Killing the CMSSM with Fittino?

GDR Terascale
IWH Heidelberg, December 2014

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Outline

Part I

Fittino and the CMSSM:

Introduction and review

Part II

The missing piece - so far:

Frequentist P-Values!

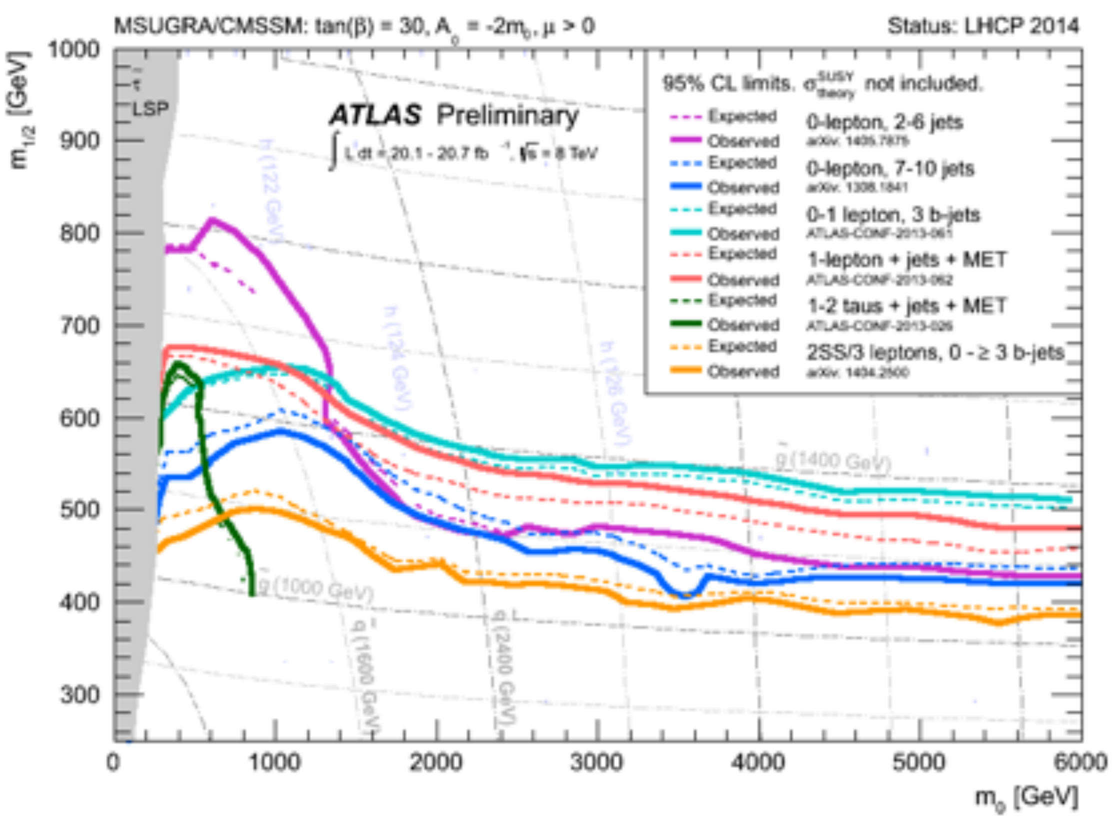
Constrained MSSM

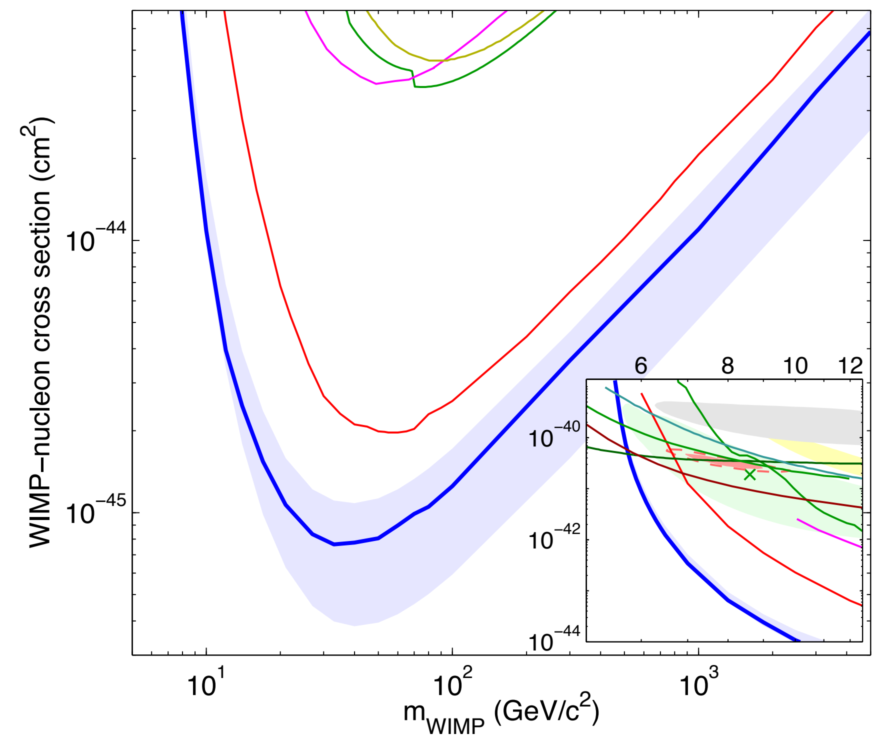
- In the most general version the MSSM introduces >100 BSM parameters.
- CMSSM as one of the simplest versions of the MSSM reduces this to just 4 continuous parameters and 1 sign:

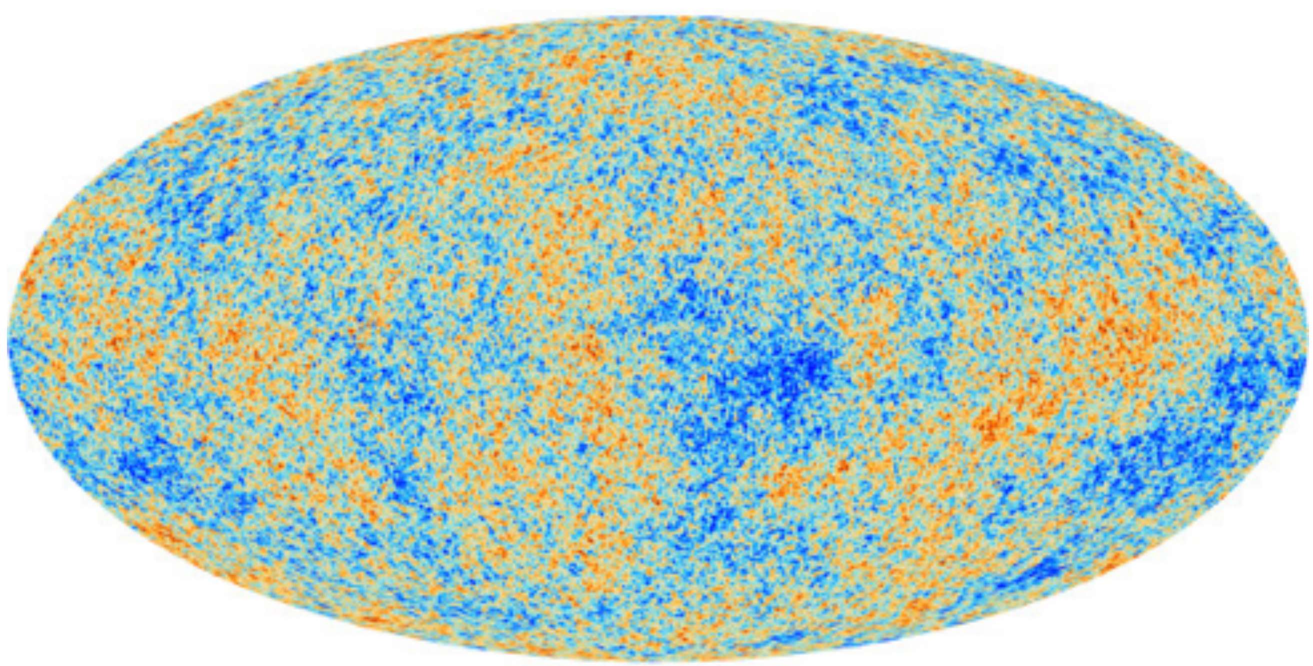
M_0	scalar mass parameter
$M_{1/2}$	gaugino mass parameter
A_0	trilinear coupling
$\tan\beta$	ratio of Higgs VEVs
sign μ	sign of Higgsino mass parameter

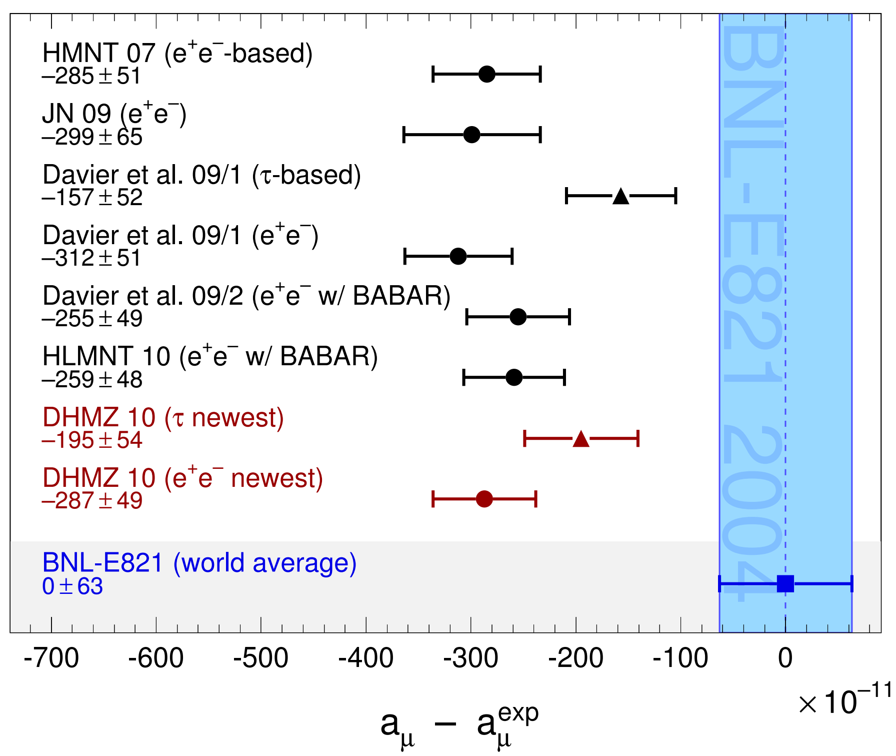
- We fix sign $\mu = 1$ and treat m_t as additional free parameter.

Probing the CMSSM

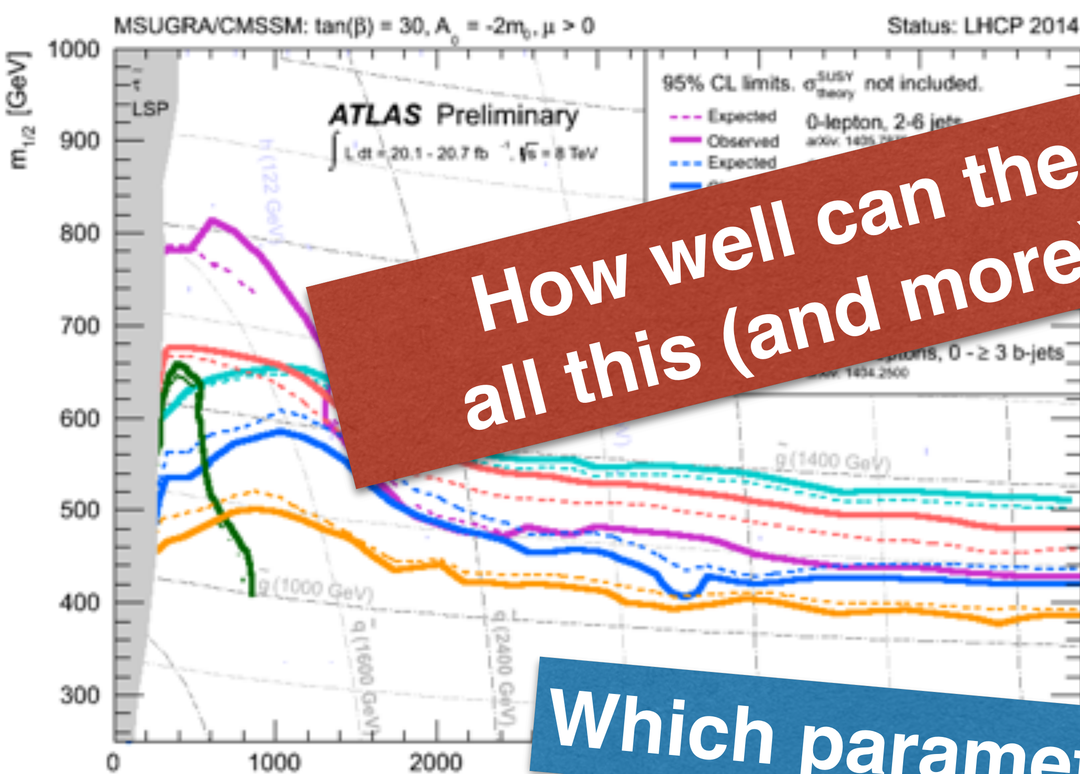




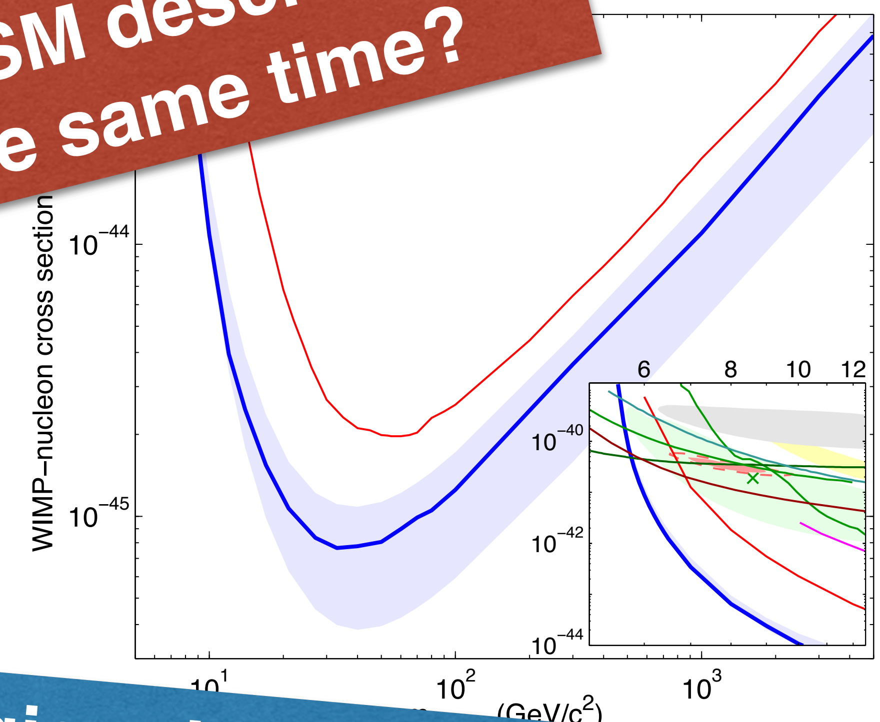




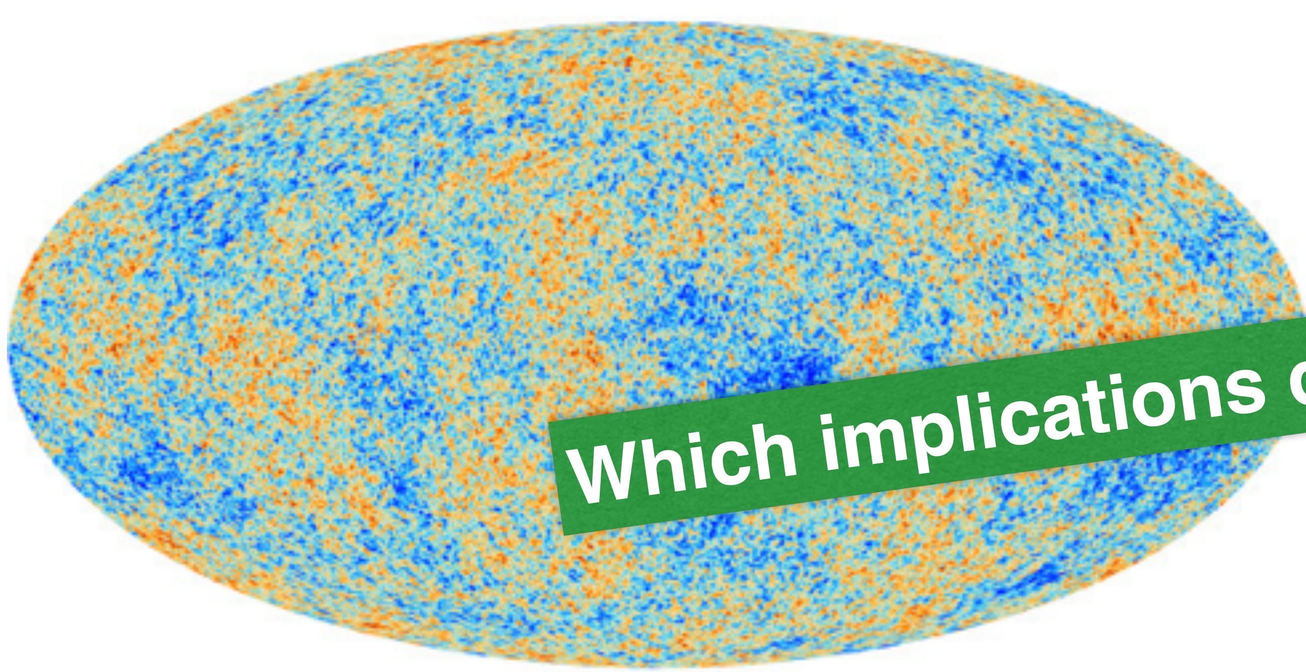
Probing the CMSSM

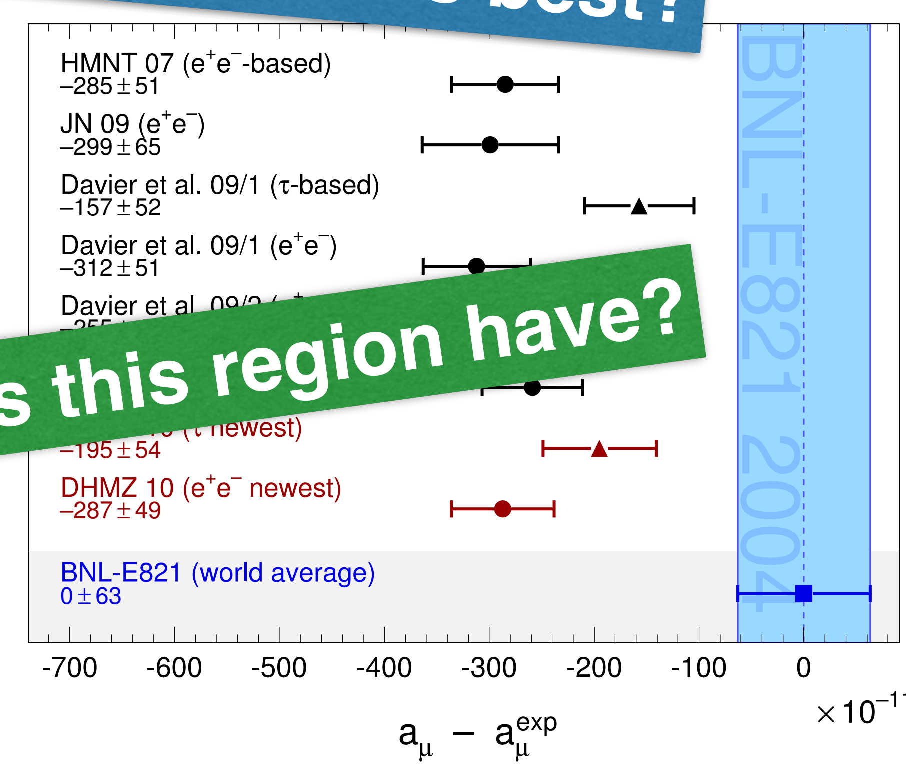


How well can the CMSSM describe all this (and more) at the same time?



