Resummation predictions for BSM particle production at the LHC

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Parts in collaboration with I. Schienbein, T. Jezo and F. Lyonnet.

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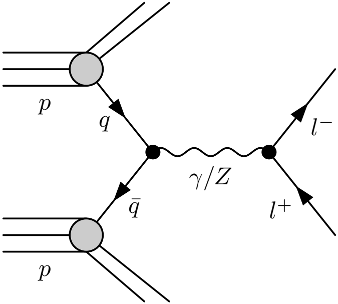
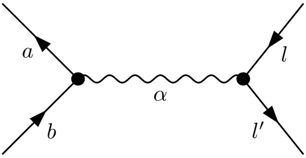
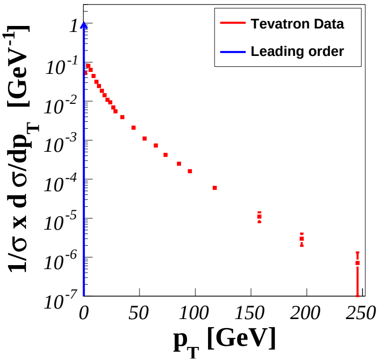
Why QCD resummation?

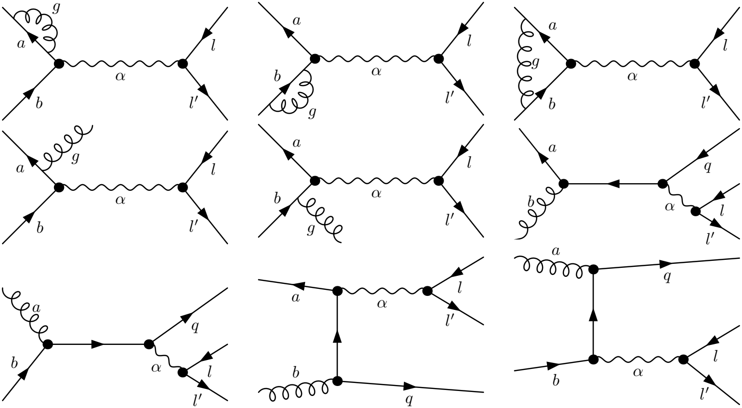


$$pp \to \gamma^*/Z + X \to l^- l^+ + X$$



$$p_{\mathrm{T}} := \sqrt{p_x^2 + p_y^2} = 0.$$





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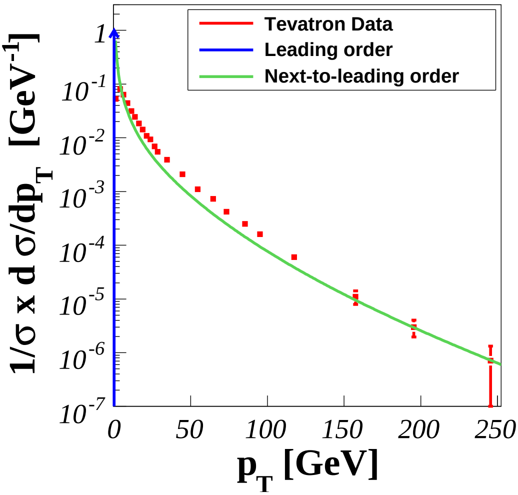
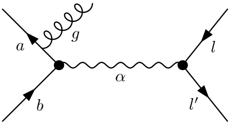
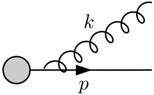


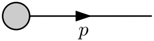
Tevatron Data
Leading order
Next-to-leading order
1/σ x dσ/dp_T [GeV^-1]
1
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
10^-7
0
50
100
150
200
250
p_T [GeV]

