Two loop Higgs masses beyond the MSSM

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LPTHE

[MDG, K. Nickel and F. Staub, 1411.0675,1411.4665, ...]

[MDG, P. Slavich in progress]

Overview

- Why go beyond the MSSM?
- Why calculate higher order corrections?
- Two loop Higgs mass in the effective potential approach
- Implementation in `SARAH` 4.4
- Results for the NMSSM
- Future direction

Experimental precision vs. theory precision

Consider the current experimental accuracy of the Higgs mass measurement:

CMS [CMS-PAS-HIG-14-009]	$125.03^{+0.26}_{-0.27}$(stat.) $^{+0.13}_{-0.15}$ (syst.) GeV
Atlas [PRD D **90**, 052004 (2014)]	125.36 ± 0.37 (stat.) ± 0.18(syst.) GeV

Gives an error of a few hundred MeV.

Compared to, in the MSSM:

- $m_h(\text{tree}) \leqslant M_Z$
- $\delta m_h^2(\text{loops}) \geqslant (86\text{GeV})^2 \gtrsim m_h^2(\text{tree})$
- Can have $\delta m_h(\text{two loops}) \lesssim$ 10 GeV $\rightarrow \delta m_h^2(\text{two loops}) \sim 15\% m_h^2$!
- While at three-loop order , have $\delta m_h \sim$ few hundred MeV,
  $\rightarrow \delta m_h^2(\text{three loops}) \lesssim 1\% m_h^2$

Not all two and three-loop corrections are known for the MSSM, but clearly the two loop corrections are particularly important for ruling models in/out/parameter extraction.

Calculations in models beyond the MSSM

There is now a large literature on precision calculations in the MSSM, too large to give references.

Up until recently, the rare models beyond the MSSM were limited to the NMSSM:

- [Degrassi and Slavich, 2009] calculated the complete one-loop corrections in the NMSSM, and the two-loop corrections proportional to $\alpha_s \alpha_t$, $\alpha_s \alpha_b$ in the effective potential approach.
- `NMSSMTools`, `NMSSMSOFTSUSY` implemented these and included the MSSM corrections proportional to the Yukawa couplings in the third family for the MSSM-like Higgs pairs.
- Recently [Muhlleitner, Nhung, Rzehak, Walz, 2014] calculated the $\alpha_s \alpha_t$, $\alpha_s \alpha_b$ corrections for the CP-violating NMSSM at zero external momentum and will implement these in `NMSSMCALC`.

For other models, the only possibility was and is `SARAH`:

- Calculates RGEs, full one-loop masses for all sparticles including momentum dependence, generates code for `SPheno`, `MicrOMEGAs`, `MadGraph`, ...

I will discuss how we have implemented the two-loop Higgs mass calculation in `SARAH`.

SARAH: a tool for BSM model builders

So what is `SARAH` ?

- `Mathematica` package written by F. Staub (with now some contributions from K. Nickel and MDG).
- Takes an input model file for any SUSY or non-SUSY model.
- Specify: gauge groups, matter content, superpotential/couplings in Lagrangian.
- Relevant for this talk: spectrum generation with `SPheno`. Produces fortran code which compiles against the `SPheno` library to generate spectrum and precision observables etc for the model.
- Will calculate two-loop RGEs, one-loop masses in $\overline{\mathrm{DR}}'$ (SUSY) or $\overline{MS}$ (non-SUSY) models. This is crucial in the following.
- Can specify input parameters at any scale: TeV, ~~SUSY~~ scale, GUT scale ...

