Large relative phase between strong and electromagnetic contributions in charmonium $\psi(3770)$ production at BES

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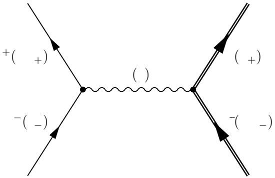
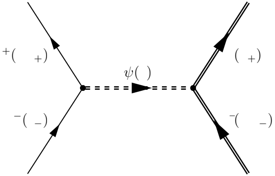
GDR PH-QCD Meeting
"Recent highlights in hadron structure"
7 october 2014

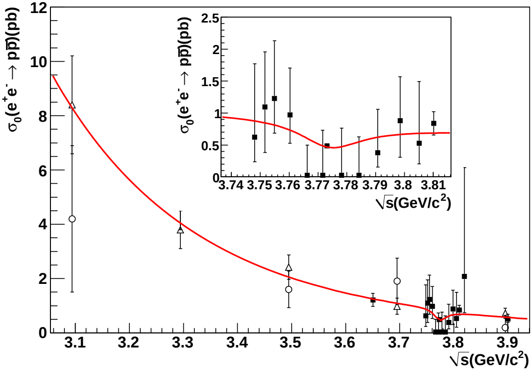
Experimental situation: Charmonium $\psi(3770)$ production at BES

Large statistics of J/ψ, $\psi(2S)$ and $\psi(3770)$ samples have been obtained in recent years by BEPCII/BESIII facility [BESIII Collaboration, M. Ablikim *et al.*, Phys.Lett. **B710**, 594 (2012)]. It provides the possibility to study many decay channels of J/ψ, $\psi(2S)$ and $\psi(3770)$ resonances.

In a profound work, BESIII has measured the phase angle ϕ between the continuum and resonant amplitudes [M. Ablikim *et al.* [BESIII Collaboration], Phys. Lett. B **735**, 101 (2014)].



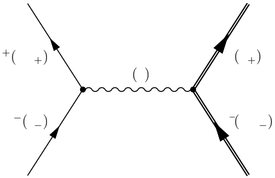


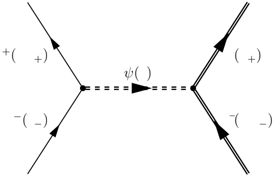


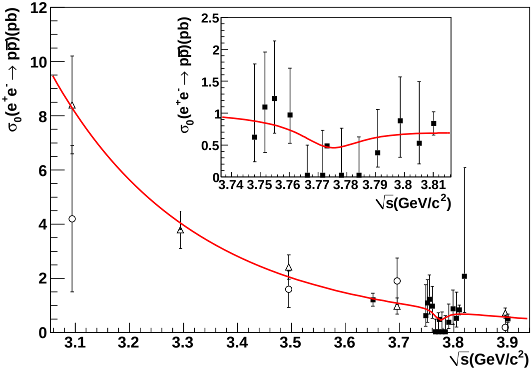
Fit parameterization was like

$$\sigma\left(s\right)=\left|A_{con}+A_{\psi}\,e^{i\phi}\right|^2=\left|\sqrt{\sigma_{con}\left(s\right)}+\sqrt{\sigma_{\psi}}\frac{M_{\psi}\Gamma_{\psi}}{s-M_{\psi}^2+iM_{\psi}\Gamma_{\psi}}e^{i\phi}\right|^2, \quad (1)$$

$$\sigma_{con}\left(s\right)=\frac{4\pi\alpha^2\beta}{3s}\left(1+\frac{2M_p^2}{s}\right)\left|G\left(s\right)\right|^2. \quad (2)$$







Experimental situation: Charmonium $\psi(3770)$ production at BES

$$\sigma(s) = \left| A_{con} + A_\psi\, e^{i\phi} \right|^2 = \left| \sqrt{\sigma_{con}(s)} + \sqrt{\sigma_\psi} \frac{M_\psi \Gamma_\psi}{s - M_\psi^2 + i M_\psi \Gamma_\psi} e^{i\phi} \right|^2, \qquad (3)$$

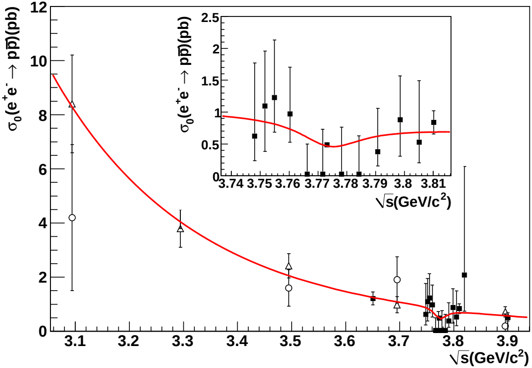
Fitting procedure found two possible solutions:

