Current Status of the Muon g-2

A.E. Dorokhov (JINR, Dubna)

In collaboration with AE Radzhabov, AS Zhevlakov (ISDCT, Irkutsk)

1 Motivation

2 Electron g-2 and fine structure constant

3 Muon g-2 : Experiment vs Standard Model

4 Hadronic contributions to the Muon g-2

Introduction

Cosmology tell us that 95% of matter is not described in text-books yet.
Dark Matter surrounds us! Where it is ?

Two search strategies:

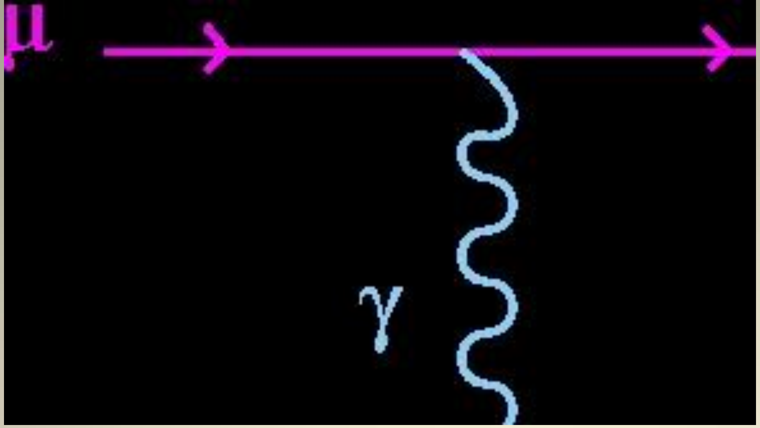
1) High energy physics to excite heavy degrees of freedom.
No any evidence till now. We live in LHC era!

2) Low energy physics to produce Rare processes in view of huge statistics.

There are some rough edges of SM.

Anomalous Magnetic Moment of the Muon $(g\text{-}2)_{\mu}$
is most famous and stable (for many years) example

That's intriguing



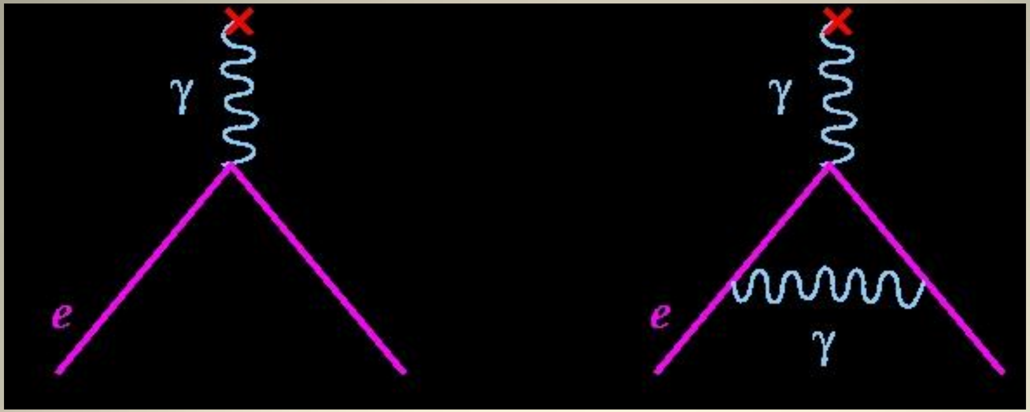
*Dirac Equation Predicts for **free** point-like spin ½ charged particle:*

$$i\hbar\frac{\partial\psi}{\partial t} = [\frac{p^2}{2m} - \frac{e}{2m}(\vec{L}+2\vec{S})\cdot\vec{B}]\psi$$

g=2, a=(g-2)/2=0 (no anomaly)

a becomes nonzero due to interactions resulting in fermion **substructure**

The lowest order radiative correction (QED)



$$\Gamma_\mu = e\gamma_\mu + a_\ell \frac{ie}{2m}\sigma_{\mu\nu}q_\nu$$

$a_l=(g_l-2)/2$

$$a = \frac{\alpha}{2\pi} = 0.001161$$

Schwinger, 1948

$$a_\mu^{exp} = 0.00119 \pm 0.00005$$

Kush, Foley, 1948

Electron AMM

To measurable level a_e arises entirely from virtual electrons and photons

$a_e^{exp} = 1\,159\,652\,180.73(0.28) \cdot 10^{-12}$ [0.24 ppb] Harvard 2008

$$a_e^{SM} = a_e(\text{QED}) + a_e(\text{hadron}) + a_e(\text{weak}),$$

$$a_e(\text{QED}) = \sum_{n=1}^{5} C_{2n} \left(\frac{\alpha}{\pi} \right)^n + \ldots$$

The theoretical error is dominated by the uncertainty in the input value of the QED coupling $\alpha \equiv e^2/(4\pi)$

$\alpha^{-1} = 137.035\,999\,1727(341)$ [0.25 ppb]

QED is at the level of the best theory ever built to describe nature

Muon AMM: BNL result vs SM

From BNL E821 g-2 experiment (1999-2006)

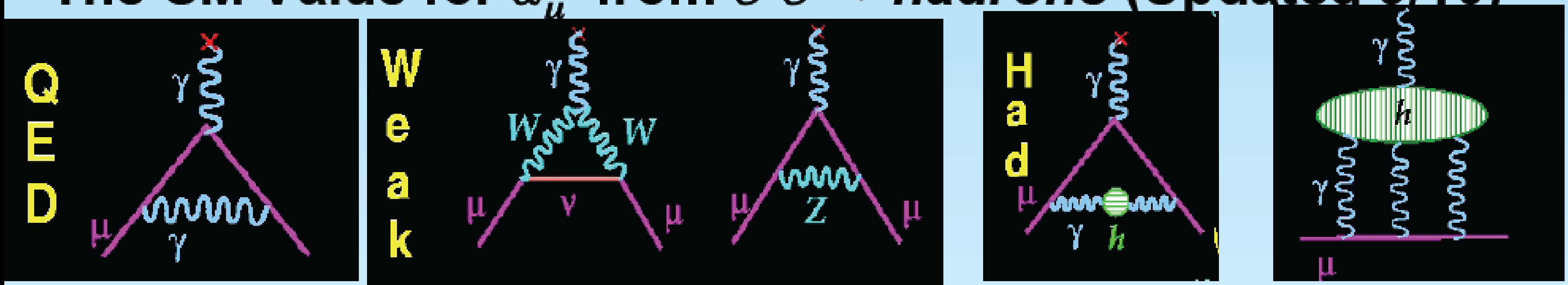
$$a_\mu^{\mathrm{BNL}} = 11\,659\,208.0(6.3) \bullet 10^{-10} \;\; (0.54\ \mathrm{ppm})$$

New Prop.
E989 at Fermilab
0.14 ppm
KEK/JParc

In Theory

$$a_\mu = \left\{ a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{EW}} + a_\mu^{\mathrm{Strong}} \right\}^{\mathrm{SM}} + ???$$

The SM Value for a_μ from $e^+e^- \to$ *hadrons* (Updated 9/10)



$$a_\mu^{\mathrm{SM}} = 11\,659\,180.2(4.9) \bullet 10^{-10}$$

From Standard Model

$$\Delta a_\mu = a_\mu^{\exp} - a_\mu^{\mathrm{SM}} = 27.8(8.0) \bullet 10^{-10} \;\; (3.6\sigma!)$$

$$a_\mu^{QED} = 11\,658\,471.8951(0.0080) \bullet 10^{-10}$$

Kinoshita&Nio 2014

plus

$$a_\mu^{EW} = 15.36(0.10) \bullet 10^{-10}$$

Czarnetski&Marciano&Vainshtein 2003
Gnendiger, Stockinger 2014

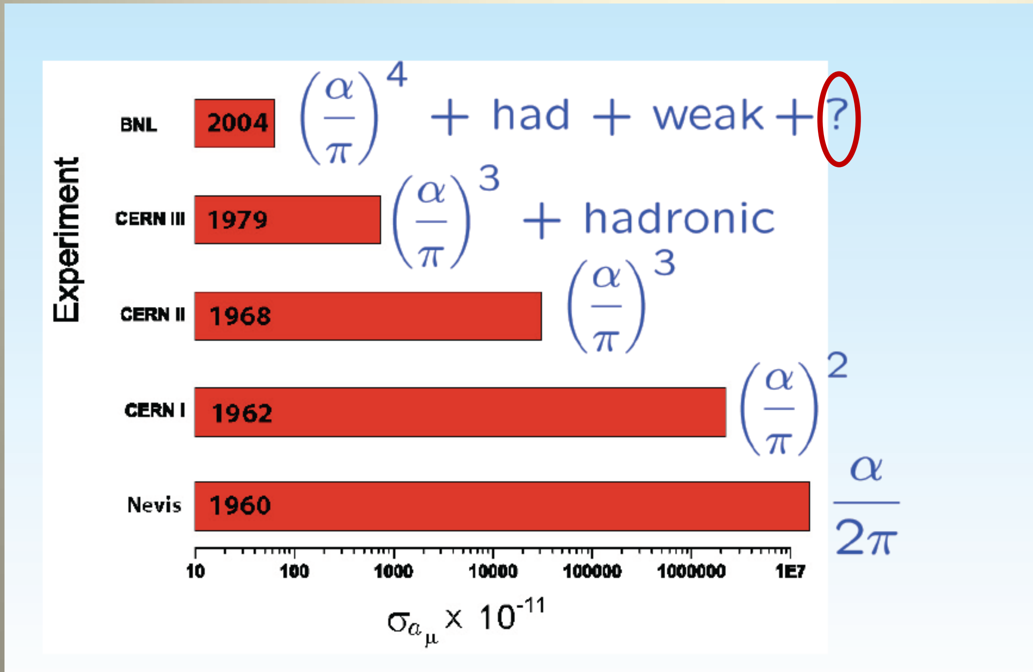
plus

the Hadronic Contribution estimated as

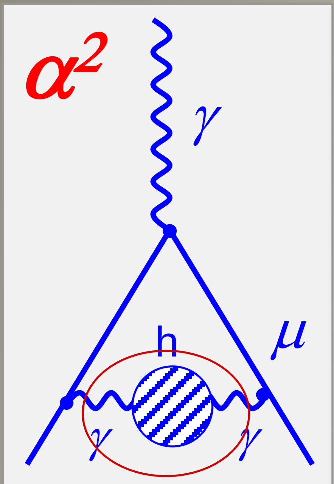
$$a_\mu^{Strong} = 693.0(4.9) \bullet 10^{-10} \quad (<1\% \text{ accuracy!})$$

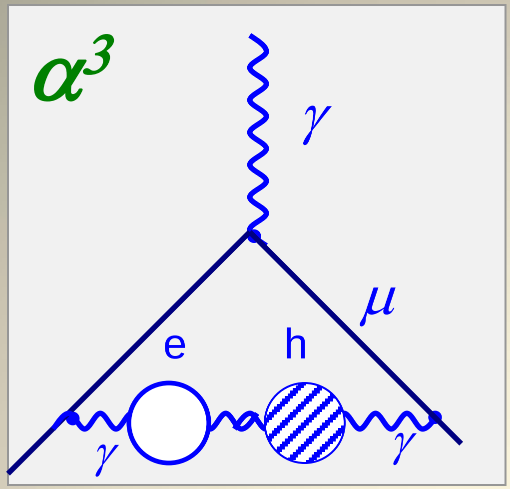
M. Davier, A. Hoecker, B. Malaescu, Z. Zhang 2010;
F. Jegerlehner, R. Szafron 2011

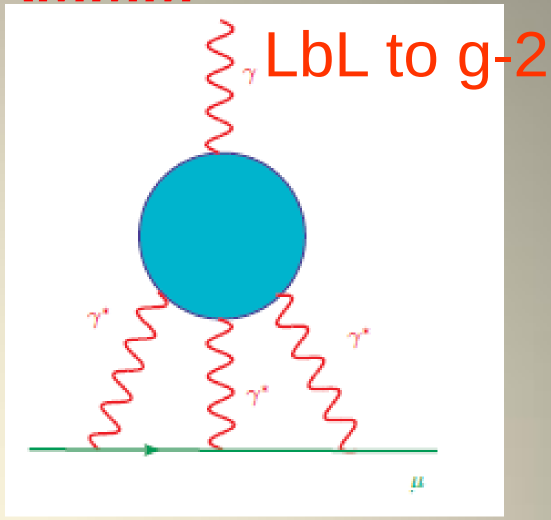
The main question how to get such accuracy from theory.