Which parameter region does this best?





Which implications does this region have?

Fittino

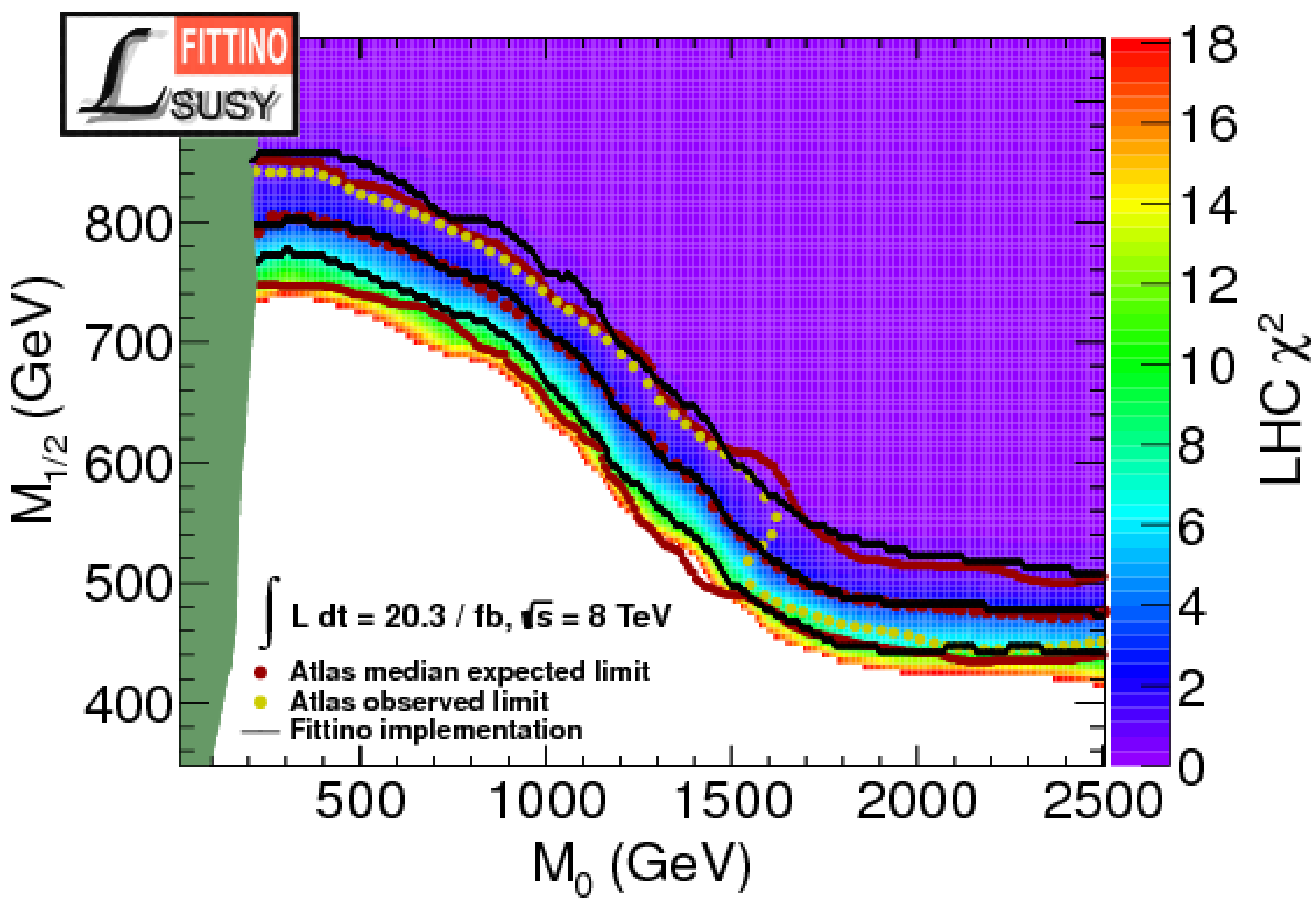
- Using the C++ program Fittino we combine a wide range of measurements sensitive to supersymmetry:
    - indirect constraints from low energy measurements
    - Higgs boson properties
    - direct searches for sparticles and BSM Higgs bosons
    - astrophysical observations

- Fittino uses
    - public codes to calculate model predictions
    - a χ^2 function to compare predictions and measurements
    - an auto-adaptive Markov Chain to sample the parameter space
    - frequentist interpretation

χ^2 contributions

At each parameter point $\vec{P}$ calculate:

$$\chi^2 = \left(\vec{O}_{\mathrm{meas}} - \vec{O}_{\mathrm{pred}}(\vec{P})\right)^T cov^{-1} \left(\vec{O}_{\mathrm{meas}} - \vec{O}_{\mathrm{pred}}(\vec{P})\right) + \chi^2_{\mathrm{limits}}$$



Optimization / Sampling

- Main method: Markov Chain Monte Carlo using Metropolis-Hastings algorithm

  $P_i \longrightarrow P_{i+1}$:

  - candidate point Q chosen according to Gaussian pdf centered around P_i
  - $$\rho = e^{-\left(\frac{\chi^2(\mathbf{Q}) - \chi^2(\mathbf{P}_i)}{2}\right)}$$

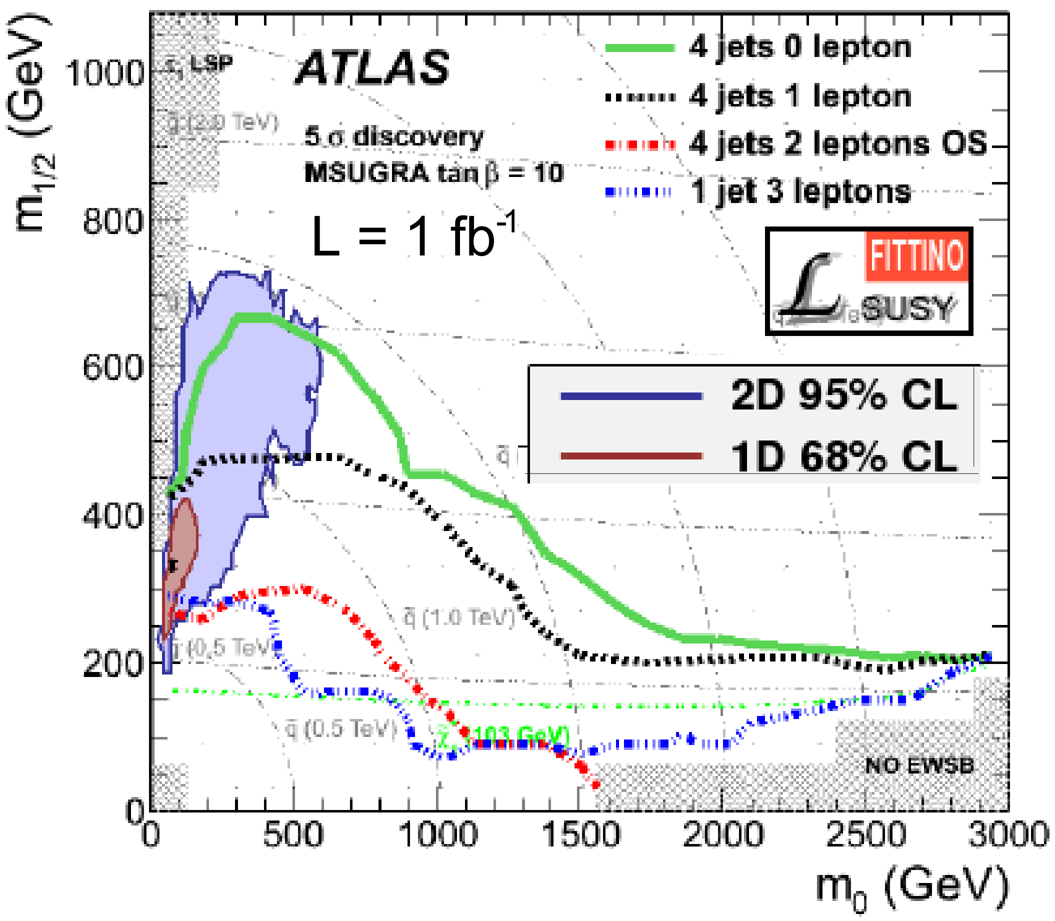
  - if $\rho > 1$: $P_{i+1} = Q$
  - else:
    - $P_{i+1} = Q$ with probability ρ
    - $P_{i+1} = P_i$ with probability $1-\rho$

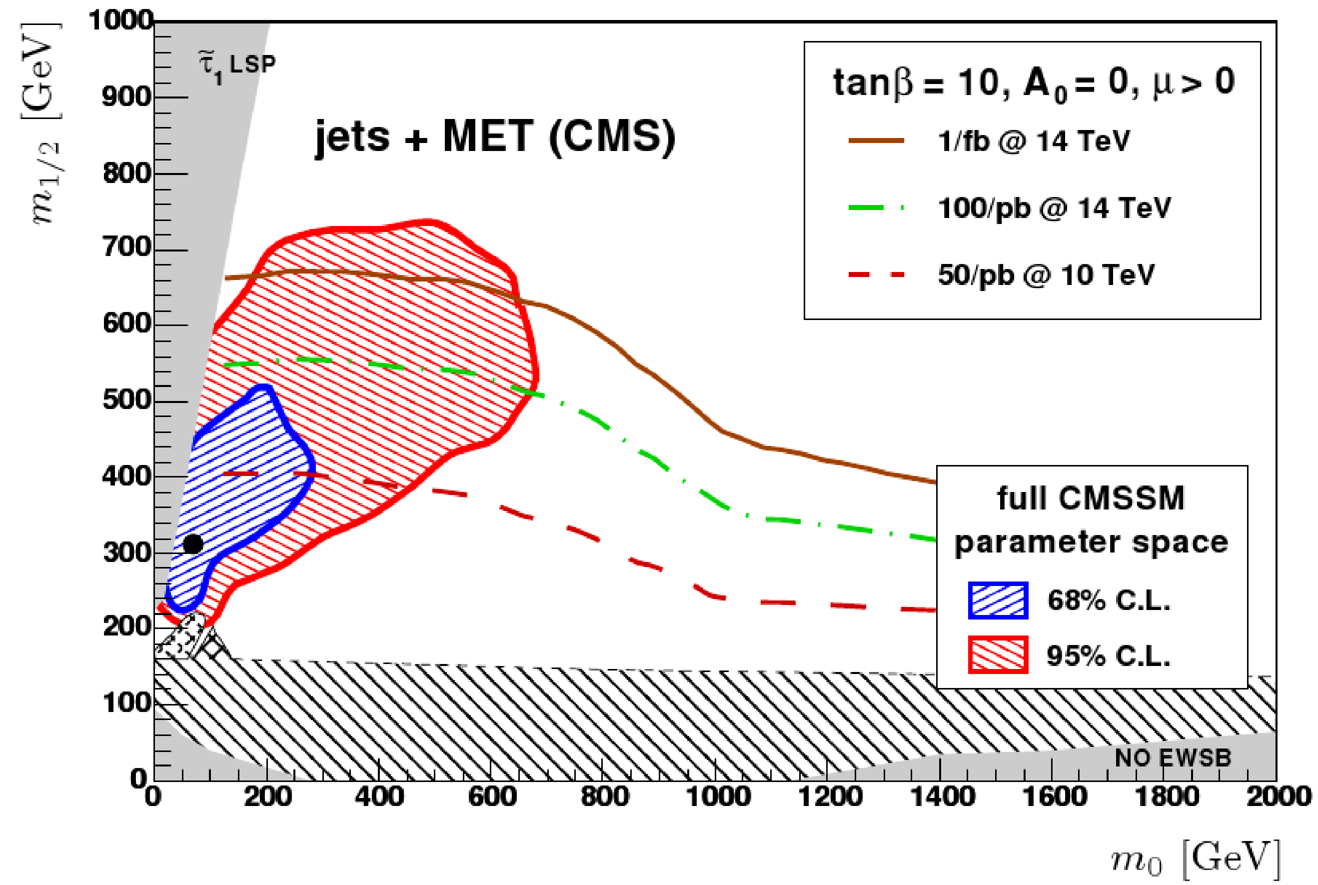
- Experimenting with many more algorithms
  - Correlated Markov Chain
  - Simulated Annealing
  - Particle Swarm
  - Genetic Algorithm

LE: Comparison with LHC potential

Bechtle, Desch, Uhlenbrock, Wienemann

Buchmüller, et al.





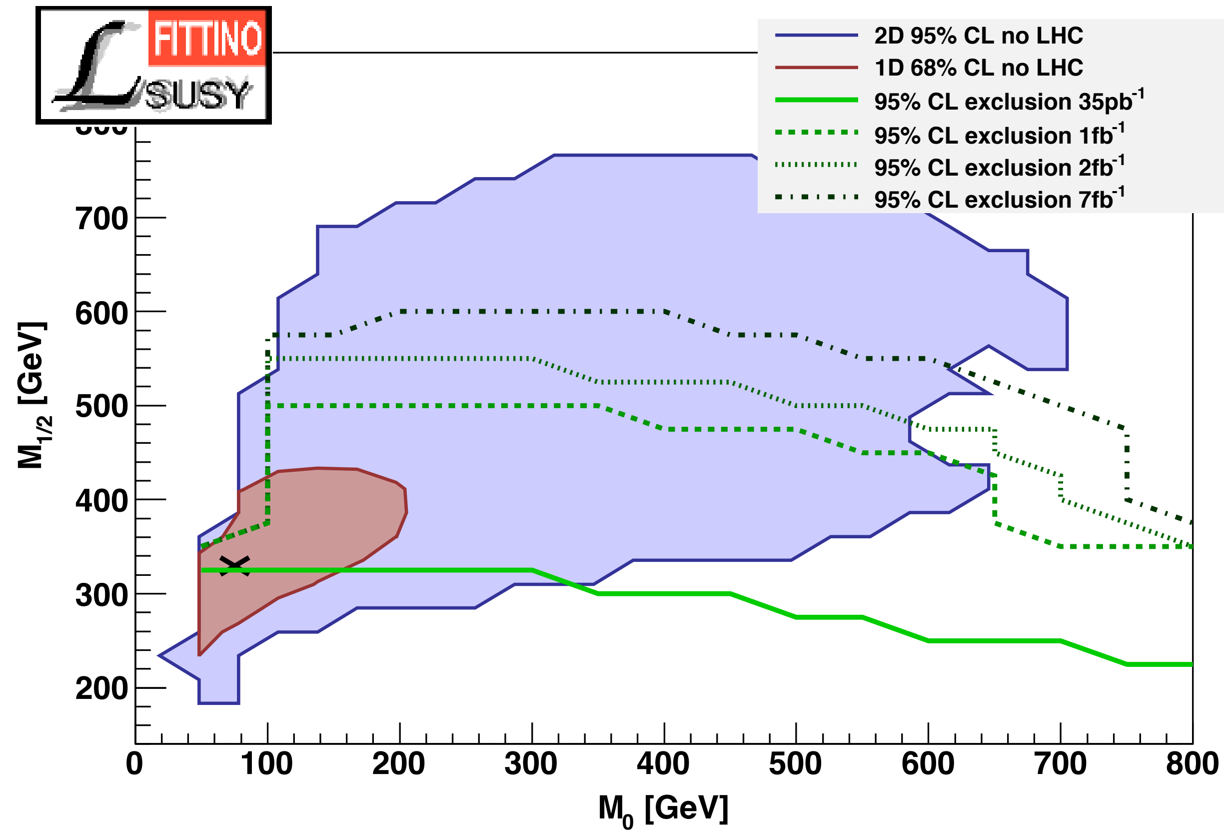
Good prospects for early BSM hints at LHC

