Structure formation
&
Clusters for Cosmology

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Transfer function I

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Notation

$$\delta(k,z) = D(z)\delta_0(k) \text{ with } D(0) = 1.$$

and:

$$P(k) = |\delta|_k^2 = D(z)^2 P_0(k)$$

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The power spectrum keeps record of the tranfer function.

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$$\delta \propto t \propto a(t)^2$$

as long as $\lambda \geq ct$ while $\lambda \leq ct$ are frozen.

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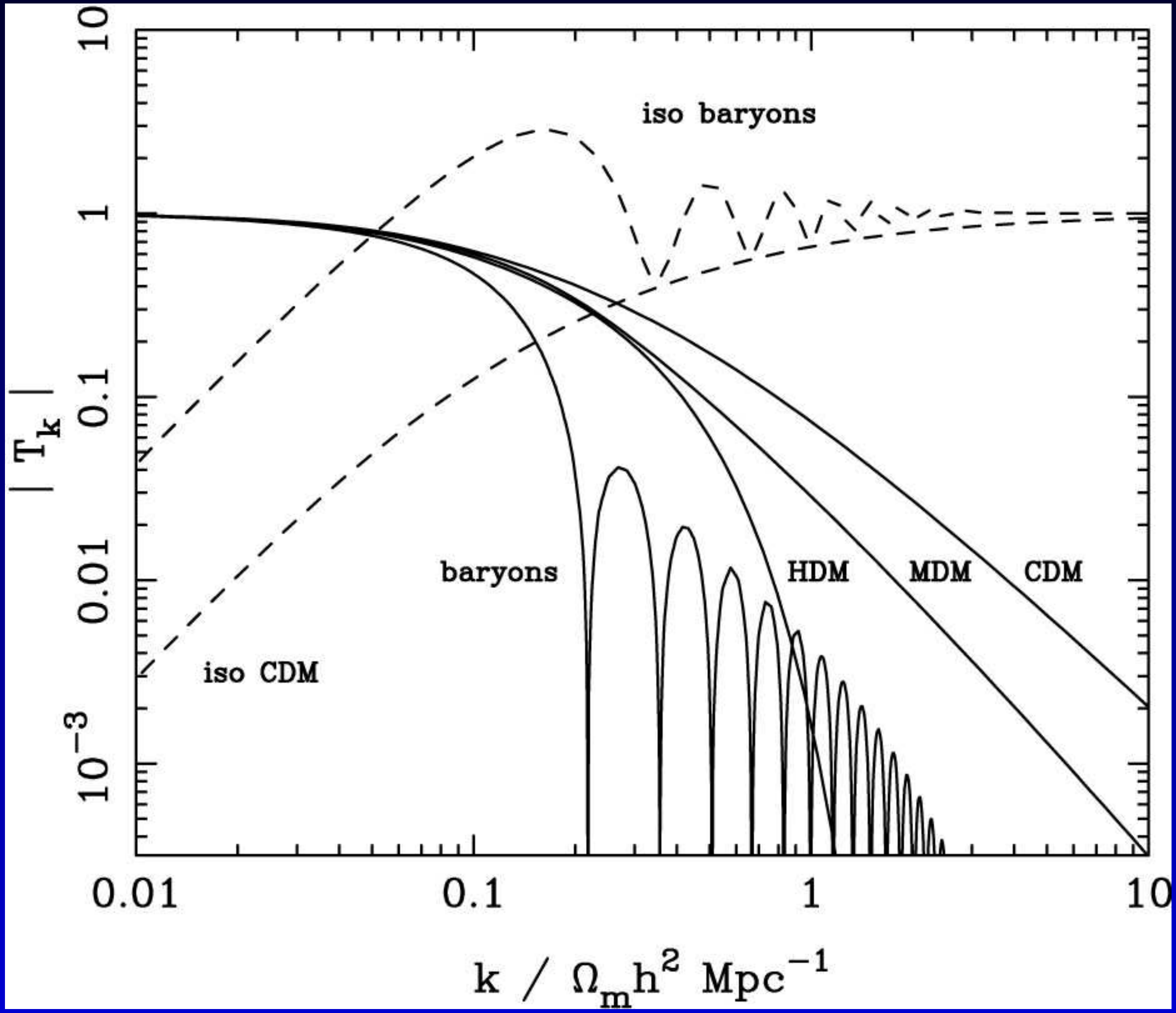
Low mass particles free-steaming.