Calculation of the Higgs mass in the effective potential approach

The Higgs mass is corrected order by order through two effects:

1. Self energy corrections

$$m^2(p^2) = m^2(\text{tree}) - \Pi(p^2)$$

2. Shifts to the tadpole conditions:

$$\begin{aligned} V =& V^{\text{tree}} + \Delta V \equiv \frac{1}{2} m_0^2 v^2 + V_\lambda^{\text{tree}} + \Delta V \\ \rightarrow 0 =& m_0^2 v + \left(\frac{\partial V_\lambda^{\text{tree}}}{\partial v} + \frac{\partial \Delta V}{\partial v} \right) \end{aligned}$$

If we take v as fixed to all orders we must shift $m^2(\text{tree})$ so that

$$\rightarrow m^2(\text{tree}) = \frac{\partial^2 V^{\text{tree}}}{\partial v^2} - \frac{1}{v} \frac{\partial \Delta V}{\partial v}.$$

One significant simplification to calculations is to take $p^2 = 0$; this is then equivalent to taking

$$\Pi(0) = -\frac{\partial^2 V}{\partial v^2}.$$

Two loop calculations in general theories

While explicit calculations for specific models have been restricted to the (N)MSSM, S. Martin in a series of pioneering papers has calculated two loop corrections in general renormalisable field theories. Amongst other papers:

- [S. Martin, 01] : general two loop effective potential in Landau gauge.
- [S. Martin, 02] : two papers on the effective potential of the MSSM and the Higgs mass.
- [S. Martin, 03] : pole masses for scalars.

We made heavy use of these results.

Goldstone boson catastrophe

One of the problems suffered in [S. Martin, 02] was the "Goldstone boson catastrophe:"

- In the Landau gauge the longitudinal components of the W and Z bosons are massless. Write

$$V \equiv V^{\text{tree}}(v^2 + G^2) + \Delta V$$
$$m^2(\text{Goldstone}) = 0 = \underbrace{\frac{1}{v}\frac{\partial V^{\text{tree}}}{\partial v}}_{\text{tree-level mass-squared}} + \frac{\partial^2 \Delta V}{\partial G^2} = -\frac{1}{v}\frac{\partial \Delta V}{\partial v} + \frac{\partial^2 \Delta V}{\partial G^2}$$

- The "tree-level" mass-squared for the would-be Goldstone boson is then generically non-zero and is what enters into the two-loop functions.
- Through RGE running this mass-squared can change sign – while the potential is well defined at two loops some derivatives of loop functions are singular!
- While there are solutions for computing the tadpoles (see [S. Martin, 14] and [Elias-Miro, Espinosa, Konstandin, 14]) they suggest the full solution for masses is to compute the pole mass at non-zero momentum.

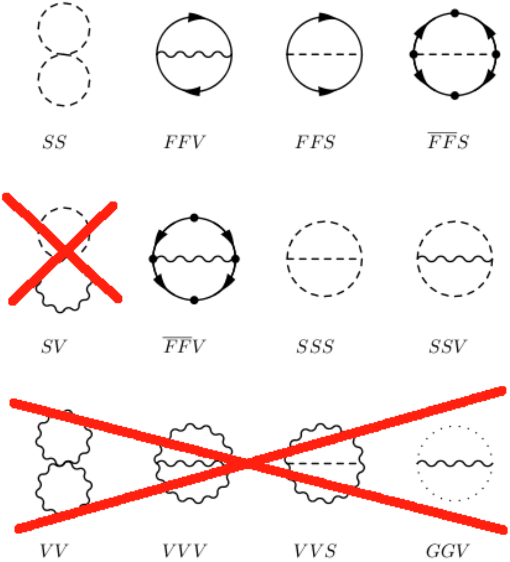
Gaugeless limit

An alternative simplification – at least for the (N)MSSM – is to go to the “gaugeless limit”: neglect the contribution of all broken gauge groups, and treat the would-be Goldstone bosons as actual Goldstone bosons!

- In the MSSM, this kills the tree-level quartic coupling, so the derivatives of the Goldstone masses w.r.t. the Higgs are zero $\rightarrow$ we do not introduce singularities in the derivatives of the potential.
- In general, the electroweak bosons have a large effect on the would-be Goldstone boson “tree-level” mass, so killing these terms simplifies or removes the problem.
- These contributions have been estimated to be of the order of hundreds of MeV in the MSSM, so are generally small.
- The two-loop calculations are greatly simplified.
- In our approach, we actually use the tree-level masses in the gaugeless limit to compute the two-loop corrections (we solve first for the full tadpole equation, and then compute the Goldstone masses with zero gauge couplings).
- In the (N)MSSM we therefore find for the neutral Goldstone boson $\frac{\partial^2 V^{\mathrm{tree}}}{G^2} = -m_Z^2 \cos^2 2\beta$.