Solution	σ_ψ (pb)	ϕ ($^\circ$)
(1)	$0.059 \pm 0.032 \pm 0.012$	$255.8 \pm 37.9 \pm 4.8$
(2)	$2.57 \pm 0.12 \pm 0.12$	$266.9 \pm 6.1 \pm 0.9$

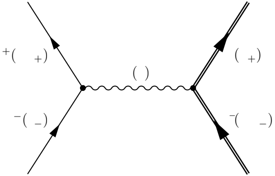
Effective proton form factor was parameterized as:

$$|G(s)| = \frac{C}{s^2 \ln^2(s/\Lambda^2)},$$

with $\Lambda = 0.3$ GeV is the QCD scale parameter and $C = (62.0 \pm 2.3)$ GeV^4.



Theoretical consideration: Born approximation



We consider the mechanisms of creation of a $p\bar{p}$ pair in electron–positron collisions which proceeds through virtual photon intermediate state

$$\mathcal{M}_B = -\frac{e^2}{s} G(s) J^e_\mu J^{p\mu}, \tag{4}$$

where lepton J^e_μ and proton J^p_μ currents have a form:

$$J^e_\mu = \bar{v}(q_+)\gamma_\mu u(q_-), \qquad J^p_\mu = \bar{u}(p_+)\gamma_\mu v(p_-),$$

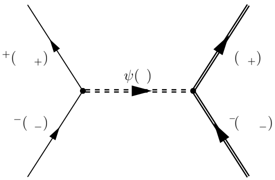
and $G(s)$ is the model–dependent proton formfactor. The corresponding contribution to the differential cross section

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2\beta}{4s}\left(2 - \beta^2\sin^2\theta\right)|G(s)|^2, \tag{5}$$

where $s = q^2 = (q_+ + q_-)^2 = 4E^2$ is the total invariant energy of the process, E is the electron beam energy in the center-of-mass reference frame, β is the proton velocity ($\beta^2 = 1 - M_p^2/E^2$) and M_p is the proton mass and the scattering angle θ is the center-of-mass reference frame angle between the 3-momenta of the initial electron $\mathbf{q}_-$ and the created proton $\mathbf{p}_+$. The total cross section then

$$\sigma_B(s) = \frac{2\pi\alpha^2\beta(3-\beta^2)}{3s}|G(s)|^2. \tag{6}$$

Theoretical consideration: Charmonium contribution



The second mechanism describes the conversion of electron–positron pair to $\psi(3770)$ with the subsequent conversion to the proton–antiproton pair. For this aim we put the whole matrix element as

$$\mathcal{M} = \mathcal{M}_B + \mathcal{M}_\psi, \tag{7}$$

where the contribution with $\psi(3770)$ intermediate state is

$$\mathcal{M}_\psi = \frac{g_e}{s - M_\psi^2 + i\sqrt{s}\Gamma_\psi} J^e_\nu J^\nu_\psi. \tag{8}$$

Here we assumed that vertex $\psi \to e^+e^-$ has the same structure as $\gamma \to e^+e^-$, i.e.:

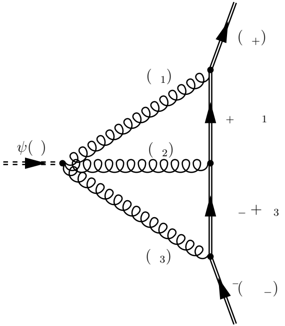
$$J^\mu_{\psi\to e^+e^-} = g_e\, J^\mu_e, \tag{9}$$

and the constant g_e is defined via $\psi \to e^+e^-$ decay

$$g_e = \sqrt{\frac{12\pi\Gamma_{\psi\to e^+e^-}}{M_\psi}} = 1.6 \cdot 10^{-3}. \tag{10}$$

We should also note that in paper [E. A. Kuraev, Y. M. Bystritskiy and E. Tomasi-Gustafsson, Nucl. Phys. A **920**, 45 (2013).] it was shown that the vertexes of this type do not generate large imaginary part and thus we can neglect their contribution to the relative phase ϕ.