Slide from 2009

Fittino Timeline

arXiv:1102.4693

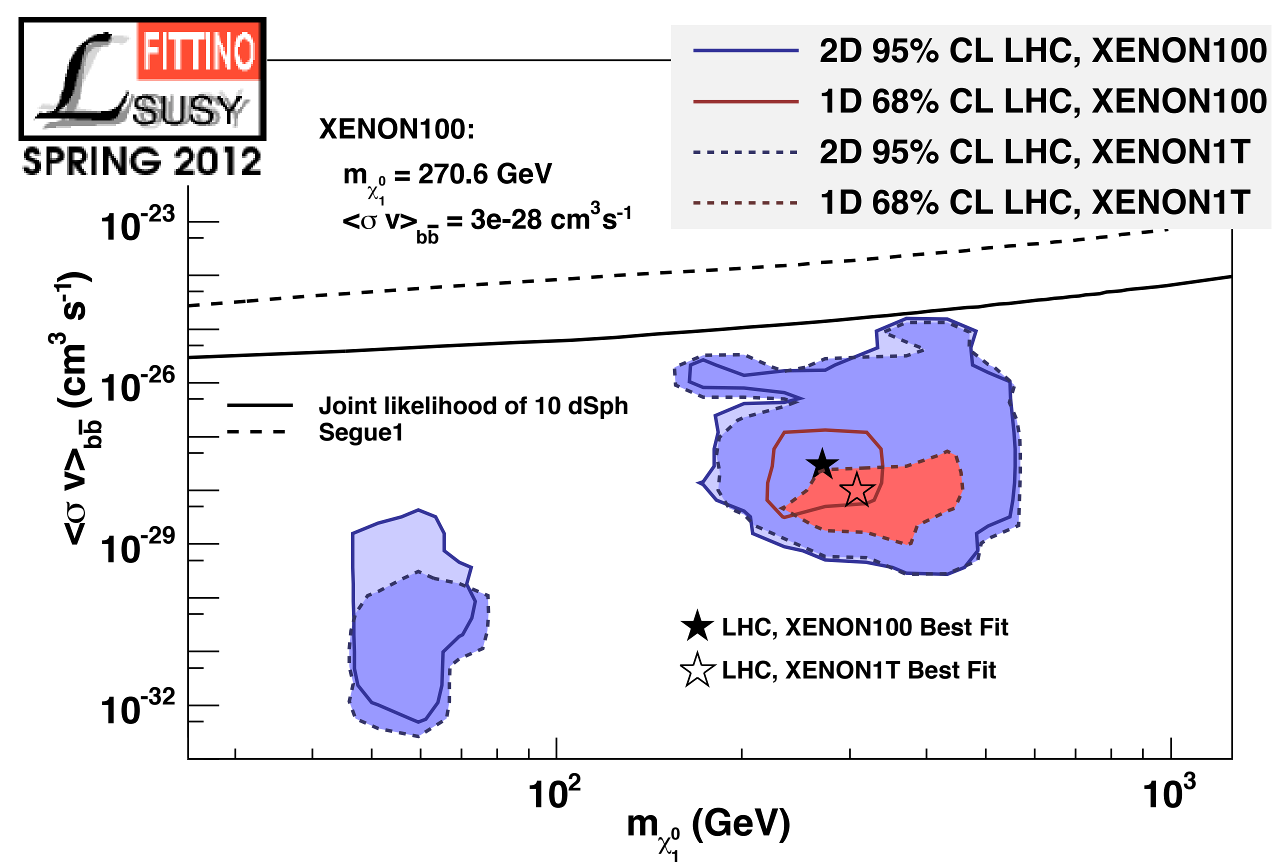
some tension building up between low energy observables and LHC



arXiv:1204.4199

increasing tension

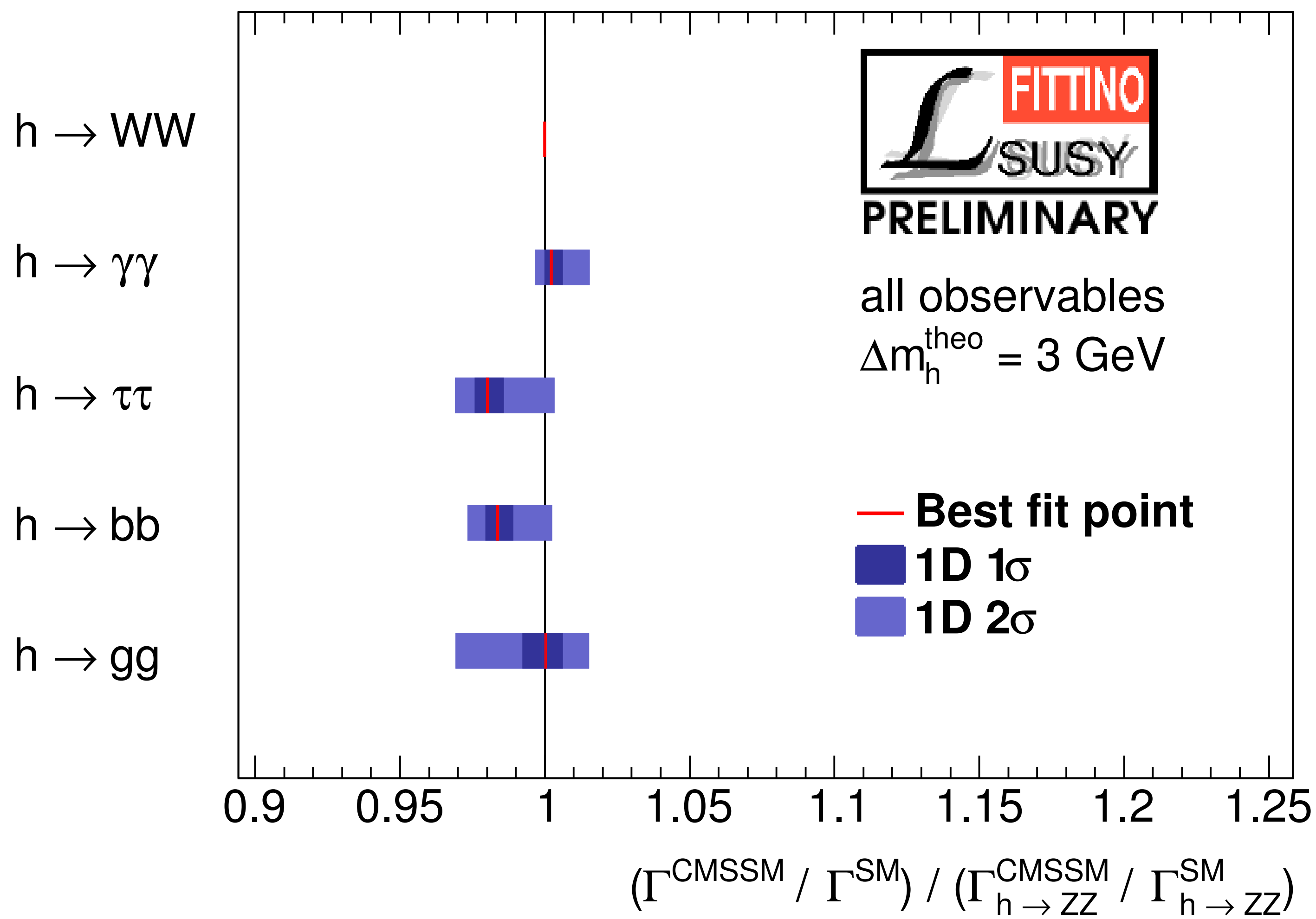
direct and indirect astrophysical detection experiments not yet sensitive to 2σ region



arXiv:1310.3045

SM like Higgs well described by CMSSM

χ^2/ndf decreases when the numerous Higgs rate measurements are taken into account



Updated measurements

Low energy observables

	Experimental value		theo uncertainty
BR($B_s \longrightarrow \mu^+\mu^-$)	(2.90 ± 0.70) x 10^{-9}	CMS + LHCb	26%
BR($B^\pm \longrightarrow \tau^\pm \nu$)	(1.14 ± 0.22) x 10^{-4}	PDG	20%
BR($b \longrightarrow s\gamma$)	(3.43 ±0.21± 0.07) x 10^{-4}	HFAG	14%
Δm_s	(17.719 ±0.036 ± 0.023) ps^{-1}	PDG	24%
$a_\mu - a_\mu^{SM}$	(28.7 ± 8.0) x 10^{-10}	Muon g-2	7%
m_t	(173.34 ± 0.27 ± 0.71) GeV	Tevatron + LHC	1 GeV
m_W	(80.385 ± 0.015) GeV	CDF + D0	0.01%
$\sin^2 \theta_{eff}$	0.2311 ± 0.00021	LEP + SLC	0.05%

Higgs boson properties and searches

- Higgs limits via HiggsBounds
- Higgs signals via HiggsSignals

Direct sparticle searches

- LEP chargino mass limit
- ATLAS MET + jets + 0 lepton search ($20fb^{-1}$)

Astrophysical observables

- We require χ_1^0 to be the LSP
- $\Omega_{CDM}h^2 = 0.1187 \pm 0.0017 \pm 0.0119_{theo}$ (Planck '13)
- Direct detection limit from LUX

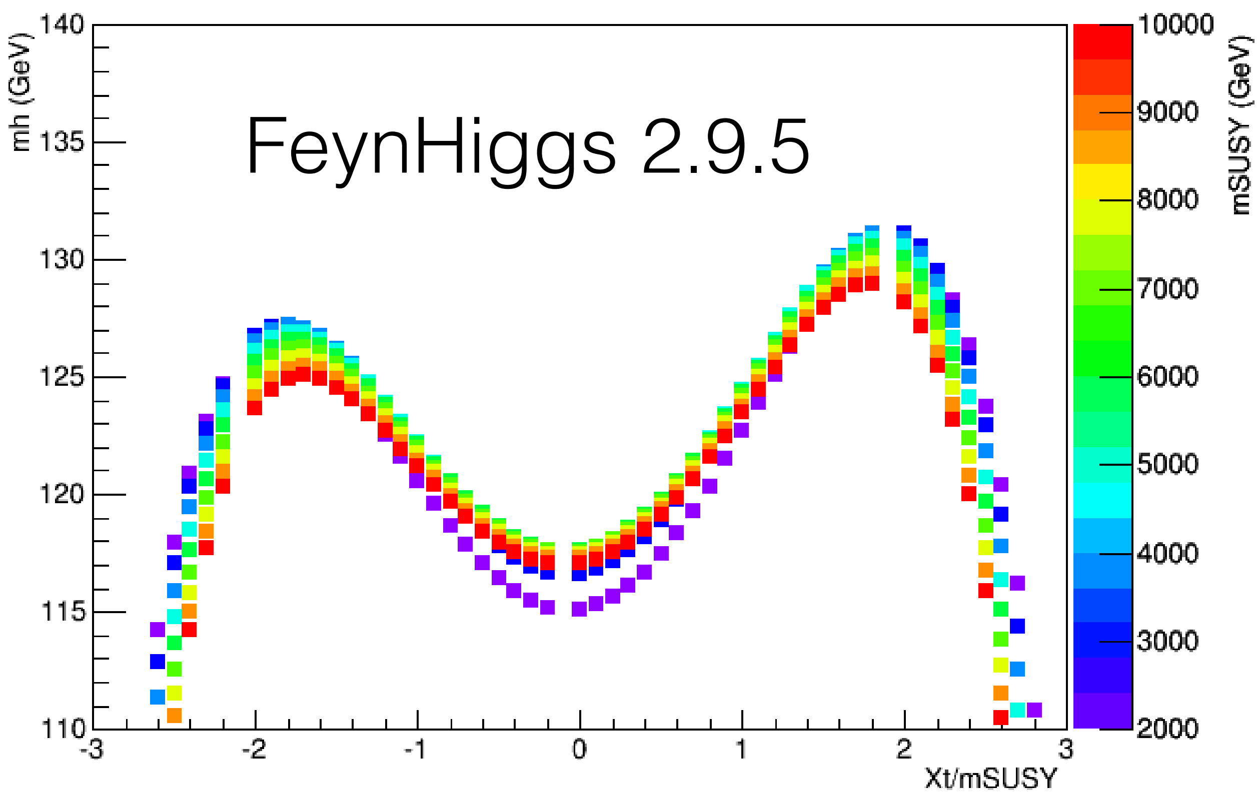
Model Predictions

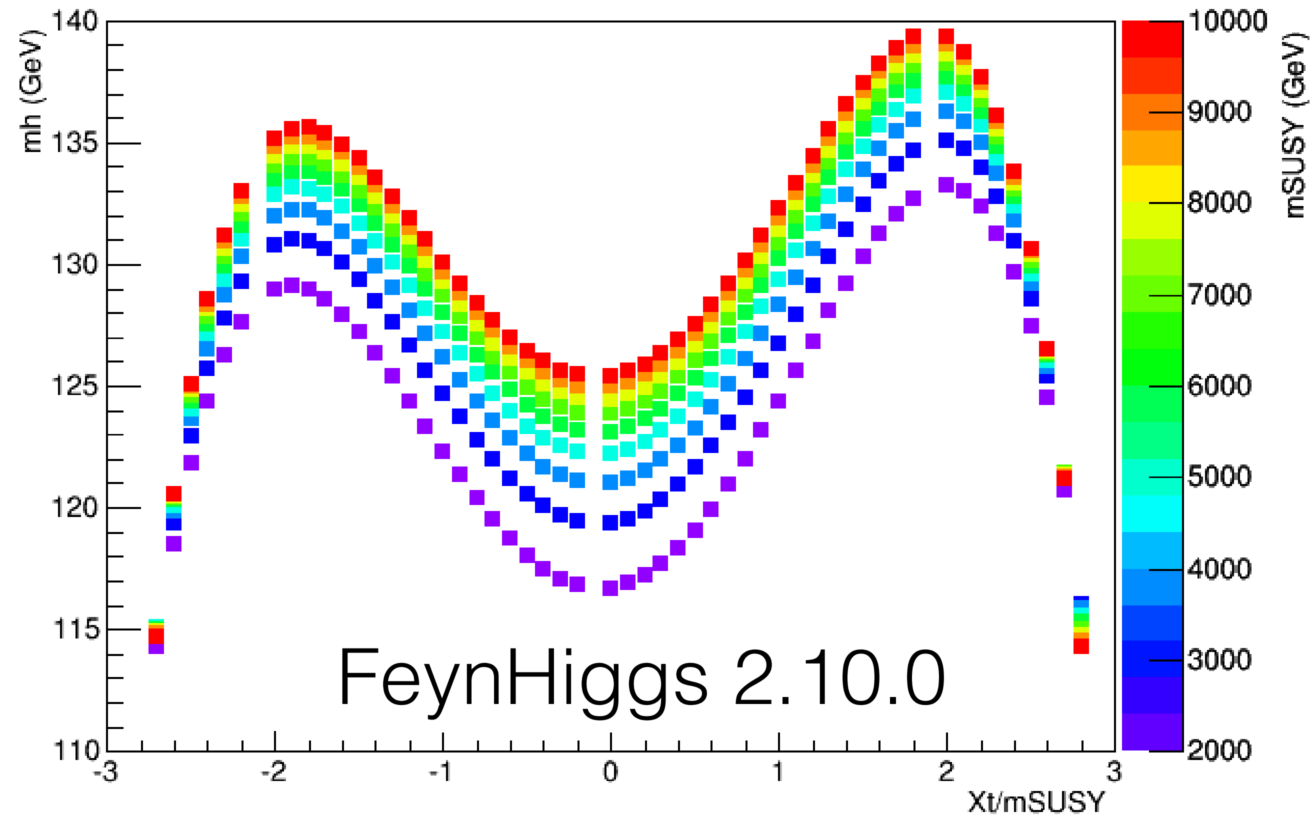
To evaluate the corresponding model predictions we use:

- SPheno for spectrum calculation
- FeynHiggs for Higgs properties, a_μ - a_μ^{SM}, $\sin^2\theta_{eff}$, m_W, Δm_s
- SuperIso for BR($B_s \longrightarrow \mu^+\mu^-$),BR($B^\pm \longrightarrow \tau^\pm\nu$),BR($b \longrightarrow s\gamma$)
- Prospino, Herwig++, Delphes for direct sparticle searches
- micrOMEGAs for dark matter relic density
- DarkSUSY via AstroFit for direct detection cross section

