Improving Estimates for $(g\text{-}2)_\mu$: Can One trust Results from Effective Lagrangians & Global Fits?

M. Benayoun
LPNHE Paris 6/7

OUTLINE

- HVP Evaluations & Effective Lagrangians
- The HLS Model, its Breaking & Scope
- The VMD Strategy for HVP Evaluations : Global Fits
- Issues with the Global Fit Method
  - χ^2 : How to deal with spectrum scale uncertainties ?
  - An Iteration Method and its Monte Carlo Checking
- Updated Global Fits to e^+e^- Annihilations
- Updated Evaluations of NP Contributions to HVP
- Updated Evaluations of the $(g-2)_\mu$ Discrepancy
- Conclusions

HVP Estimates & Effective Lagrangians

- **Non Perturbative** **contributions to Hadronic VP :**

$$a_\mu(H_i) = \frac{1}{4\pi^3} \int_{s_{th}}^{s_{cut}} ds\, K(s)\, \sigma(e^+e^- \to H_i, s)$$ ⇐ **Measured** Xsection

- **Effective Lagrangians imply physics correlations among the** $e^+e^- \to H_i \ \ (i=1, \dots\dots)$

- **EL cross-sections : fed through a global fit**

  → **(param. values & error covariance matrix) :**

Measured Xsection → **Model** Xsection

The HLS Model & Breaking

e^+e^- data handling framework : HLS Lagrangian

M. Harada & K. Yamawaki Phys. Rep. 381 (2003) 1

- **equiped with two breaking schemes:**
- **BKY mechanism :**
  (SU_2 & SU_3 brk)

M.Bando *et al*. Nucl. Phys. B259 (1985) 493

M.Benayoun *et al*. Phys. Rev. D58 (1998) 074006

M.Hashimoto Phys. Rev. D54 (1996) 5611

- **vector meson mixing :**
  (s-dependent)

M.Benayoun *et al*. EPJ C55 (2008) 199

M.Benayoun *et al*. EPJ C65 (2010) 211

- **Latest Model Status :**

M.Benayoun *et al*. EPJ C72 (2012) 1848

HLS : A Global VMD Model (I)

- The (Broken) Hidden Local Symmetry (BHLS) model :
  - Unified VMD framework which encompasses

$e^+e^- \rightarrow \pi\pi$ /KKbar / $\pi\gamma$ / $\eta\gamma$ / $\pi\pi\pi$ & $\tau \rightarrow \pi\pi\nu_\tau$

& *$PV\gamma$, $P\gamma\gamma$ decays* & *$\eta/\eta' \rightarrow \gamma\pi\pi/\gamma\gamma$* & ...

  - ***BHLS :: (almost) an empty shell :***

[α_{em}, G_F, f_π, V_{ud}, V_{us}, m_π's, m_K's, m_{η}, , $m_{\eta'}$]

  - **Main Limitation :**

✓ Up to the ≈ ϕ mass region (≈ 1.05 GeV)

HLS : A Global VMD Model (I)

- The (Broken) Hidden Local Symmetry (BHLS) model :
  - ➢ Unified VMD framework which encompasses

$e^+e^- \to \pi\pi$ /KKbar /$\pi\gamma$ /$\eta\gamma$ /$\pi\pi\pi$ & $\tau \to \pi\pi\nu_\tau$
& *PVγ, Pγγ decays* & *η/η' → γππ/γγ* & ...

  - ➢ *BHLS :: (almost) an empty shell :*
    [α_{em}, G_F, f_π, V_{ud}, V_{us}, m_π's, m_K's, m_η, $m_{\eta'}$]
  - ➢ Main Limitation :

✓ Up to the ≈ ϕ mass region (≈ 1.05 GeV)

HLS : A Global VMD Model (I)

- The (Broken) Hidden Local Symmetry (BHLS) model :
  - Unified VMD framework which encompasses

$e^+e^- \to \pi\pi$ /KKbar / $\pi\gamma$ / $\eta\gamma$ / $\pi\pi\pi$ & $\tau \to \pi\pi\nu_\tau$
& *PVγ, Pγγ decays* & *η/η' → γππ/γγ* & ...

  - *BHLS :: (almost) an empty shell :*
    [α_{em}, G_F, f_π, V_{ud}, V_{us}, m_π's, m_K's, m_η, $m_{\eta'}$]

M.Benayoun *et al*. EPJ C72 (2012) 1848

- Main Limitation :

✓ Up to the ≈ ϕ mass region (≈ 1.05 GeV)

HLS : A Global VMD Model (II)

- **BHLS correlates several physics channels :**

$e^+e^- \to \pi\pi$ / KKbar / $\pi\gamma$ / $\eta\gamma$ / $\pi\pi\pi$ & $\tau \to \pi\pi\nu_\tau$

& $PV\gamma$, $P\gamma\gamma$ decays & $\phi \to \pi\pi$ (Br ratio and phase)

1. **BHLS : *overconstrained* & numerically driven by *more than 40 data sets***
2. **New paradigm : statistics on any channel ($\pi^0\gamma$, τ) ≈ additional statistics for any other ($\pi^+\pi^-$ / $\eta\gamma$)**
3. **All available exp. data sets about these channels are not necessarily consistent within BHLS**

VMD Strategy for HVP Estimates

- Perform a global fit :: if successful then

- 1/ VMD correlations are fulfilled by DATA

- 2/ HLS form factors & fit parameters values
  & errors covariance matrix should lead to

better estimates of HVP contributions to g-2 for
$\pi^+\pi^-$/ K^+K^-/ $K_L K_S$/$\pi\gamma$ /$\eta\gamma$ /$\eta'\gamma$ /$\pi\pi\pi$ up to 1.05 GeV

VMD Strategy for HVP Estimates

- Perform a global fit : if successful then

- 1/ VMD correlations are fulfilled by DATA

- 2/ HLS form factors & fit parameters values
  & errors covariance matrix should lead to

better estimates of HVP contributions to g-2 for
$\pi^+\pi^-$/ K^+K^-/ $K_L K_S$/$\pi\gamma$ /$\eta\gamma$ /$\eta'\gamma$ /$\pi\pi\pi$ up to 1.05 GeV

Can One trust Global Fits?

- **Outcome of a Minimization Tool : χ^2 (& MINUIT)**

implemented using *assumptions* on :

- **Error Covariance Matrices (metrics of χ^2 distance)**
- **Global Scale Uncertainties (possibly s-dependent)**
- **Th. models (Non-linear parameter dependence)**

Even if fits are 100% successful :

check if numerical conclusions can be trusted

How to Check Global Fits?

- Several Expectations :
  - ✓ Parameter residuals OK : Unbiased
  - ✓ Parameter Pulls OK : Gaussians G(m=0, σ=1)
  - ✓ Probability distributions OK: Uniform on [0,1]
- → Fit parameters values & Fit Error Cov. Matrix OK

So : Any derived info. $X_0+\Delta X$ (val./err.) OK

BUT truth should be known → MC methods

How to Check Global Fit Methods?

- Several Expectations :
  - ✓ Parameter residuals OK : Unbiased
  - ✓ Parameter Pulls OK : Gaussians G(m=0, σ=1)
  - ✓ Probability distributions OK: Uniform on [0,1]
- → Fit parameters values & Fit Error Cov. Matrix OK

  So : Any derived info. $X_0 \pm \Delta X$ (val./err.) VALID

  BUT truth should be known → MC methods

How to Check Global Fit Methods?

- Several Expectations :
  - ✓ Parameter residuals OK : Unbiased
  - ✓ Parameter Pulls OK : Gaussians G(m=0, σ=1)
  - ✓ Probability distributions OK: Uniform on [0,1]
- → Fit parameters values & Fit Error Cov. Matrix OK

So : Any derived info. $X_0 \pm \Delta X$ (val./err.) VALID

BUT truth should be known → MC methods

How to Check Global Fit Methods?

- Several Expectations :
  - ✓ Fit parameter residuals OK : Unbiased
  - ✓ parameter Pulls OK : Gaussians G(m=0, σ=1)
  - ✓ Probability distributions OK: Uniform on [0,1]
- → Fit parameters values & Fit Error Cov. Matrix OK

So : Any derived info. $X_0 \pm \Delta X$ (val./err.) VALID

BUT truth should be known → MC methods

χ^2 Function : Global Scale Issues

- Spectrum (**E**) subject to **one** scale uncertainty

λ_E [G(0,σ)] and stat. err. cov. V_E :

$$\chi_E^2 = [\, m_E - M - \lambda_E A \,]^T V_E^{-1} [\, m_E - M - \lambda_E A \,] + \lambda_E^2 / \sigma^2$$

data

model

Global scale

« Penalty term »

Stat. & uncorrel. syst

If no global scale :

$$\chi_E^2 = [\, m_E - M \,]^T V_E^{-1} [\, m_E - M \,]$$

what about A : Specific to E? Common to {E}?

s-dependent Global Scale Factors

- several independent scale factors (necessarily s-dependent) affect the spectrum (E)
- The α^{th} scale factor : $\lambda_\alpha = \mu_\alpha (0,1)\, \sigma_\alpha (s)$
- Define the vectors $B_\alpha(s) = A(s)\ \sigma_\alpha(s)$
- then

$$\chi_E^2 = \left[m_E - M - \mu_\alpha B_\alpha \right]^T V_E^{-1} \left[m_E - M - \mu_\beta B_\beta \right] + \mu_\alpha \mu_\beta \delta_{\alpha\beta}$$

M.Benayoun *et al* . **EPJ C73**(2013)2453

« Penalty term »

Scale Uncertainty(ies)

M. Benayoun *et al* EPJ C73 (2013)2453

- Minimize :

$$\chi^2 = [\, m - M - \lambda A \,]^T V^{-1} [\, m - M - \lambda A \,] + \lambda^2 / \sigma^2$$

•Solving for λ ($\partial \chi^2 / \partial \lambda = 0$) leads to:

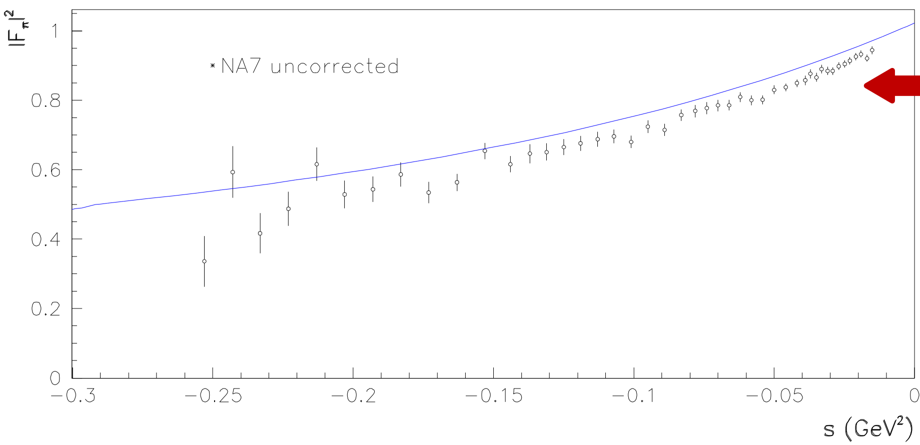
$$\chi^2 \equiv [\, m - M \,]^T \left[V + \sigma^2 A A^T \right]^{-1} [\, m - M \,]$$

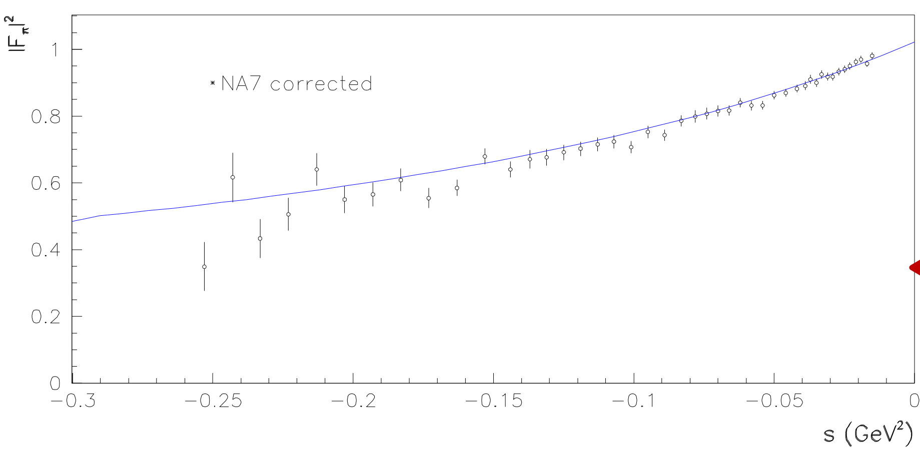
•with : $\lambda = \left\{ A^T V^{-1} [\, m - M \,] \right\} / \left\{ A^T V^{-1} A + \frac{1}{\sigma^2} \right\}$

How to choose A ?

How to check A ?

NA7 Residuals ($\chi^2/N \approx 2$)!





$$[\, m - M \,]$$

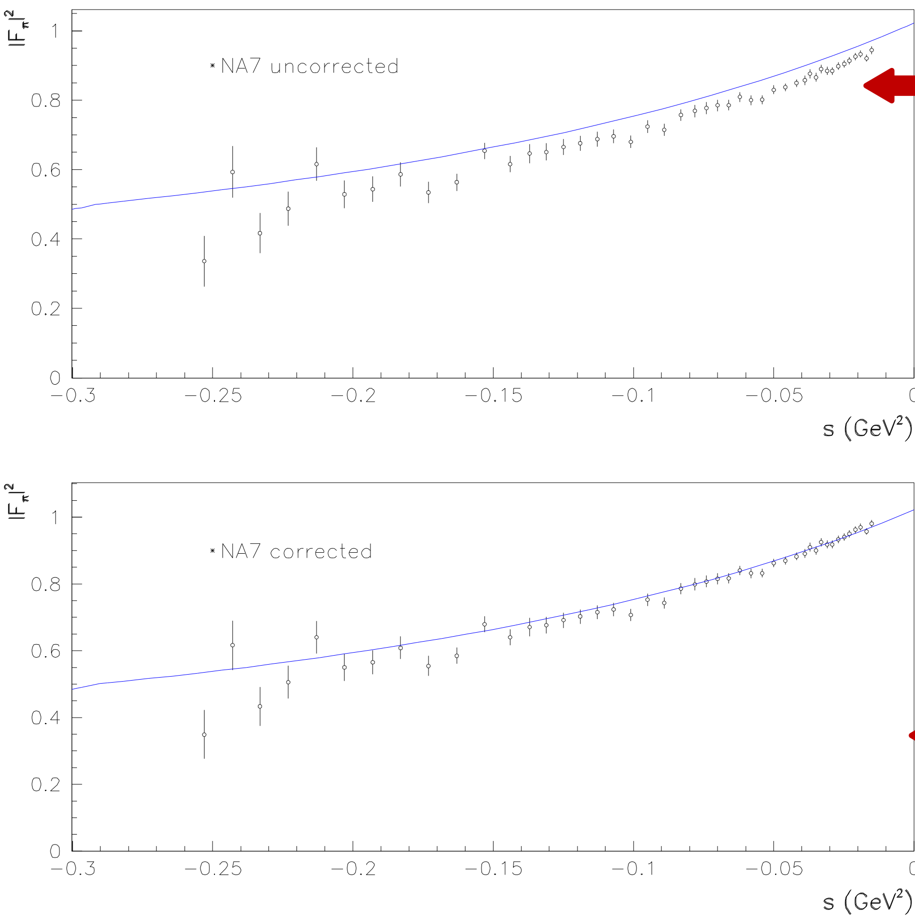
spacelike

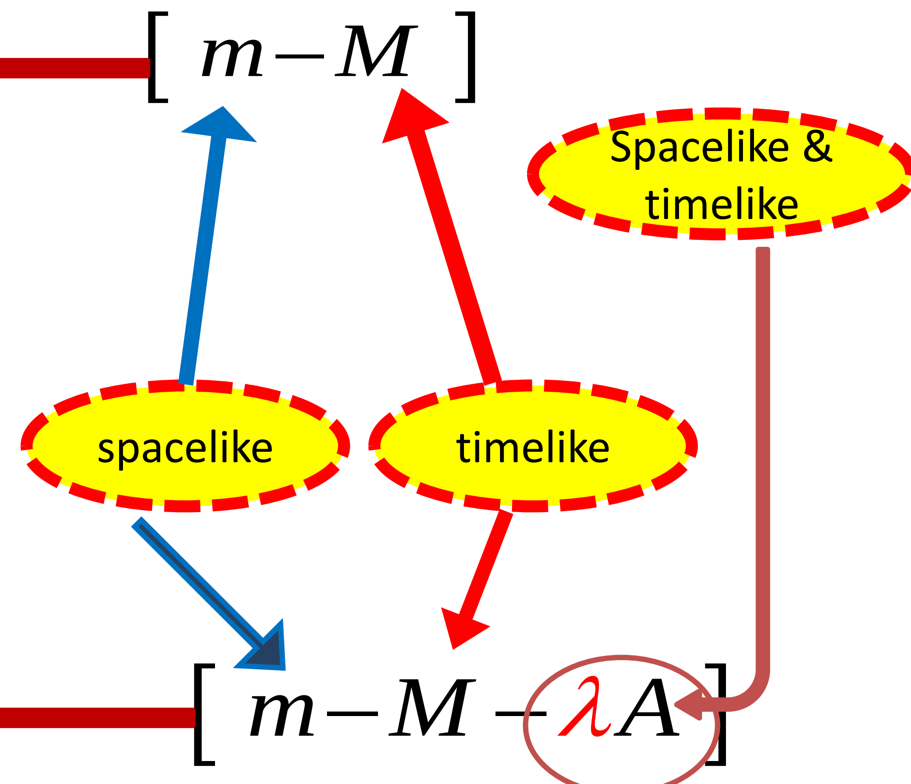
timelike

$$[\, m - M - \lambda A \,]$$

M. Benayoun *et al* EPJ C73 (2013)2453

NA7 Residuals ($\chi^2/N\approx2$)!





