

Valentina De Romeri¹

Impact of sterile neutrinos in lepton flavour violating processes

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GDR Terascale
Heidelberg

In collaboration with Asmaa Abada², Ana Teixeira¹,
Stephane Monteil¹ and Jean Orloff¹

1) Laboratoire de Physique Corpusculaire Clermont-Ferrand
2) Laboratoire de Physique Theorique, Orsay



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 - Inverse Seesaw (ISS)
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 - Neutrinoless double beta decay
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 - LFV Z decays at a high luminosity Z factory
- **Conclusions**

Neutrino physics

As of today, data favour a **three-active neutrinos oscillation** framework, with very accurate measurements of the solar and atmospheric parameters.

Neutrino oscillation parameters have been inferred by detecting neutrinos coming from the Sun, the Earth's atmosphere, nuclear reactors and accelerator beams.

Although in the last years many pieces of the puzzle have been found, the **neutrino physics picture is far from being complete.**

parameter	best fit $\pm 1\sigma$	2σ range	3σ range
Δm_{21}^2 [10^{-5}eV^2]	$7.60^{+0.19}_{-0.18}$	7.26–7.99	7.11–8.18
$ \Delta m_{31}^2 $ [10^{-3}eV^2] (NH)	$2.48^{+0.05}_{-0.07}$	2.35–2.59	2.30–2.65
$ \Delta m_{31}^2 $ [10^{-3}eV^2] (IH)	$2.38^{+0.05}_{-0.06}$	2.26–2.48	2.20–2.54
$\sin^2 \theta_{12}/10^{-1}$	3.23 ± 0.16	2.92–3.57	2.78–3.75
$\theta_{12}/^\circ$	34.6 ± 1.0	32.7–36.7	31.8–37.8
$\sin^2 \theta_{23}/10^{-1}$ (NH)	$5.67^{+0.32}_{-1.28}$ ^a	4.13–6.23	3.92 – 6.43
$\theta_{23}/^\circ$	$48.9^{+1.9}_{-7.4}$	40.0–52.1	38.8–53.3
$\sin^2 \theta_{23}/10^{-1}$ (IH)	$5.73^{+0.25}_{-0.43}$	4.32–6.21	4.03–6.40
$\theta_{23}/^\circ$	$49.2^{+1.5}_{-2.5}$	41.1–52.0	39.4–53.1
$\sin^2 \theta_{13}/10^{-2}$ (NH)	2.34 ± 0.20	1.95–2.74	1.77–2.94
$\theta_{13}/^\circ$	8.8 ± 0.4	8.0–9.5	7.7–9.9
$\sin^2 \theta_{13}/10^{-2}$ (IH)	2.40 ± 0.19	2.02–2.78	1.83–2.97
$\theta_{13}/^\circ$	8.9 ± 0.4	8.2–9.6	7.8–9.9
δ/π (NH)	$1.34^{+0.64}_{-0.38}$	0.0–2.0	0.0–2.0
$\delta/^\circ$	241^{+115}_{-68}	0–360	0–360
δ/π (IH)	$1.48^{+0.34}_{-0.32}$	0.0–0.14 & 0.81–2.0	0.0–2.0
$\delta/^\circ$	266^{+61}_{-58}	0–25 & 146–360	0–360

^aThere is a local minimum in the first octant, $\sin^2 \theta_{23} = 0.467$ with $\Delta\chi^2 = 0.28$ with respect to the global minimum

(Forero et al., [arXiv:1405.7540](https://arxiv.org/abs/1405.7540))

Solar $\nu_e \rightarrow \nu_{\mu,\tau}$ SNO, BOREXino, Super-Kamiokande, GALLEX/GNO, SAGE, Homestake, Kamiokande

Atmospheric $\nu_\mu \rightarrow \nu_\tau$, LBL Accelerator ν_μ disappearance, LBL Accelerator IMB, MACRO, Soudan-2, Kamiokande, Super-Kamiokande

LBL Accelerator ($\nu_\mu \rightarrow \nu_e$), LBL Reactor ($\bar{\nu}$ disappearance) T2K, MINOS Daya Bay, RENO, Double Chooz

SBL Accelerator (ν_μ ($\bar{\nu}_\mu$) $\rightarrow \nu_e$ ($\bar{\nu}_e$)), SBL Reactor ($\bar{\nu}$ disappearance) LSND, MiniBooNE, ++ Solar: GALLEX, SAGE++Bugey, ILL, Rovno..

Neutrino physics open questions



Among the missing ingredients there are:

- **Absolute mass scale** (Tritium β decays: $m_{\nu_e} < 2.05 \text{ eV}$, Cosmology: $\sum m_{\nu_i} < 0.66 \text{ eV}$ (CMB), $\sum m_{\nu_i} < 0.23 \text{ eV}$ (CMB+BAO+WMAP polarization data+high-resolution CMB experiments and flat Universe)) (Troitsk and Mainz, Planck 2013)
- **Majorana** versus Dirac nature ($0\nu\beta\beta$ decay) (KamLAND-Zen, EXO-200, Gerda)
- The mass ordering (normal or inverted "hierarchy") (matter effects in sun and long baseline oscillations, T2K, NOvA...)
- Is there CP violation in the lepton sector?
- Are there extra **sterile** states?
- What is the underlying mechanism responsible for the generation of their masses?

In the SM, neutrinos are strictly **massless**:

- absence of RH neutrino fields \Rightarrow no Dirac mass term (no renormalizable mass term)
- no Higgs triplet \Rightarrow no Majorana mass term (would break the electroweak gauge symmetry, because it is not invariant under the weak isospin symmetry; does not conserve the lepton number L)

Massive neutrinos require BSM physics

Several models of neutrino mass generation:

- Seesaw mechanism: Type-I, Type-II, Type-III, low-scale seesaws (**Inverse seesaw**, Linear seesaw, ν MSM) etc ...
- Radiative models

...

(Minkowski 77, Gell-Mann Ramond Slansky 80, Glashow, Yanagida 79, Mohapatra Senjanovic 80, Lazarides Shafi Wetterich 81, Schechter-Valle, 80 & 82, Mohapatra Senjanovic 80, Lazarides 80, Foot 88, Ma, Hambye et al., Bajc, Senjanovic, Lin, Abada et al., Notari et al...)

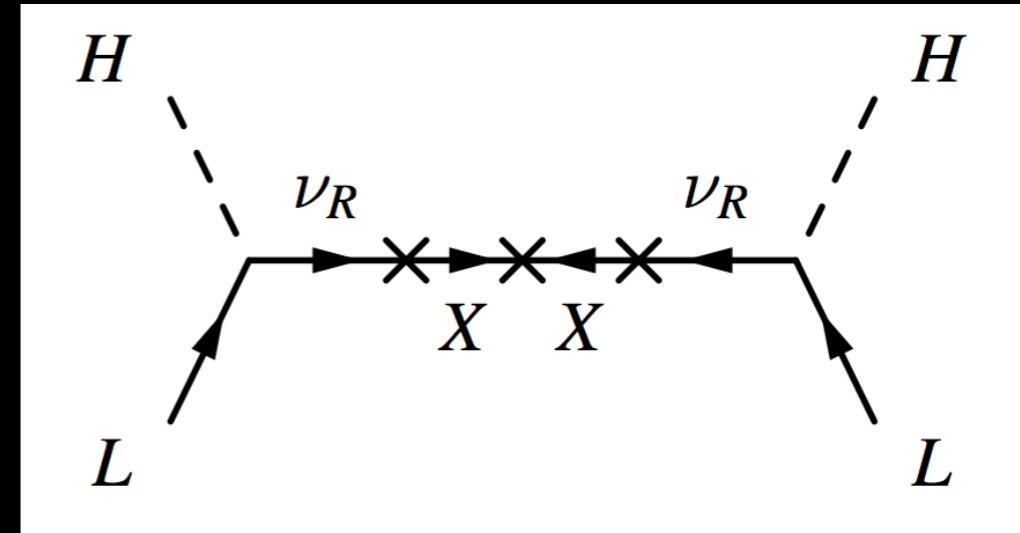
Inverse seesaw

(Mohapatra & Valle, 1986)

Add three generations of SM singlet pairs, ν_R and X (with $L=+1$)

Inverse seesaw basis (ν_L, ν_R, X)

$$M^\nu = \begin{pmatrix} 0 & m_D & 0 \\ m_D^T & 0 & M_R \\ 0 & M_R^T & \mu_X \end{pmatrix}$$



After EWSB the effective light neutrino masses are given by

$$m_\nu = m_D (M_R^T)^{-1} \mu_X (M_R)^{-1} m_D^T$$

$Y_\nu \sim O(1)$ and $M_R \sim 1\text{TeV}$ testable at the colliders and low energy experiments.

Large mixings (active-sterile) and light sterile neutrinos are possible

Sterile neutrinos

From the **invisible decay width of the Z boson** [LEP]:

⇒ extra neutrinos must be sterile (=EW singlets) or cannot be a Z decay product

Any singlet fermion that mixes with the SM neutrinos

- Right-handed neutrinos
- Other singlet fermions

Sterile neutrinos are SM gauge singlets - only interact via their mixing with the active ones

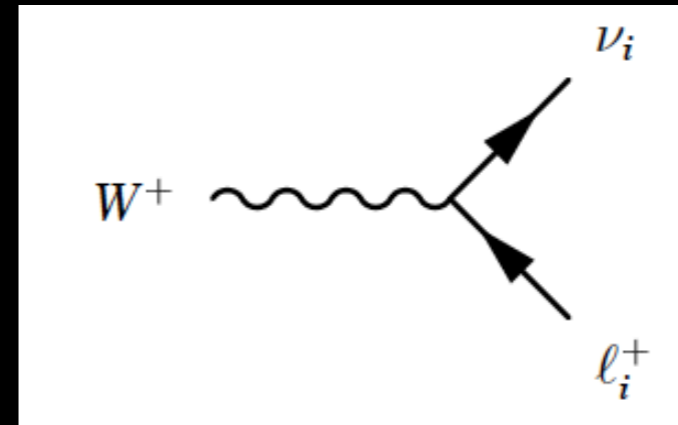
Several oscillation results or **anomalies** (reactor antineutrino anomaly, LSND, MiniBooNe...) cannot be explained within 3-flavor oscillations

⇒ need at least an extra neutrino

Other motivations for sterile neutrinos from **cosmology**, e.g. keV sterile neutrino as warm dark matter or to explain pulsar velocities

Active-sterile mixing

Leptonic charged currents can be modified due to the mixing with the steriles.



Active-sterile mixing

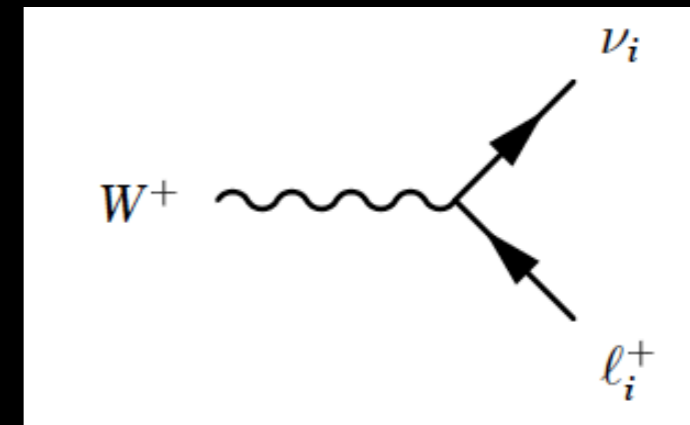
Leptonic charged currents can be modified due to the mixing with the steriles.

Standard case (3 flavors):

$$\nu_i = e, \mu, \tau$$

$$\nu_i = \text{flavor eigenstate} = \sum_{a_i} U_{a_i}^{\text{PMNS}} \nu_a$$

$$\nu_a = \text{mass eigenstates, } a = 1, 2, 3$$



Add sterile neutrinos:

$$-\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} U^{ji} \bar{l}_j \gamma^\mu P_L \nu_i W_\mu^- + \text{c.c.}$$

$$\nu_i = \sum_{a_i} U_{a_i} \nu_a, \quad a = 1, 2, 3, 4 \dots 9 \dots n_\nu \quad U = \text{extended matrix, } j=1 \dots 3, \quad i=1 \dots n_\nu$$

If $n_\nu > 3$, $U \neq U_{\text{PMNS}} \rightarrow$ the 3×3 sub matrix is **not unitary**

$$U_{\text{PMNS}} \rightarrow \tilde{U}_{\text{PMNS}} = (\mathbb{1} - \eta) U_{\text{PMNS}}$$

(see also: [Fernandez-Martinez et al. 2007](#), [Gavela et al. 2009](#), [Abada et al. 2014](#), [Arganda et al. 2014](#))

Experimental constraints

The deviations from unitarity and the possibility of having steriles as final decay products, might induce departures from the SM expectations.