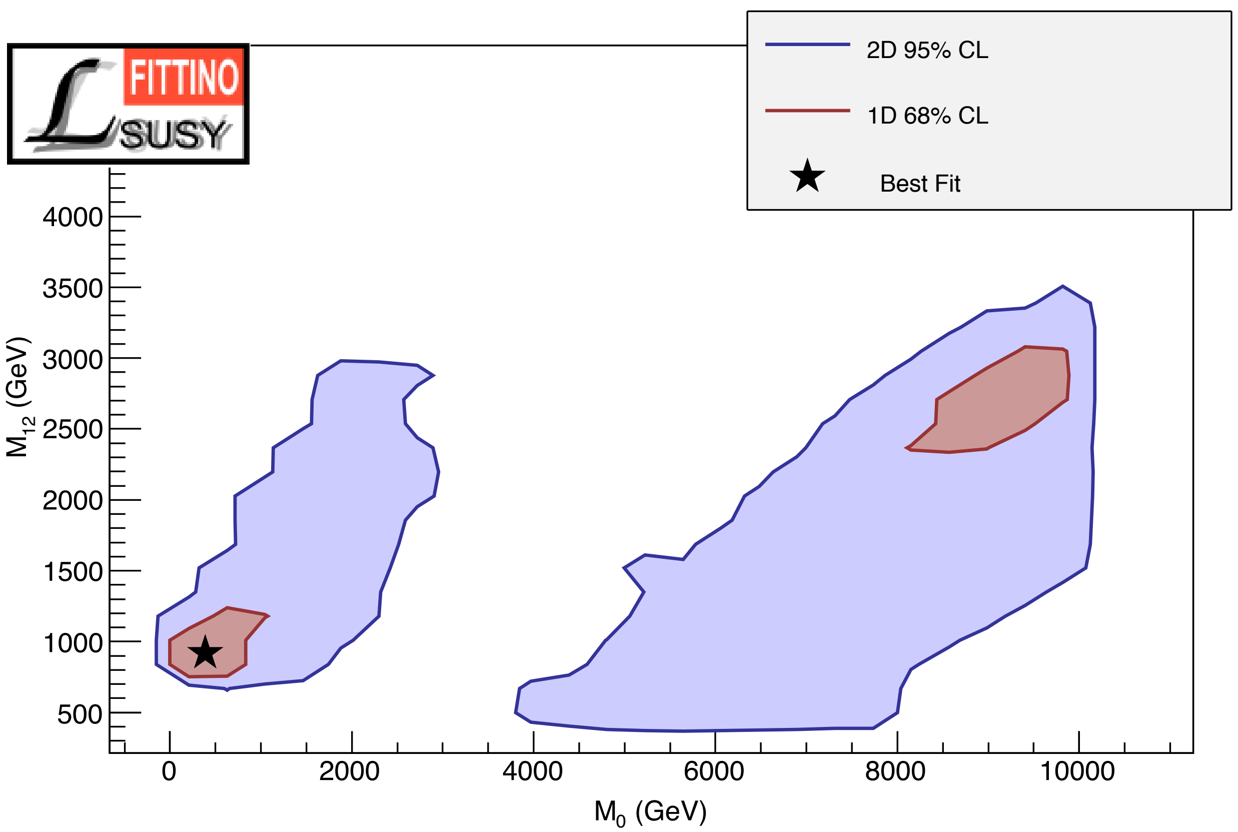
Impact of new Higgs mass calculation

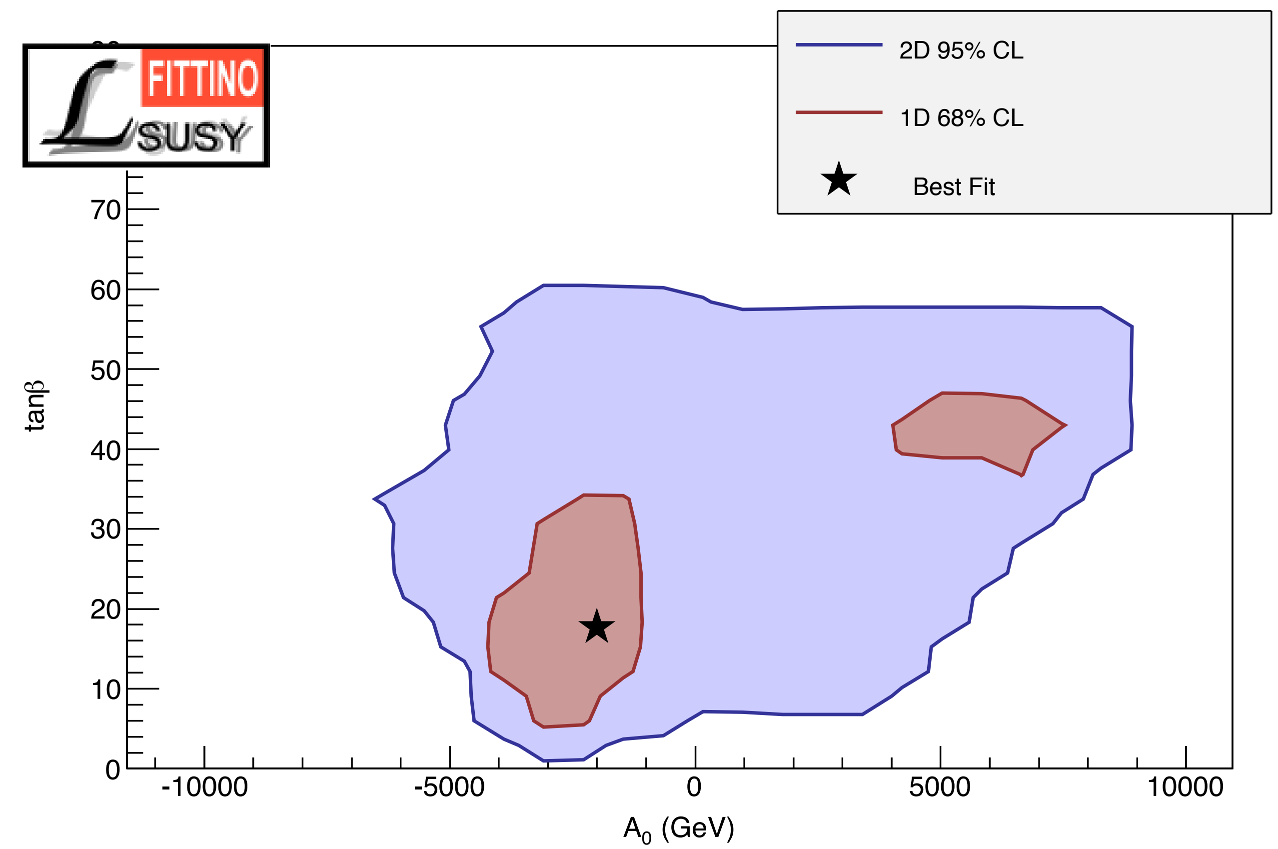
- Of course there are also improvements on the theory side
- The new Higgs mass calculation contained in FeynHiggs 2.10.0 makes it significantly “easier” to reach high Higgs masses





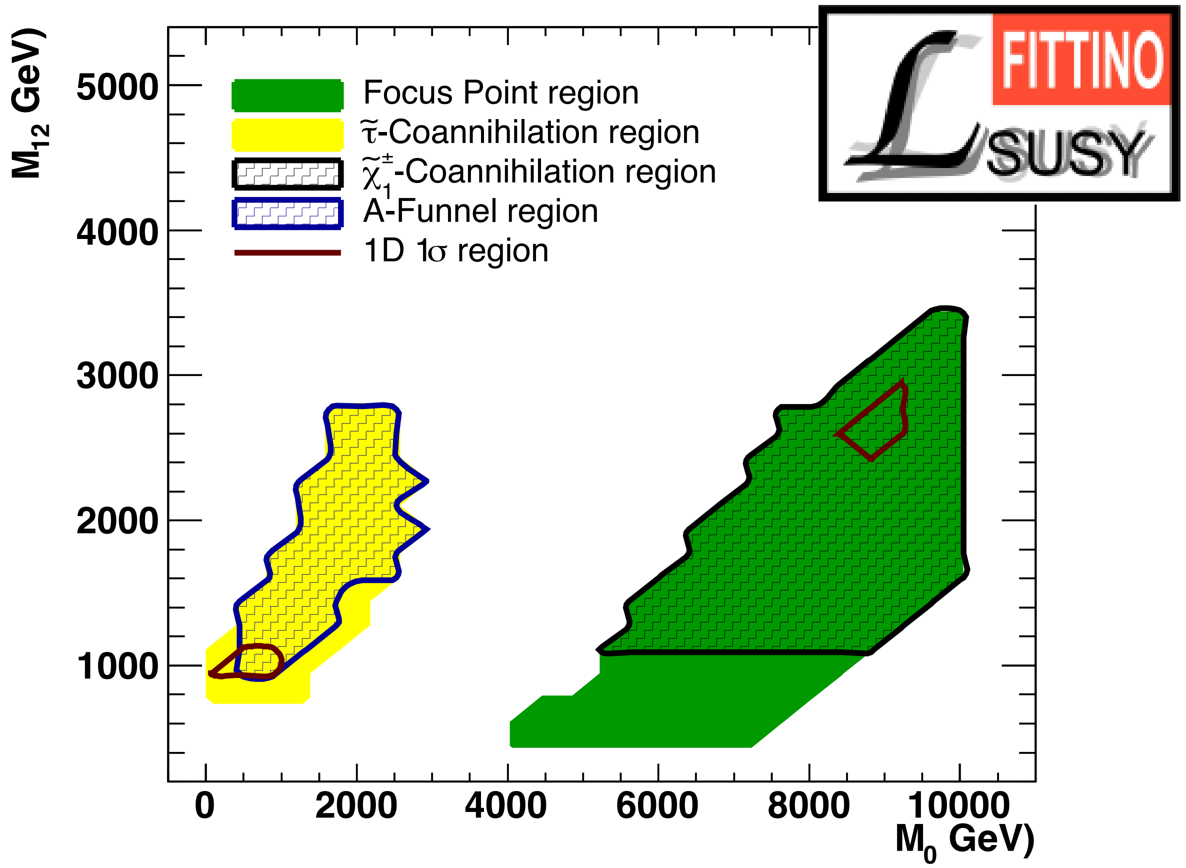
Preferred parameter region

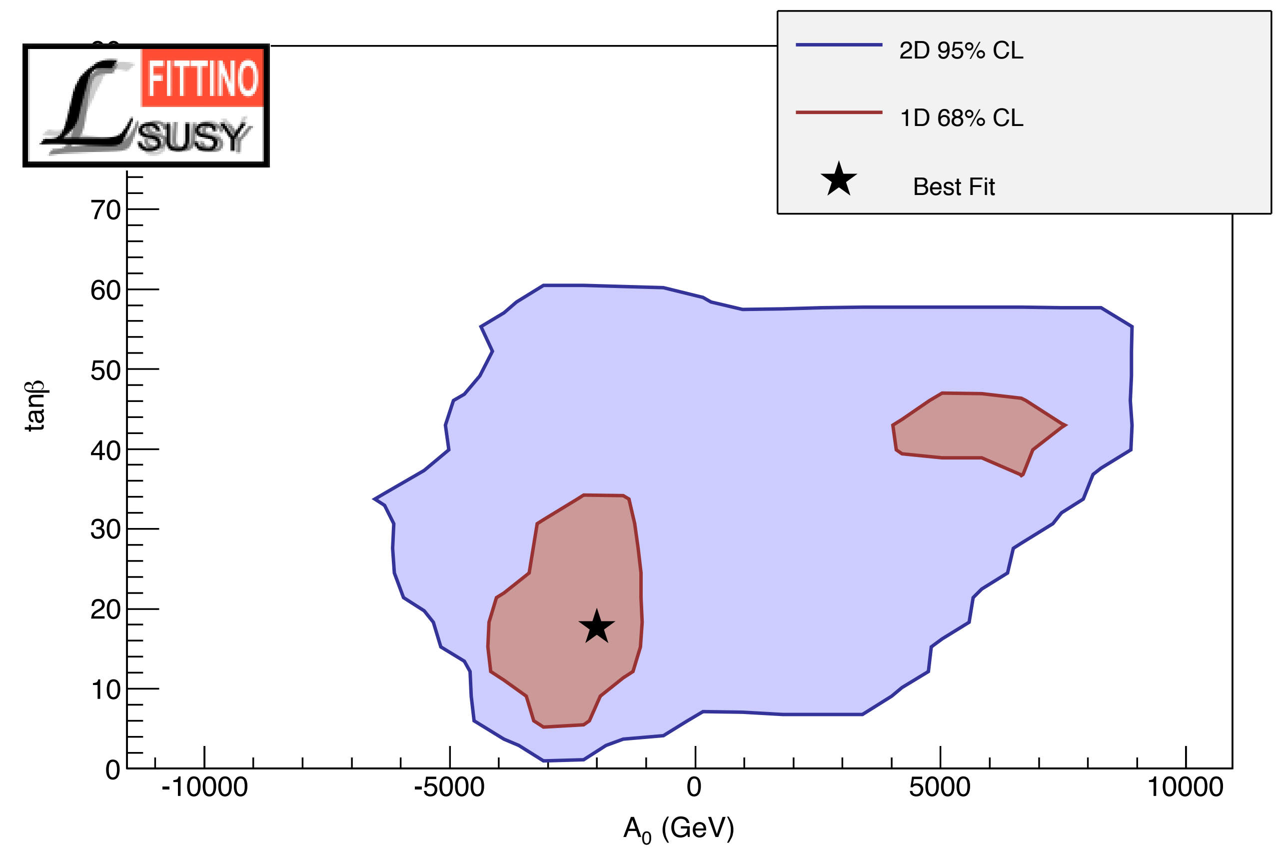




- χ^2/ndf = 30.4/22
- High mass region allowed at 1D 1σ due to new Higgs mass calculation

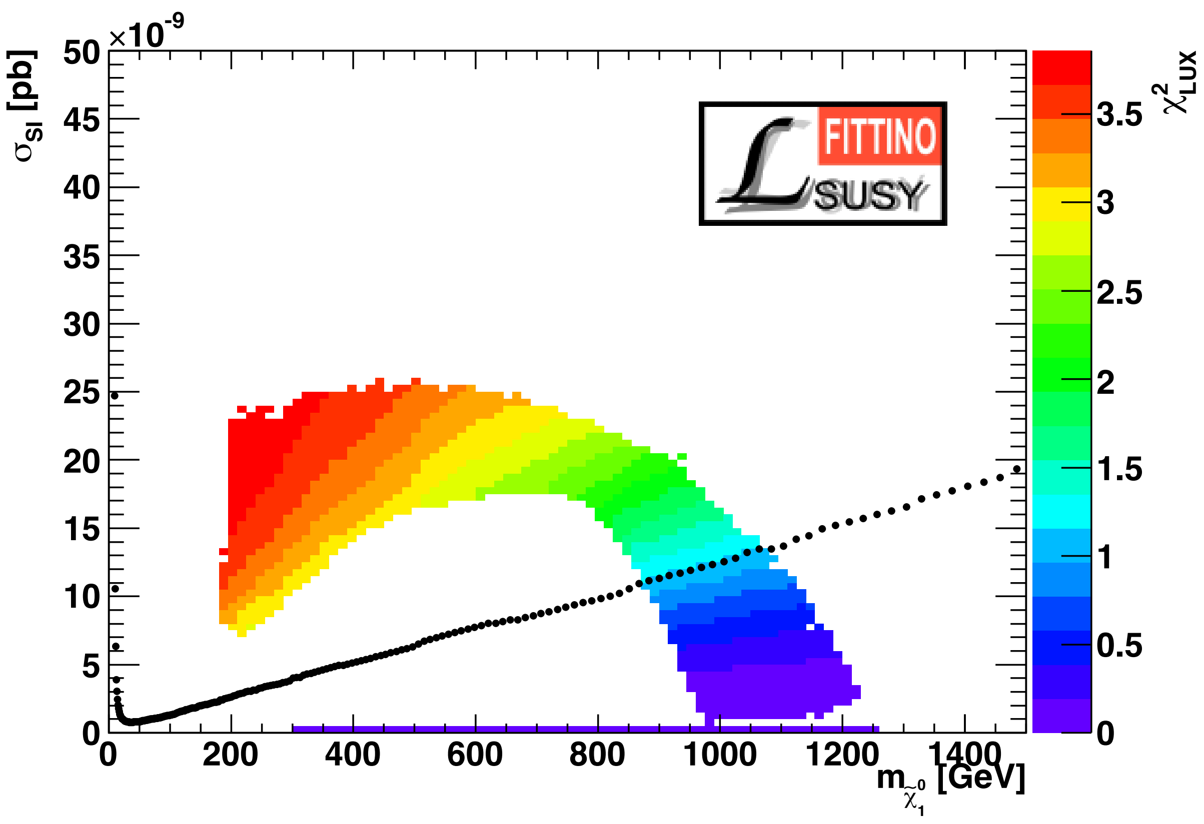
Preferred parameter region

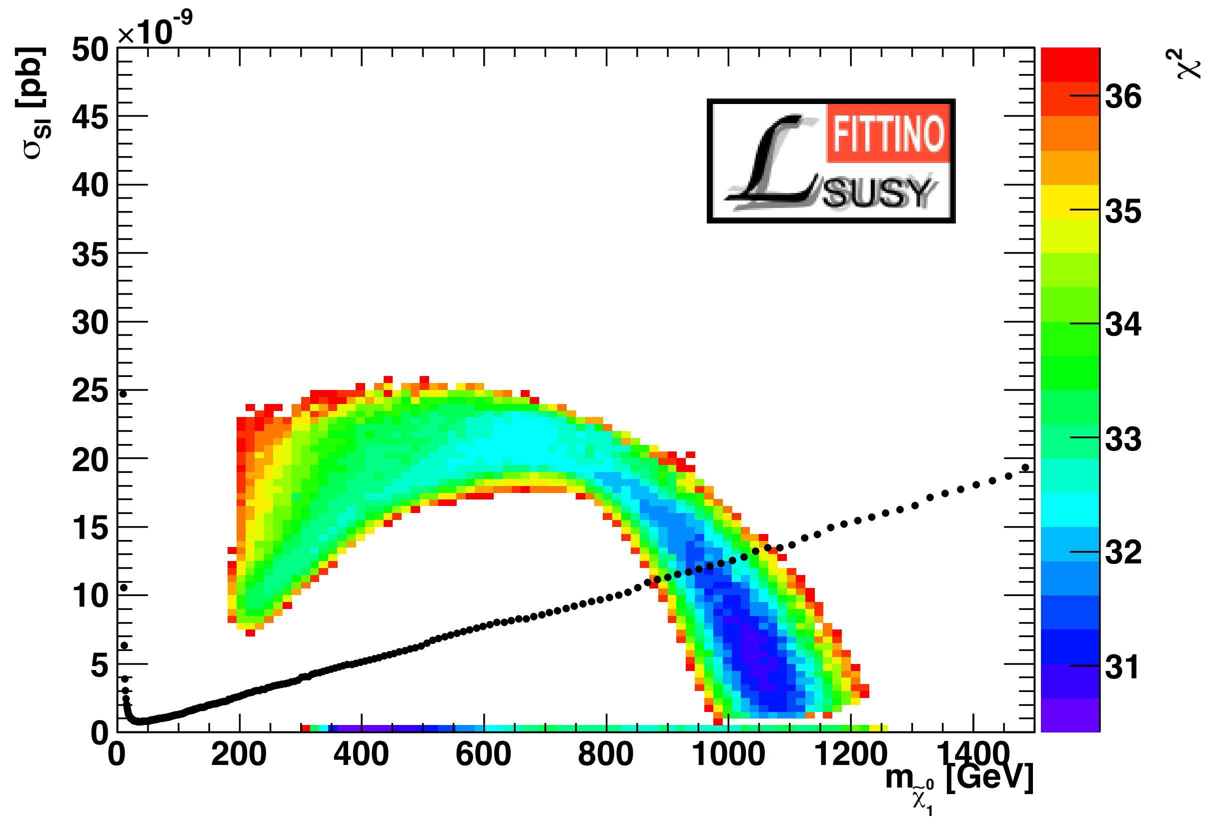




- χ^2/ndf = 30.4/22
- High mass region allowed at 1D 1σ due to new Higgs mass calculation

