

Structure formation & Clusters for Cosmology

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Primordial Universe: $\frac{\delta\rho}{\rho} \ll 1$

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mais $\delta h \ll 1$ so Newton dynamics is enough.

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CMB:

$$\frac{\delta T}{T} \sim \delta h \sim \frac{\sigma^2}{c^2} \sim 10^{-5}$$

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but by CMB!

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Minimal assumption: gravity should be active.

Perturbations I

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peculiar velocity

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Fluid equations:

$$\begin{aligned}\frac{d\rho}{dt} + \nabla(\rho u) &= 0 \\ \rho \frac{du}{dt} + \rho(u \cdot \nabla)u &= -\nabla p - \rho \nabla \Phi \\ \nabla^2 \Phi &= 4\pi G \rho\end{aligned}$$

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New coordinates: $x, v, \rho = (1 + \delta)\bar{\rho},$
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$$\frac{\partial \delta}{\partial t} + \frac{1}{a}\nabla((1 + \delta)v) = 0$$

$$\frac{\partial v}{\partial t} + \frac{1}{a}(v \cdot \nabla)v = -\frac{1}{\bar{\rho}a}\nabla p - \nabla\phi$$

$$\nabla^2\phi = 4\pi G\bar{\rho}a^2\delta$$

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No term coming directly from uniform background
(but through $a(t)$).

Linearization

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Neglecting pressure term:

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No spatial derivates anymore.

Modes

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The solution can be written:

$\delta(x, t) = D(t)f(x)$ with:

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The general solution can be written:

$$\delta(x, t) = D(t)\delta_0(x)$$

with $D(t_0) = 1$.

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General case (arbitrary Ω_m but $\Lambda = 0$) has analytical solution: one growing mode and one decaying mode.

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Pressure not coupled to matter.

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that is :

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Growing mode is frozen.

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But let's us use the newtonion regime approximation :

- $\rho + 3p/c^2$ is the source of gravity.
- Pressure comes in the relativistic equations of motion.

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Equation for perturbations becomes:

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\Rightarrow linear fluctuations are damped.

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becomes ($\delta p = c_S^2 \delta \rho$):

$$\frac{\partial^2 \delta}{\partial t^2} + 2\frac{\dot{a}}{a}\frac{\partial \delta}{\partial t} = \left(\frac{c_S}{a}\right)^2 \nabla^2 \delta + 4\pi G \bar{\rho} \delta$$

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$$\frac{\partial^2 \delta_k}{\partial t^2} + 2\frac{\dot{a}}{a}\frac{\partial \delta_k}{\partial t} = [4\pi G \bar{\rho} - \left(k \frac{c_S}{a}\right)^2] \delta_k$$

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so:

- $\lambda \gg \lambda_J$ growing mode, no p effect.
- $\lambda \ll \lambda_J$ damped oscillations.

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with $z_{eq} \sim 23000\Omega h^2$ (3 light neutrinos)

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$p \simeq p_r = \rho_r c^2 / 3$ so:

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 $p \simeq p_r = \rho_r c^2 / 3$ so:

$$c_S = \left(\frac{\partial p}{\partial \rho} \right)^{1/2} = \frac{c}{\sqrt{3}} \left(\frac{1}{1 + \frac{\partial \rho_b}{\partial \rho_r}} \right)^{1/2}$$

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Three regimes:

Jeans Mass evolution II

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- $z \gg z_{eq}$ and $c_S \sim \frac{c}{\sqrt{3}}$
- $z_{rec} \leq z \leq z_{eq}$ one has $c_S \sim \frac{c}{\sqrt{3}} \left(\frac{1+z_{eq}}{1+z} \right)^{1/2}$

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Three regimes:

- $z \gg z_{eq}$ and $c_S \sim \frac{c}{\sqrt{3}}$
- $z_{rec} \leq z \leq z_{eq}$ one has $c_S \sim \frac{c}{\sqrt{3}} \left(\frac{1+z_{eq}}{1+z} \right)^{1/2}$
- $z \leq z_{rec}$ No coupling anymore: $c_S \sim \left(\frac{3kT_m}{m} \right)^{1/2} \propto T_r$

Jeans Mass evolution III

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Numerically:

For the Jeans' mass: $M_J \simeq \frac{4\pi}{3}\rho_m\lambda_J^3$

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- $z \gg z_{eq}$
 $M_J \sim 310^{15}(\Omega h^2)^{-2}/(1+z/1+z_{eq})^3 M_\odot$

Jeans Mass evolution III

Numerically:

- $z \gg z_{eq}$ $c_S \sim \frac{c}{\sqrt{3}}$
- $z_{rec} \leq z \leq z_{eq}$ $c_S \simeq 210^8(1+z/1+z_{eq})^{1/2}$ m/s

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Numerically:

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- $z_{rec} \leq z \leq z_{eq}$ $c_S \simeq 210^8(1 + z/1 + z_{eq})^{1/2}$ m/s
- $z \leq z_{rec}$ $c_S \simeq 510^5(1 + z/1 + z_{eq})^{1/2}$ m/s

For the Jeans' mass: $M_J \simeq \frac{4\pi}{3}\rho_m\lambda_J^3$

- $z \gg z_{eq}$
 $M_J \sim 310^{15}(\Omega h^2)^{-2}/(1 + z/1 + z_{eq})^3 M_\odot$
- $z_{rec} \leq z \leq z_{eq}$ $M_J \sim 310^{15}(\Omega h^2)^{-2} M_\odot$
- $z \leq z_{rec}$ $M_J \sim 510^4(\Omega h^2)^{-1/2} M_\odot$

Jeans Mass evolution IV