M. Benayoun *et al* EPJ C73 (2013)2453

An Effect of Scale Uncertainty

- *Experimental* quantity ⟷ M ?

m? **or** m-λA ?

- Contribution to HVP :

$$\int K(s)\, M(s) \ =?\ \int K(s)\, m(s)$$

An Effect of Scale Uncertainty

- *Experimental* quantity ⟷ M ?

m? or m-λA ?

- Contribution to HVP :

$$\int K(s)\, M(s) \;=?\; \int K(s)\, m(s)$$

$$a_\mu(H_i) = \frac{1}{4\pi^3} \int_{s_{th}}^{s_{cut}} ds\, K(s)\, \sigma(e^+e^- \to H_i, s)$$

An Effect of Scale Uncertainty

- *Experimental* quantity ⟷ M ?

  m? or m-λA ?

- Contribution to HVP should be corrected

$$\int K(s)\, M(s) = \int K(s)\, m(s) - \lambda \int K(s)\, A(s)$$

- Correction Evaluation requires λ & A(s)

  → fits!

An Effect of Scale Uncertainty

- *Experimental* quantity ⟷ M ?

  m? or $m-\lambda A$?

- Contribution to HVP should be corrected

$$\int K(s)\, M(s) = \int K(s)\, m(s) - \lambda \int K(s)\, A(s)$$

- Correction Evaluation requires λ & A(s)

  → but fits provide M(s) directly

How to choose/check A?

- The best choice is $A = M_{truth}$

  M_{truth} is unknown !

G. D'Agostini NIM A346 (1994)306

- $A = m$ may be not optimum:

M.Benayoun *et al* . EPJ C73(2013)2453

→ biased(?) information → How to unbias?

- A solution : Iterative Method

R.D.Ball *et al* JHEP 1005 (2010)075

iteration 0 : $A = m$ → it=0 fit. func. : M_0

iteration 1 : $A = M_0$ → it=1 fit. func. : M_1

ETC..... up to convergence $M_n = M_{truth}$

- Also $A=M$ (varying) if some good starting point

Global Fit of Toy Monte-Carlo Samples

- Choosing theoretical function(s) $f_{th}(s)$
- Generate N_{rep} replicas of N_{exp} spectra built using $f_{th}(s)$ together with :
  - a *given* statistical covariance matrix **V**
  - *given* scale uncertainties
- Fitting the N_{rep} set of N_{exp} spectra (MINUIT)

Global Fit of Toy Monte-Carlo Samples

- **Th. functions [f(s)] : exponential, logarithm, polynomials, BW & combinations**

➢ ***report on***

$$f(s) = \frac{g}{(s-a)^2 + b^2} + c + ds + es^2$$

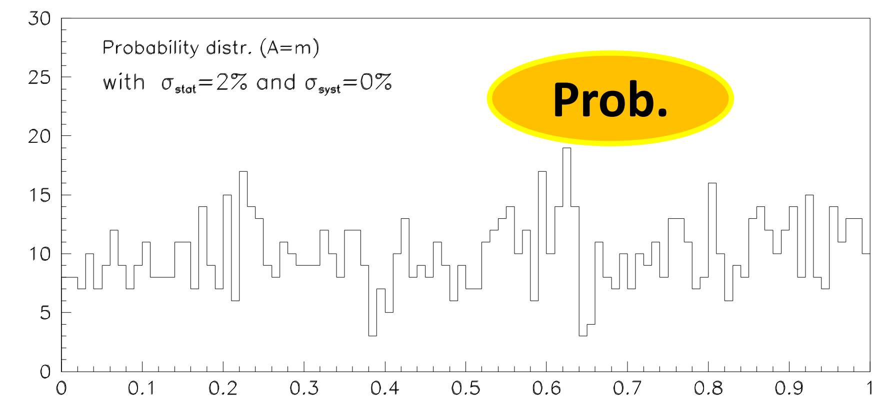
- **Fit the N_{rep} replicas of the N_{exp} spectra**

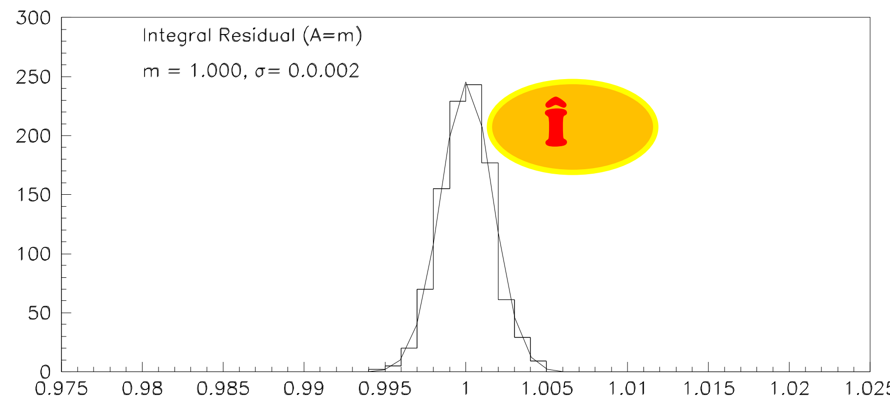
➢ **Check Residuals, pulls, prob. distributions**

➢ **Check ratio Î of Integrals for $f_{fit}(s)$ & $f_{th}(s)$ (~ a_μ)**

Non Linear Effects

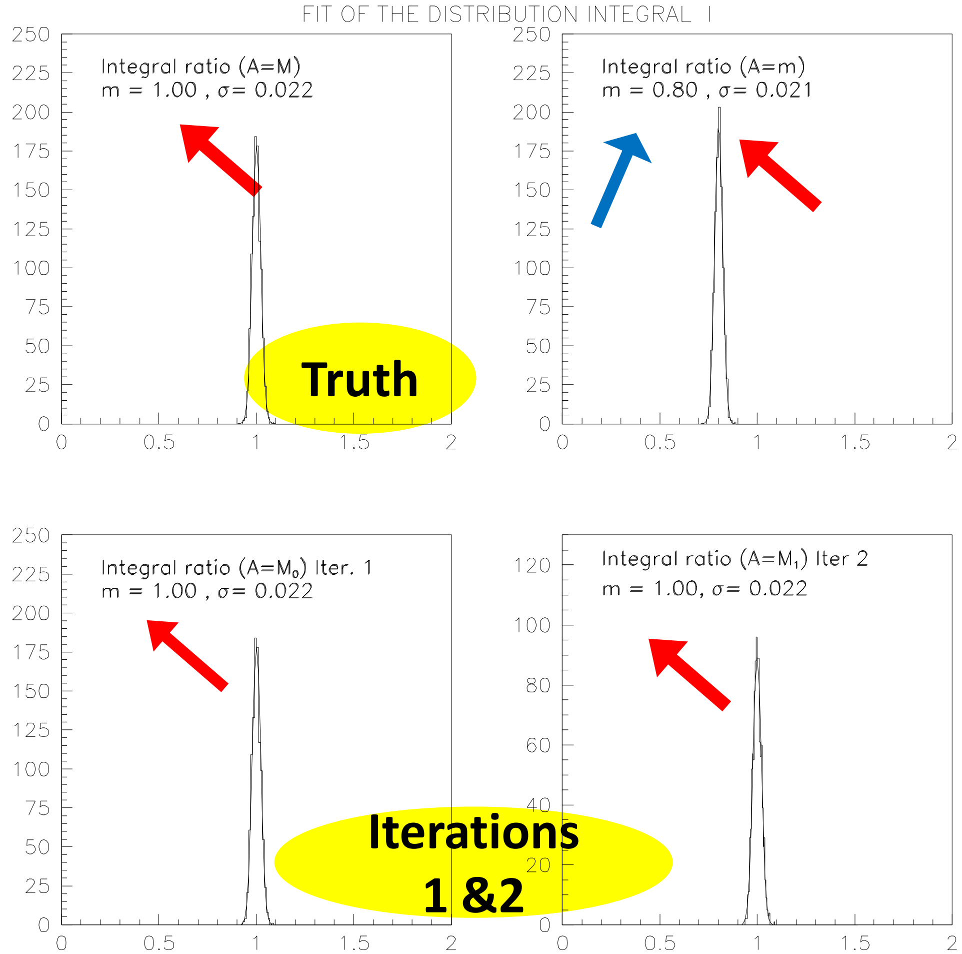
- $N_{exp} = 5 / N_{rep} = 1000$
- σ_{stat}=2% , σ_{scale}=0%
- χ^2 does not depend on A
- < î > =1
- Proba : (m=0.5, σ=1/√12)
- All pulls G(0,1)
- Errors : Parabolic ≡ MINOS (Migrad/MINUIT)





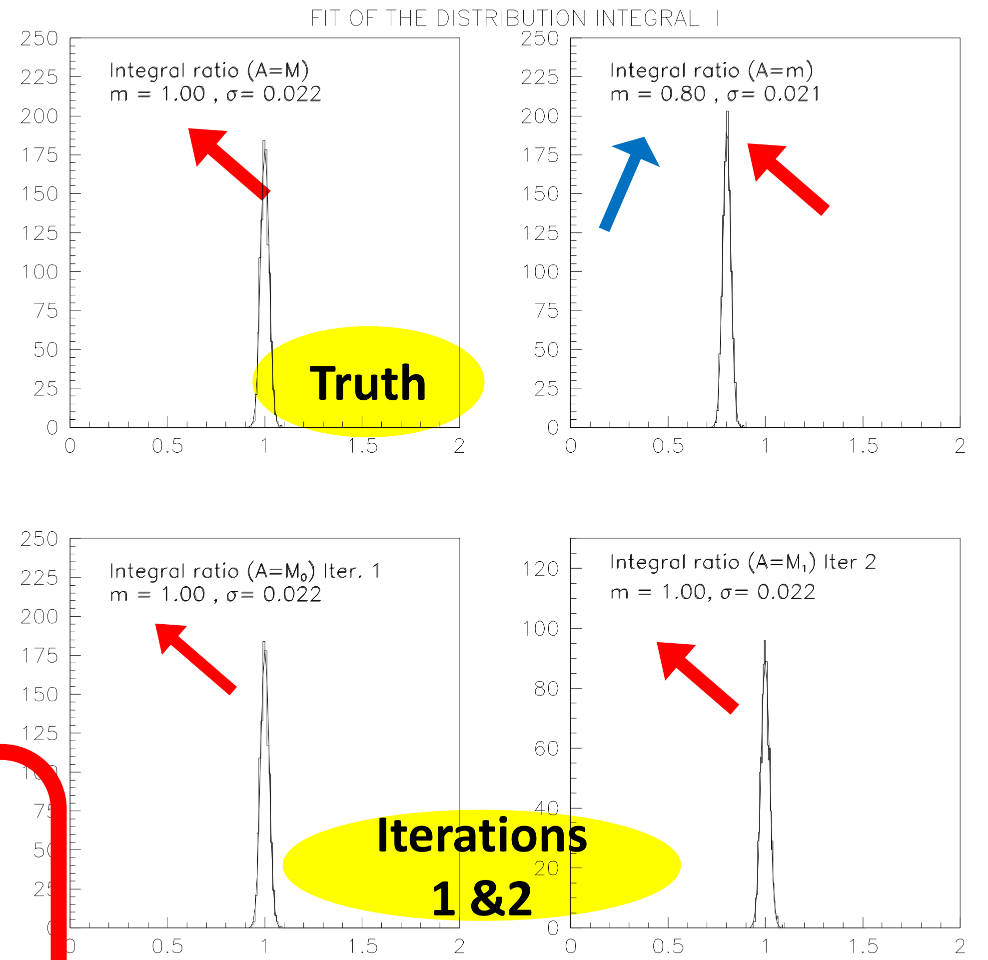
The Integral ratios ↑ (I)

- $N_{exp} = 5/N_{rep} = 1000$
- Each : σ_{stat}=3% σ_{scale}=5%
- A=M_{truth} : fit OK
- A=m : fit biased (20%!)
- A=M_0 : fit derives M_1
- A=M_1 : fit derives M_2



The Integral ratios (I)

- $N_{exp} = 5 / N_{rep} = 1000$
- Each : $\sigma_{stat}=3\%$ $\sigma_{scale}=5\%$
- $A=M_{truth}$: fit OK
- $A=m$: fit biased (20%!)
- $A=M_0$: fit derives M_1
- $A=M_1$: fit derives M_2



Converges at iteration #1

R.D.Ball *et al* JHEP 1005 (2010)075

The Integral ratios (II)

samples free of scale error

(parenthesis)

- $N_{exp} = 5$ some with
- $\sigma_{stat}=3\%$ & $\sigma_{scale}=5\%$
- ++ 1 or 2 samples with
- $\sigma_{stat}=6\%$ & $\sigma_{scale}=0\%$

