrik,[1] B. Spaun,[1] A. C. Vutha,[4] A. D. West[3]

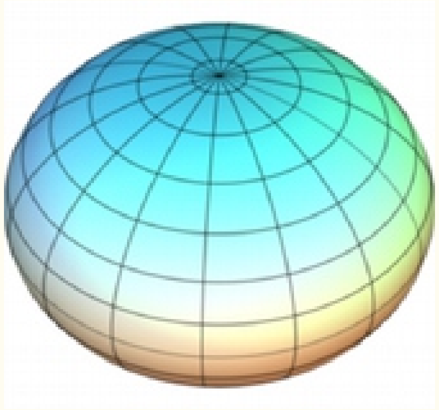
What will be on the cover when d_e discovered?

Analogous discussion about the shape of the earth (18th century): which theory of gravity is correct?

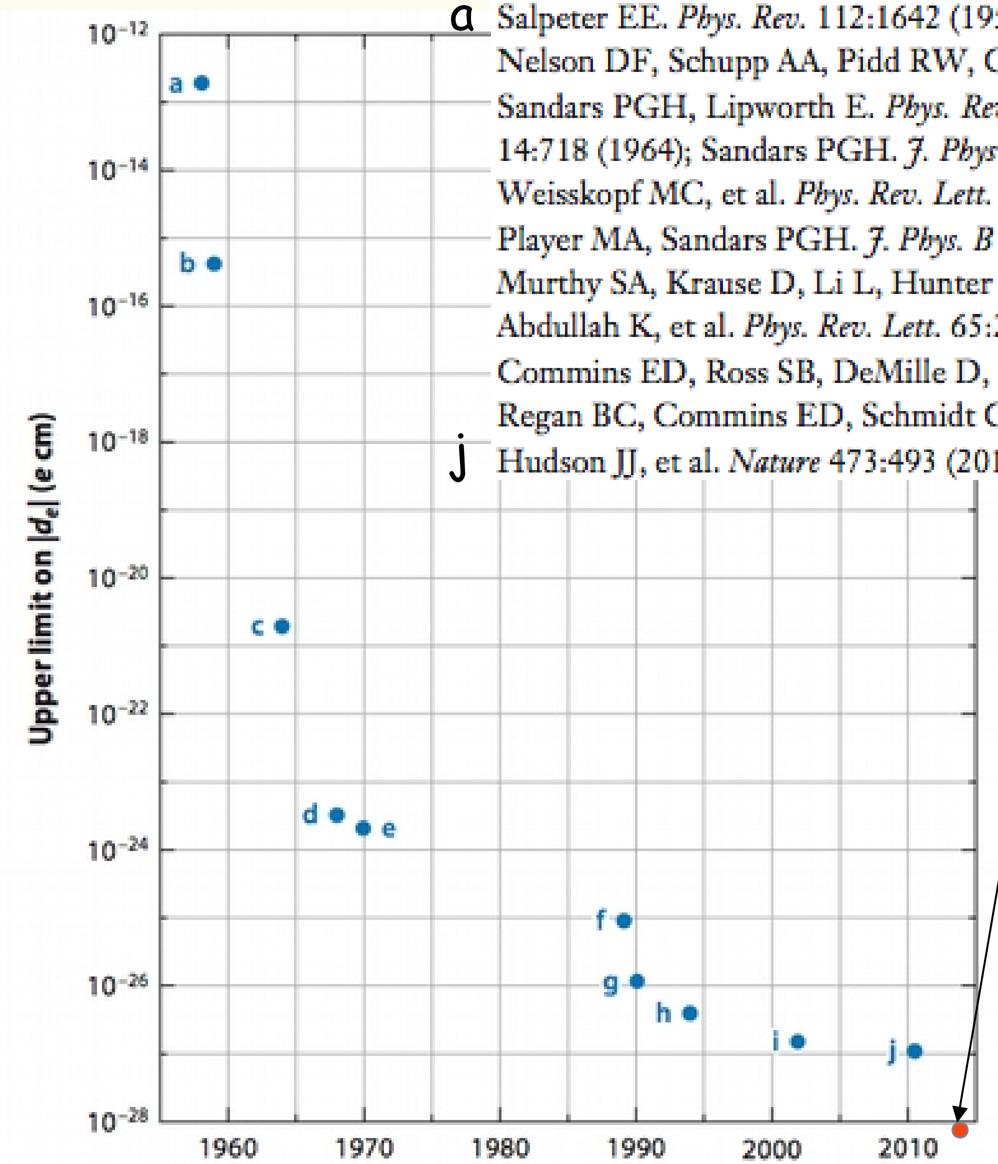
Descartes:



Newton:



