



# Search for $t\bar{t}H$ events using the Matrix Element Method

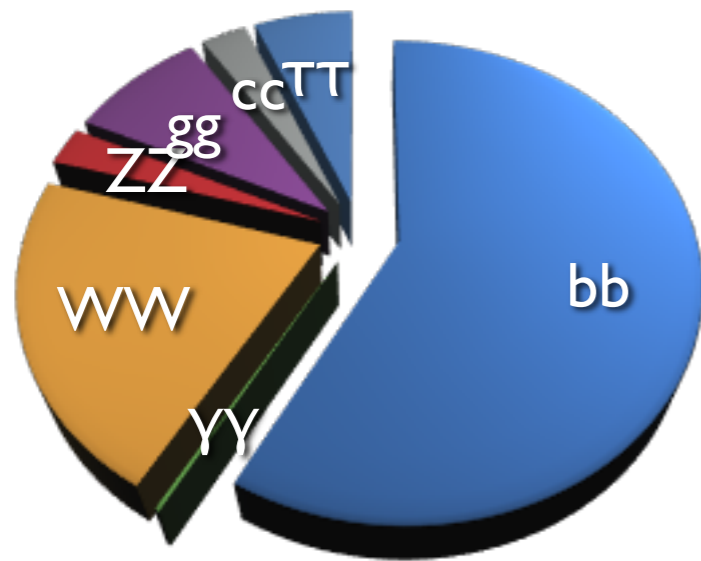
Lorenzo Bianchini  
IPP (ETH Zurich)

**GDR Terascale @ Heidelberg, 12 Dec. 2014**



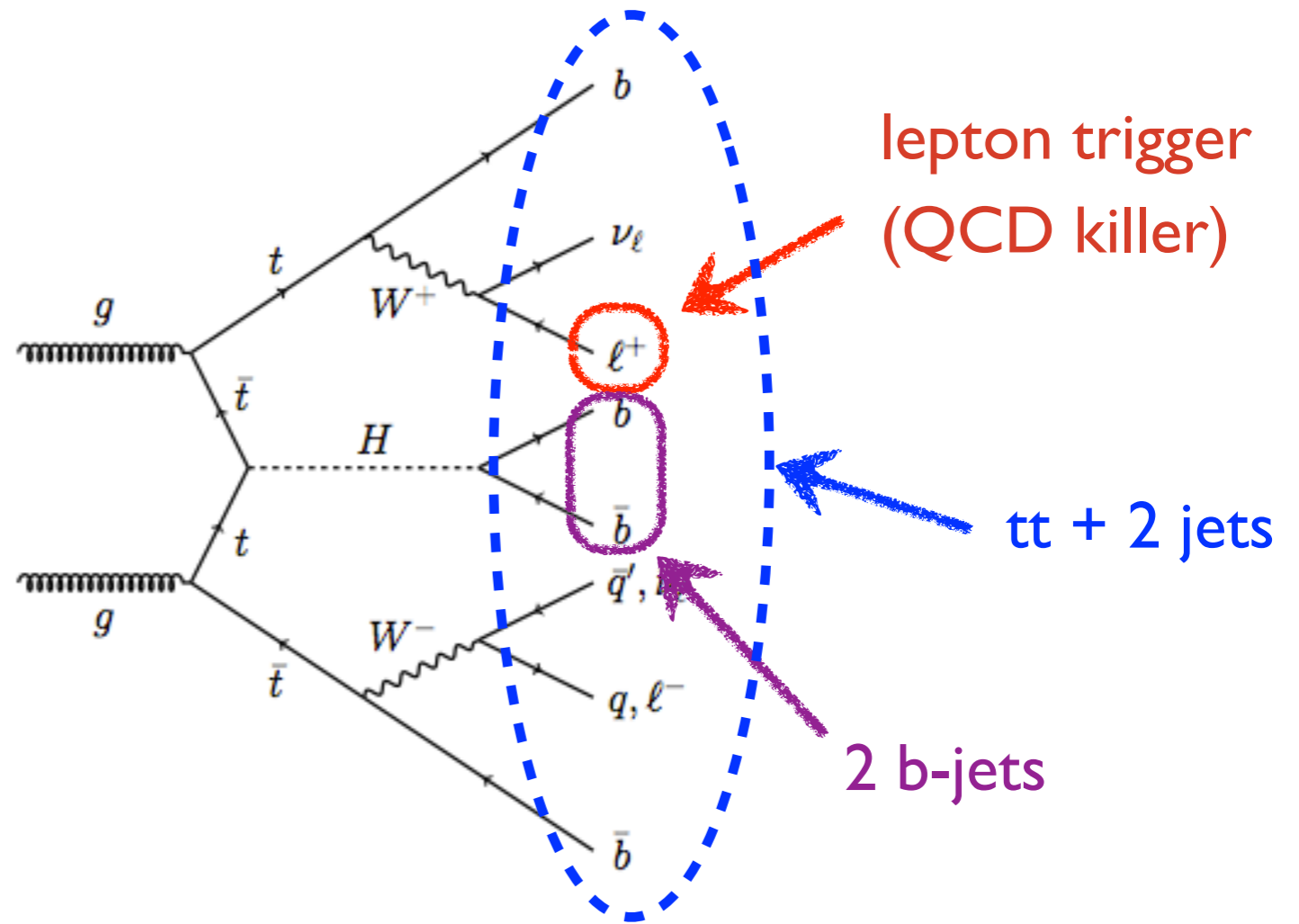
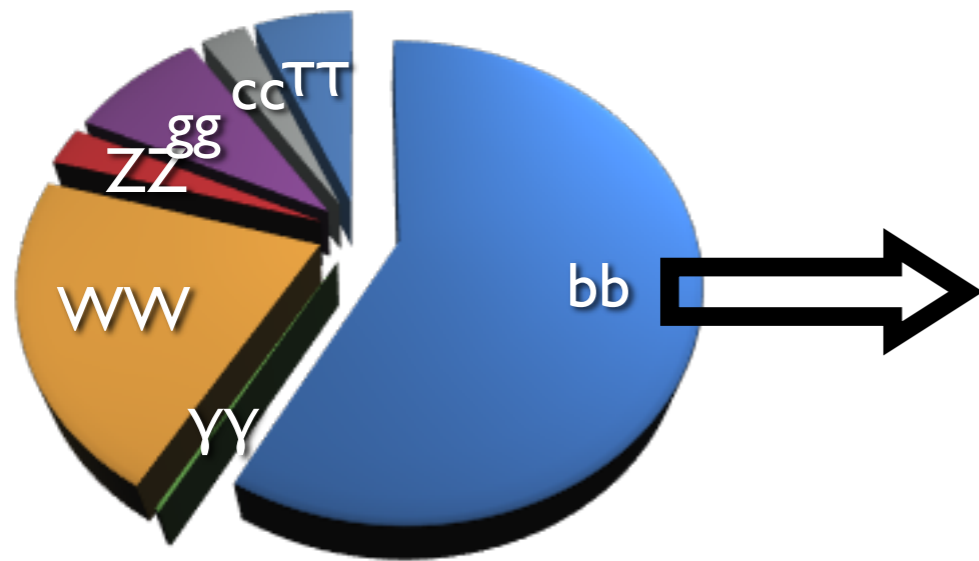
# Tackling $t\bar{t}H$ , $H \rightarrow bb$ final states

$$\sigma_{t\bar{t}H} \times \underbrace{\text{BR}(H \rightarrow X)} \propto |y_t|^2$$



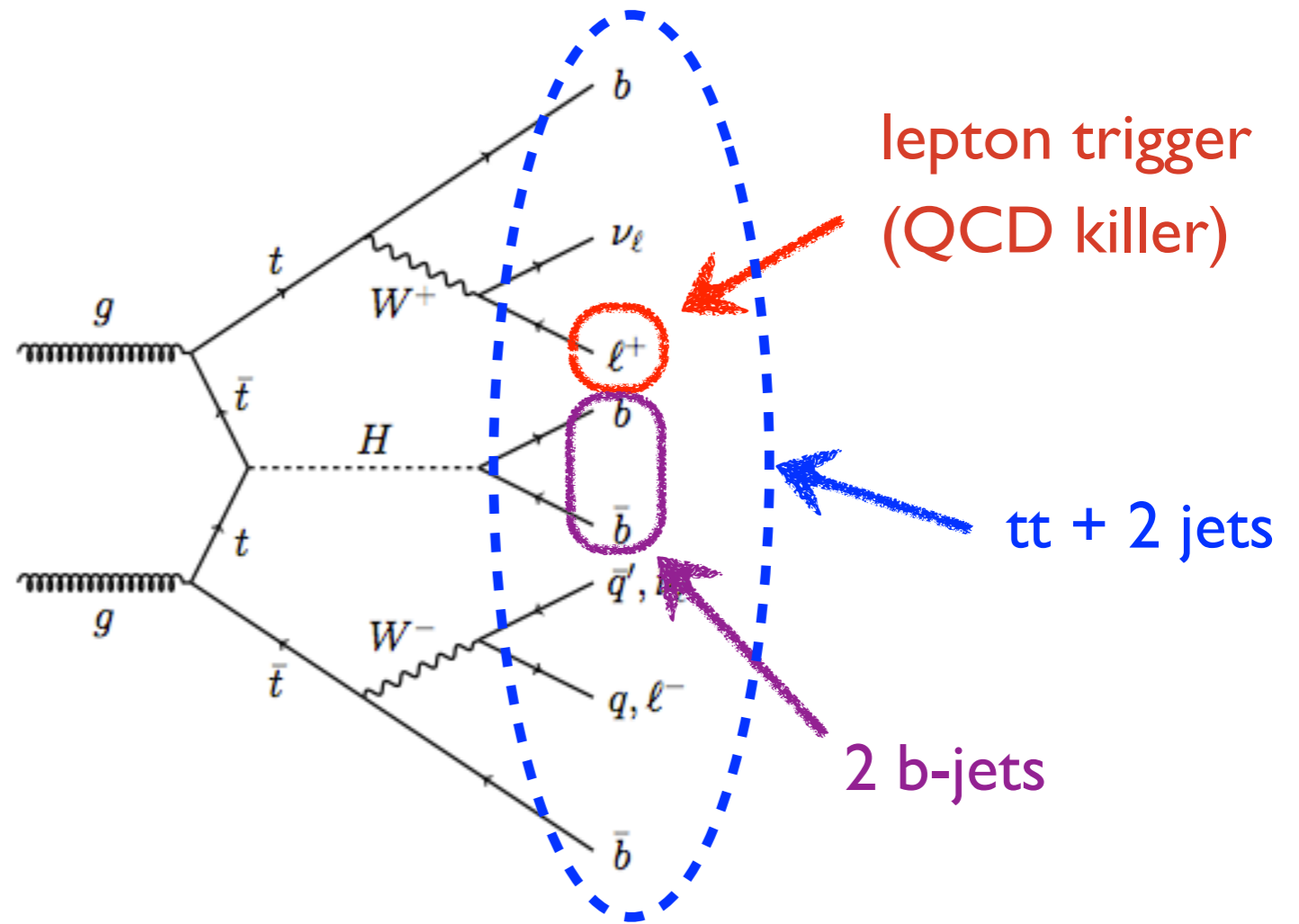
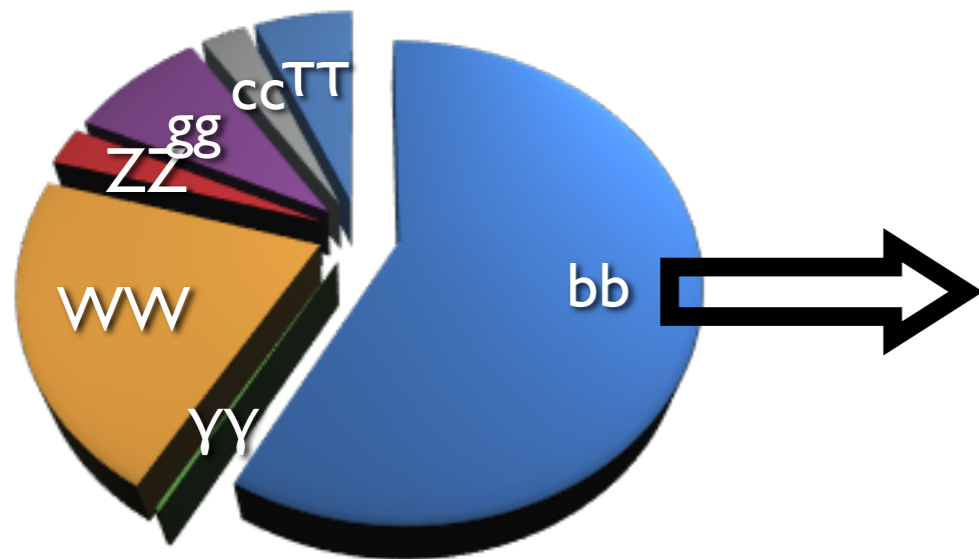
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# Tackling $t\bar{t}H$ , $H \rightarrow b\bar{b}$ final states

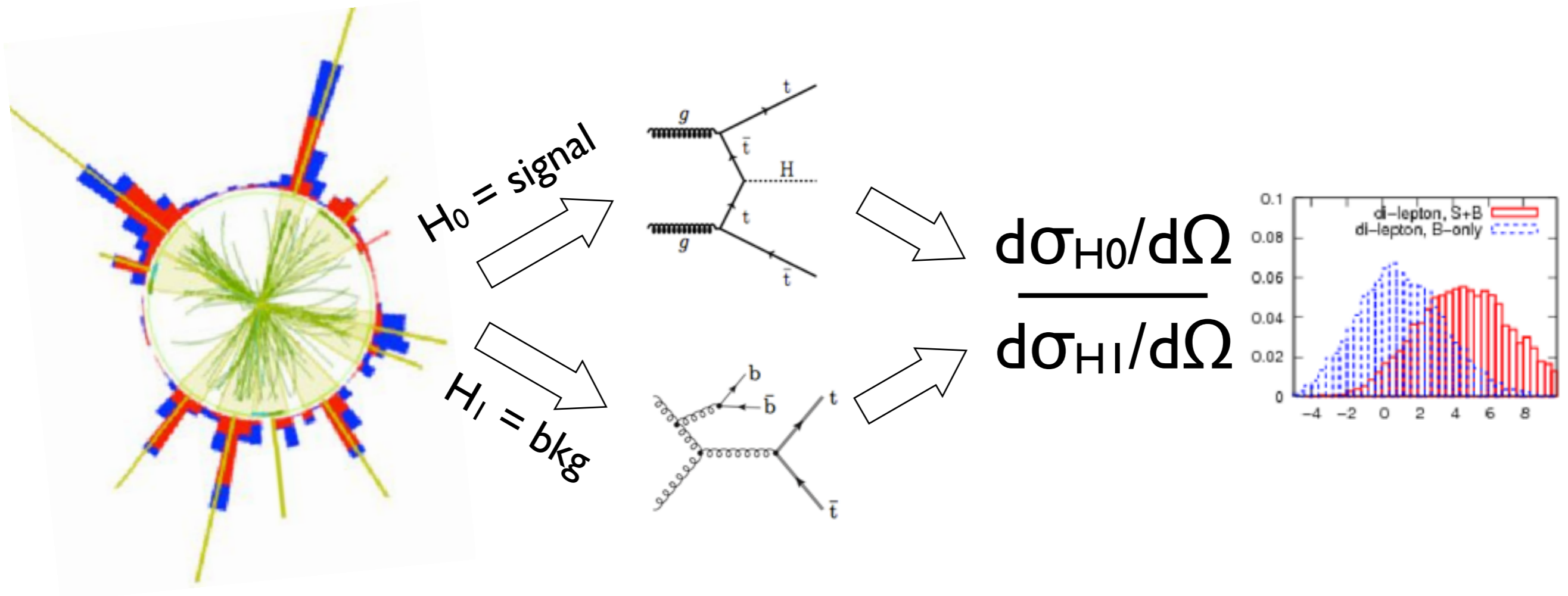
$$\sigma_{t\bar{t}H} \times \text{BR}(H \rightarrow X) \propto |y_t|^2$$



- **$t\bar{t}+b\bar{b}$  background** irreducible and plagued by large unc. ( $\approx 35\%$  @NLO)
  - ▶ counting experiment not feasible
  - ▶ extra handles:  $m_{b\bar{b}}$  spectrum
  - ▶ separable using mass peak ?
- Not so easy: b's from top quarks complicate mass peak extraction
  - ▶ **combinatorial** self-background  $\Rightarrow$  need for **multidimensional** analyses

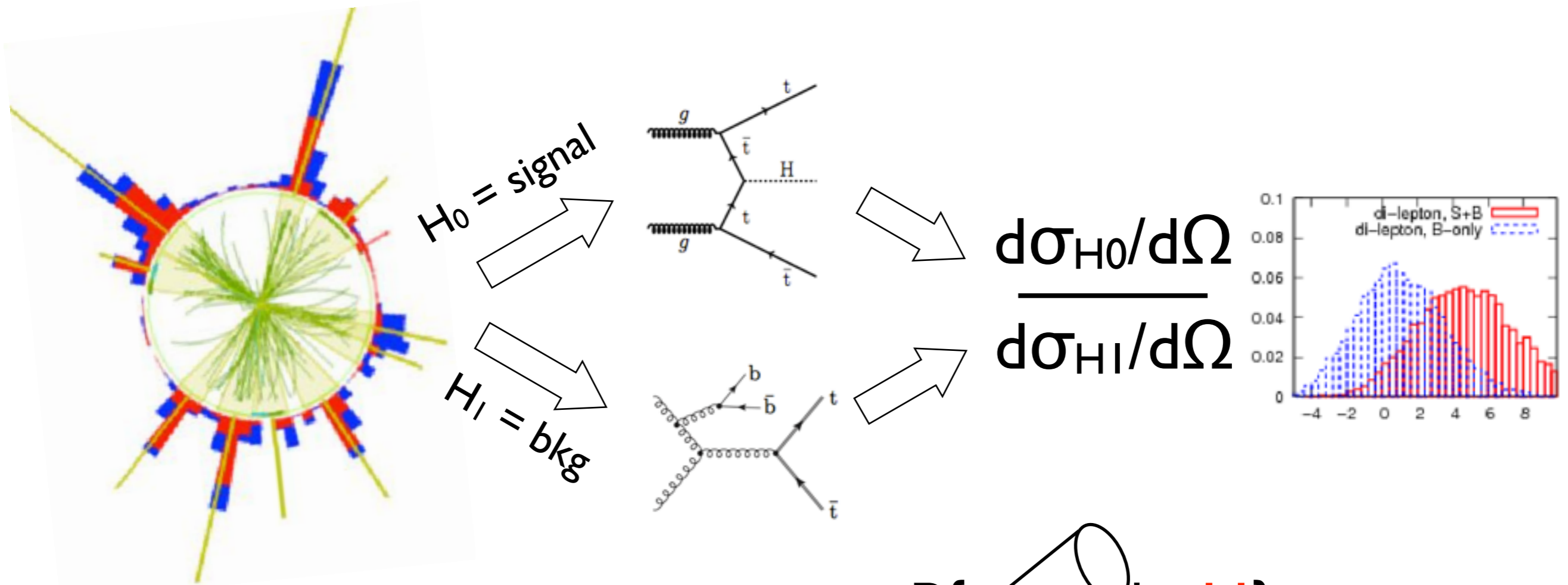
# A possible analysis strategy

Matrix Element Method (**MEM**) suited to the task



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Matrix Element Method (**MEM**) suited to the task



Virtues of the MEM:

- ▶ combinatorial issue properly addressed:
- ▶ optimal usage of kinematics and dynamics for S/B separation (**Neyman lemma**)