Transfer function: IV



Comoving sound horizon size I

RW metric:

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For a sound wave:

$$\chi_S(a) = \int_0^a a^{-1}(t) c_S dt = \int_0^a \frac{c}{\sqrt{3}} \frac{da}{a(t)^2 H(a)\sqrt{1 + \frac{3}{4}\frac{\Omega_b}{\Omega_r}}}$$

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- z_* imprint in CMB (peak)
- z_d imprint in $P(k)$ (BAO)

Non linear regime

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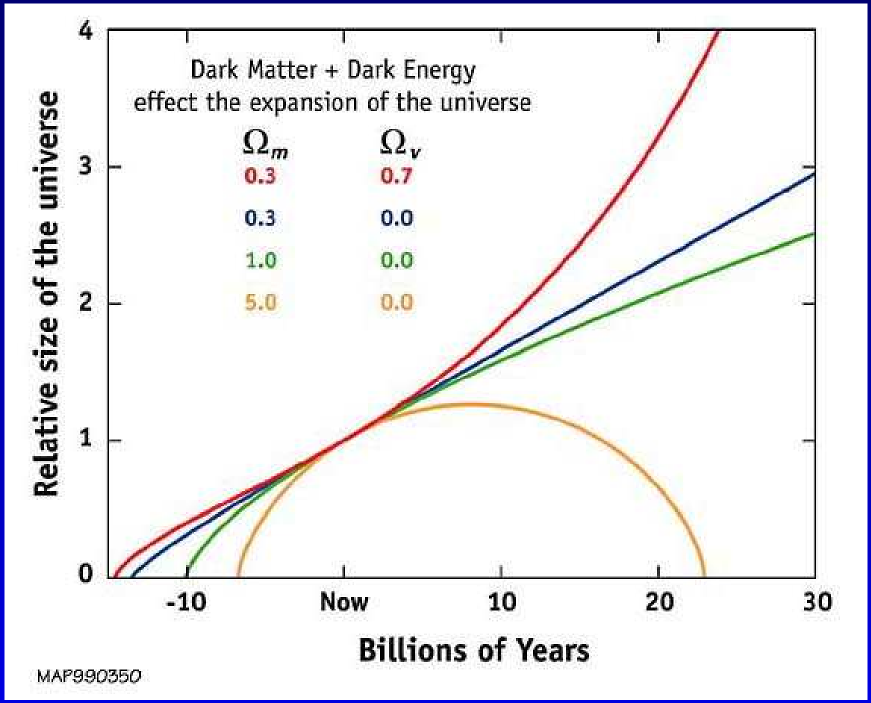
Spherical model (Lemaître, 1933)
Newtonian problem. Solution already seen:



Spherical Perturbation I

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$$
\begin{aligned}
\tilde{H}_0\, t &= \frac{\tilde{\Omega}_0}{2(\tilde{\Omega}_0 - 1)^{3/2}}(\phi - \sin(\phi)) \\
R(t) &= \frac{\tilde{\Omega}_0 \tilde{R}_0}{2(\tilde{\Omega}_0 - 1)}(1 - \cos(\phi))
\end{aligned}
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Density at maximum:

$$\tilde{\rho} = \tilde{\rho}_0 \left(\frac{\tilde{R}_0}{\tilde{R}}\right)^3$$

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i.e.

$$\tilde{\rho}_m = \frac{3\pi^2}{32\pi G t_m^2}$$

with : $1 + \Delta_m = \frac{\tilde{\rho}_m}{\rho}$ and $\rho = \frac{1}{6\pi G t^2}$ (EdS)

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$$\Delta_m = \frac{9}{16}\pi^2 - 1. \simeq 4.55$$

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let's estimate the linear expected amplitude at viriliza-tion.