1. Neutrino oscillation parameters (seesaw approximation and PMNS)
2. Unitarity constraints
3. Electroweak precision data
4. LHC data (invisible decays)
5. Leptonic and semileptonic meson decays (B and D)
6. Laboratory bounds: direct searches for sterile neutrinos
7. Lepton flavor violation ($\mu \rightarrow e \gamma$)
8. Neutrinoless double beta decay
9. Cosmological bounds on sterile neutrinos

Experimental constraints

1. Neutrino oscillation parameters (seesaw approximation and PMNS)

2. Unitarity constraints Non-standard neutrino interactions with matter can be generated by NP. $U_{3\times 3} = (1 - \eta)U_{PMNS}$ effective theory approach

(Antusch et al., 2009)

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(Del Aguila et al., 2008, Atre et al., 2009)

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4. LHC data (invisible decays) decay modes of the Higgs boson
 $h \rightarrow \nu_R \nu_L$ relevant for sterile neutrino masses ~ 100 GeV

(Bhupal Dev et al., 2012,
P. Bandyopadhyay et al,2012,
Cely et al., 2013,Arganda et
al. 2014)

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5. Leptonic and semileptonic meson decays (K,B and D) $\Gamma(P \rightarrow l\nu)$ with $P = K,D,B$ with one or two neutrinos in the final state
(J. Beringer et al. ,PDG, 2013)

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6. Laboratory bounds: direct searches for sterile neutrinos e.g. $\pi^\pm \rightarrow \mu^\pm \nu_s$, the lepton spectrum would show a monochromatic line.
(Atre et al. 2009, Kusenko et al. 2009)

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7. Lepton flavor violation ($\mu \rightarrow e \gamma$) $Br(\mu \rightarrow e\gamma)_{MEG} = 0.57 \times 10^{-12}$

(Ilakovac and Pilaftsis, 1995, Deppisch and Valle, 2005)

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 9. Neutrinoless double beta decay $m_\nu^{\beta\beta} = \sum_i U_{ei}^2 m_i \leq (140 - 700) meV$ (EXO-200, KamLAND-Zen, GERDA, CUORICINO)
- (see also: Blennow et al. 2010, Lopez-Pavon et al. 2013, Abada et al. 2014)
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10. Cosmological bounds on sterile neutrinos Large scale structure, Lyman- α , BBN, CMB, X-ray constraints (from $\nu_i \rightarrow \nu_j \gamma$), SN1987a
(Smirnov et al. 2006, Kusenko 2009, Gelmini 2010)

**Numerical analysis:
Inverse Seesaw and
Effective “3+1” model**

Inverse Seesaw

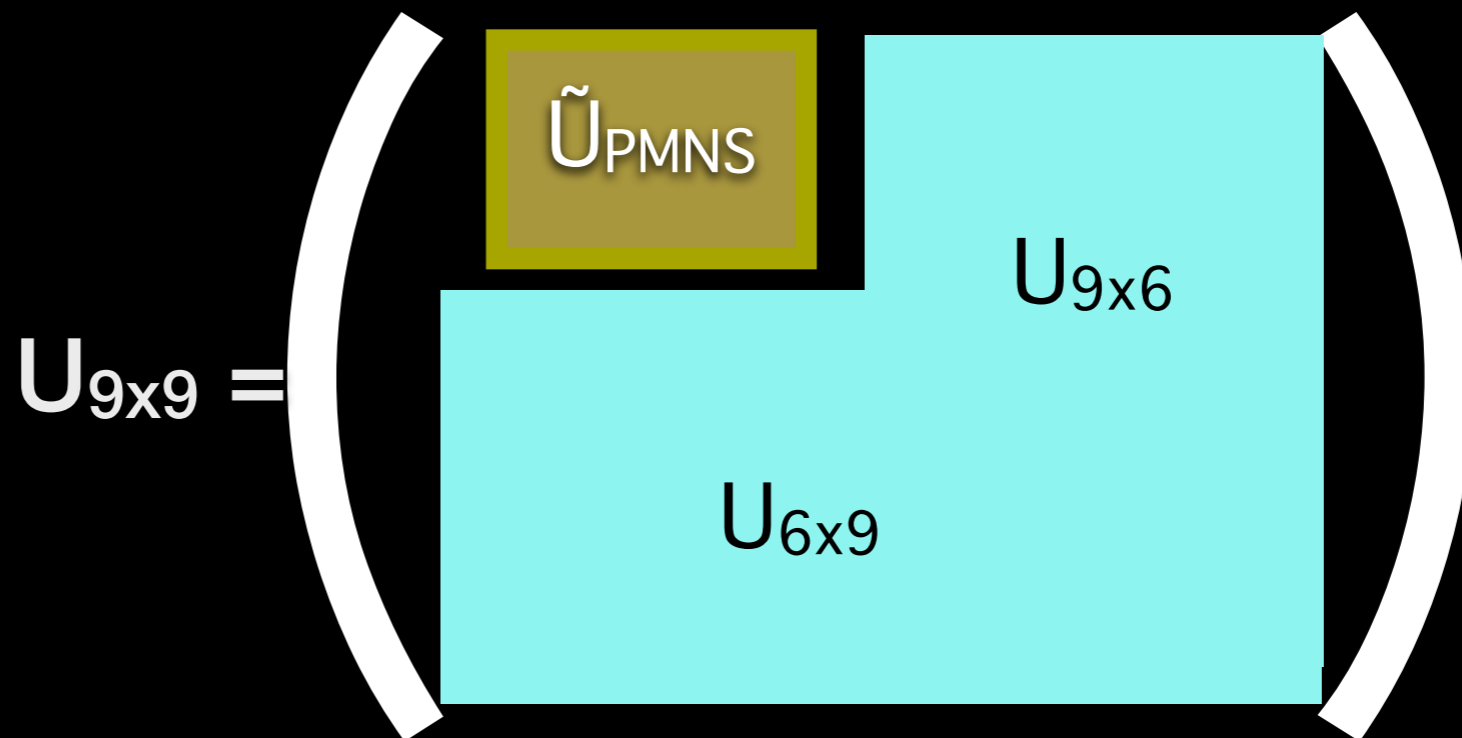
couplings Y_ν can be written using a modified Casas-Ibarra parametrization

$$Y_\nu = \frac{\sqrt{2}}{v} D^\dagger \text{diag}(\sqrt{M}) R \text{diag}(\sqrt{m_\nu}) U_{\text{PMNS}}^\dagger \quad M = M_R \frac{1}{\mu_X} M_R^T$$

basis (ν_L, ν_R, X)

$$M^\nu = \begin{pmatrix} 0 & m_D & 0 \\ m_D^T & 0 & M_R \\ 0 & M_R^T & \mu_X \end{pmatrix}$$

diagonalised by 9x9 complex matrix U_ν



Parameters:

- M_R (real, diagonal) $M_R = (0.1 \text{ MeV}, 10^6 \text{ GeV})$
- μ_X (complex, symmetric) $\mu_X = (0.01 \text{ eV}, 1 \text{ MeV})$
- R_{mat} (rotation, complex)
- 2 Majorana and 1 Dirac phases from U_{PMNS}
- Normal (NH) / Inverted (IH) hierarchy

Effective model: 3+1

Add a sterile state \rightarrow 3 new mixing angles active-sterile

$$U_{4 \times 4} = R_{34} \cdot R_{24} \cdot R_{14} \cdot \boxed{R_{23} \cdot R_{13} \cdot R_{12}} U_{\text{PMNS}}$$

$$U_{4 \times 4} = \left(\begin{array}{c|c} \tilde{U}_{\text{PMNS}} & \begin{array}{c} U_{eS} \\ U_{\mu S} \end{array} \\ \hline \begin{array}{cc} U_{Se} & U_{S\mu} \end{array} & U_{\tau S} \end{array} \right)$$

Parameters:

- $\theta_{14}, \theta_{24}, \theta_{34}$
- 3 Majorana and 3 Dirac phases
- Normal (NH) / Inverted (IH) hierarchy

**Lepton flavor
conserving observables:
lepton magnetic moments and
neutrinoless double beta decay**

Lepton magnetic moments

The Dirac theory predicts a magnetic dipole moment in the presence of an external magnetic field, for any lepton ($l=e,\mu,\tau$)

with gyromagnetic ratio $g_l = 2$

$$\vec{M} = g_l \frac{q}{2m_l} \vec{S}$$

Quantum loop effects lead to a small calculable deviation, which is parametrized by the anomalous magnetic moment ($g-2$)

$$g_l = 2(1 + a_l)$$

$$a_l = a_l^{QED} + a_l^{EW} + a_l^{had} + a_l^{NP}$$

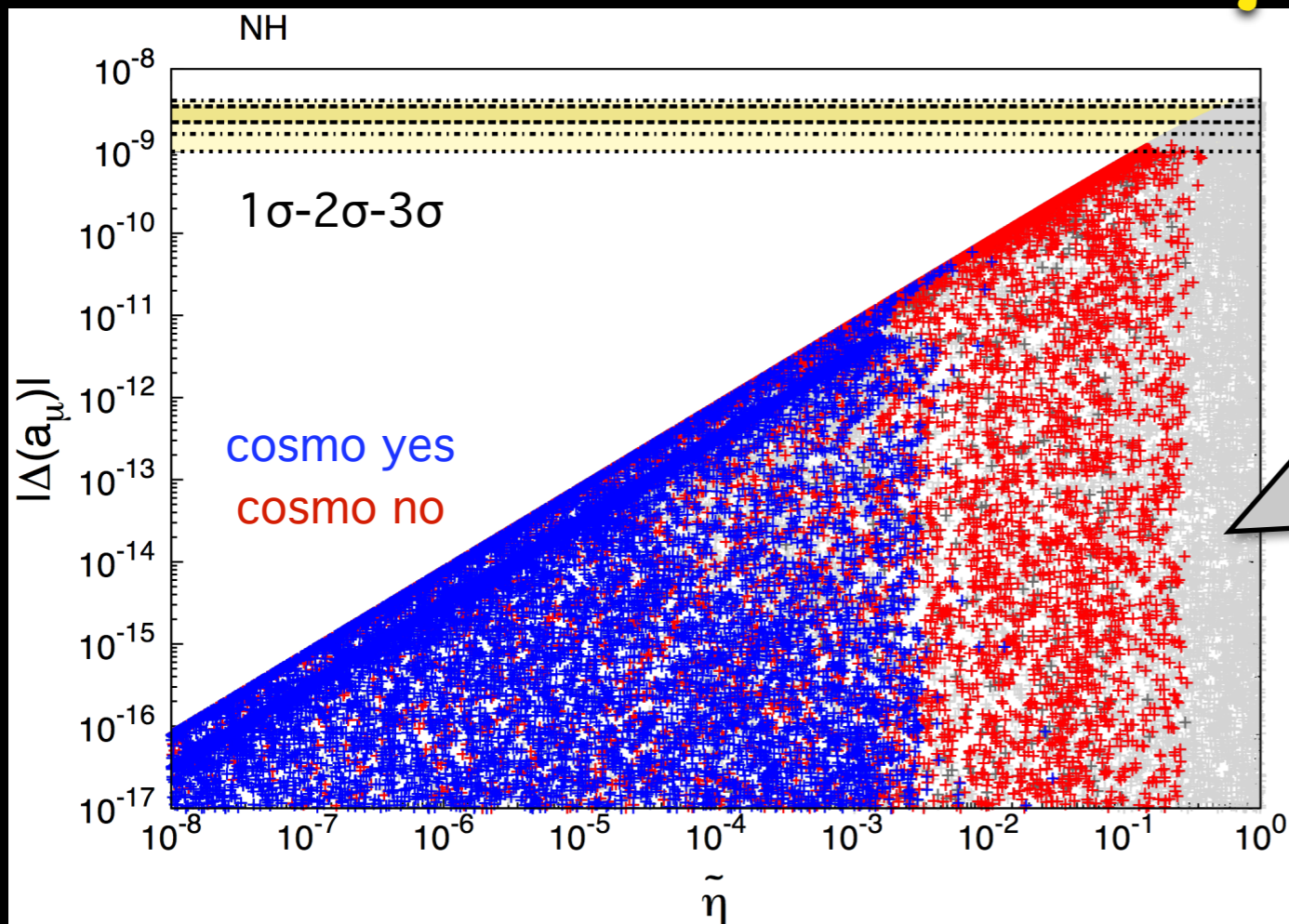
$$\Delta a_e = a_e^{exp} - a_e^{SM} = -10.5(8.1) \times 10^{-13}$$

$$\Delta a_\mu = a_\mu^{exp} - a_\mu^{SM} = 288(63)(49) \times 10^{-11}$$

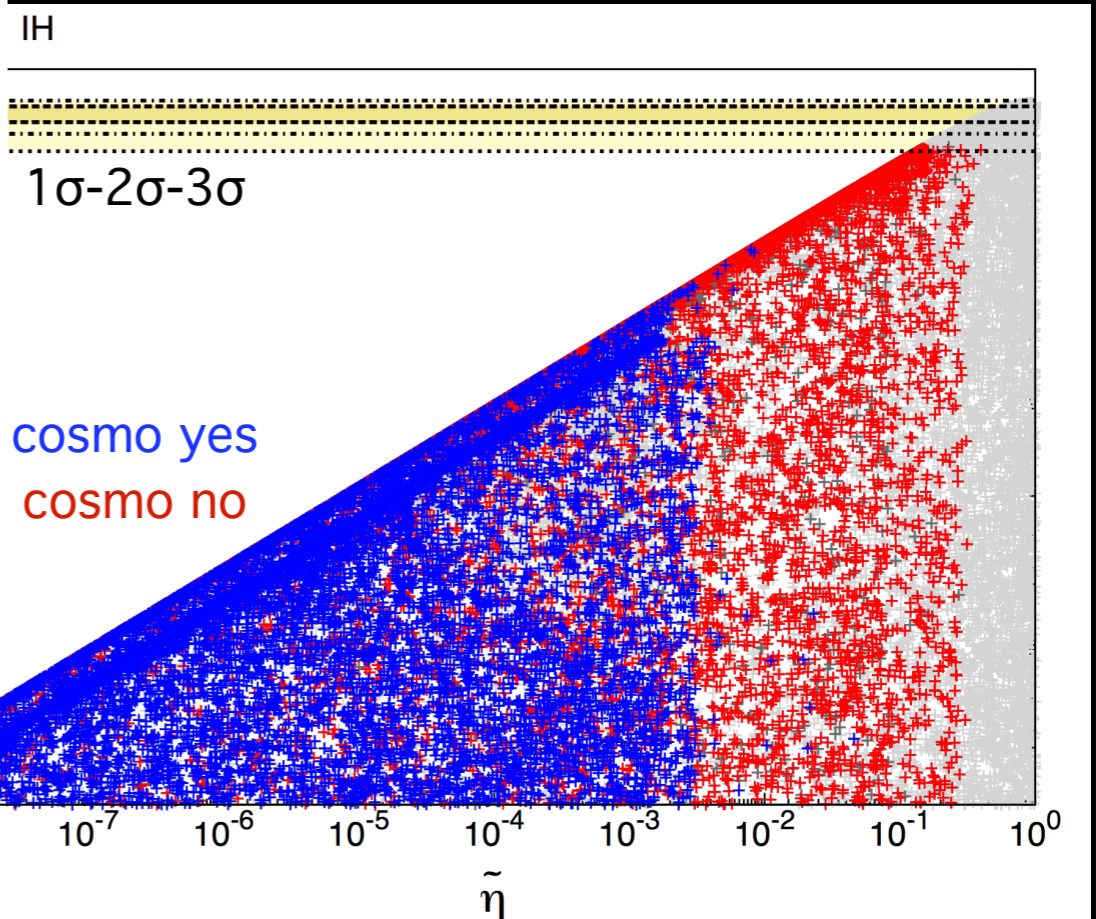
(J. Beringer et al. PDG, 2013)

Effective "3+1": a_μ

$\tilde{\eta} = 1 - \det(\tilde{U}_{\text{PMNS}})$
measures the deviation from
unitarity.