Theoretical consideration: Charmonium current



The current J_ψ^ν which describes the transition of $\psi(3770)$ with momentum $q = 2p$ into proton–antiproton pair via three gluon intermediate state has the form:

$$
\begin{aligned}
J_\psi^\nu = {} & R\,\alpha_s^3 g_{col} \times \\
& \times \int \frac{d^4k_1\, d^4k_2\, d^4k_3}{k_1^2\, k_2^2\, k_3^2\, ((p_+ - k_1)^2 - M_p^2)((p_- - k_3)^2 - M_p^2)} \times \\
& \times \delta(q - k_1 - k_2 - k_3) \left[\bar{u}(p_+)\hat{O}^\nu v(p_-)\right], \qquad (11)
\end{aligned}
$$

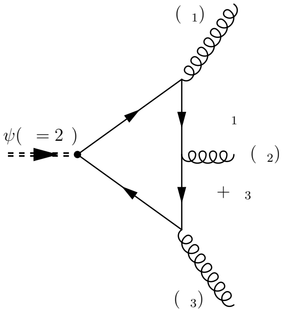
where α_s is the strong interaction coupling which is associated with each gluon line and $\hat{O}^\nu$ is

$$\hat{O}_\nu = \hat{O}_\nu^{\alpha\beta\gamma} \gamma_\alpha (\hat{p}_+ - \hat{k}_1 + M_p) \gamma_\beta (-\hat{p}_- + \hat{k}_3 + M_p) \gamma_\gamma. \qquad (12)$$

Color factor

$$g_{col} = \langle p|\,(3/4) d^{abc} t_a t_b t_c\,|p\rangle = 5/6 \qquad (13)$$

describes the interaction of gluons with quarks of the proton. The quantity R is connected with wave function of $\psi(3770)$ and is described below.



Let us consider the conversion of the bound state with quantum numbers $J^{PC} = 1^{--}$ to three real massless vector bosons. Similar problem was solved years ago for the problem of ortho-positronium decay [I. Pomeranchuk, Dokl. Akad. Nauk SSSR **60**, 213 (1948) and A. Ore and J. Powell, Phys.Rev. **75**, 1696 (1949)]. For the case of ortho-positronium Ops decay, we start from matrix element of the process:

$$Ops \to \gamma(k_1) + \gamma(k_2) + \gamma(k_3), \tag{14}$$

which has the form:

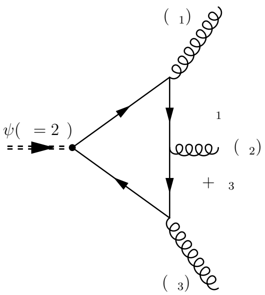
$$\mathcal{M}_{Ops} = A\frac{1}{m_e^4} O_\sigma^{\mu\nu\lambda} e_\mu(k_1) e_\nu(k_2) e_\lambda(k_3) \epsilon_\sigma(q), \tag{15}$$

where $e(k_i)$ and $\epsilon(q)$ are the polarization vectors of photons and the ortho-opositronium respectively. The quantity A includes the information on the wave function of ortho-positronium. Operator

$$O_\sigma^{\mu\nu\lambda} e_\mu(k_1) e_\nu(k_2) e_\lambda(k_3) = \frac{1}{4}\mathrm{Sp}\left[\hat{Q}(\hat{p} + m_e)\gamma_\sigma(\hat{p} - m_e)\right]; \tag{16}$$

$$\hat{Q} = \frac{1}{x_1 x_3}\hat{e}_3(-\hat{p} + \hat{k}_3 + m_e)\hat{e}_2(\hat{p} - \hat{k}_1 + m_e)\hat{e}_1 + \text{cyclic permutations}, \tag{17}$$

describes the electron loop.



Using the amplitude $\mathcal{M}_{Ops}$ we obtain for the decay width

$$\Gamma_{Ops} = \frac{1}{12m_e} \int \sum_{spins} |\mathcal{M}_{Ops}|^2 \cdot \frac{m_e^2\pi^2}{(2\pi)^5} d^2x \cdot \frac{(4\pi\alpha)^3}{3!} = \\ = \frac{128}{9} m_e A^2(\pi^2 - 9)\alpha^3. \tag{18}$$