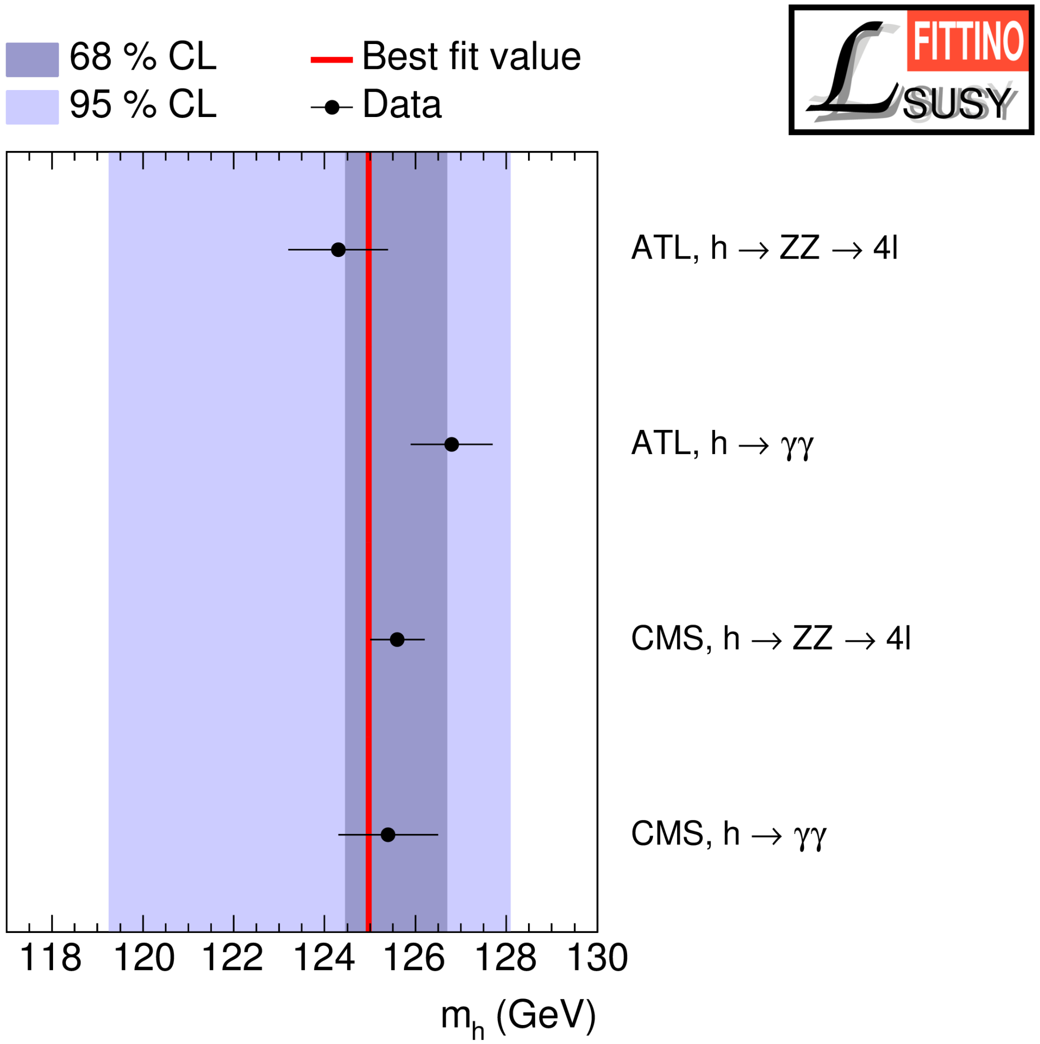
Impact of LUX experiment

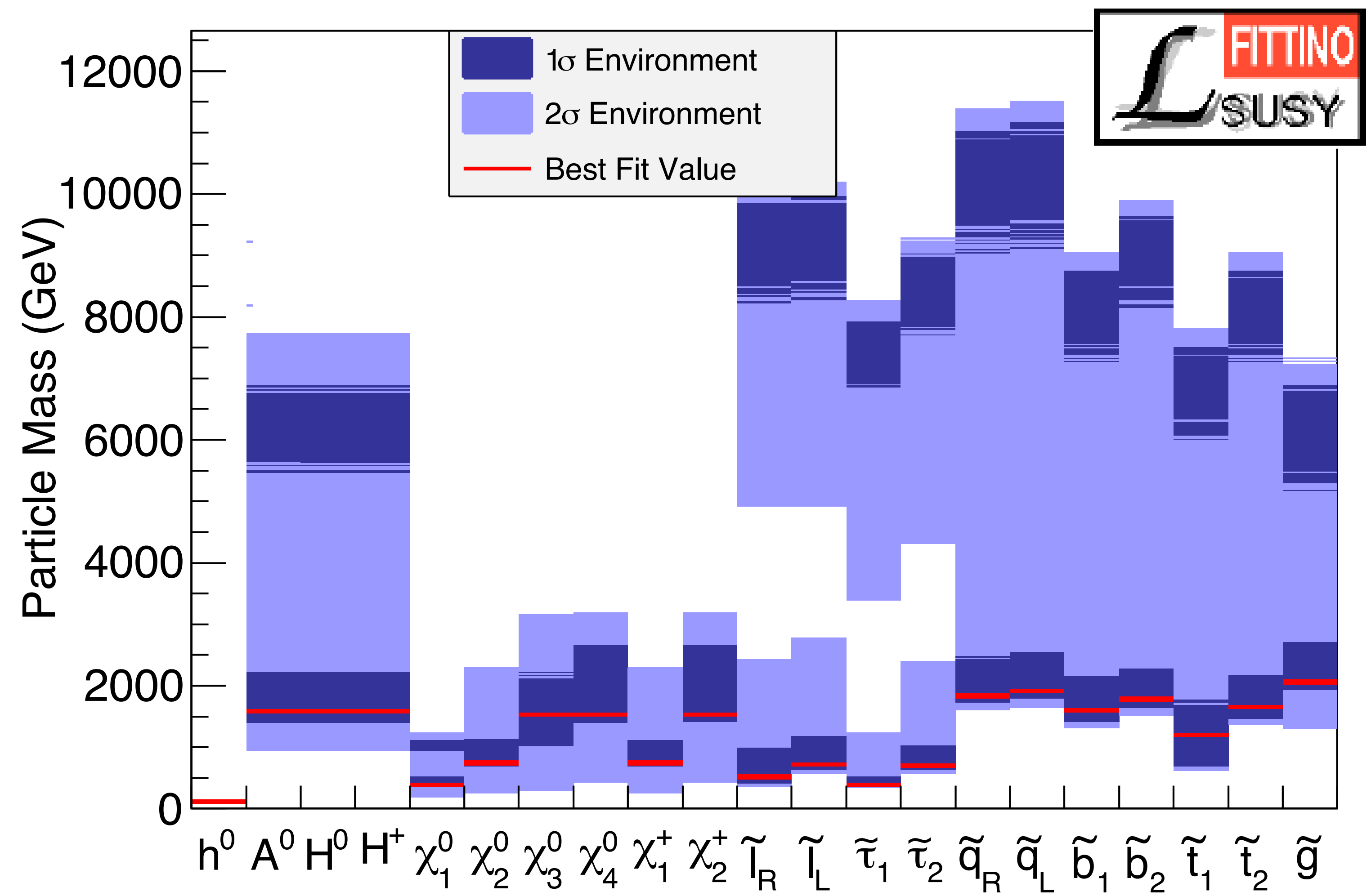




- LUX contributing significantly to χ^2 in 2σ region
- Starting to probe 1σ region

Predicted mass spectrum





- Higgs mass measurements well described by CMSSM
- squark and gluino masses at best fit point about 2 TeV
- But now also masses of 10 TeV allowed at 1σ

Summary of part I

- In the CMSSM there is some tension between low energy observables and exclusions from LHC

- The CMSSM is in agreement with astrophysical measurements but on the other hand no convincing direct or indirect detection hints are found

- A SM like Higgs is well described by the CMSSM with large particle masses but no BSM Higgs sector is found

CMSSM doesn’t look very attractive anymore…

.. but can we exclude it?

How well does the CMSSM describe the data quantitatively?

p-value

If the best fit point is realized in nature

fitting the model to the measurements

how probable is it to get

a minimal χ^2 at least as bad as the one observed?

Difficulties

- If our χ2 - function would be χ2 - distributed we could just look up the integral

$$\int_{\chi^2_{\mathrm{min}}}^{\infty} P_{\chi^2_{\mathrm{ndf}}}(x)\, dx$$

- Unfortunately this is not necessarily true because of:
  - Non - linear dependence of observables on parameters
  - Non - gaussian uncertainties
- Thus also χ^2/ndf isn't the appropriate goodness-of-fit measure

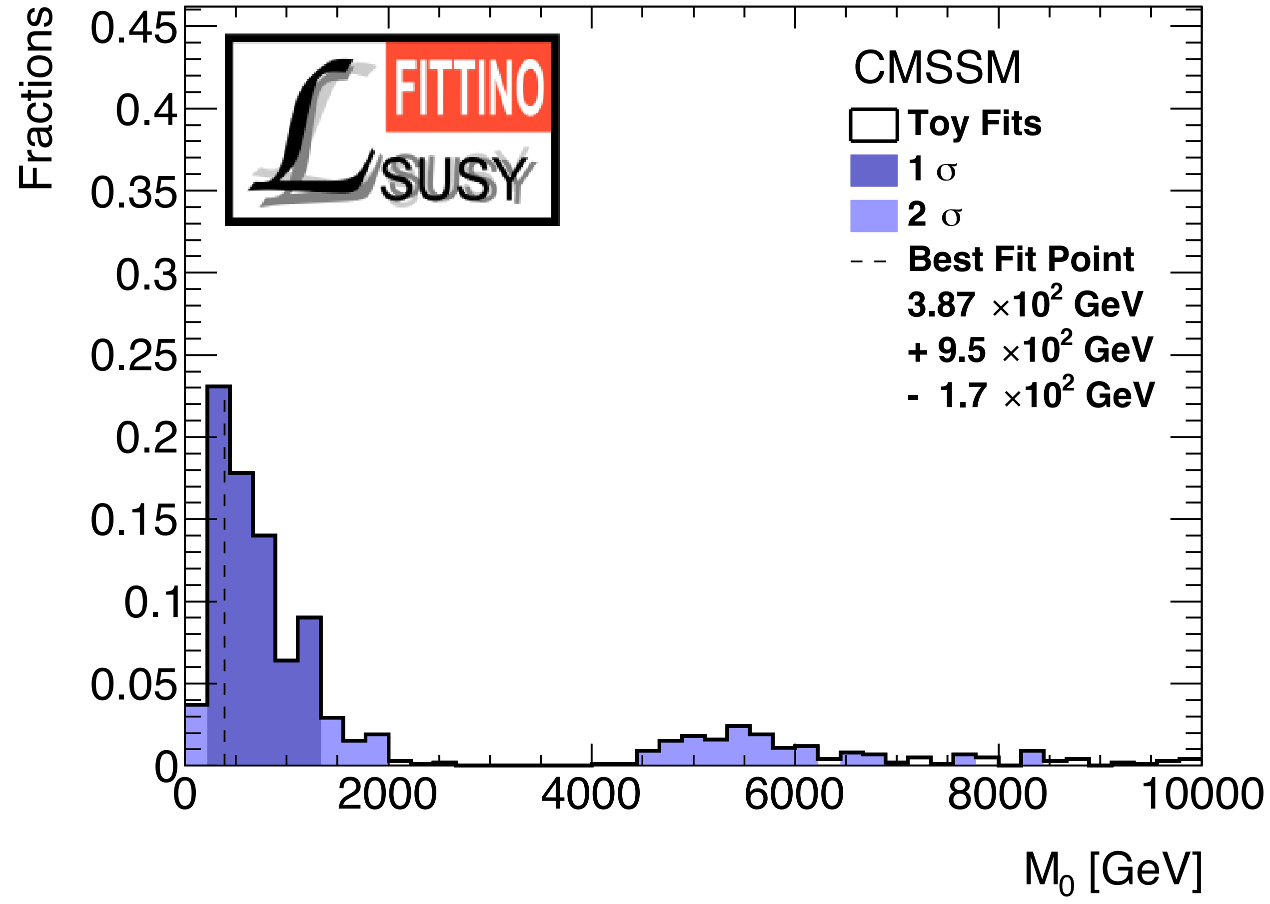
How well does the CMSSM describe the data quantitatively?

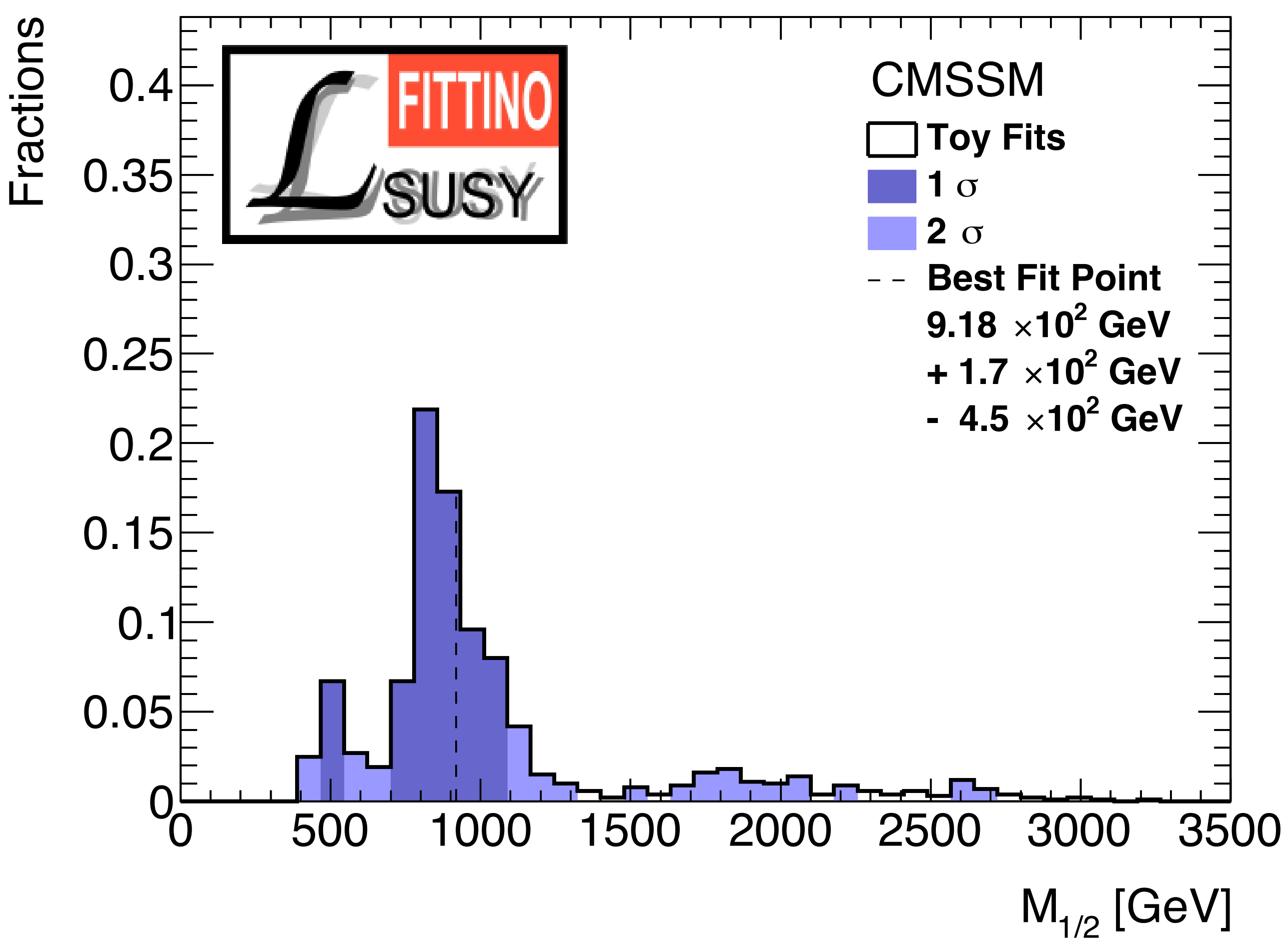
p-value	Toy fits
If the best fit point is realized in nature	Smearing observables around the best fit prediction
fitting the model to the measurements	and fitting the model to each of these toy measurements
how probable is it to get	how often do you get