History of the electron EDM

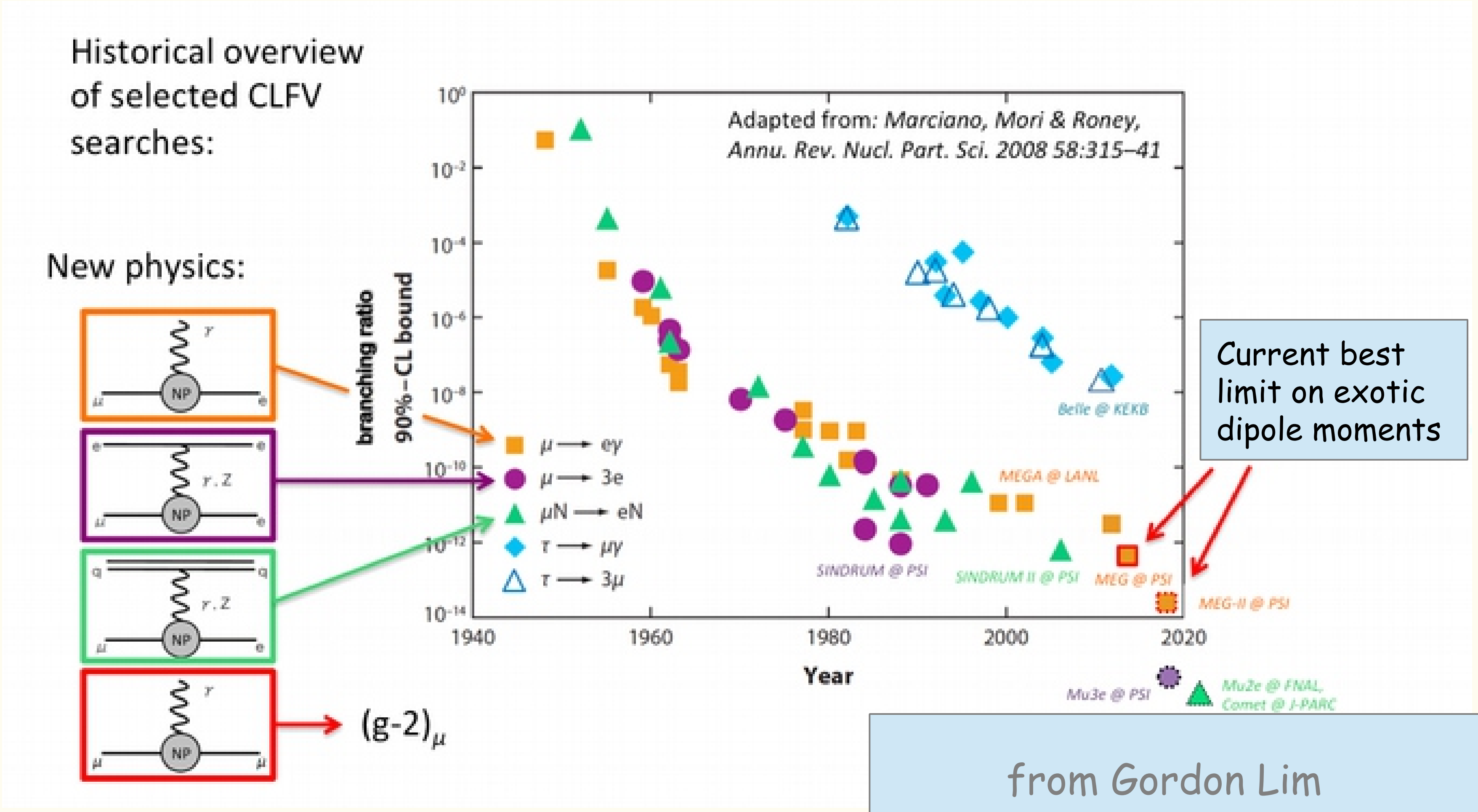


a Salpeter EE. *Phys. Rev.* 112:1642 (1958)
Nelson DF, Schupp AA, Pidd RW, Crane HR. *Phys. Rev. Lett.* 2:492 (1959)
Sandars PGH, Lipworth E. *Phys. Rev. Lett.* 13:529 (1964); Sandars PGH, Lipworth E. *Phys. Rev. Lett.* 14:718 (1964); Sandars PGH. *J. Phys. B* 1:494 (1968); Sandars PGH. *J. Phys. B* 1:511 (1968)
Weisskopf MC, et al. *Phys. Rev. Lett.* 19:741 (1967); Weisskopf MC, et al. *Phys. Rev.* 174:125 (1968)
Player MA, Sandars PGH. *J. Phys. B* 3:1620 (1970)
Murthy SA, Krause D, Li L, Hunter LR. *Phys. Rev. Lett.* 63:965 (1989)
Abdullah K, et al. *Phys. Rev. Lett.* 65:2347 (1990)
Commins ED, Ross SB, DeMille D, Regan BC. *Phys. Rev. A* 50:2960 (1994)
Regan BC, Commins ED, Schmidt CJ, DeMille D. *Phys. Rev. Lett.* 88:071805 (2002)
j Hudson JJ, et al. *Nature* 473:493 (2011)

Baron et al. Science 343:269 (2014)

from Commins: Electron spin and its history

Efforts in LFV searches



from Gordon Lim

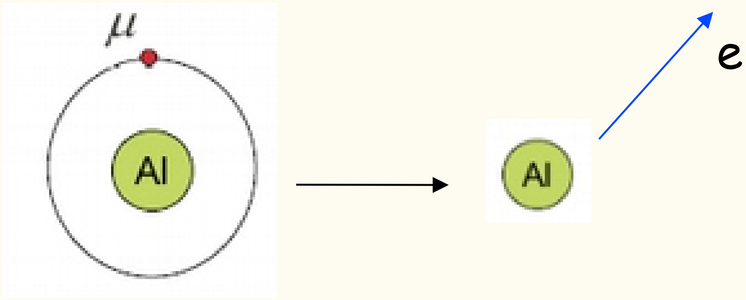
Conversion: probes also non-dipole interactions

So far, we have only talked about dipole interactions.
There are also vectors and scalars.

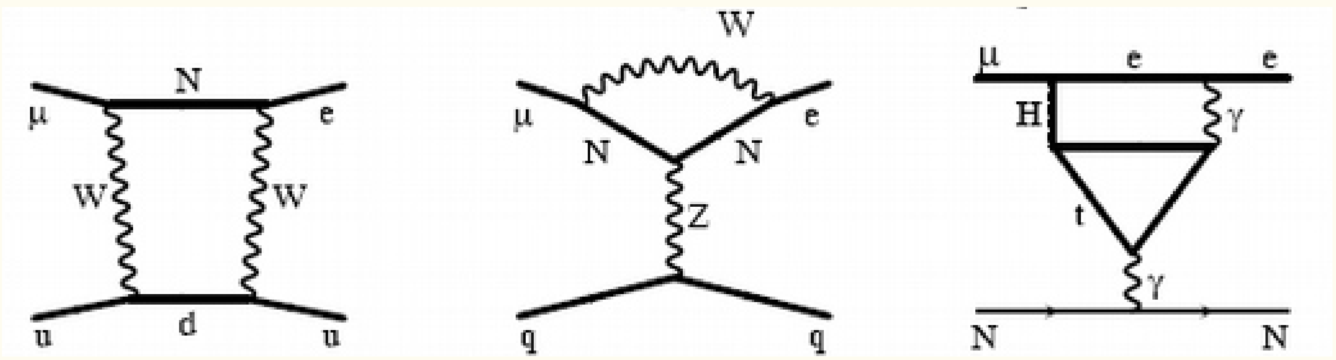
They are not (directly) probed by processes with external photons,
by gauge invariance requirements.