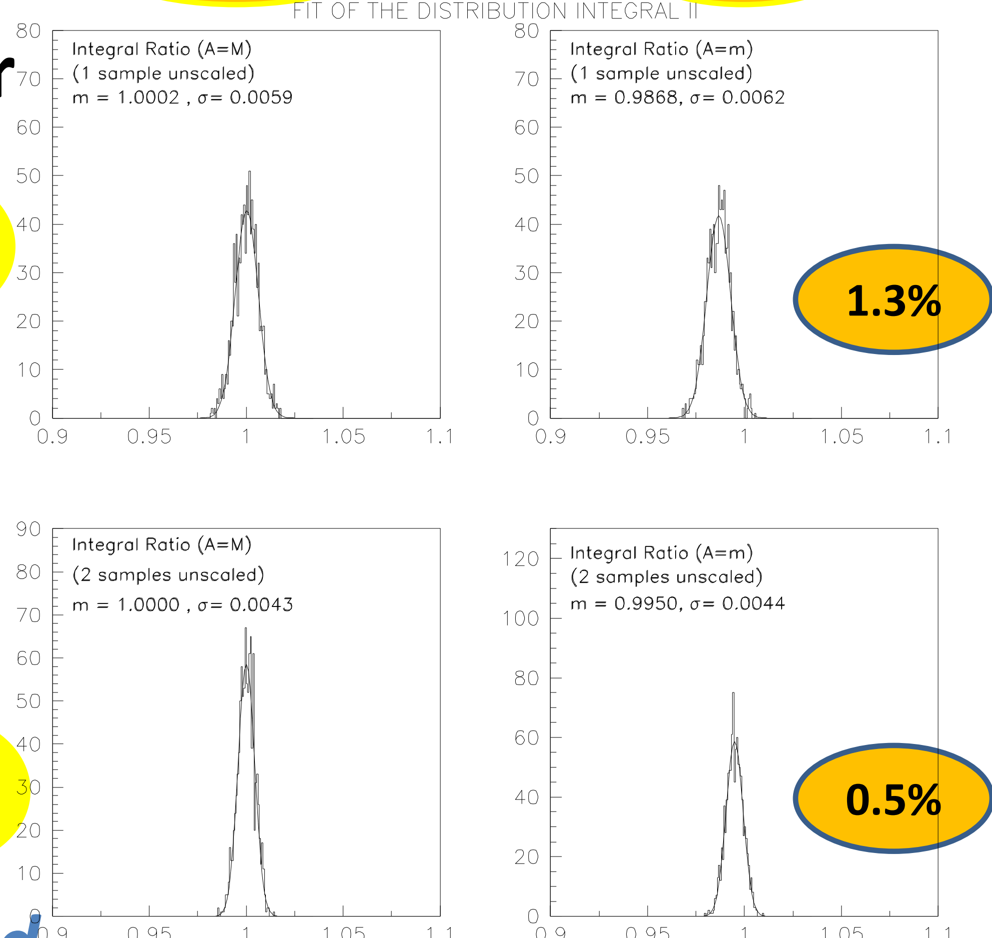
4+1

3+2

- $A=M_{truth}$: fit OK
- A=m : bias *much reduced*

$A=M_{truth}$

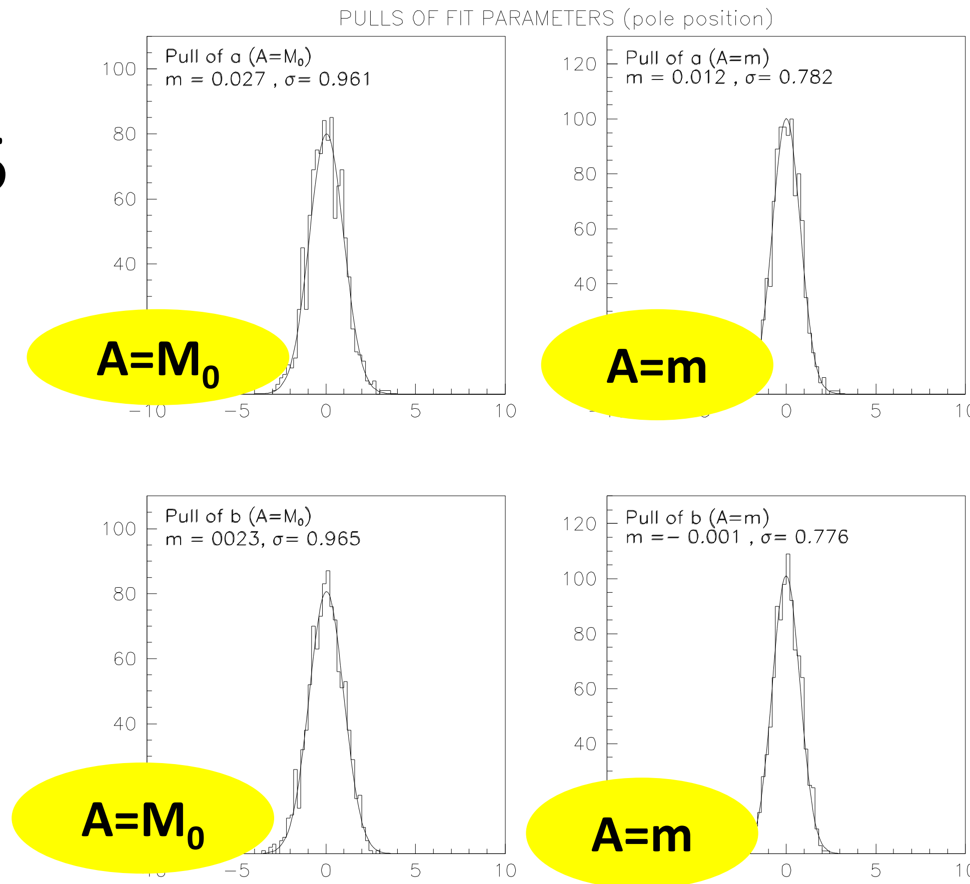
A=m



Fit Parameter Pulls I

- A=m $\sigma_{pull} \approx 0.80$
- $A=M_0$ $\sigma_{pull} \geq 0.95$

- **Peaks shrink (param. a & b)**



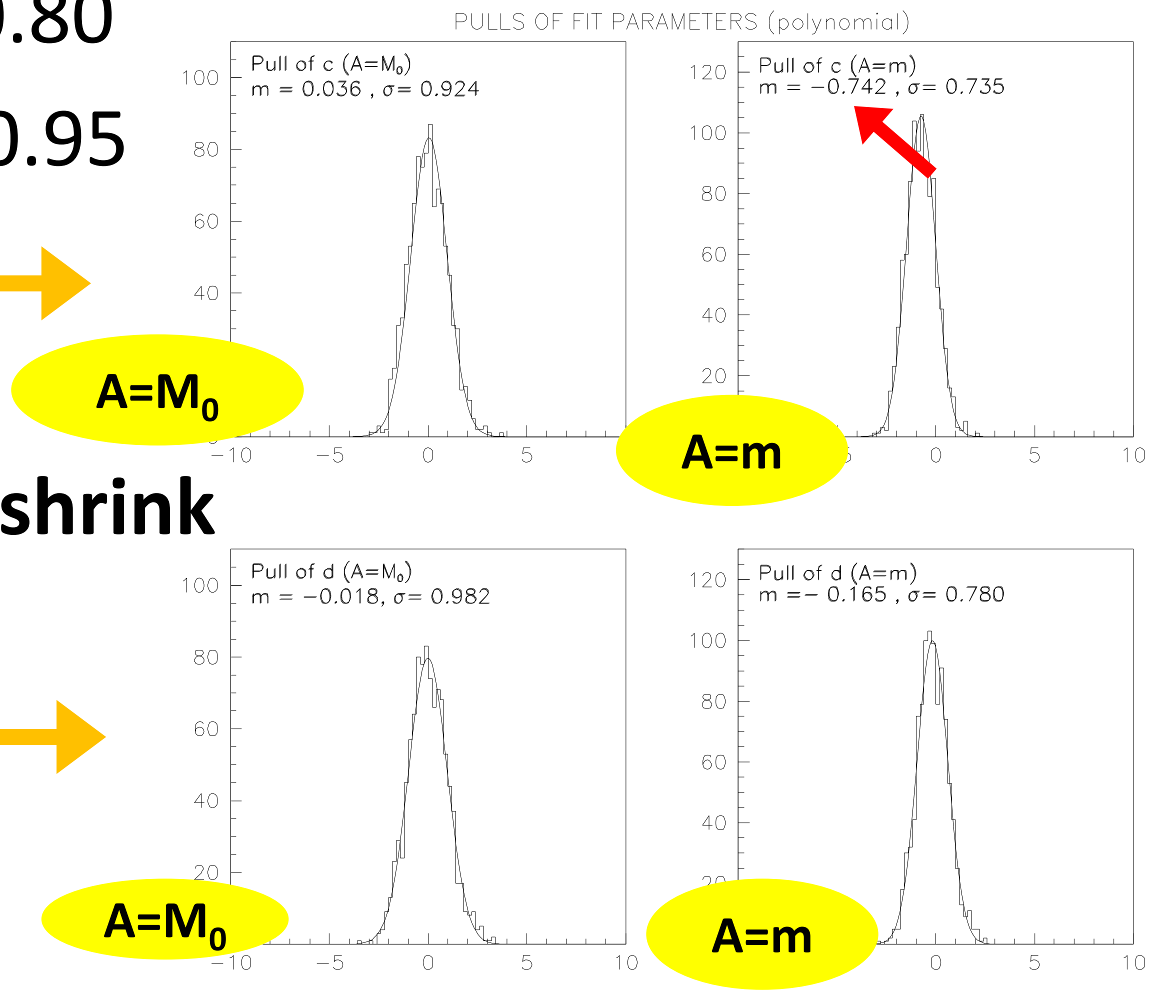
Fit Parameter Pulls II

- A=m $\sigma_{pull} \approx 0.80$
- A=M_0 $\sigma_{pull} \geq 0.95$

Const.term

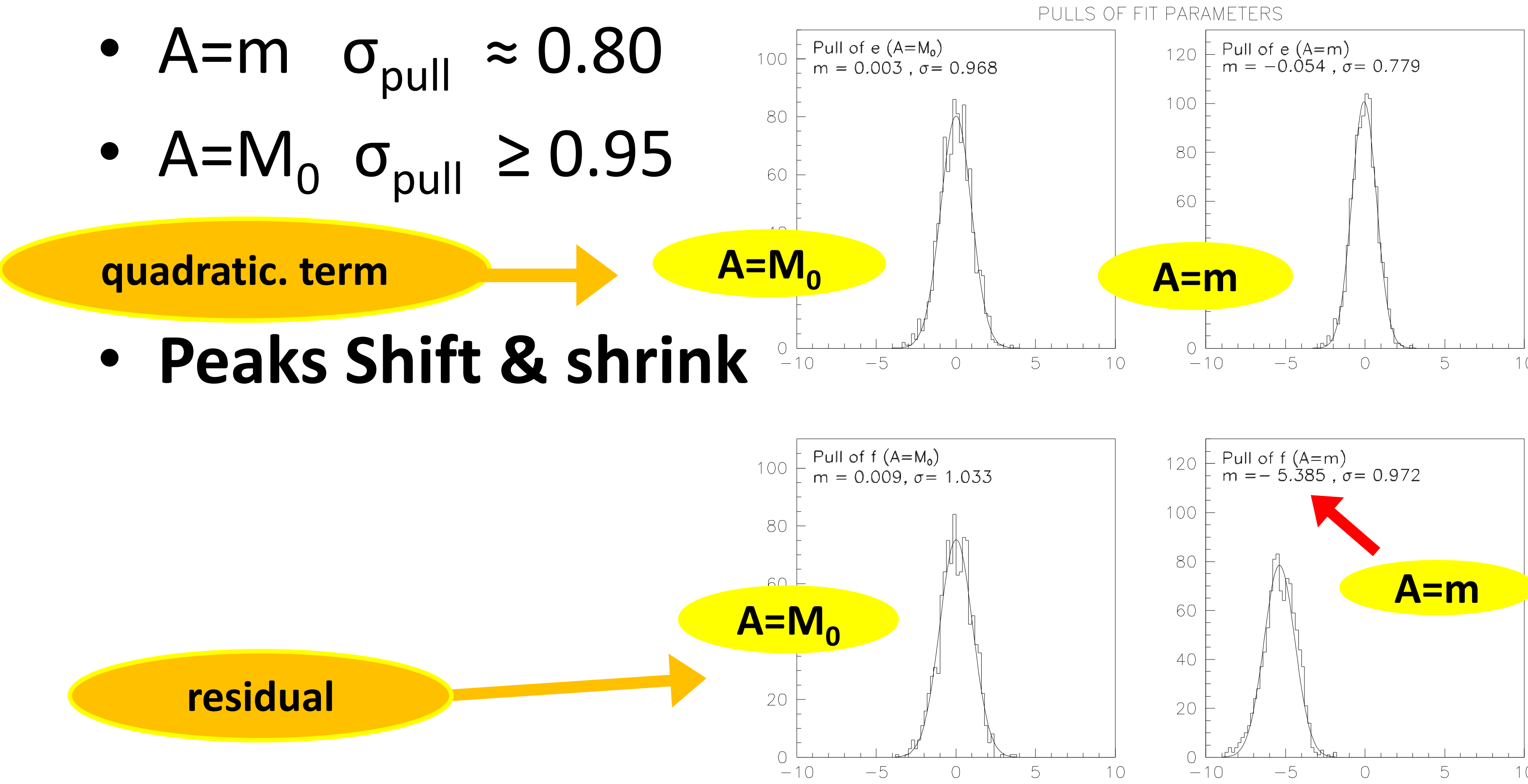
- **Peaks Shift & shrink**

Linear term



Fit Parameter Pulls III

- A=m $\sigma_{pull} \approx 0.80$
- A=M_0 $\sigma_{pull} \geq 0.95$
- **Peaks Shift & shrink**