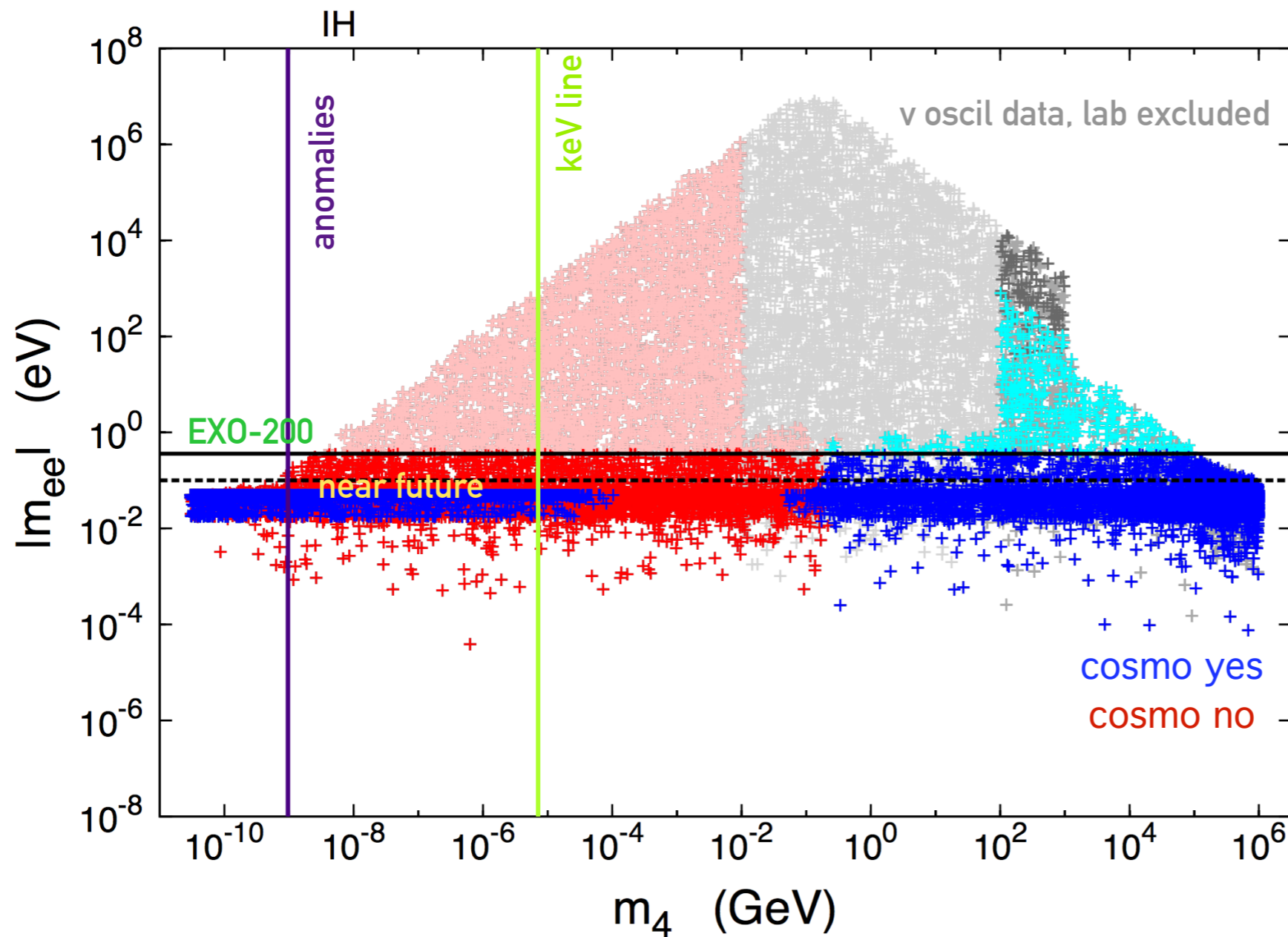


mainly excluded by
 ν oscillation data
and lab bounds



- Constraint from active neutrino oscillations (entries of U_{PMNS}) rules out most solutions with large $\hat{\eta}$

Effective "3+1": $0\nu\beta\beta$ decay

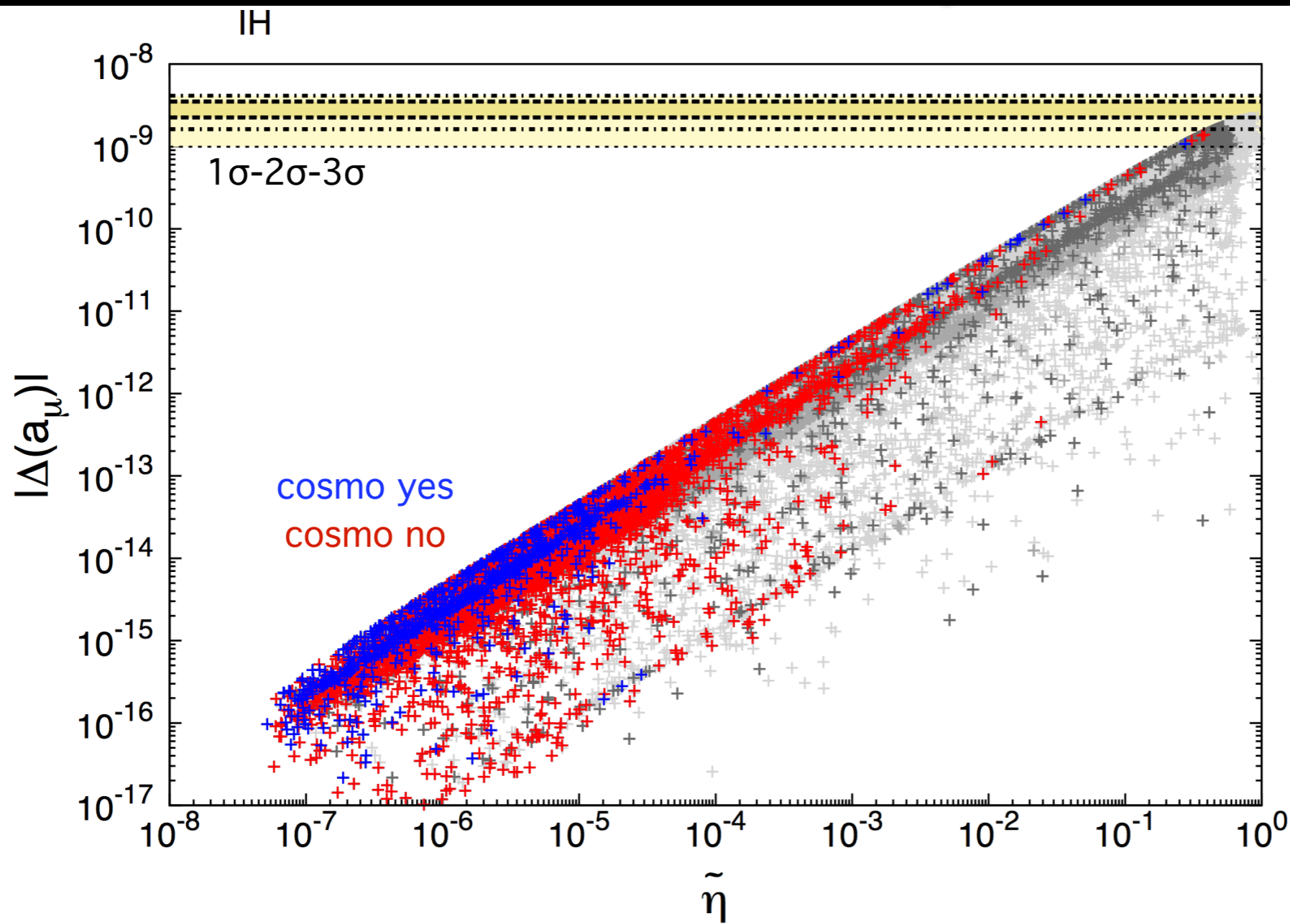


$$m_{ee} \simeq \sum_{i=1}^4 U_{ei}^2 p^2 \frac{m_i}{p^2 - m_i^2}$$

p : momentum exchanged in the process
 $(p^2 \sim - (100 \text{ MeV})^2$
 virtual momentum of the neutrino)

We also studied effective masses $|m_{\mu\mu}|$ and $|m_{e\mu}|$, no significant contribution.

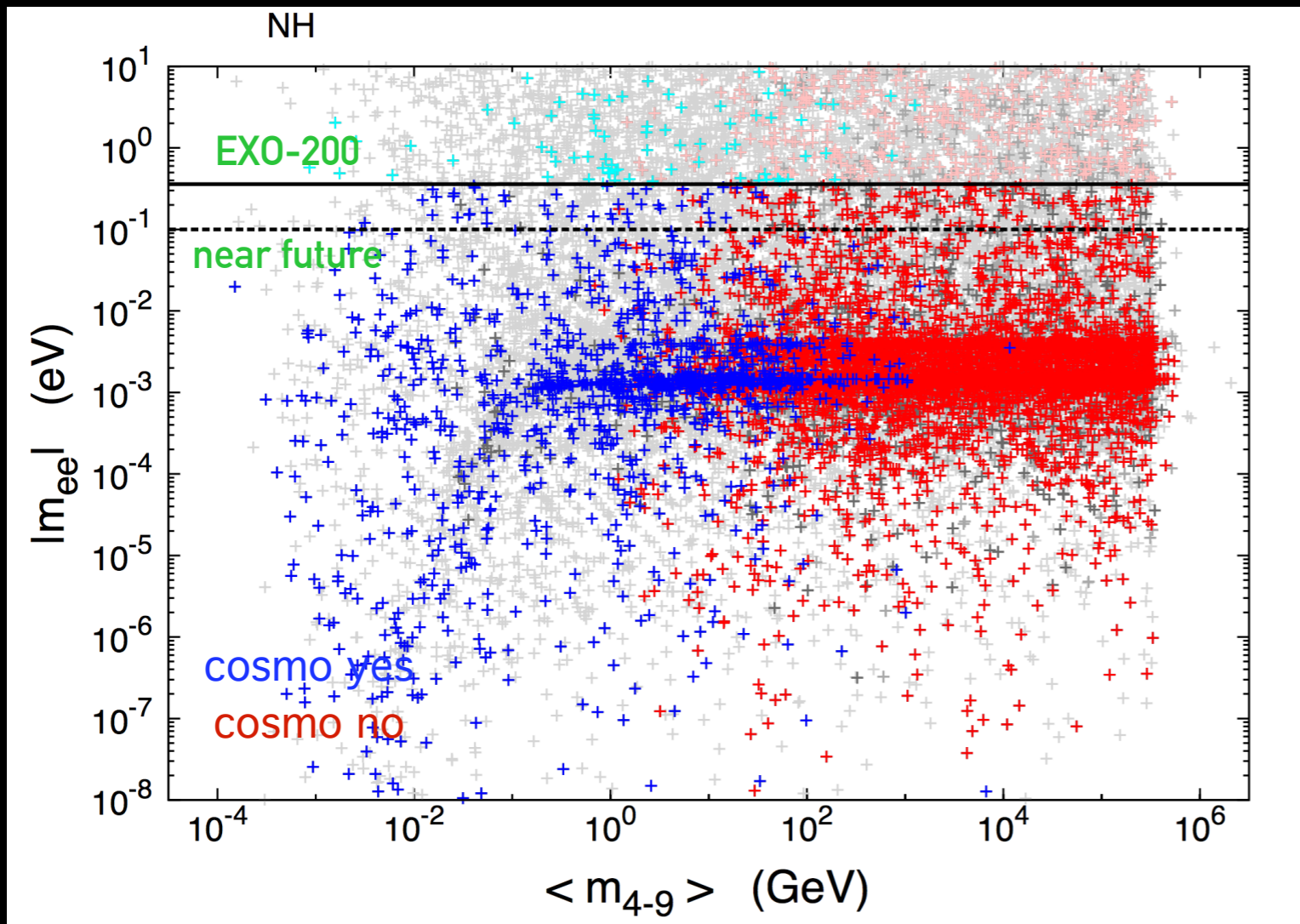
ISS: a_μ



$\tilde{\eta} = 1 - \det(\tilde{U}_{\text{PMNS}})$
measures the deviation from
unitarity.

For large $\tilde{\eta}$ we can get points with
 a_μ within 3σ of the expected value

ISS: $0\nu\beta\beta$ decay



p : momentum exchanged in the process

$m_s \ll |p|$: in this regime the effective mass goes to zero

$$m_{ee} \simeq \sum_{i=1}^9 U_{ei}^2 p^2 \frac{m_i}{p^2 - m_i^2}$$

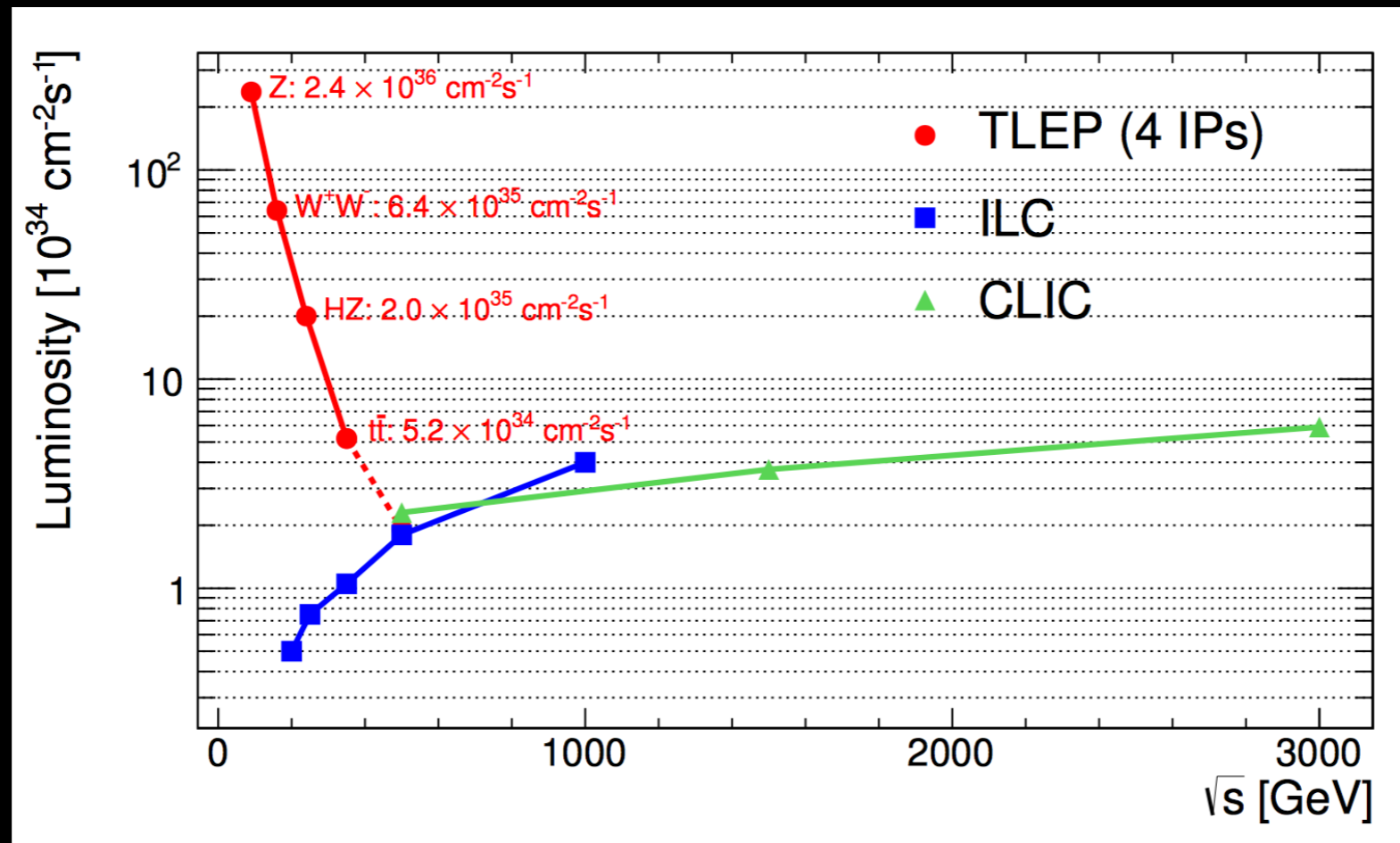
$m_s \approx |p|$: the contribution of the pseudo-Dirac states becomes more important, and can induce sizeable effects to m_{ee}

$m_s \gg |p|$: in this regime the heavy states decouple, and the contributions to m_{ee} only arise from the 3 light neutrino states.