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$$\tilde{\rho} = \frac{8\rho_m}{(1-\cos\psi)^3} = \frac{64\rho_m}{\psi^6(1-\psi^2/4)}$$

$$t = \frac{t_m}{\pi}(\psi - \sin\psi) = \frac{t_m}{\pi}\frac{\psi^3}{6}[1 - \frac{\psi^2}{20}]$$

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so:

$$\psi^6 = \left(\frac{6\pi t}{t_m}\right)^2 [1 + \frac{\psi^2}{10}]$$

Virialization IV

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and

$$\tilde{\rho} = (1 + \frac{\psi^2}{4} - \frac{\psi^2}{10}) \frac{64 \rho_m t_m^2}{(6\pi)^2 t^2}$$

$$= (1 + \frac{\psi^2}{4} - \frac{\psi^2}{10}) \frac{64}{36\pi^2} \frac{3\pi^2}{32\pi G t^2} = \rho(1 + \frac{3\psi^2}{20})$$

Virialization IV

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so with : $\tilde{\rho} = \rho(1+\delta)$

$$\delta = \frac{3}{20}\psi^2 = \frac{3}{20}\left(\frac{6\pi t}{t_m}\right)^{2/3} = \frac{3(6\pi)^{2/3}}{20}\frac{1+z_m}{1+z}$$

Conclusion

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$$\delta_m = \frac{3(6\pi)^{2/3}}{20}(1+z_m) = 1.06(1+z_m)\mathrm{when}\Delta_m \simeq 4.5$$

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For $z < z_v$, $\Delta = 177 \left(\frac{1+z_v}{1+z}\right)^3$

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so that M and z are the only two numbers to characterize a cluster (Δ is set by the cosmology...or by the cosmologist!)

Velocity dispersion II

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$$\rho(r) = \frac{\sigma^2}{2\pi G r}$$

with $< v^2 >= \sigma_x^2 + \sigma_y^2 + \sigma_z^2 = 3\sigma^2$

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$$\sigma = \sqrt{\frac{GM}{r}}$$

Velocity dispersion III

Numerically:

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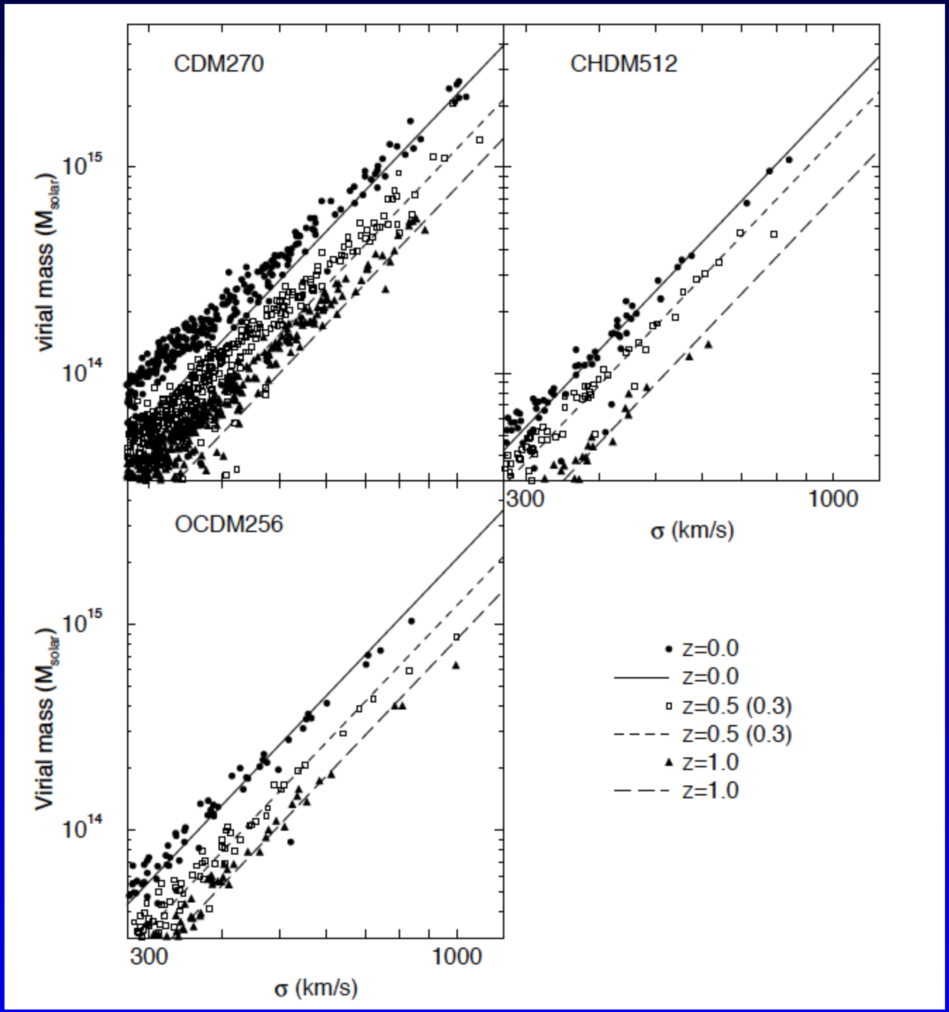
Scaling laws (dependene on mass and redshift).