New process: muon-electron conversion

(as well as mu --> eee)

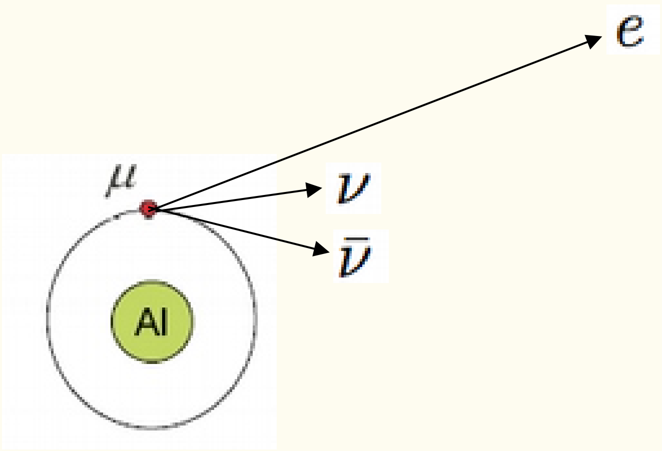


Variety of mechanisms:



Background for the conversion search

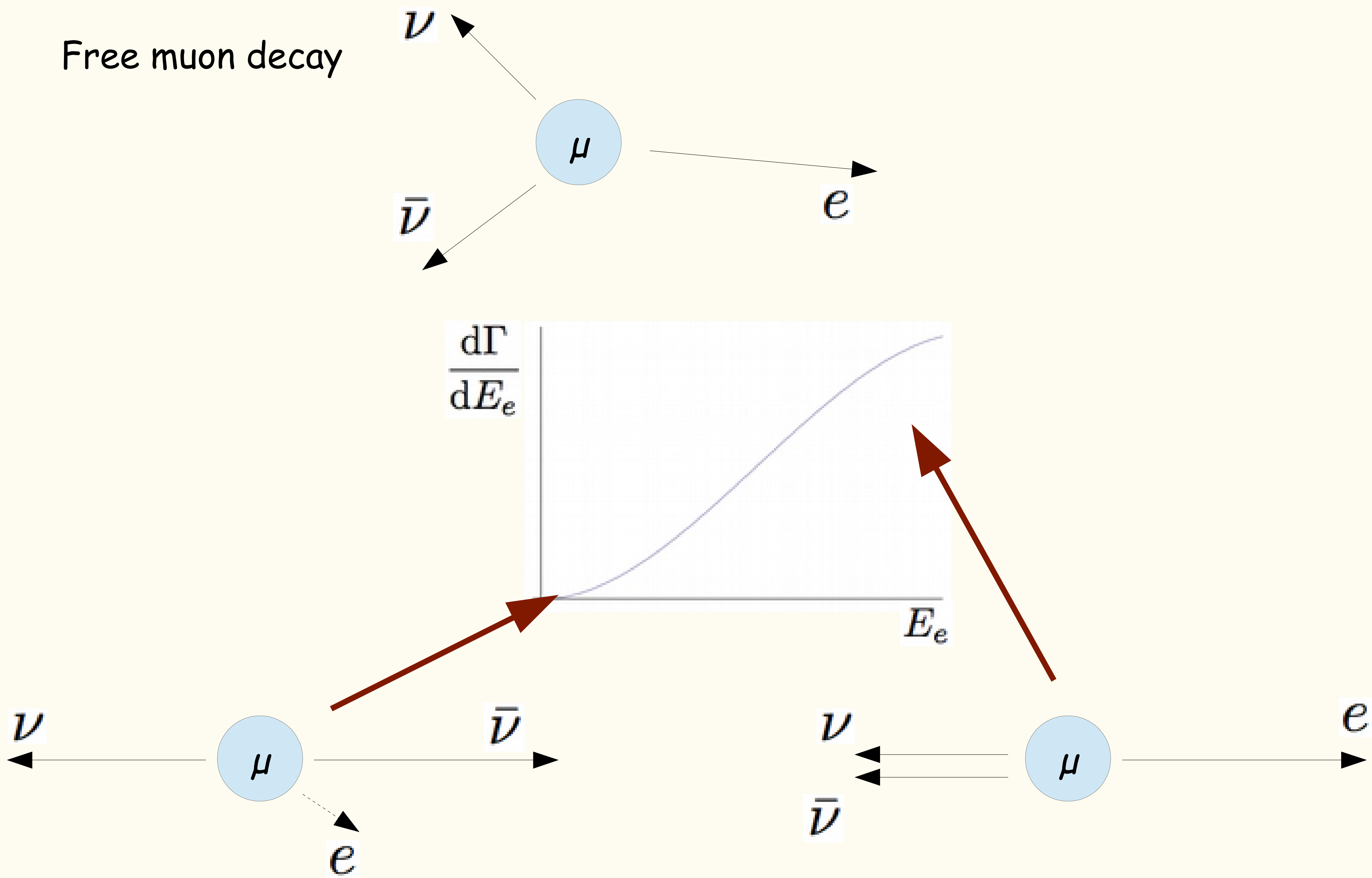
Normal decay of the muon bound in the atom
can produce high-energy electron,



Spectrum has to be well understood.