The origin of large logarithms





$$\propto g_s \frac{p_\mu}{p^0 k^0 (1-\cos\theta)}$$



Divergences

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\omega} = 1 + a_\mathrm{S}(L^2 + L + 1) + a_\mathrm{S}^2(L^4 + L^3 + L^2 + L + 1) + \ldots$$

where L represents a potentially large logarithmic term:

$$\omega = p_\mathrm{T}^2 \rightsquigarrow L = \frac{1}{p_\mathrm{T}^2} \log \frac{M^2}{p_\mathrm{T}^2}$$

$$\omega = M^2 \rightsquigarrow L = \left(\frac{\log(1-z)}{1-z}\right)_+$$

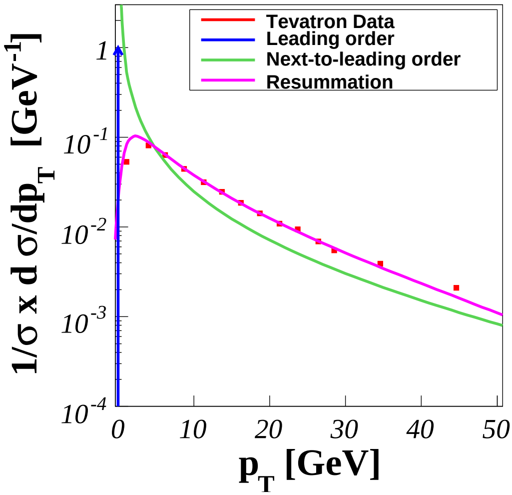
with

$$z := \frac{M^2}{s}, \qquad p_\mathrm{T} := \sqrt{p_x^2 + p_y^2}.$$

LO	1				
NLO	$\alpha_{\mathrm{S}} L^2$	$\alpha_{\mathrm{S}} L$	α_{S}		
NNLO	$\alpha_{\mathrm{S}}^2 L^4$	$\alpha_{\mathrm{S}}^2 L^3$	$\alpha_{\mathrm{S}}^2 L^2$	$\alpha_{\mathrm{S}}^2 L$	α_{S}^2
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

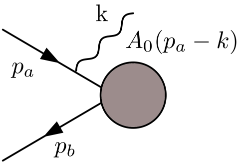
$$\frac{\mathrm{d}\sigma_{\mathrm{res}}}{\mathrm{d}\omega} = C(a_{\mathrm{S}}) \exp\left(L g_1(a_{\mathrm{S}} L) + g_2(a_{\mathrm{S}} L) + a_{\mathrm{S}} g_3(a_{\mathrm{S}} L) + \ldots\right) + R(a_{\mathrm{S}})$$

- $C(a_{\mathrm{S}}), R(a_{\mathrm{S}})$ are well-behaved and computable perturbatively.
- The different g_i resum up to a given order.


Tevatron Data
Leading order
Next-to-leading order
Resummation
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
0
10
20
30
40
50
$1/\sigma \times d\sigma/dp_T$ [GeV^{-1}]
p_T [GeV]

Warmup: QED resummation

QED resummation



$$\mathcal{M} = \bar{v}(p_b) A_0(p_a - k) \frac{\mathrm{i}(\not{p}_a - \not{k} + m)}{(p_a - k)^2 - m^2} (-\mathrm{i}eQ_f) \not{\epsilon}(k) u(p_a)$$

$$= \bar{v}(p_b) A_0(p_a) \frac{\mathrm{i}(\not{p}_a + m)}{-2p_a \cdot k} (-\mathrm{i}eQ_f) \gamma^\mu u(p_a) \epsilon_\mu(k)$$

$$= \mathcal{M}_{p \to p'} (-eQ_f) \frac{p_a \cdot \epsilon}{p_a \cdot k}$$

Generalization to *n*-photon emission:

$$\mathrm{i}\mathcal{M}_{p \to p' + n\gamma} = \mathrm{i}\mathcal{M}_{p \to p'} \frac{e^n}{n!} \sum_{i=1}^{l} \prod_{r=1}^{n} \eta_i \frac{p_i \cdot \epsilon_r}{p_i \cdot k_r}$$

QED resummation

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{p\to p'+\gamma} = \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{p\to p'} \times \int_{|\vec{k}|\leq \Delta E} \frac{\mathrm{d}^3 k}{(2\pi)^3 2E} e^2 \left(\frac{-p_a^2}{p_a\cdot k} + \frac{-p_b^2}{p_b\cdot k} + \frac{2p_a\cdot p_b}{(p_a\cdot k)(p_b\cdot k)}\right)$$

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right) = \left(\frac{\mathrm{d}\sigma_0}{\mathrm{d}\Omega}\right)\sum_{n=0}^{\infty}\frac{Y^n}{n!} = \left(\frac{\mathrm{d}\sigma_0}{\mathrm{d}\Omega}\right)\exp\left(Y\right)$$

Including virtual corrections in soft limit:[1]

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right) = \left(\frac{\mathrm{d}\sigma_0}{\mathrm{d}\Omega}\right)\exp\left(Y + 2X\right).$$

[1] For full treatment see D. Yennie et al., *Ann. Phys.* **13**, 379 (1961).