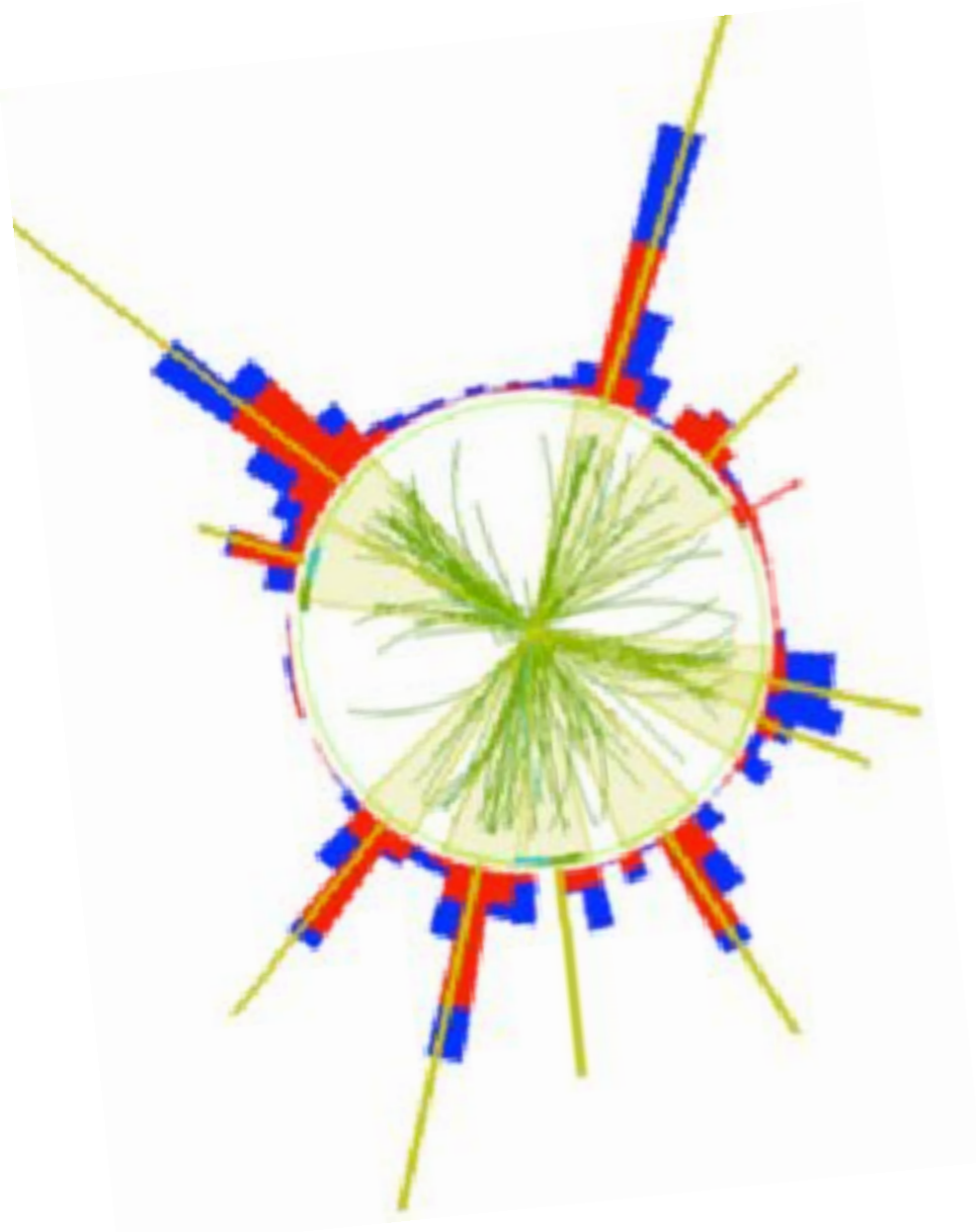
$$P\left\{ \begin{array}{c} \text{cone} \\ \text{cone} \\ \text{cone} \end{array} \middle| t, H \right\} = P\left\{ \begin{array}{c} \text{cone} \\ \text{cone} \\ \text{cone} \end{array} \middle| \begin{array}{c} H \\ t \\ t \end{array} \right\} + P\left\{ \begin{array}{c} \text{cone} \\ \text{cone} \\ \text{cone} \end{array} \middle| \begin{array}{c} t \\ H \\ H \end{array} \right\}$$

# Master formula

$$d\sigma_{S/B}/d\vec{Y} = w_{S/B}(\vec{Y}) = \int d\Phi_{\mathbf{X}} dx_a dx_b f(x_a, x_b) \delta^4(x_a P_a + x_b P_b - \vec{X}) |\mathcal{M}_{S/B}(\vec{X})|^2 w(\vec{Y}, \vec{X})$$

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reconstructed jet and  
lepton momenta

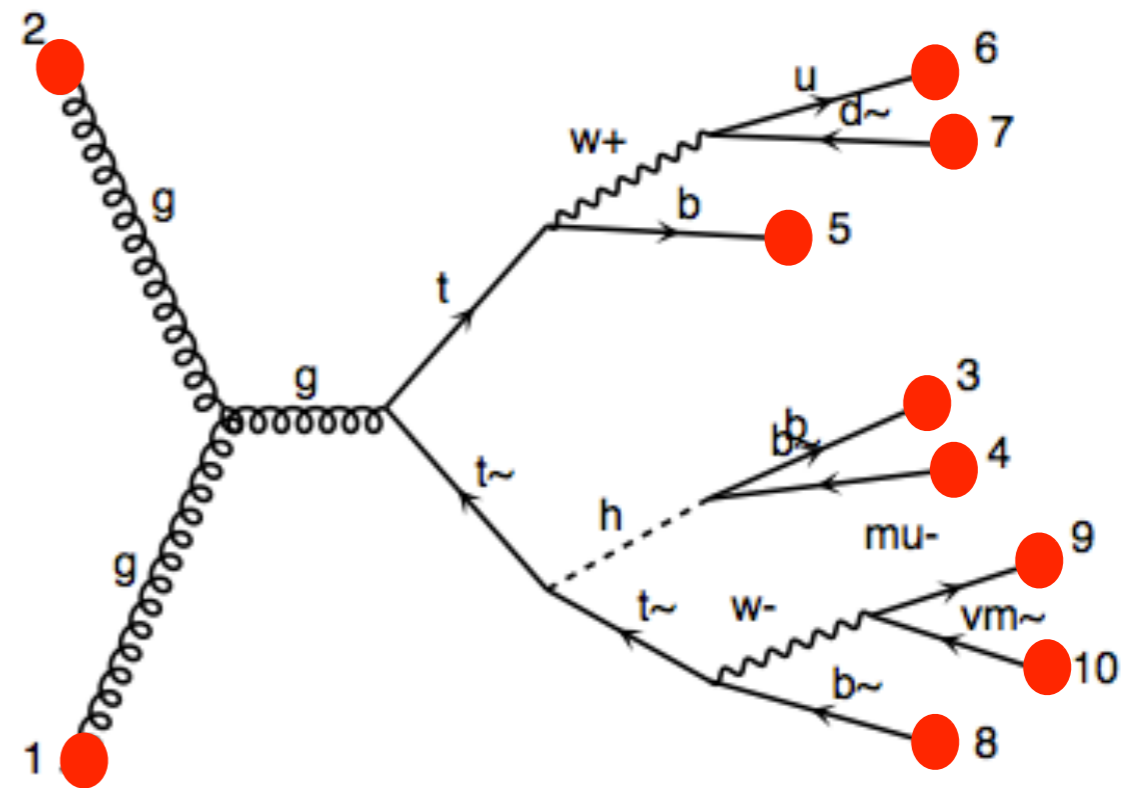


# Master formula

$$d\sigma_{S/B}/d\vec{Y} = w_{S/B}(\vec{Y}) =$$

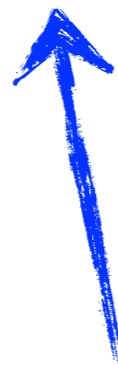
$$\int d\Phi_{\mathbf{x}} dx_a dx_b f(x_a, x_b) \delta^4(x_a P_a + x_b P_b - \vec{\mathbf{X}}) |\mathcal{M}_{S/B}(\vec{\mathbf{X}})|^2 w(\vec{Y}, \vec{\mathbf{X}})$$

generated particles;  
numerical integration  
(VEGAS)

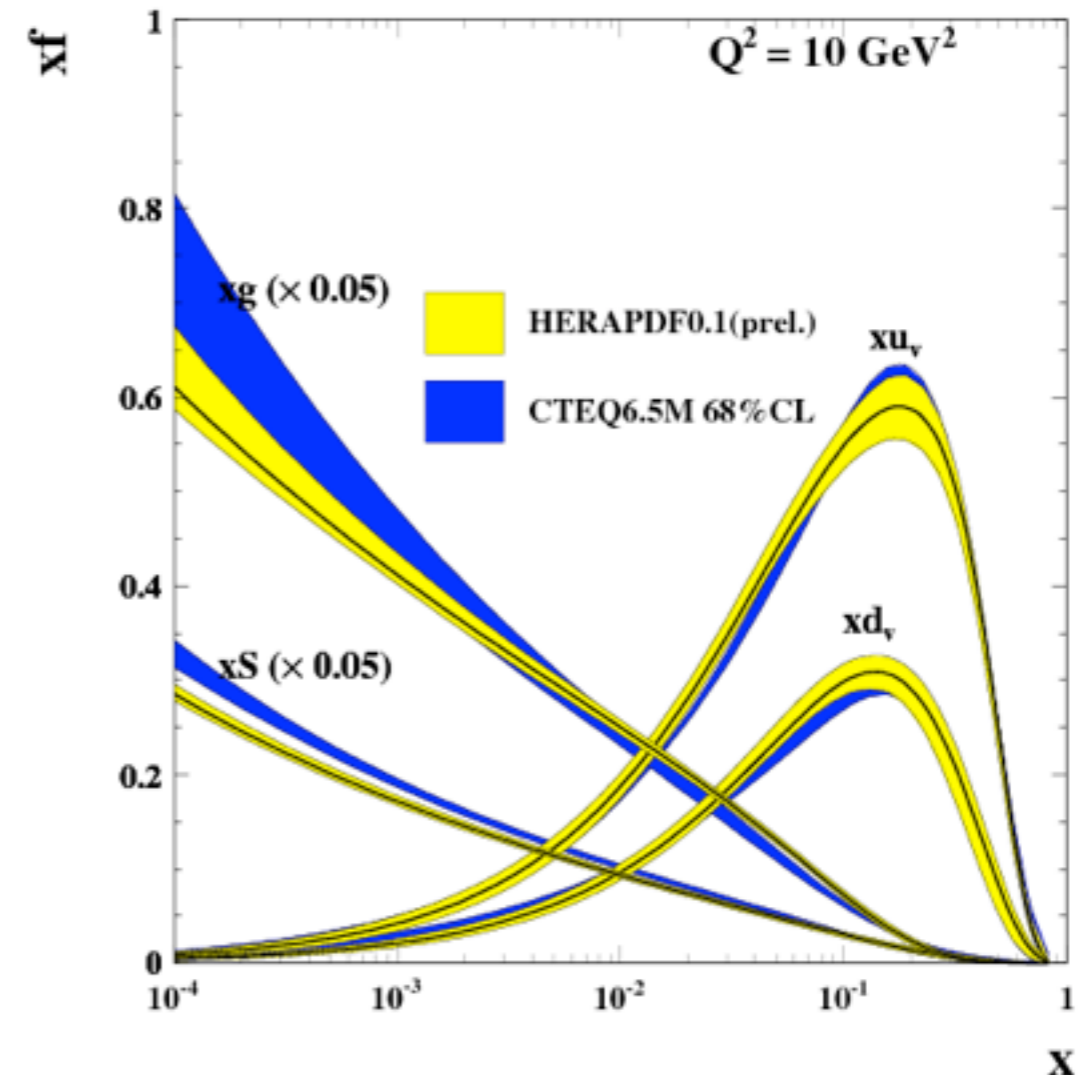


# Master formula

$$d\sigma_{S/B}/d\vec{Y} = w_{S/B}(\vec{Y}) = \int d\Phi_{\mathbf{X}} dx_a dx_b \boxed{f(x_a, x_b)} \delta^4(x_a P_a + x_b P_b - \vec{X}) |\mathcal{M}_{S/B}(\vec{X})|^2 W(\vec{Y}, \vec{X})$$

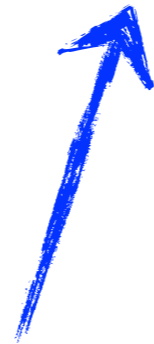


PDF from LHAPDF  
(CTEQ6.5m set)



# Master formula

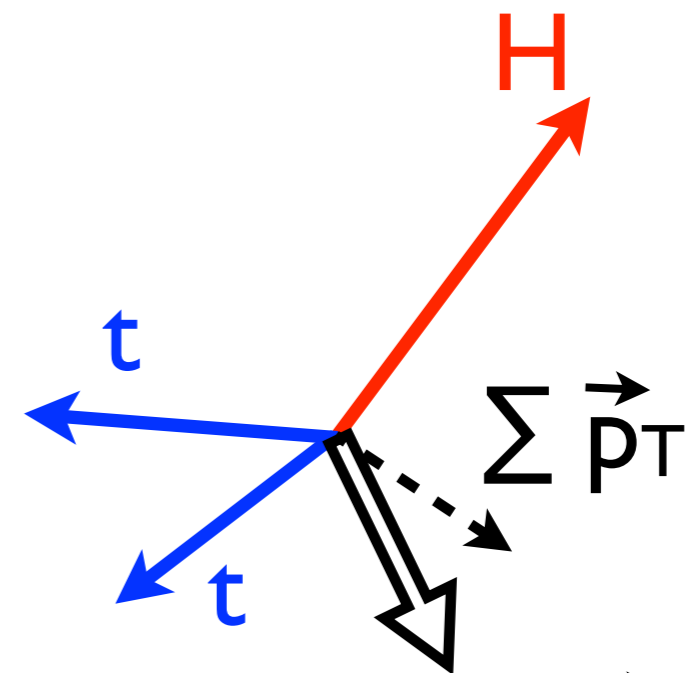
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Momentum balancing

N.B.:  $\sum \vec{p}_T = 0$  @ LO

Here:  $p_T$  balance not imposed.  
 $\sum \vec{p}_T$  constrained to measured recoil via a TF



$$\text{recoil} = -\vec{jets} - \vec{lep} - \vec{E}_T^{\text{miss}}$$



# Master formula

$$d\sigma_{S/B}/d\vec{Y} = w_{S/B}(\vec{Y}) = \int d\Phi_{\mathbf{x}} dx_a dx_b f(x_a, x_b) \delta^4(x_a P_a + x_b P_b - \vec{X}) |\mathcal{M}_{S/B}(\vec{X})|^2 w(\vec{Y}, \vec{X})$$

$$|\mathcal{M}(\mathbf{x})|^2 \propto \mathcal{M}_{ME}(\mathbf{x}) \cdot \mathcal{M}_{BW}(\mathbf{x}) \cdot \mathcal{M}_{\Gamma}(\mathbf{x})$$

calculated by  
OpenLoops  
(LO in  $\alpha_s$ )

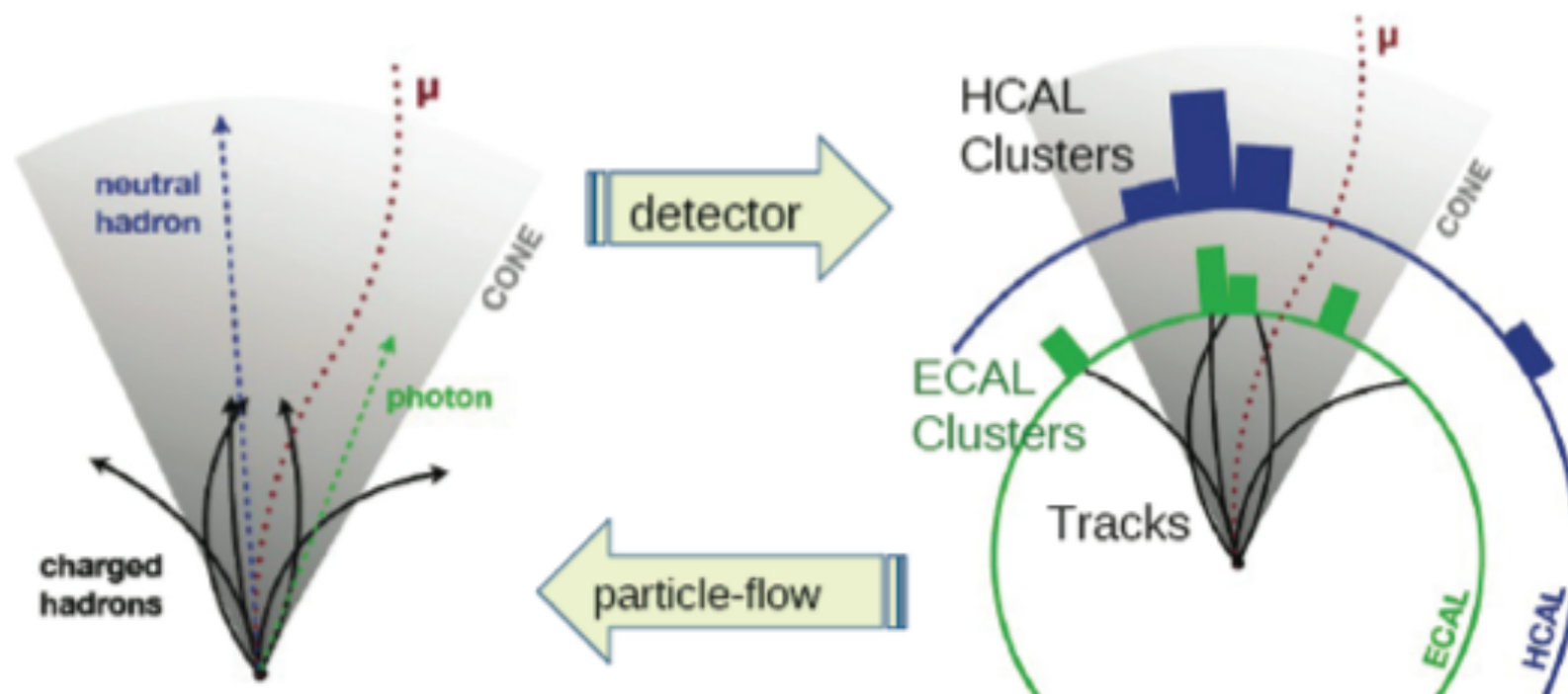
Narrow-width  
approximation

analytical  
(w/o spin-  
correlations)

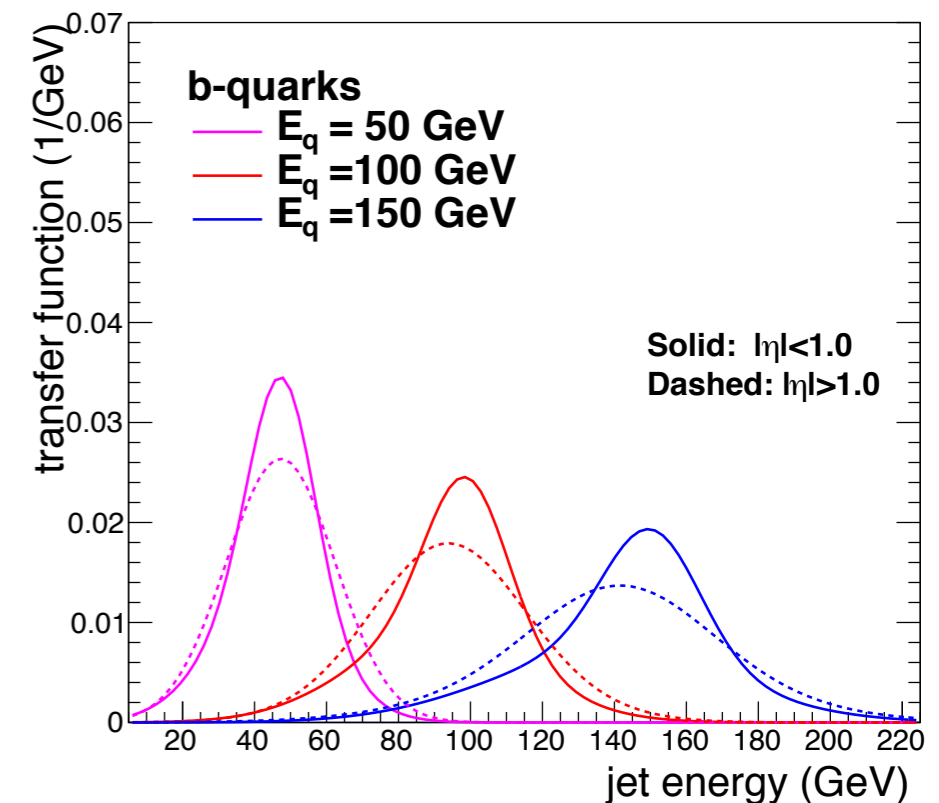
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## Transfer function