Velocity dispersion IV

Numerically: good agreement with numerical simulations (Bryan and Norman, 1998):



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so that:

$$T_x = A_{TM} M_{15}^{2/3}(1+z)(\Omega_m \Delta/178)^{1/3}$$

The mass function

Inspired from Press and Schechter (1974)
The density field $\rho(x)$ has to be smoothed:

$$\tilde{\delta}(x) = \int \delta(x+u) W_R(u) du$$

and

$$\overline{\tilde{\delta}^2(x)} = \sigma^2(R)$$

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For a top hat window (!):

$$M(R) = \frac{4\pi}{3} R^3 \overline{\rho}$$

The mass function

dV will be included in a NL object with mass greater than M if included in a fluctuation of radius $> R$ and witch is satisfying the non linear criteria ($\delta > \delta_{NL}$).

$$\int_M^{+\infty} mn(m)dm = \overline{\rho} \int \mathcal{F}_\delta(\delta)s(\delta)d\delta \sim \overline{\rho} \int_{\delta_{NL}}^{+\infty} \mathcal{F}_\delta(\delta)d\delta$$

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for a sharp threshold:

$$\int_M^{+\infty} mn(m)dm = \overline{\rho} \int_{\nu_{NL}}^{+\infty} \mathcal{F}(\nu)d\nu$$

The mass function

Following the spherical model:

$$\nu_{NL} = \frac{\delta_{NL}}{\sigma(M)}$$

Just derive against M:

$$N(M) = -\frac{\rho}{M^2\sigma(M)}\delta_{NL}\frac{\ln\sigma}{\ln M}\mathcal{F}(\nu_{NL})$$

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and test it against numerical simulations...