Diagrams

Here is the set of two-loop diagrams we consider (mangled from [S. Martin, 01])



Two-loop calculation

Three methods of mass calculation:

1. Calculate the effective potential and numerically take the first and second derivatives. This was advocated for the MSSM in [S. Martin, 02] .
2. Numerically take the derivatives of the parameters entering the potential, and use analytic expressions for the derivatives of the loop functions themselves, e.g.

$$
\begin{aligned}
V^{(2\text{ loop})} &\supset \sum_{ijk} (\lambda^{ijk})^2 f_{SSS}(m_i^2, m_j^2, m_k^2) \\
\rightarrow \frac{\partial V^{(2\text{ loop})}}{\partial v} &\supset \sum_{ijk} 2\lambda^{ijk} \frac{\partial \lambda^{ijk}}{\partial v} f_{SSS}(m_i^2, m_j^2, m_k^2) \\
&\quad + 3(\lambda^{ijk})^2 \frac{\partial f_{SSS}(m_i^2, m_j^2, m_k^2)}{\partial m_i^2} \frac{\partial m_i^2}{\partial v}
\end{aligned}
$$

3. Use analytic expressions for the derivatives, equivalent to a diagrammatic calculation.

The first two methods are currently implemented in the public version of `SARAH`. The third method is in beta testing; it will be announced along with the analytic expressions in a paper soon.

Validation

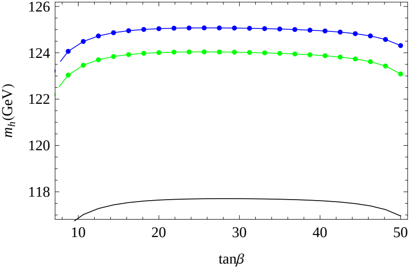
To validate our code we compared with

- $\alpha_s \alpha_t$ corrections in the MSSM
- Full $\alpha_s(\alpha_t + \alpha_b) + (\alpha_t + \alpha_b + \alpha_\tau)^2$ MSSM results in the tools of P. Slavich.
- $\alpha_s \alpha_t$ corrections in the NMSSM by P. Slavich.
- $\alpha_s \alpha_t$ corrections in Dirac gaugino models by MDG and P. Slavich (currently private code, will be made available in `SARAH` in future).

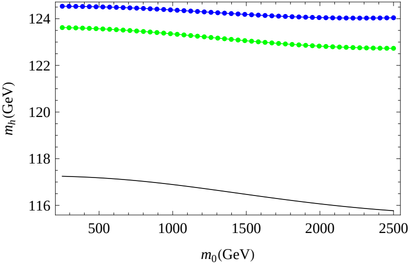
These are the only examples available!

Comparison plots for the CMSSM

Comparing with the full routines for the MSSM (solid lines vs dots for our code):



$m_0 = M_{1/2} = 1$ TeV, $\mu > 0$, $A_0 = -2$TeV



$M_{1/2} = 1$ TeV
$\tan\beta = 10$, $\mu > 0$, $A_0 = -2$TeV

Clearly we find excellent agreement!

Results for the NMSSM

Shall consider the NMSSM with $\mathbb{Z}_3$ symmetry:

$$W = \lambda S H_d \cdot H_u + \frac{\kappa}{3} S^3$$

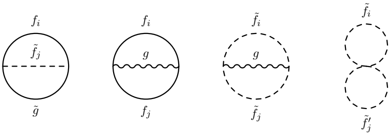
In the past, since only incomplete results were available, the best approach was to combine the NMSSM $\alpha_s \alpha_t$ results with the MSSM Yukawa couplings:

$$\Delta M^2(\text{two loops}) \sim \alpha_s(\alpha_t + \alpha_b) \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix} + (\alpha_t + \alpha_b + \alpha_\tau)^2 \begin{pmatrix} \Delta M^2_{11}(MSSM) & \Delta M^2_{12}(MSSM) & 0 \\ \Delta M^2_{21}(MSSM) & \Delta M^2_{22}(MSSM) & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

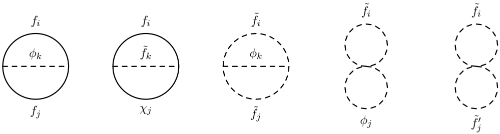
- We can now check the validity of this approximation.
- Of course, expect that the approximation is only useful for the non-singlet-like Higgses.