CMS Simulation  $\sqrt{s}=8$  TeV



# Implementation

SL

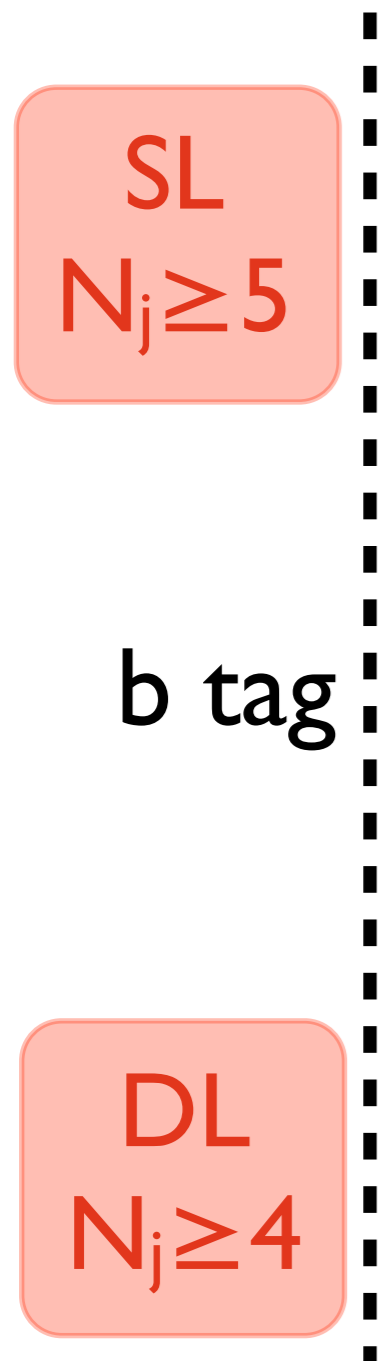
Categorize events as to  
aid evaluation of ME at LO

**category**  $\Leftrightarrow$  **event interpretation**

DL



# Implementation



Reduction of  $V$ +jets and  $tt$ +jets requires cut on  
**number of jets, b tagging**

# Implementation

assignment based on jet permutation w/ largest btag

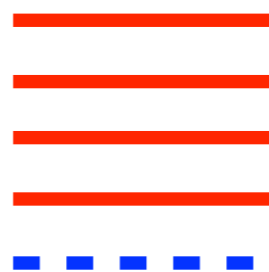
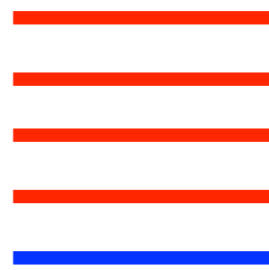
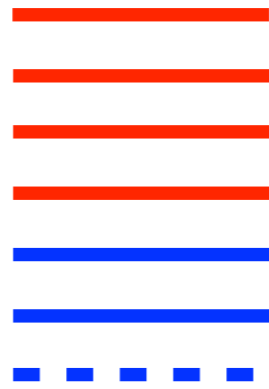


- tagged
- untagged
- - - extra jets

SL  
 $N_j \geq 5$

b tag

DL  
 $N_j \geq 4$



# Implementation

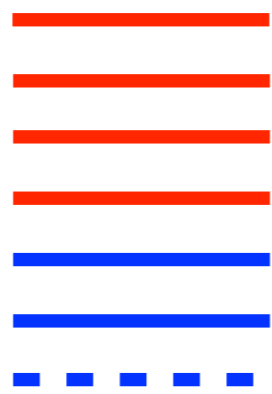
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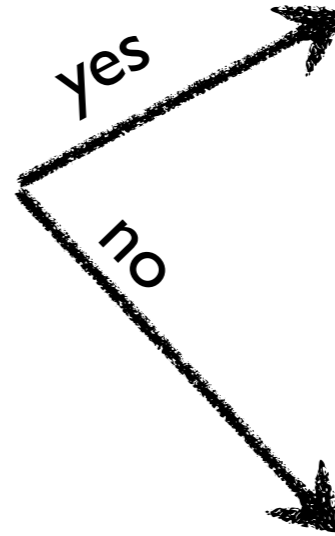
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}  $|M_{uu}-m_W| < 20 \text{ GeV} ?$

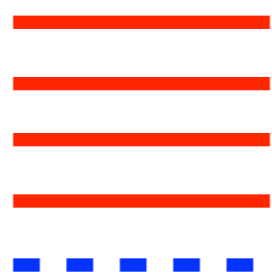
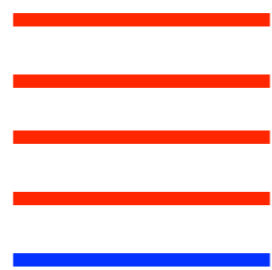


SL Cat. 1

all top/H quarks reconstructed

SL Cat. 2

one W-quark missed;  
extra gluon(s) from ISR  
 $\Rightarrow$ integrate over missing quark





# Implementation

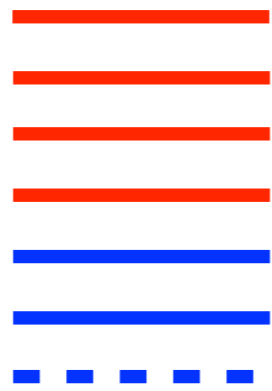
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— untagged  
- - - extra jets

SL  
 $N_j \geq 5$

b tag

DL  
 $N_j \geq 4$



}  $|M_{uu}-m_W| < 20 \text{ GeV} ?$

yes

no

SL Cat. 1

all top/H quarks reconstructed

SL Cat. 2

one W-quark missed;  
extra gluon(s) from ISR  
 $\Rightarrow$ integrate over missing quark

SL Cat. 3

one W-quark missed  
no extra-radiation  
 $\Rightarrow$ integrate over missing quark

# Implementation

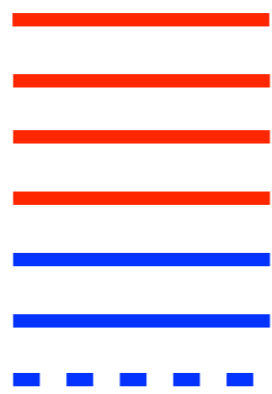
assignment based on jet permutation w/ largest btag


  
 — tagged
   
 — untagged
   
 - - - extra jets

SL  
 $N_j \geq 5$

b tag

DL  
 $N_j \geq 4$



$|M_{uu} - m_W| < 10 \text{ GeV} ?$

yes

no

SL Cat.1

all top/H quarks reconstructed

SL Cat.2

one W-quark missed;  
extra gluon(s) from ISR  
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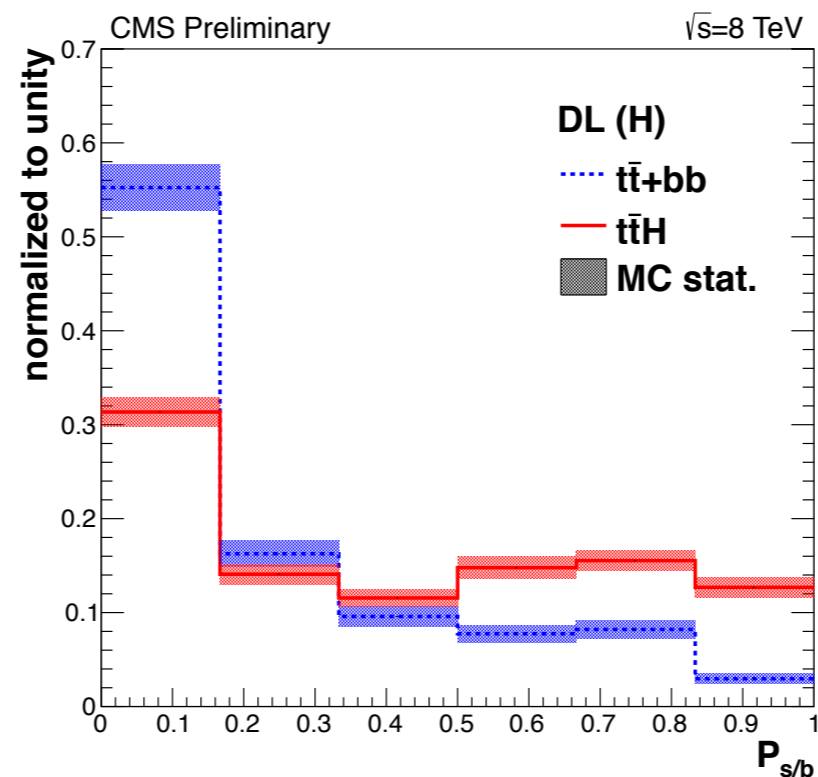
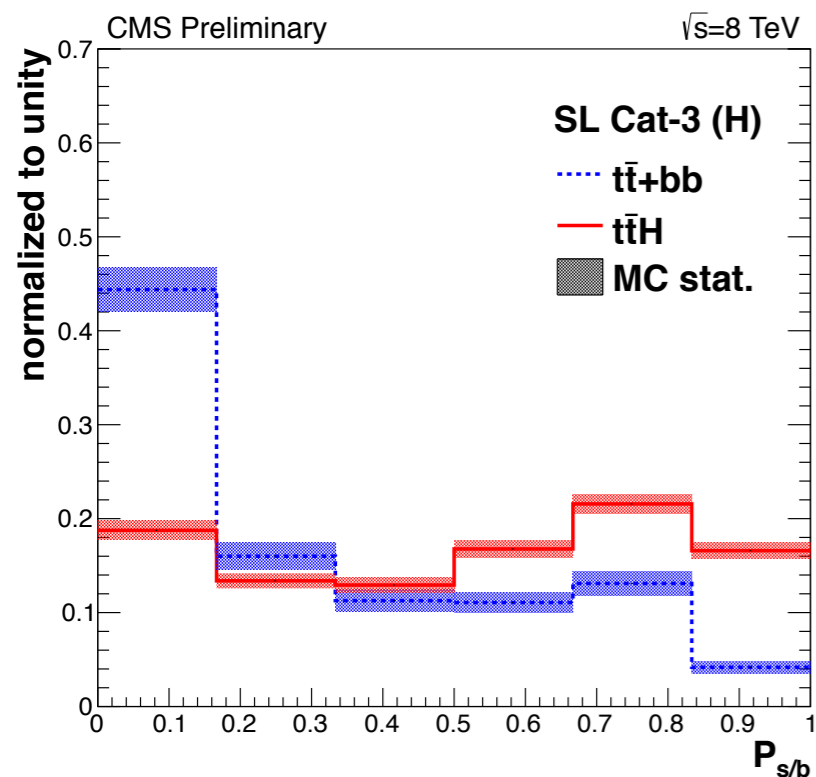
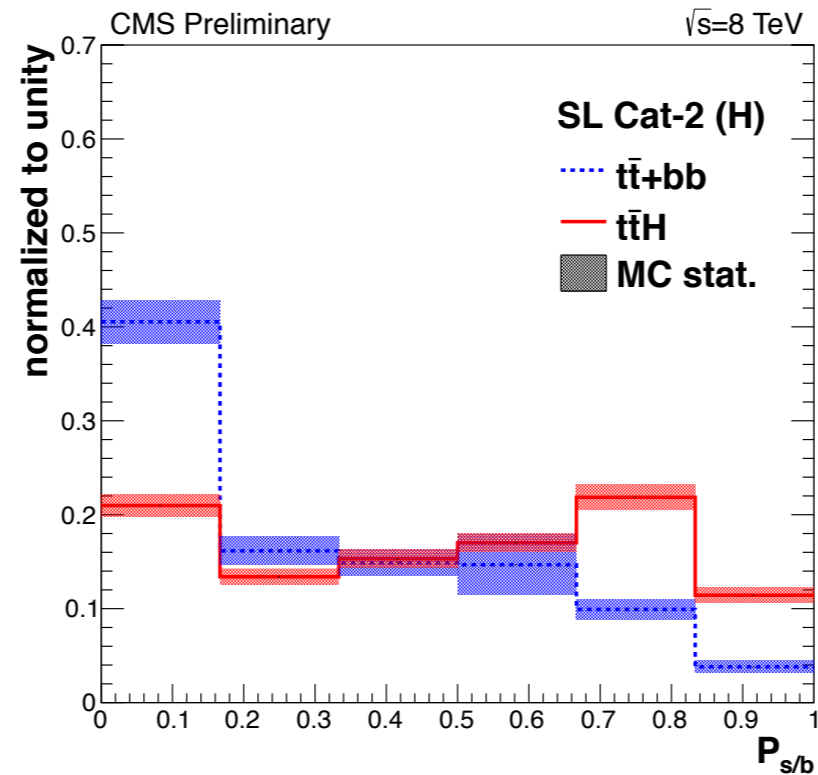
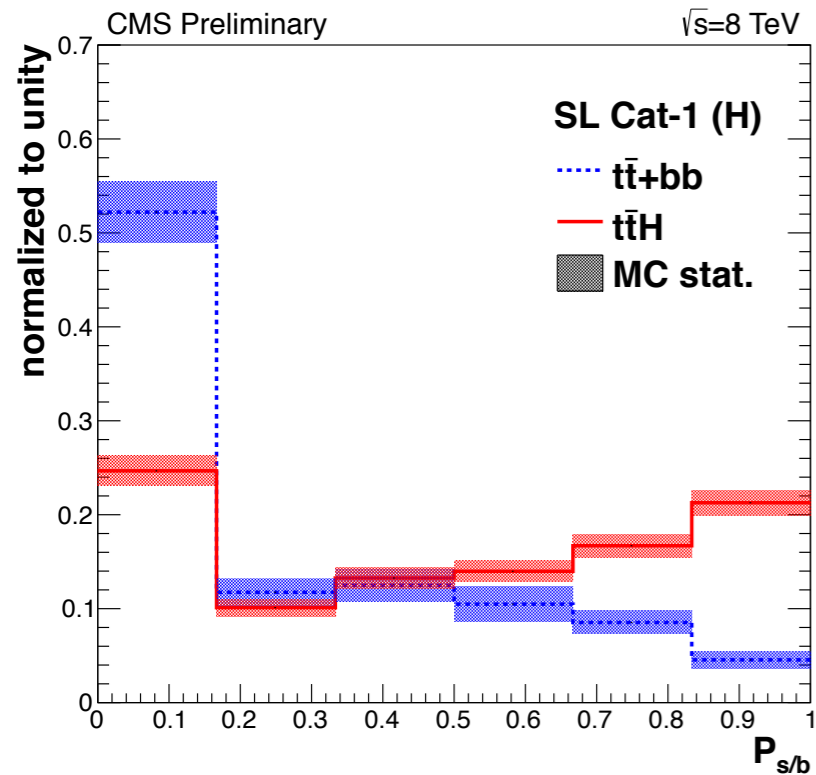
SL Cat.3

one W-quark missed  
no extra-radiation  
 $\Rightarrow$ integrate over missing quark

DL

all top/H quarks reconstructed

# Shape comparison: $t\bar{t}+bb$ vs $t\bar{t}H$



## In situ validation of shapes

- ▶ sidebands with low  $b$  tagging score
- ▶ in signal region, validate low-sensitivity version of  $P_{s/b}$
- one random permutation only
- testing different values of  $m_H$

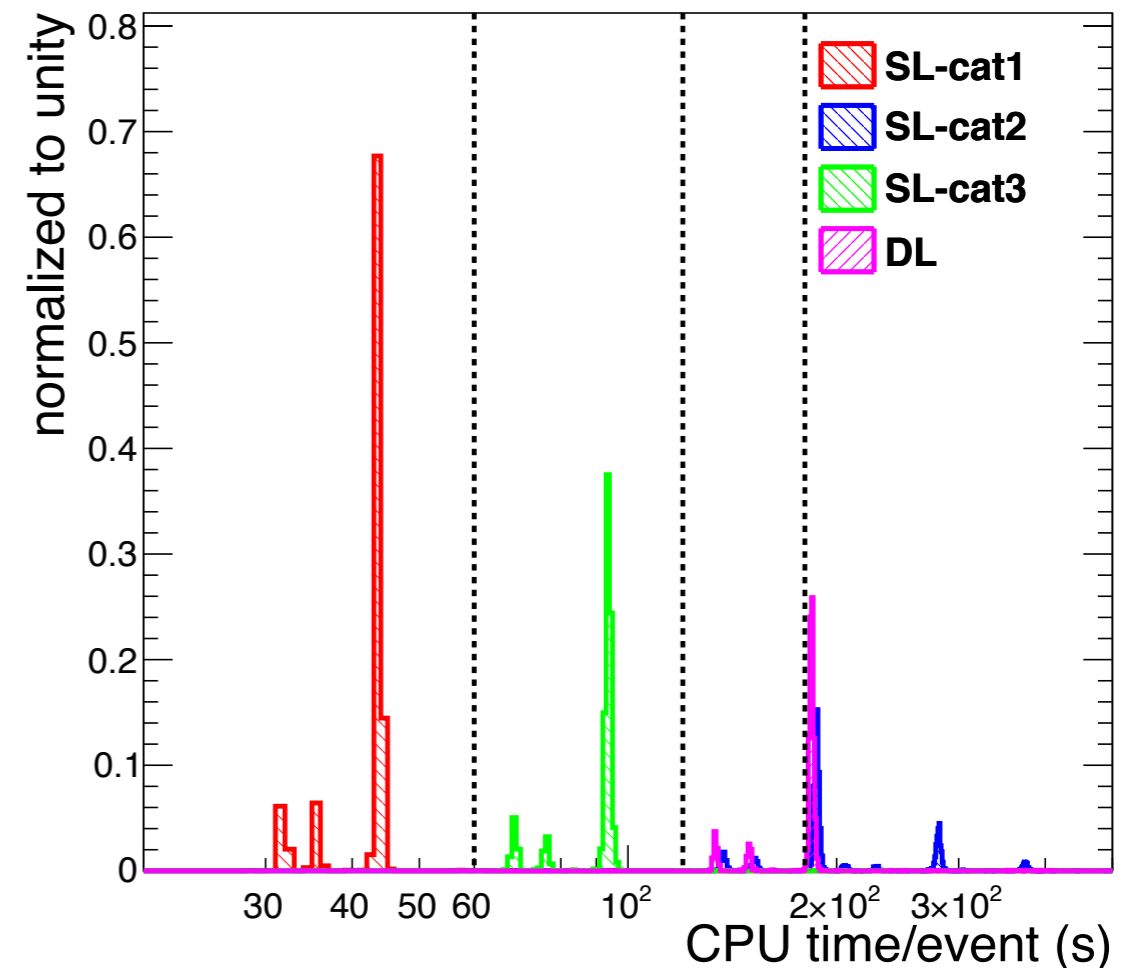
$$P_{s/b} = (1 + w_B/w_S)^{-1}$$

# FAQ: is CPU time an issue ?

Not really; compromise between **performance** and **timing**:

- ▶ run only on the good “events”
- ▶ filter-out permutations using b tagging ( $6! \rightarrow 4!$ )
- ▶ test one background hypothesis only
  - optimize separation against  $tt+bb$
- ▶ parallelize (by event) as much as possible
- ▶ neglect spin/correlations
- ▶ JEC/JER systematics: bookkeep VEGAS grid result from “nominal” for faster evaluation
- ▶ ...