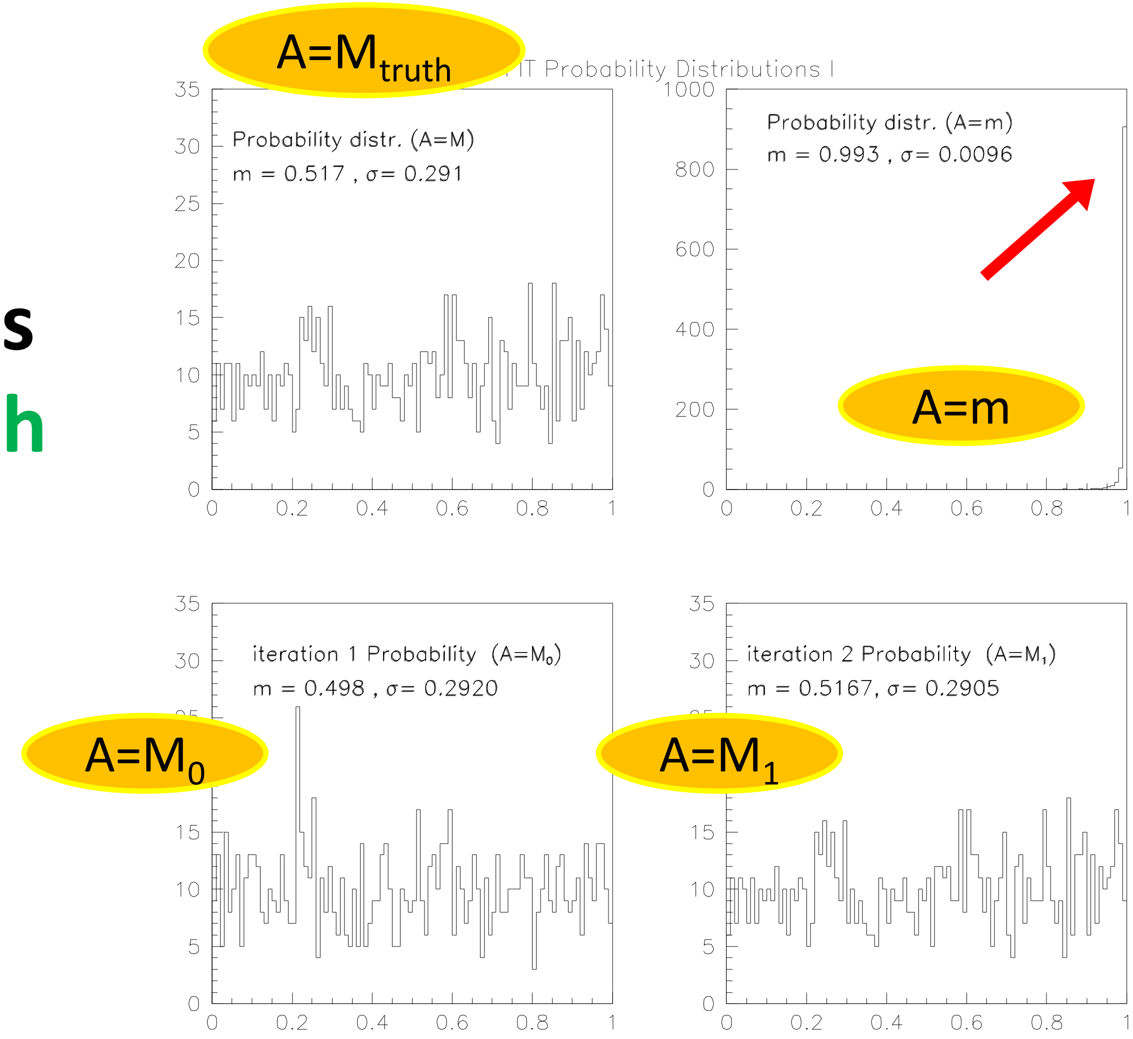


Fit Probabilities

- A=m **peaked**
- **A=M/M$_0$/M$_1$/**

Flat distributions

for iter. & truth



Fit Probabilities II

- **Flat distributions**

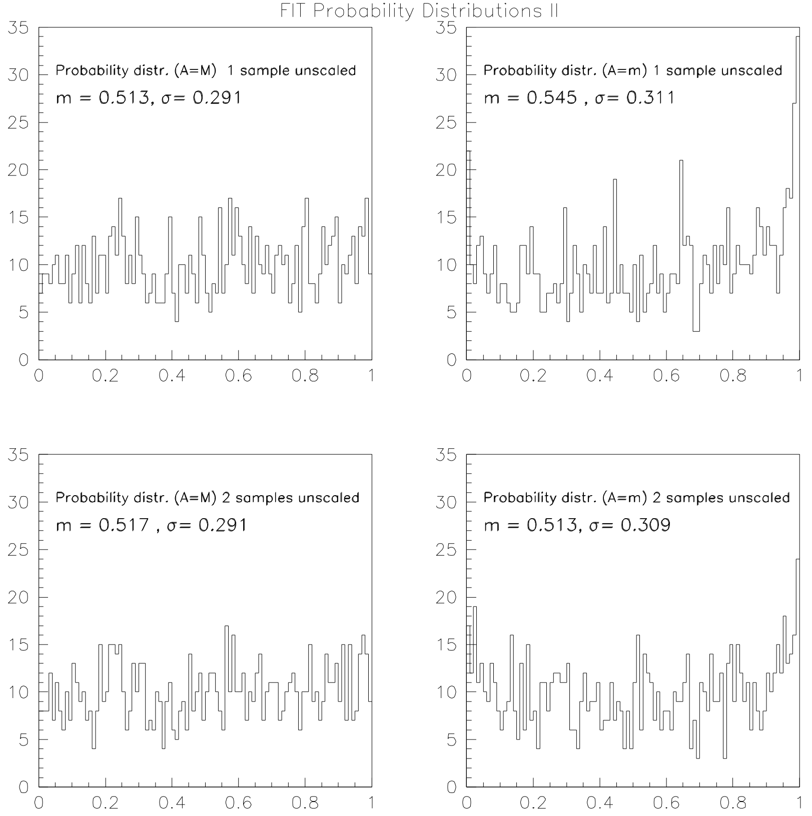
(without iterations)

A=M_{truth}

A=m

4+1

3+2

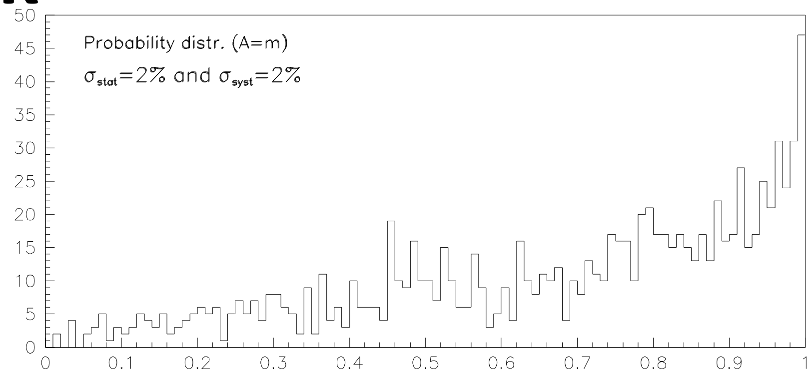


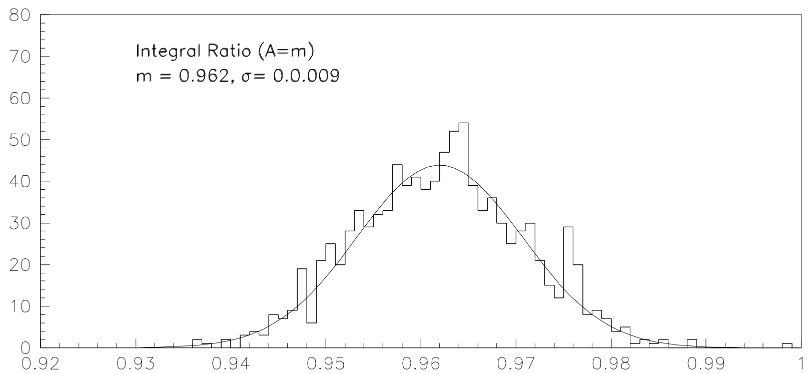
CASE : σ_{stat} =2% & σ_{syst}=2%

- smooth probability peak
- $A=M/M_0/M_1/$

Flat distributions

4% bias **(20%)**





Can One trust Global Model/Fits?

- From Toy MC studies one may conclude :
  - **Bias & shrink exist with amplitudes depending on the relative magnitudes of σ_{stat} & σ_{syst}**
  - **but a few data samples with $\sigma_{syst} << \sigma_{stat}$ sharply limit bias & shrinkage**
  - **Running 1 iteration allows always full recovery**
  - **Global fit of the largest set of data samples should give more robust estimates**

Can One trust Global Model/Fits?

- From Toy MC studies one may conclude :
- **Bias & shrink exist with amplitudes depending on the relative magnitudes of σ_{stat} & σ_{syst}**
- **but a few data samples with $\sigma_{syst} << \sigma_{stat}$ sharply limit bias & shrinkage**
- **Running 1 iteration allows always full recovery**
- **Iterated Global fits of the largest set of data samples should give more robust estimates (central & rms)**

[Iterated] GLOBAL FITS

- **Global Fits (+ iterate) of the data samples for**

$\tau \rightarrow \pi\pi\, \nu_\tau$, $e^+e^- \rightarrow \pi^+\pi^-$/ K^+K^-/ $K_L K_S$/$\pi\gamma$/$\eta\gamma$/$\pi\pi\pi$

(probability , average χ^2/N)

- **Discard $\pi\pi\pi$ data in ϕ region (conf. B)**
- **Fitting from thresholds to 1. GeV/c (τ, $\pi\pi$, $\pi\pi\pi$)**
- **Fitting from thresholds to 1.05 GeV/c**
- **Identify samples not consistent within BHLS**

$\pi^+ \pi^-$ Spectra : NSK, KLOE, BaBar

- Several measurements of the $\pi^+ \pi^-$ spectrum

i. CMD2, SND

CMD2: Phys. Lett. **B648** (2007) 28, JETP Lett. **84** (2006) 413
SND: JETP **103** (2006) 380

ii. KLOE

KLOE08 : AIP Conf. Proc. **1182** (2009) 665 *

KLOE10: Phys. Lett. **B700** (2011) 102
KLOE12: Phys. Lett. **B720** (2013)336

iii. BaBar

BaBar : Phys. Rev. Lett. **103** (2009) 231801 *
Phys. Rev . **D86** (2012) 032013

exhibit conflicting behaviors within global fits

M. Benayoun ***et al*** **EPJ C73** (2013)2453

Fitting $\pi^+\pi^-$ Data using τ Samples

Fit Cond. ($\chi^2/N_{\pi+\pi-}$)	KLOE08(60)	KLOE10(75)	KLOE12(60)	NSK(127/209)	BaBar(250) (trunc)
Single ($\chi^2/N_{\pi+\pi-}$)	1.64 59 %	0.96 97%	1.02 97 %	0.96 [0.83] 97 % [99%]	1.15 74%
Comb 1 χ^2/N : 1.28(11%)		1.02	1.48	1.18[0.96]	1.35
Comb 2 χ^2/N: 1.06(97%)		1.02	1.05	1.10[0.89]	
Comb 3 χ^2/N: 0.98 (96%)		0.97	1.00		

Global Fit Results with τ & NSK

Data Set (#data points)	$\chi2$ (NSK+τ)	X^2 (NSK+KLOE)+τ
Decays (10)	8.4	9.2
New Timelike (127)	122.3	139.7
Old Timelike (82)	50.4	46.2
$\pi^0\gamma$ (86)	64.0	64.2
$\eta\gamma$ (182)	120.1	120.8
$\pi^+\pi^-\pi^0$ (99)	102.3	101.8
K^+K^- (36)	29.9	29.9
K_LK_S (119)	119.3	119.1
τ ALEPH (37)	19.5	19.3
τ CLEO (29)	35.6	36.4
τ Belle (19)	28.3	30.9
X^2/dof // Probability	701/801 // 99.5%	857/936// 97%

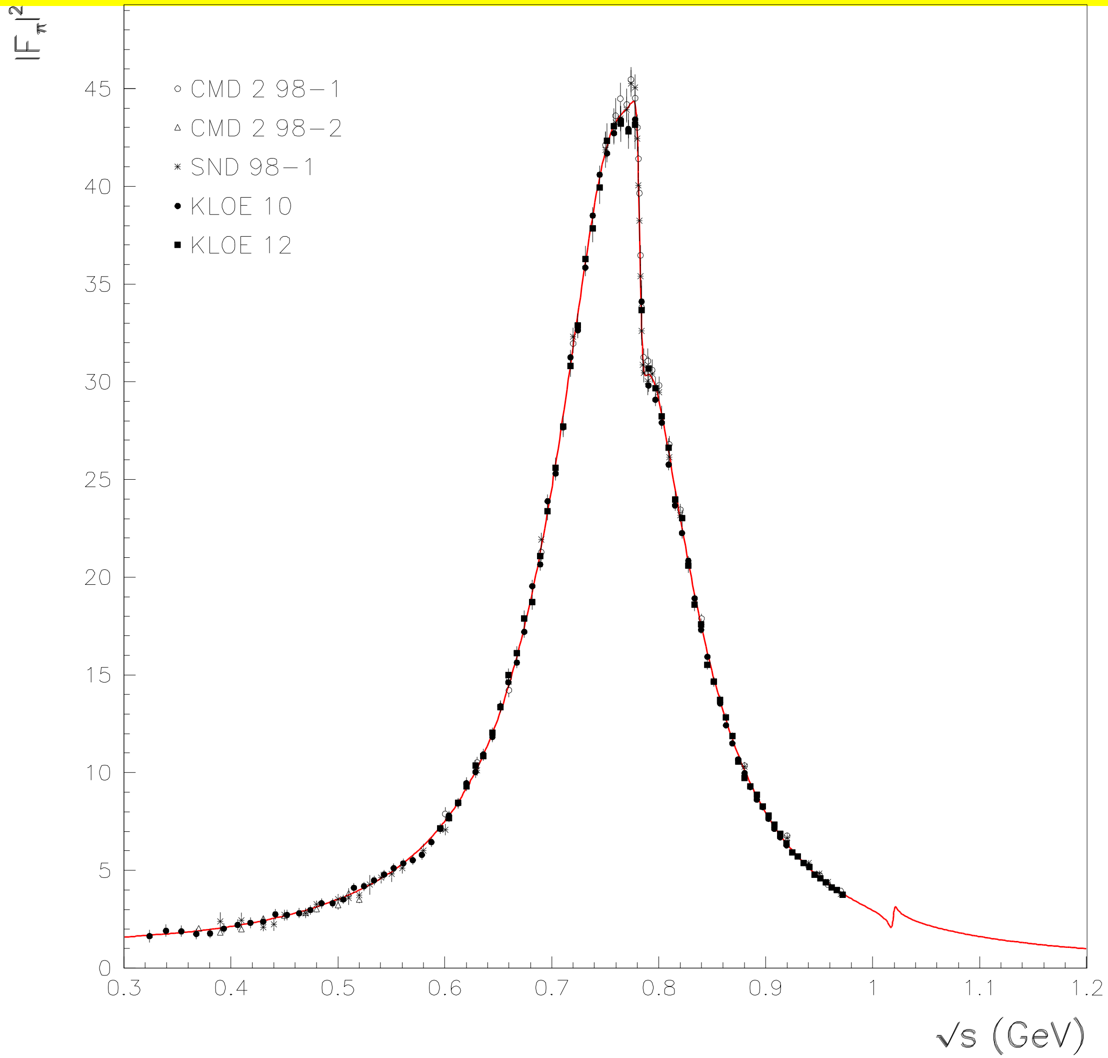
GLOBAL FITS : Check ππ data sets

- Fits with each ππ data set in isolation
  (scan/KLOE's/BaBar) → **select on Prob.**

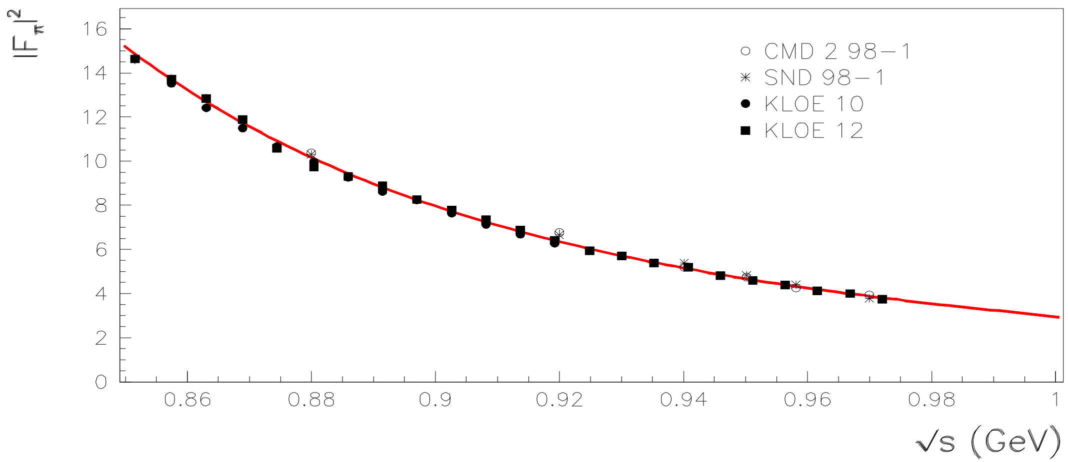
- **All other channels always included in fits**
  ($\tau \to \pi\pi \nu_\tau$, $e^+e^- \to$ K^+K^-/ $K_L K_S$/$\pi \gamma$/$\eta\gamma$/$\pi \pi \pi$)

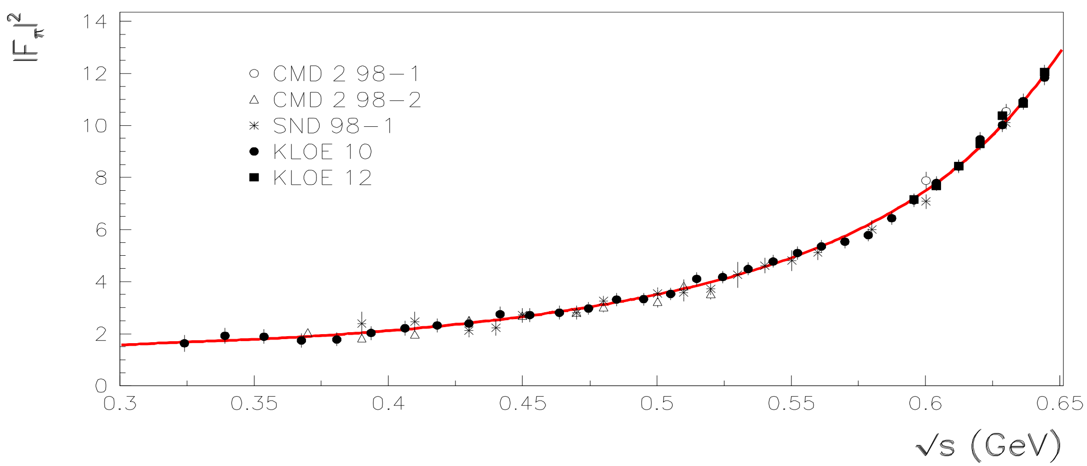
Consistent $\pi^+\pi^-$ data sets for Global Treatment :
CMD2 & SND & KLOE10 & KLOE12