The very basics of QCD resummation

Hadronic cross section in Mellin space

Distribution from QCD factorization

$$M^2 \frac{\mathrm{d}^2\sigma_{AB}}{\mathrm{d}M^2\mathrm{d}p_{\mathrm{T}}^2}(\tau := \frac{M^2}{S}) = \sum_{ab} \int \mathrm{d}x_a \mathrm{d}x_b \mathrm{d}z \, x_a f_{a/A}(x_a, \mu^2) x_b f_{b/B}(x_b, \mu^2)$$
$$\times \, z\hat{\sigma}_{ab}(z, M^2, M^2/p_{\mathrm{T}}^2, M^2/\mu^2)\delta(\tau - x_a x_b z)$$

Mellin transform

$$\tilde{F}(N) \equiv F(N) := \int_0^1 \mathrm{d}x \, x^{N-1} F(x)$$

$$M^2 \frac{\mathrm{d}\sigma_{AB}}{\mathrm{d}M^2\mathrm{d}p_{\mathrm{T}}^2}(N-1) = \sum_{ab} f_{a/A}(N, \mu^2) f_{b/B}(N, \mu^2) \hat{\sigma}_{ab}(N, M^2, p_{\mathrm{T}}^2, \mu^2)$$

Factorization[1]

$$M^2 \frac{\mathrm{d}\sigma_{ab}}{\mathrm{d}M^2}(N-1) = \psi_{a/a}(N,\mu^2)\psi_{b/b}(N,\mu^2) \times H_{ab}(M^2,\mu^2) S_{ab}(N,M,\mu^2).$$

- Hard function H:
  - Perturbatively computable: $H_{ab}(M^2, M^2/\mu^2) = \sum_{n=0}^{\infty} a_{\mathrm{S}}^n H_{ab}^{(n)}(M^2, M^2/\mu^2)$
- Parton-in-parton distributions ψ:
  - Satisfy the evolution equation: $\frac{\partial \psi_{a/a}(N,M^2)}{\partial \log M^2} = \gamma_a(a_{\mathrm{S}}(M^2))\psi_{a/a}(N,M^2)$

[1] Collins, Soper, Sterman, *Nucl. Phys.* **B261**, 104 (1985).

Threshold resummation

The final formula reads:

$$\hat{\sigma}_{ab}(N, M^2, \mu^2) = \mathcal{H}_{ab}(M^2, \mu^2) \exp(\mathcal{G}_{ab}(N, M^2, \mu^2)),$$

with

$$\mathcal{H}_{ab}^{(0)}(M^2, \mu^2) = H_{ab}^{(0)}(M^2, \mu^2)$$
$$\mathcal{H}_{ab}^{(1)}(M^2, \mu^2) = H_{ab}^{(1)}(M^2, \mu^2) + \frac{\pi^2}{6}(A_a^{(1)} + A_b^{(1)})H_{ab}^{(0)}(M^2)$$

$$\mathcal{G}_{ab}(N, M^2, \mu^2) = \log \bar{N} g_{ab}^{(1)}(\lambda) + g_{ab}^{(2)}(\lambda, \mu^2) + a_{\mathrm{S}} g_{ab}^{(3)}(\lambda, \mu^2) + \cdots$$

Still. . .

- Double counting
- Inverse Mellin transform

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$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

BSM at the LHC

A QFT is defined by:

- Symmetries

- Particle content

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  - Gauge symmetries (internal)
  - Spacetime symmetries (external)
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    - $\to$ New gauge groups (U(1), SU(2), ...), GUTs
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  - $\rightarrow$ Right handed neutrinos, ...