Strong contributions to Muon AMMM







$$a_\mu^{\mathrm{HVP}} = (692.3 \pm 4.2) \bullet 10^{-10}$$

Hadronic Vacuum polarization

(Davier, Hoecker, Malaescu,Zhang 2011; Hagiwara, Martin, Teubner 2011)

$$a_\mu^{\mathrm{LbL}} = (10.5 \pm 2.6) \bullet 10^{-10}$$

Hadronic Light-by-Light Scattering

(AED, A.Radzhabov, A.Zhevlakov 11-14; C.Fischer, T. Goecke, R.Williams 11-13)

Hadronic Vacuum Polarization
contributes 99%
and half of error
Fixed by Experiment

$$a_\mu^{(2)\mathrm{hvp}} = \frac{\alpha^2}{3\pi^2} \int_{4m_\pi^2}^{\infty} ds \quad \frac{K(s)}{s} \quad R^{(0)}(s)$$

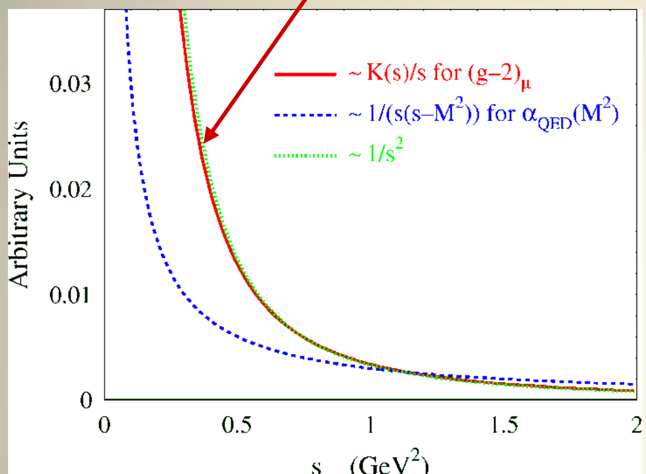
Light-by-light process
contributes 1%
and half of error

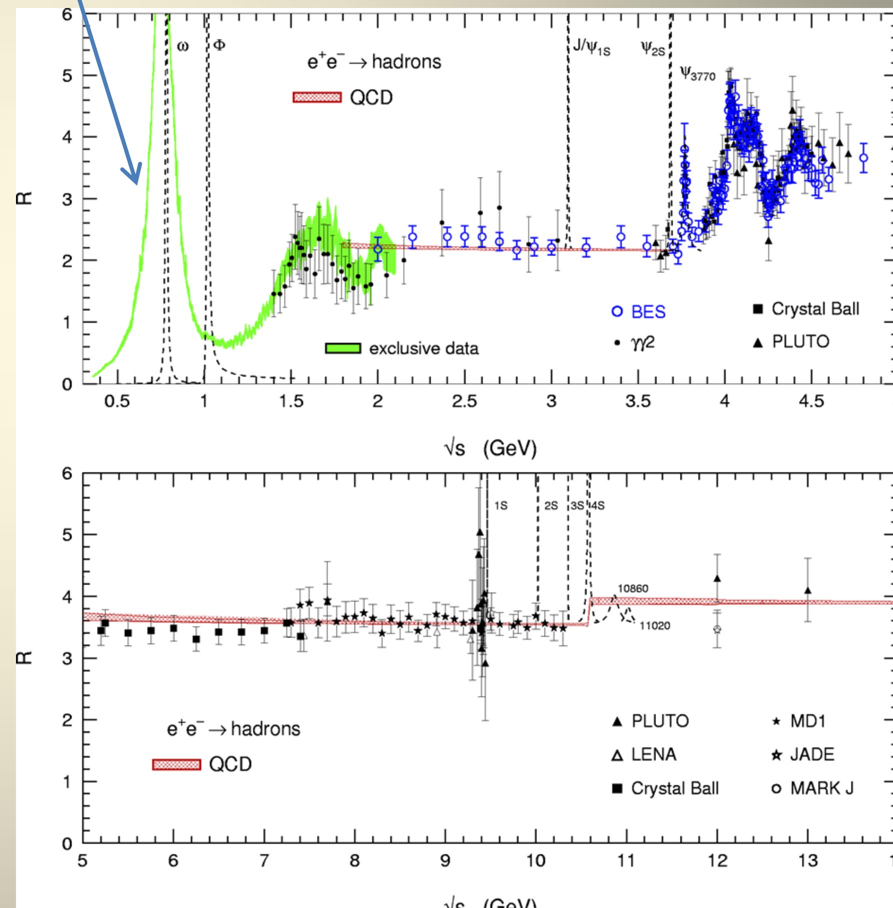
Model Dependent

II. Leading Order Hadronic contributions

$$a_\mu^{had} = \frac{\alpha^2}{3\pi^2} \int_{4m_\pi^2}^{\infty} ds \, \frac{K(s)}{s} \, R^{(0)}(s)$$

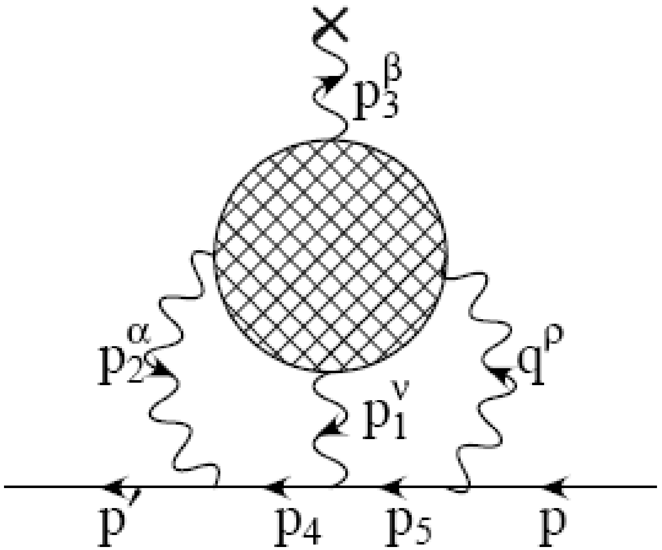
Dispersion relation, uses unitarity (optical theorem) and analyticity (Bouchiat and Michel, 1961)





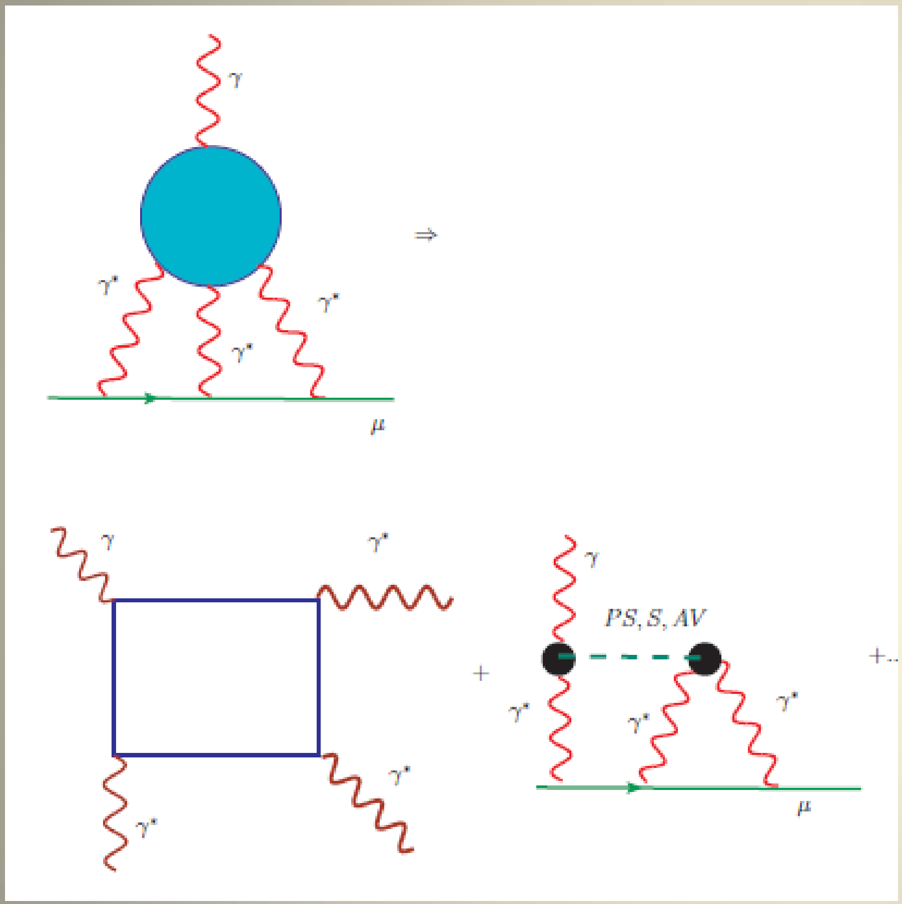
$$R(s) = \frac{\sigma\left[e^+e^- \rightarrow \text{hadrons}\right]}{\sigma\left[e^+e^- \rightarrow \mu^+\mu^-\right]}$$

Hadronic light-by-light contribution to muon $g-2$

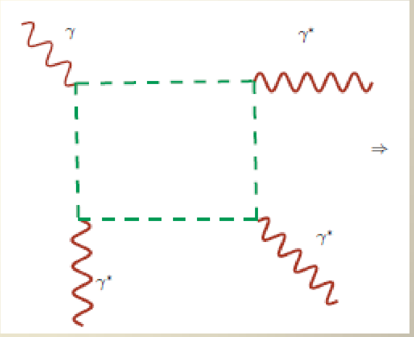


$$\mathcal{M} = |e|^7 A_\beta \int \frac{\mathrm{d}^4 p_1}{(2\pi)^4} \int \frac{\mathrm{d}^4 p_2}{(2\pi)^4} \frac{1}{q^2\, p_1^2\, p_2^2 (p_4^2 - m^2)\,(p_5^2 - m^2)}$$
$$\times \underline{\Pi^{\rho\nu\alpha\beta}(p_1, p_2, p_3)}\, \bar{u}(p')\gamma_\alpha(\not{p}_4 + m)\gamma_\nu(\not{p}_5 + m)\gamma_\rho u(p)$$

Structure of hadronic LbL contribution



Hierarchy in
a) 1/Nc
b) M μ /(4 π f $_{\pi}$)