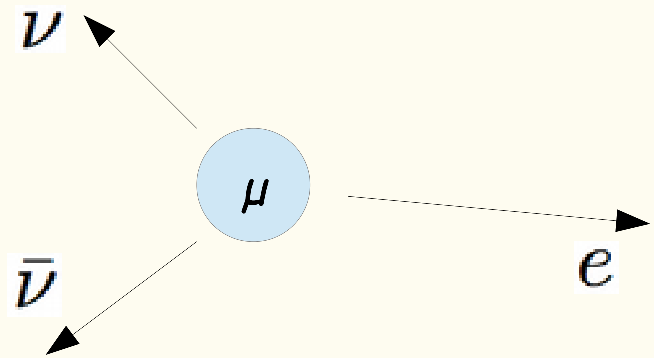
Muon decay: electron energy spectrum

Free muon decay

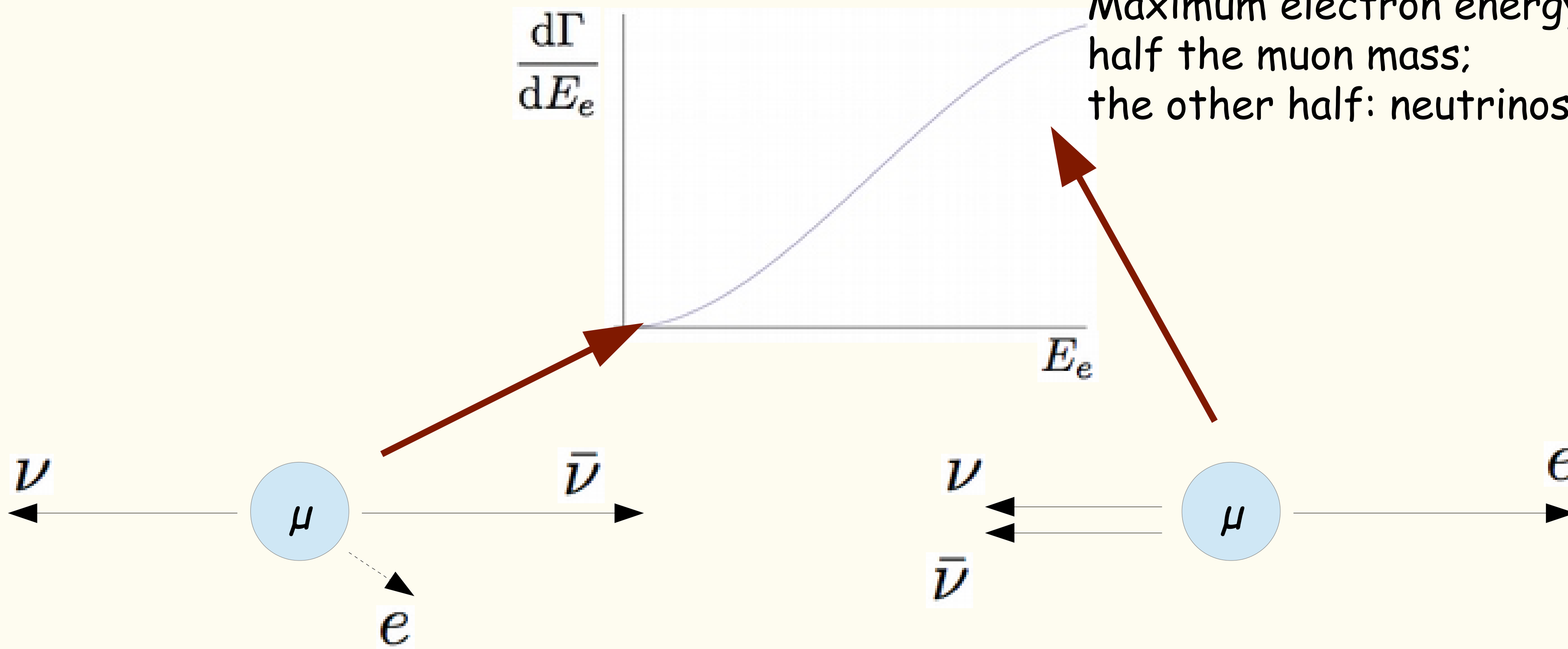


Muon decay: electron energy spectrum

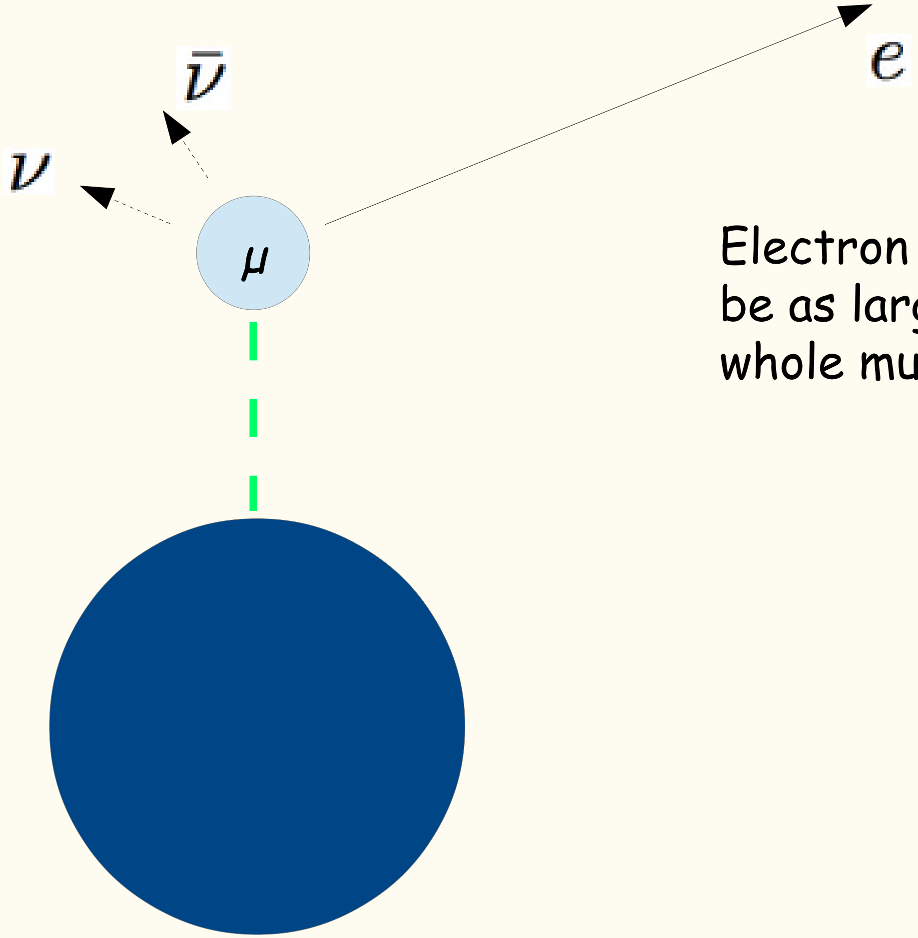
Free muon decay



Maximum electron energy:
half the muon mass;
the other half: neutrinos.