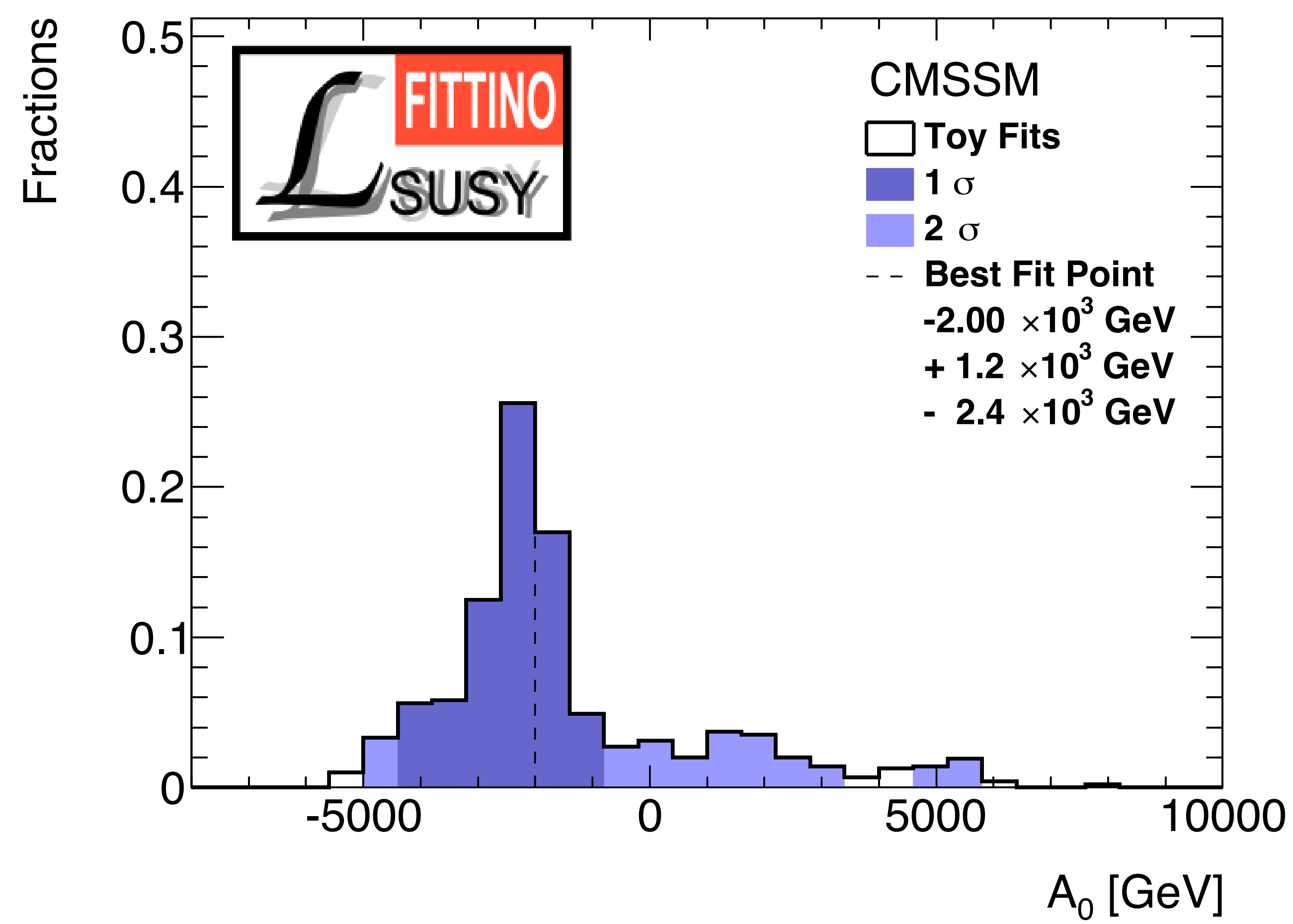
a minimal χ^2 at least as bad as the one observed?

- Very common in HEP
- Hasn't been done in global SUSY fits (extremely CPU intensive)

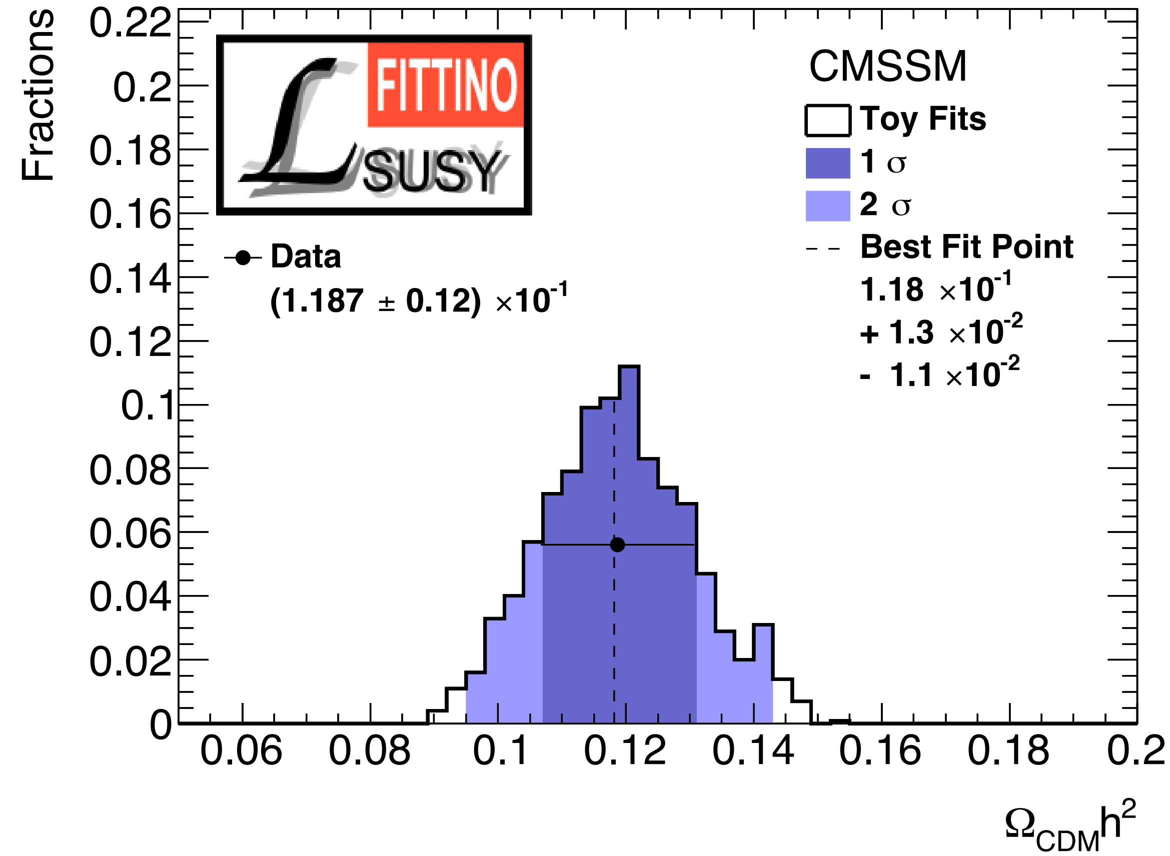
We repeat the fit described above **1000** times with smeared observables and get these best fit points.

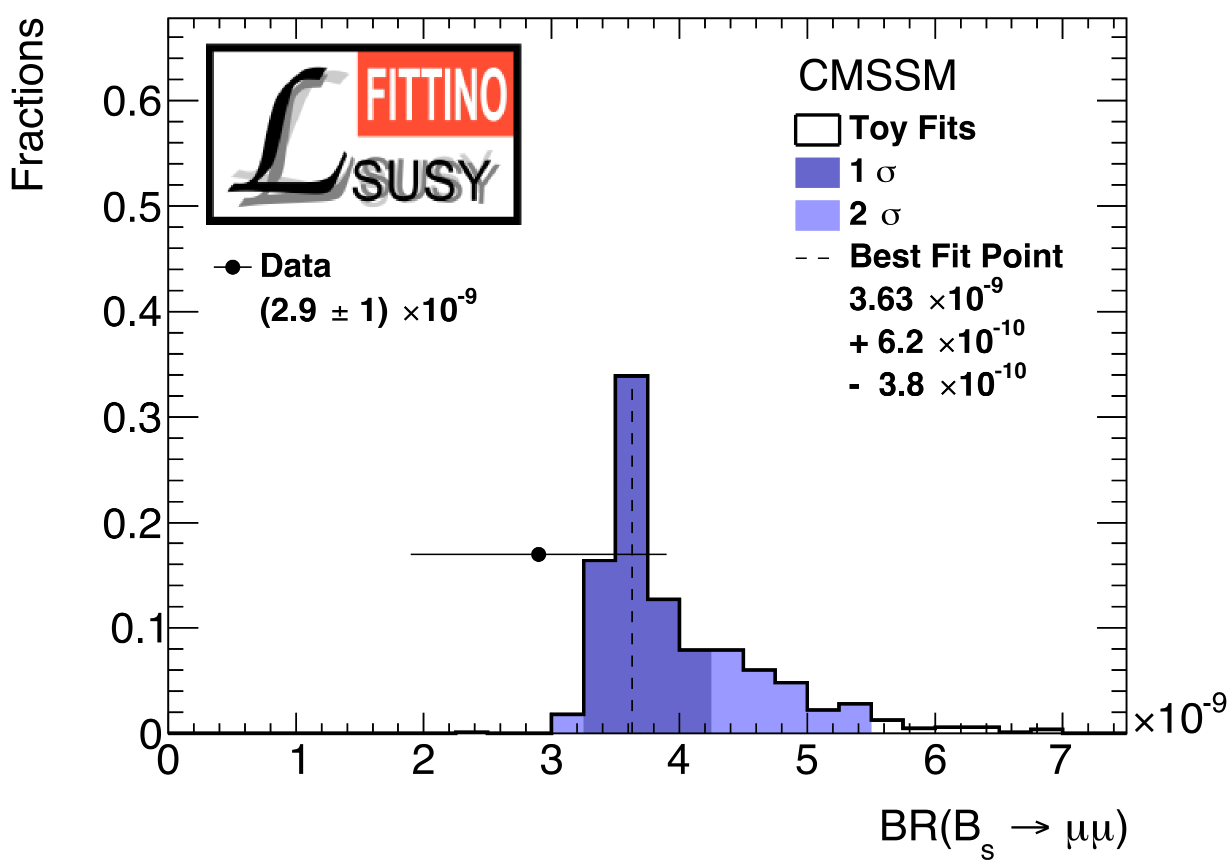


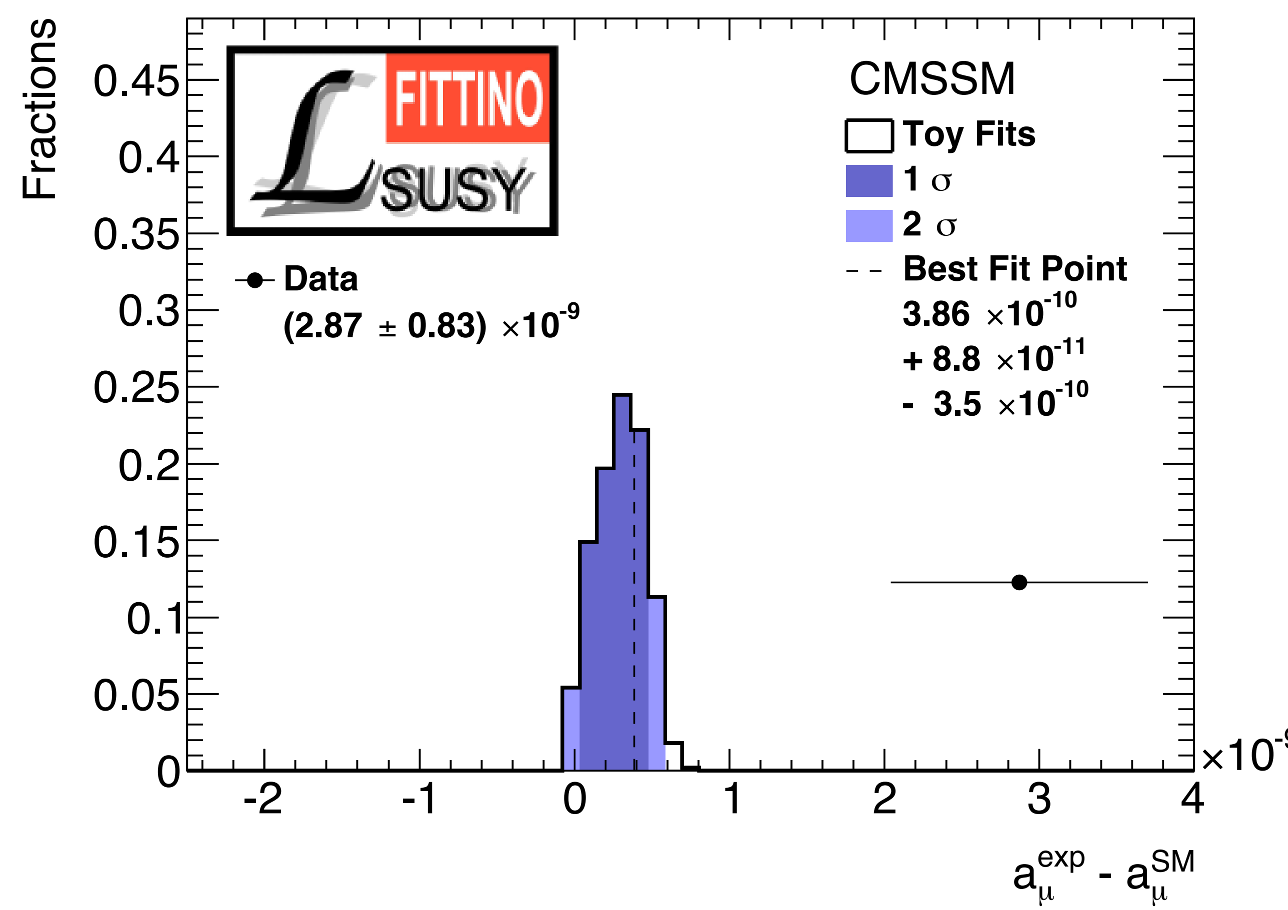




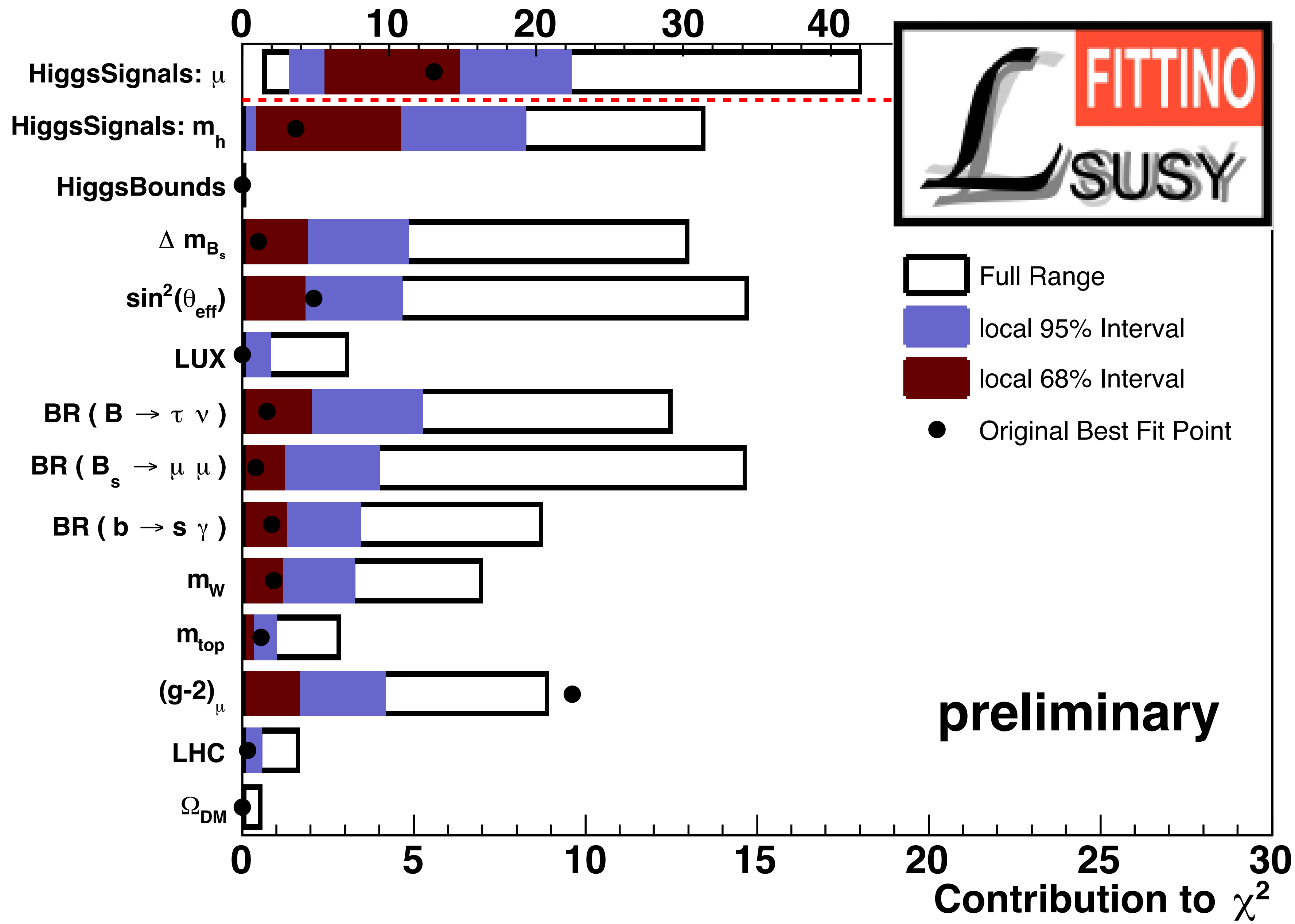
Corresponding predictions of observable values at the best fit points.