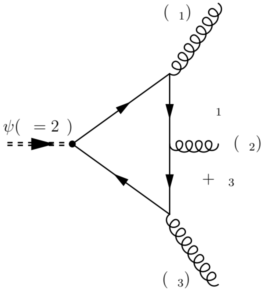
Comparing this value with the known result

$$\Gamma_{Ops} = \frac{2m_e}{9\pi}(\pi^2 - 9)\alpha^6, \tag{19}$$

we conclude that

$$A = \frac{\alpha^{3/2}}{8\sqrt{\pi}}. \tag{20}$$

Theoretical consideration: Vertex $\psi \to 3g$



For the case of decay of $\psi(3770)$ to three gluons with the subsequent turning them to hadrons we define the amplitude in the similar form:

$$\mathcal{M}_{\psi\to 3g} = R(4\pi\alpha_s)^{3/2}\frac{1}{4}d^{abc}\, e^a_\mu(k_1)e^b_\nu(k_2)e^c_\lambda(k_3)\times \times \frac{1}{m_c^4}O^{\mu\nu\lambda}_\sigma \varepsilon^\sigma(q), \tag{21}$$

with $q = 2p$ and $\varepsilon(q)$ are the momentum and the polarization vector of $\psi(3770)$. The decay width then reads as:

$$\Gamma_{\psi\to 3g} = \frac{320}{9}M_\psi R^2(\pi^2 - 9)\alpha_s^3. \tag{22}$$

And comparing this result with the known one [T. Appelquist and H. Politzer, Phys.Rev.Lett. **34**, 43 (1975) and W. Kwong, P. B. Mackenzie, R. Rosenfeld, and J. L. Rosner, Phys.Rev. **D37**, 3210 (1988)]:

$$\Gamma_{\psi\to 3g} = \frac{160 M_\psi}{2187\pi}(\pi^2 - 9)\alpha_s^6, \tag{23}$$

we conclude that

$$R = \frac{\alpha_s^{3/2}}{9\sqrt{6\pi}} \approx 1.595\cdot 10^{-5}, \tag{24}$$

if one assumes that $\alpha_s \approx 0.26$. Note that both A and R are real.

Thus the contribution to the total cross section arising from the interference of relevant amplitudes has the form

$$\delta\sigma_\psi = \frac{1}{8s} 2\,\mathrm{Re}\left[\sum_{\text{spins}} \int \mathcal{M}_B^* \, \mathcal{M}_\psi \, d\Phi_2\right], \tag{25}$$

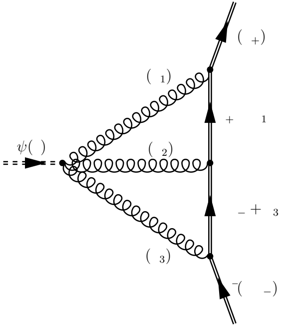
where two–particle phase volume $d\Phi_2$ is

$$d\Phi_2 = \frac{d^3p_+}{2E_+}\frac{d^3p_-}{2E_-}\frac{1}{4\pi^2}\delta^4(q - p_+ - p_-) = \frac{\beta}{16\pi}d\cos\theta, \tag{26}$$

and θ is again the angle between the directions of initial electron $\mathbf{q}_-$ and the produced proton $\mathbf{p}_+$. To perform the summation over spin states we use the method of invariant integration:

$$\sum_{\text{spins}} \int d\Gamma_2 \, J_\mu^{p*} J_\nu^\psi = \frac{1}{3}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\int d\Gamma_2 \sum_{\text{spins}} J_\alpha^{p*} J_\psi^\alpha. \tag{27}$$

Theoretical consideration: Interference of Born and Charmonium amplitudes



Thus we get for the contribution to the total cross section

$$\delta\sigma_\psi = C_\psi \mathrm{Re}\left(\frac{A_\psi(s)}{s - M_\psi^2 + i\sqrt{s}\Gamma_\psi}\right), \tag{28}$$

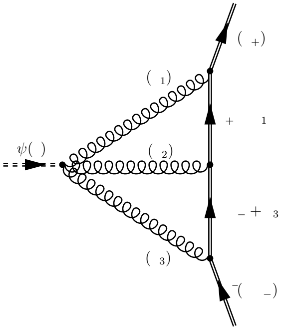
where C_ψ is the constant

$$C_\psi = -\frac{10}{9\pi^4}\alpha\alpha_s^3 g_e R, \tag{29}$$

and quantity $A_\psi(s)$ describes the vertex $\psi \to 3g \to p\bar{p}$:

$$A_\psi(s) = \frac{1}{4s^3}\int d\Phi_2 \int \frac{dk_1}{k_1^2}\frac{dk_2}{k_2^2}\frac{dk_3}{k_3^2}\delta(q - k_1 - k_2 - k_3)\frac{Sp}{(p_+k_1)(p_-k_2)}. \tag{30}$$