CMS Simulation  $\sqrt{s}=8$  TeV



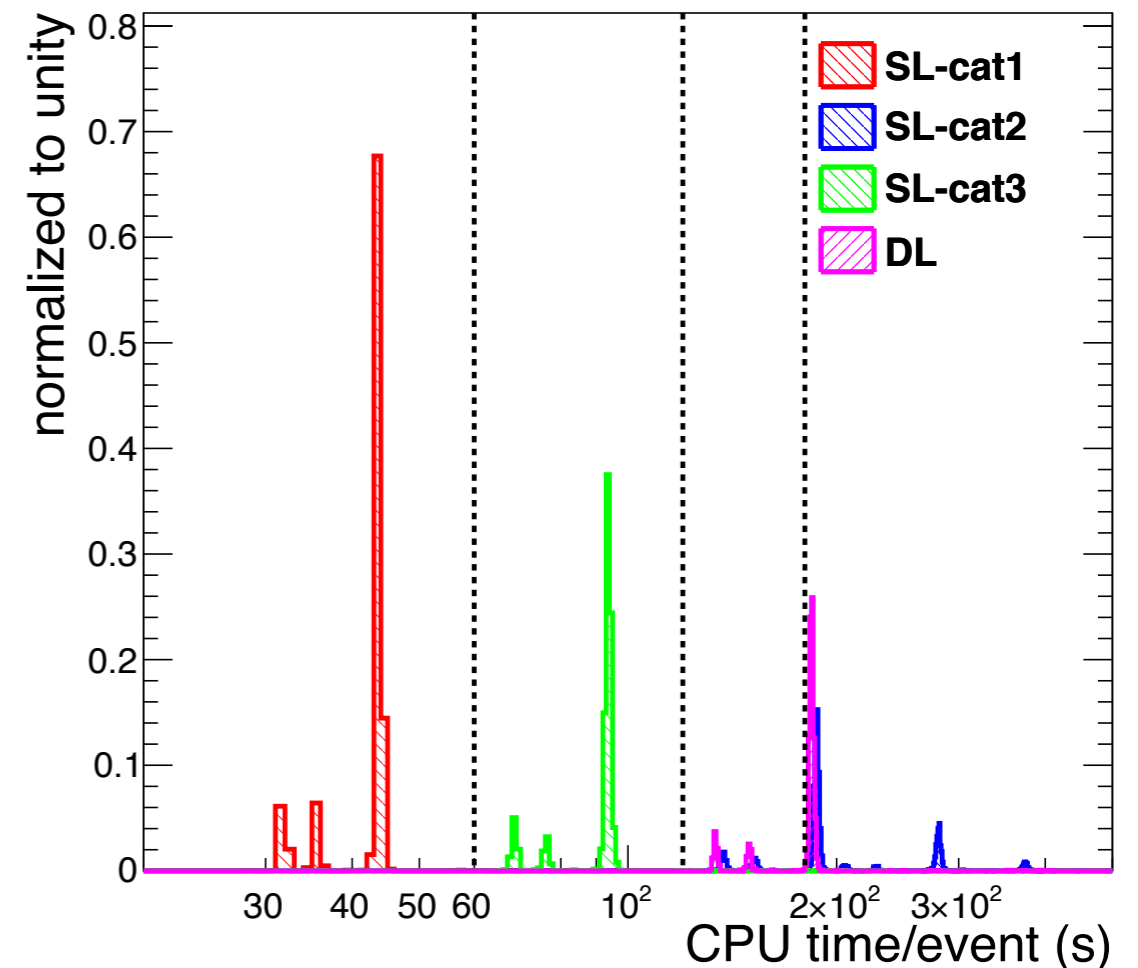
Number of variables	4 (+1)	6 (+1)	5 (+1)
Number of iterations	5	5	5
Function calls	2000	4000	10000
Numerical precision (mode of $\sigma_w/w$ )	0.8%	1.2%	0.8%
CPU-time per integral (mean)	0.5 (1.5) s	1.1 (3.2) s	2.3 (6.2) s
Time budget for $ \mathcal{M} _{ME}^2$	30% (80%)	30% (80%)	30% (80%)

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Number of iterations	5	5	5
Number of events			
Time budget for $ \mathcal{M} _{ME}$	30% (80%)	30% (80%)	30% (80%)

⇒ complete analysis round in ~10h using the batch system of a T3



# Overview of systematic uncertainties

ttH modeling: PYTHIA

tt+jets: MadGraph ( $\leq 3$  partons)+PYTHIA

50% normalization uncertainty on tt+HF

$\alpha_s$  scale uncertainty

JEC/JER, b tagging

top  $p_T$  modeling  
statistical uncertainty

nuisance	treatment
luminosity	lnN 2.6%
ID/trigger	lnN 2-4%
ttH	lnN 12%
tt+bb	lnN 50%
tt+cc	lnN 50%
tt+b	lnN 50%
QCD scale	lnN 17-3%
fact/renorm. scale	shapes (tt + 1p/2p/3p/bb/b/jj/cc)
PDF	lnN 3-9%
JES	shape
JER	shape
btagging	8 shapes
top $p_T$ model	shape
MC stat.	shape (all bins)

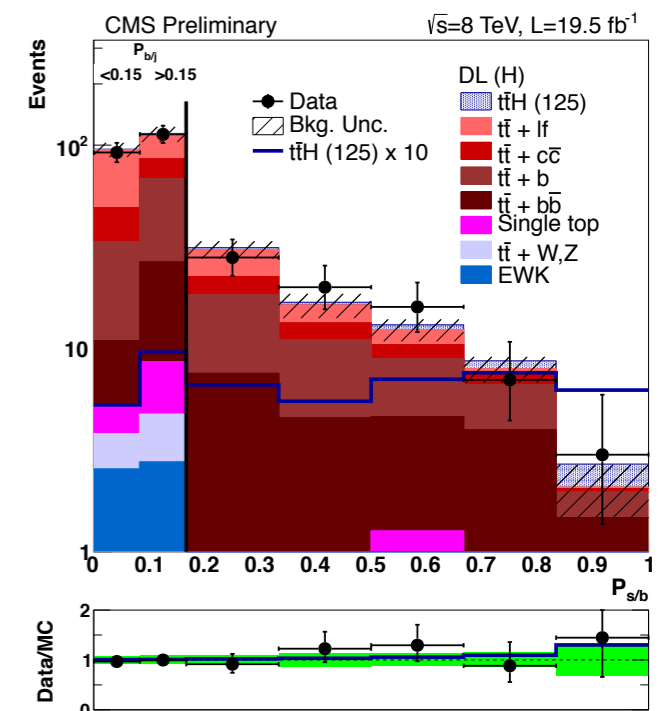
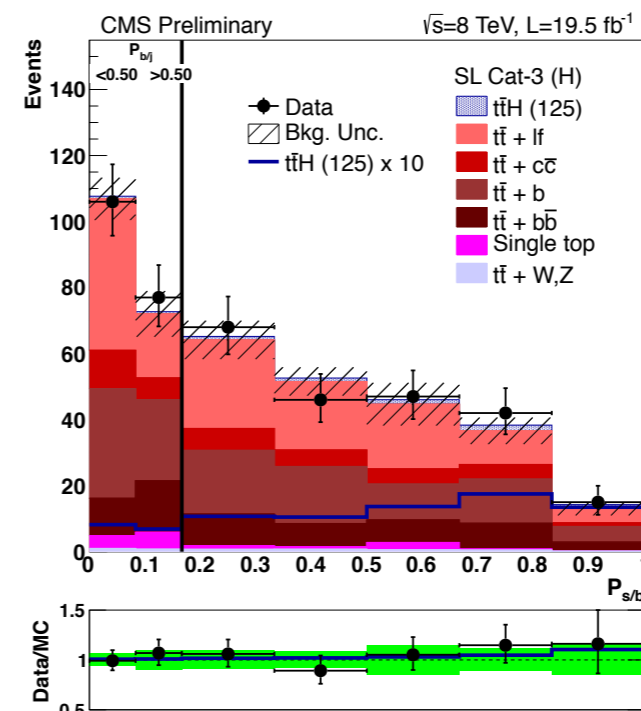
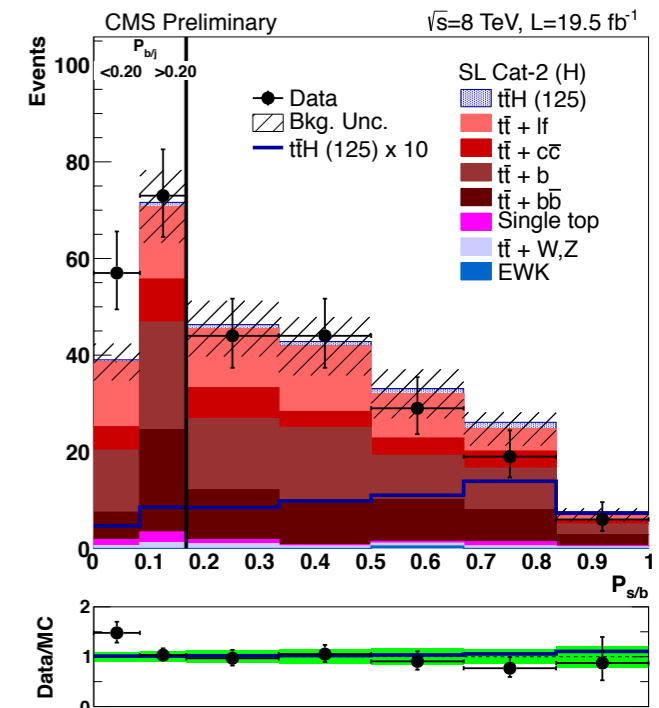
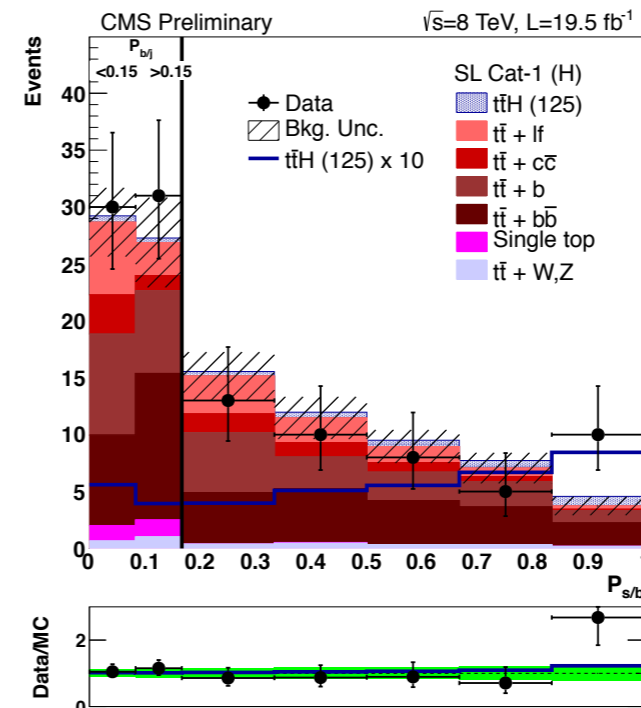
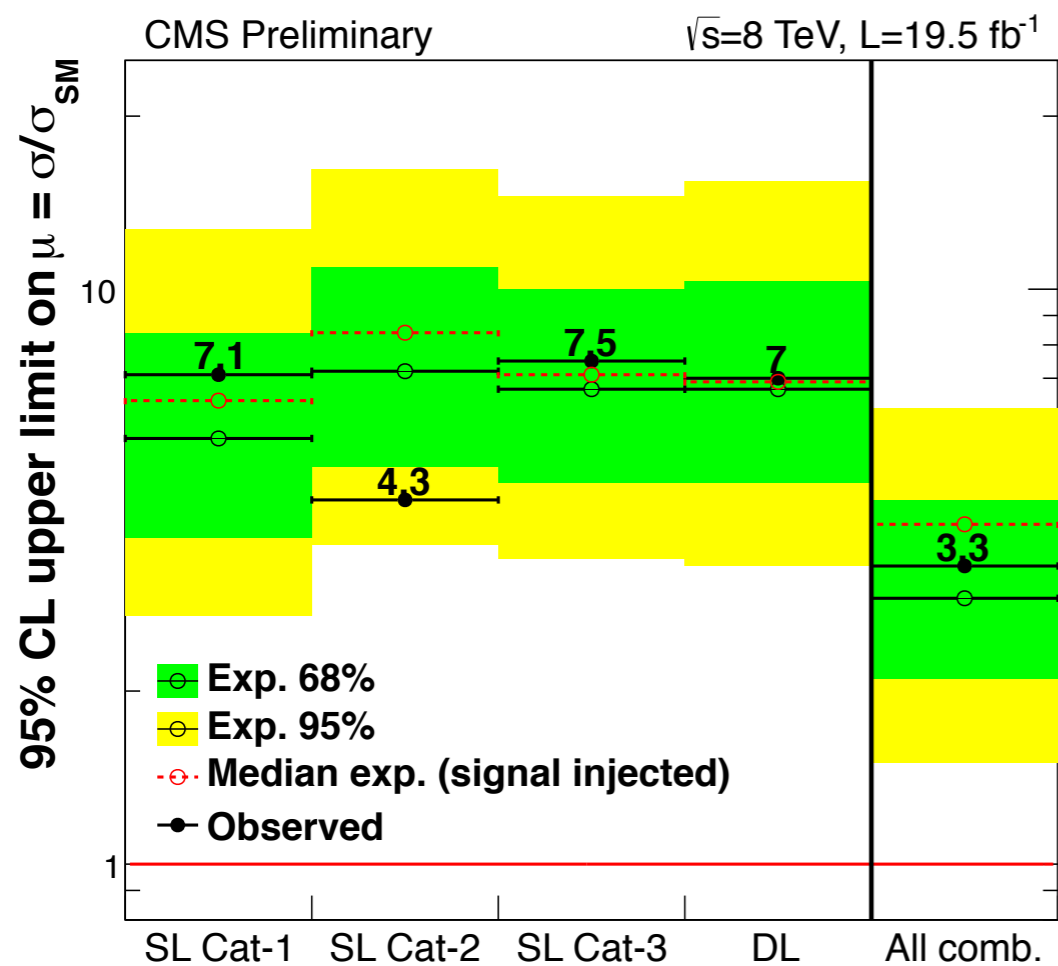
lnN = normalization uncertainty  
shape = vertical morphing

# Results (8 TeV)

## Combined fit to ME discriminant:

Median Exp. 95% CL	Median Exp. (signal injected)	Obs.
$\mu < 2.9$	$\mu < 3.9$	$< 3.3$

▶ Best-fit value  $\mu_{t\bar{t}H} = 0.7 \pm 1.4$



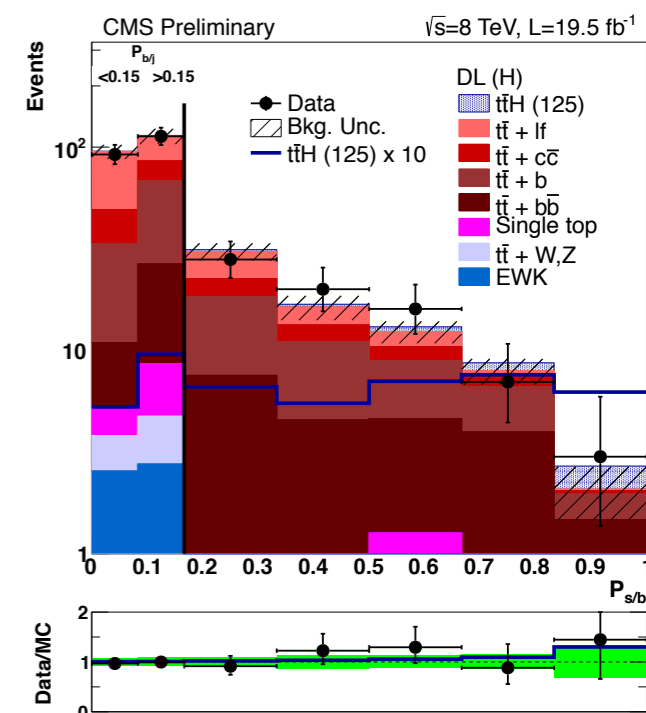
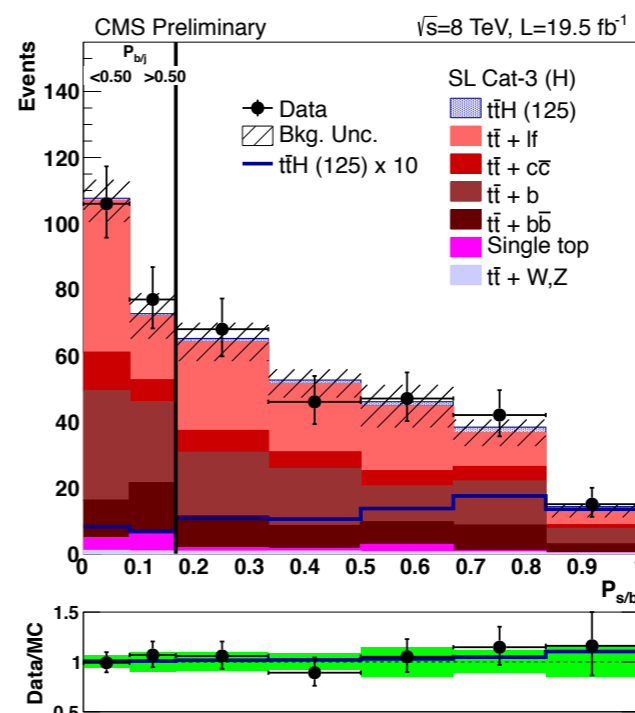
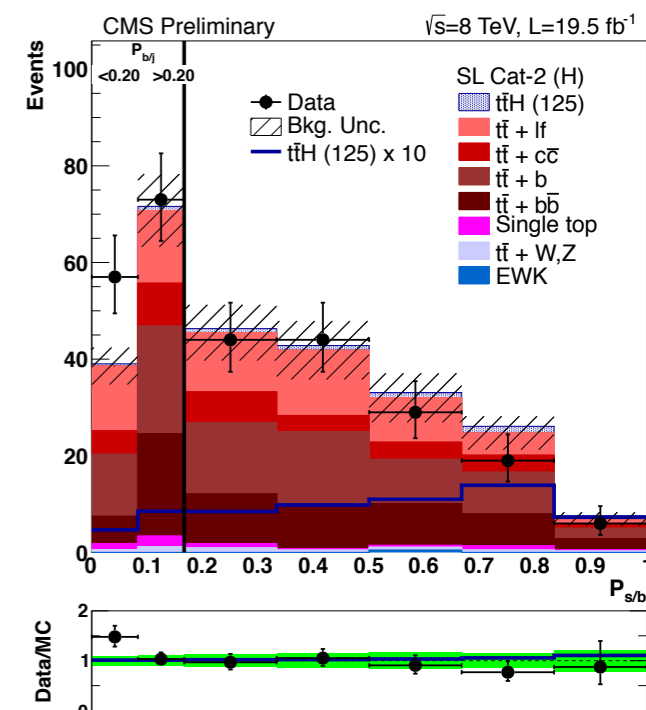
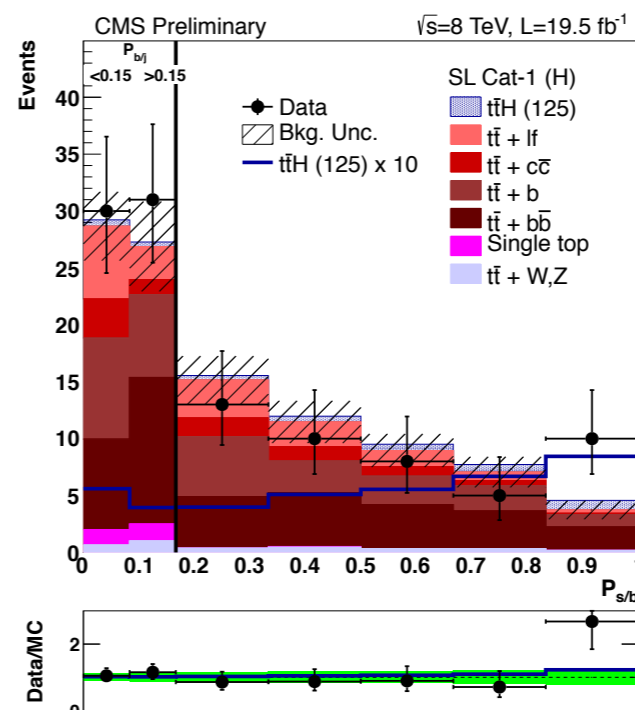
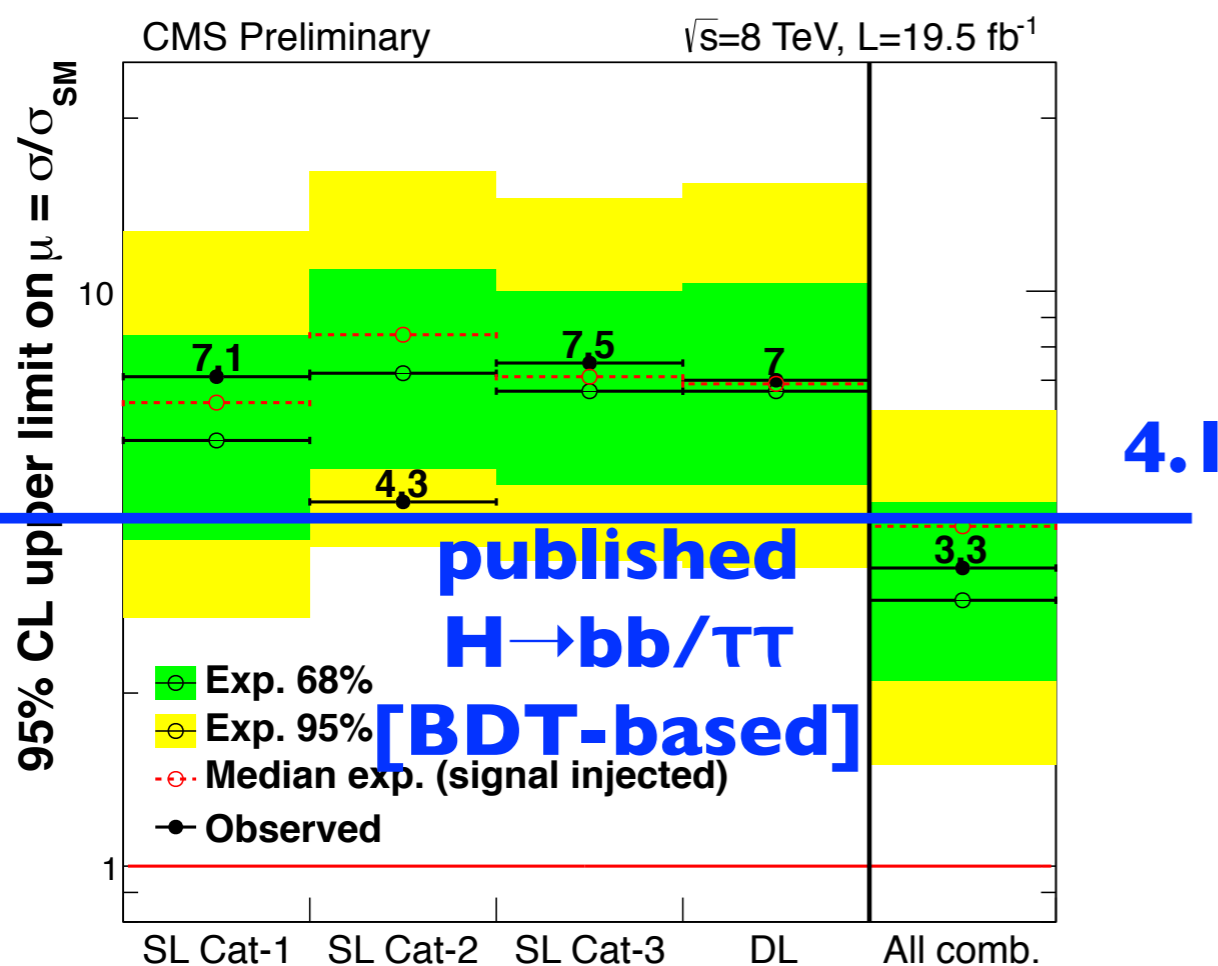
CMS-PAS-HIG-14-010