- $0\nu\beta\beta$ decay excludes some solutions
- points within the reach of actual and near-future experiments

Lepton flavor violating observables:
LFV Z decays
at a high luminosity Z-factory

Future circular (and linear) colliders



Instantaneous luminosity expected at FCC-ee, in a configuration with four interaction points operating simultaneously, as a function of the centre-of-mass energy.

FCC-ee is designed to provide e⁺e⁻ collisions in the beam energy range of 40 to 175 GeV.

What would we like see with 10¹² Z?

New physics effects in rare Z decays

In the SM with lepton mixing (U_{PMNS}) the theoretical predictions are:

$$BR(Z \rightarrow e^{\pm} \mu^{\mp}) \sim BR(Z \rightarrow e^{\pm} \tau^{\mp}) \sim 10^{-54}$$

$$BR(Z \rightarrow \mu^{\pm} \tau^{\mp}) \sim 4 \times 10^{-60}$$

The detection of a rare decay as $Z \rightarrow l_i^{\mp} l_j^{\pm}$ ($i \neq j$) would serve as an indisputable evidence of **new physics**

Current limits:

$$BR(Z \rightarrow e^{\mp} \mu^{\pm}) < 1.7 \times 10^{-6}$$

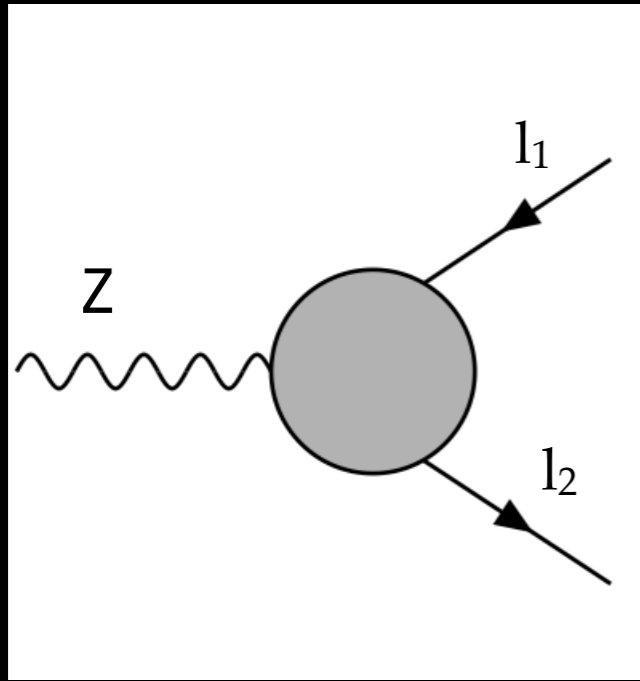
$$BR(Z \rightarrow e^{\mp} \tau^{\pm}) < 9.8 \times 10^{-6}$$

$$BR(Z \rightarrow \mu^{\mp} \tau^{\pm}) < 1.2 \times 10^{-5}$$

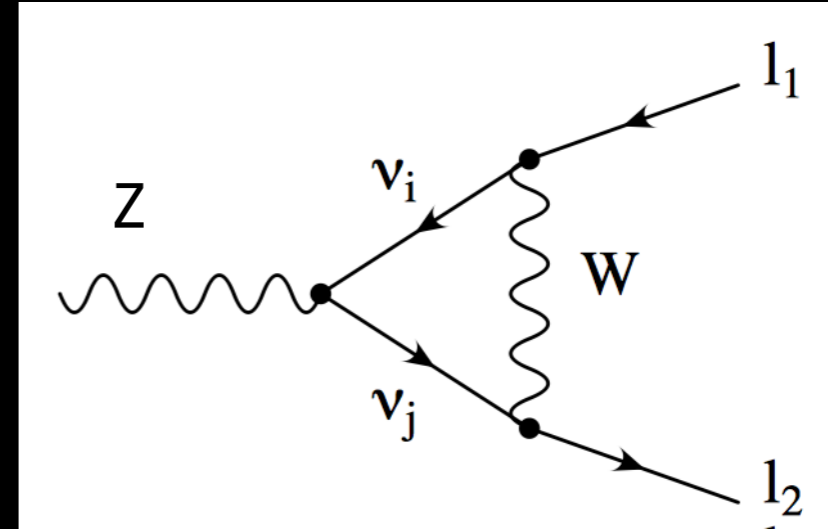


$$Br(Z \rightarrow e\mu) < 7.5 \cdot 10^{-7}$$

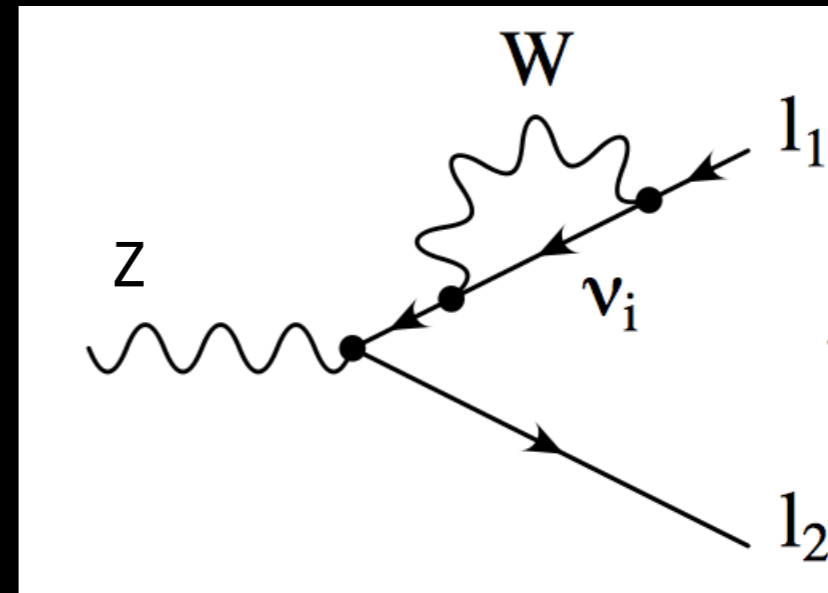
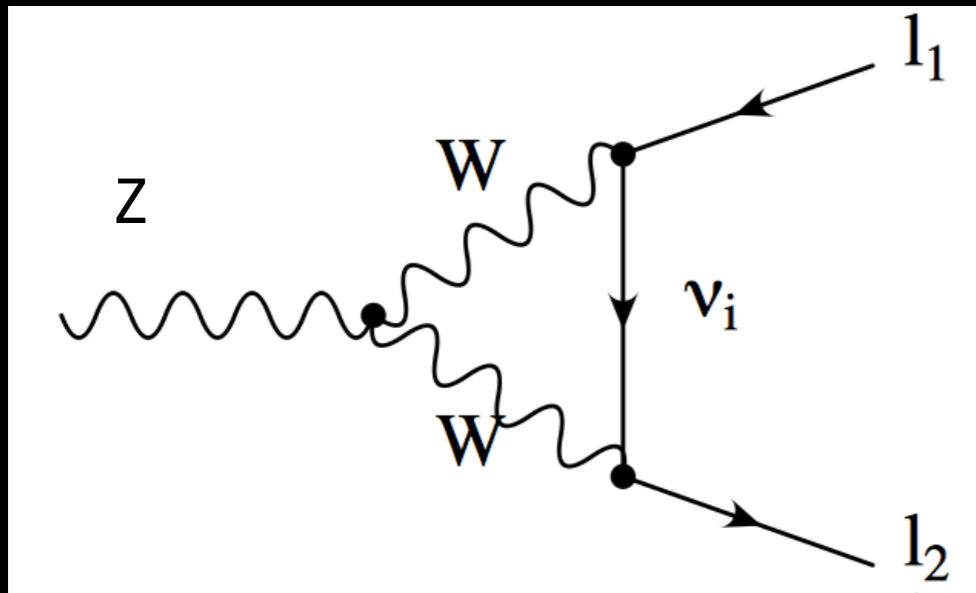
OPAL Collaboration, R. Akers et al., Z. Phys. C67 (1995) 555-564.
L3 Collaboration, O. Adriani et al., Phys. Lett. B316 (1993) 427.
DELPHI Collaboration, P. Abreu et al., Z. Phys. C73 (1997) 243.
ATLAS, CERN-PH-EP-2014-195 (2014)



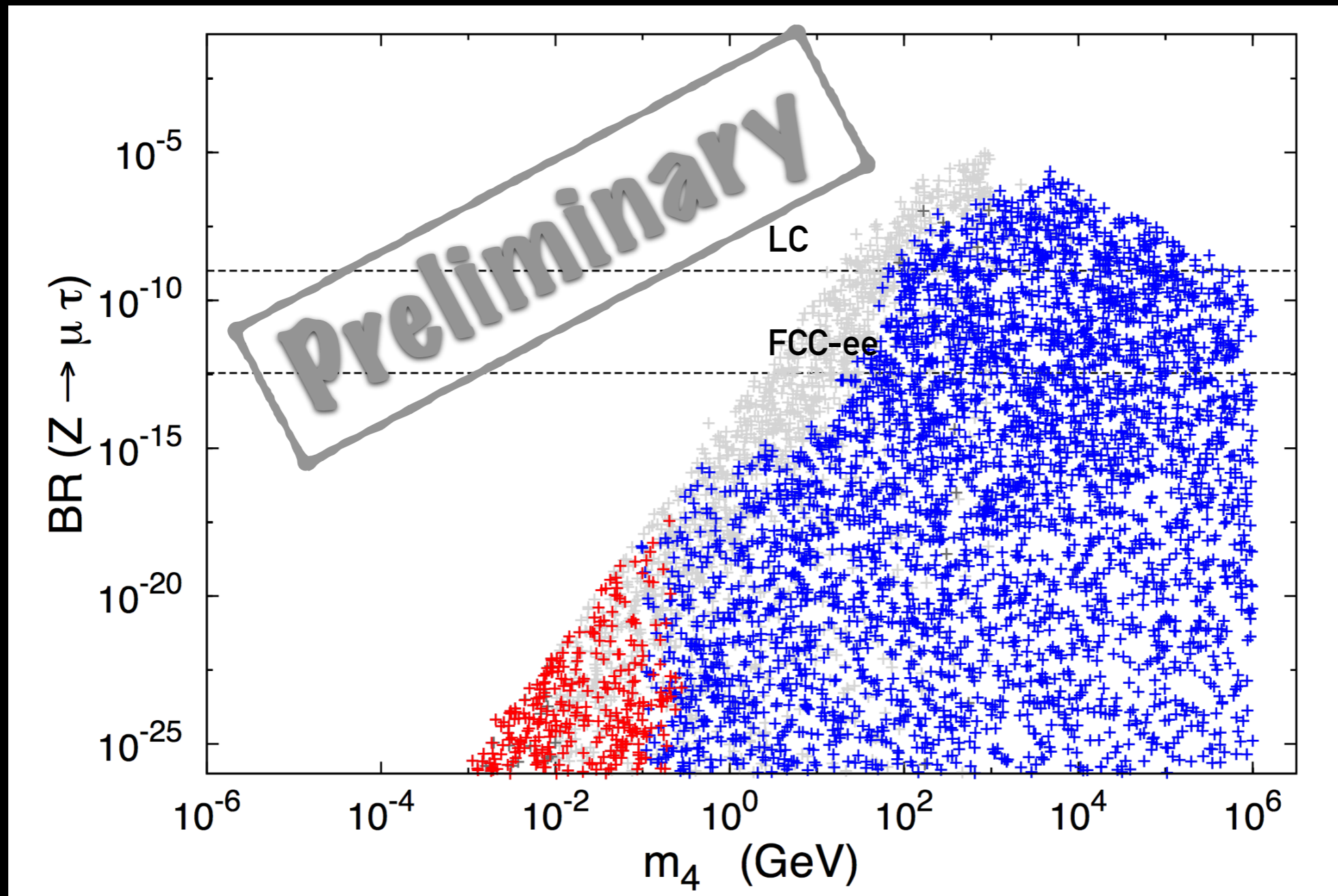
ν_i are physical states, $i = 3+N$



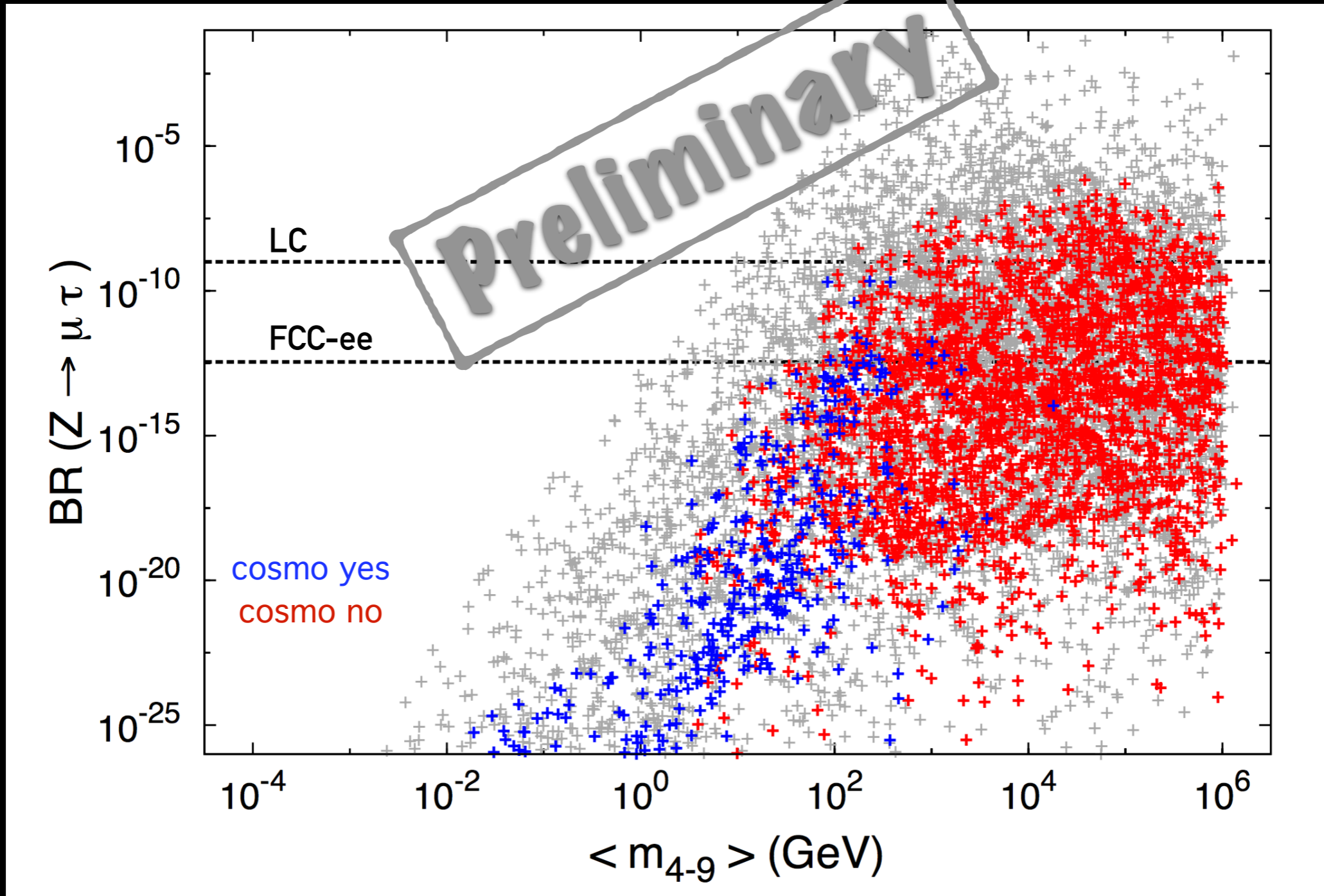
$N = \text{extra Majorana states}$
 $(m \sim 10^{-10} - 10^3 \text{ GeV})$