Theoretical consideration: Imaginary part calculation



Our approach consists in calculation of the s-channel discontinuity of $A_\psi(s)$ with the subsequent restoration of real part with the use of dispersion relation. For this aim we use the Cutkosky rule for gluon propagators

$$\frac{1}{k^2} \to (-i\pi)\delta(k^2)\theta(k_0).$$

This allows us integrate over phase volume of three gluon intermediate state as

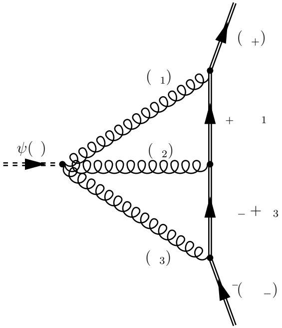
$$dk_1\, dk_2\, dk_3\, \delta(k_1^2)\, \delta(k_2^2)\, \delta(k_3^2)\, \delta^4(q - k_1 - k_2 - k_3) = s\, dx_1\, dx_2\, \frac{dc_1 dc_2}{\sqrt{D}}, \tag{31}$$

where

$$D = 1 - c_1^2 - c_2^2 - c^2 + 2c_1c_2c > 0, \tag{32}$$

and $c_{1,2} \equiv \cos(\mathbf{p}_+, \mathbf{k_{1,2}})$. and c is the cosine of the angle between directions $\mathbf{k_1}$ and $\mathbf{k_2}$.

Theoretical consideration: Imaginary part calculation



So we obtain s-channel discontinuity of $A_\psi(s)$ in the form:

$$\text{Im}\left(A_\psi(s)\right) = -\frac{4\pi^6}{s^4}\frac{\left(s-4M_p^2\right)^{1/2}}{8\pi\sqrt{s}}\int\limits_0^1 dx_1 \int\limits_{1-x_1}^1 dx_2 \times$$

$$\times \int \frac{dc_1 dc_2}{\sqrt{D}}\frac{Sp}{C_1 C_2} \equiv F(\beta), \qquad (33)$$

where $C_1 = x_1(1-\beta c_1)$ and $C_2 = x_2(1+\beta c_2)$ and the integration over phase volume $d\gamma$ is performed in the kinematical region where $D > 0$.

Theoretical consideration: Real part calculation

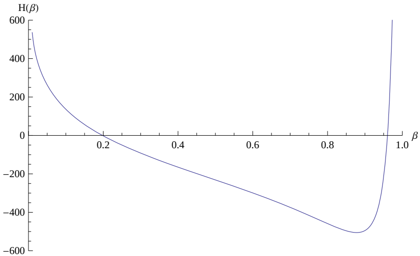
Real part of the amplitude $A_\psi(s)$ we restore with the use of dispersion relation. As we are interested in the energy region close to the mass of resonance, we use some trick to restore the real part of $A_\psi(s)$ by means of dispersion relations. For this aim we do a replacement

$$A_\psi(s) \to g(s) = \frac{4M_p^2}{s} A_\psi(s). \tag{34}$$

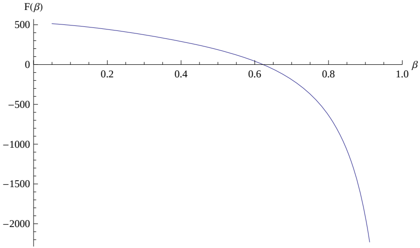
Thus using the Cauchy theorem for function $g(s)$ one obtains the following dispersion relation for the real part of $A_\psi(s)$:

$$\begin{aligned}
\mathrm{Re}\left(A_\psi\left(s\left(\beta\right)\right)\right) &= \frac{\mathcal{P}}{\pi}\int_0^1 \frac{d\beta_1^2}{\beta_1^2-\beta^2}\mathrm{Im}\left(A_\psi\left(s\left(\beta_1\right)\right)\right) = \\
&= \frac{1}{\pi}\mathrm{Im}\left(A_\psi\left(s\left(\beta\right)\right)\right)\ln\frac{1-\beta^2}{\beta^2} + \\
&+ \frac{1}{\pi}\int_0^1 \frac{d\beta_1^2}{\beta_1^2-\beta^2}\left[\mathrm{Im}\left(A_\psi\left(s\left(\beta_1\right)\right)\right) - \mathrm{Im}\left(A_\psi\left(s\left(\beta\right)\right)\right)\right].
\end{aligned} \tag{35}$$