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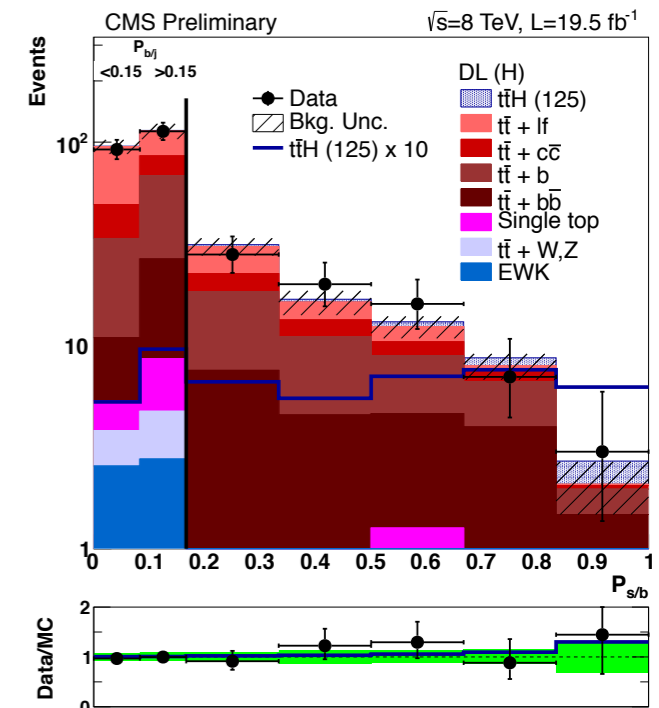
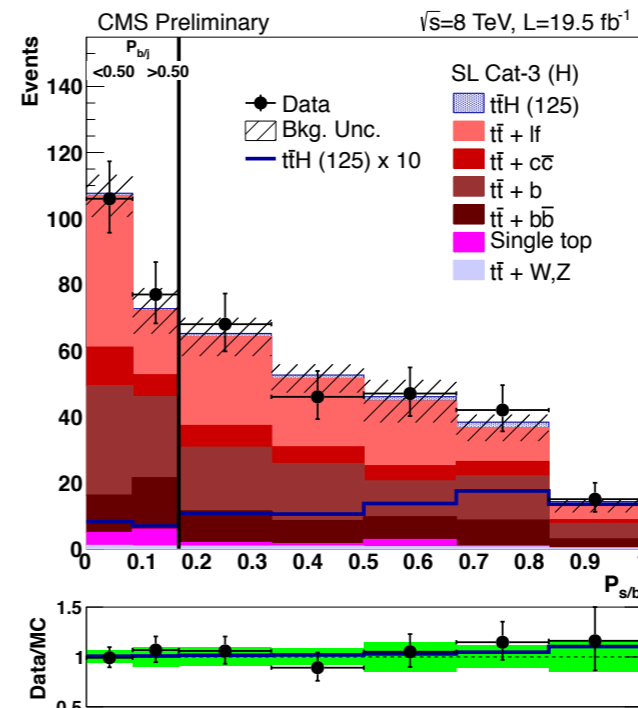
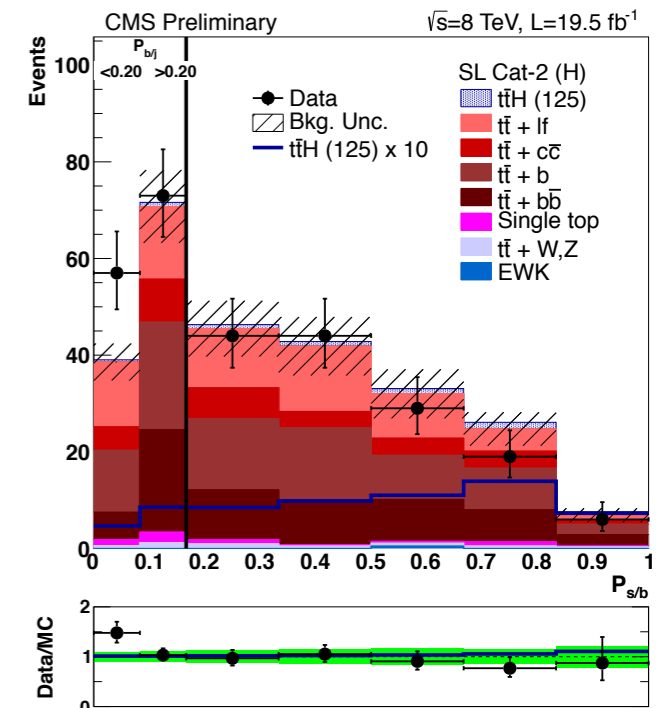
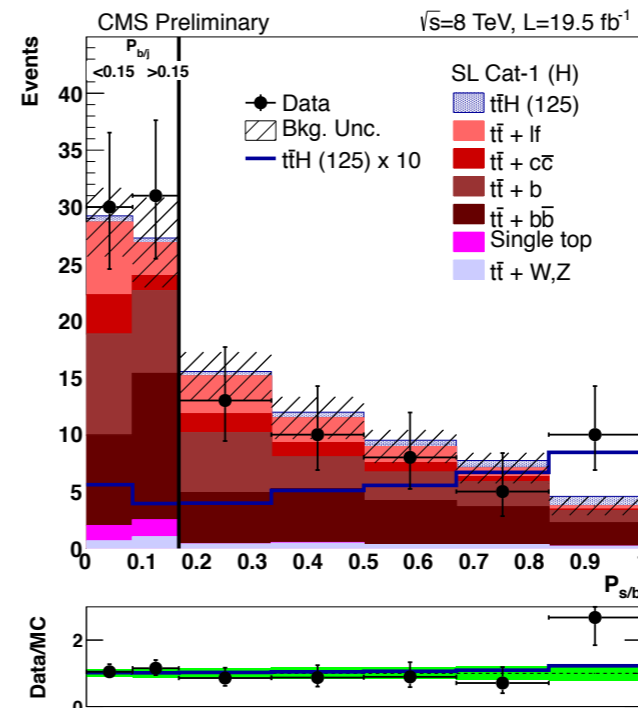
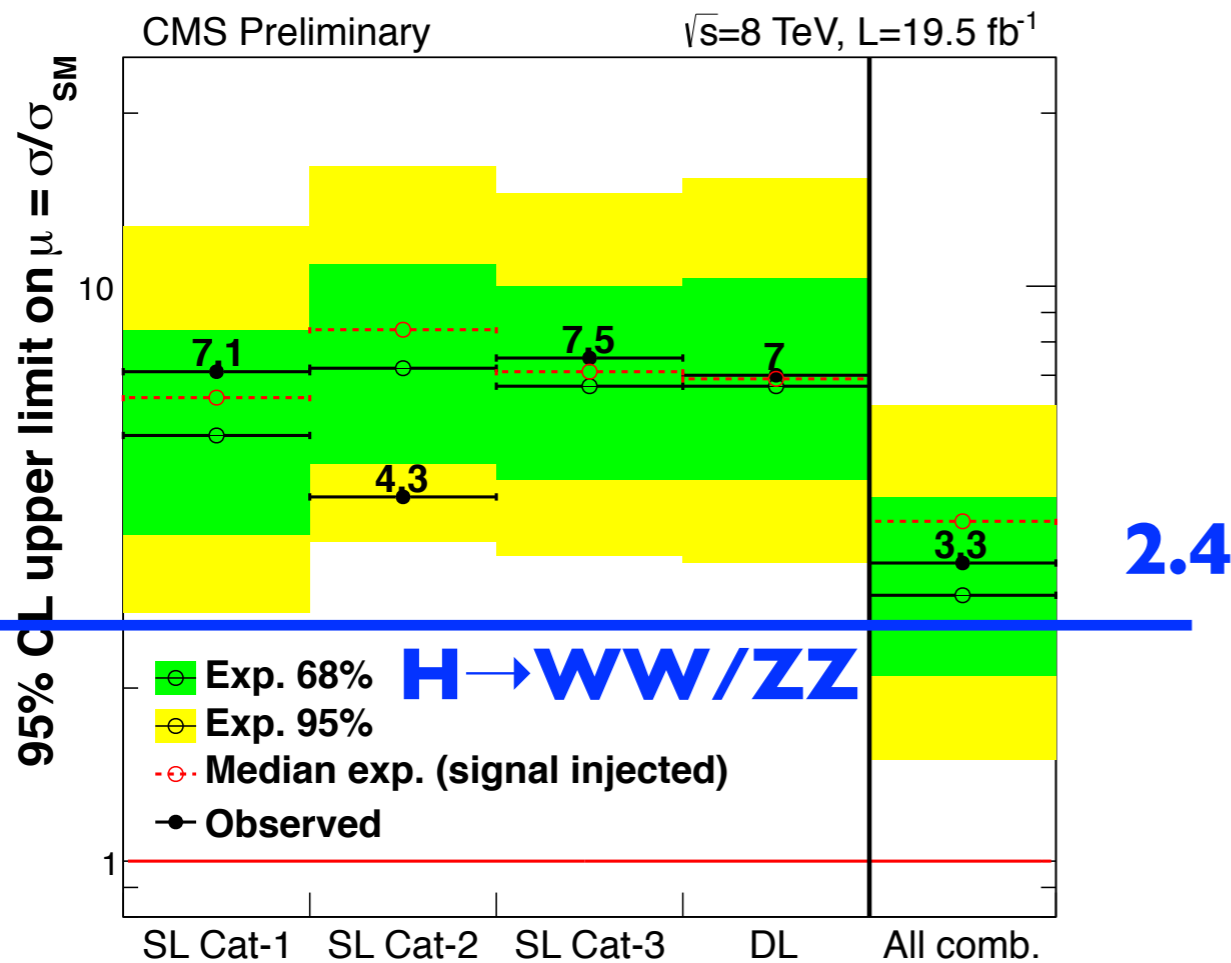
CMS-PAS-HIG-14-010

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**CMS-PAS-HIG-14-010**

# Summary & outlook

- First  $t\bar{t}H$  search based on the MEM at a collider
  - ▶ new algorithm developed and optimized for CMS
    - 20-30% better expected limit compared to previously published analysis
    - CPU & human-time sustainable
    - Run I: results in agreement with SM expectation ( $\Delta\mu/\mu \sim 1.4$ )
- The 13 TeV challenge is behind the corner
  - ▶ with more data, theoretical uncertainties will become relevant
    - use of NLO programs will make analysis more solid
    - definition proper sidebands for background estimation/calibration
    - $t\bar{t}Z(\rightarrow b\bar{b})$  as a standard candle?
  - ▶ further improvements:
    - spin-correlations as an extra handle (?)
    - inclusion of more event topologies (?)
    - inclusion of boosted top tagging variables (?)



*Thanks for your attention*

*Back up*

# Motivation

Determination of top-quark Yukawa **coupling** ( $y_t$ ) is a major goal

- ▶ gather direct evidence of Higgs coupling to up-type fermions
- ▶ implication on EWSB

**Cross sections** of top+Higgs channels can unravel value/sign of  $y_t$

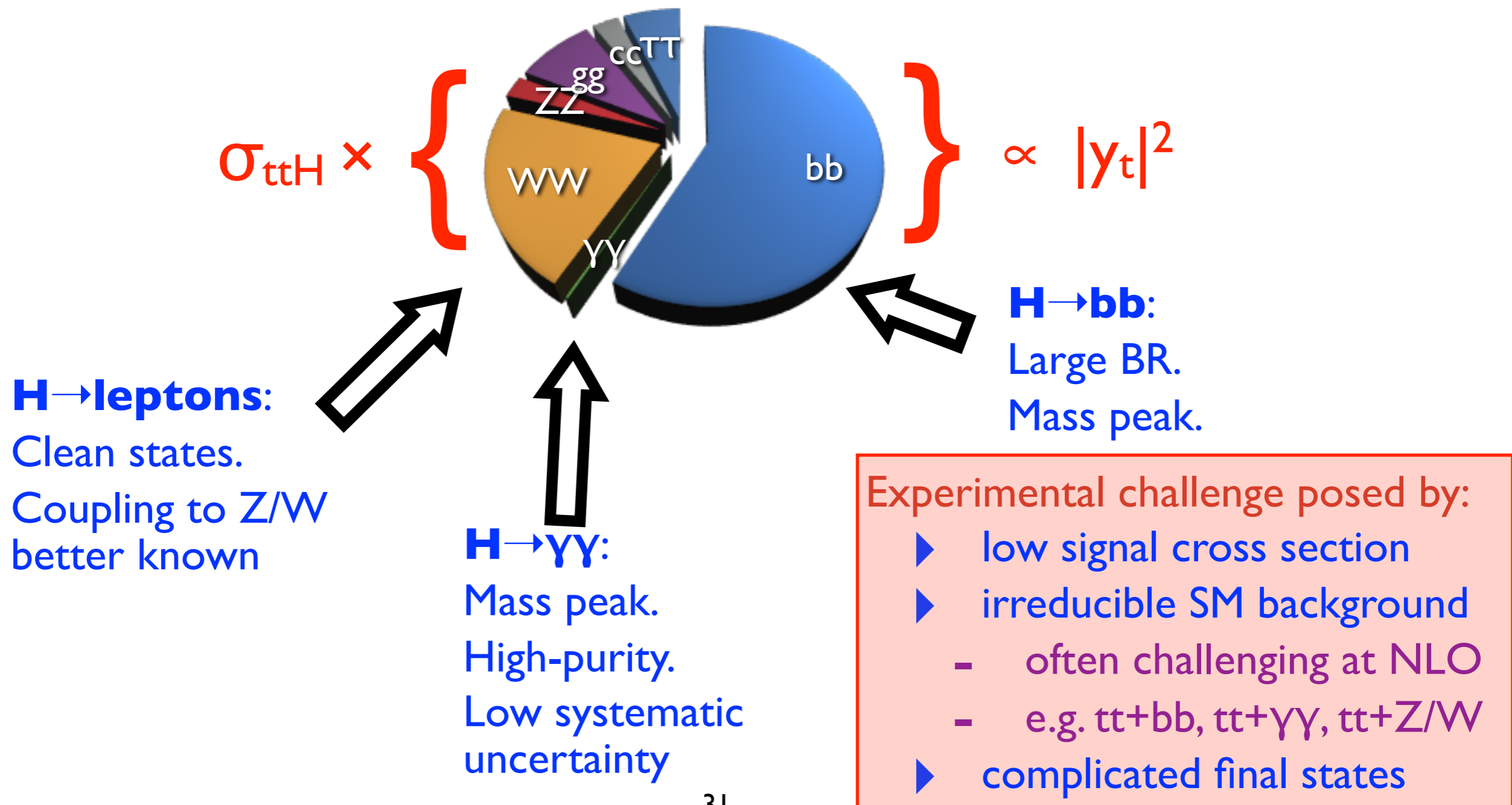
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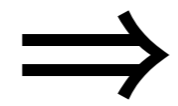
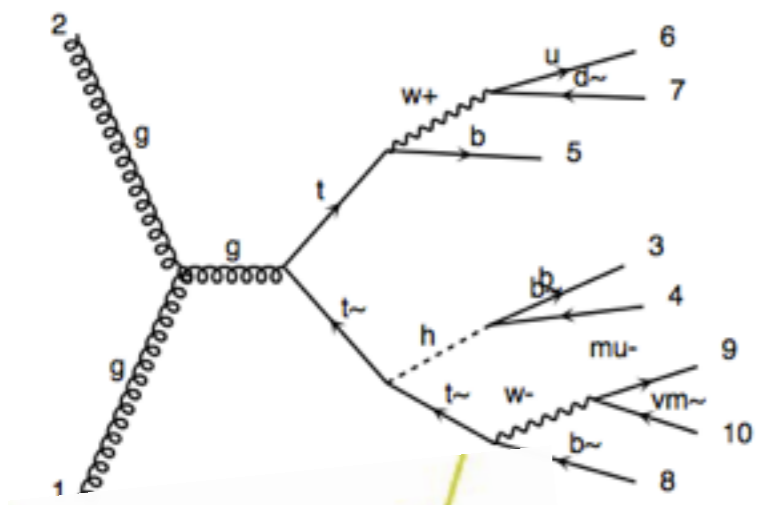
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- ▶ gather direct evidence of Higgs coupling to up-type fermions
- ▶ implication on EWSB

