Dark matter dynamics: a numerical perspective

Stéphane Colombi
IAP

Vlasov-Poisson equations

Self-gravitating collisionless fluid
f: phase-space density

$$\frac{\partial f}{\partial t} + \mathbf{u}.\frac{\partial f}{\partial \mathbf{r}} - \frac{\partial \psi}{\partial \mathbf{r}}.\frac{\partial f}{\partial \mathbf{u}} = 0,$$

$$\Delta\psi = 4\pi G\rho, \quad \rho(\mathbf{r}) = \int f(\mathbf{r},\mathbf{u},t)\mathrm{d}^3 v$$

In the expanding Universe:

Superconformal time

$$\mathrm{d}\tau \equiv \frac{\mathrm{d}t}{a^2},$$

Comoving coordinates **x**

$$\mathbf{r} = a\mathbf{x}, \quad \mathbf{p} = \frac{\mathrm{d}\mathbf{x}}{\mathrm{d}\tau} = a^2\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} = a\mathbf{v}$$

Peculiar velocity **v**

$$\mathbf{v} = \mathbf{u} - H\mathbf{r}$$

$$\frac{\partial f}{\partial \tau} + \mathbf{p}.\frac{\partial f}{\partial \mathbf{x}} - \frac{\partial \phi}{\partial \mathbf{x}}.\frac{\partial f}{\partial \mathbf{p}} = 0,$$

$$\Delta_{\mathbf{x}}\phi = 4\pi G a^4(\rho - \bar{\rho}), \quad \rho(\mathbf{x}) = a^{-3}\int f(\mathbf{x},\mathbf{p},\tau)\mathrm{d}^3 p$$

Lagrangian equation of motion:

$$\frac{\mathrm{d}^2\mathbf{x}}{\mathrm{d}\tau^2} = -\nabla_{\mathbf{x}}\phi.$$

The *N*-body approach

f(**x**,**p**,t) sampled with *N* particles of mass *m* in a volume of size *L*
Mass resolution of the simulation fixed by *N*

Each particle is a small smooth profile of size *ε* to soften small scale interactions : *ε* defines the **spatial resolution** of the simulation

The main difference between various *N*-body techniques is the way Poisson equation is resolved.