Effective Model Approach

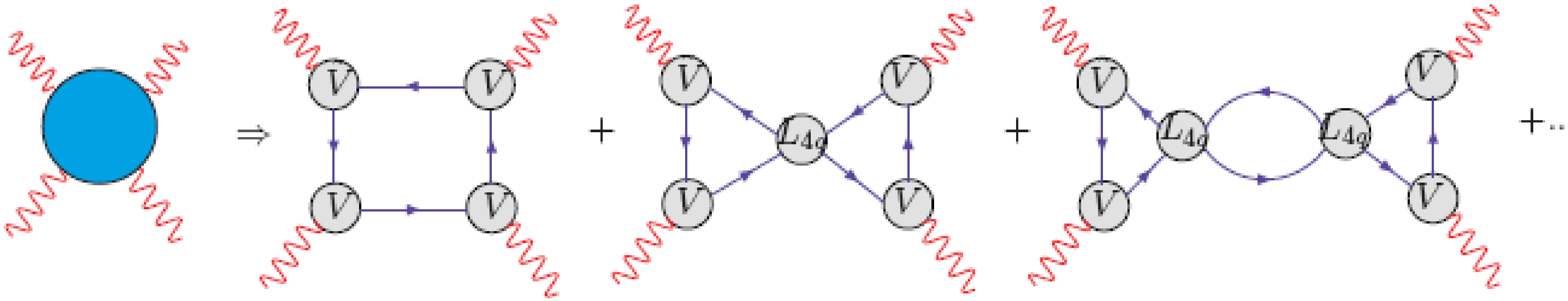
AED, W. Broniowski PRD (08'),
AED, A. Radzhabov, A. Zhevlakov (11'—14')

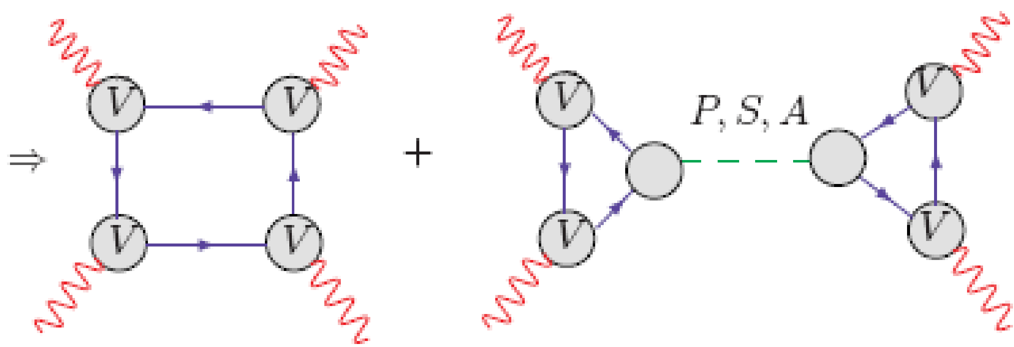
$$\mathcal{L} = \bar{q}(x)(i\hat{\partial} - m_c)q(x) + \frac{G}{2}[J_S^a(x)J_S^a(x) + J_P^a(x)J_P^a(x)] - \frac{H}{4}T_{abc}[J_S^a(x)J_S^b(x)J_S^c(x) - 3J_S^a(x)J_P^b(x)J_P^c(x)], \quad (1)$$

$$J_M^a(x) = \int d^4x_1 d^4x_2\, f(x_1)f(x_2) \times \times \overline{Q}(x - x_1, x)\,\Gamma_M^a Q(x, x + x_2),$$

$$Q(x,y) = \mathcal{P}\exp\left\{i\int_x^y dz^\mu V_\mu^a(z)\,T^a\right\} q(y)$$

Leading 1/Nc contribution



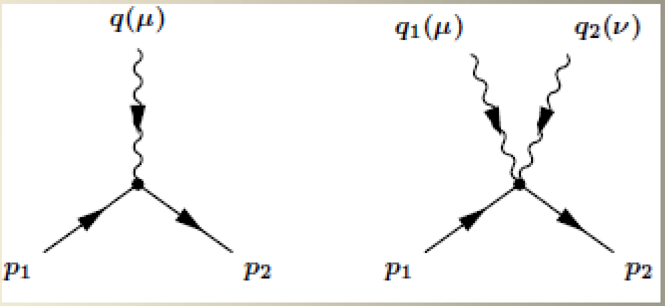


Nonperturbative QCD is simulated by Nonlocal Chiral Quark model

Quark Propagator

$$\frac{\not k}{k^2} \Rightarrow S(k) = \frac{\not k + m(k^2)}{D(k^2)} \xrightarrow{k^2 \to \infty} \frac{\not k}{k^2}$$

Quark-Photon Vertex



$$\gamma_\mu \Rightarrow \Gamma_\mu = \gamma_\mu + \Delta\Gamma_\mu(k,k') \xrightarrow{k^2 \to \infty} \gamma_\mu, \text{ where } \Delta\Gamma_\mu(k,k')$$

guarantes WTI $(k' = k + q)$: $q_\mu \Gamma_\mu = S^{-1}(k') - S^{-1}(k)$

Quark-Pion vertex

$$\frac{1}{f_\pi}\gamma_5 \Rightarrow \Gamma_\pi = \frac{1}{f_\pi}\gamma_5 F(k,k') \overset{\substack{k'^2 \to \infty \\ k^2 \to \infty}}{=} 0$$

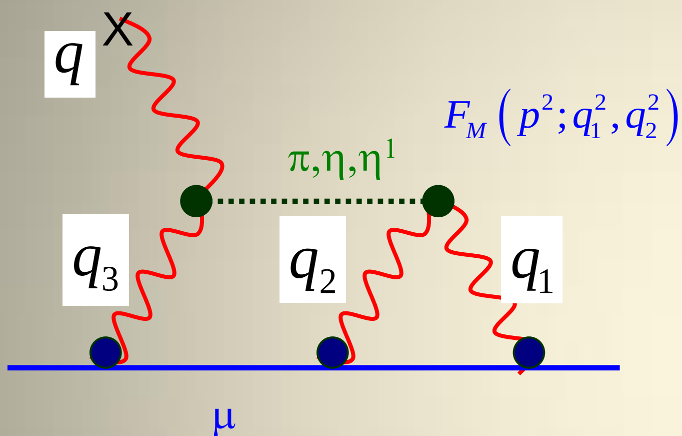
The vertex F is equivalent of the light-cone pion WF

$m(k^2)$ is related to nonlocal quark condensate and thus $m(k^2) \approx M_q e^{-C(k^2)^a}$

We use for the Dynamical Quark Mass

$$m(k^2) = M_q \exp(-2\Lambda k^2)$$

A) Meson exchange LbL contribution – "Goat" diagram



$$a_\mu^{\rm LbL,PS} = -\frac{2\alpha^3}{3\pi^2}\int_0^\infty dq_1^2 \int_0^\infty dq_2^2 \int_{-1}^{1} dt\sqrt{1-t^2}\frac{1}{q_3^2}\times$$

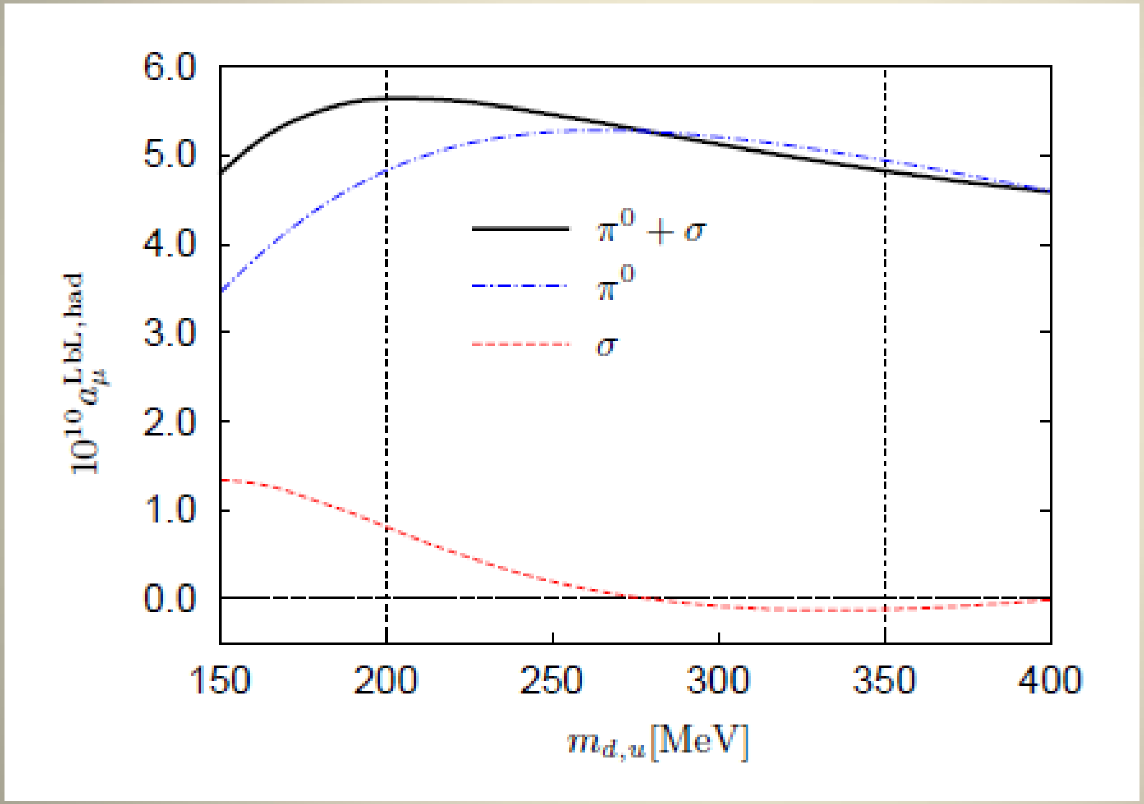
$$\times \sum_{a=\pi^0,\eta,\eta'}\Big[2\frac{\mathrm{F}_{a^*\gamma^*\gamma^*}\left(q_2^2;q_1^2,q_3^2\right)\mathrm{F}_{a^*\gamma^*\gamma}\left(q_2^2;q_2^2,0\right)}{q_2^2+M_a^2}I$$

$$+\frac{\mathrm{F}_{a^*\gamma^*\gamma^*}\left(q_3^2;q_1^2,q_2^2\right)\mathrm{F}_{a^*\gamma^*\gamma}\left(q_3^2;q_3^2,0\right)}{q_3^2+M_a^2}I_2\Big], \quad ($$

Phenomenological and QCD Constraints are used to reduce Model Dependence

Sum of PS(π,η,η') and S(σ,a0(980),f0(980)) exchange contributions to a $_{\mu}$

AED, AE Radzhabov, AS Zhevlakov (11'—14')