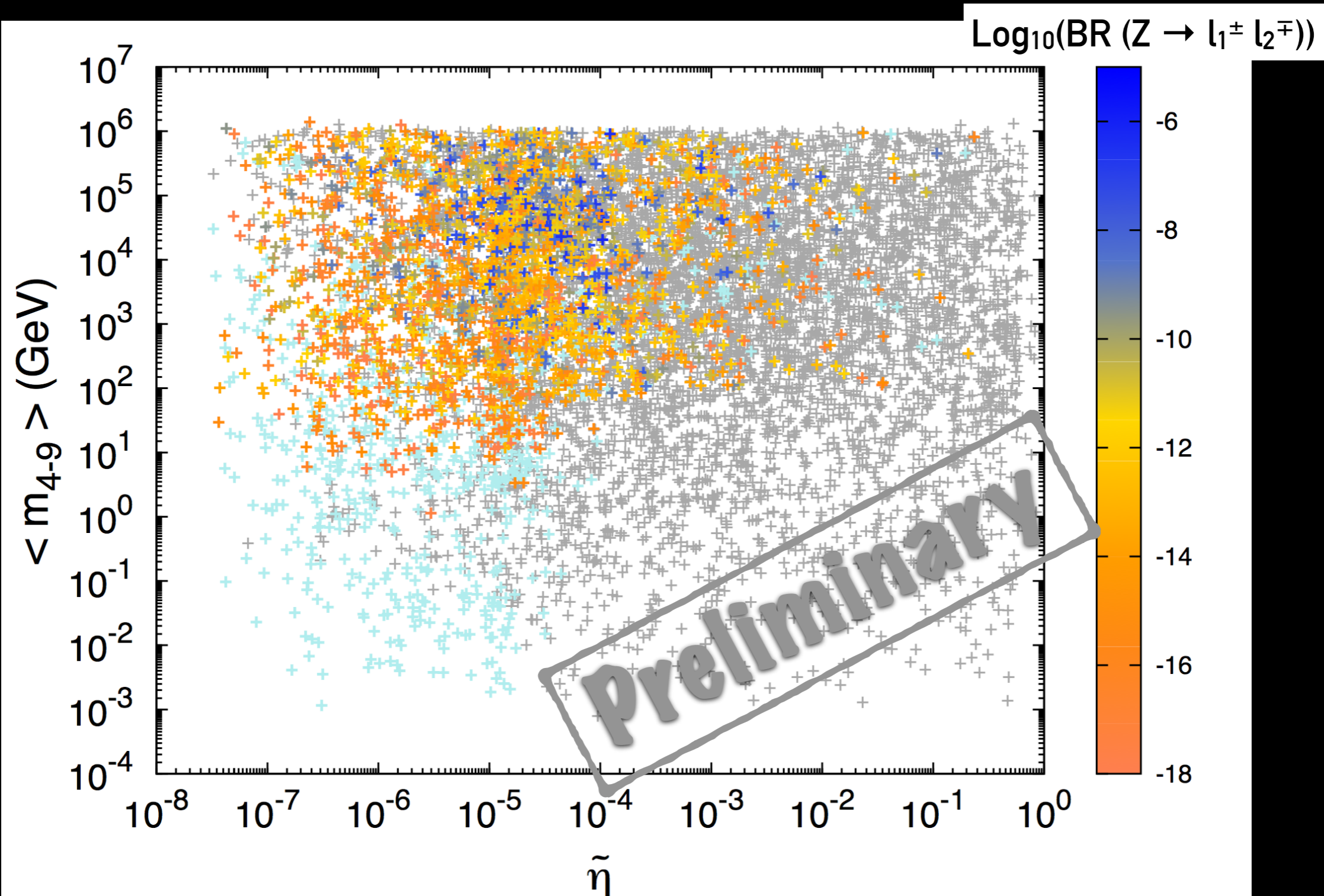
"3+1" model: $Z \rightarrow \mu^\pm \tau^\mp$



ISS: $Z \rightarrow \mu^\pm \tau^\mp$



ISS: summary plot



Conclusions

We have considered **two extensions** of the SM (ISS and 3+1) which add to the particle content of the SM one or more sterile neutrinos.

We have investigated the **contribution of the sterile states** to the anomalous magnetic moment of the leptons in these two classes of models and discussed them taking into account a number of **experimental and theoretical constraints**.

Even if the scale of such NP is low, its **contribution** to the anomalous magnetic moment of the leptons **is generically smaller** than the errors in theoretical calculation. However, **for large η** (deviation from unitarity) we can get solutions within 3σ of the expectation.

The **largest mixing angles (active-sterile)** which would give a sizeable contribution to the muon $g-2$ are indeed **strongly constrained** by other EW observables, among which $0\nu\beta\beta$.

Concerning rare LFV Z decays, we have seen that a **future high-luminosity Z factory** may have the power to probe charged LFV in models with sterile neutrinos. (to appear soon!!)

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Even if the scale of such M_{ν} is **small**, its **contribution** to the anomalous magnetic moment of the leptons is **generically smaller** than the errors in theoretical calculation. However, **for large η** (deviation from unitarity) we can get solutions within 3σ of the expectation.

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BACKUP

Neutrino masses and mixings

parameter	best fit $\pm 1\sigma$	2σ range	3σ range
Δm_{21}^2 [10^{-5}eV^2]	$7.60^{+0.19}_{-0.18}$	7.26–7.99	7.11–8.18
$ \Delta m_{31}^2 $ [10^{-3}eV^2] (NH)	$2.48^{+0.05}_{-0.07}$	2.35–2.59	2.30–2.65
$ \Delta m_{31}^2 $ [10^{-3}eV^2] (IH)	$2.38^{+0.05}_{-0.06}$	2.26–2.48	2.20–2.54
$\sin^2 \theta_{12}/10^{-1}$	3.23 ± 0.16	2.92–3.57	2.78–3.75
$\theta_{12}/^\circ$	34.6 ± 1.0	32.7–36.7	31.8–37.8
$\sin^2 \theta_{23}/10^{-1}$ (NH)	$5.67^{+0.32}_{-1.28}$ ^a	4.13–6.23	3.92 – 6.43
$\theta_{23}/^\circ$	$48.9^{+1.9}_{-7.4}$	40.0–52.1	38.8–53.3
$\sin^2 \theta_{23}/10^{-1}$ (IH)	$5.73^{+0.25}_{-0.43}$	4.32–6.21	4.03–6.40
$\theta_{23}/^\circ$	$49.2^{+1.5}_{-2.5}$	41.1–52.0	39.4–53.1
$\sin^2 \theta_{13}/10^{-2}$ (NH)	2.34 ± 0.20	1.95–2.74	1.77–2.94
$\theta_{13}/^\circ$	8.8 ± 0.4	8.0–9.5	7.7–9.9
$\sin^2 \theta_{13}/10^{-2}$ (IH)	2.40 ± 0.19	2.02–2.78	1.83–2.97
$\theta_{13}/^\circ$	8.9 ± 0.4	8.2–9.6	7.8–9.9
δ/π (NH)	$1.34^{+0.64}_{-0.38}$	0.0–2.0	0.0–2.0
$\delta/^\circ$	241^{+115}_{-68}	0–360	0–360
δ/π (IH)	$1.48^{+0.34}_{-0.32}$	0.0–0.14 & 0.81–2.0	0.0–2.0
$\delta/^\circ$	266^{+61}_{-58}	0–25 & 146–360	0–360

^aThere is a local minimum in the first octant, $\sin^2 \theta_{23} = 0.467$ with $\Delta\chi^2 = 0.28$ with respect to the global minimum

Super-K $\rightarrow \theta_{\text{Atm}}$

MINOS $\rightarrow m_{\text{Atm}}^2$

Solar data $\rightarrow \theta_{\odot}$

KamLAND $\rightarrow m_{\odot}^2$

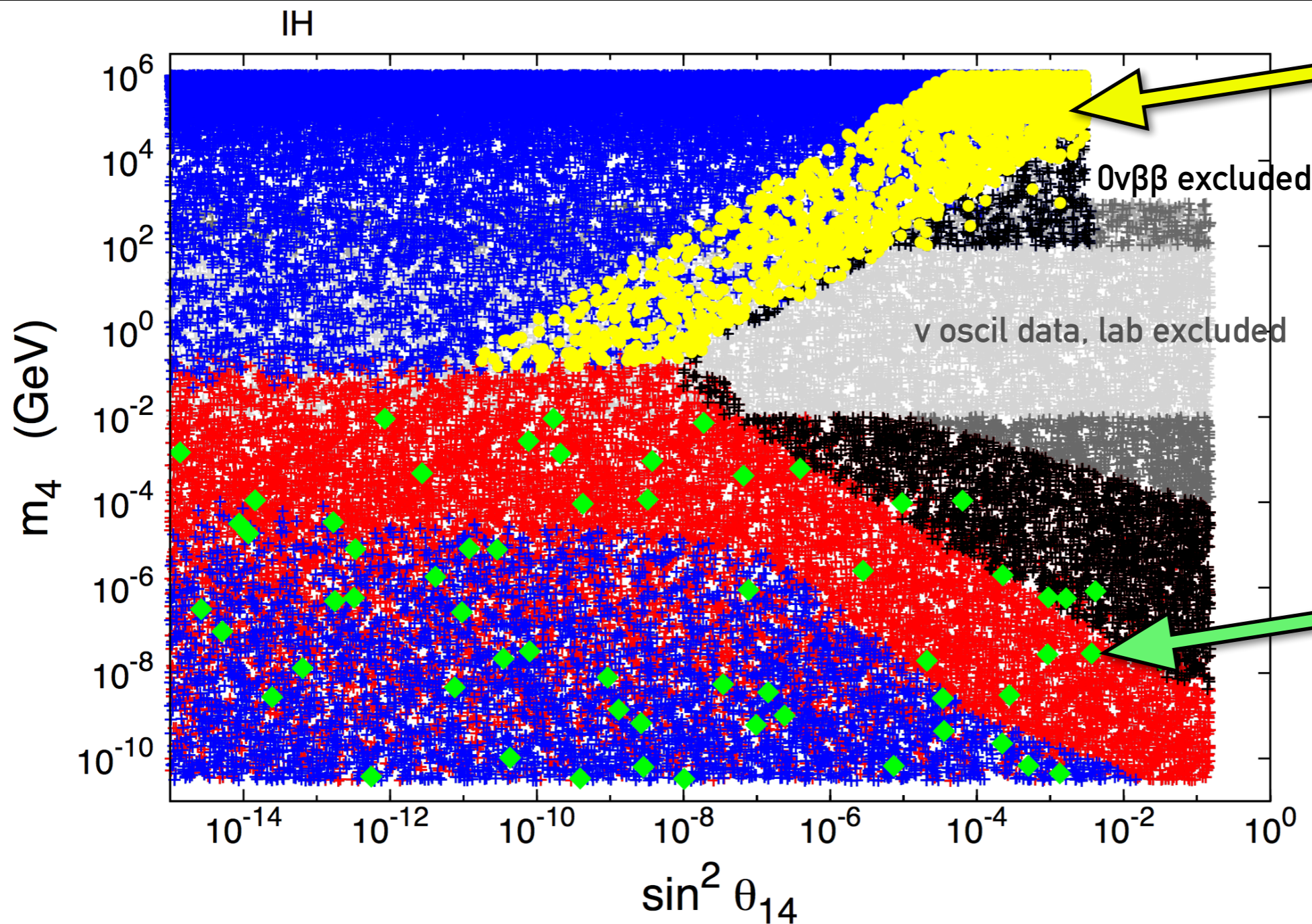
D-Chooz, Daya-Bay, Reno, T2K $\rightarrow \theta_{13}$

(Forero, Tortola, Valle 2014)

(Troitsk and Mainz, Planck 2013)

- **Absolute mass scale** (Tritium β decays: $m_{\nu e} < 2.05 \text{eV}$, Cosmology: $\sum m_{\nu i} < 0.66 \text{eV}$ (CMB), $\sum m_{\nu i} < 0.23 \text{eV}$ (CMB+BAO+WMAP polarization data+high-resolution CMB experiments and flat Universe)) (KamLAND-Zen, EXO-200, Gerda)
- **Majorana** versus Dirac nature ($0\nu\beta\beta$ decay)
- Which **hierarchy**: Normal or inverted? (matter effects in sun and long baseline oscillations, T2K, NOvA...)
- Is there CP violation in the lepton sector?
- Are there extra **sterile** states?