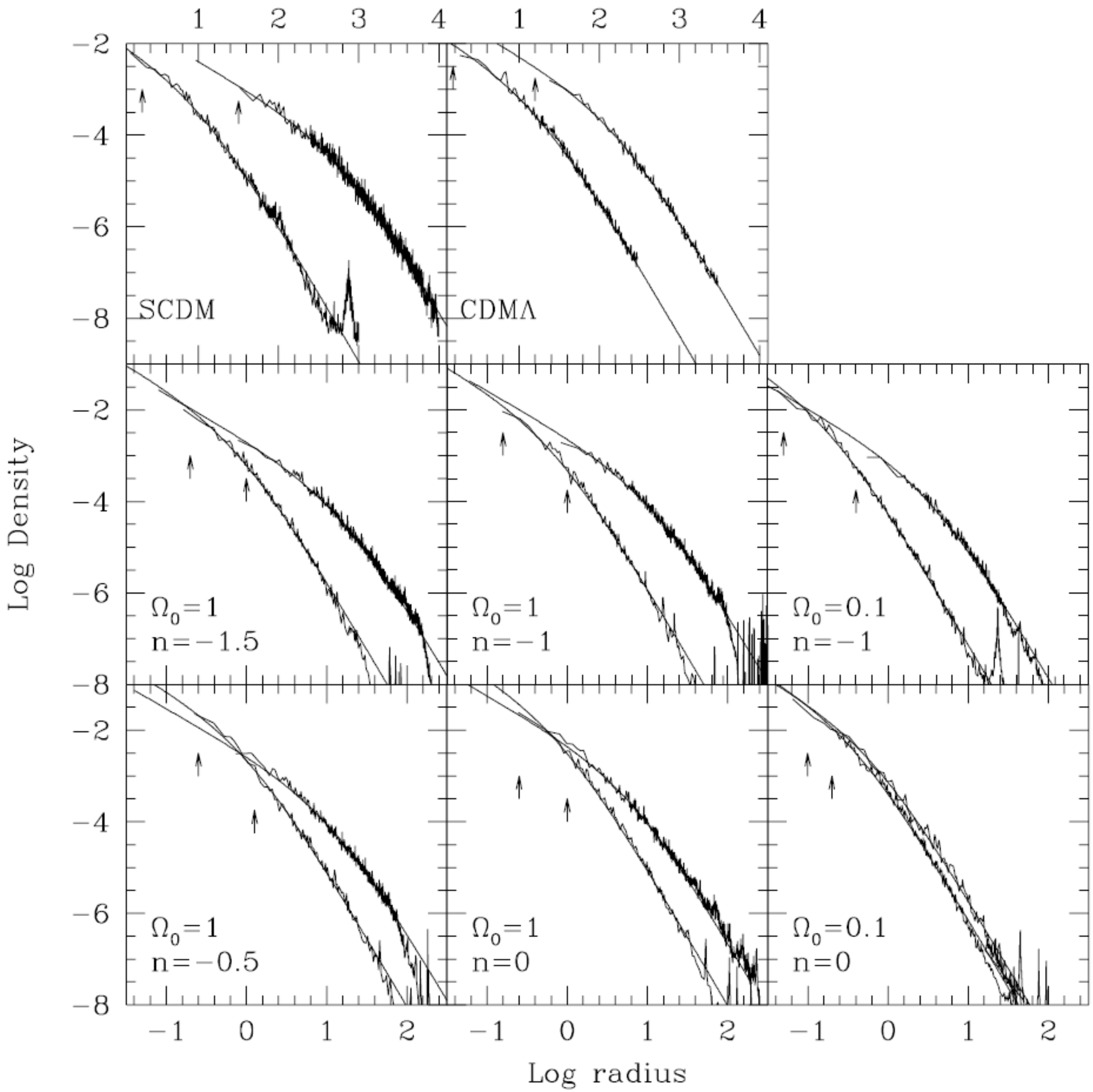
Dark matter halo simulations

The Aquarius simulation
Springel et al. 2008

The famous NFW profile

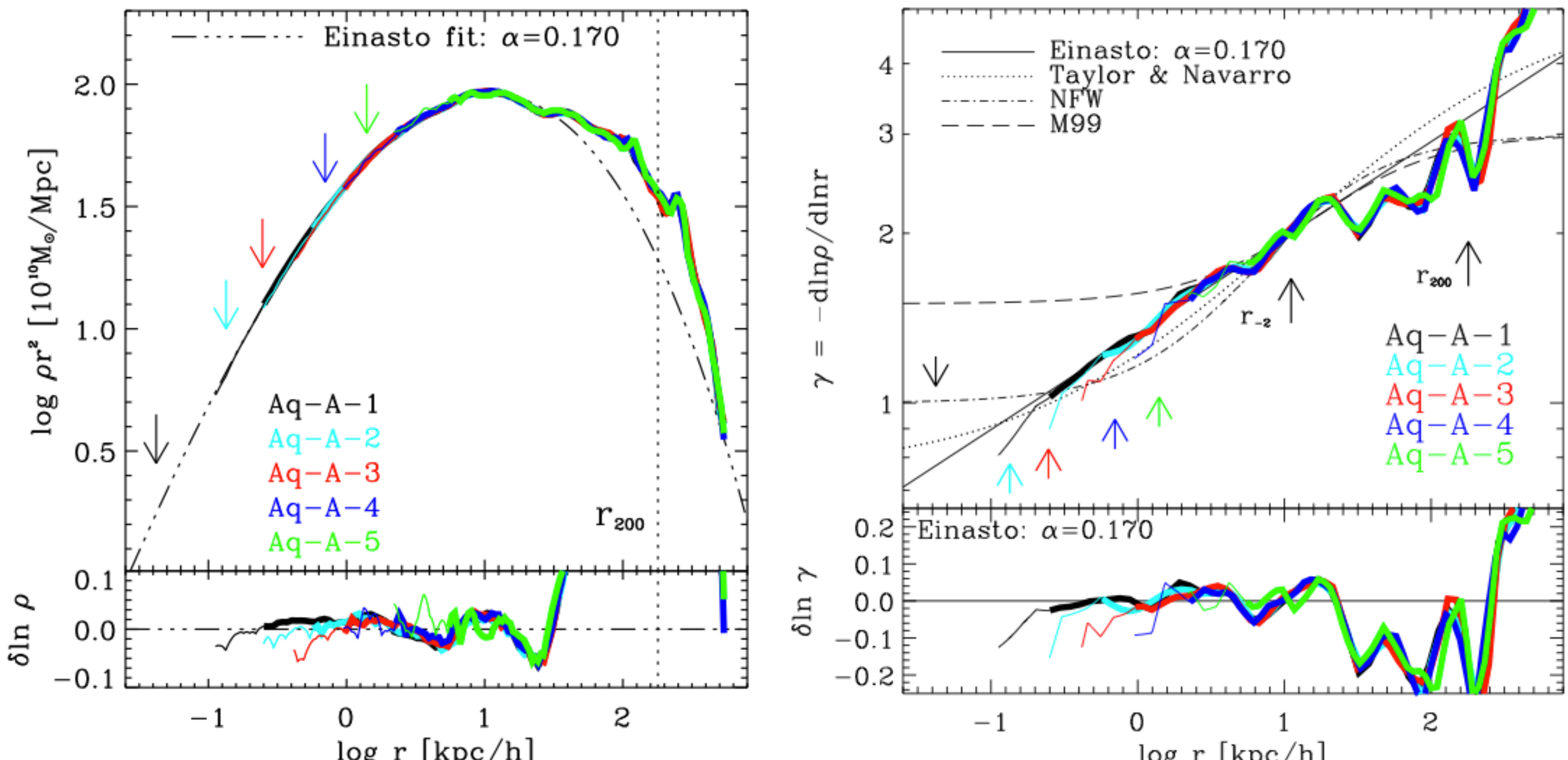
$$\rho(r) = \frac{\rho_s}{(r/r_s)(1 + r/r_s)^2}$$

Navarro, Frenk & White, 1996, ApJ 462, 563; 1997, ApJ 490, 493



A recent improvement: Einasto profile which works for sub-structures (Springel et al. 2008, MNRAS 391, 1685) and the main halo (Navarro et al. 2010, MNRAS 402, 21)