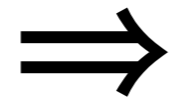
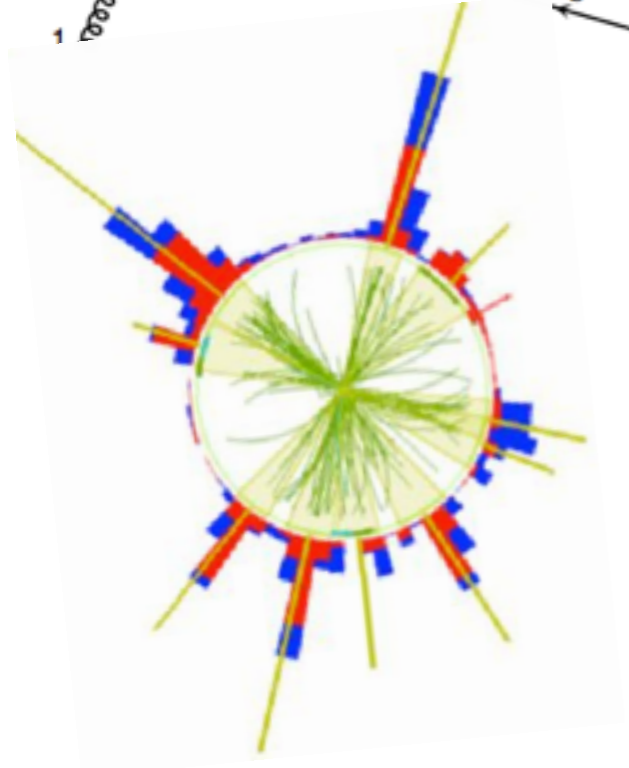
**Cross sections** of top+Higgs channels can unravel value/sign of  $y_t$



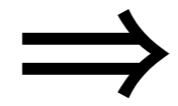
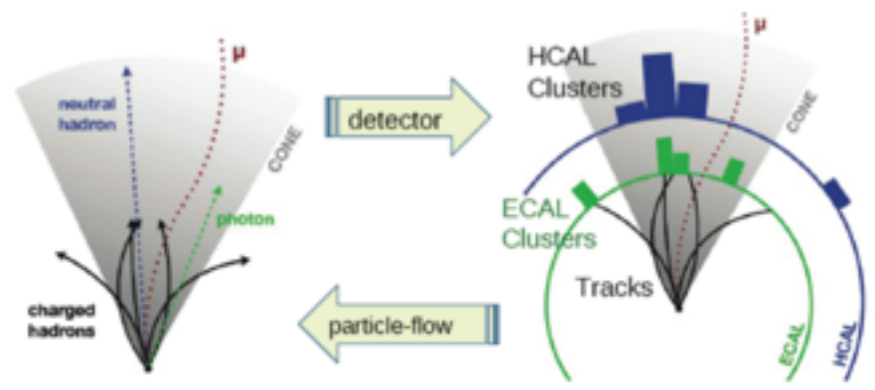
# Building bricks



$\vec{X}$  = (generated particles)  
 $\mathcal{M}(\vec{X})$  = scattering amplitude



$\vec{Y}$  = (reconstructed particles)



“metric”  $W(\vec{Y}, \vec{X})$



# Dimensional reduction

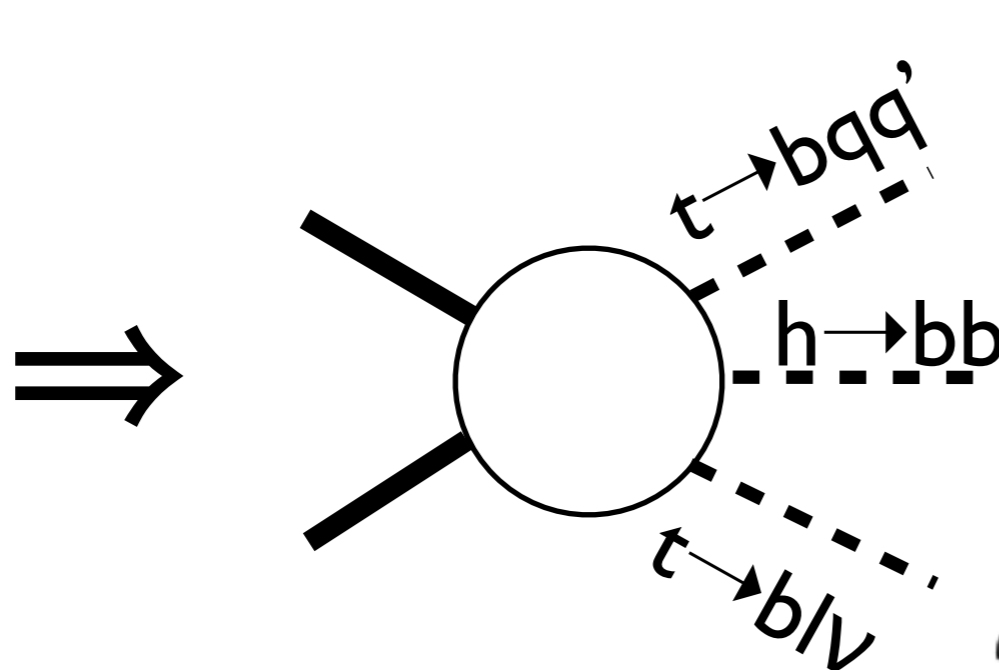
- Factorize integration over final-state particles via

$$d\Phi_n(P; p_1, \dots, p_n) = d\Phi_j(q; p_1, \dots, p_j) \times d\Phi_{n-j+1}(P; q, p_{j+1}, \dots, p_n) (2\pi)^3 dq^2$$

- Narrow-width approx:  $\frac{1}{(t^2 - M_t^2) + \Gamma_t^2 M_t^2} \rightarrow \frac{1}{(M_t \Gamma_t)^2} \delta(t^2 - M_t^2)$

- Diff. decay amp. from MC:  $|\mathcal{M}(t \rightarrow bqq')|^2 \propto \frac{1}{\Gamma_t} \frac{M_t}{|\vec{q}^*| |\vec{b}|} \frac{d\Gamma}{d\Omega_q^* d\Omega_b}$

- Assume lepton and jet direction perfectly measured



$$d\Phi_{tq} \propto \frac{2E_{bq} E_q E_{q'}^2}{M_W^2 E_{wq}} \frac{1}{\left| \frac{\beta_{wq}}{\beta_{bq}} \hat{e}_{wq} \cdot \hat{e}_{bq} - 1 \right|} dE_q$$

$$d\Phi_h \propto \frac{E_{b2}}{\left| \frac{\beta_{b1}}{\beta_{b2}} \hat{e}_{b1} \cdot \hat{e}_{b2} - 1 \right|} dE_{b1}$$

$$d\Phi_{t\ell} \propto \frac{2E_{b\ell} \hat{E}_\ell E_\nu^2}{M_W^2 E_{w\ell}} \frac{1}{\left| \frac{\beta_{w\ell}}{\beta_{b\ell}} \hat{e}_{w\ell} \cdot \hat{e}_{b\ell} - 1 \right|} d\Omega_\nu$$

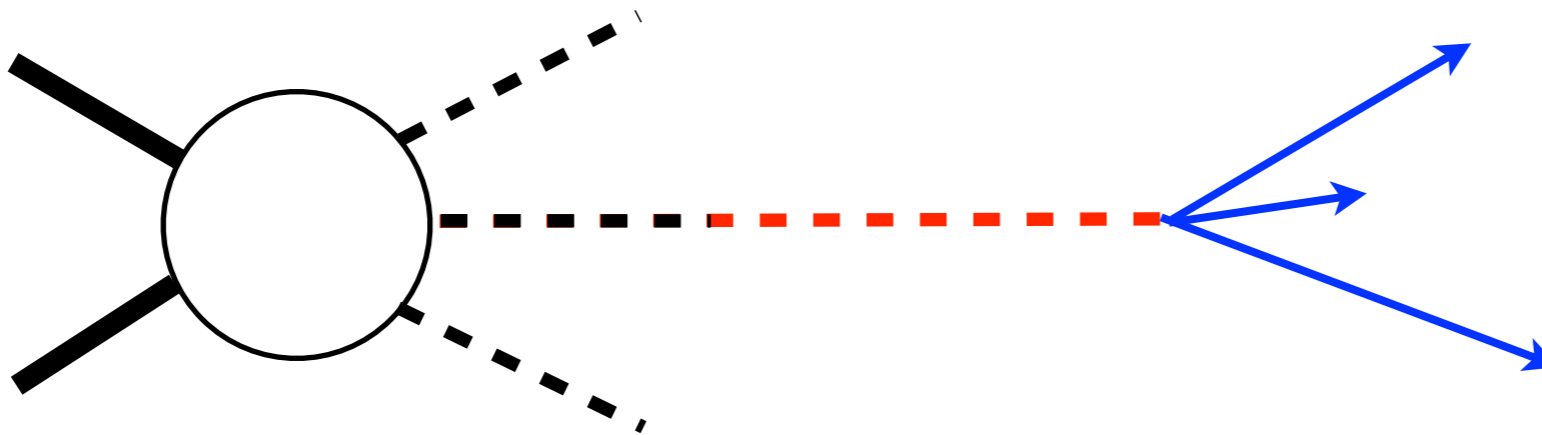
# The Matrix Element Method (MEM)

Given event  $j$ , find its probability  $w_j(\cdot | H_i)$  under hypothesized process  $H_i$

- ▶  $w_j$  is a function of detector-level observables  $\mathbf{Y}$  (technically, a differential prob. density on  $\mathbf{Y}$ )
- ▶ can depend parametrically on unknown model parameters  $\lambda$
- ▶ is the relative weight with which a MC generator of process  $H_i$  would generate ev.  $j$ 
  - for  $N \gg 1$  events,  $(\sum^N w_j / N) \rightarrow 1$
- ▶ N.B.: a HEP process  $H_i$  is known ( $\Leftrightarrow$  can be simulated) fully differentially only to some approximation (typically not better than NLO). Conversely, nature is “to all orders” :
  - a certain approximation is implicit in the method
  - this does not invalidate the method *per se*, rather makes it not optimal

# The algorithm: basic principles

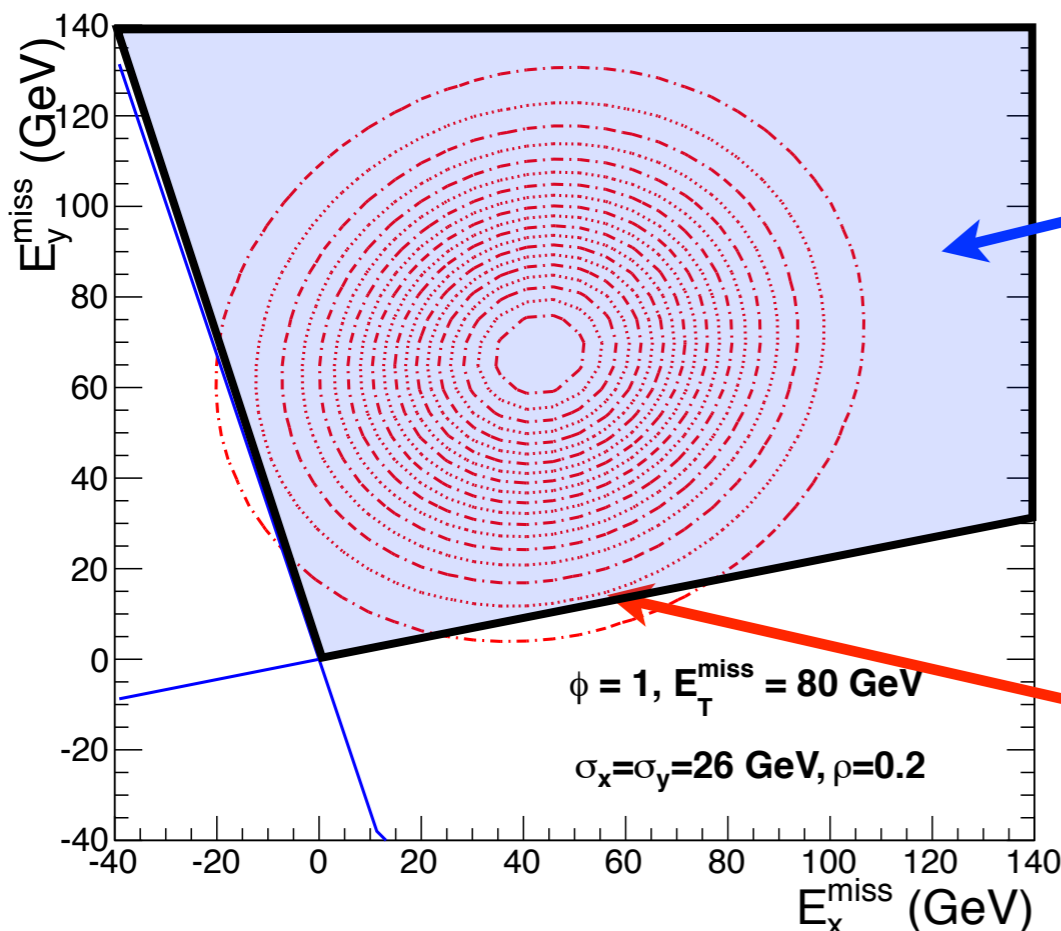
- Factorize the reaction  $pp \rightarrow tt+(bb) \rightarrow \Omega$  as a 3-steps process:
  - ▶  $gg \rightarrow 3$  on-shell intermediate particles  $\propto |\mathcal{M}(g g \rightarrow t t H)|^2$
  - ▶ intermediate particles propagate:  $\propto [(q^2-M^2)^2 - M^2\Gamma^2]^{-1}$
  - ▶ intermediate particles decay  $\propto \Gamma^{-1} d\Gamma/d\Omega$



- This way:
  - ▶ no need to evaluate CPU-intensive  $2 \rightarrow 8$  amplitudes [only  $2 \rightarrow 3(4)$ ]
  - ▶ spin-correlations and polarizations neglected
  - ▶ cross-check with MadWeight

# $P_T$ balance

- Event-by-event constraint to the *measured* recoil  $\rho = -\sum \mathbf{p}_T^{\text{vis}} - \mathbf{E}_T^{\text{miss}}$  via transfer function
  - ▶ for each phase-space point, boost so that  $\mathbf{P}_T = \mathbf{0}$ , and evaluate  $|\mathcal{M}|^2$
- N.B.: at present, we instead constrain  $v$ 's  $\mathbf{p}_T$  to  $\mathbf{E}_T^{\text{miss}}$  and the quark energy to jet energy
  - ▶ not optimal because  $E_T^{\text{miss}}$  correlated w/ jet energy.



$v$ 's direction integrated over blue plane w/ prior:

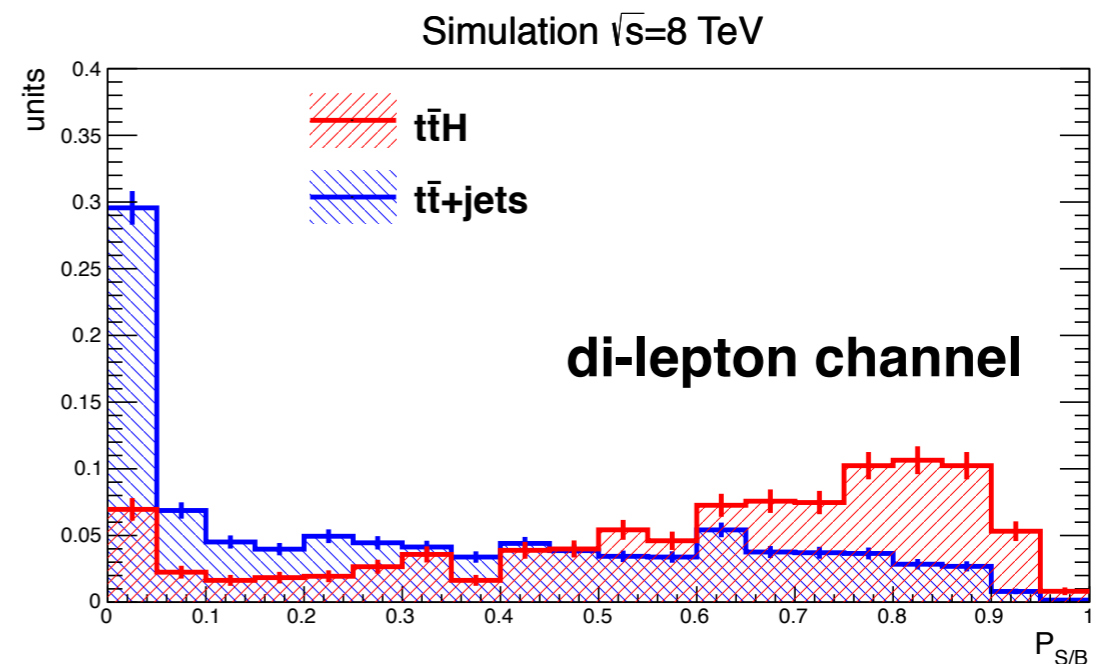
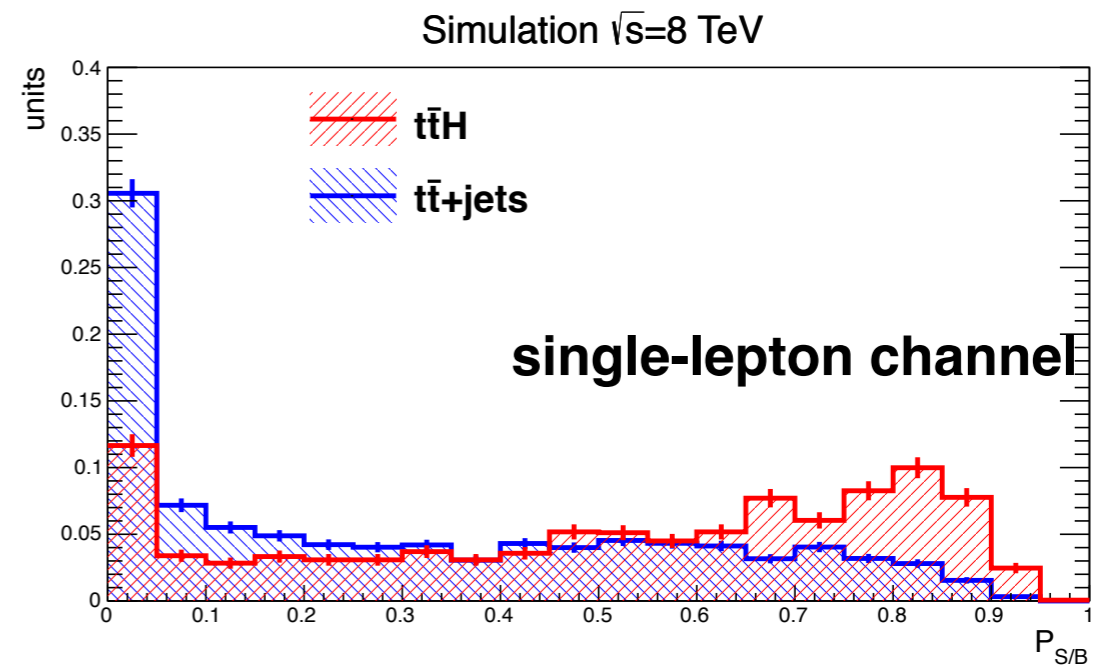
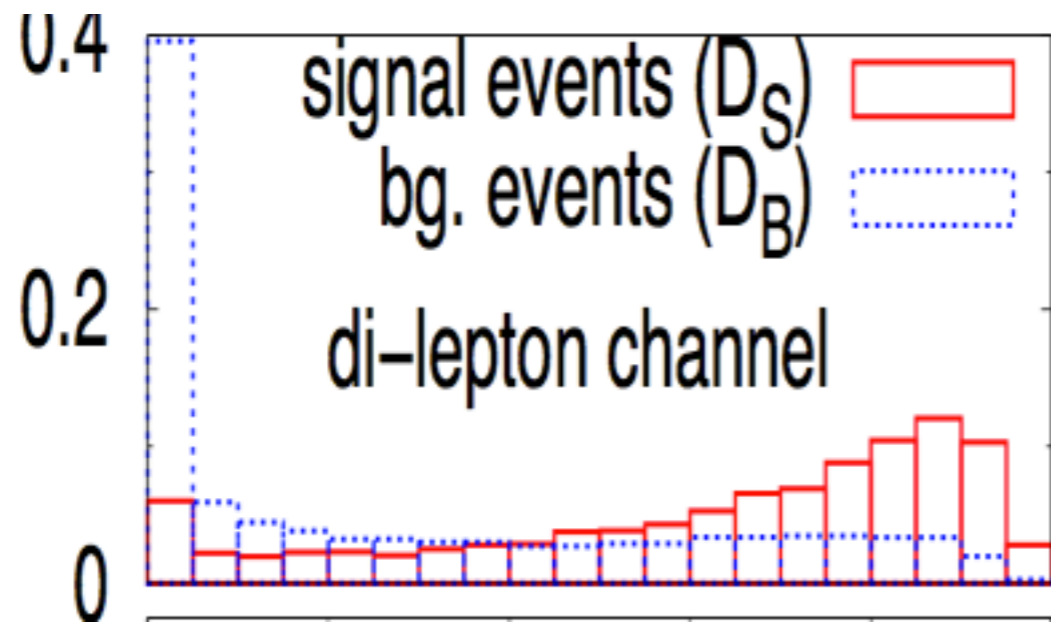
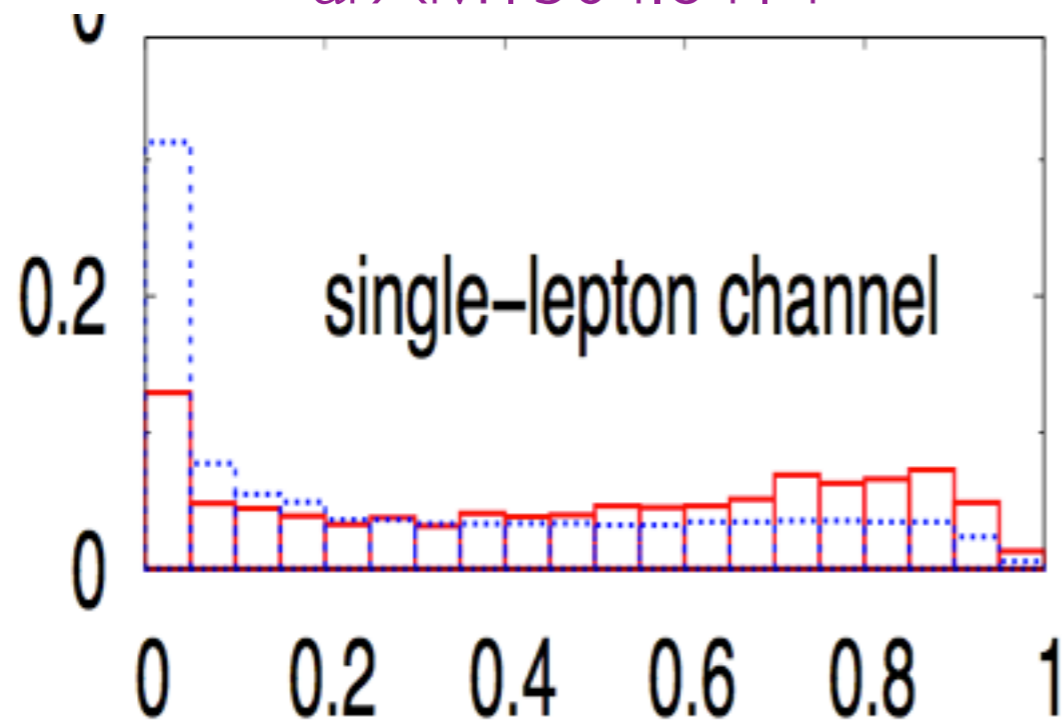
$$\frac{1}{2\pi\sqrt{|V_{x,y}|}} \exp\left\{-\frac{1}{2}(\vec{E}_T^{\text{miss}} - \sum \vec{v}_T)^T V_{x,y}^{-1} (\vec{E}_T^{\text{miss}} - \sum \vec{v}_T)\right\}$$