Diagrams

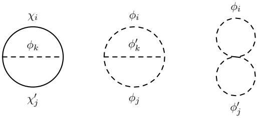
The diagrams already computed:



MSSM-like diagrams:



New diagrams computed for the first time by our code:



Heavy singlet case

When the singlet-like Higgs is heavier than 125 GeV and λ is moderate, find small corrections from the Yukawa-coupling part:

$$m_0 = 1.4 \text{ TeV} \qquad M_{1/2} = 1.4 \text{ TeV} \qquad \tan\beta = 2.9 \qquad A_0 = -1.35 \text{ TeV}$$
$$\lambda = 0.56 \qquad \kappa = 0.33 \qquad A_\lambda = -390 \text{ GeV} \qquad A_\kappa = -280 \text{ GeV} \qquad \mu_{\text{eff}} = 200 \text{ GeV}$$

	Tree	one-loop	two-loop $(\alpha_s(\alpha_b + \alpha_t))$	full two-loop
m_{h_1}	93.8	117.6 (+ 25.4%)	126.1 (+7.2%)	124.7 (-1.1%)
m_{h_2}	214.5	209.2 (-2.4%)	209.2 ($\pm$ 0%)	208.7 (-0.2%)
m_{h_3}	555.5	541.9 (-2.4%)	542.3 (+0.1%)	541.4 (-0.2%)

NB the second Higgs h_2 is mostly singlet-like, with the MSSM-like heavy Higgs being the heaviest (h_3).
Here the approximation works well.

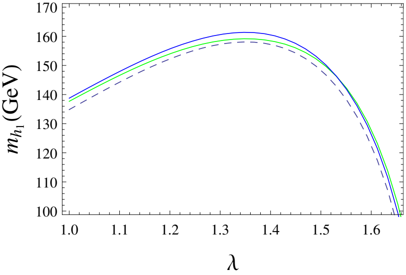
Large λ

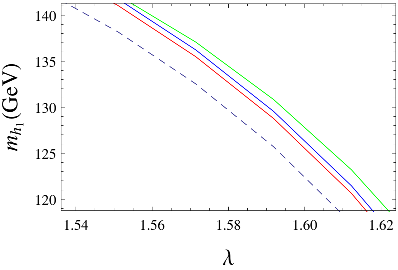
$\lambda = 1.6$ $\kappa = 1.6$ $\tan\beta = 3$ $T_\lambda = 600$ GeV $T_\kappa = -2650$ GeV $\mu_{eff} = 614$

	Tree	one-loop	two-loop ($\alpha_s(\alpha_b + \alpha_t)$)	full two-loop
m_{h_1}	144.8	122.6 (-15.3%)	126.5 (+3.2%)	128.0 (+1.2%)
m_{h_2}	713.2	745.9 (+4.6%)	745.8 (+0.0%)	747.9 (+0.3%)
m_{h_3}	1454.5	1421.1 (-2.3%)	1420.1 (-0.1%)	1420.3 (+0.0%)

Here the corrections have changed sign: the NMSSM-specific corrections dominate the MSSM ones.

Heavy singlet II





Blue: $\alpha_s \alpha_t$, Red: MSSM $(\alpha_t + \alpha_b + \alpha_\tau)^2$ included, Green: full SARAH result.
Clearly the approximation gives misleading results for large λ

Light singlet

When the singlet-like scalar is ligher than 125 GeV – a case particularly interesting for experimental searches – the SM-like Higgs has a tree-level boost. However, we expect existing 2-loop results to be inadequate because they do not include the new singlet or the other new states!