$$\ln(\rho(r)/\rho_{-2}) = (-2/\alpha)[(r/r_{-2})^{\alpha} - 1]$$

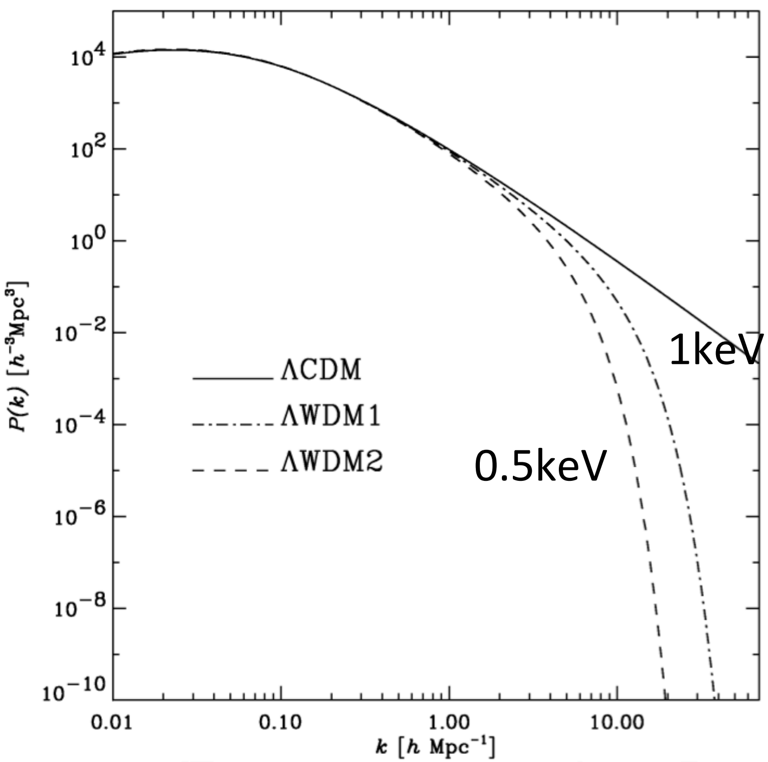


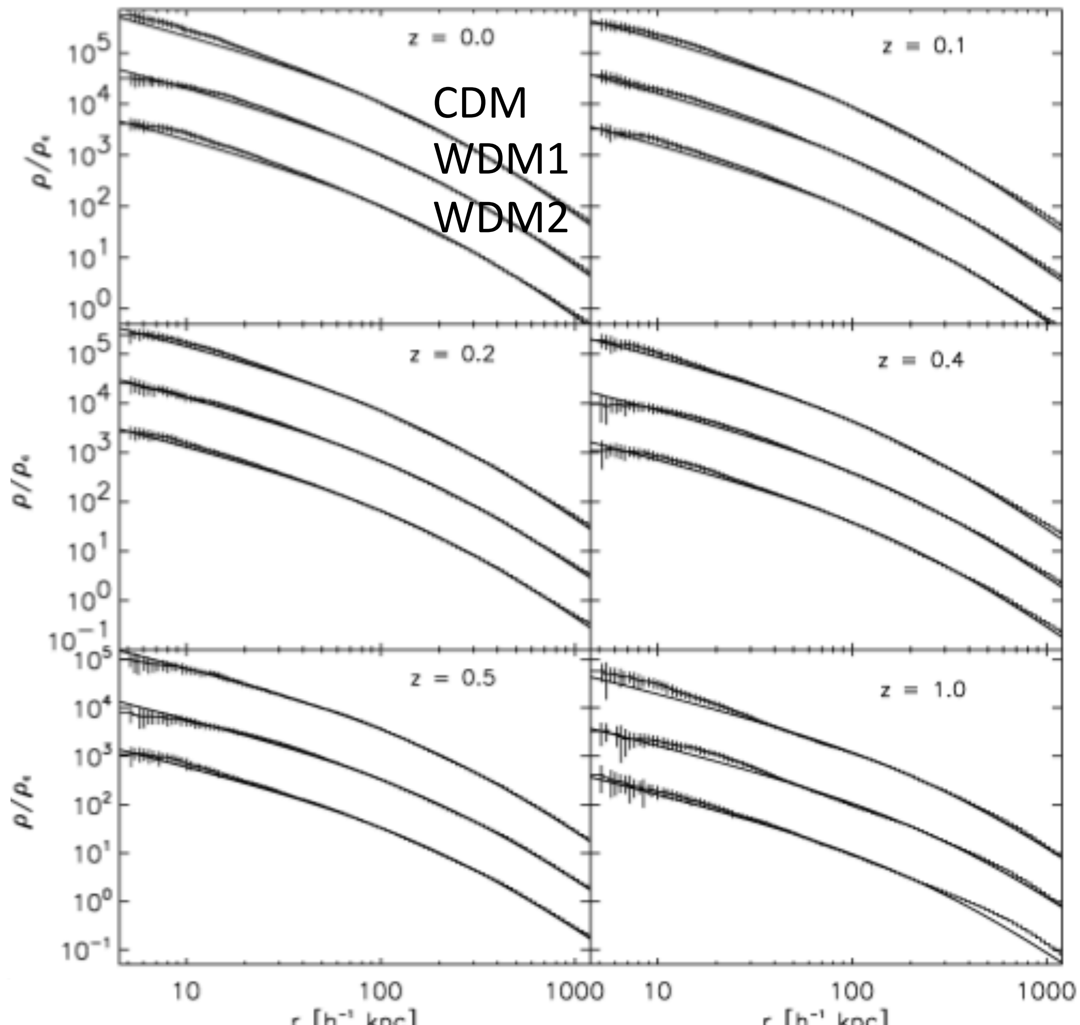
Navarro et al. 2010

NFW works also approximately for warm dark matter

Avila-Reese et al., 2001, ApJ 559, 516
Knebe et al., 2002, MNRAS 329, 813