GLOBAL FIT : ππ spectrum

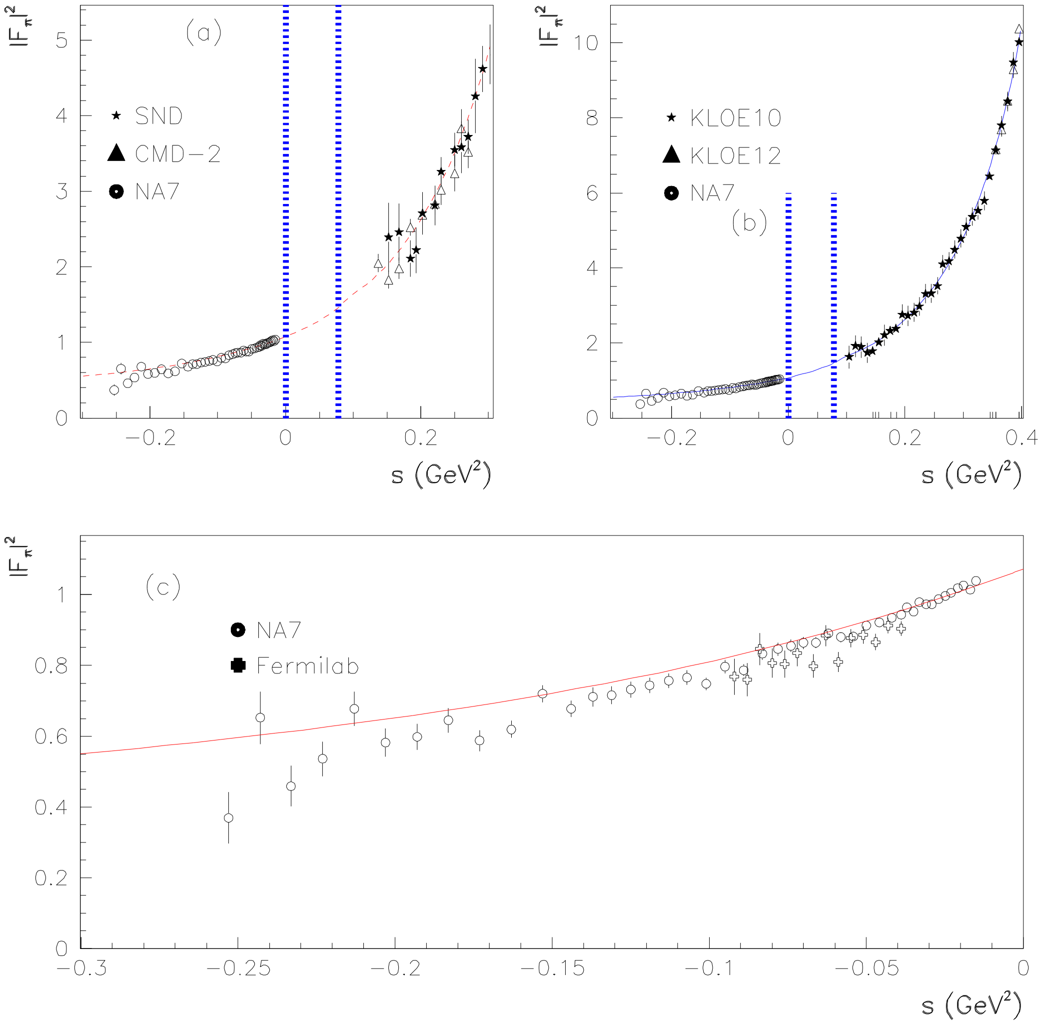


Global Fits : Side Regions





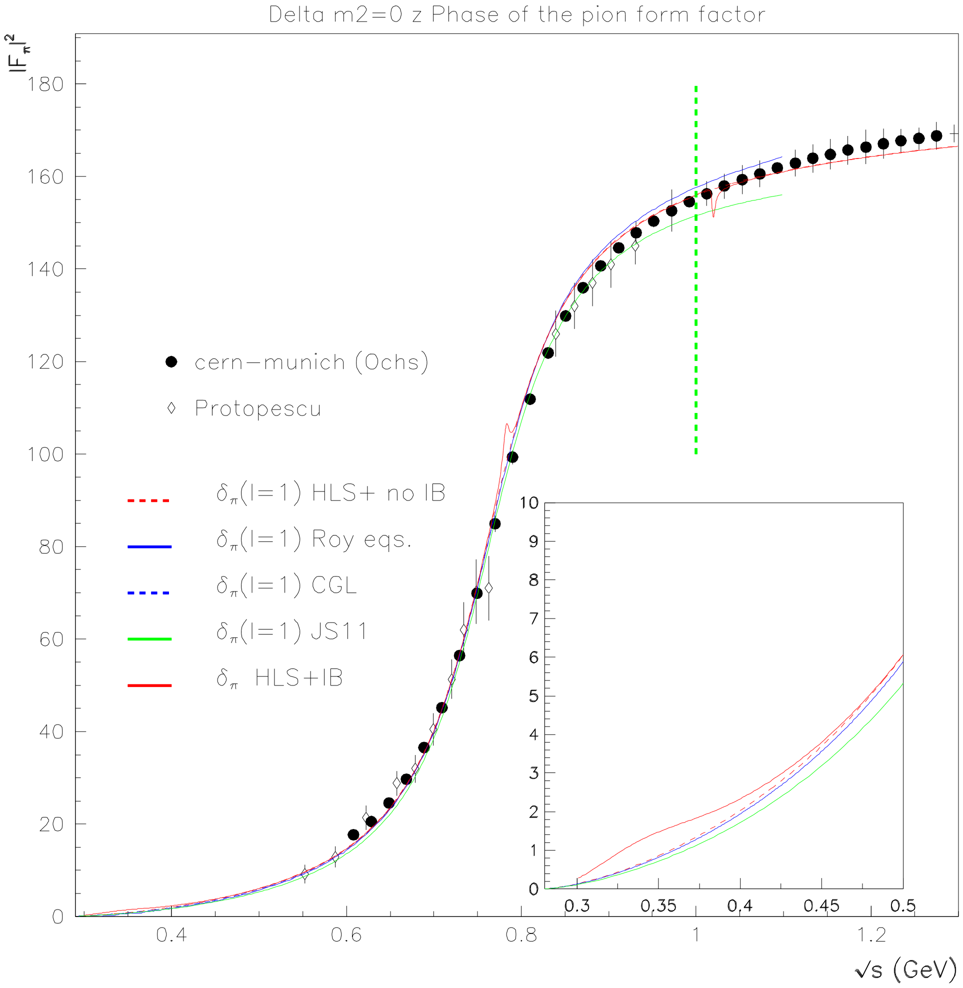
The Spacelike & threshold Regions



Threshold region

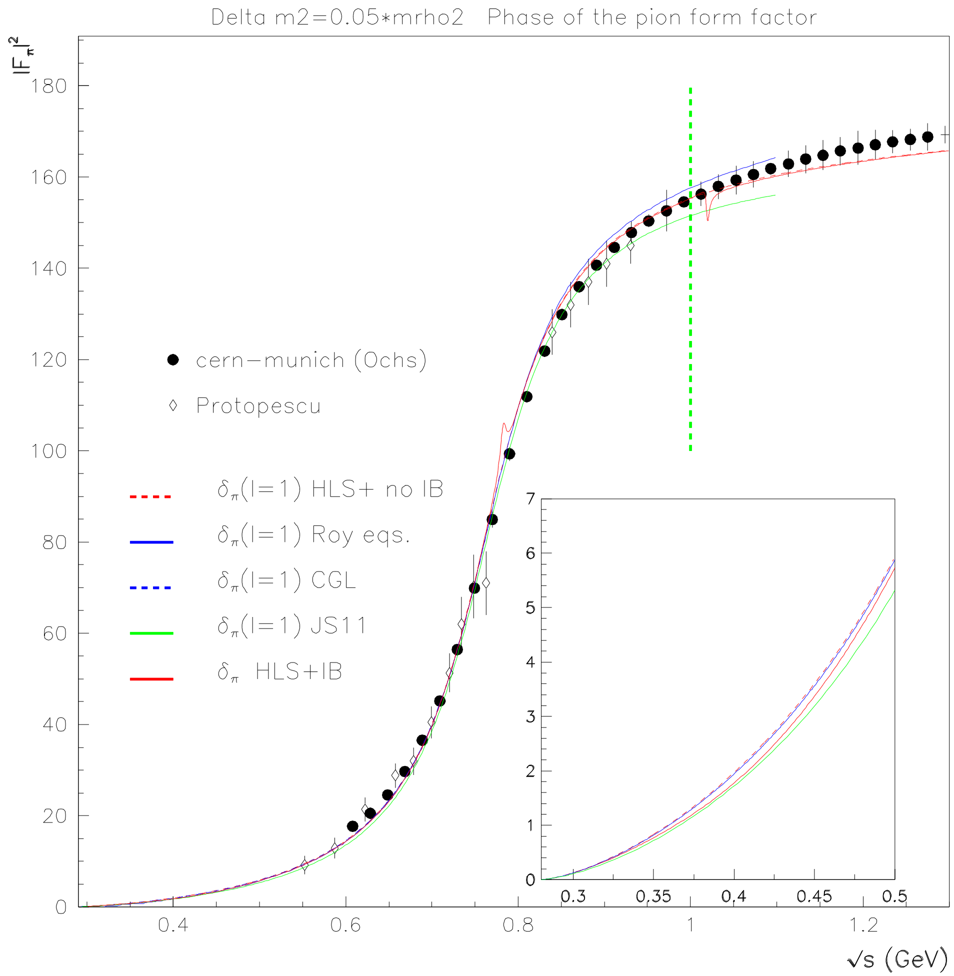
Extrapotation to s<0

Predicted Phase shift (I)



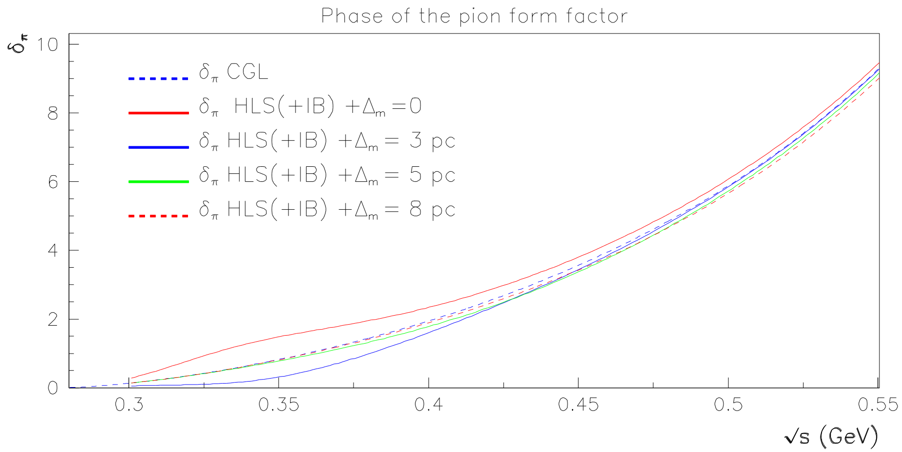
$$\left[\frac{m_\omega^{HK}}{m_\rho^{HK}}\right]^2 = 1.00$$

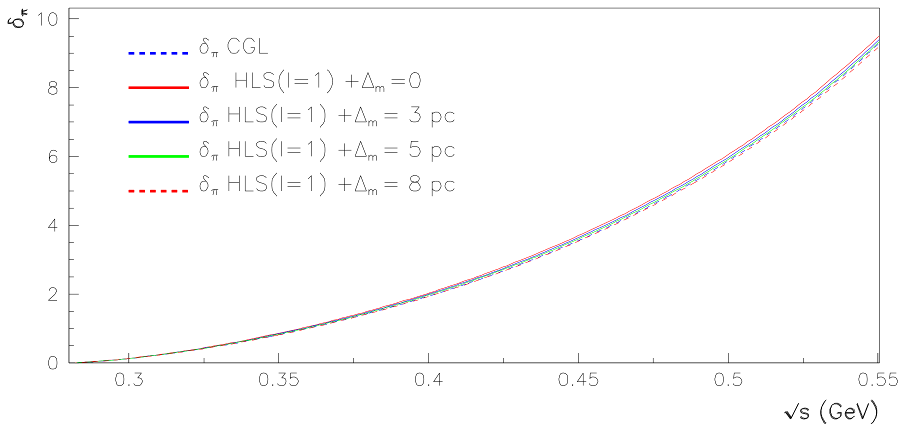
Predicted Phase shift (II)



$$\left[\frac{m_\omega^{HK}}{m_\rho^{HK}}\right]^2 = 1.05$$

Threshold Behavior -> NSB



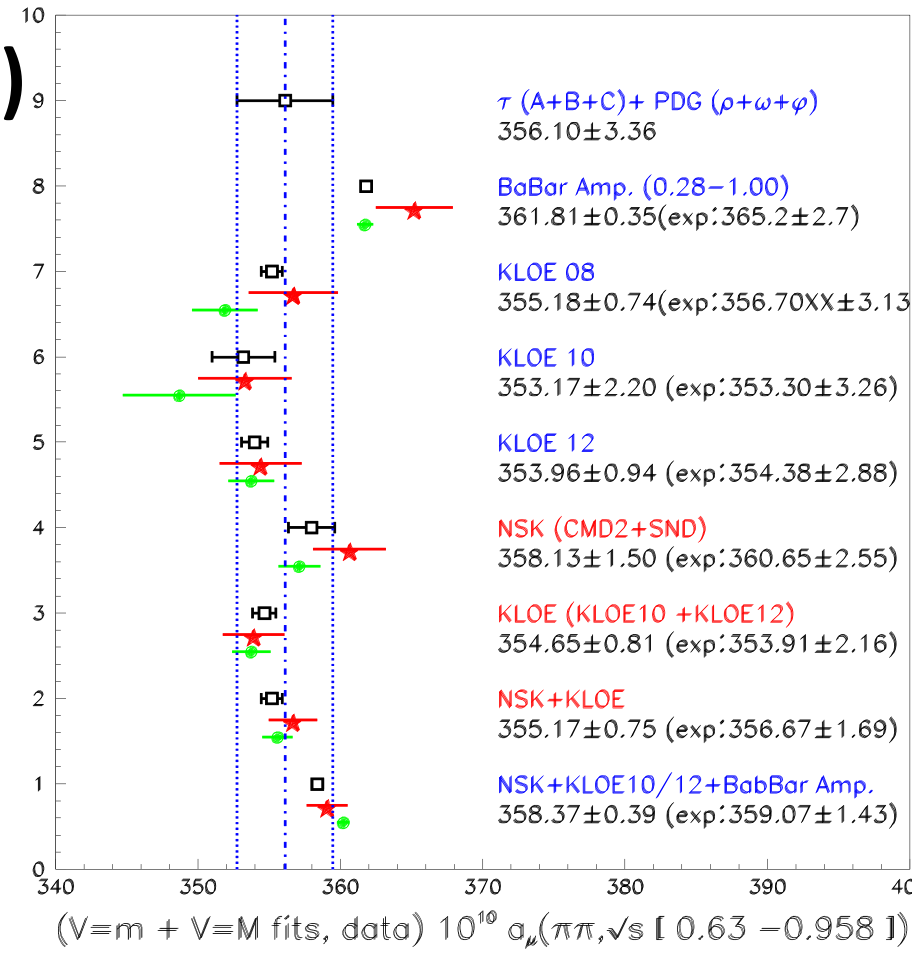


$$\alpha(s)=\frac{\varepsilon_1(s)}{\left[m_\rho^{HK}\right]^2+\Pi_{\pi\pi}^{\rho}(s)-\left[m_\omega^{HK}\right]^2}$$