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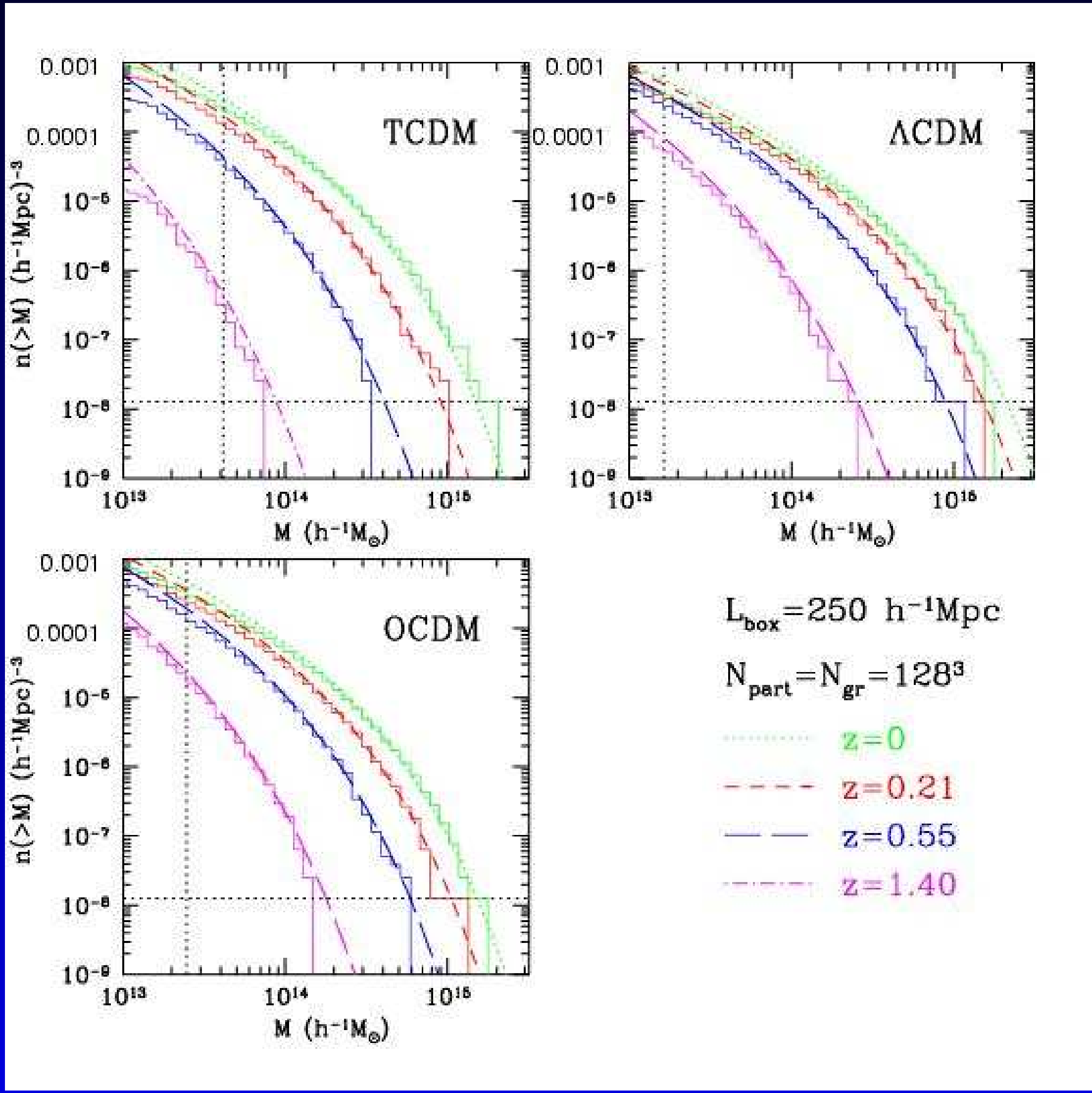

It actually works! (Borgani et al., 2000)

More accurate N(m)

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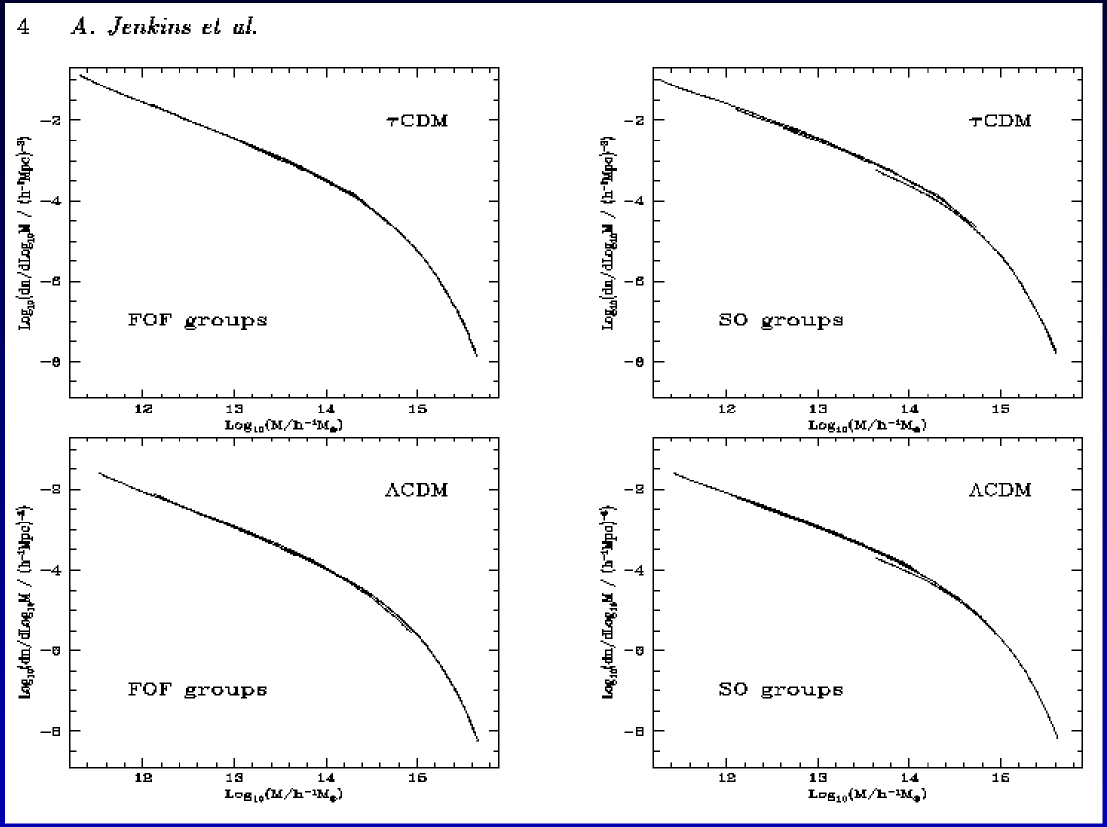