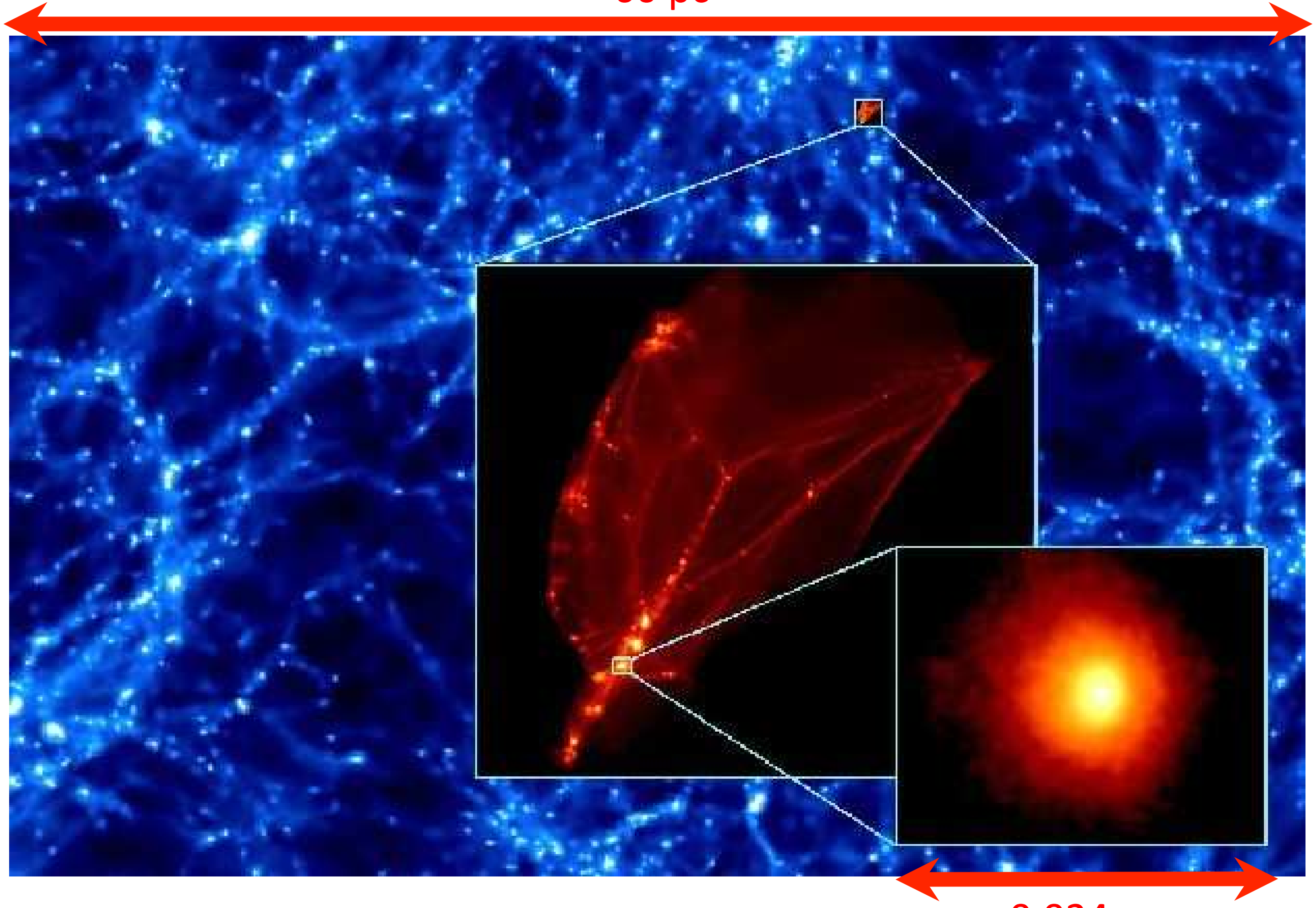
Knebe et al. 2002





The smallest dark matter halos : earth mass, solar system size

60 pc

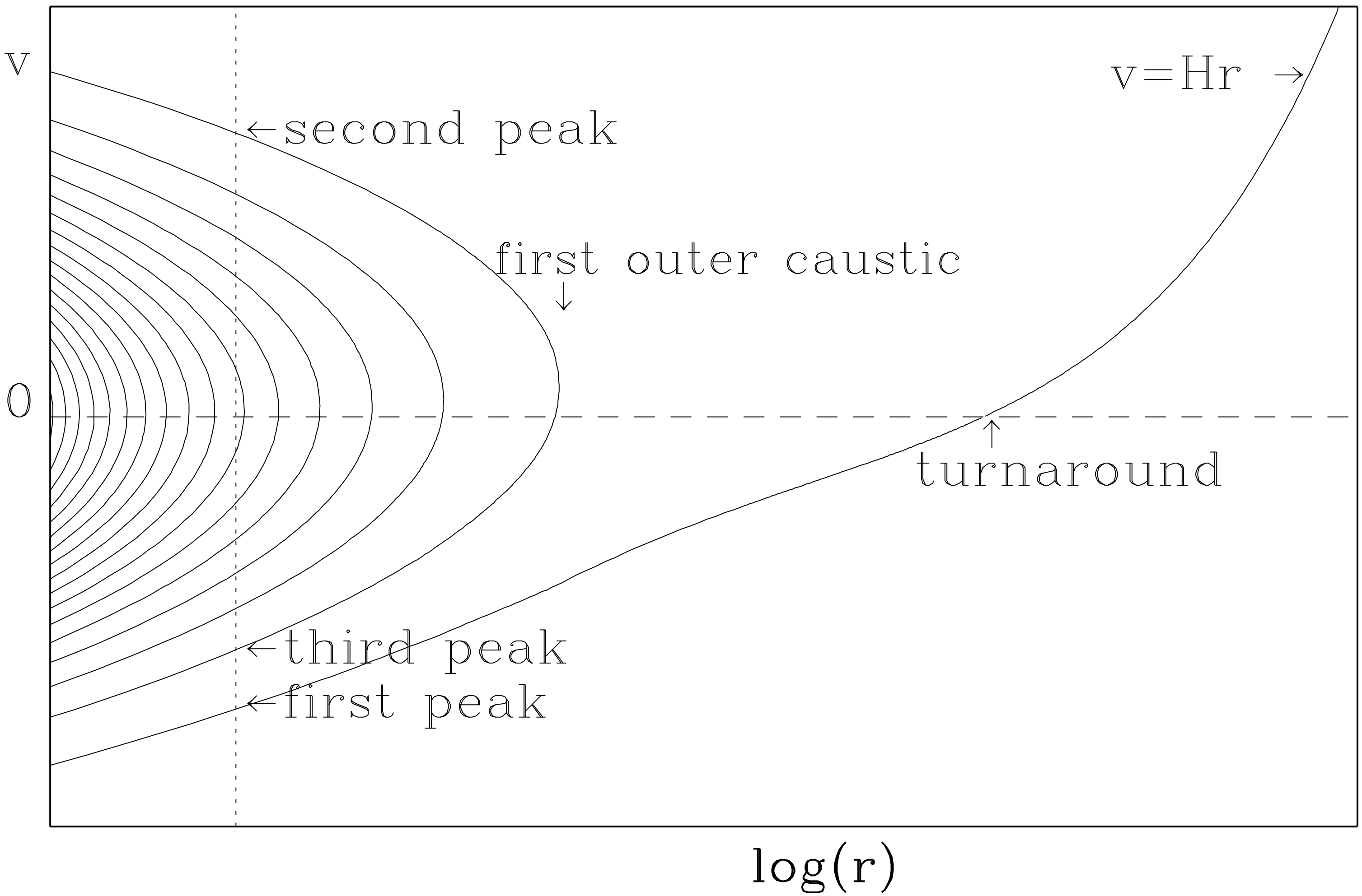


0.024 pc

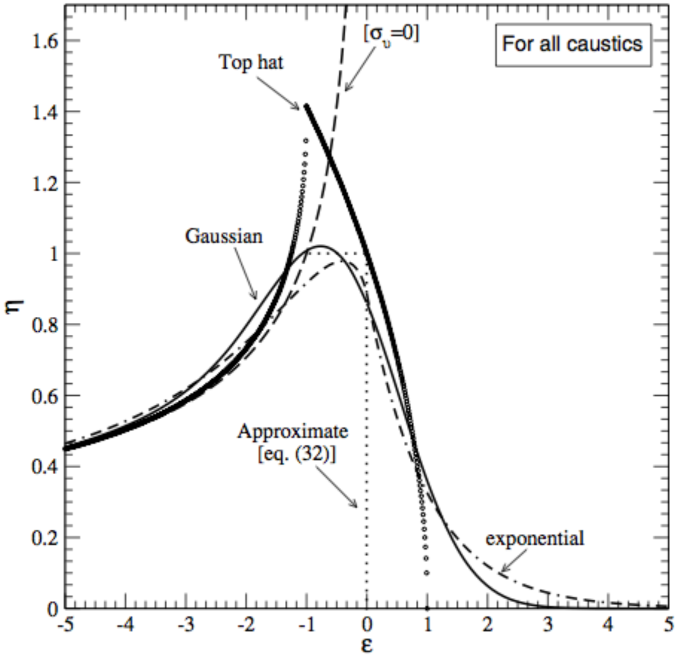
Earth mass halo with cuspy density profile