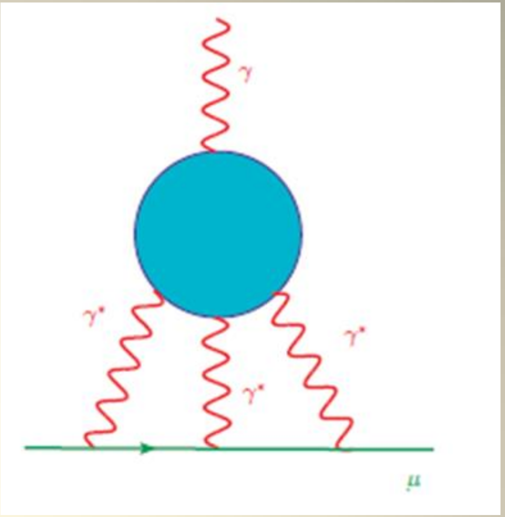


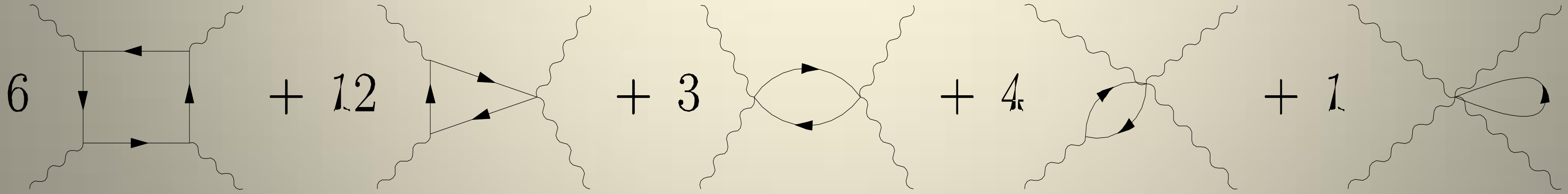
$$a_\mu^{\mathrm{LbL,PS+}S} = (6.19 \pm 0.95) \cdot 10^{-10}$$

B) Contribution of Dynamical Quark Box to a $_\mu$

$$a_\mu^{\mathrm{HLbL}} = \frac{1}{48m_\mu}\mathrm{Tr}[(\hat{p}+m_\mu)[\gamma^\rho,\gamma^\sigma](\hat{p}+m_\mu)\Pi_{\rho\sigma}(p,p)]$$

$$\Pi_{\rho\sigma}(p',p) = -ie^6\int\frac{d^4q_1}{(2\pi)^4}\int\frac{d^4q_2}{(2\pi)^4}\frac{1}{q_1^2q_2^2(q_1+q_2-k)^2} \times\gamma^\mu\frac{\hat{p}'-\hat{q}_1+m_\mu}{(p'-q_1)^2-m_\mu^2}\gamma^\nu\frac{\hat{p}-\hat{q}_1-\hat{q}_2+m_\mu}{(p-q_1-q_2)^2-m_\mu^2}\gamma^\lambda \times\frac{\partial}{\partial k^\rho}\Pi_{\mu\nu\lambda\sigma}(q_1,q_2,k-q_1-q_2),$$