95% CL isocontour

$V_{x,y} = E_T^{\text{miss}}$  cov. matrix

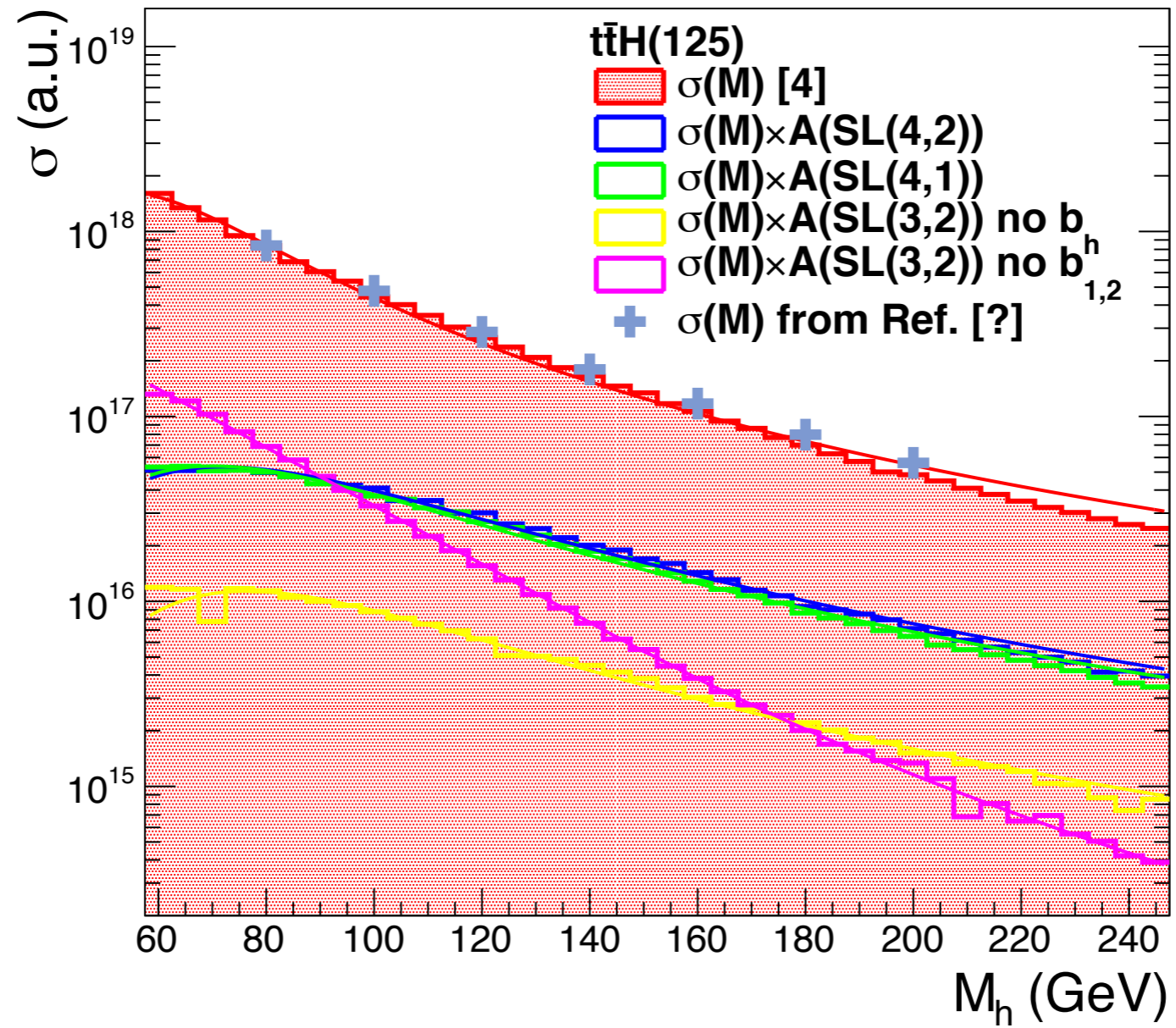
# Comparing with MadWeight

arXiv:1304.6414

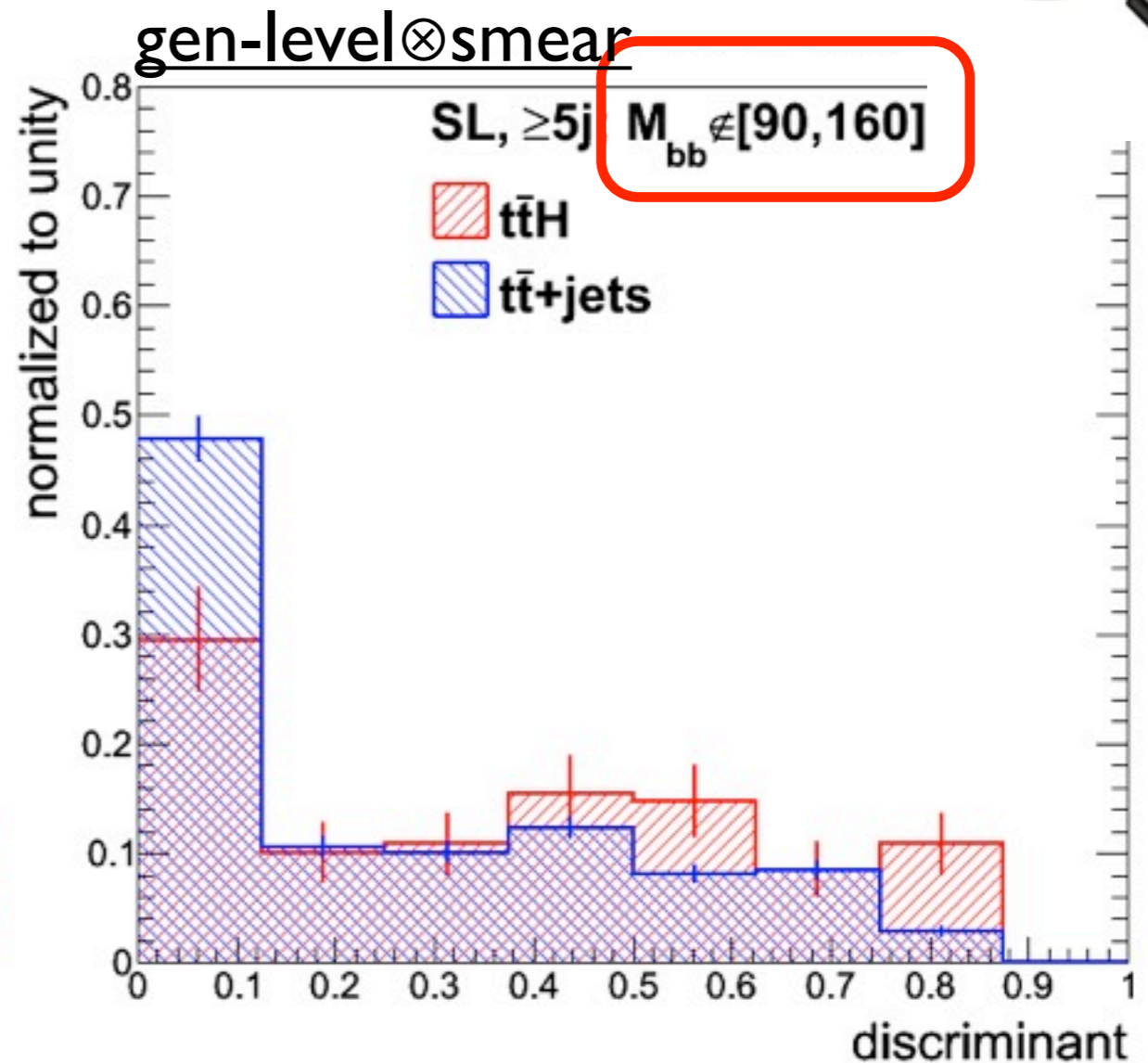
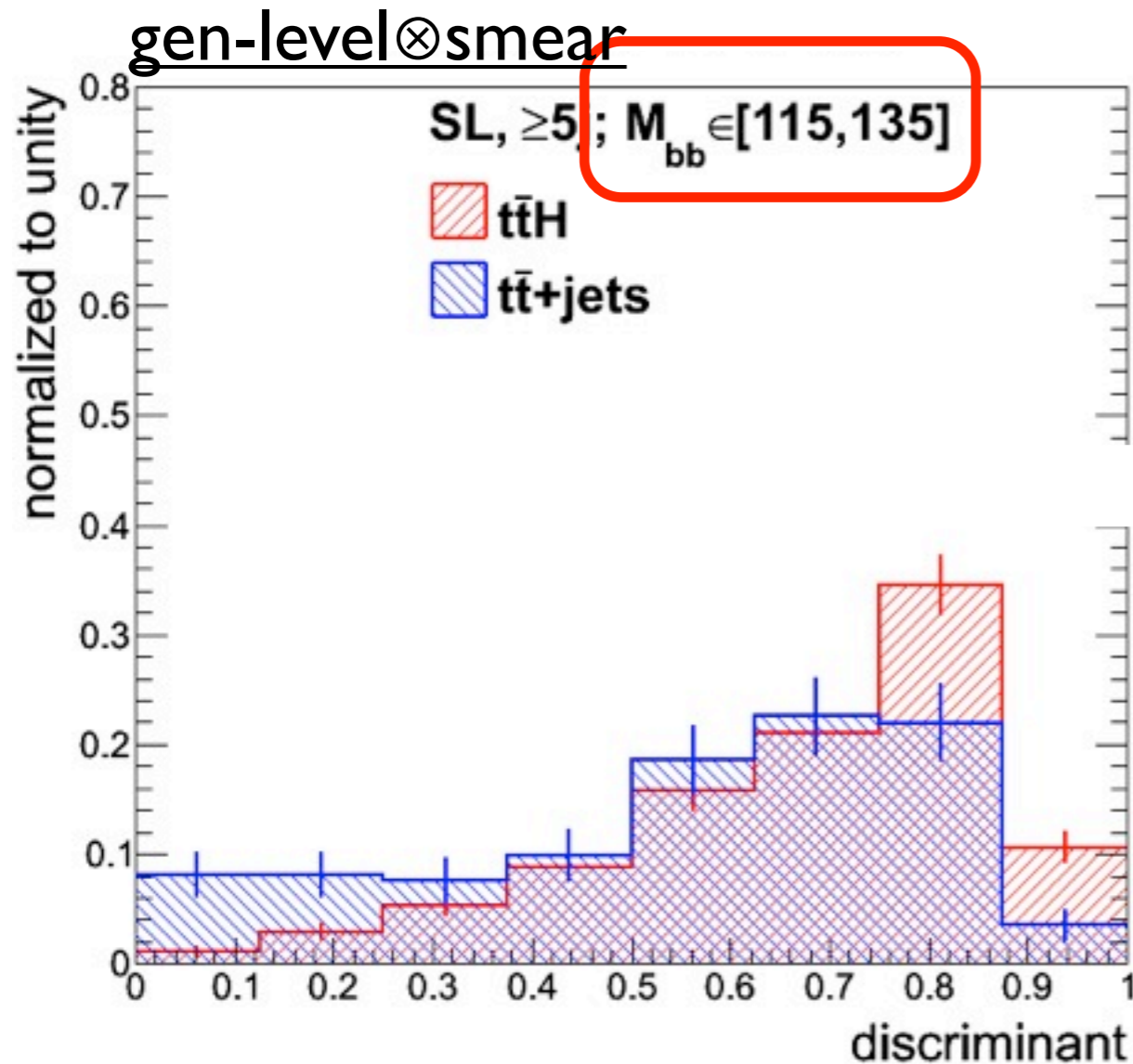




# Cross-section



# Digression I: the Higgs mass

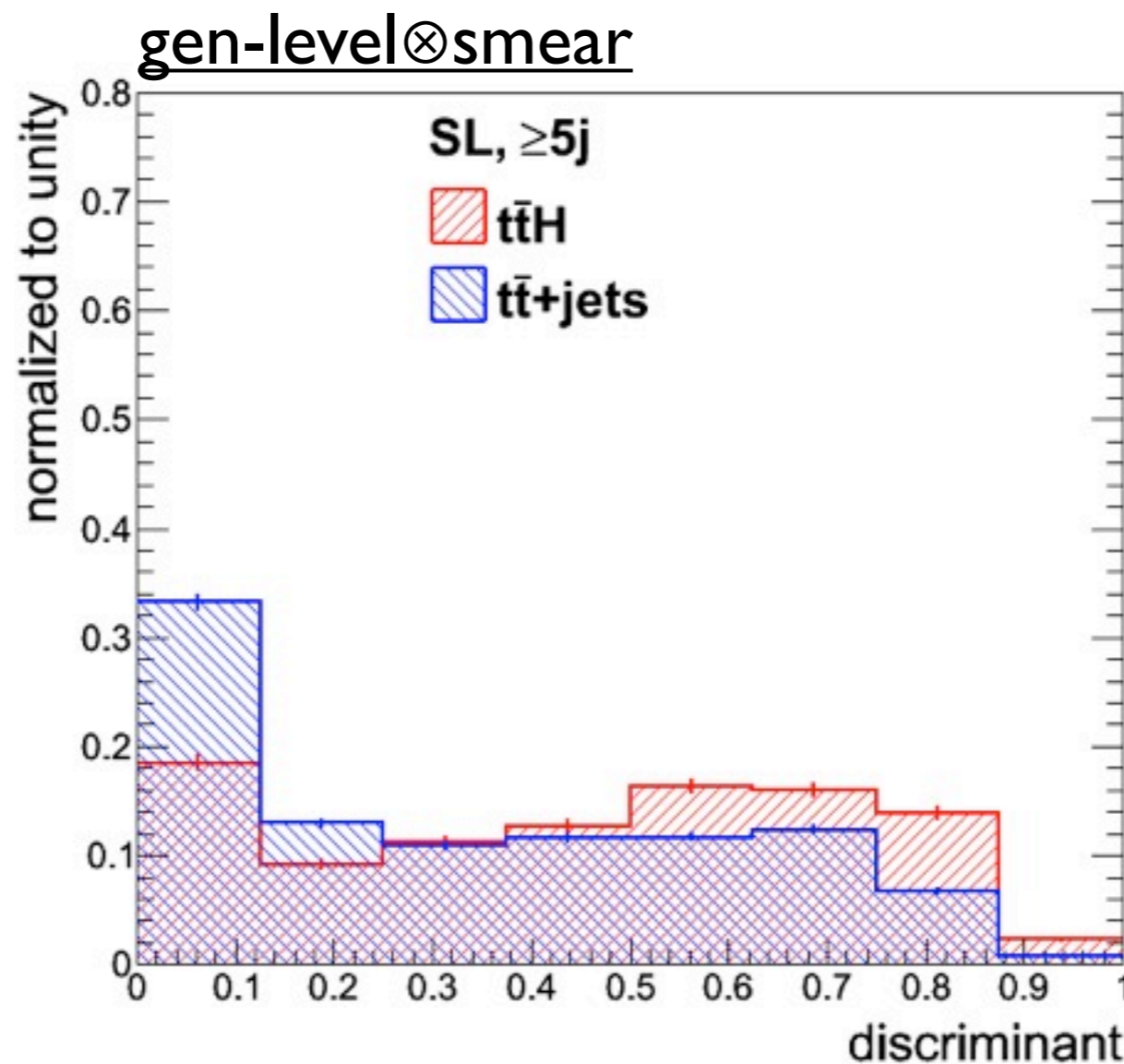


- $ttbb$  events w/  $M(bb) \approx 125$  indeed look like  $ttH$ !
  - ▶ but not identical  $\Leftrightarrow$  the ME is sensitive also to the other variables
- $ttH$  events w/  $M(bb) \neq 125$  (e.g. poor resolution) undistinguishable from  $ttbb$