Diemand, Moore, Stadel, 2005, Nature 433, 389

The importance of Caustics: spherical collapse



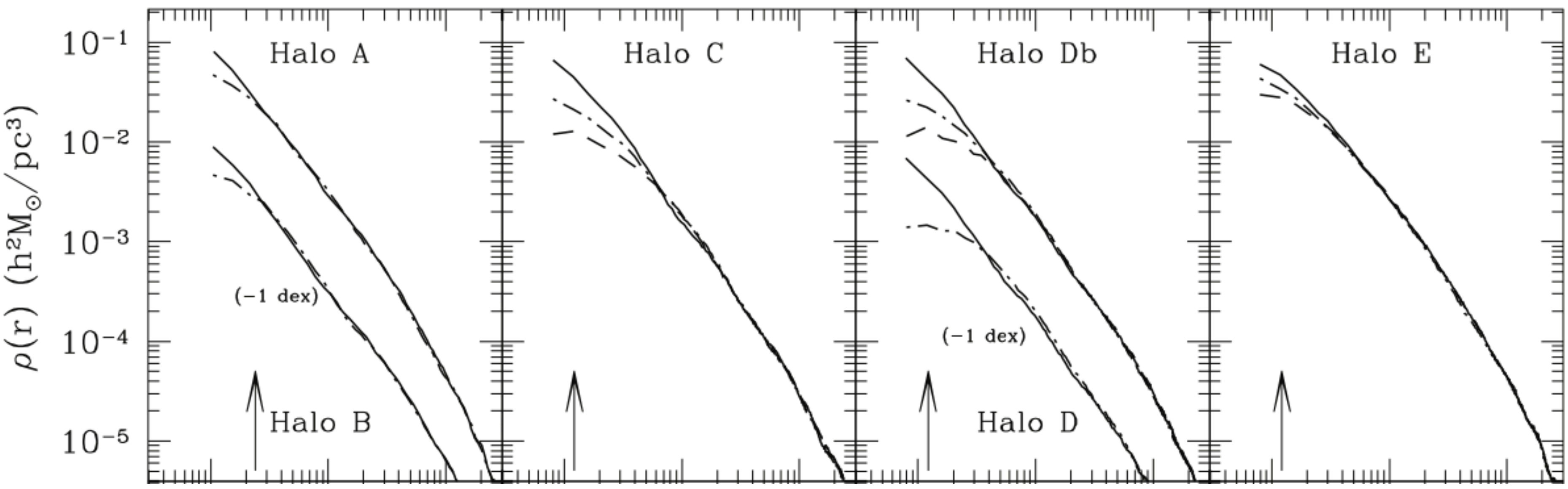
Sikivie, Tkachev & Wang 1997, PhRvD 56, 1863



Effect of velocity dispersion on a density caustic

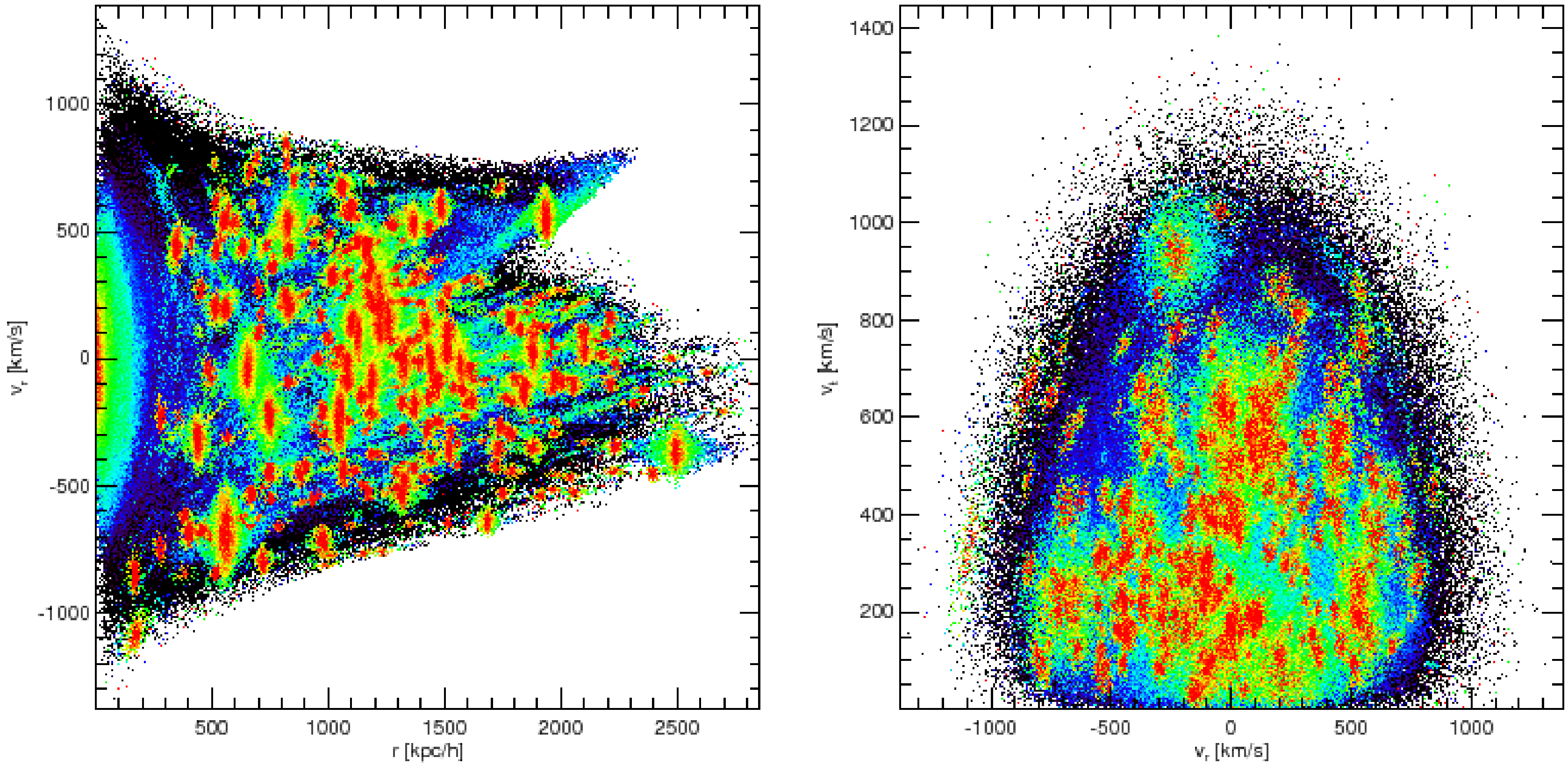
Mohayaee & Shandarin (2006, MNRAS 366, 1217)

On the structure of the centre of the halo in WDM Colín et al., 2008



The complexity of Phase-space

As a results caustics are expected to be much diluted



Phase-space density of a dark matter halo

Maciejewski et al. 2009, MNRAS 393, 703

The need for an accurate modeling of the local Universe for a proper description of the dark matter dynamics in the Local Group of galaxies

IAP : interesting works of
Lavaux 2010, MNRAS 406, 1007
Peirani 2010, MNRAS 407, 1487