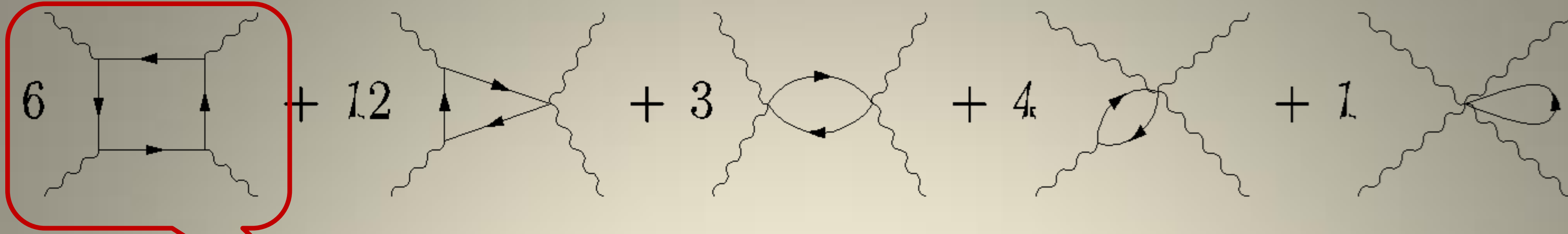




$$a^{Box} = \iint_0^\infty dQ_1 dQ_2 \rho(Q_1,Q_2)$$

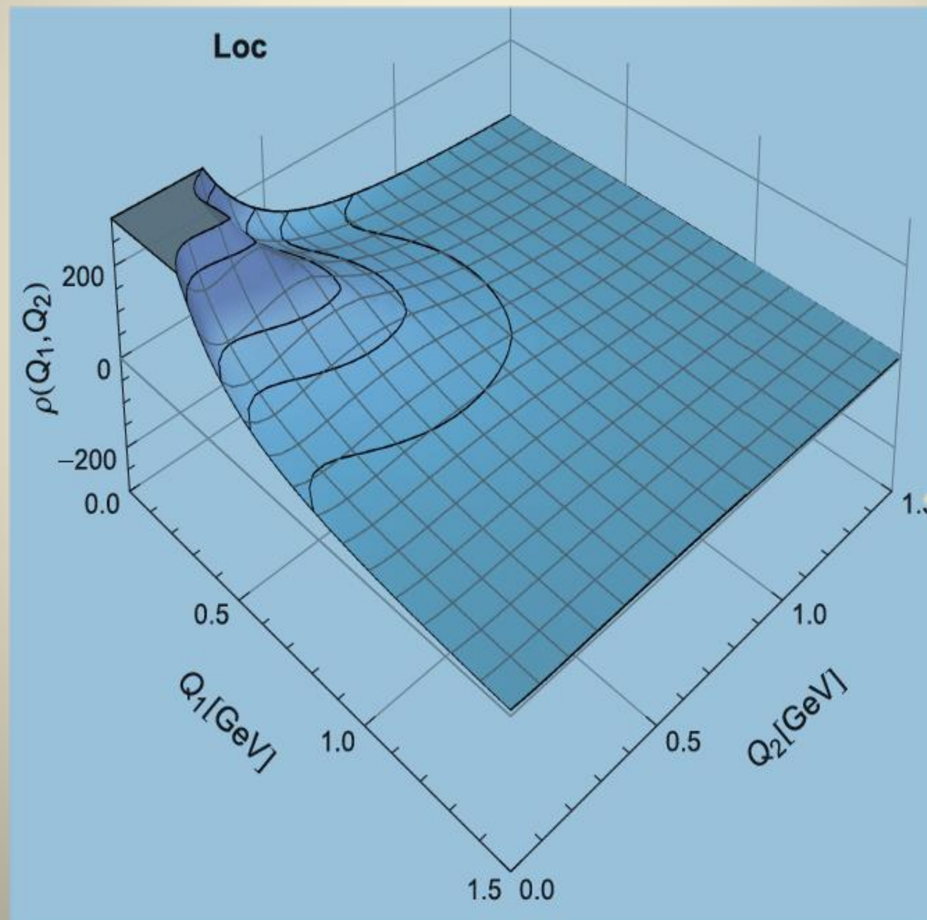
$Q_4 \to 0,$

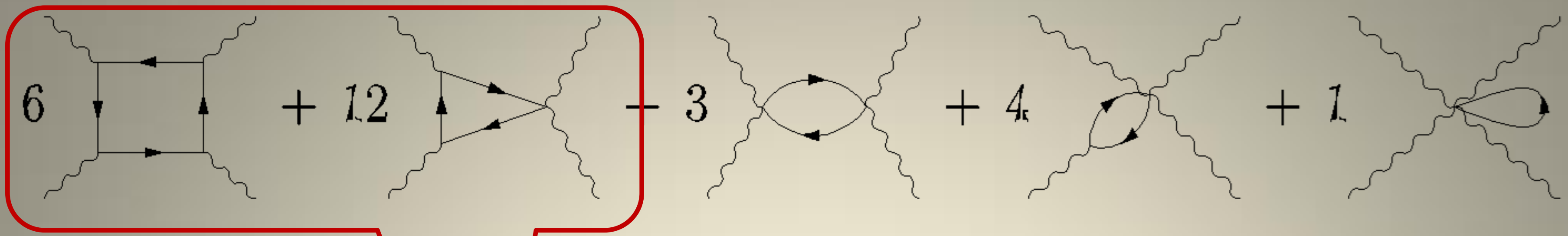
Q3= – Q2 –Q1



1) Monotonously decreasing at large Q

2) Rapidly growing at small Q

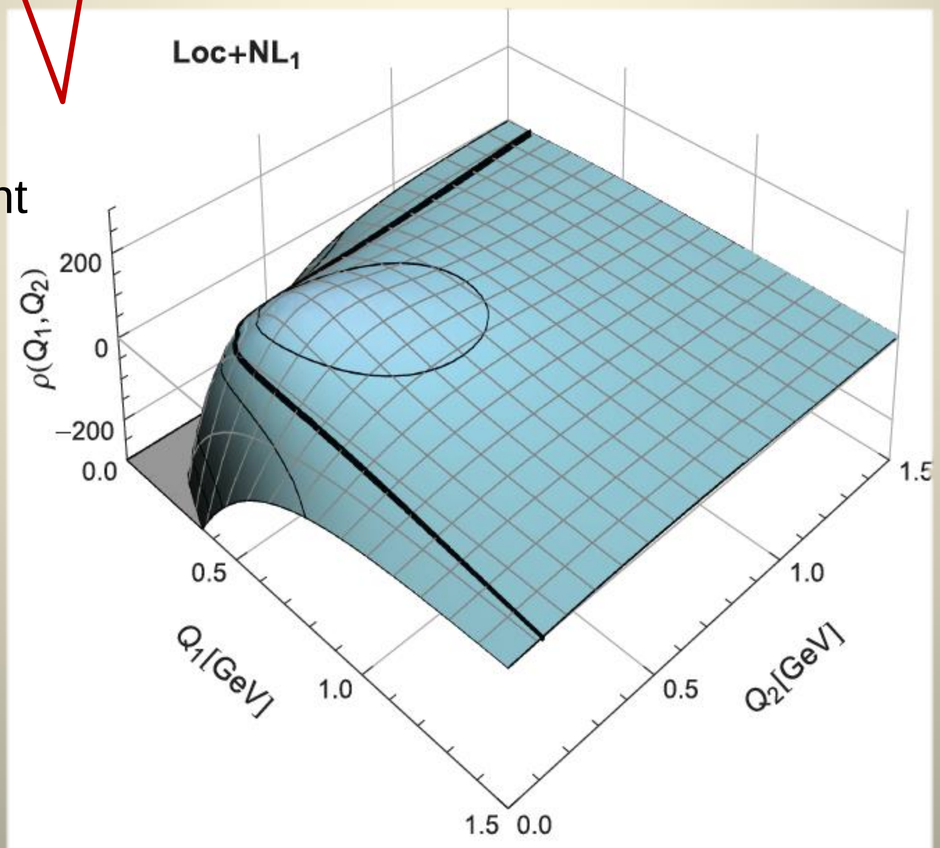


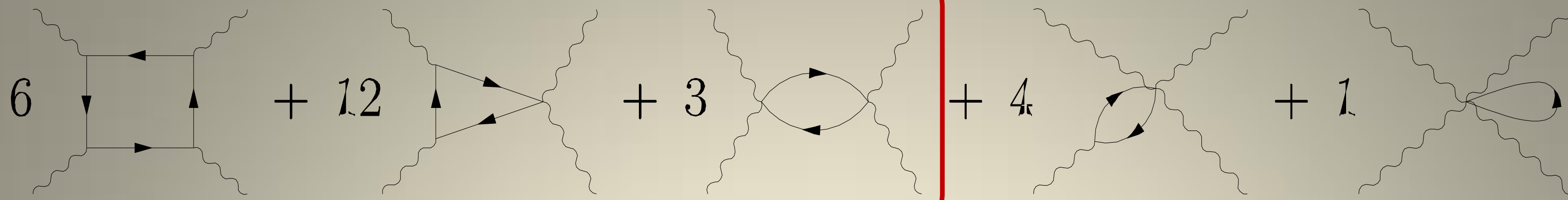


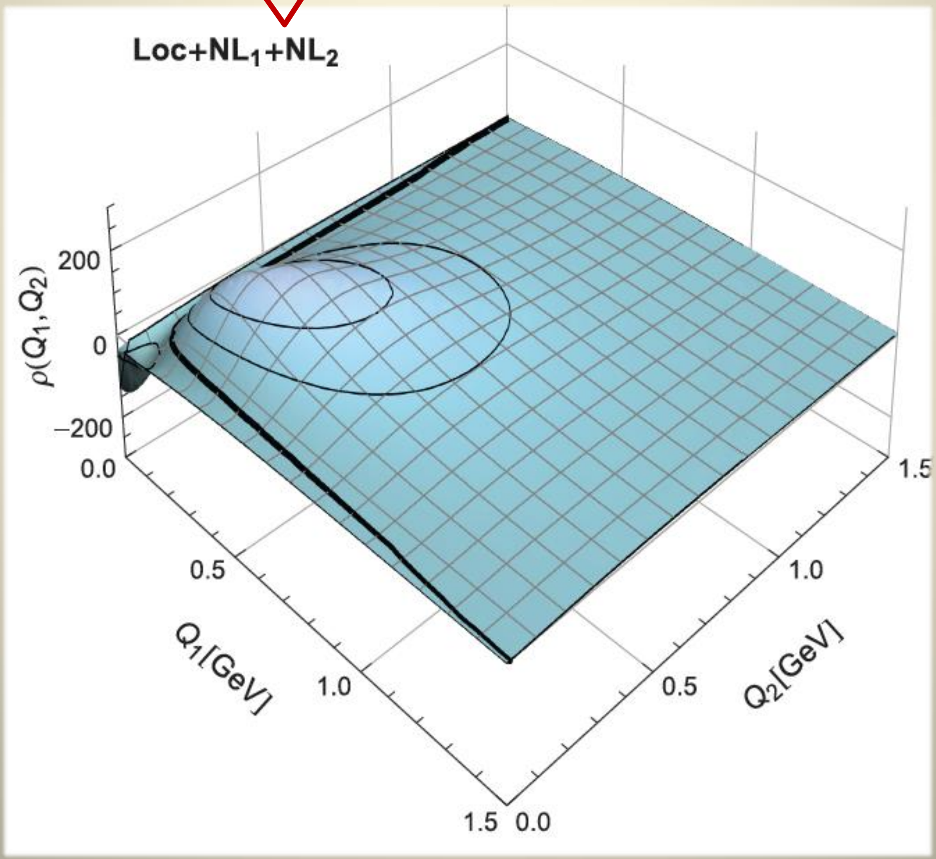
1) Completely different structure

2) Changes sign at small Q

3) Island with Hill appears

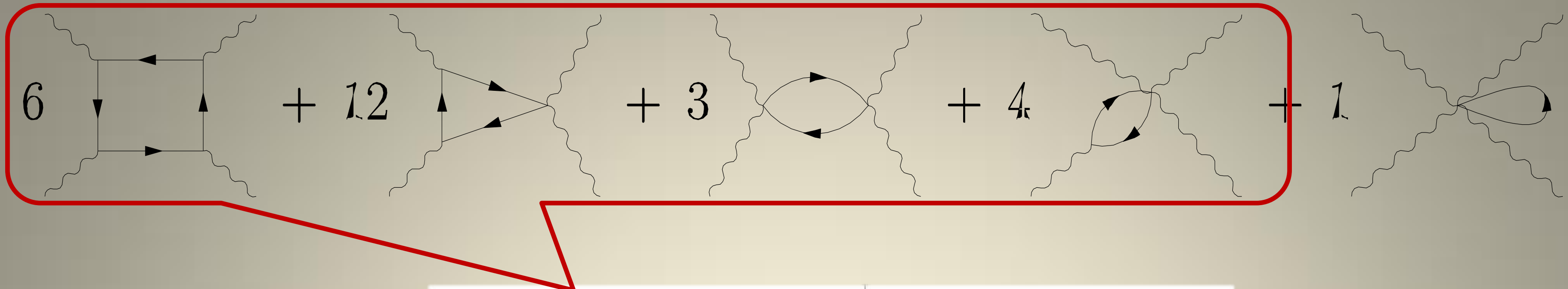






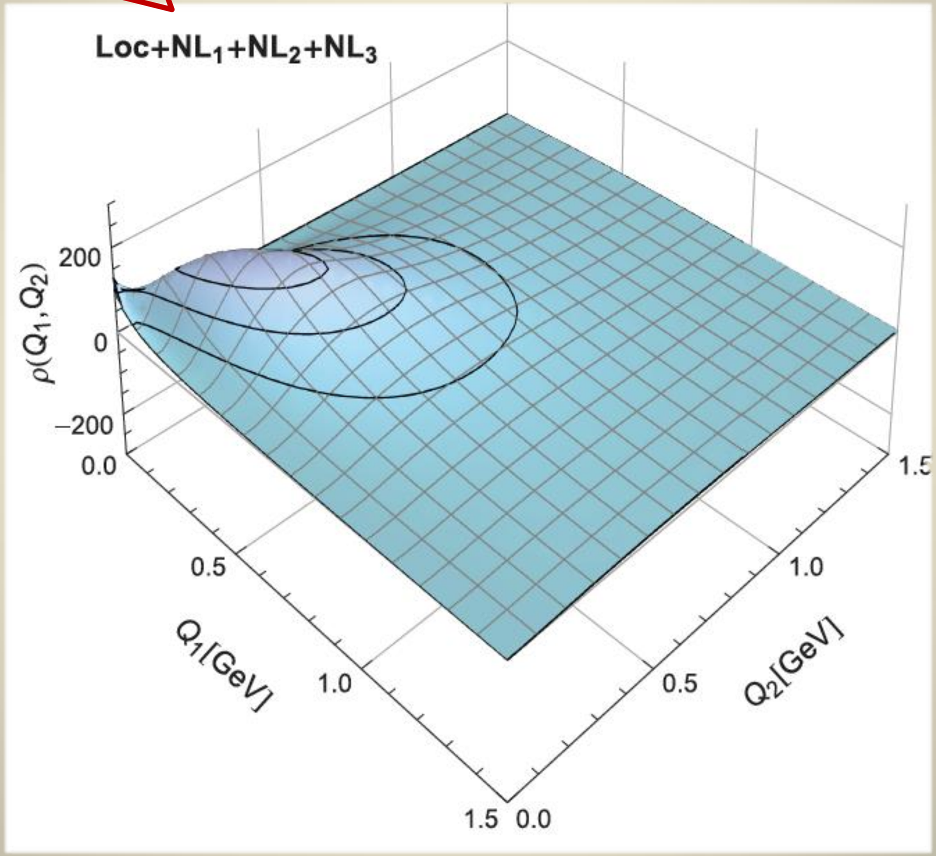
1) Becomes more Regular

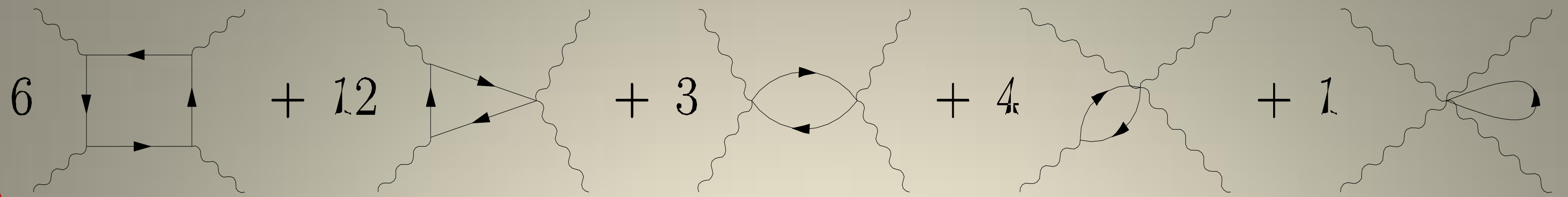
2) Island with Hill grows



1) Becomes more Regular

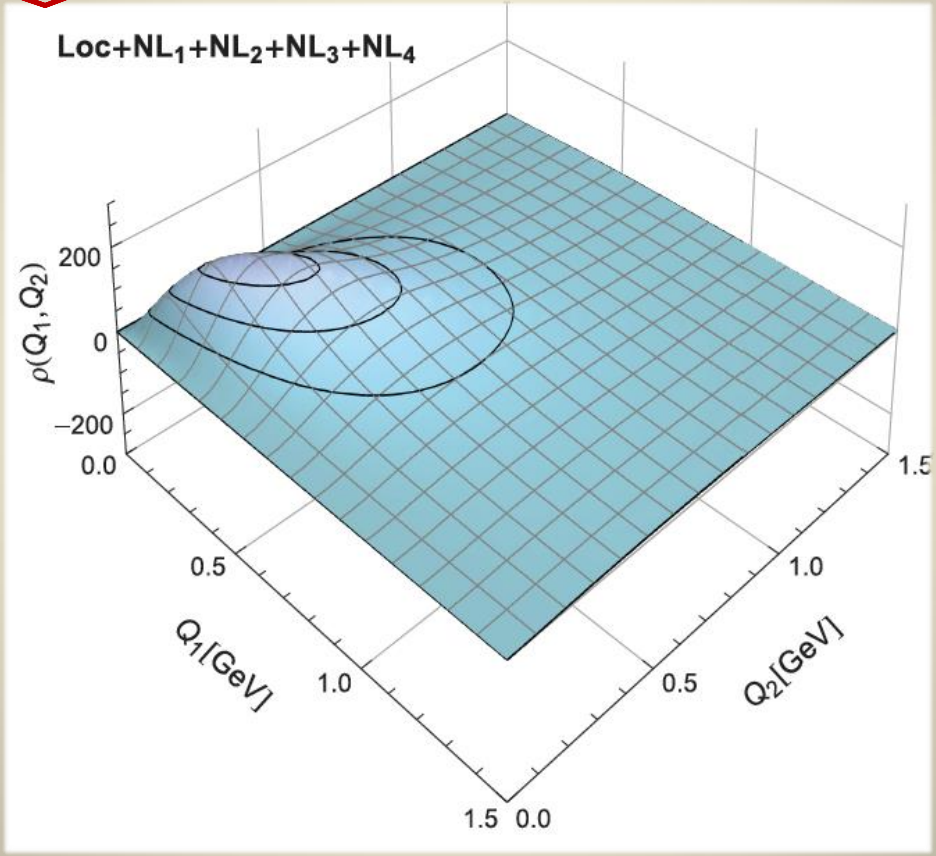
2) Island with Hill grows





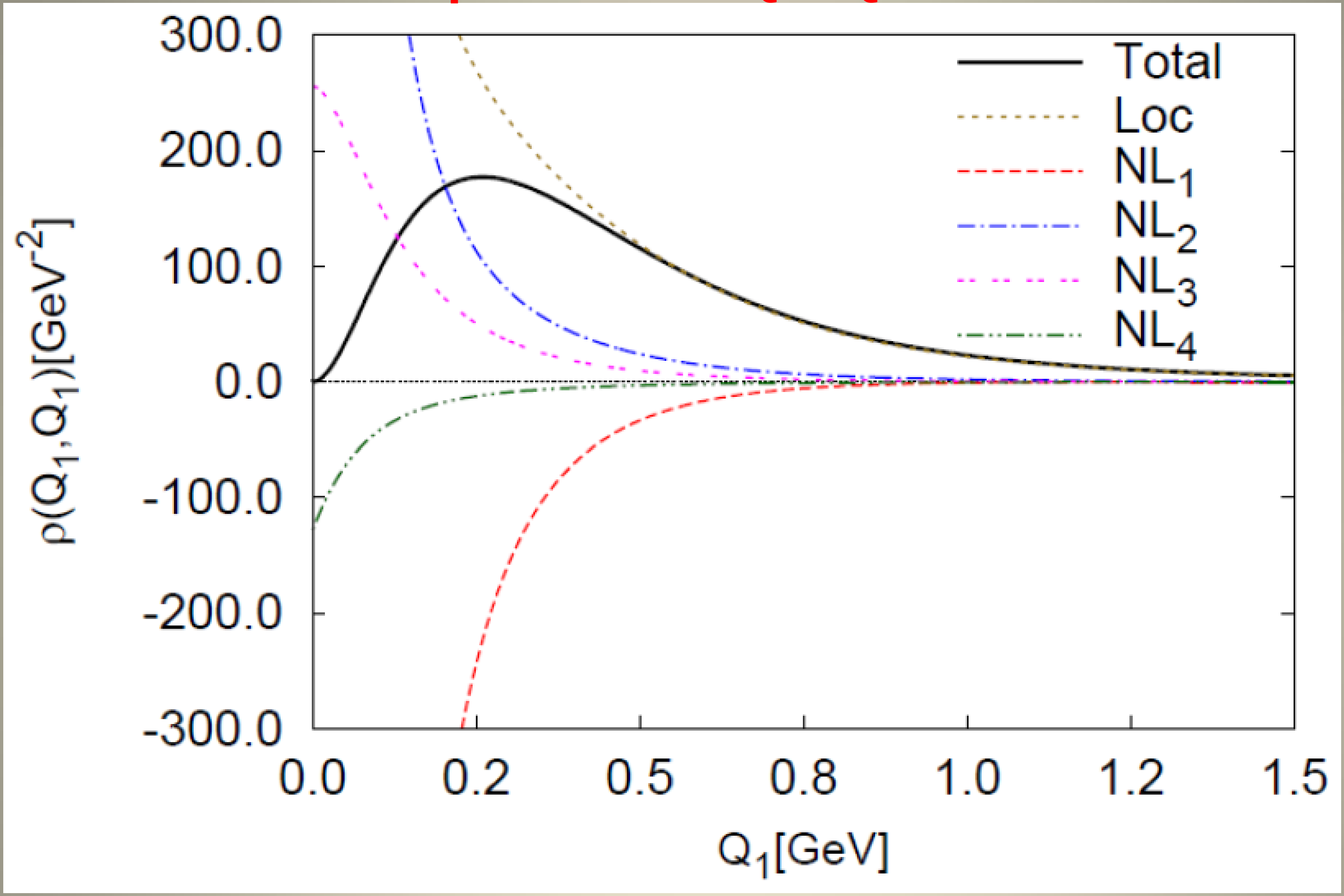
1) Completely Regular

2) With clear Island and Hill



$$a^{Box} = \iint_0^\infty dQ_1 dQ_2 \rho(Q_1, Q_2)$$

Profile of previous at Q2=Q1

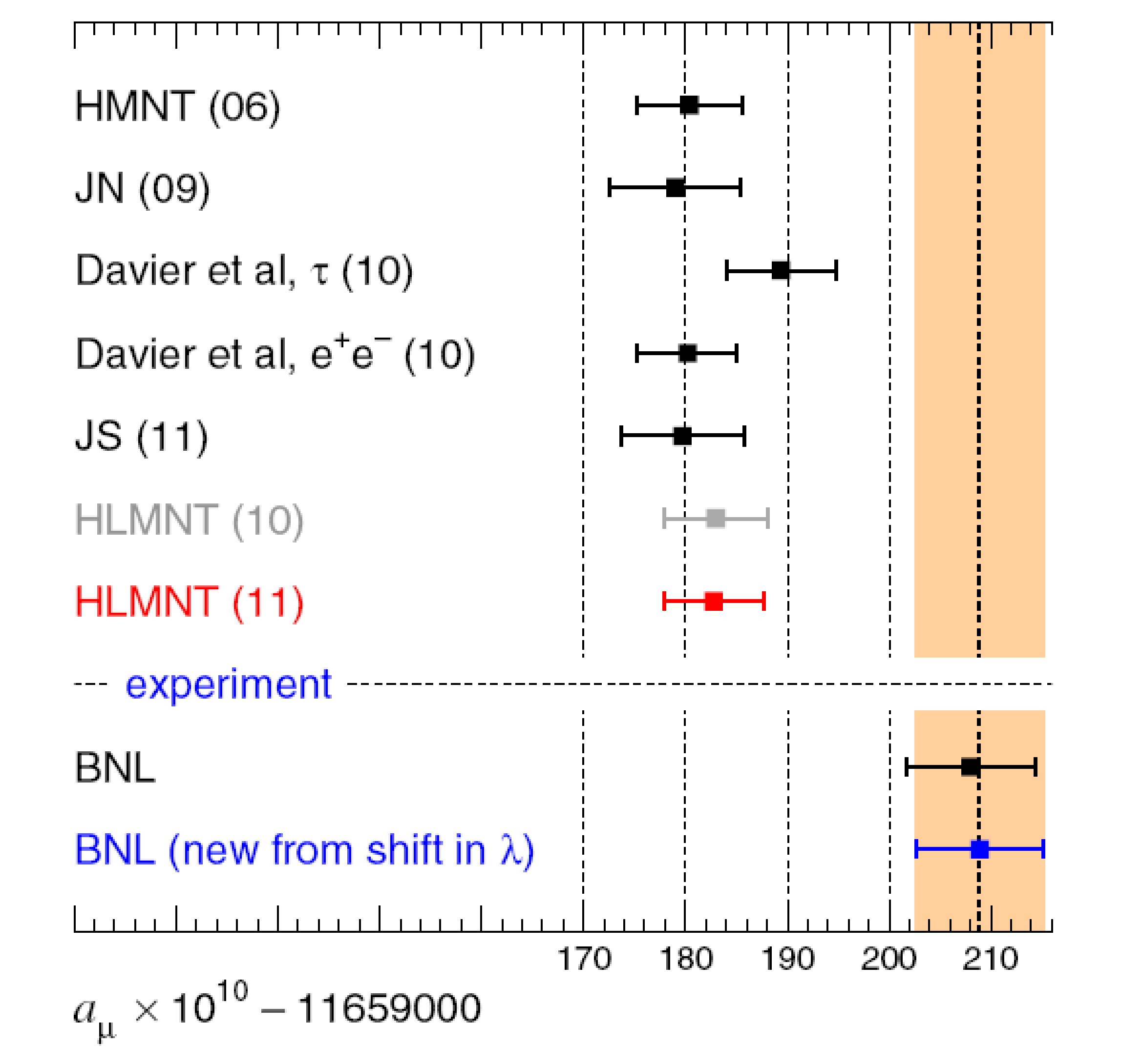