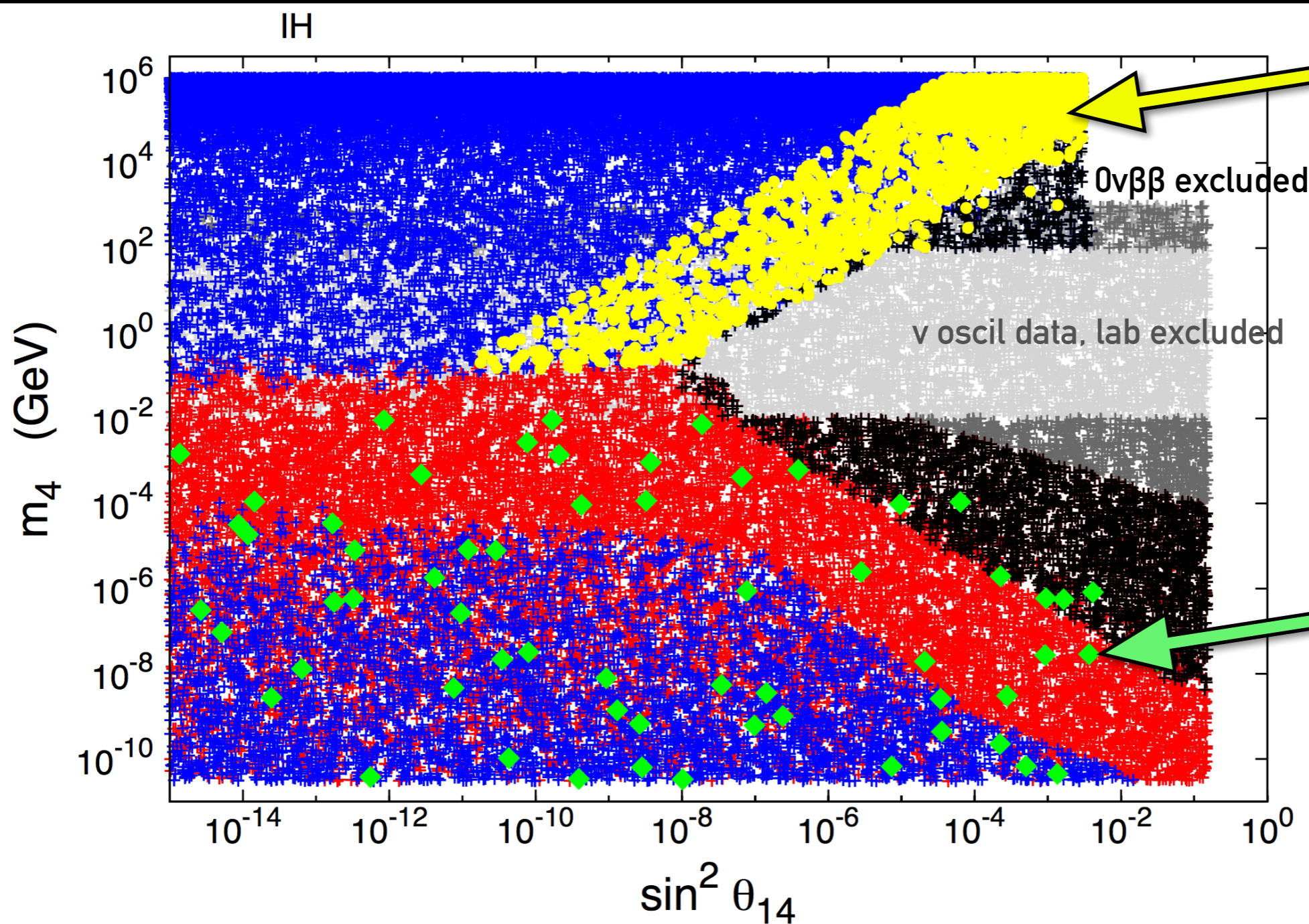
Effective case



solutions within reach of near future $0\nu\beta\beta$ experiments

solutions within 3σ of a_μ

Effective "3+1": summary plot



solutions within reach of near future $0\nu\beta\beta$ experiments

solutions within 3σ of a_μ

In the SM, neutrinos are strictly **massless**:

- absence of RH neutrino fields \Rightarrow no Dirac mass term (no renormalizable mass term)
- nor Higgs triplet \Rightarrow no Majorana mass term (would break the electroweak gauge symmetry, because it is not invariant under the weak isospin symmetry; does not conserve the lepton number L)

Massive neutrinos require BSM physics

Several models of neutrino mass generation:

- Seesaw mechanism: Type-I, Type-II, Type-III, low-scale seesaws (**Inverse seesaw**, Linear seesaw) etc ...
- Radiative models

...

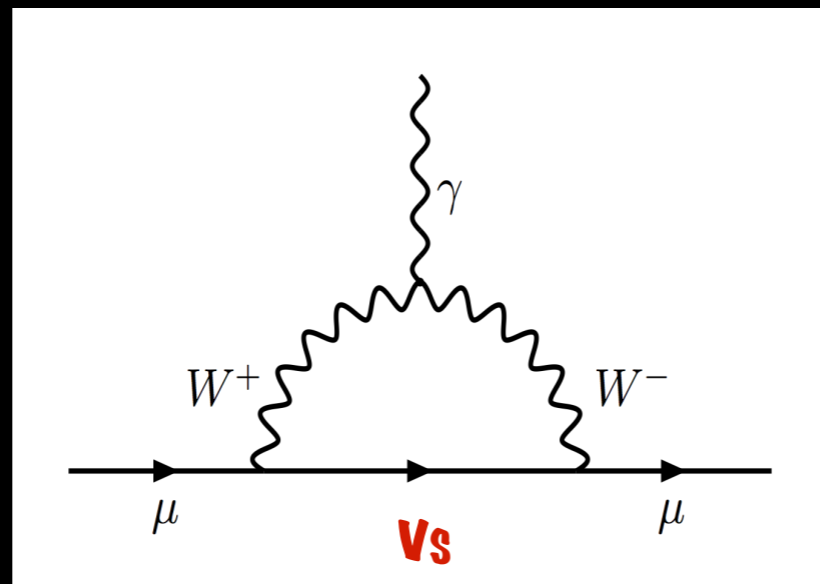
(Minkowski 77, Gell-Mann Ramond Slansky 80, Glashow, Yanagida 79, Mohapatra Senjanovic 80, Lazarides Shafi Wetterich 81, Schechter-Valle, 80 & 82, Mohapatra Senjanovic 80, Lazarides 80, Foot 88,...)

Sterile states contribution to a_l

One-loop diagram involving weak gauge bosons contributes to the e.m. form factors with the sterile states

$$\mathcal{L}_{CC} = \frac{g}{2} (U_{li}^* \bar{\nu}_i \gamma^\alpha W_\alpha^+ P_L l + U_{li} \bar{l} \gamma^\alpha W_\alpha^- P_L \nu_i)$$

$$a_\mu^\nu = \frac{G_F}{\sqrt{2}} \frac{m_\mu^2}{8\pi^2} \sum_{i=1}^9 U_{\mu i}^* U_{\mu i} f((m_{\nu_i}/M_W)^2)$$

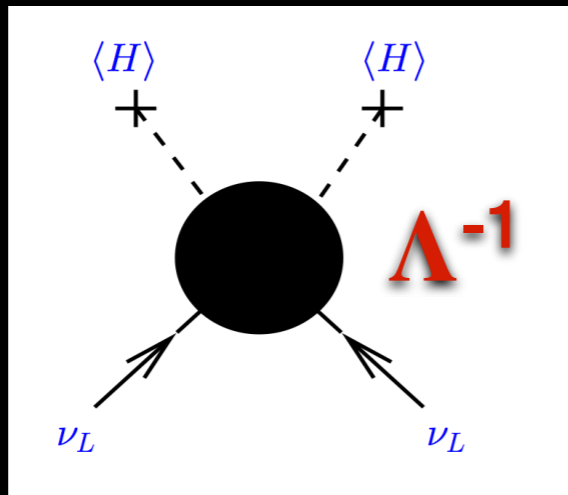


We address the impact of the **modified charged current vertex** on the **magnetic moments of leptons** and other observables (e.g. **$0\nu\beta\beta$ decay**), assuming that all NP effects are encoded in the modified leptonic weak current vertices and do not affect the hadronic sector.

Majorana neutrinos

If Lepton Number is Violated:

The lowest order operator, which generates Majorana neutrino masses is the Weinberg's d=5 operator (WO)



$$\mathcal{L} \ni \frac{LLHH}{\Lambda}$$

S. Weinberg, Phys. Rev. Lett. 43, 1566 (1979)

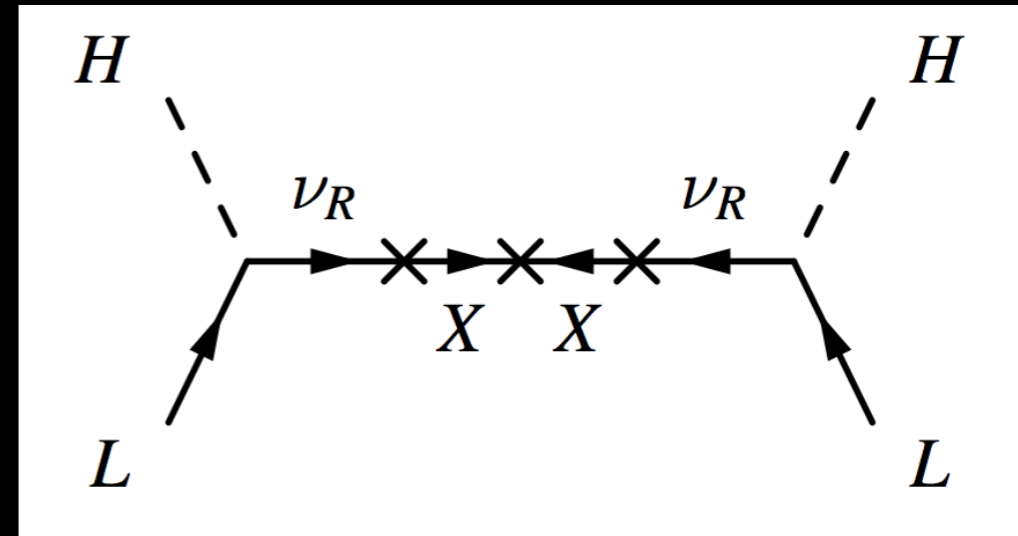
After EWSB takes place, through the nonzero vev v , Majorana neutrino masses are induced

$$m_\nu \sim Y^2 \frac{v^2}{\Lambda}$$

small neutrino masses by making Λ very large and/or with Y small
The exchange of heavy messenger states provides a simple way to generate the WO.

Inverse seesaw basis (ν_L, ν_R, X)

$$M^\nu = \begin{pmatrix} 0 & m_D & 0 \\ m_D^T & 0 & M_R \\ 0 & M_R^T & \mu_X \end{pmatrix}$$



$$m_\nu = m_D (M_R^T)^{-1} \mu_X (M_R)^{-1} m_D^T$$

$$m_\nu \approx \frac{m_D^2 \mu_X}{m_D^2 + M_R^2}$$

$$m_{1,2} \approx \mp \sqrt{m_D^2 + M_R^2} + \frac{M_R^2 \mu_X}{2(m_D^2 + M_R^2)}$$

The photon-lepton vertex

The electromagnetic current describing the coupling of spin 1/2 fermion to the em. field

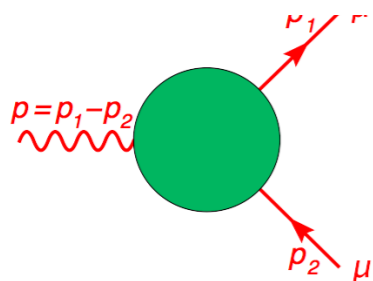
$$\begin{aligned} \bar{u}(p_1)\Gamma_\mu(p_1, p_2)u(p_2) &= \\ &= \bar{u}(p_1) \left[F_1(k^2)\gamma_\mu + \frac{i}{2m_l} F_2(k^2)\sigma_{\mu\nu}k^\nu - \right. \\ &\quad \left. F_3(k^2)\gamma_5\sigma_{\mu\nu}k^\nu + F_4(k^2)(k^2\gamma_\mu - 2m_l k_\mu)\gamma_5 \right] \end{aligned}$$

$F_1(k^2) \rightarrow$ Dirac form factor , $F_1(0) = 1$ (gauge invariance)

$F_2(k^2) \rightarrow$ Pauli form factor, In the on-shell limit of the photon field $a_l \equiv F_2(0)$ corresponds to an anomalous magnetic moment

$F_3(k^2) \rightarrow$ /P, /T $d_l \equiv e_l F_3(0)$ corresponds to an electric dipole moment

$F_4(k^2) \rightarrow$ /P



$$= \bar{u}(p_1) \left[\gamma^\mu F_E(p^2) + i \frac{\sigma^{\mu\nu} p_\nu}{2m_\mu} F_M(p^2) \right] u(p_2)$$

Sterile states contribution to a_l

One-loop diagram involving weak gauge bosons contributes to the e.m. form factors with the sterile states

$$\mathcal{L}_{CC} = \frac{g}{2} (U_{li}^* \bar{\nu}_i \gamma^\alpha W_\alpha^+ P_L l + U_{li} \bar{l} \gamma^\alpha W_\alpha^- P_L \nu_i)$$

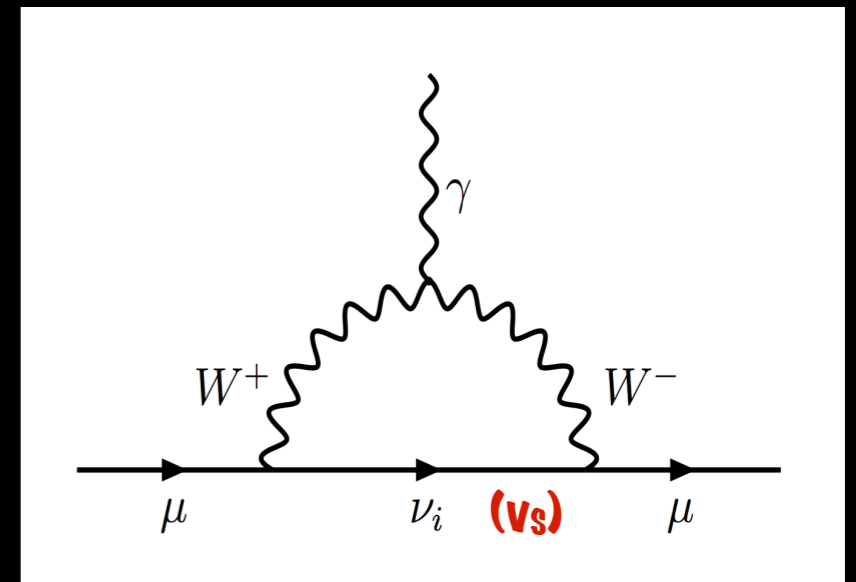
$$a_\mu^\nu = \frac{G_F}{\sqrt{2}} \frac{m_\mu^2}{8\pi^2} \sum_{i=1}^9 U_{\mu i}^{\nu*} U_{\mu i}^\nu f\left(\left(m_{\nu_i}/M_W\right)^2\right)$$

Unitary gauge

$$f(x_{\nu_i}) = \frac{10 - 43x + 78x^2 - 49x^3 + 4x^4 + 18x^3 \ln(x)}{3(1-x)^4}$$

$$x_{\nu_i} = \left(\frac{m_{\nu_i}}{M_W}\right)^2$$

$$J_n(x) = \int_0^1 d\alpha \frac{\alpha^n}{1 - \alpha(1-x)} \quad I_n(x) = \int_0^1 d\alpha \frac{\alpha^n}{x + \alpha(1-x)}$$



In a general gauge, extra diagrams appear where one or both of the W lines are replaced by the unphysical Higgs boson H which is absorbed by W in the unitary gauge.