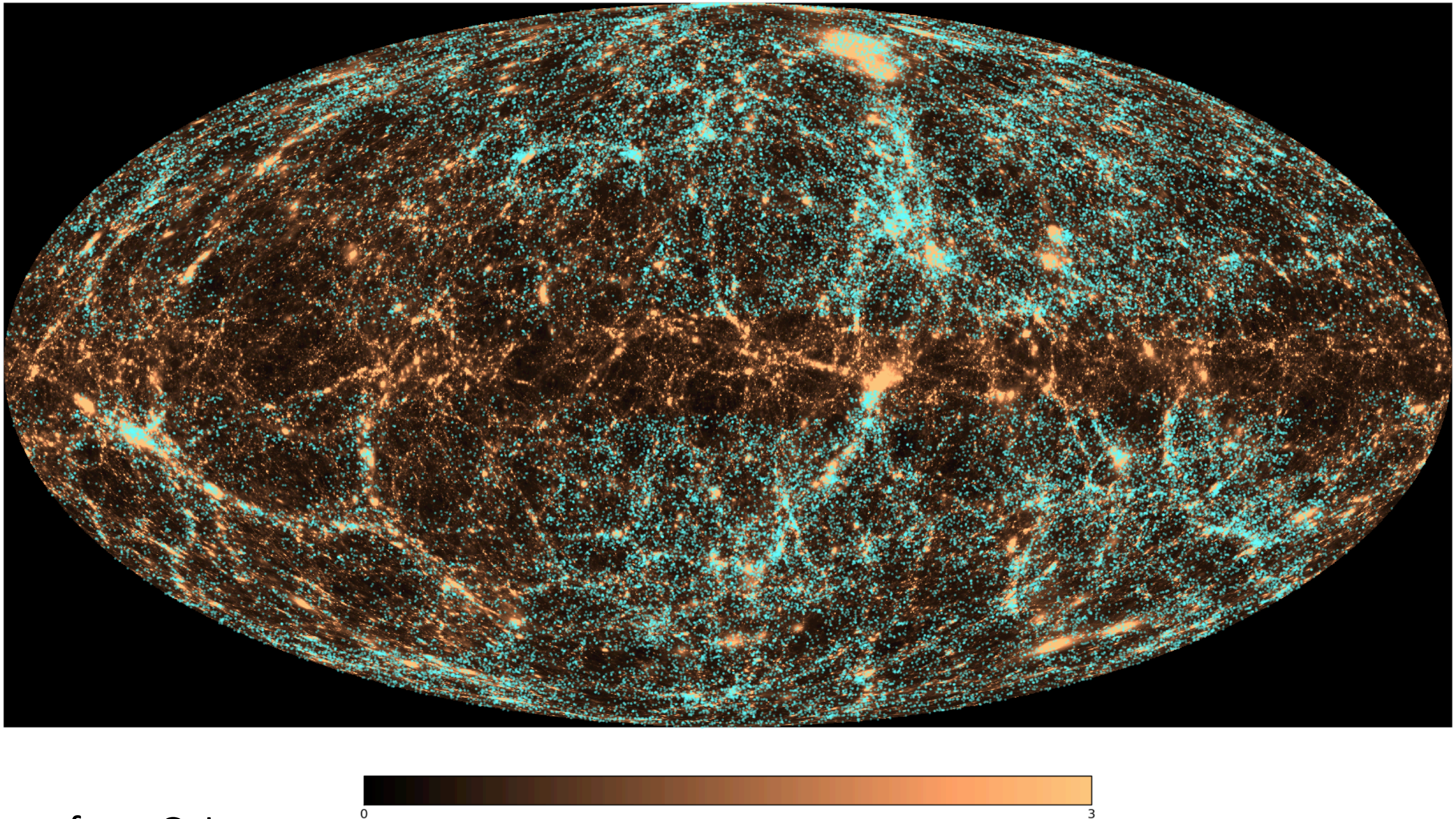
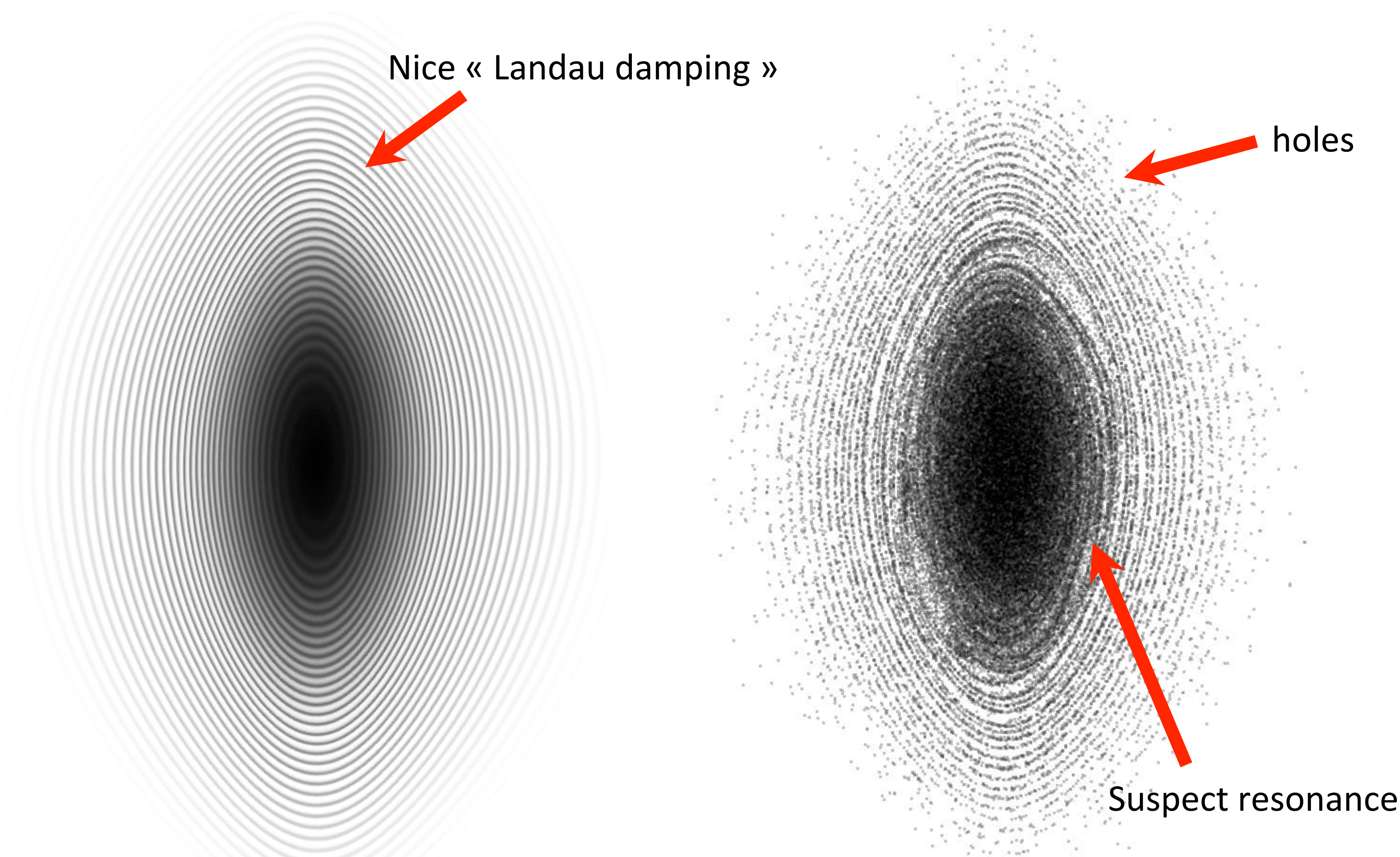