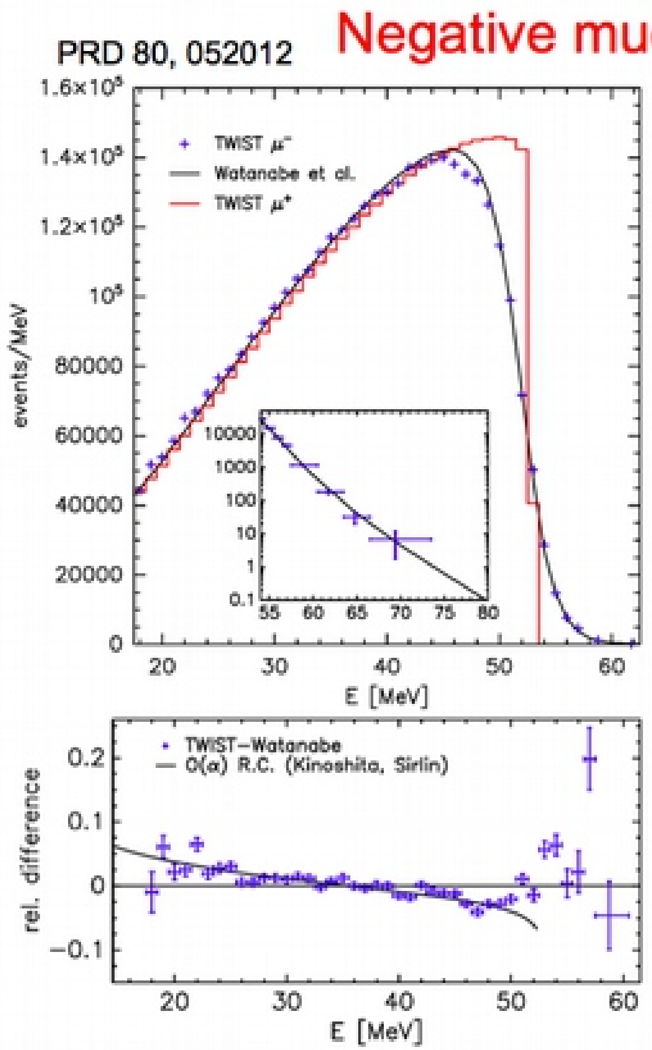


Electron spectrum in a mu-decay near nucleus



Electron energy can be as large as the whole muon mass

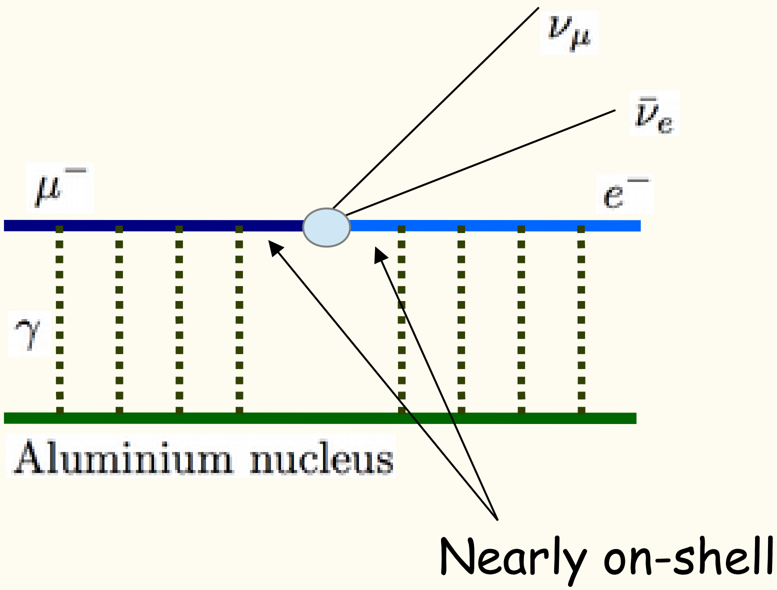
Bound muon decay has been measured: TWIST/TRIUMF



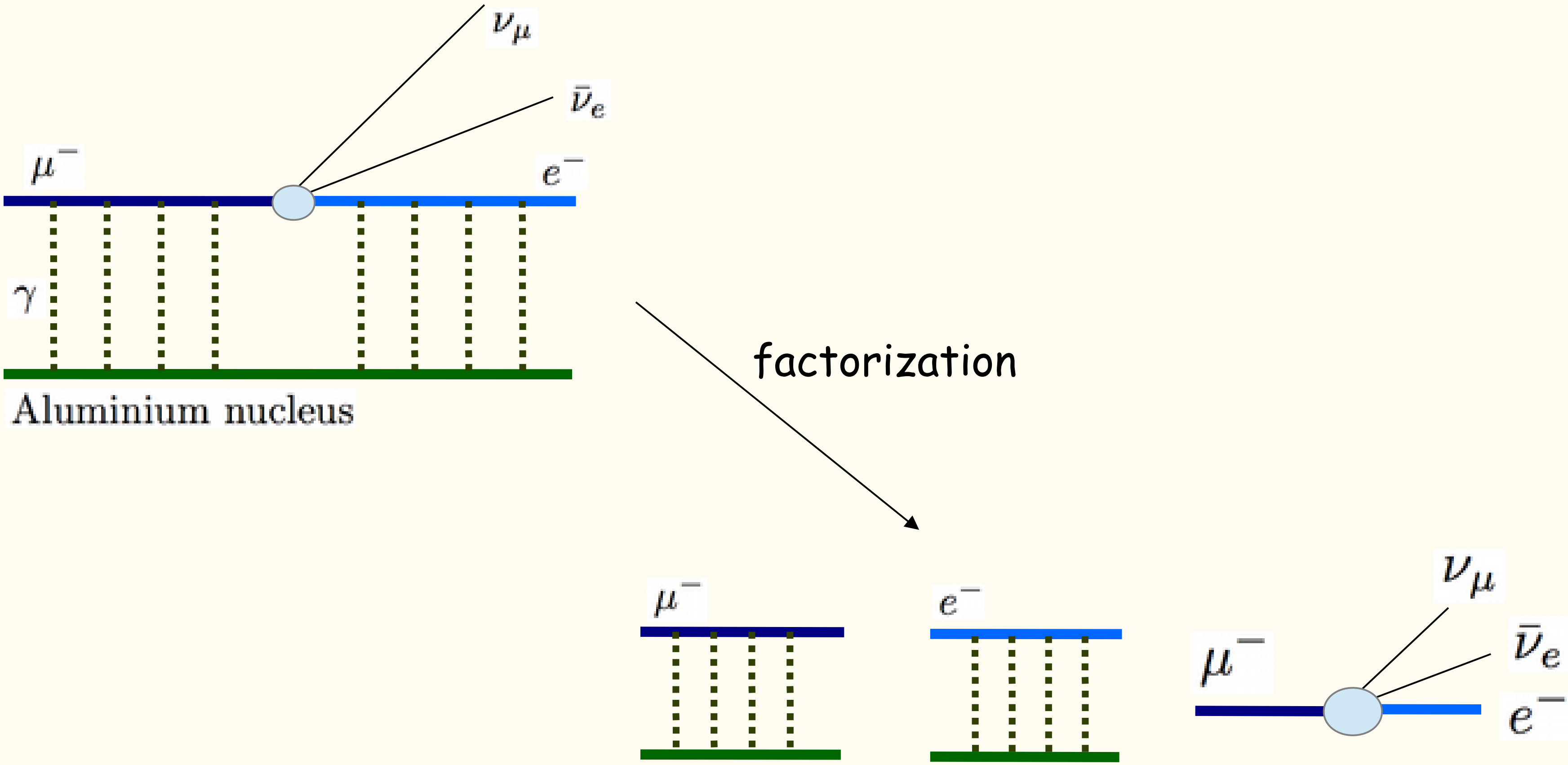
- Future $\mu \rightarrow e$ conversion experiments plan to study negative muons bound to Al
- Most precise measurement ever of the muon decay-in-orbit spectrum
- Theoretical predictions include higher-order contributions from the muon+nucleus potential
- Need to include the $O(\alpha)$ radiative corrections that arise from the interaction between the muon and the outgoing electron