$$\Delta_m=\frac{\left[m_\omega^{HK}\right]^2}{\left[m_\rho^{HK}\right]^2}-1$$

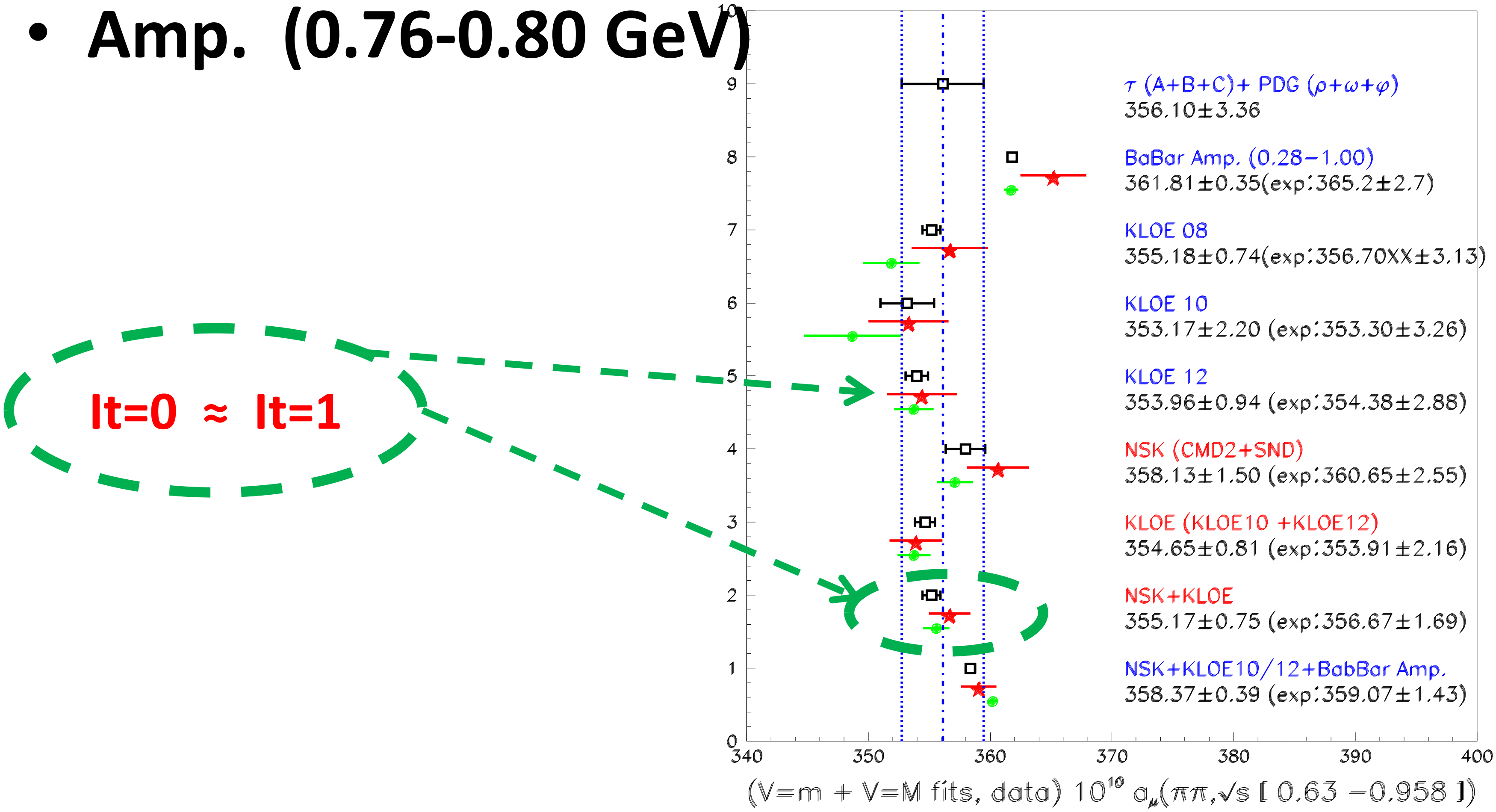
a_μ ($\pi^+\pi^-$, $\sqrt{s}$=[0.630,0.958] GeV)

- Data and BHLS estimates
- Amp. : (0.76-0.80 GeV)
- Green : A = m (it=0)
- Black : A = M_0 (it=1)



a_μ ($\pi^+\pi^-$, $\sqrt{s}$=[0.630,0.958] GeV)

- **Data and BHLS estimates**
- **Amp. (0.76-0.80 GeV)**



HVP contribution ($E \leq 1.05$ GeV)

- $\pi^+\pi^-$: CMD2/SND/KLOE10/KLOE12 /OLYA/CMD

Channel	A=m	A=M_0	A=M (variable)	Direct Estimate
$\pi^+\pi^-$	494.57 ± 1.48	494.02 ±1.11	493.77 ± 1.03	(498.53 ± 3.73)scan (494.50 ± 3.13)isr
$\pi^0 \gamma$	4.53 ± 0.04	4.54 ± 0.04	4.54 ± 0.04	3.35 ± 0.11
$\eta \gamma$	0.64 ± 0.005	0.64 ± 0.005	0.64 ± 0.005	0.48 ± 0.01
$\eta' \gamma$	0.00 ± 0.00	0.00 ± 0.00	0.00 ± 0.00	---
$\pi^+ \pi^- \pi^0$	38.94 ± 0.58	38.96 ± 0.58	39.97 ± 0.57	43.24 ± 1.47
$K_L K_S$	11.56 ± 0.08	11.56 ± 0.08	11.56 ± 0.08	12.31 ± 0.33
K^+K^-	16.78 ± 0.21	16.77 ± 0.21	16.76 ± 0.21	17.88 ± 0.54
Total up to 1.05 GeV	567.03 ± 1.60	566.49 ± 1.27	566.25 ± 1.20	(575.79 ± 4.06)scan (571.76 ± 3.52)isr

HVP contribution (E≤ 1.05 GeV)

- Central values ≈ coincides with isr (NSK+KLOE)

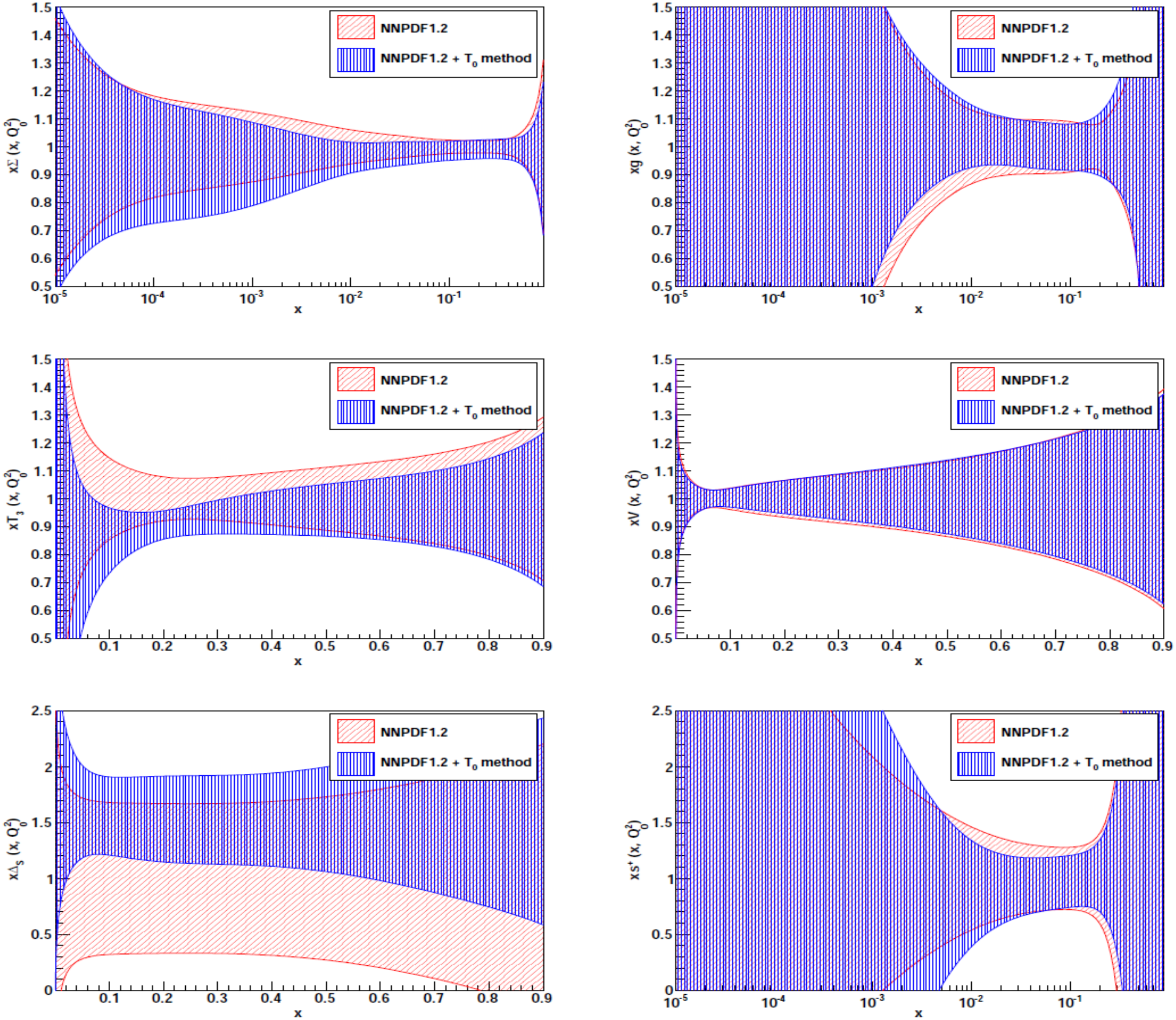
Channel	A=m	A=M_0	A=M (variable)	Direct Estimate
$\pi^+\pi^-$	494.57 ± 1.48	494.02 ±1.11	493.77 ± 1.03	(498.53 ± 3.73)scan (494.50 ± 3.13)isr
$\pi^0\,\gamma$	4.53 ± 0.04	4.54 ± 0.04	4.54 ± 0.04	3.35 ± 0.11
$\eta\,\gamma$	0.64 ± 0.005	0.64 ± 0.005	0.64 ± 0.005	0.48 ± 0.01
$\eta'\,\gamma$	0.00 ± 0.00	0.00 ± 0.00	0.00 ± 0.00	---
$\pi^+\,\pi^-\,\pi^0$	38.94 ± 0.58	38.96 ± 0.58	39.97 ± 0.57	43.24 ± 1.47
$K_L\,K_S$	11.56 ± 0.08	11.56 ± 0.08	11.56 ± 0.08	12.31 ± 0.33
K^+K^-	16.78 ± 0.21	16.77 ± 0.21	16.76 ± 0.21	17.88 ± 0.54
Total up to 1.05 GeV	567.03 ± 1.60	566.49 ± 1.27	566.25 ± 1.20	(575.79 ± 4.06)scan (571.76 ± 3.52)isr

HVP contribution (E≤ 1.05 GeV)

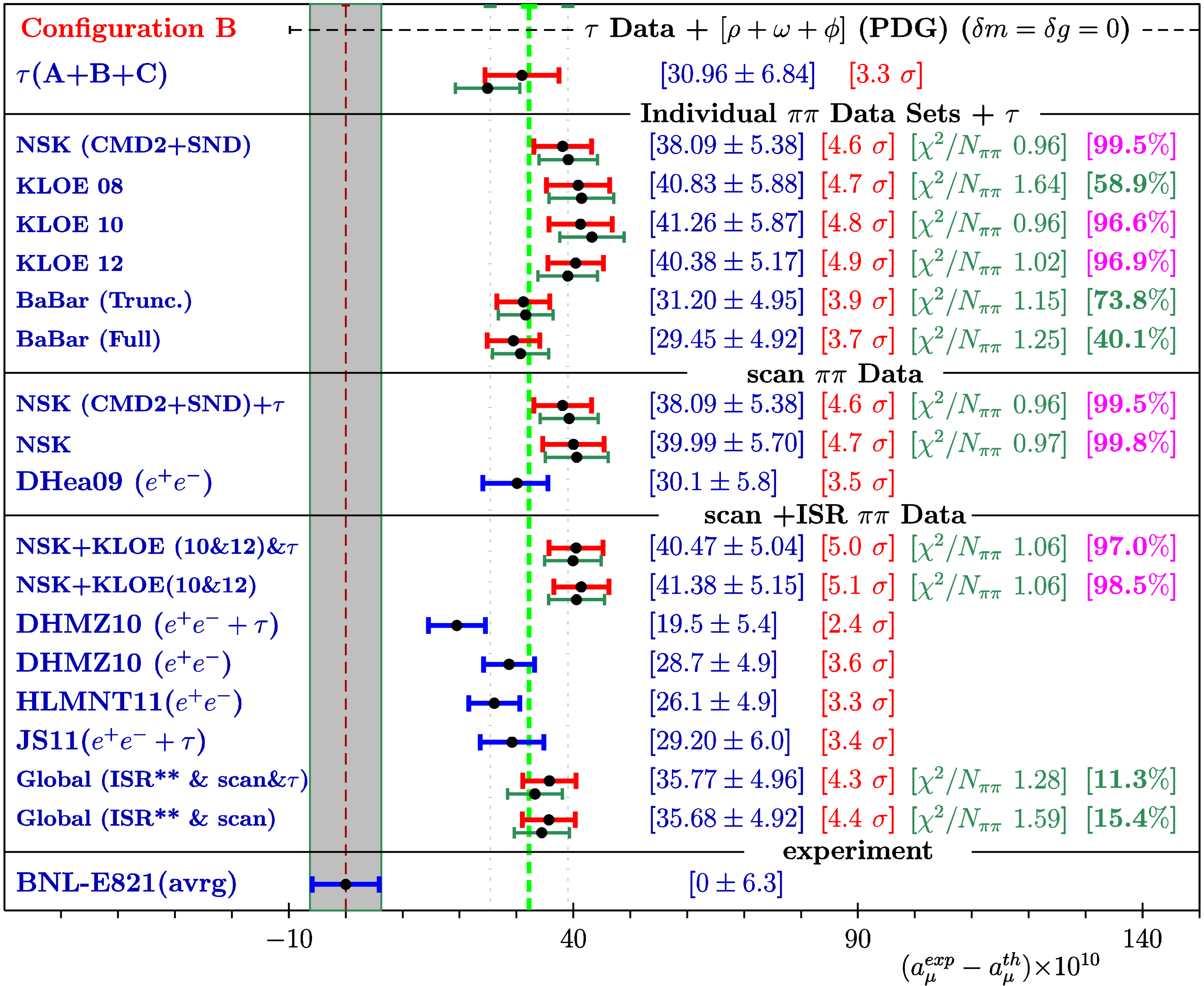
- uncertainties improved by ≈2.5 to 3

Channel	A=m	A=M_0	A=M (variable)	Direct Estimate
$\pi^+\pi^-$	494.57 ± 1.48	494.02 ±1.11	493.77 ± 1.03	(498.53 ± 3.73)scan (494.50 ± 3.13)isr
$\pi^0\,\gamma$	4.53 ± 0.04	4.54 ± 0.04	4.54 ± 0.04	3.35 ± 0.11
$\eta\,\gamma$	0.64 ± 0.005	0.64 ± 0.005	0.64 ± 0.005	0.48 ± 0.01
$\eta'\,\gamma$	0.00 ± 0.00	0.00 ± 0.00	0.00 ± 0.00	---
$\pi^+\,\pi^-\,\pi^0$	38.94 ± 0.58	38.96 ± 0.58	39.97 ± 0.57	43.24 ± 1.47
$K_L\,K_S$	11.56 ± 0.08	11.56 ± 0.08	11.56 ± 0.08	12.31 ± 0.33
K^+K^-	16.78 ± 0.21	16.77 ± 0.21	16.76 ± 0.21	17.88 ± 0.54
Total up to 1.05 GeV	567.03 ± 1.60	566.49 ± 1.27	566.25 ± 1.20	(575.79 ± 4.06)scan (571.76 ± 3.52)isr