1) LET at Q=0
2) pQCD at large Q

This is due to Gauge Invariance and Spontaneous breaking of Chiral Symmetry

Estimates of Hadronic Contributions in different Approaches

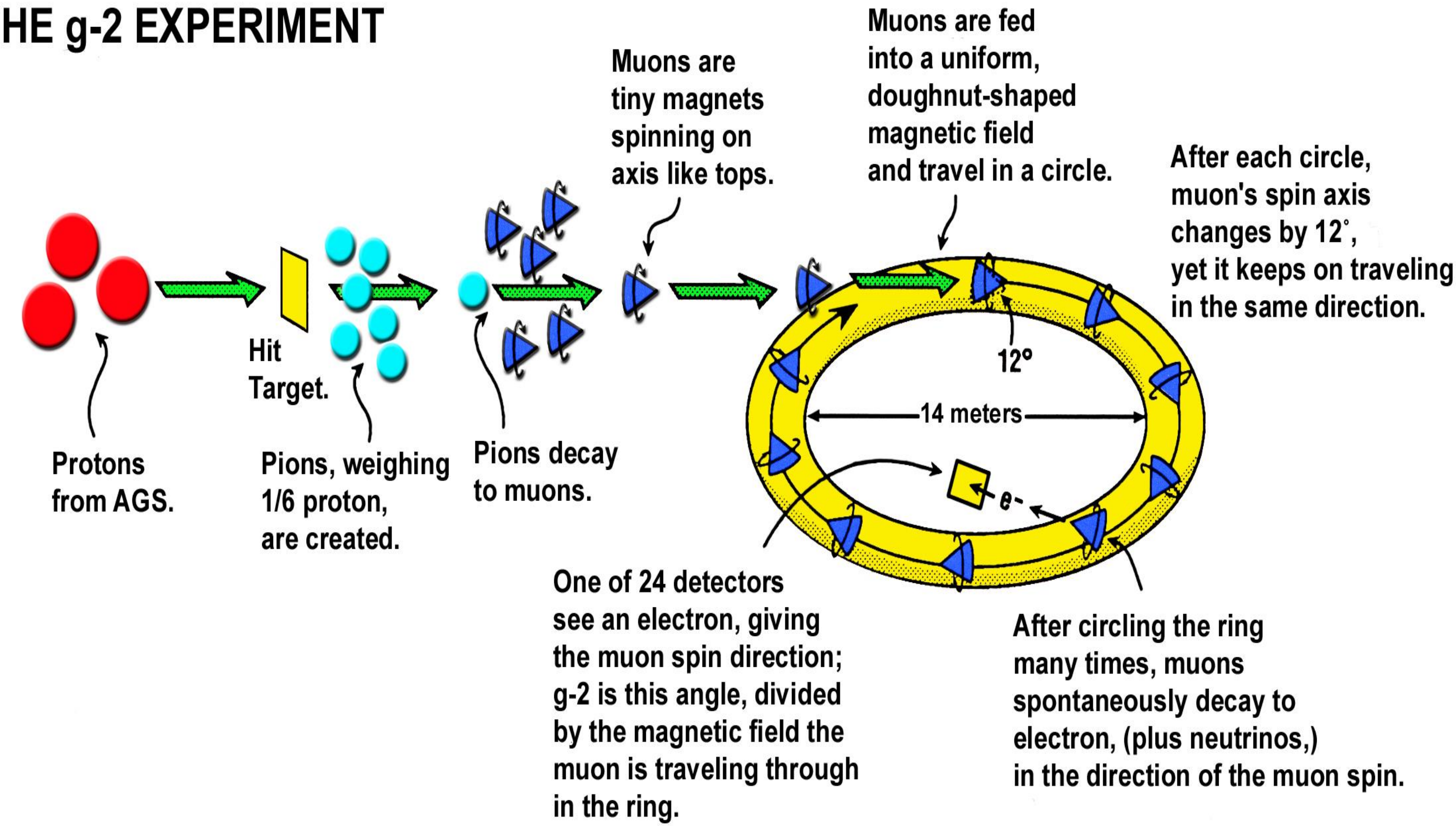
Model	π^0	PS (π^0, η, η')	S (σ, f_0, a_0)	AV	Quark loop	$\pi, K-$ loops	Total
VMD (Hayakawa [24])	5.74(0.36)	8.27(0.64)		0.17(0.10)	0.97(1.11)	−0.45(0.81)	8.96(1.54)
ENJL (Bijnens [25])	5.58(0.05)	8.5(1.3)	−0.68(0.2)	0.25(0.1)	2.1(0.3)	−1.9(1.3)	8.3(3.2)
LMD+V (Knecht [26])	5.8(1.0)	8.3(1.2)					8.0(4.0)
Q-box (Pivovarov [32])					14.05		14.05
LENJL (Bartos [31])	8.18(1.65)	9.55(1.7)	1.23(0.24)				10.77(1.68)
(LMD+V)′(Melnikov [27])	7.65(1.0)	11.4(1.0)		2.2(0.5)		0(10)	13.6(0.25)
NχQM (Dorokhov [36–38])	5.01(0.37)	5.85(0.87)	0.34(0.48)		11.0(0.9)		16.8(1.25)
oLMDV (Nyffeler [28])	7.2(1.2)	9.9(1.6)	−0.7(0.2)	2.2(0.5)	2.1(0.3)	−1.9(1.3)	11.6(0.4)
DS (Goecke [39])	5.75(0.69)	8.07(1.2)			10.7(0.2)		18.8(0.4)
CχQM (Greynat [35])	6.8(0.3)	6.8(0.3)			8.2(0.6)		15.0(0.3)