From Carl A. Gagliardi/TWIST

Decay of a muon bound in aluminium



Decay of a muon bound in aluminium



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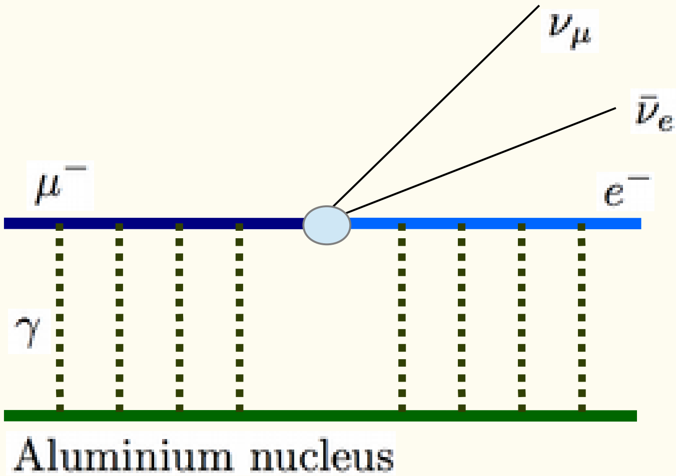
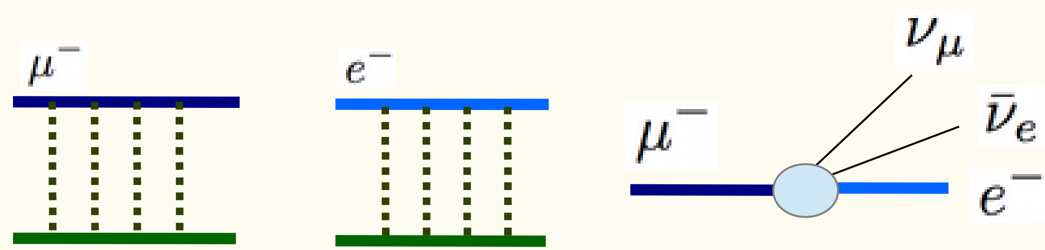


factorization



Free muon decay rate, with all corrections!

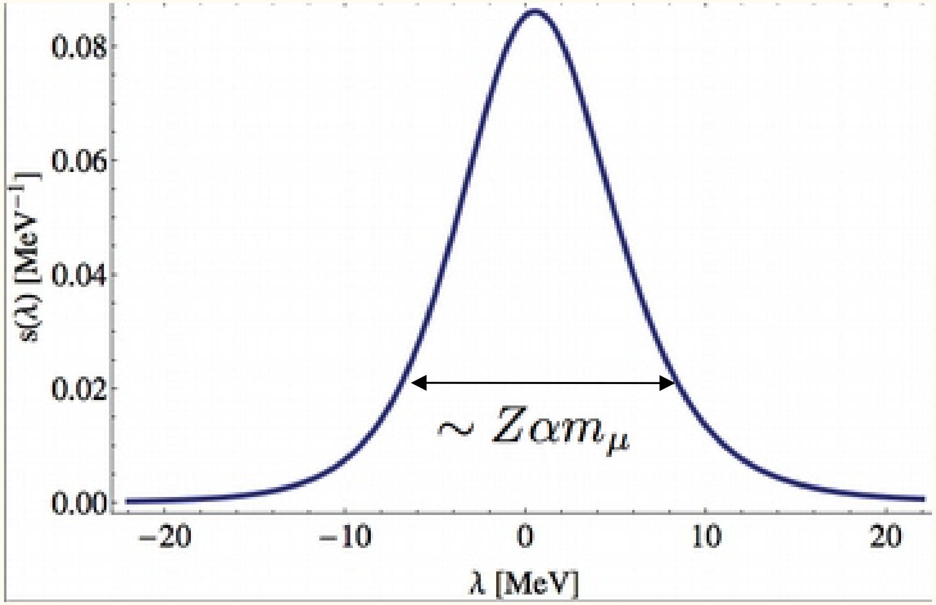
$$\frac{\mathrm{d}\Gamma}{\mathrm{d}E_e} = \int \mathrm{d}\lambda\, s(\lambda) \frac{\mathrm{d}\Gamma_{\text{free}}}{\mathrm{d}z} \frac{\mathrm{d}z}{\mathrm{d}E_e}\bigg|_{z\to z(\lambda)}$$

$$z(\lambda) = \frac{2(E_e+\lambda)+(Z\alpha)^2 m_\mu}{m_\mu+\lambda}$$

AC, M. Dowling, X. Garcia i Tormo, W. Marciano, R. Szafron

Shape function

$$s(\lambda) = \int \frac{d^3k}{(2\pi)^3} \psi_g^\star\left(\vec{k}\right) \delta(\lambda + \vec{n}\cdot\vec{k}) \psi_g\left(\vec{k}\right)$$

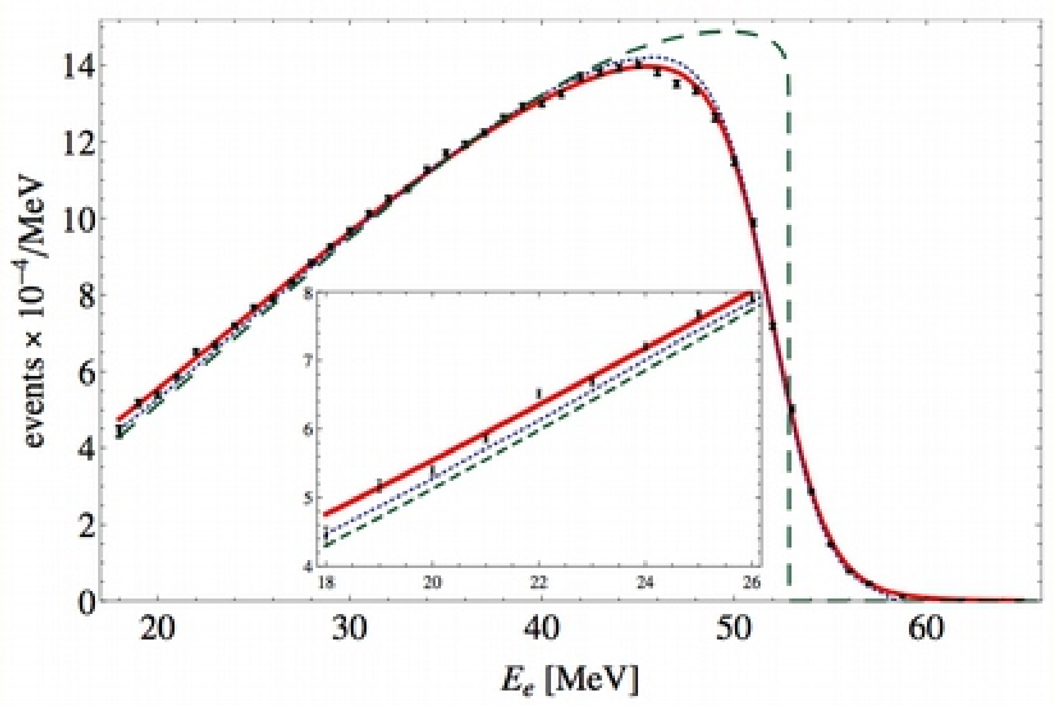


Previously used in heavy mesons, where it cannot be computed from first principles, but can be experimentally accessed.