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New gauge bosons at the LHC

Phenomenology of new gauge bosons

New spin-1 particles must be gauge bosons associated with SSB.
$\rightarrow$ The breaking mechanism determines the charges and (up to mixing terms) the couplings of the theory.

$$Z'_\mu(g_u^L \bar{u}_L \gamma^\mu u_L + g_d^L \bar{d}_L \gamma^\mu d_L + g_u^R \bar{u}_R \gamma^\mu u_R + g_d^R \bar{d}_R \gamma^\mu d_R + g_\nu^L \bar{\nu}_L \gamma^\mu \nu_L + g_e^L \bar{e}_L \gamma^\mu e_L + g_e^R \bar{e}_R \gamma^\mu e_R)$$

$$\frac{W'^+_\mu}{\sqrt{2}} \left[\bar{u}_i (C^R_{q,ij} P_R + C^L_{q,ij} P_L) \gamma^\mu_j + \bar{\nu}_i (C^R_{l,ij} P_R + C^L_{l,ij} P_L) \gamma^\mu e_j) \right]$$

The sequential standard model (SSM)

- Defined to have the same couplings as the SM.
- Widely used by experimental collaborations.

G(221) models

- Many BSM theories include an additional $\mathrm{SU}(2)$ symmetry.
  $\rightarrow$ They lead to Z' as well as W' bosons.
- The most studied is the left-right model.

The symmetry $\mathrm{SU}(2)_1 \times \mathrm{SU}(2)_2 \times \mathrm{U}(1)_X$ must break to the SM:

$$\text{BP I: } \mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_2 \times \mathrm{U}(1)_X \rightarrow \mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_Y$$
$$\text{BP II: } \mathrm{SU}(2)_1 \times \mathrm{SU}(2)_2 \times \mathrm{U}(1)_Y \rightarrow \mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_Y$$

Model	$\mathrm{SU}(2)_1$	$\mathrm{SU}(2)_2$	$\mathrm{U}(1)_X$ quarks	$\mathrm{U}(1)_X$ leptons
Left-right	$(u_\mathrm{L}, d_\mathrm{L}), (\nu_\mathrm{L}, e_\mathrm{L})$	$(u_\mathrm{R}, d_\mathrm{R}), (\nu_\mathrm{R}, e_\mathrm{R})$	1/6	-1/2
Leptophobic	$(u_\mathrm{L}, d_\mathrm{L}), (\nu_\mathrm{L}, e_\mathrm{L})$	$(u_\mathrm{R}, d_\mathrm{R})$	1/6	Y_SM
Hadrophobic	$(u_\mathrm{L}, d_\mathrm{L}), (\nu_\mathrm{L}, e_\mathrm{L})$	$(\nu_\mathrm{R}, e_\mathrm{R})$	Y_SM	-1/2
Fermiophobic	$(u_\mathrm{L}, d_\mathrm{L}), (\nu_\mathrm{L}, e_\mathrm{L})$		Y_SM	Y_SM
Ununified	$(u_\mathrm{L}, d_\mathrm{L})$	$(\nu_\mathrm{L}, e_\mathrm{L})$	Y_SM	Y_SM
Nonuniversal	$(u_\mathrm{L}, d_\mathrm{L})_{1,2}, (\nu_\mathrm{L}, e_\mathrm{L})_{1,2}$	$(u_\mathrm{L}, d_\mathrm{L})_3, (\nu_\mathrm{L}, e_\mathrm{L})_3$	Y_SM	Y_SM

SSM cross sections

Boson	Mass	$\sqrt{S}$	$\sigma_{\rm LO}$ (ab)	$\sigma_{\rm NLO}$ (ab)	$\sigma_{\rm res}$ (ab)
Z'	3 TeV	8 TeV	$54.5^{+9.3}_{-7.5}$	$62.9^{+6.9+6.4}_{-6.0-2.6}$	$69.7^{+1.2+7.1}_{-0.6-2.8}$
W'^{+}	3 TeV	8 TeV	$310.1^{+55.4}_{-44.4}$	$275.7^{+31.3+40.0}_{-27.1-29.5}$	$310.7^{+1.8+43.2}_{-0.0-32.0}$
W'^{-}	3 TeV	8 TeV	$94.4^{+17.1}_{-13.7}$	$122.2^{+13.0+19.3}_{-11.5-11.4}$	$136.6^{+0.9+21.4}_{-0.2-12.4}$