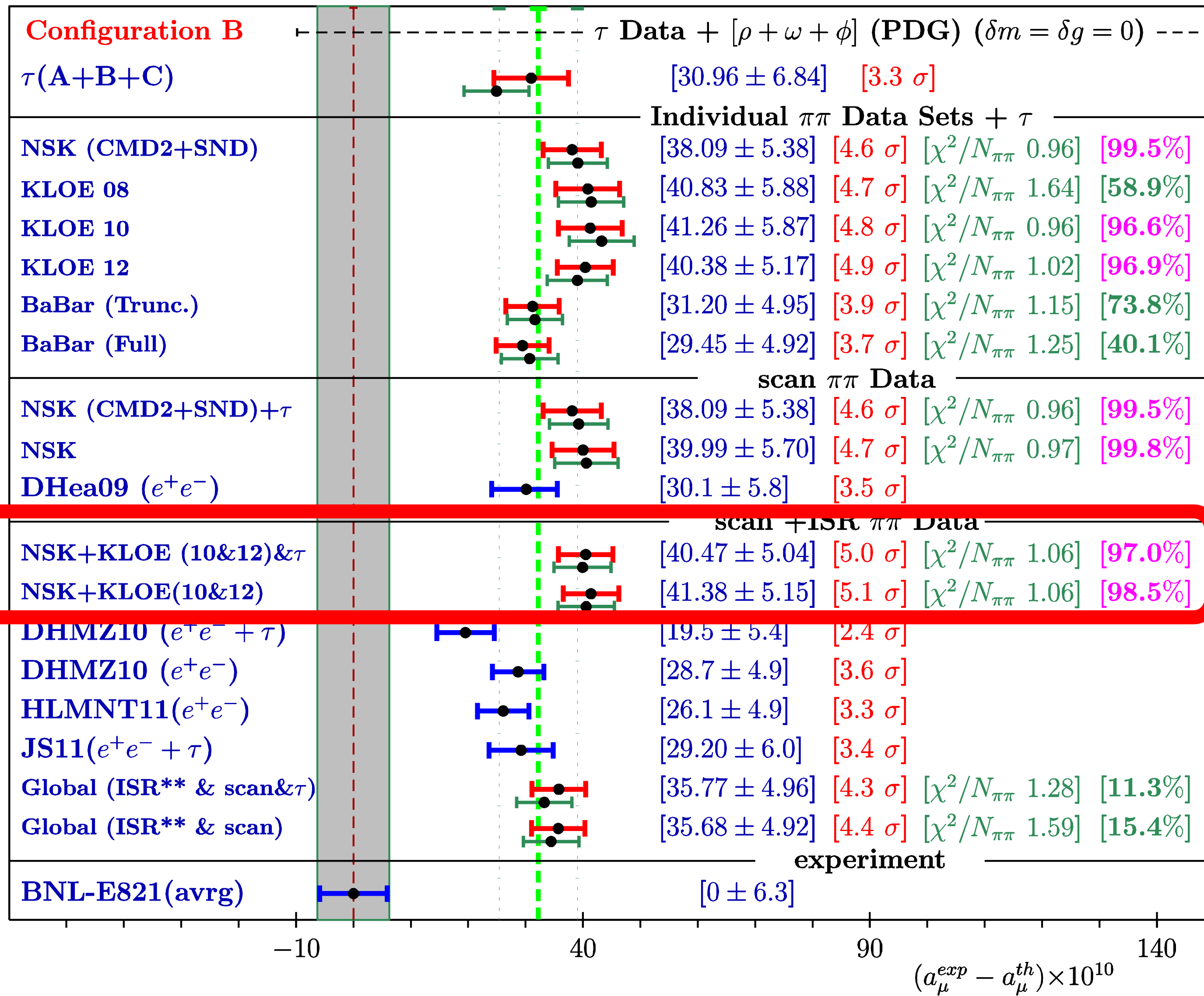
R.D.Ball *et al* JHEP 1005 (2010)075



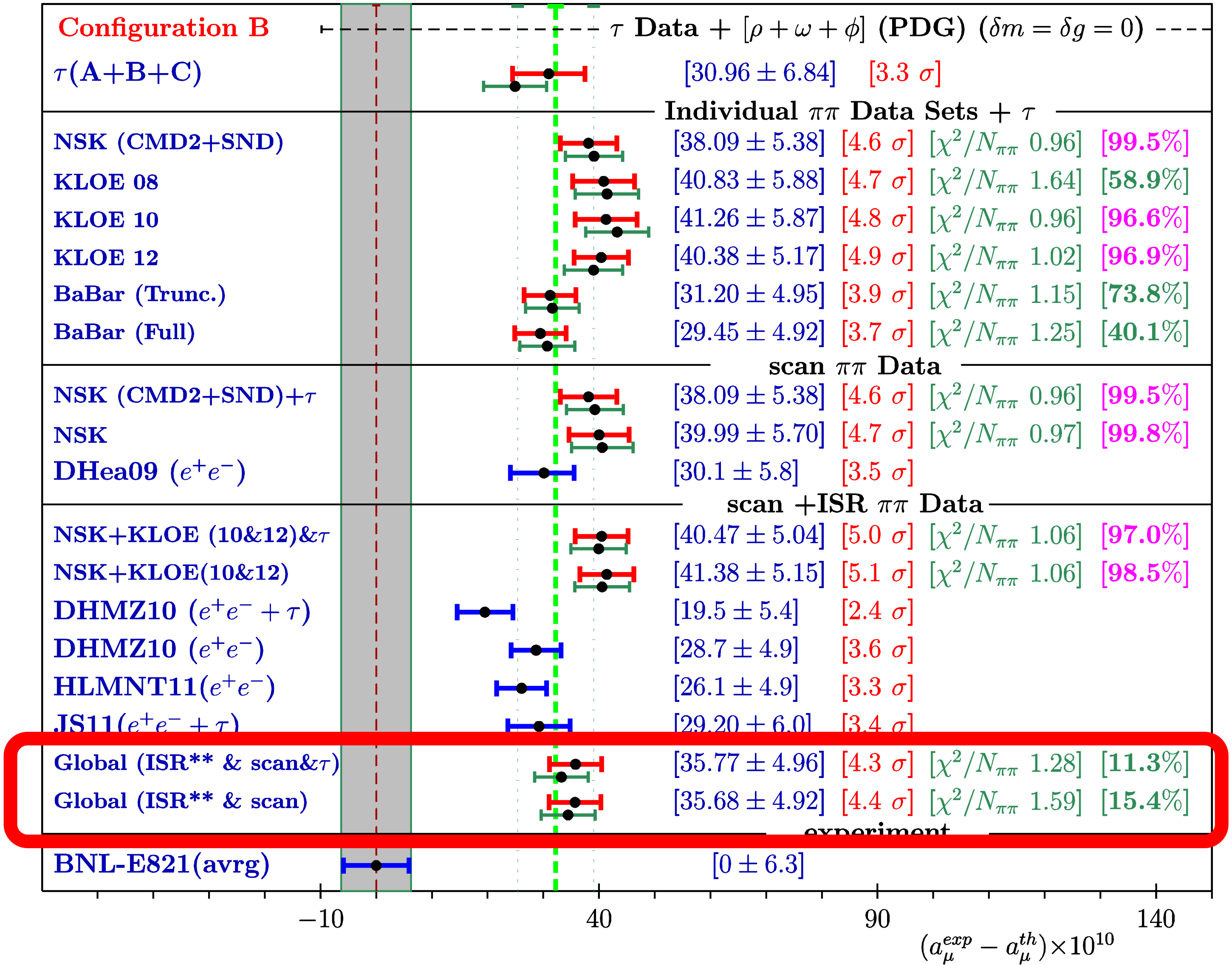
g-2 Estimates & Discrepancy



g-2 Estimates & Discrepancy



g-2 Estimates & Discrepancy



++ Additional Systematics

$$F_\pi(s) \underset{s\to 0}{\approx} 1 + a\,s + b\,s^2$$

a = 1.8 GeV^{-2} b = 4.2 GeV^{-4}

syst. : shift $a_\mu(\pi\pi)$ => ±2. 10^{-10}

NSK+KLOE (B)

$$10^{10}\times\left[a_\mu^{\exp}-a_\mu^{th}\right] = 40.47 + \begin{bmatrix}+0.6\\-1.3\end{bmatrix}_\phi + \begin{bmatrix}+0.9\\-0.0\end{bmatrix}_\tau + \begin{bmatrix}+2.0\\-2.0\end{bmatrix}_{s=0} \pm 5.04_{th} \pm 6.30_{\exp}$$

NSK+KLOE+ BaBar (trunc)

$$10^{10}\times\left[a_\mu^{\exp}-a_\mu^{th}\right] = 35.68 + \begin{bmatrix}+0.6\\-1.3\end{bmatrix}_\phi + \begin{bmatrix}+0.1\\-0.0\end{bmatrix}_\tau + \begin{bmatrix}+2.0\\-2.0\end{bmatrix}_{s=0} \pm 4.92_{th} \pm 6.30_{\exp}$$

significance for Δ a_μ > 4.6/4.2 σ

Conclusions

1/ The upgraded HLS model → good simultaneous fit of

$e^+e^- \to \pi\pi$ / K^+K^- / $K_L K_S$ / $\pi\gamma$ / $\eta\gamma$ / $\pi\pi\pi$ ($\sqrt{s} \leq 1.05$ GeV)

2/ Iterating global fits is shown to drop out biases

3/ Iterated global fit improves HVP uncertainty by ≈3 !

4/ Good quality data samples with $\sigma_{syst} << \sigma_{stat}$: Helpful

4/ The discrepancy with BNL g-2 value is $\Delta a_\mu > 4.6/4.2\ \sigma$

5/ Can one trust global fit methods?

Conclusions

1/ The upgraded HLS model → good simultaneous fit of
$e^+e^- \to \pi\pi / K^+K^- / K_L K_S / \pi\gamma / \eta\gamma / \pi\pi\pi$ ($\sqrt{s} \leq 1.05$ GeV)

2/ Iterating global fits is shown to drop out biases

3/ Iterated global fit improves HVP uncertainty by ≈3 !

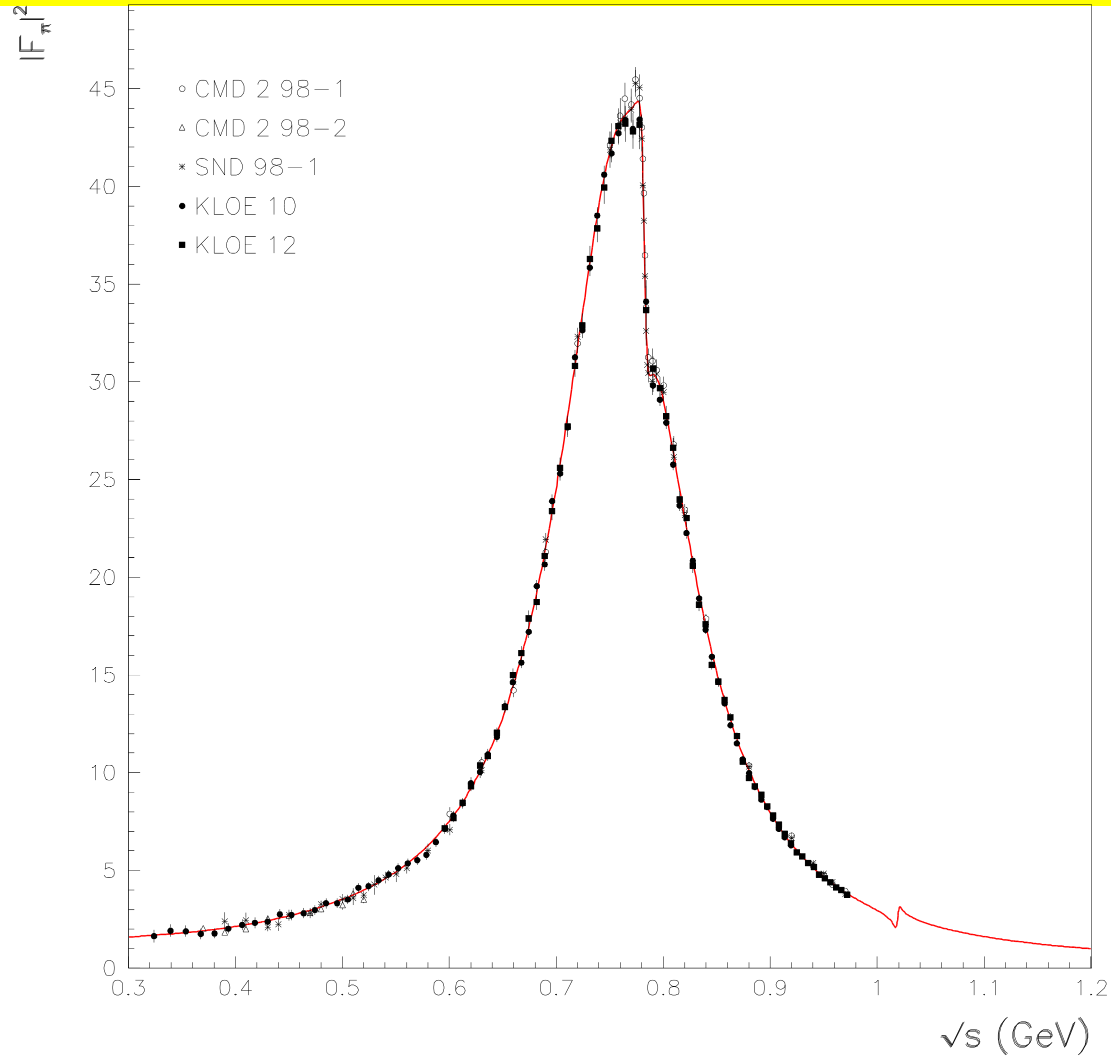
4/ Good quality data samples with $\sigma_{syst} << \sigma_{stat}$: Helpful

4/ The discrepancy with BNL g-2 value is $\Delta a_\mu > 4.6/4.2\ \sigma$

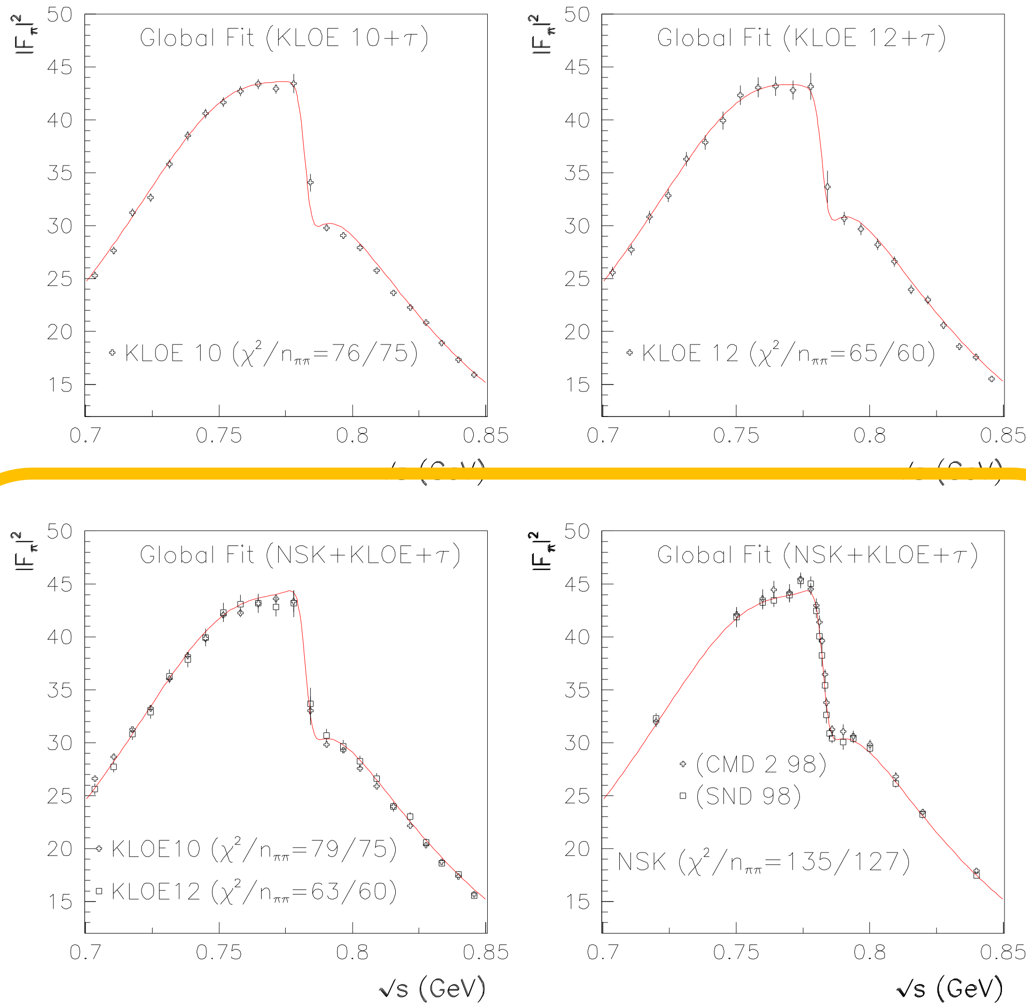
5/ One can trust *iterated* global fit methods