Mannel, Neubert, Bigi, Shifman, Uraltsev, Vainshtein

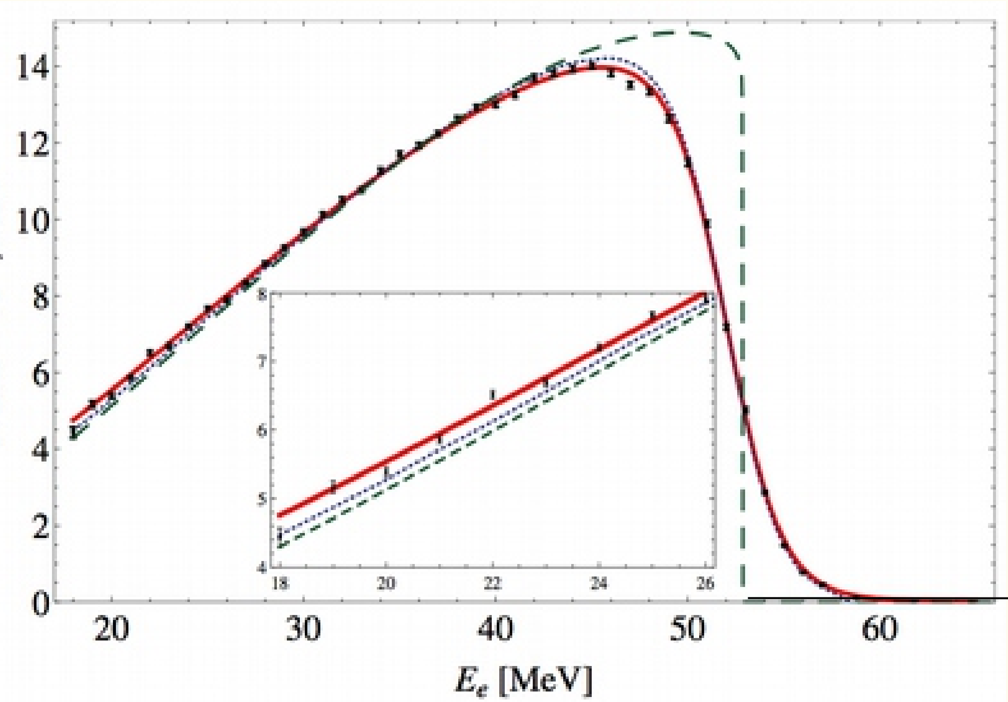
AC, M. Dowling, X. Garcia i Tormo, W. Marciano, R. Szafron

Result: spectrum of the bound muon decay



AC, M. Dowling, X. Garcia i Tormo, W. Marciano, R. Szafron

What about the upper half of the spectrum?



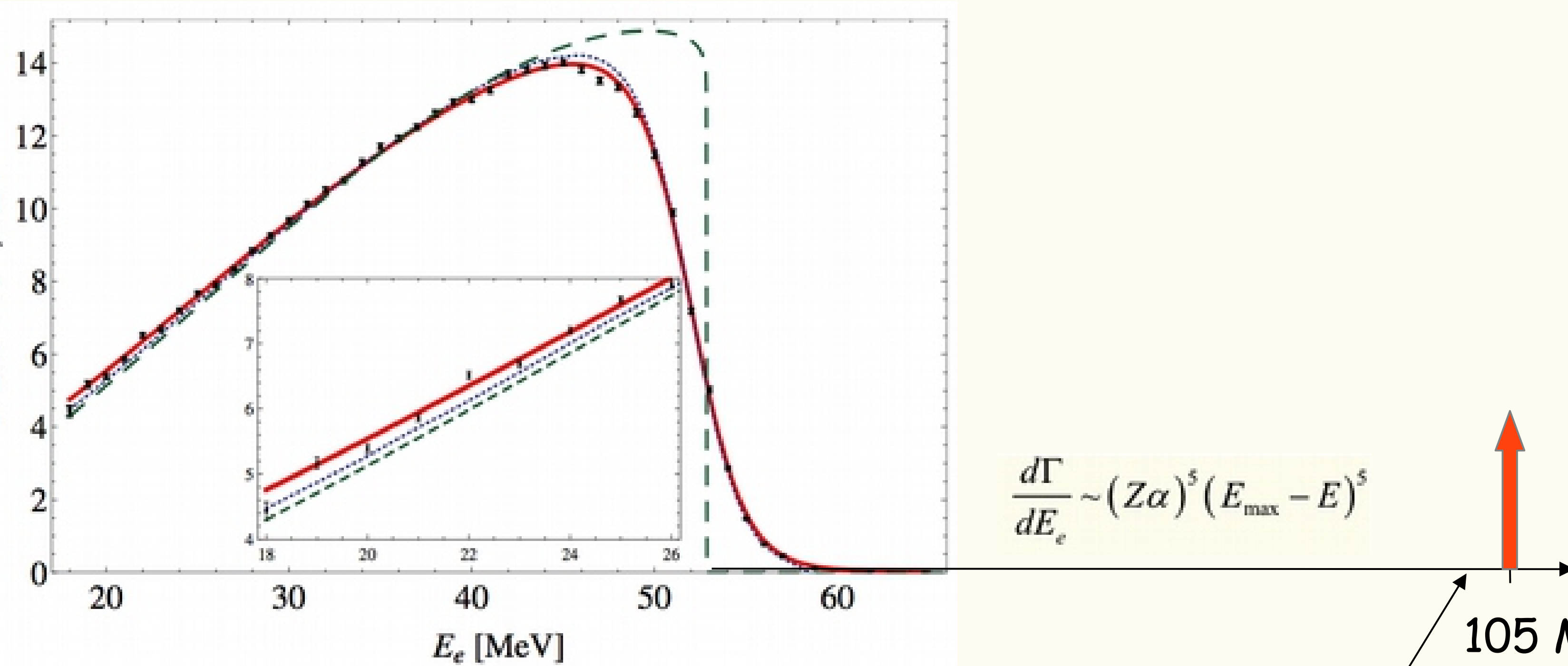
$$\frac{m_\mu}{2} \lesssim E_e \lesssim m_\mu$$

$$\frac{d\Gamma}{dE_e} \sim (Z\alpha)^5 (E_{\max} - E)^5$$

105 MeV

It seems to be VERY suppressed; so who cares?