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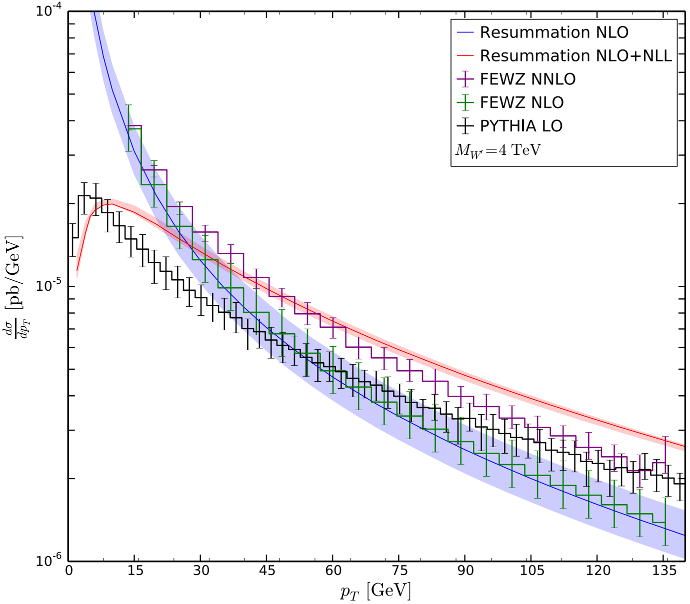
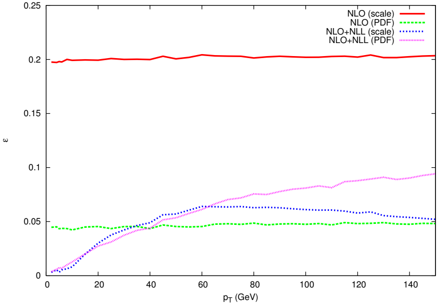
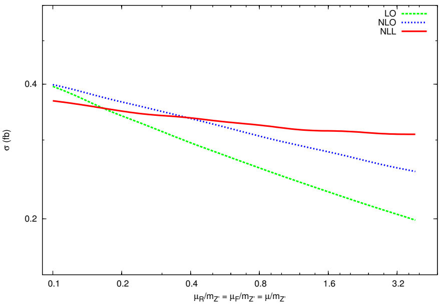
SSM: Transverse momentum distributions

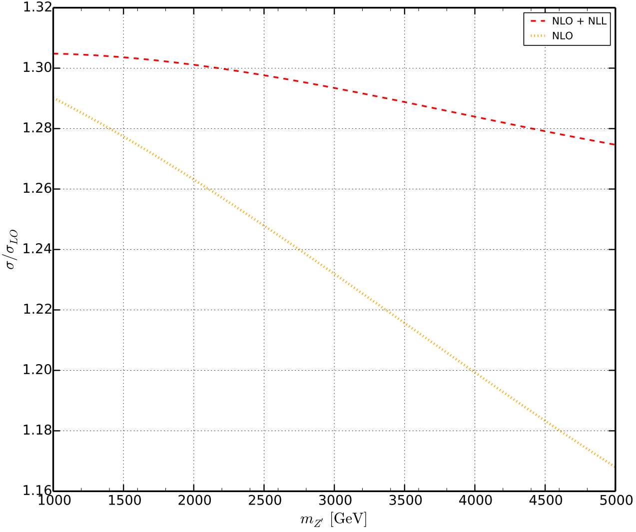


Uncertainties





SSM: Mass-dependence



Parting words

Conclusions

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  - Restores convergence in critical kinematical regions
  - Improves overall precision
  - Resummation corrections become more important for larger masses

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  - Restores convergence in critical kinematical regions
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Code

Code is open source and available at
`http://www.resummino.org/`.

Thank you for your attention.