BACKUP

GLOBAL FIT : All ππ spectrum

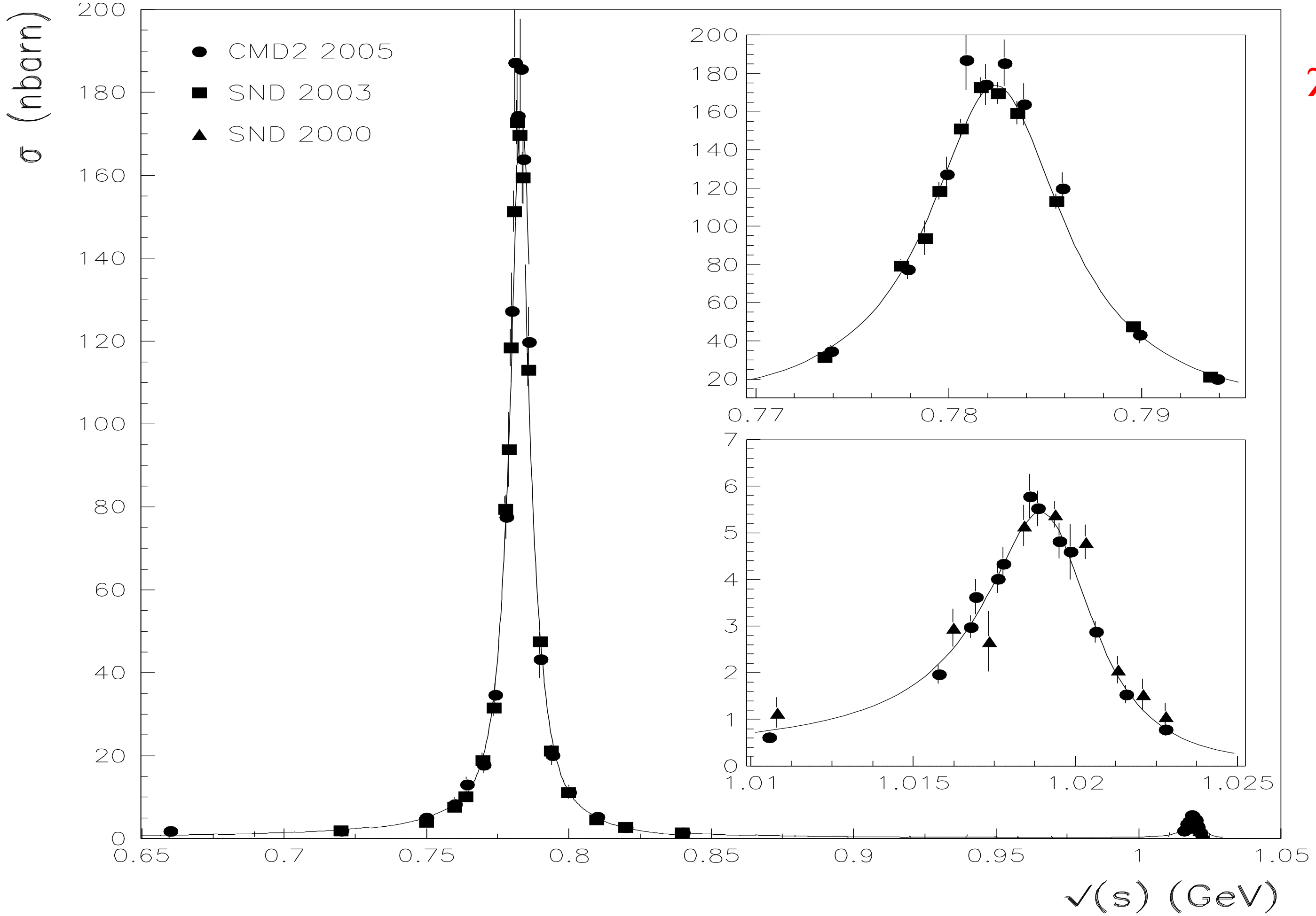


Global Fits : Top



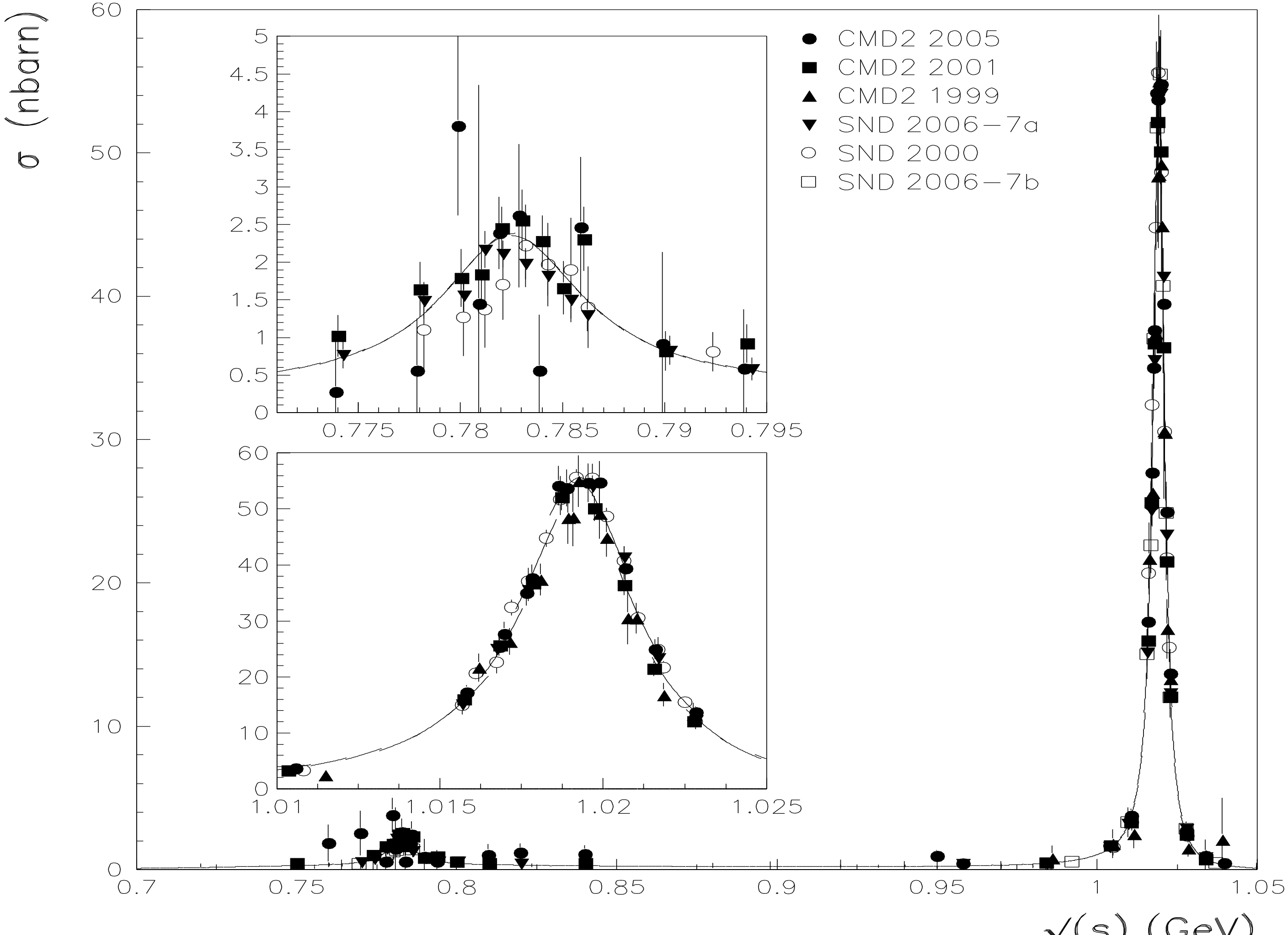
Prefered Solution

$e^+e^- \rightarrow \pi^0 \gamma$



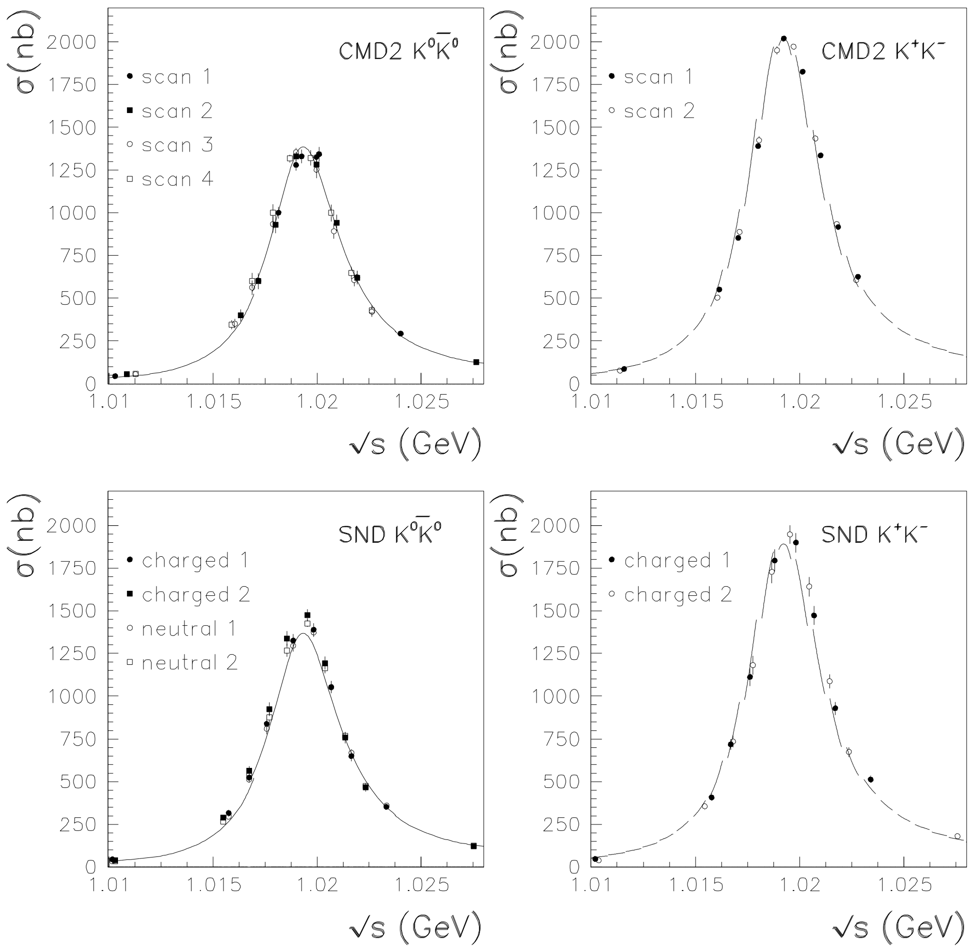
$$\chi^2 / N_p = 64/86$$

$e^+e^- \rightarrow \eta\gamma$ Data



$$\chi^2 / N_p = 121/182$$

$e^+e^- \rightarrow$ K Kbar Data



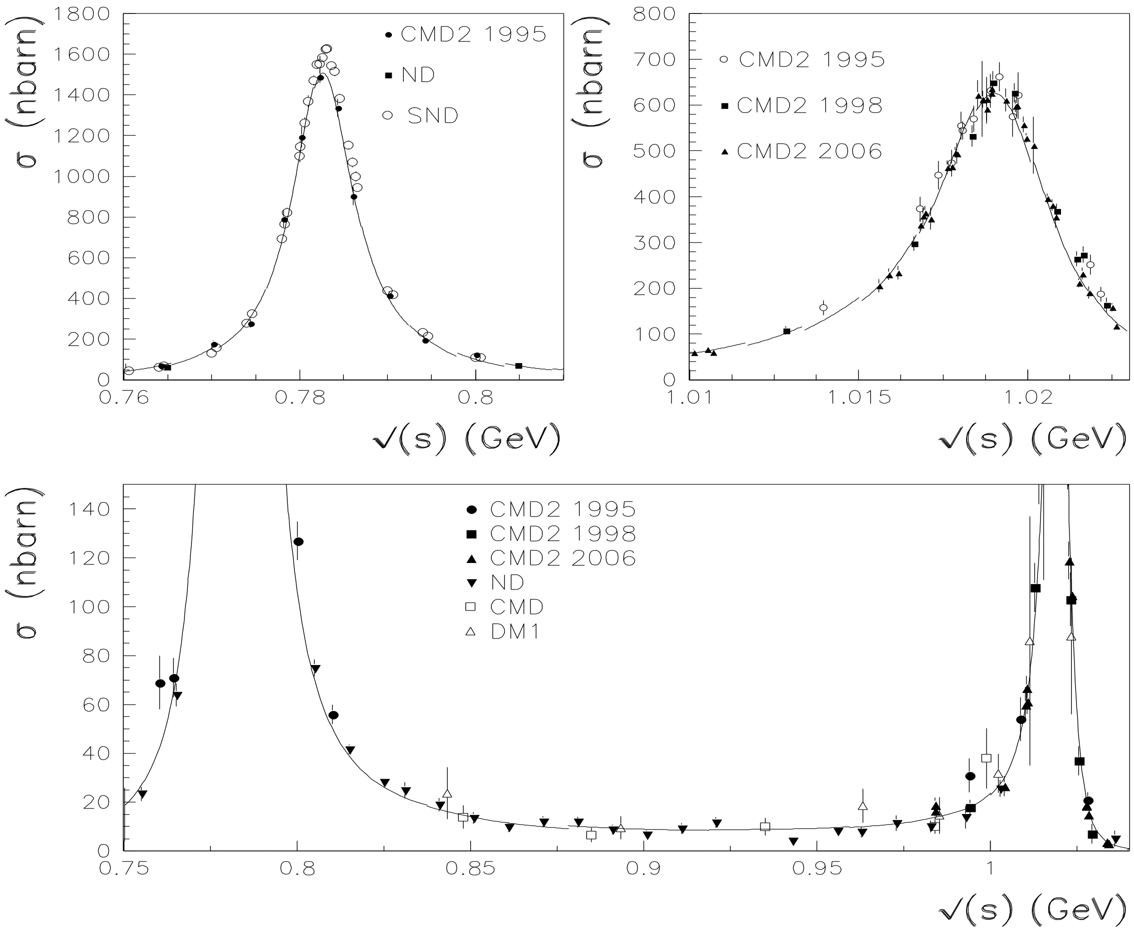
$K^+ K^-$

$$\chi^2/N_p = 30/36$$

$K_L K_S$

$$\chi^2/N_p = 119/119$$

3-pion Data



$$\chi^2 / N_p = 279/179$$