Image from G. Lavaux

NOTE : *N*-body simulations are noisy

Example: phase-space of a 1D simulation with Gaussian initial conditions



“Exact” solution with the Waterbag method
Colombi & Touma (2008, 2014)

N-body

The direct approach

○ **Aim:** to solve directly Vlasov-Poisson equations in phase-space, without *N*-body relaxation and artificial instabilities due to noise

○ **Possible now with Petaflopic supercomputers**

○ **Methods:**

- *The waterbag method*: Robert & Berk (1967), Jain (1971), Cuperman et al. (1971), Colombi & Touma (2008; 2014)
- *The semi-Lagrangian splitting scheme* of Cheng & Knorr (1976) and its numerous extensions
- *Hydrodynamics* in phase-space: for instance, standard upwind schemes, e.g. PPM, but in phase-space (Arber & Vann 2002)
- *Finite element* method: Zaki et al. (1988)
- *Lattice dynamics*: fully symplectic discrete algorithm of Syer & Tremaine (1995)
- *Hybrid:* the spherical solver of Rasio et al. (1989)
- *Metric transport :* the "cloudy" method of Alard & Colombi (2005)

Direct phase-space solvers: example: the cold case

The goal in the cold case: to follow directly the evolution of a 3D phase-space sheet in 6D phase-space

E.g. the preliminary investigations of Hahn, Abel & Kaehler, 2013, MNRAS 434, 1171

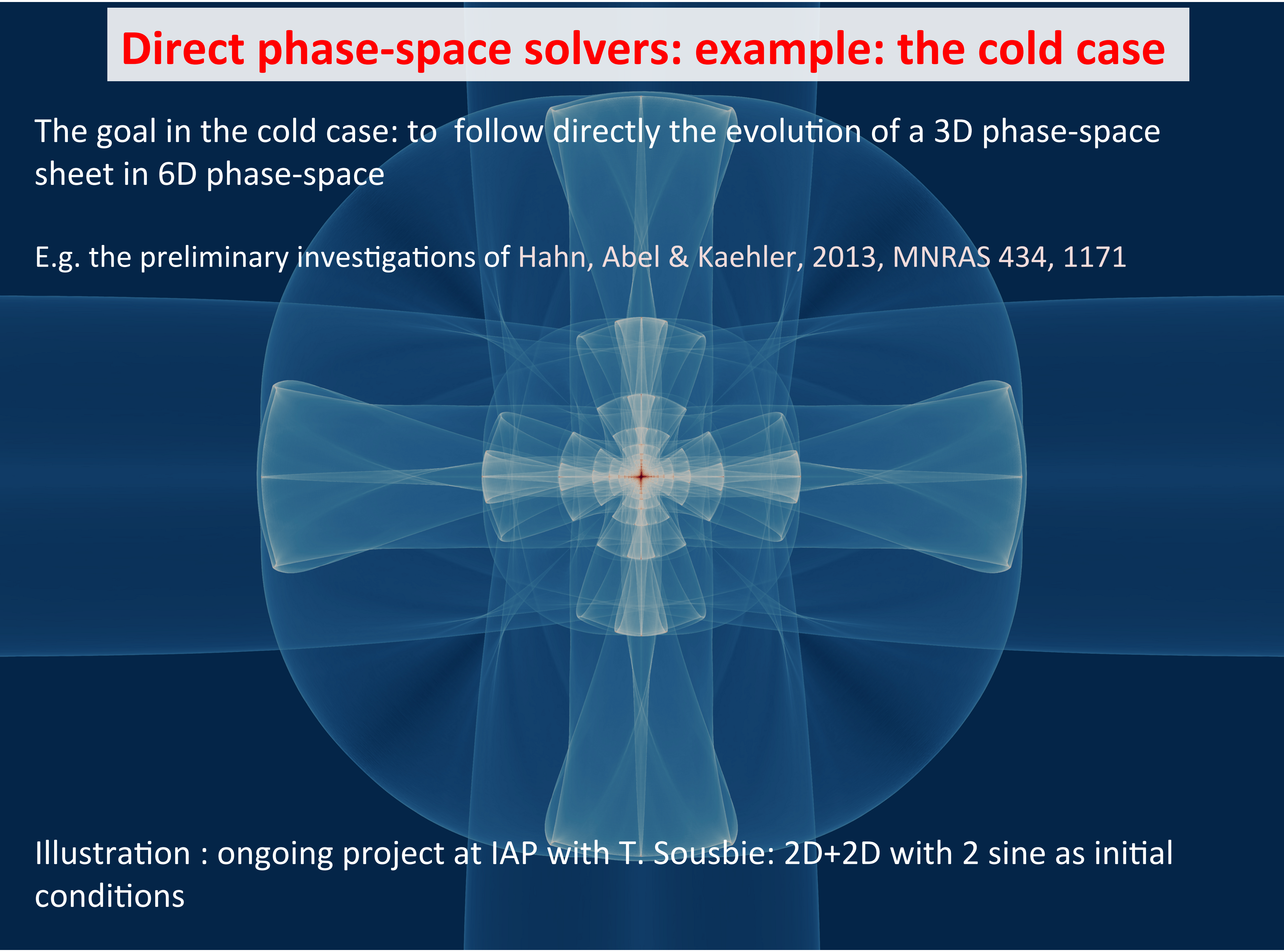


Illustration : ongoing project at IAP with T. Sousbie: 2D+2D with 2 sine as initial conditions