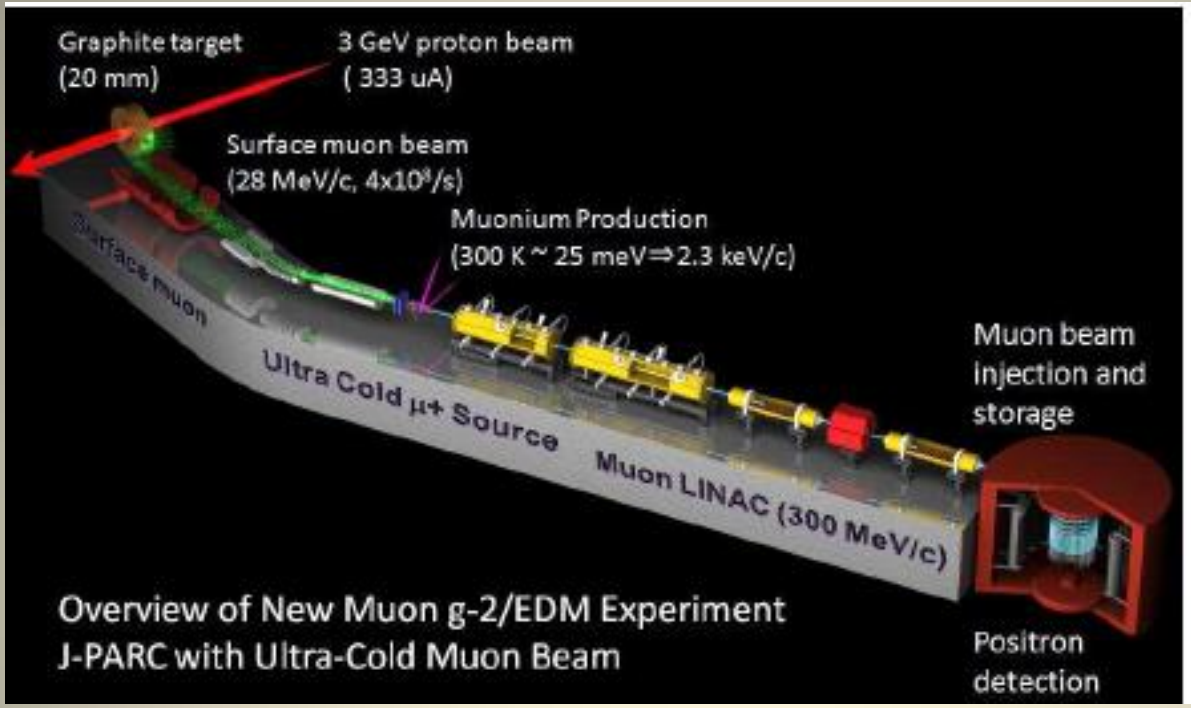
Our results indicate that the LbL is underestimated
And discrepancy may be less than 3 sigma

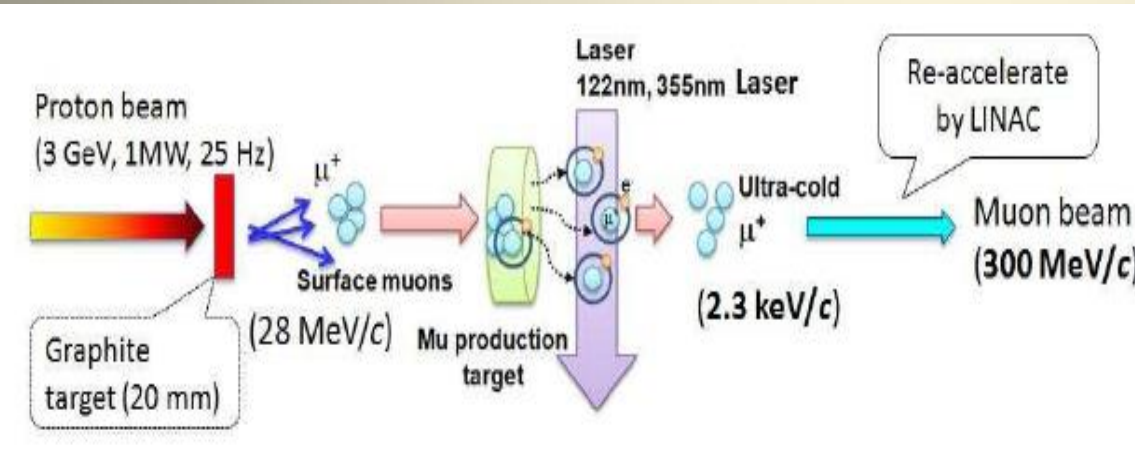
$$\Delta a_\mu \bullet 10^{+10} = a_\mu^{\exp} - a_\mu^{SM} = 27.8(8.0) \Rightarrow \approx 23$$

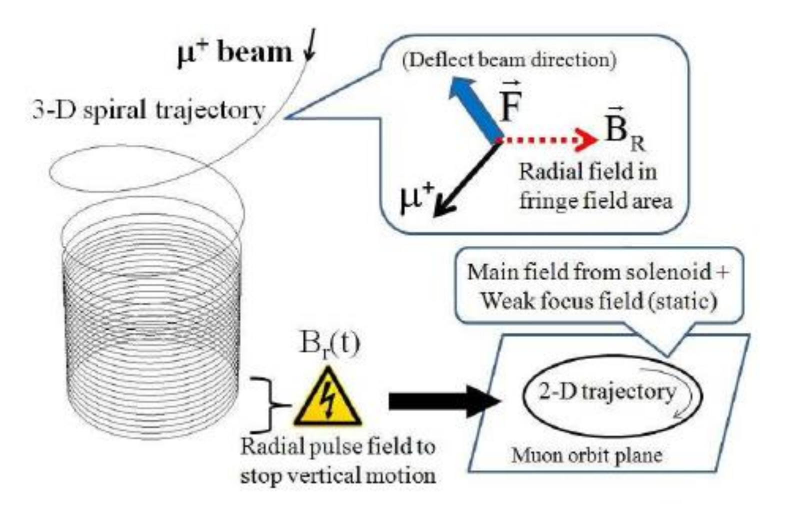
LIFE OF A MUON: THE g-2 EXPERIMENT



Precise measurment of muon g-2/EDM at JPARC

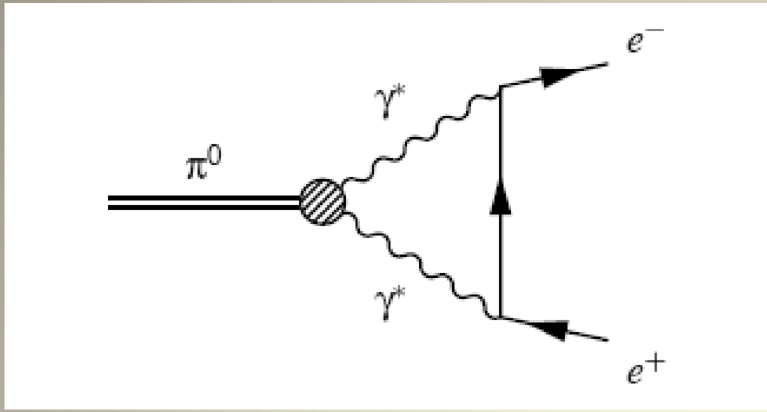






Rare Pion Decay $\pi^0 \to e^+e^{--}$ *from KTeV*

PRD (2007)



Lowest order diagram

One of the simplest process for THEORY

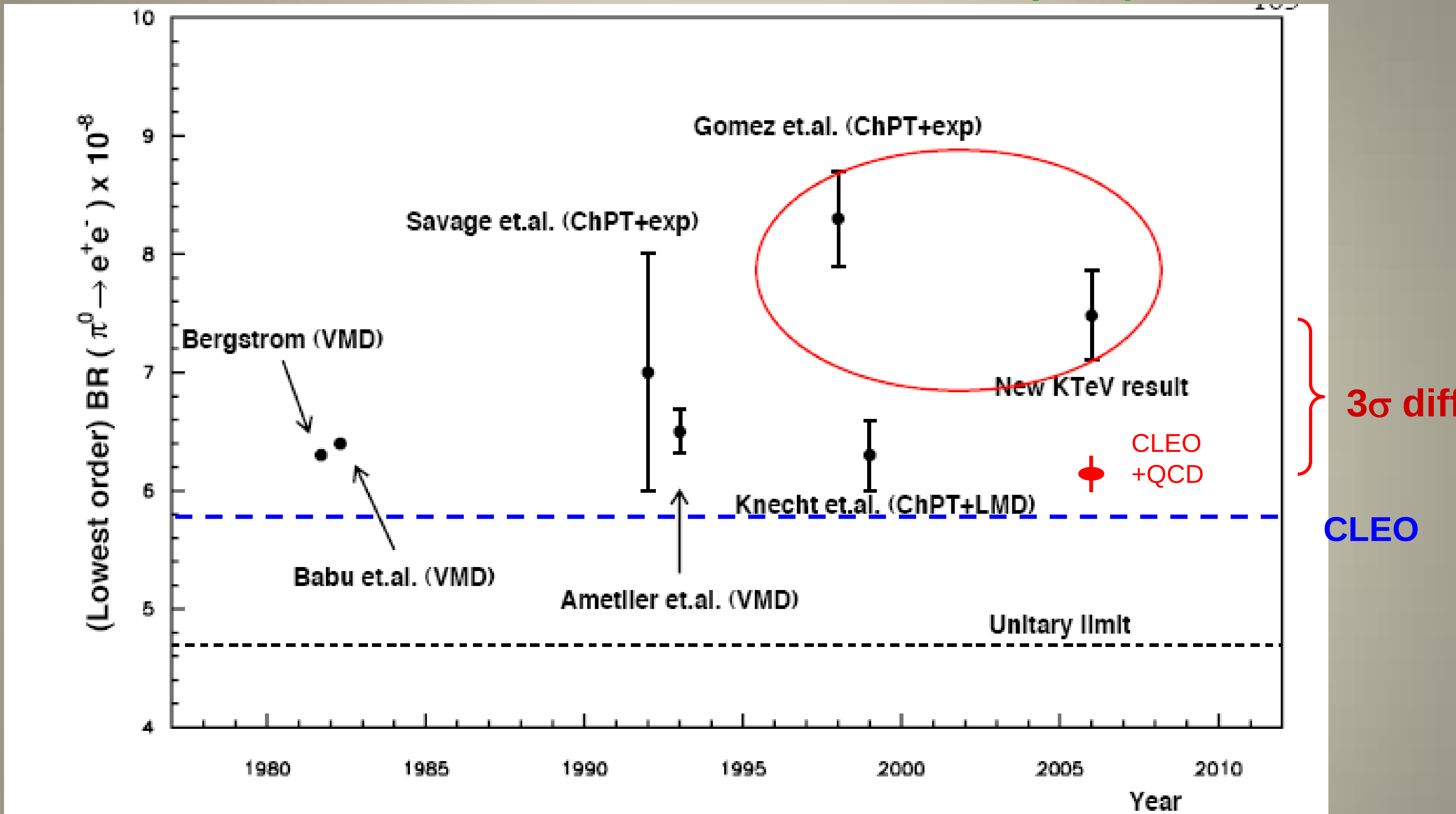
From KTeV E799-II EXPERIMENT at Fermilab experiment (1997-2007)

$$B^{\text{KTeV}}_{\pi \to e^+ e^-} = \left(7.48 \pm 0.29 \pm 0.25\right) \cdot 10^{-8}$$

99-00' set,

The result is based on observation of 794 candidate $\pi^0 \to e^+e^-$ events using $K_L \to 3\pi_0$ as a source of tagged π^0s.