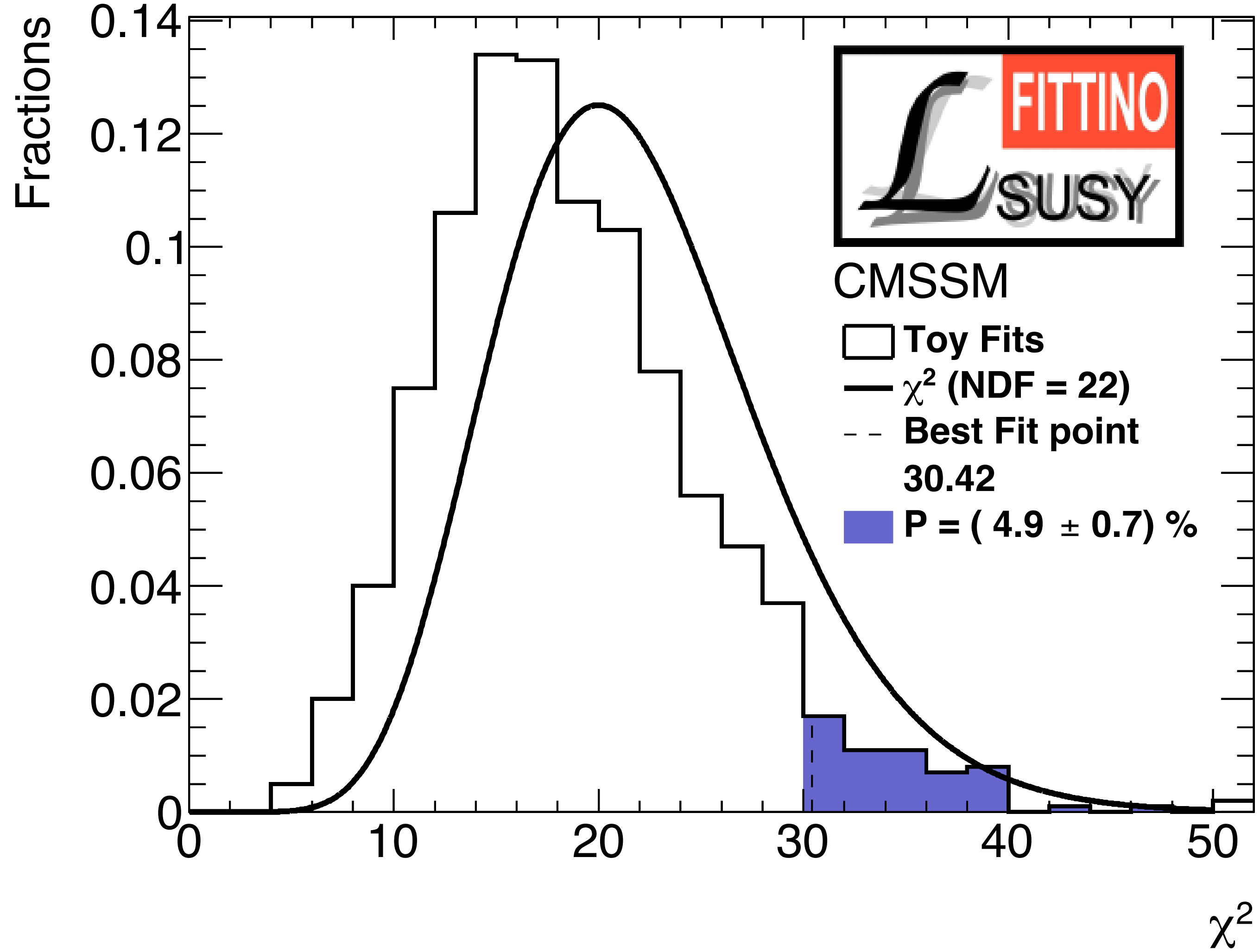




This corresponds to these individual χ2 contributions in the toy fits …



…and results in this p-Value!



- χ^2/ndf overestimates goodness of fit.

Impact of g-2 and Higgs rates

- Without g-2: P = (51 ± 3) %

  Low P-Value of baseline fit due to incompatibility of g-2 measurement with large sparticle masses

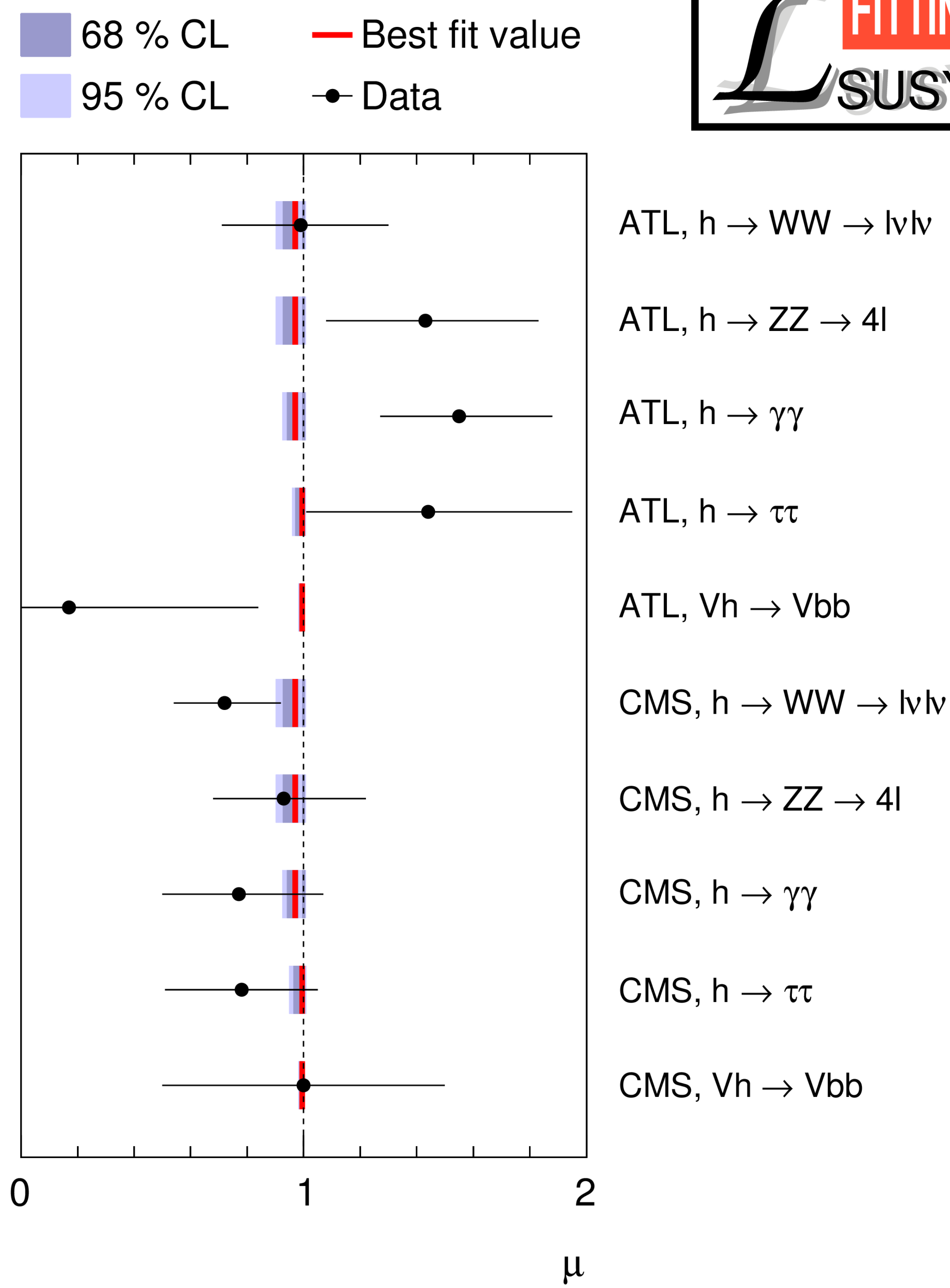
Impact of g-2 and Higgs rates

- **Without g-2: P = (51 ± 3) %**

  Low P-Value of baseline fit due to incompatibility of g-2 measurement with large sparticle masses

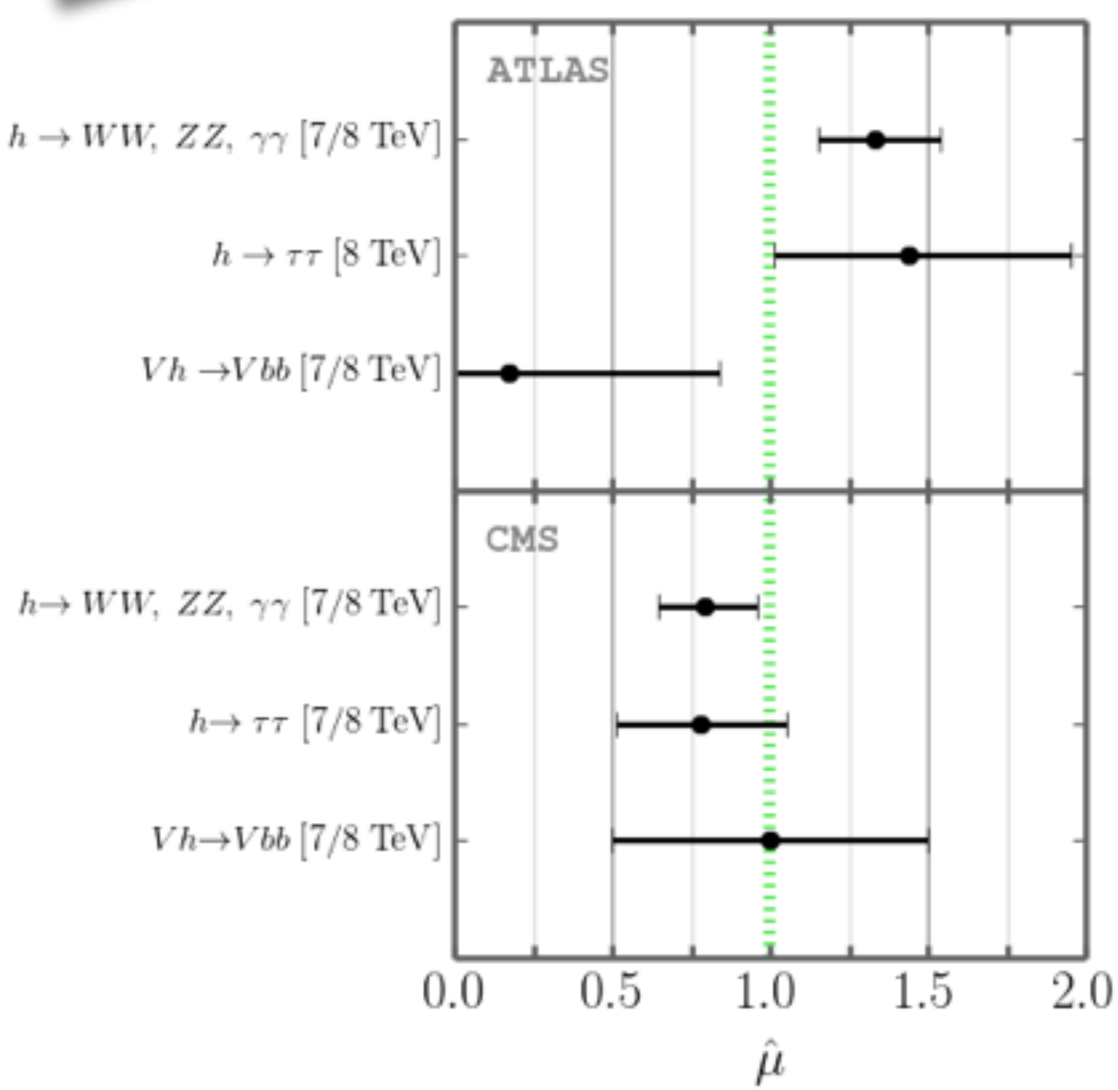
- **Without Higgs rates: P = (1.3 ± 0.4) %**
  - Higgs rates in decoupling limit very SM-like
  - LHC not able to distinguish from SM
  - Inclusion of Higgs rates improves fit quality despite some tension between ATLAS and CMS measurements

  (summer '14 results not included)



Impact of Higgs input parametrization

SmallObsSet

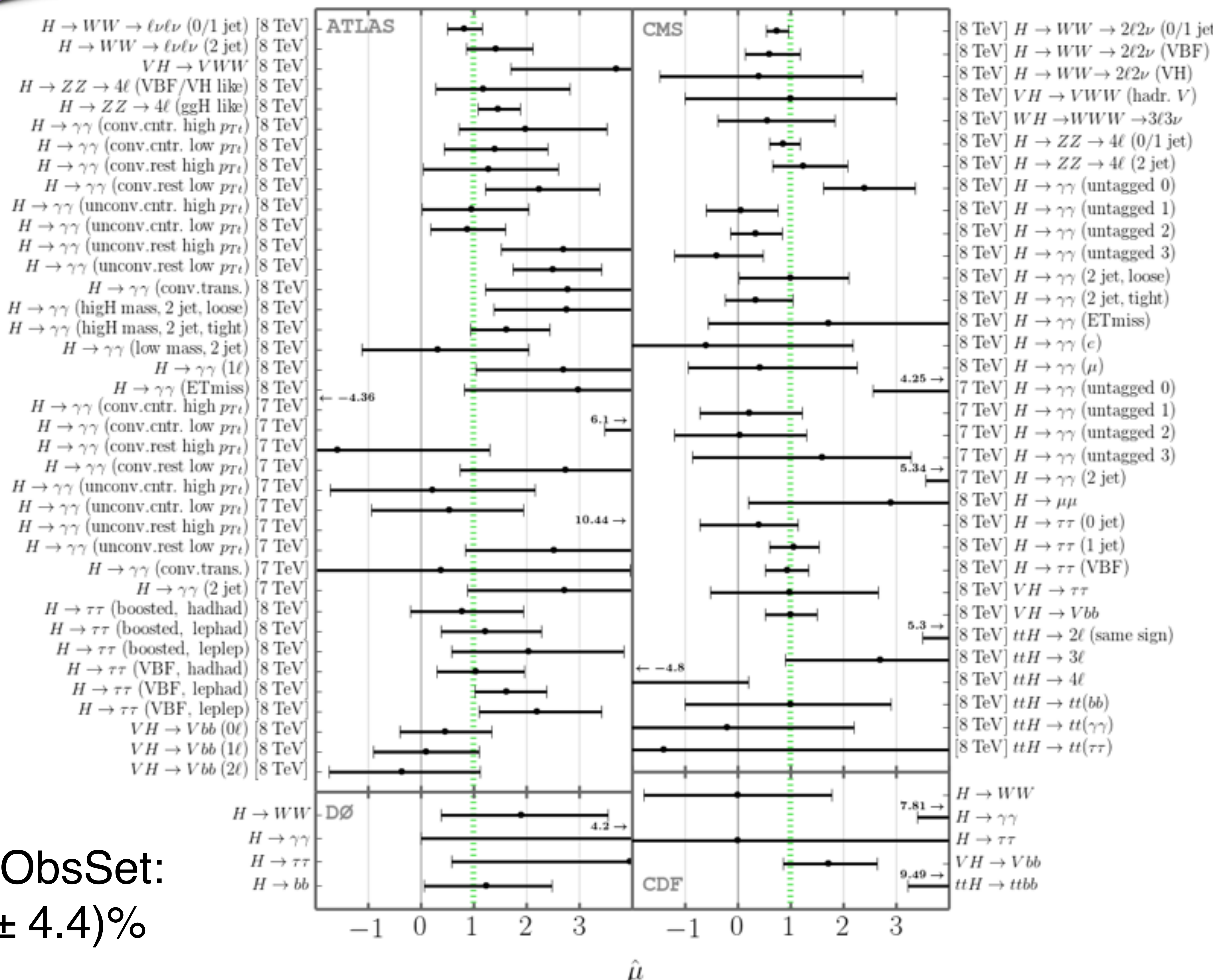


+ 2 mass measurements

With SmallObsSet: P = (1.9 ± 0.4)%

CMSSM punished for the common trend of the disagreement between ATLAS and CMS measurements in the three h —> VV channels.

LargeObsSet



With LargeObsSet:
$P = (41.6 \pm 4.4)\%$

+ 4 mass measurements

Example: Impact of split measurements

Measurements $(x_i, \sigma_i), i = 1..N$

Model predictions $a_i(\mathbf{P}), i = 1..N$ $a_n \equiv \cdots \equiv a_N$

Example: Impact of split measurements

Measurements $(x_i, \sigma_i), i = 1..N$

$$\chi^2_{\text{split}} = \sum_{i=1}^{N} \left(\frac{x_i - a_i}{\sigma_i} \right)^2$$

Model predictions $a_i(\mathbf{P}), i = 1..N$ $\quad a_n \equiv \cdots \equiv a_N$

Example: Impact of split measurements

Measurements $(x_i, \sigma_i), i = 1..N$

$$\chi^2_{\text{data}} = \sum_{i=n}^{N} \left(\frac{x_i - \bar{x}}{\sigma_i} \right)^2$$

$$\chi^2_{\text{split}} = \sum_{i=1}^{N} \left(\frac{x_i - a_i}{\sigma_i} \right)^2$$

Combination $(\bar{x}, \sigma)$

$$\chi^2_{\text{combined}} = \sum_{i=1}^{n-1} \left(\frac{x_i - a_i}{\sigma_i} \right)^2 + \left(\frac{\bar{x} - a_n}{\sigma} \right)^2$$

Model predictions $a_i(\mathbf{P}), i = 1..N \quad a_n \equiv \cdots \equiv a_N$

$$\chi^2_{\mathrm{split}} = \chi^2_{\mathrm{combined}} + \chi^2_{\mathrm{data}}$$

$$\frac{\chi^2_{\mathrm{split}}}{\mathrm{ndf}_{\mathrm{split}}} = \frac{\chi^2_{\mathrm{data}}}{\mathrm{ndf}_{\mathrm{data}} + \mathrm{n}} + \frac{\chi^2_{\mathrm{combined}}}{\mathrm{ndf}_{\mathrm{combined}} + \mathrm{N} - \mathrm{n}}$$

The more uncombined measurements are used

- the less depends the p-value on the agreement between data and model
- the more depends the p-value on the agreement within the data.