The real life : the very complex physics of baryons

The Mare Nostrum simulation (2007) : a product of the HORIZON project

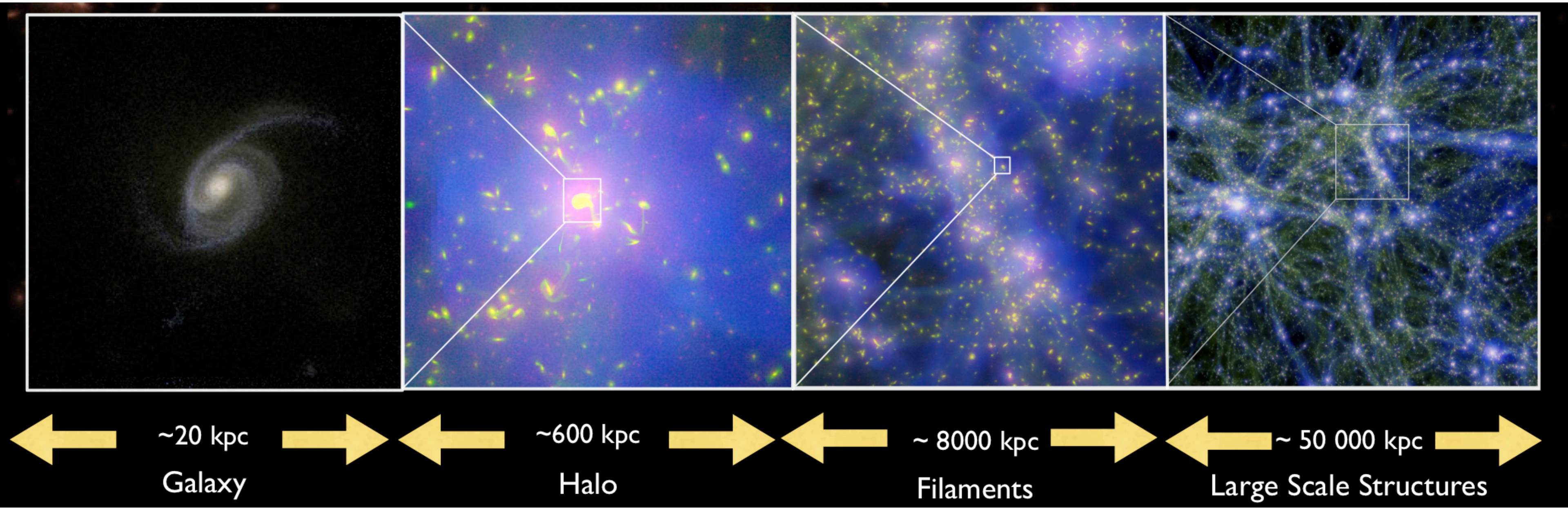
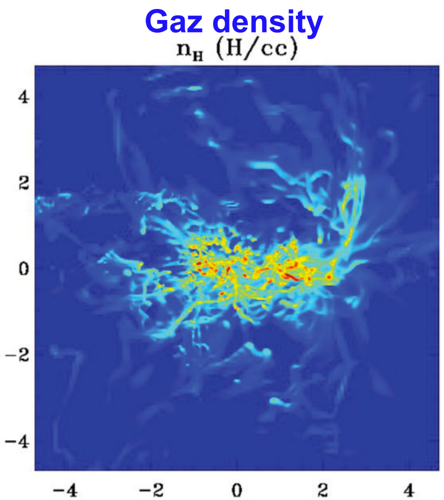
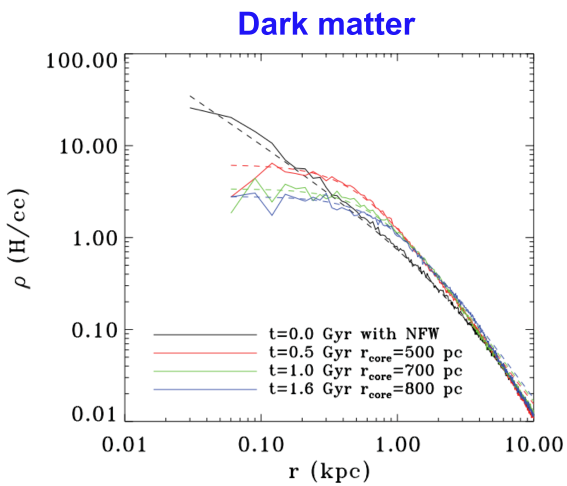


Image from C. Pichon

Movie : the HORIZON-AGN project : Dubois, Pichon, Peirani et al.

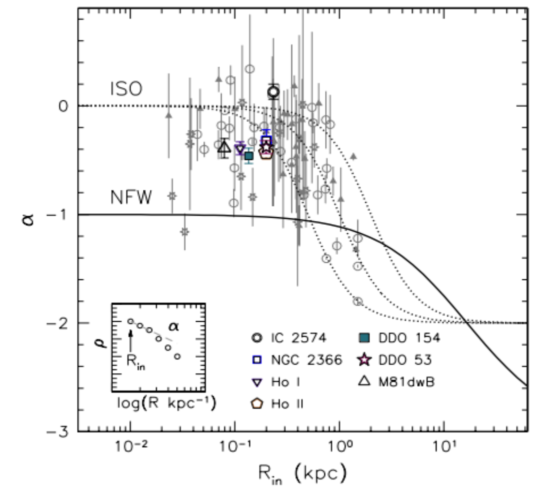
Transformation of cusps in cores by feedback processes



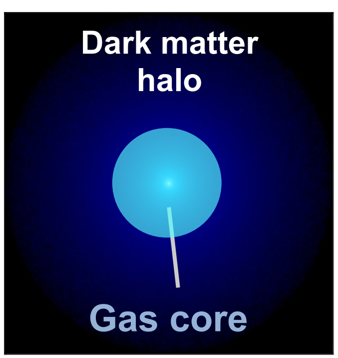


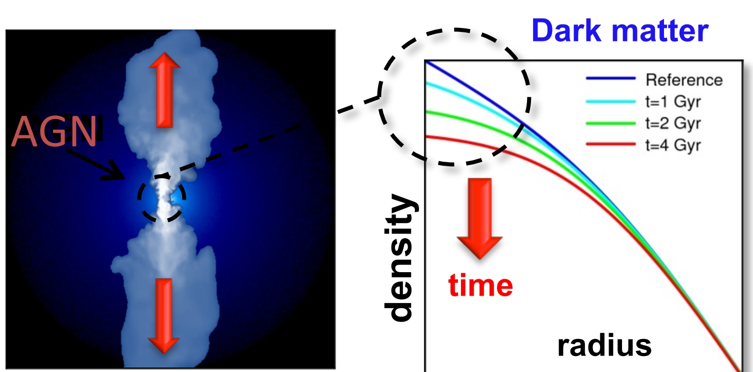
Teyssier, Pontzen, Dubois, Read, 2013

Observed dwarf galaxies



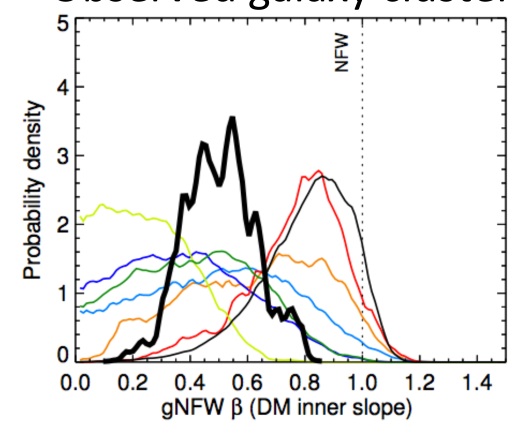
Pontzen et al, 2014





Peirani, Kay, Silk, 2008

Observed galaxy clusters



Pontzen et al, 2014

Slide provided by Y. Dubois