The diagrams with two or more scalar couplings to the muon line are suppressed by an extra factor of m_μ^2/M_W^2 and can be discarded. This is true already at the one-loop level, where one neglects the diagrams with the Higgs boson loop and with two Goldstone boson couplings to the muon. Making this approximation and taking advantage of the mirror symmetry mentioned above reduces the number of relevant diagrams to about 240 in the linear 't Hooft-Feynman gauge. This number can be almost halved by choosing a non-linear gauge in which the $\gamma W^\pm G^\mp$ vertex vanishes.

(C. Gnendiger, D. Stöckinger, H. Stöckinger-Kim, 2013)

Lepton magnetic moments

ELECTRON

$$a_e^{SM} = 1159652181.78(77) \times 10^{-12}$$

(Aoyama T, Hayakawa M, Kinoshita T, Nio M, 2012)

has played the central role in testing the validity of quantum electrodynamics (QED)

$$a_e^{exp} = 1159652180.73(0.28) \times 10^{-12}$$

(D. Hanneke, S. Fogwell Hoogerheide, G. Gabrielse, 2011)

MUON

$$a_\mu^{SM} = 116591803(1)(42)(26) \times 10^{-11}$$

(Aoyama T, Hayakawa M, Kinoshita T, Nio M, 2012)

EW LO had HO had

$$a_\mu^{exp} = 11659209.1(5.4)(3.3) \times 10^{-10}$$

(P.J. Mohr, B.N. Taylor, and D.B. Newell, CODATA, 2012)

statist system

TAU

$$a_\tau^{SM} = 11721(5) \times 10^{-8}$$

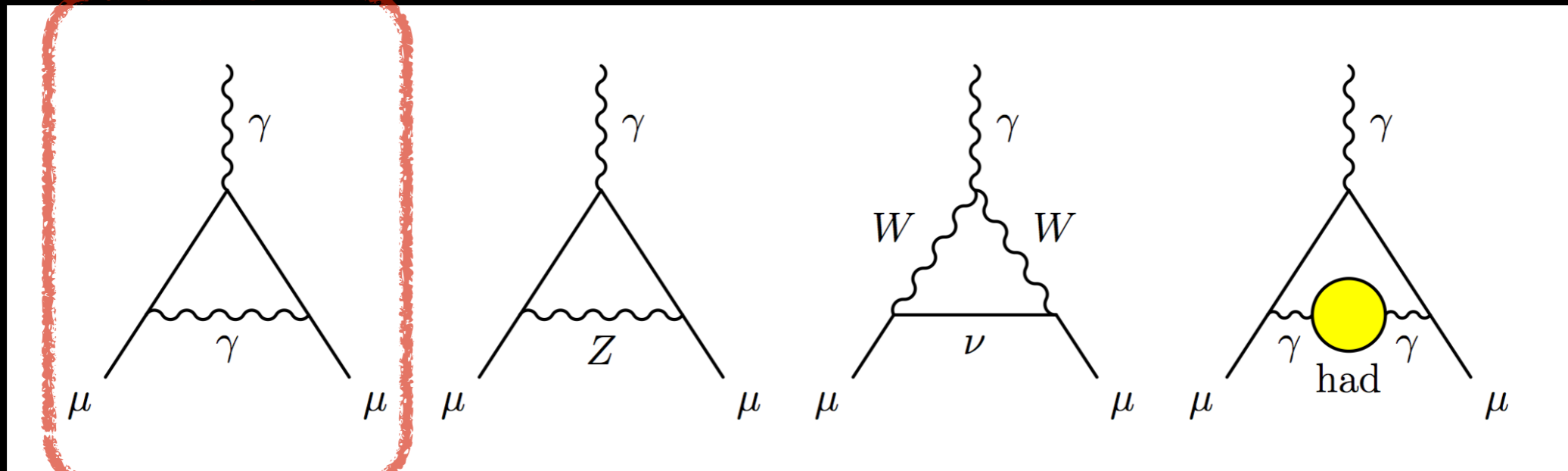
(Passera, 2007)

$$-0.052 < a_\tau^{exp} < 0.013$$

(Abdallah et al., DELPHI 2012)

Theoretical contributions to a_μ

QED



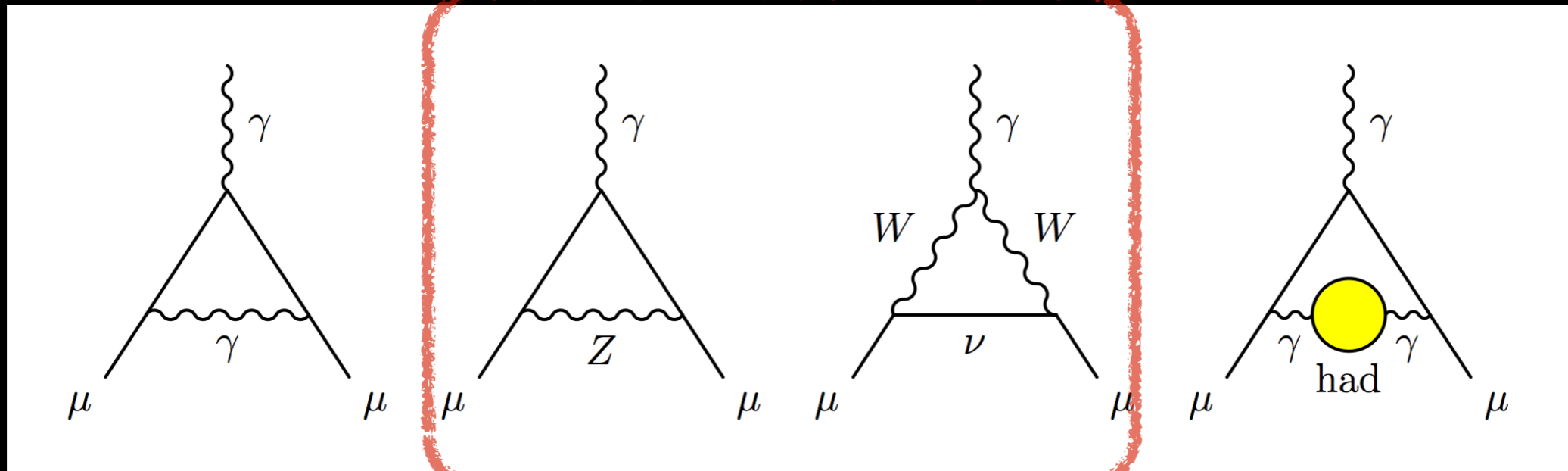
QED Schwinger 1948

$$a_\mu^{\text{QED}} = \frac{\alpha}{2\pi} + 0.765857410(27) \left(\frac{\alpha}{\pi}\right)^2 + 24.05050964(43) \left(\frac{\alpha}{\pi}\right)^3 + 130.8055(80) \left(\frac{\alpha}{\pi}\right)^4 + 663(20) \left(\frac{\alpha}{\pi}\right)^5 + \dots$$

Complete 5 loop result (Aoyama T, Hayakawa M, Kinoshita T, Nio M, 2012)
Dominated by $\alpha/2\pi$

Theoretical contributions to a_μ

EW



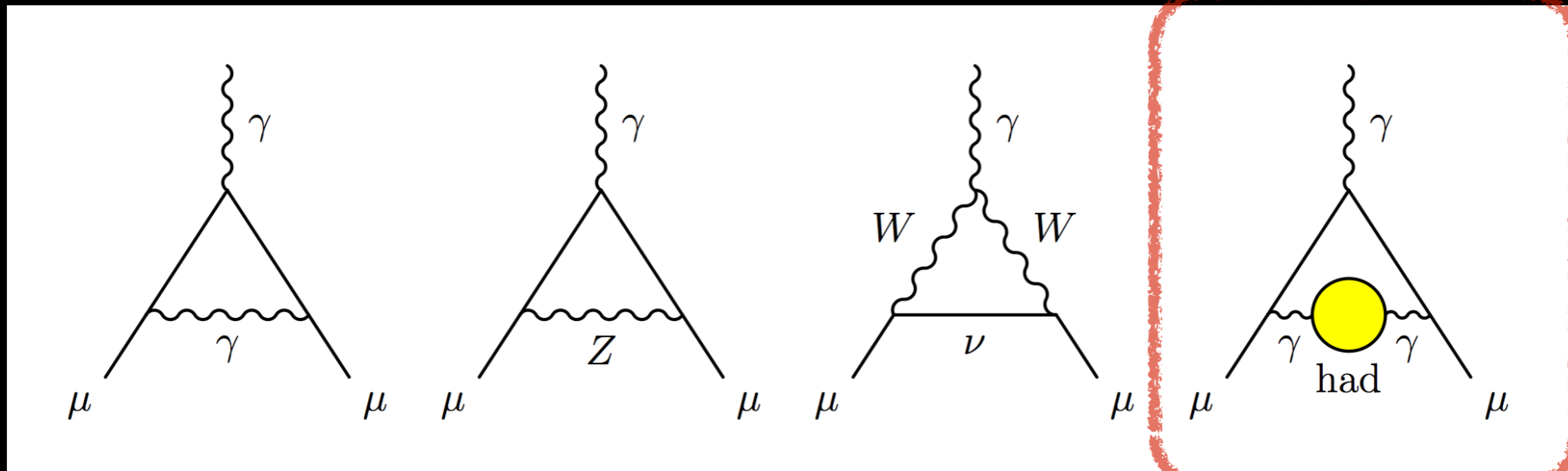
The EW part is known to 2-loops (two-loop corrections are relatively large and negative)

For a Higgs boson mass of ≈ 126 GeV: $a^{\text{EW}}[2\text{-loop}] = -41.2(1.0) \times 10^{-11}$
(C. Gnendiger, D. Stöckinger, H. Stöckinger-Kim, 2013)

1loop: Jackiv, Weinberg; Bars, Yoshimura; Altarelli, Cabibbo, Maiani; Bardeen, Gastmans, Lautrup; Fujikawa Lee, Sanda. Kukhto et al. '92, Czarnecki, Krause, Marciano '95; Knecht, Peris, Perrottet, de Rafael '02; Czarnecki, Marciano & Vainshtein '02; Degrassi & Giudice '98; Heinemeyer, Stockinger & Weiglein '04 Gribouk & Czarnecki '05

Theoretical contributions to a_μ

HADRONIC



Main theoretical uncertainties (can be reduced with lattice QCD improvements)

Dispersion relation approach to evaluate the lowest-order $O(\alpha^2)$ hadronic vacuum polarization contribution from corresponding cross section measurements ($\sigma(e^+e^- \rightarrow \text{hadrons})$)

(Bouchiat & Michel 1961, Gourdin & de Rafael 1969)

Experimental constraints

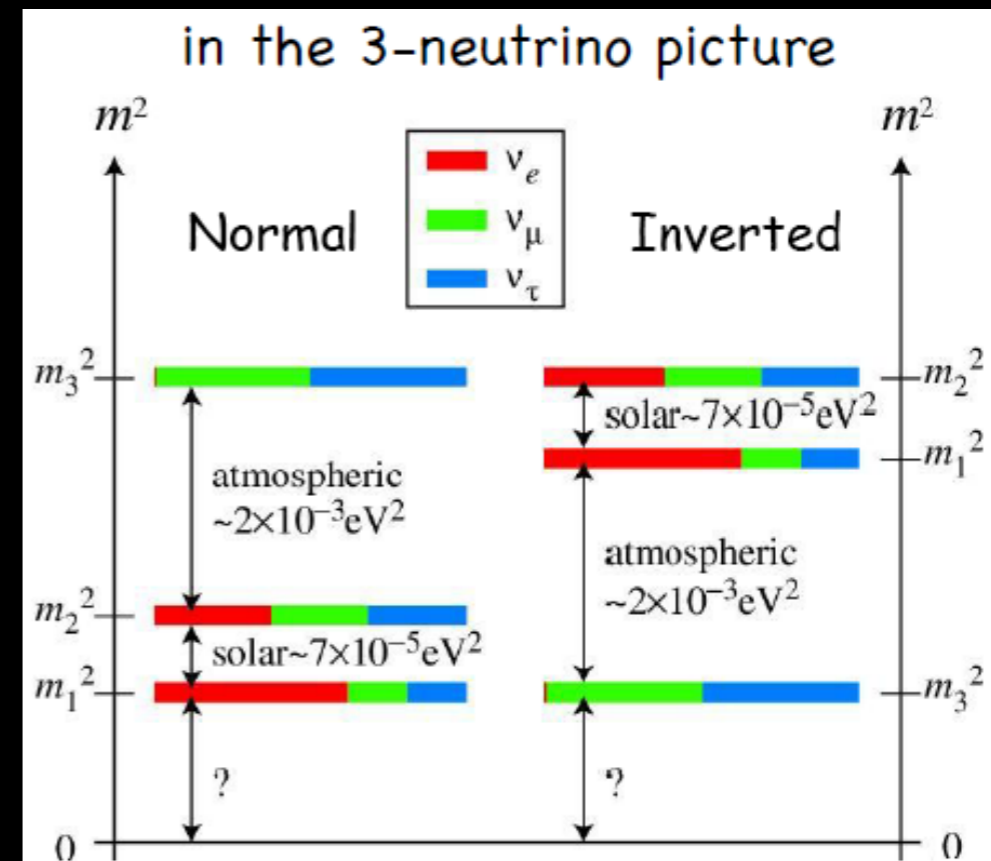
The deviations from unitarity and the possibility of having steriles as final decay products, might induce departures from the SM expectations.