Especially, for n fixed and N —> ∞ :

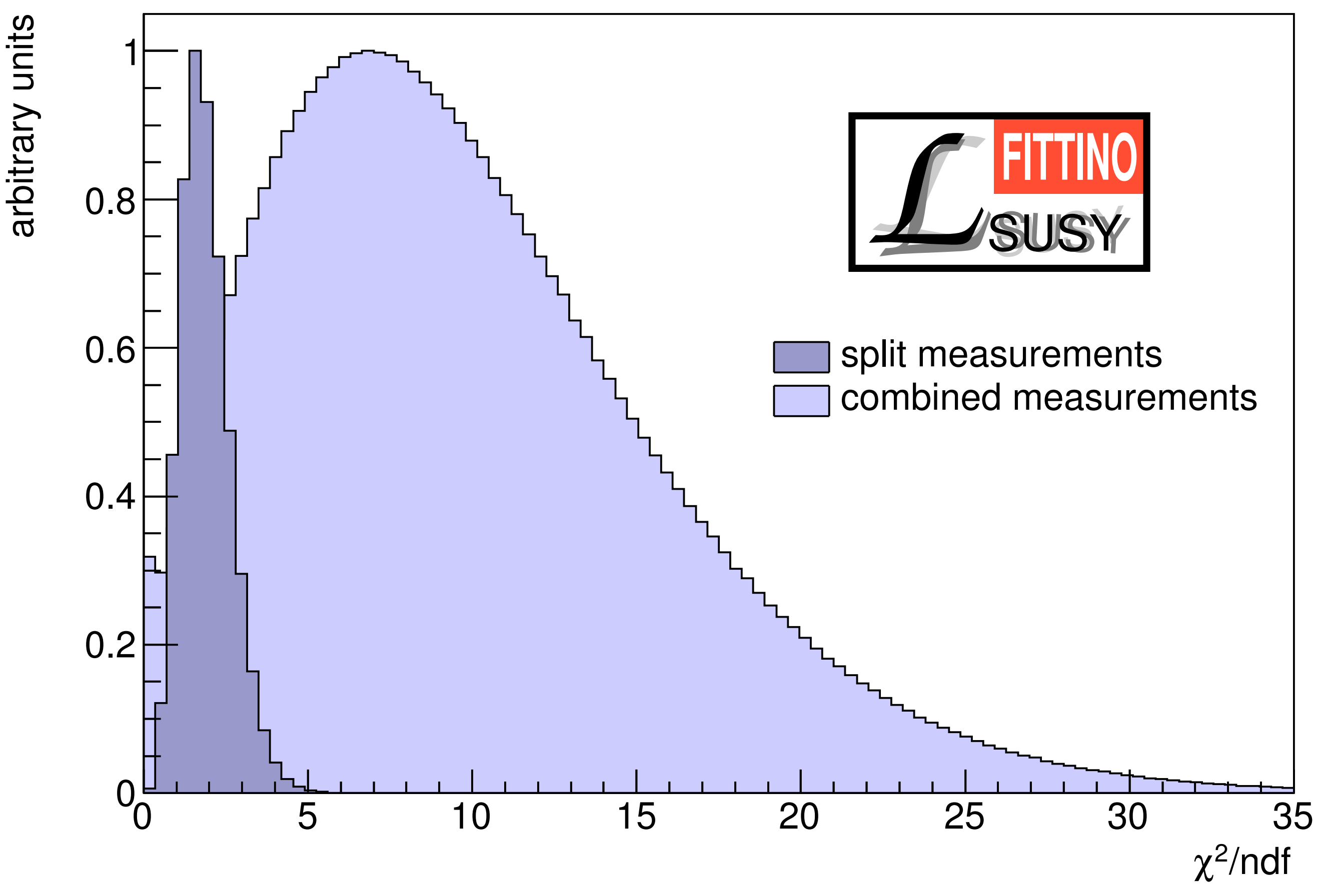
$$\frac{\chi^2_{\mathrm{split}}}{\mathrm{ndf}_{\mathrm{split}}} = \frac{\chi^2_{\mathrm{data}}}{\mathrm{ndf}_{\mathrm{data}}}$$

- Agreement within the data is improbable to be significantly bad
- $$\frac{\chi^2_{\mathrm{data}}}{\mathrm{ndf}_{\mathrm{data}}} = 1$$ expected
- Most of the time p-value will get larger when using uncombined measurements hiding deviations between model and data

Numerical example:

- n=1
- N=10
- 3σ deviation between true value and model prediction

“Dilution of the p-value”

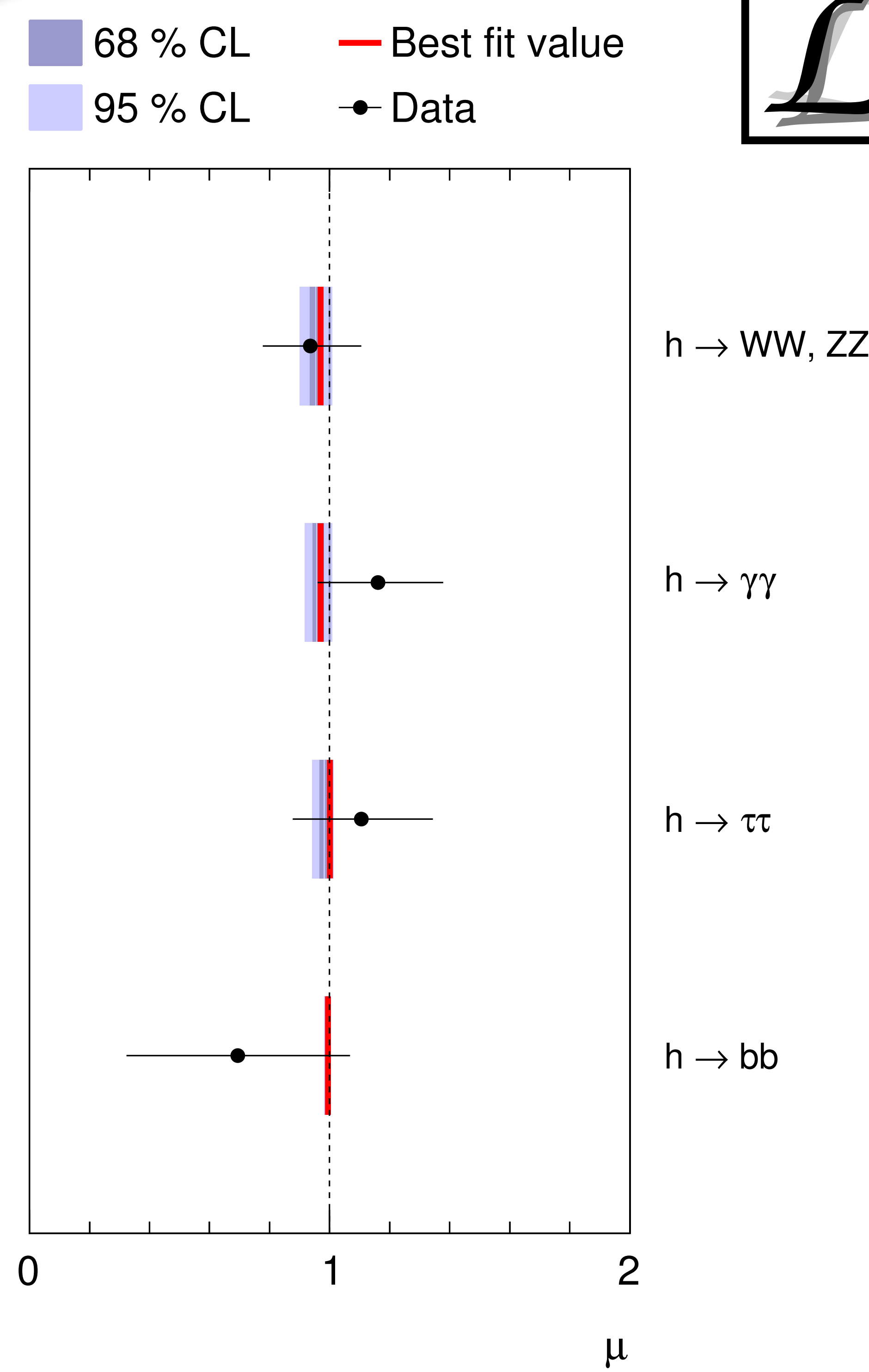


Effect very well visible for LargeObsSet.

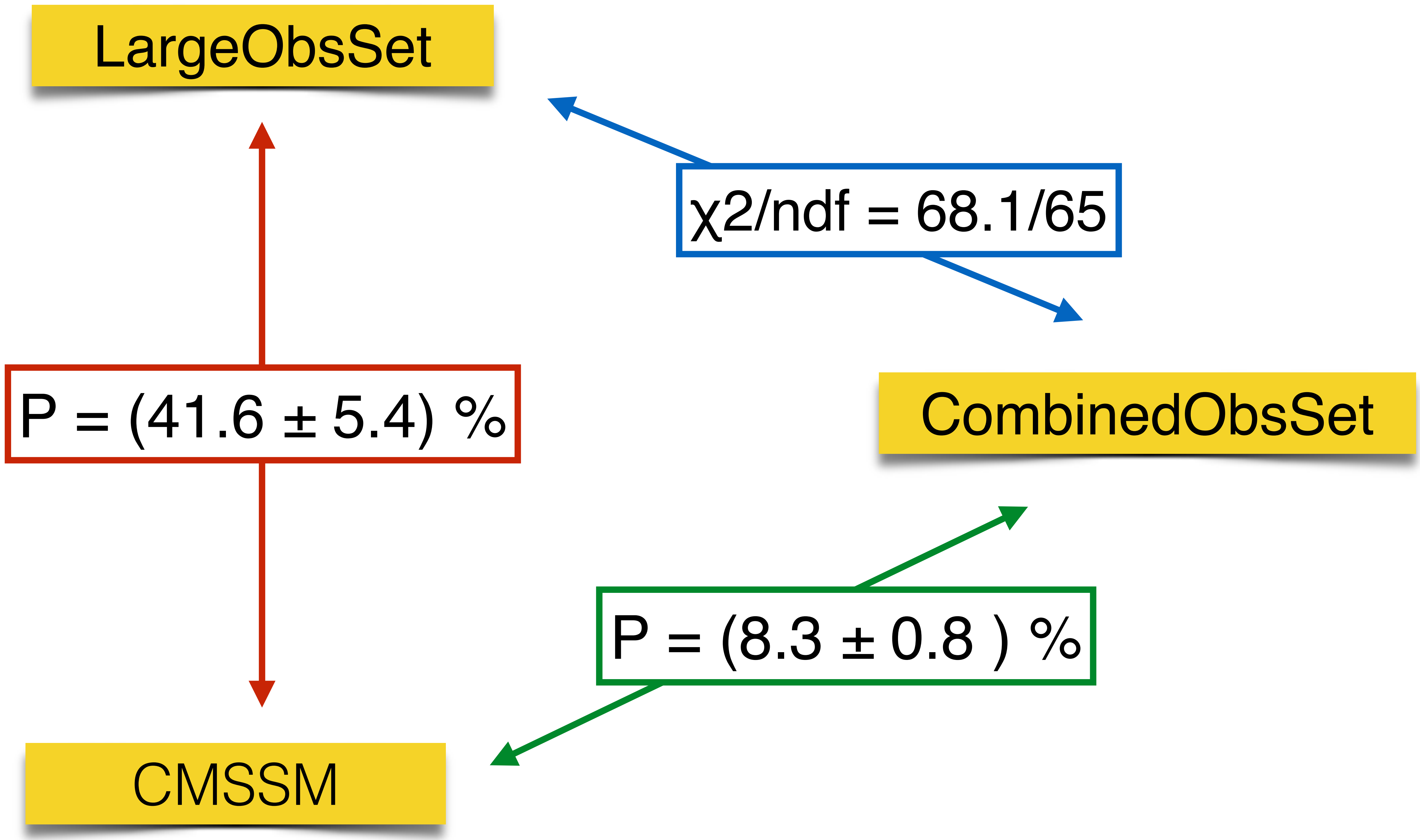
ATLAS + CMS combination

CombinedObsSet

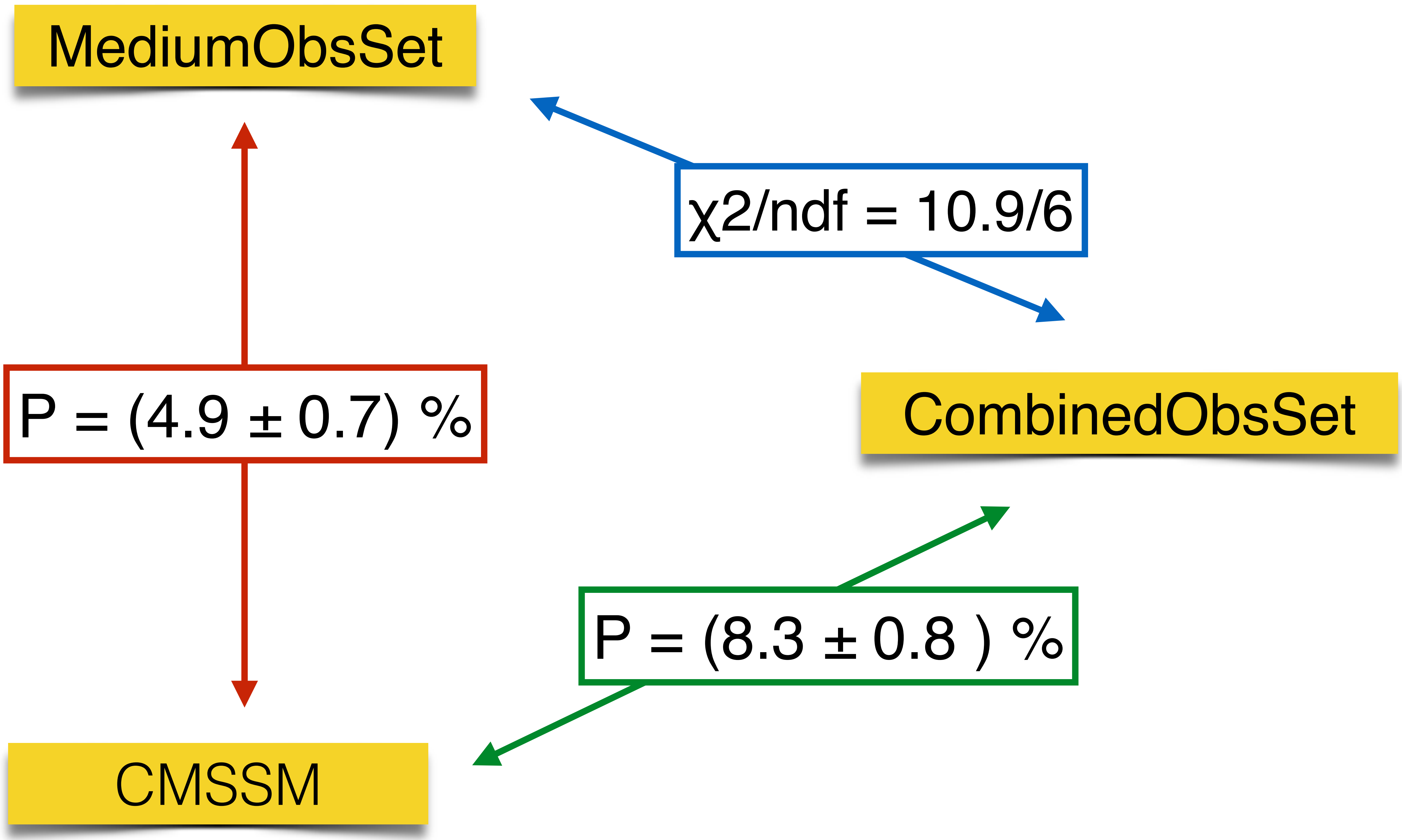
- On the other hand if there is some tension within the data, the innocent model is punished for that (MediumObsSet, SmallObsSet)
- In order to incorporate our assumption that ATLAS and CMS measured the same Higgs boson we produce a private ATLAS+CMS combination.
- We also assume that custodial symmetry is preserved but do not assume that h —> γγ is connected to h—>WW and h—>ZZ.



+ 1 mass measurement



Impact of agreement within the data on p-Value has been removed by doing the combination first.



Impact of agreement within the data on p-Value has been removed by doing the combination first.

Is the CMSSM excluded?

[arXiv:1410.6035]

preliminary

	χ^2/ndf	naive p-Value (%)	p-Value (%)
ObsSet without Higgs rates	15.5/9	7.8	1.3 ± 0.4
SmallObsSet	27.1/16	4.0	1.9 ± 0.4
MediumObsSet	30.4/22	10.8	4.9 ± 0.7
CombinedObsSet	17.5/13	17.7	8.3 ± 0.8
LargeObsSet	101.1/92	24.3	41.6 ± 4.4
MediumObsSet without g-2	18.1/21	64	51 ± 3

The CMSSM - a zombie?

Summary of part II and outlook

- For the first time p-values for a SUSY model have been calculated using global toy fits
- This gives an appropriate measure for the agreement between the model and the selected data
- p-value depends on (Higgs) observable parametrization
- Using our favorite Higgs parametrization based on a private ATLAS + CMS combination we find a p-Value of (8.3±0.8)% for the CMSSM.
- Applying the method to more general models which e.g. decouple the electroweak and strong sector will quantify how much better they perform.