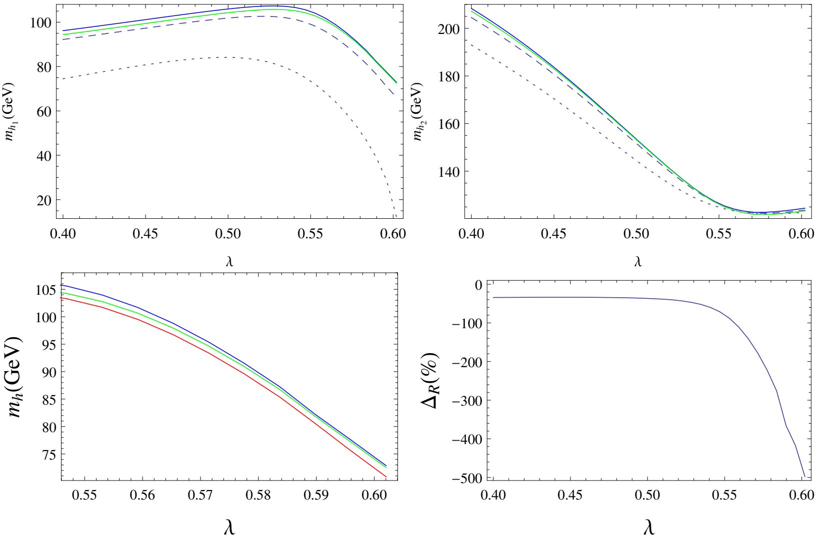
$$\lambda = 0.596 \;\; \kappa = 0.596 \;\; T_\lambda = -27 \text{ GeV} \;\; T_\kappa = -240 \text{ GeV} \;\; \mu_{\text{eff}} = 130 \text{ GeV}$$

$$T_t = -3050 \text{ GeV} \; T_b = T_\tau = -1000 \text{ GeV}$$

$$m^2_{\tilde{Q},33} = 9.0 \cdot 10^5 \text{ GeV}^2 \; m^2_{\tilde{U},33} = 1.05 \cdot 10^6 \text{ GeV}^2$$

	Tree	one-loop	two-loop ($\alpha_s(\alpha_b + \alpha_t)$)	full two-loop
m_{h_1}	19.4	67.8 (+249.5%)	74.5 (+9.9%)	74.2 (-0.4%)
m_{h_2}	122.7	123.5 (+0.7%)	124.3 (+0.6%)	123.3 (-0.8%)
m_{h_3}	177.4	188.2 (+6.1%)	192.7 (+2.3%)	191.1 (-0.8%)

Light singlet error



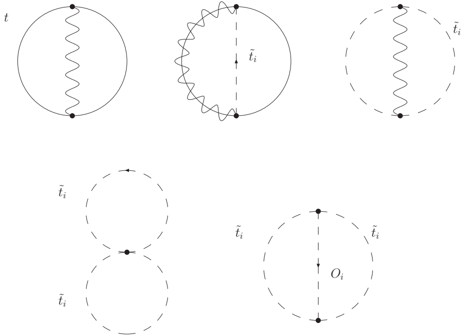
Clearly the MSSM approximation fails here!

Conclusions: NMSSM

- We have considered the standard NMSSM at small $\tan\beta$ – where the tree-level effects are largest, and the two-loop effects are expected in general to be small.
- However, even in this case we find that generally, while the $\alpha_s \alpha_t$ corrections are a reasonable approximation to the full result, including the MSSM Yukawa corrections can actually make the approximation <u>worse</u>!
- `SARAH`should really be considered now an indispensible tool for studying the NMSSM Higgs masses (at least until we port a standalone code to be included in other spectrum generators).

Dirac gauginos

As mentioned above, have analytic results for the $\alpha_s \alpha_t$ contributions universal to all Dirac gaugino models, and a code (written with P. Slavich).



Find that the effects can be large!
Analytic results and study of full corrections in specific models to come ...

Conclusions

- We now have a tool to compute the Higgs mass to two loop precision for <u>any</u> SUSY model.
- For the MSSM, there might be cases where we provide an improvement over existing tools.
- For the NMSSM, there are certainly regions of the parameter space where previously existing tools are inaccurate.
- We have opened up a new line of work to improve upon this! Many future directions ...