AED, M. Ivanov PRD (2007)



What is next? It would be very desirable if Others will confirm KTeV result
Also, $\eta \to \mu\mu$ pair decay is very perspective

Other P →l+l– decays

A.D., M. Ivanov, S. Kovalenko (Phys Lett 2009)

TABLE I: Values of the branchings $B(P \to l^+l^-)$ obtained in our approach and compared with the available experimental results.

R_0	Unitary bound	CLEO bound	CLEO+OPE	This work	Experiment	BES III (1 year)
$R_0(\pi^0 \to e^+e^-) \times 10^8$	≥ 4.69	$\geq 5.85 \pm 0.03$	6.23 ± 0.12	6.26	7.49 ± 0.38 [1]	*WASA@COSY*
$R_0(\eta \to \mu^+\mu^-) \times 10^6$	≥ 4.36	$\leq 6.23 \pm 0.12$	5.12 ± 0.27	4.64	5.8 ± 0.8 [20, 21]	0.08
$R_0(\eta \to e^+e^-) \times 10^9$	≥ 1.78	$\geq 4.33 \pm 0.02$	4.60 ± 0.09	5.24	$\leq 2.7 \cdot 10^4$ [22]	$0.7 \cdot 10^2$
$R_0(\eta' \to \mu^+\mu^-) \times 10^7$	≥ 1.35	$\leq 1.44 \pm 0.01$	1.364 ± 0.010	1.30		0.8
$R_0(\eta' \to e^+e^-) \times 10^{10}$	≥ 0.36	$\geq 1.121 \pm 0.004$	1.178 ± 0.014	1.86		$0.7 \cdot 10^3$

π->ee will be available soon from WASA@COSY

Mass power corrections are visible for η(η’) decays

BESIII for one year will get for η ,η’->ll the limit $0.7*10^{-7}$

Summary

1) Study of Electron AMM provides very precise value for the QED coupling α

2) Study of Muon AMM is sensitive to effects of SM and NP

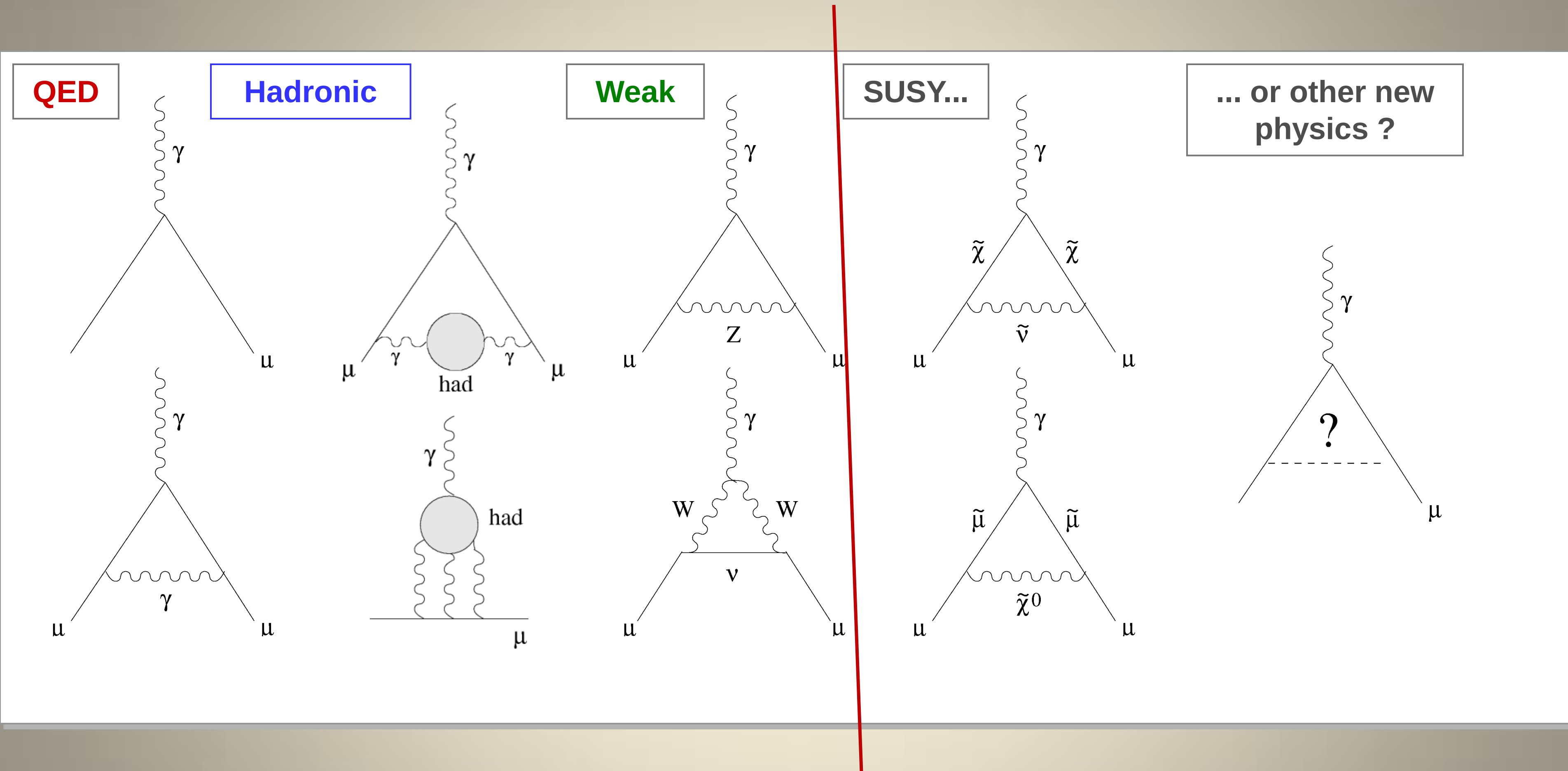
3) At present there is 3.4σ disagreement between SM and BNL experiment. New experiments at FNAL and Jparc are promising

4) New experiments at VEPP2000, KLOE2, BESS III on cross section will further diminish the error for HVP contribution

5) The account of full kinematic dependence of meson-two-photon vertex reduces the value for the meson exchange LbL contribution

6) Dynamical quark box contribution make total result bigger than in previous estimates

Anomalous Magnetic Moment in SM and beyond



Basic of Standard Model

Results on PS meson exchange LbL contribution

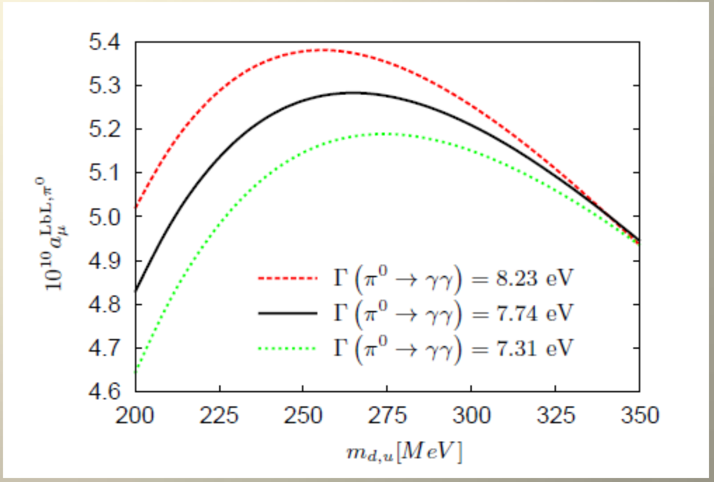
AED, AE Radzhabov, AS Zhevlakov, EPJC (2011)

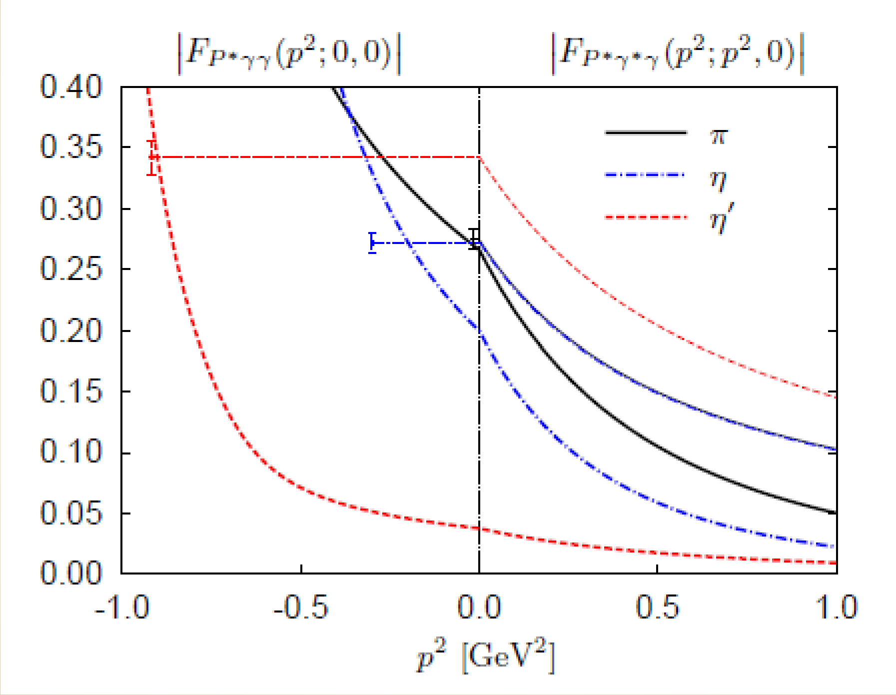
Model	π^0	η	η'	$\pi^0+\eta+\eta'$
VMD [6]	5.74	1.34	1.19	8.27(0.64)
ENJL [11]	5.6			8.5(1.3)
LMD+V, VMD [7]	5.8(1.0)	1.3(0.1)	1.2(0.1)	8.3(1.2)
NJL [12]	8.18(1.65)	0.56(0.13)	0.80(0.17)	9.55(1.66)
(LMD+V)′,VMD[8]	7.97	1.8	1.8	11.6(1.0)
NχQM [13]	6.5(0.2)			
HM [16]	6.9	2.7	1.1	10.7
DIP, VMD [10]	6.54(0.25)			
DSE [15]	5.75(0.69)	1.36(0.30)	0.96(0.21)	8.07(1.20)
This work (NχQM)	5.01(0.37)	0.54	0.30	5.85

Our results are systematically lower!

Why?

Because we use full kinematical Dependence of the photon-meson vertices!





$|F_{P^*\gamma\gamma}(p^2;0,0)|$
$|F_{P^*\gamma^*\gamma}(p^2;p^2,0)|$
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
-1.0
-0.5
0.0
0.5
1.0
p^2 [GeV^2]
π
η
η'