Jenkins et al. (2001)

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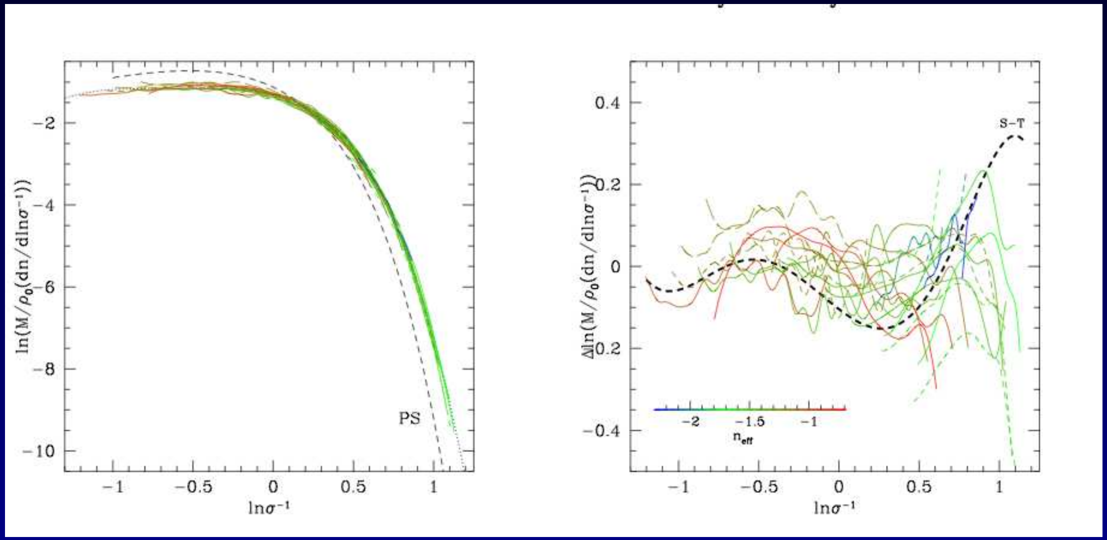