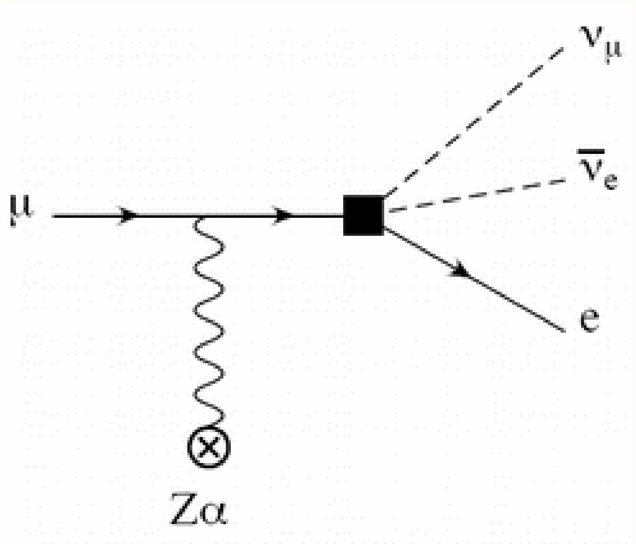
What about the upper half of the spectrum?

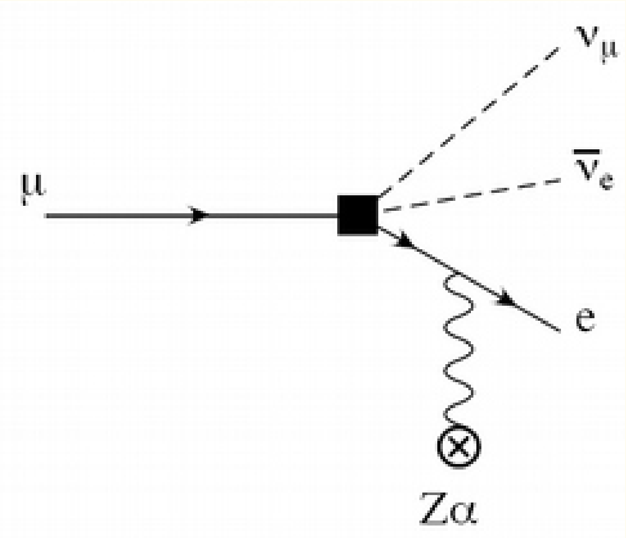


$$\frac{d\Gamma}{dE_e} \sim (Z\alpha)^5 (E_{max} - E)^5$$

It is the main background for the expected conversion signal

Origin of the $(E_{max}-E)^5$ suppression



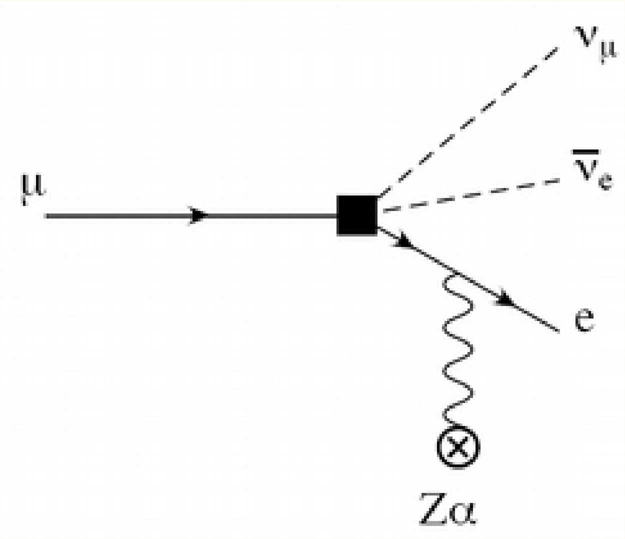


Neutrinos get no energy;
The nucleus balances electron's momentum, takes no energy.
Near the end point:

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}E_e} \sim |\psi(0)|^2 (Z\alpha)^2 \frac{\mathrm{d}^3\nu_e}{\nu_e}\frac{\mathrm{d}^3\nu_\mu}{\nu_\mu}\delta\left(E_{\max}-E_e-\nu_e-\nu_\mu\right) \mathrm{Tr}\ldots \not{\nu}_e \ldots \not{\nu}_\mu$$

$$\sim (Z\alpha)^5 \left(E_{\max}-E_e\right)^5$$

Next step: radiative corrections to the electron spectrum

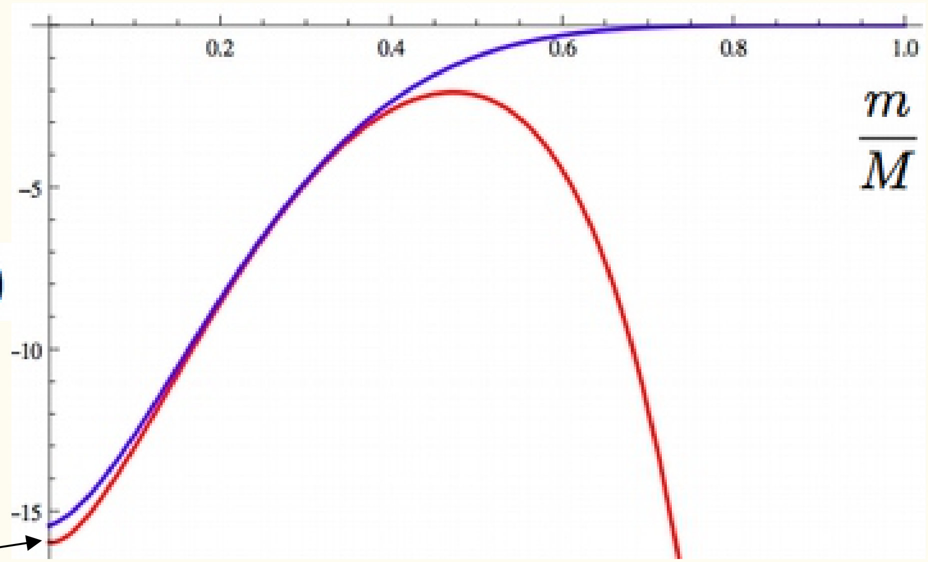


Competing effects:
- vacuum polarization in the hard photon; and
- self-energy and real radiation

Ultimate goal: smooth matching of all energy regions, from the bound electron at low energy to the end-point.

Free muon lifetime

α^2 correction to
$\Gamma\left(\mu(M) \to e\left(m\right)\nu\bar{\nu}\right)$



exact

Note: the blue curve is designed for m ~ M,
but is good even for m << M.

So we want to exploit the expansion around m = M
to get α^3

M. Dowling, with J. Piclum, AC, M. Czakon

Conclusions

Great opportunities for precise theoretical studies in low-energy physics:

- bound electron *g*-factor: complements a vigorous experimental program in Mainz, Heidelberg and GSI Darmstadt

- muon decay: background for μ-e conversion searches; theoretically interesting

- many atomic/molecular structure studies needed, eg for electric dipole searches