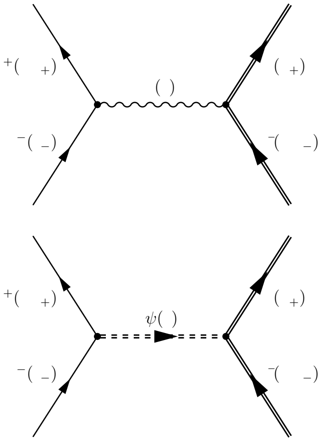
Here we present the numerical results for real and imaginary parts of the amplitude $A_\psi\left(s\left(\beta\right)\right)$.



$$H(\beta) = \mathrm{Re}\left(A_\psi\left(s\left(\beta\right)\right)\right)$$



$$F(\beta) = \mathrm{Im}\left(A_\psi\left(s\left(\beta\right)\right)\right)$$



To compare with the experimental result of BES first we calculate the relative phase ϕ between electromagnetic (Born) and charmonium channels.

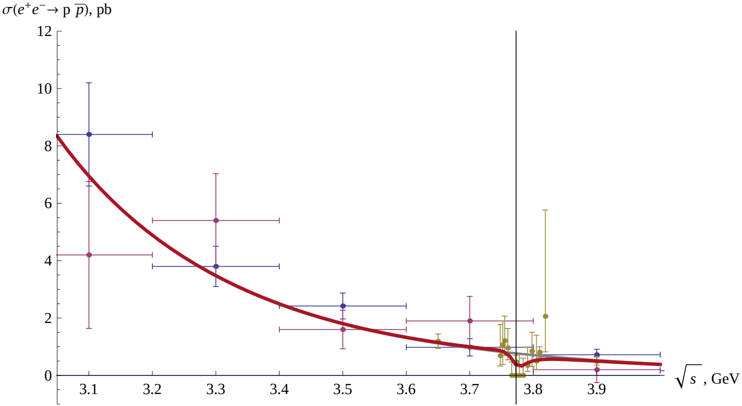
$$\phi = \arctan\left(\frac{\text{Im}\left(A_\psi\left(M_\psi^2\right)\right)}{\text{Re}\left(A_\psi\left(M_\psi^2\right)\right)}\right) + 180^o = \\ = 81^o + 180^o = 261^o. \tag{36}$$

This estimate should be compared with BES fitting result [M. Ablikim *et al.* [BESIII Collaboration], Phys. Lett. B **735**, 101 (2014)]:

Solution	σ_ψ (pb)	ϕ ($^\circ$)
(1)	$0.059 \pm 0.032 \pm 0.012$	$255.8 \pm 37.9 \pm 4.8$
(2)	$2.57 \pm 0.12 \pm 0.12$	$266.9 \pm 6.1 \pm 0.9$

Comparison with the experiment: Total cross section

Besides we can compare the result for total cross section. Here is the figures of with data presented in [M. Ablikim *et al.* [BESIII Collaboration], Phys. Lett. B **735**, 101 (2014)] with the curves estimated by the model described above.



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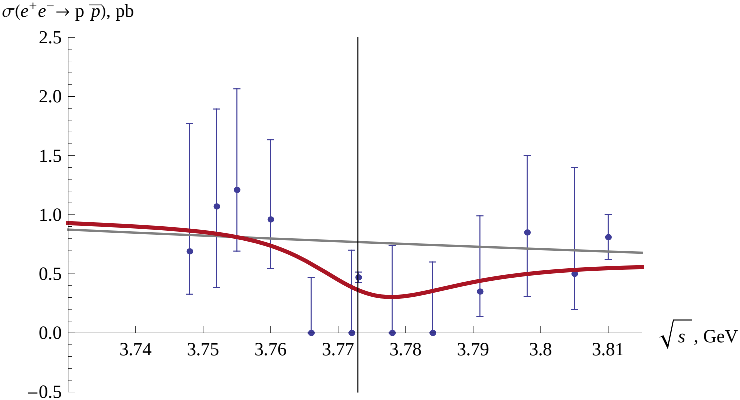

Conclusions

- Three gluon mechanism of Charmonium $\psi(3770)$ conversion into proton–antiproton pair generates large phase with respect to pure electromagnetic (Born) contribution.