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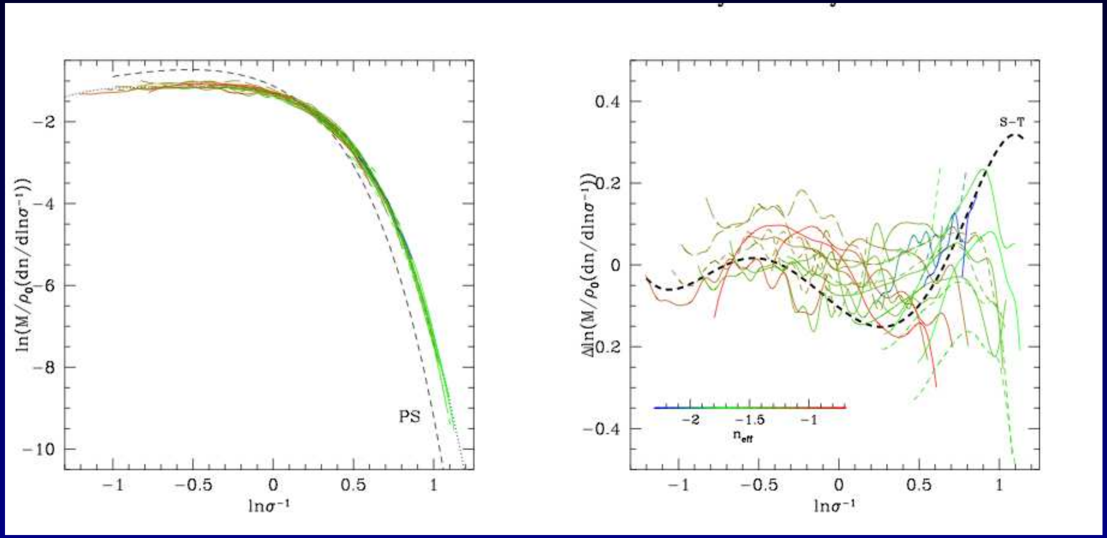

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Universal mass function.

e-PS formula

ST (Steth & Tormen, 1999) expression for $\mathcal{F}$:

$$\mathcal{F}(\nu) = \sqrt{\frac{2A}{\pi}} C \exp(-0.5A\nu^2)(1. + (1./(A\nu)^2)^Q)$$

with $A = 0.707$ $C = 0.3222$ $Q = 0.3$.

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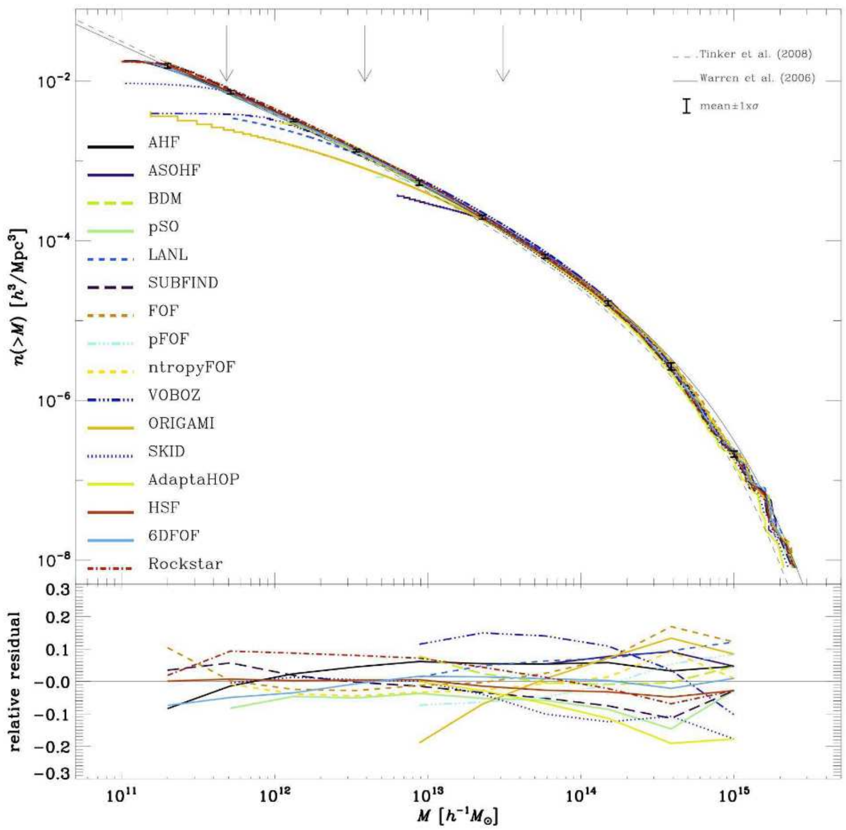

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- Allows to investigate structure formation. History of individual structure is missing: merging tree $\rightarrow$ semi-analytical method “SAM” in order to model galaxy formation : assembly/evolution.
- Warning: data come through “light” which is coming from baryons and this was almost not discussed in these lectures...