# Digression II: wrong hypothesis



- If the event does not fulfill the *tested* ME hypo, the weight is broadly distributed
  - ▶ yet,  $t\bar{t}H$  remains slightly more “signal-like”



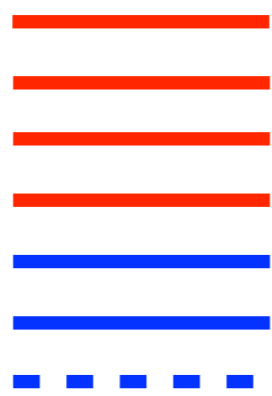
# Categories

assignment based on jet permutation w/ largest btag

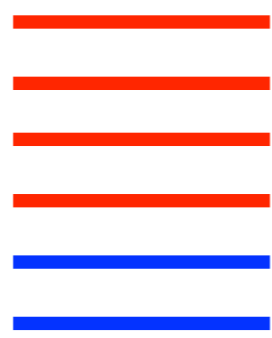

  
 — tagged  
 — untagged  
 - - - extra jets

SL  
 $N_j \geq 5$

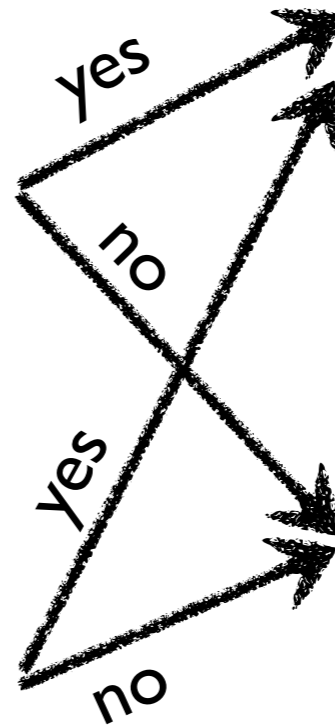
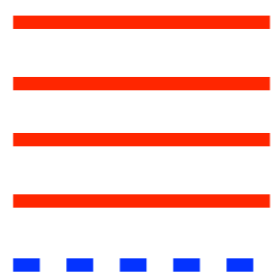
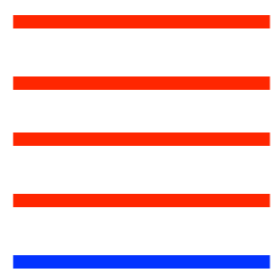
DL  
 $N_j \geq 4$



$|M_{uu}-m_W| < 10 \text{ GeV} ?$



$|M_{uu}-m_W| < 20 \text{ GeV} ?$



SL Cat. 1

all top/H quarks reconstructed

SL Cat. 2

one W-quark missed;  
extra gluon(s) from ISR  
 $\Rightarrow$ integrate over missing quark

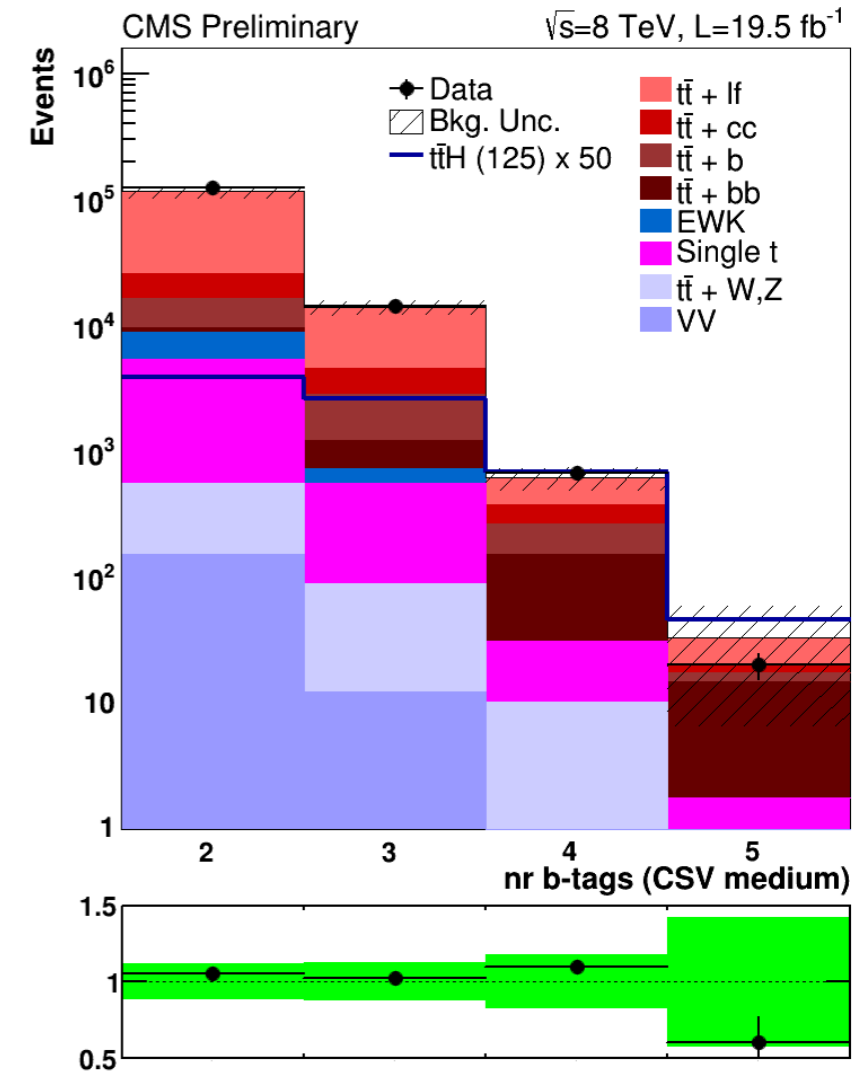
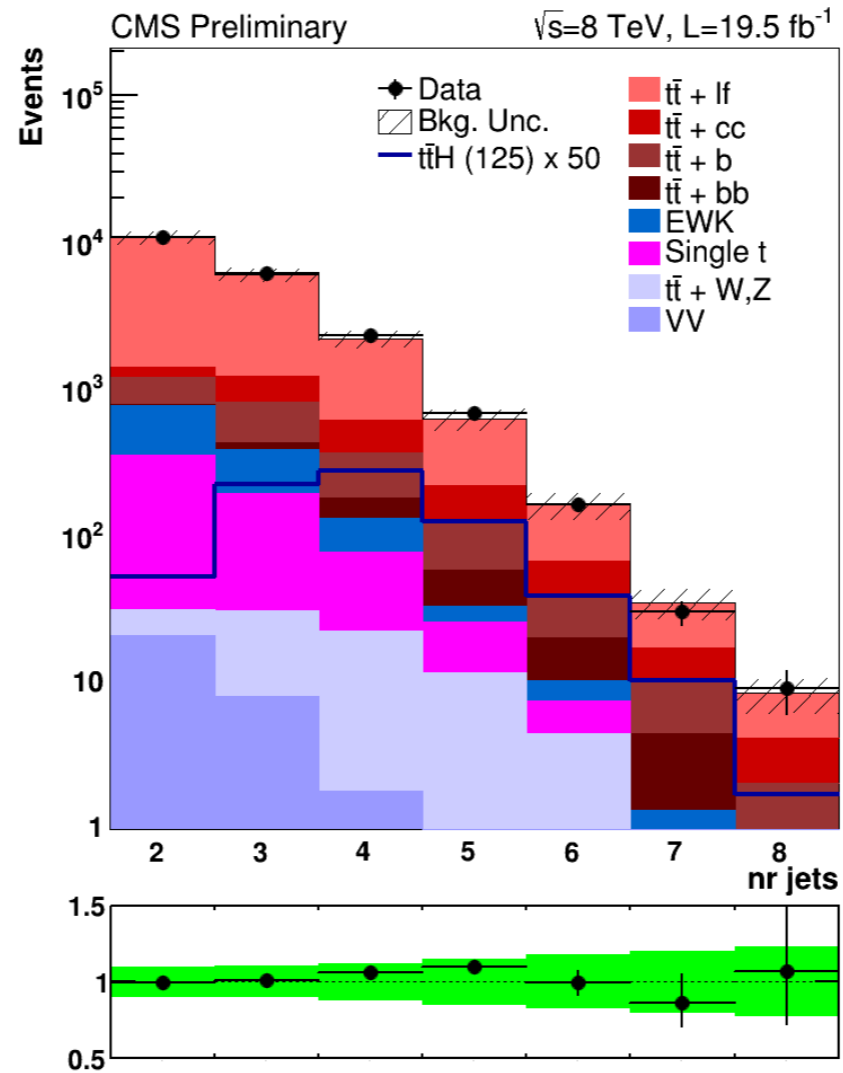
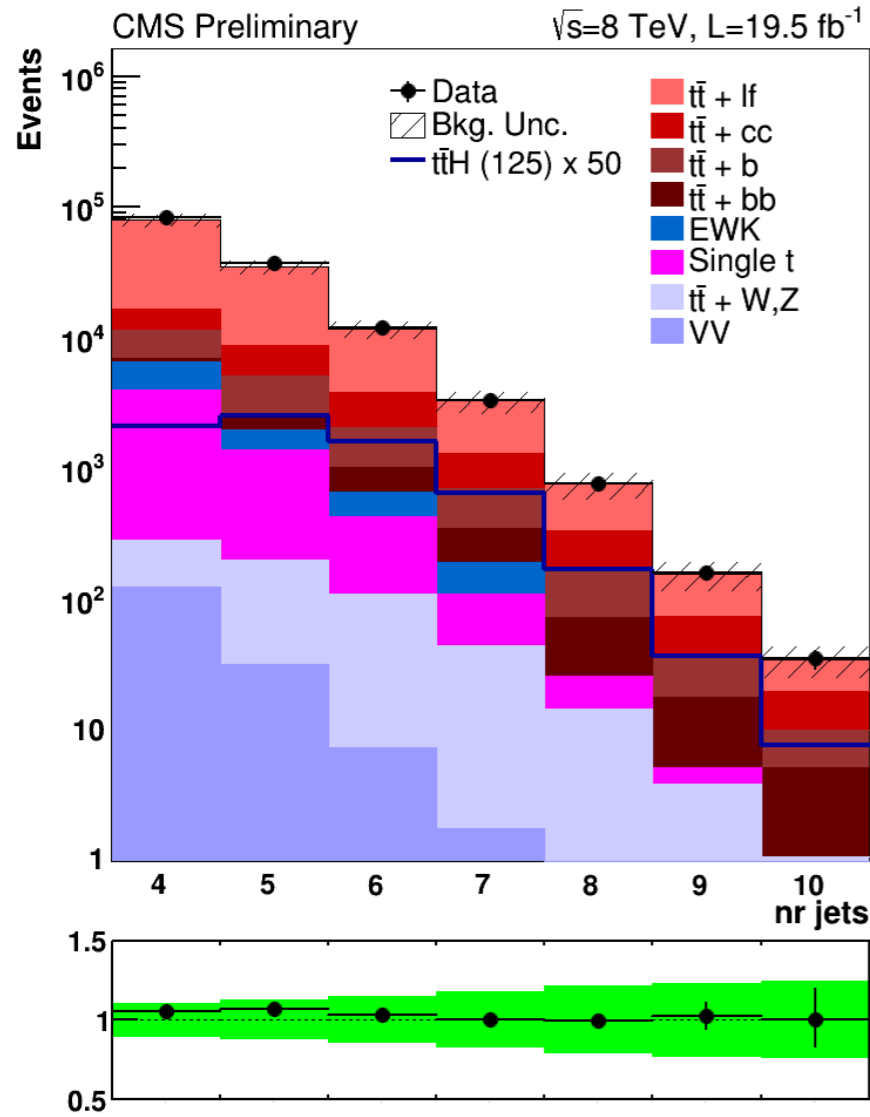
SL Cat. 3

one W-quark missed  
no extra-radiation  
 $\Rightarrow$ integrate over missing quark

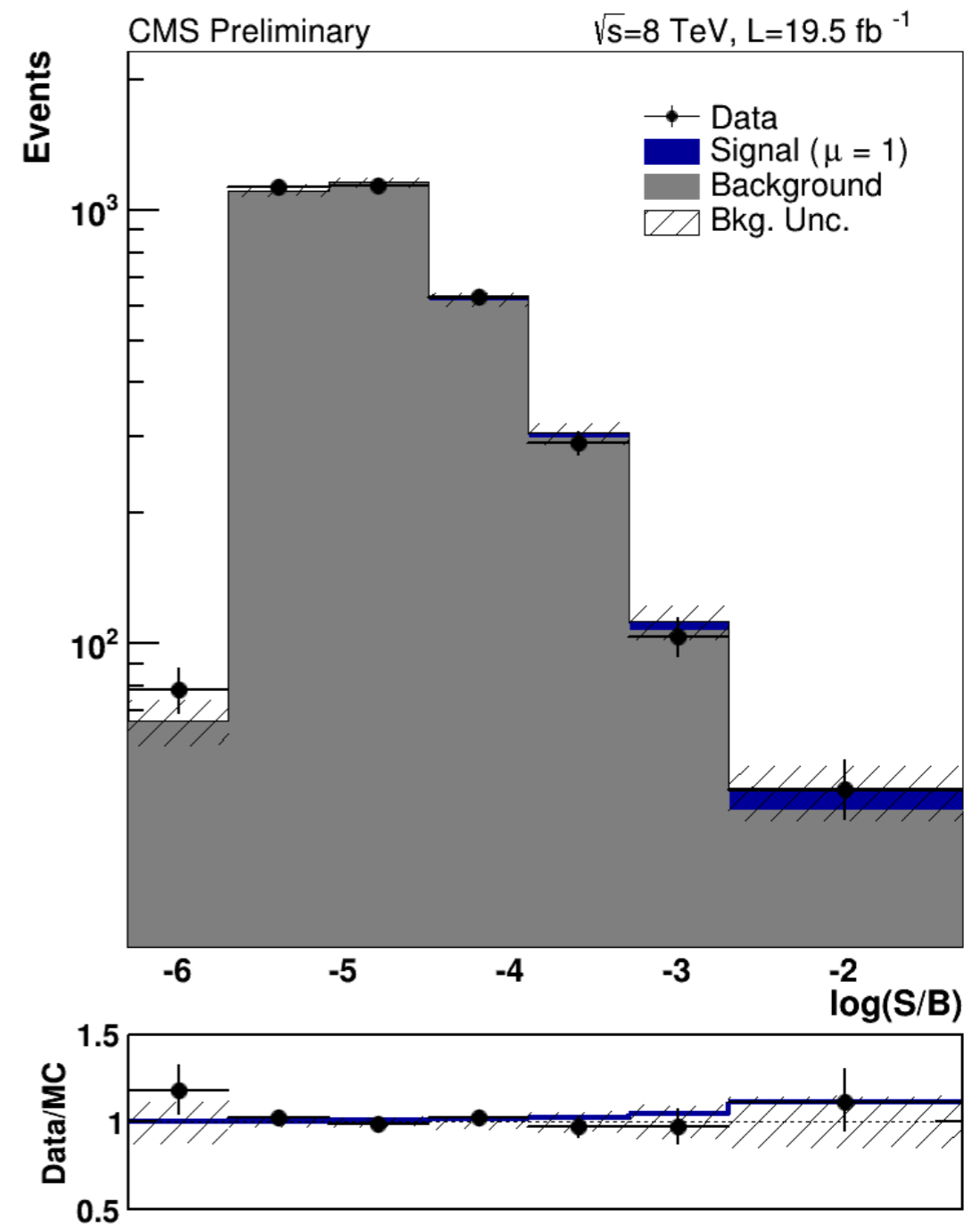
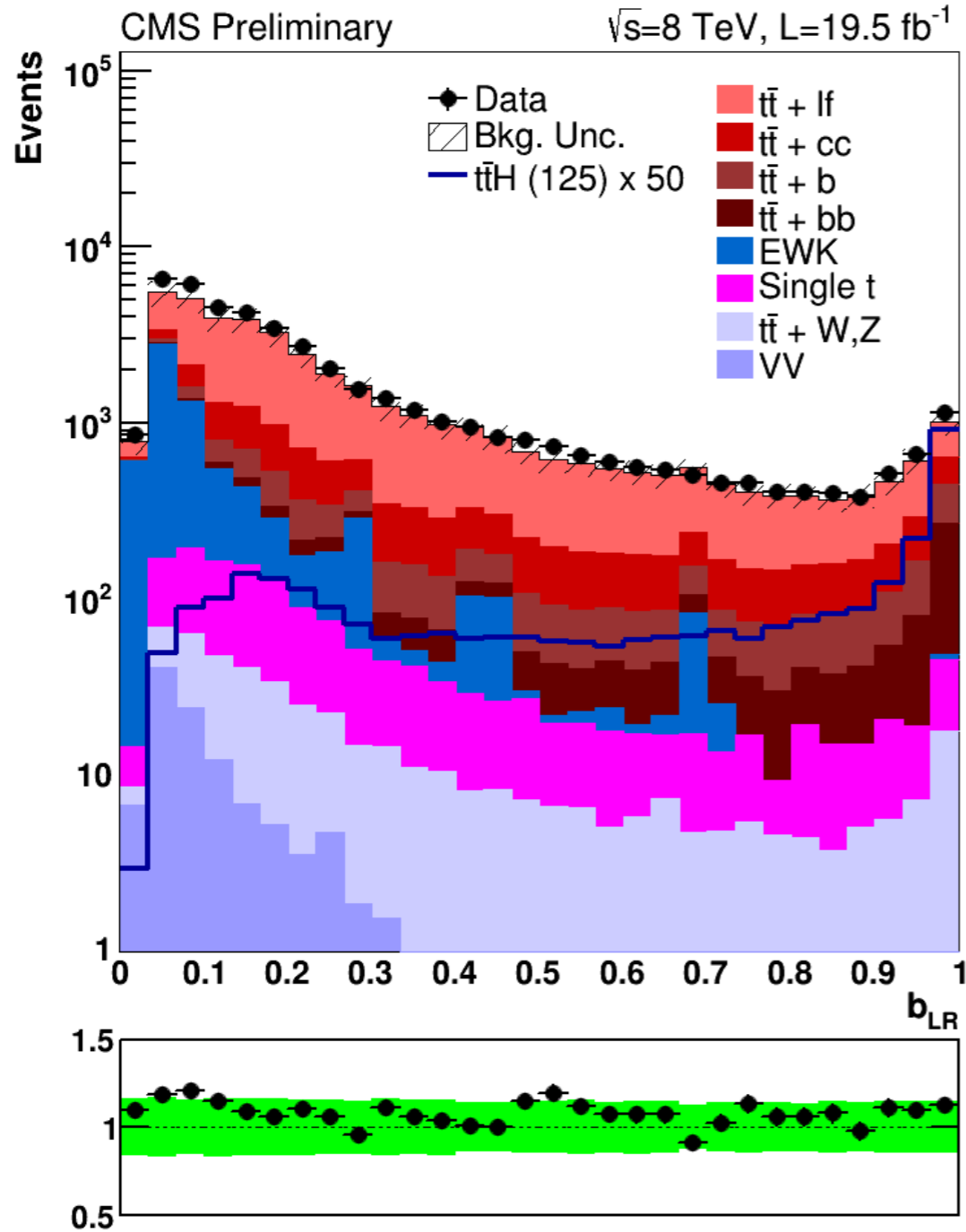
DL

all top/H quarks reconstructed

# Control plots



# Control plots



$$\mathcal{L}_{bbbb}(\xi_1, \dots, \xi_6) \equiv \sum_{\{i_1, \dots, i_6\}} f_b(\xi_{i_1}) \cdot f_b(\xi_{i_2}) \cdot f_b(\xi_{i_3}) \cdot f_b(\xi_{i_4}) \cdot f_u(\xi_{i_5}) \cdot f_u(\xi_{i_6})$$