1. Neutrino oscillation parameters (seesaw approximation and PMNS)
2. Unitarity constraints
3. Electroweak precision data
4. LHC data (invisible decays)
5. Leptonic and semileptonic meson decays (K,B and D)
6. Laboratory bounds: direct searches for sterile neutrinos
7. Lepton flavor violation ($\mu \rightarrow e \gamma$)
8. Neutrinoless double beta decay
9. Cosmological bounds on sterile neutrinos

Experimental constraints

1. Neutrino oscillation parameters (seesaw approximation and PMNS)

parameter	best fit $\pm 1\sigma$	2σ	3σ
Δm_{21}^2 [10^{-5}eV^2]	7.62 ± 0.19	7.27–8.01	7.12–8.20
Δm_{31}^2 [10^{-3}eV^2]	$2.53^{+0.08}_{-0.10}$ $-(2.40^{+0.10}_{-0.07})$	2.34 – 2.69 $-(2.25 - 2.59)$	2.26 – 2.77 $-(2.15 - 2.68)$
$\sin^2 \theta_{12}$	$0.320^{+0.015}_{-0.017}$	0.29–0.35	0.27–0.37
$\sin^2 \theta_{23}$	$0.49^{+0.08}_{-0.05}$ $0.53^{+0.05}_{-0.07}$	0.41–0.62 0.42–0.62	0.39–0.64
$\sin^2 \theta_{13}$	$0.026^{+0.003}_{-0.004}$ $0.027^{+0.003}_{-0.004}$	0.019–0.033 0.020–0.034	0.015–0.036 0.016–0.037
δ	$(0.83^{+0.54}_{-0.64})\pi$ $0.07\pi^a$	$0 - 2\pi$	$0 - 2\pi$



(Forero, Tortola, Valle 2012)

We fix active neutrino masses and mixings in order to reproduce neutrino oscillation data, with normal and inverted hierarchy

Experimental constraints

1. Neutrino oscillation parameters (seesaw approximation and PMNS)
2. Unitarity constraints (Antusch et al., 2009)

Non-standard neutrino interactions with matter can be generated by NP BSM.

$$U_{3\times 3} = (1 - \eta)U_{PMNS}$$

Strongly constrained if $m_N > \Lambda_{EW}$

When **singlet fermions** (RH neutrinos) with Y couplings and a (Majorana) mass matrix are introduced, this can in general lead to two effective operators at tree-level: the $W0$ (LN violating) and the dim-6 operator which contributes to the kinetic energy of the neutrinos and induces non-unitarity of the leptonic mixing matrix.

After diagonalising and normalising the neutrino kinetic terms, a **non-unitary lepton mixing** matrix is produced from this operator.

Experimental Bounds

1. Neutrino oscillation parameters (seesaw approximation and PMNS)

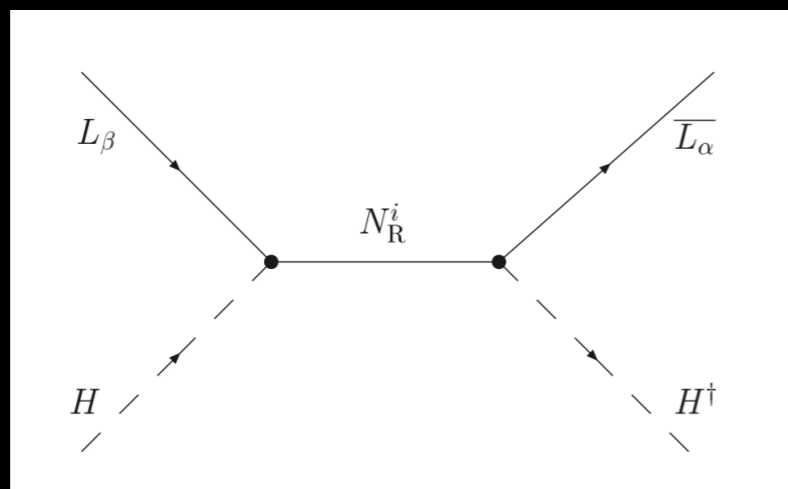
2. Unitarity constraints (Antusch et al., 2009)

When singlet fermions (RH neutrinos) with Y couplings and a (Majorana) mass matrix are introduced, this can in general lead to two effective operators at tree-level: the WO (LN violating) and the dim-6 operator which contributes to the kinetic energy of the neutrinos and induces non-unitarity of the leptonic mixing matrix.

$$\mathcal{L}_{kin}^{d=6} = -c_{\alpha\beta}^{d=6,kin} (\bar{L}_\alpha \cdot H^\dagger) i\not{\partial} (H \cdot L_\beta)$$

After diagonalising and normalising the neutrino kinetic terms, a **non-unitary lepton mixing** matrix is produced from this operator.

$$\mathcal{L}_{int}^Y = -Y_{\alpha i}^* (\bar{L}_\alpha \cdot H^\dagger) N_R^i + \text{H.c.}$$



- No new interactions of four charged fermions
- No cancellations between diagrams with different messenger particles
- Tree-level generation of the NSIs through dimension 6 and 8 operators
- Electroweak symmetry breaking is realised via the Higgs mechanism

Experimental constraints

1. Neutrino oscillation parameters (seesaw approximation and PMNS)
2. Unitarity constraints

3. Electroweak precision data (Del Aguila et al., 2008, Atre et al., 2009)

The presence of singlet neutrinos can affect the **electroweak precision observables** via tree-level as well as loop contributions, as a consequence of non-unitarity of the active neutrino mixing matrix. The couplings of the light neutrinos to the Z and W bosons are suppressed with respect to their SM values, reducing the tensions:

- LEP measurement of the invisible Z-decay width is two sigma below the value expected in the SM;
 $\Gamma_{\text{SM}}(Z \rightarrow \nu\nu) = (501.69 \pm 0.06) \text{ MeV}$, $\Gamma_{\text{Exp}}(Z \rightarrow \nu\nu) = (499.0 \pm 1.5) \text{ MeV}$
- The neutral-to-charged-current ratio in neutrino scattering experiments is three sigma below the value expected in the SM - NuTeV anomaly;
- The input parameters of the ew fit and the experimentally observed value of the W boson mass (derived from other SM parameters)

invisible and leptonic Z-decay widths, the Weinberg angle and the values of g_L and g_R

Apply to sterile neutrino masses $\gtrsim 1 \text{ TeV}$

Experimental constraints

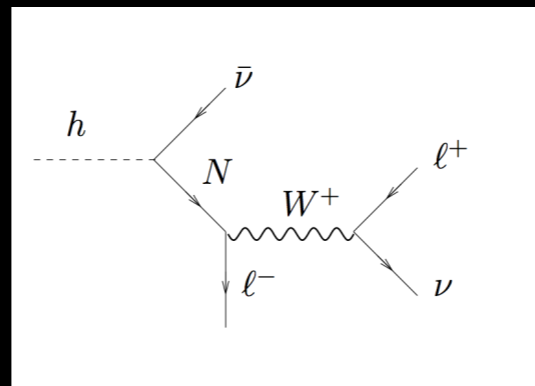
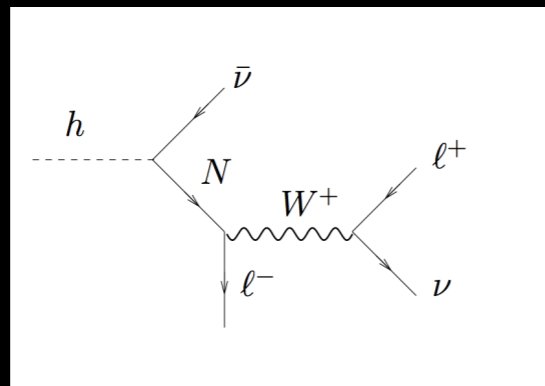
1. Neutrino oscillation parameters (seesaw approximation and PMNS)
2. Unitarity constraints
3. Electroweak precision data

4. LHC data (decay modes of the Higgs boson) (Bhupal Dev et al., 2012, P. Bandyopadhyay et al, 2012, Cely et al., 2013)

$h \rightarrow \nu_R \nu_L$ relevant for sterile neutrino masses ~ 100 GeV

Bounds on the Dirac Yukawa couplings of the neutrinos in seesaw models using the LHC data on Higgs decays for the case where the SM singlet heavy leptons needed for the seesaw mechanism have masses in the 100 GeV range.

Such scenario with large Yukawa couplings is natural in ISS models since the small neutrino mass owes its origin to a small Majorana mass of a new set of singlet fermions.



Higgs decay modes into $ll\nu\nu$ mediated by the ISS couplings

Experimental constraints

1. Neutrino oscillation parameters (seesaw approximation and PMNS)
2. Unitarity constraints
3. Electroweak precision data
4. LHC data (invisible decays)
5. Leptonic meson decays (B and D) (J. Beringer et al. ,PDG, 2013)

Decays of pseudoscalar mesons into leptons, whose dominant contributions arise from tree-level W mediated exchanges.

$\Gamma(P \rightarrow l\nu)$ with $P = D, B$ with one or two neutrinos in the final state

⚠ The theoretical prediction of some decays can be plagued by hadronic matrix element uncertainties

K decays

Why do not apply $K \rightarrow l\nu$ as a laboratory constraint?

In order to use this channel as a lab channel we need also to have $K \rightarrow \pi l\nu$ since in $K \rightarrow l\nu$ one needs V_{us} and f_k (decay constant). In order to have V_{us} you need f_k and viceversa.

People then use $K \rightarrow \pi l\nu$ which depends on V_{us} and $F(0)$ (a form factor at zero recoil).

$K \rightarrow l\nu$ cannot be a "lab constraint" since it depends on another measurement or a global fit of CKM (which calls for many channels)

The other point is that even if we assume to know V_{us} perfectly, then this decay is not free from soft photon contributions (a photon issued for instance by the charged lepton). This call for radiative contributions (loop).

Study of the tree-level enhancement to the violation of lepton flavor universality in light meson decays arising from modified $Wl\nu$ couplings. (Abada et al. 2014)

Experimental constraints

1. Neutrino oscillation parameters (seesaw approximation and PMNS)
2. Unitarity constraints
3. Electroweak precision data
4. LHC data (invisible decays)
5. Leptonic and semileptonic meson decays (K,B and D)
6. Laboratory bounds: direct searches for sterile neutrinos

(Atre et al. 2009, Kusenko et al. 2009)

A very powerful probe of the mixing of heavy neutrinos with both ν_e and ν_μ are peak searches in leptonic decays of pions and kaons.

If a heavy neutrino is produced in such decays (e.g. $\pi^\pm \rightarrow \mu^\pm \nu_s$), the lepton spectrum would show a monochromatic line.

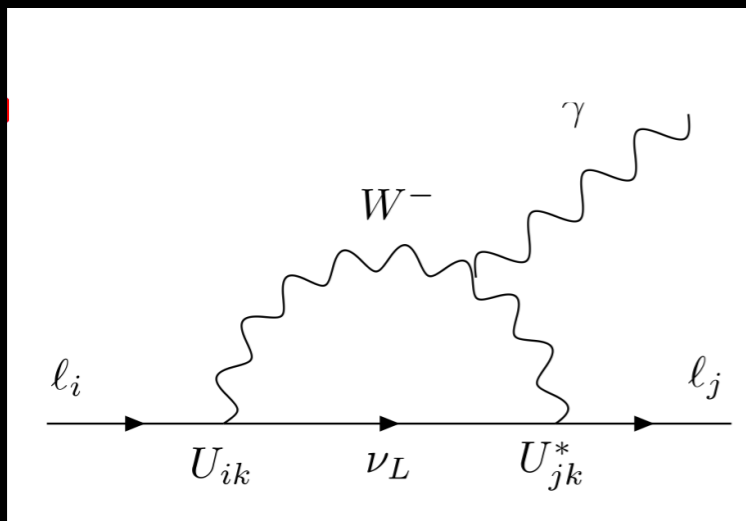
Experimental constraints

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7. Lepton flavor violation ($\mu \rightarrow e \gamma$) (Ilakovac and Pilaftsis, 1995, Deppisch and Valle, 2005)

$$Br(\mu \rightarrow e \gamma) = \frac{a_W^3 s_W^2 m_\mu^5}{256 \pi^2 m_W^4 \Gamma_\mu} \left| \sum_k U_{ek} U_{\mu k}^* G_\gamma \left(\frac{m_{\nu k}^2}{m_W^2} \right) \right|^2$$

$$Br(\mu \rightarrow e \gamma)_{MEG} = 0.57 \times 10^{-12}$$

(MEG, 2013)



Experimental constraints

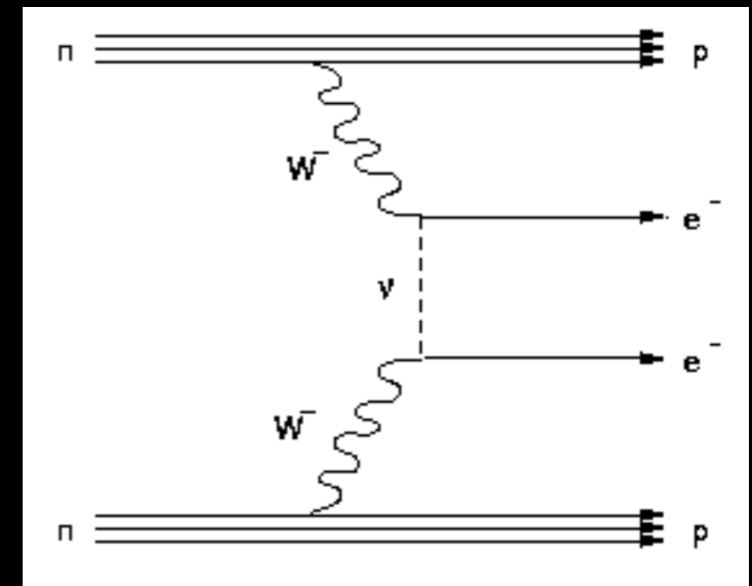
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8. Neutrinoless double beta decay

Most well studied among $\Delta L = 2$ processes

$$m_{\nu}^{\beta\beta} = \sum_i U_{ei}^2 m_i \leq (140 - 700) \text{meV}$$

(EXO-200, KamLAND-Zen, GERDA, CUORICINO)



Experimental constraints

1. Neutrino oscillation parameters (seesaw approximation and PMNS)
2. Unitarity constraints
3. Electroweak precision data
4. LHC data (invisible decays)
5. Leptonic and semileptonic meson decays (K,B and D)
6. Lepton flavor violation ($\mu \rightarrow e \gamma$)
7. Laboratory bounds: direct searches for sterile neutrinos
8. Neutrinoless double beta decay

9. Cosmological bounds on sterile neutrinos

(Smirnov et al. 2006
Kusenko 2009, Gelmini 2010)

- Large scale structure
- Lyman- α
- BBN
- CMB
- X-ray constraints (from $\nu_i \rightarrow \nu_j \gamma$)
- SN1987a

some cosmological bounds can be evaded with a non-standard cosmology
(e.g. low reheating temperature < 1 GeV)

Experimental constraints

1. Neutrino oscillation parameters (seesaw approximation and PMNS)

2. Unitarity constraints (Antusch et al., 2009) Non-standard neutrino interactions with matter can be generated by NP. $U_{3 \times 3} = (1 - \eta)U_{PMNS}$ Strongly constrained if $m_s > \Lambda_{EW}$

3. Electroweak precision data (Del Aguila et al., 2008, Atre et al., 2009) invisible and leptonic Z-decay widths, the Weinberg angle and the values of g_L and g_R

4. LHC data (invisible decays) (Bhupal Dev et al., 2012, P. Bandyopadhyay et al., 2012, Cely et al., 2013) decay modes of the Higgs boson $h \rightarrow \nu_R \nu_L$ relevant for sterile neutrino masses ~ 100 GeV

5. Leptonic and semileptonic meson decays (B, D and K) (J. Beringer et al., PDG, 2013) $\Gamma(P \rightarrow l\nu)$ with $P = D, B$ with one or two neutrinos in the final state

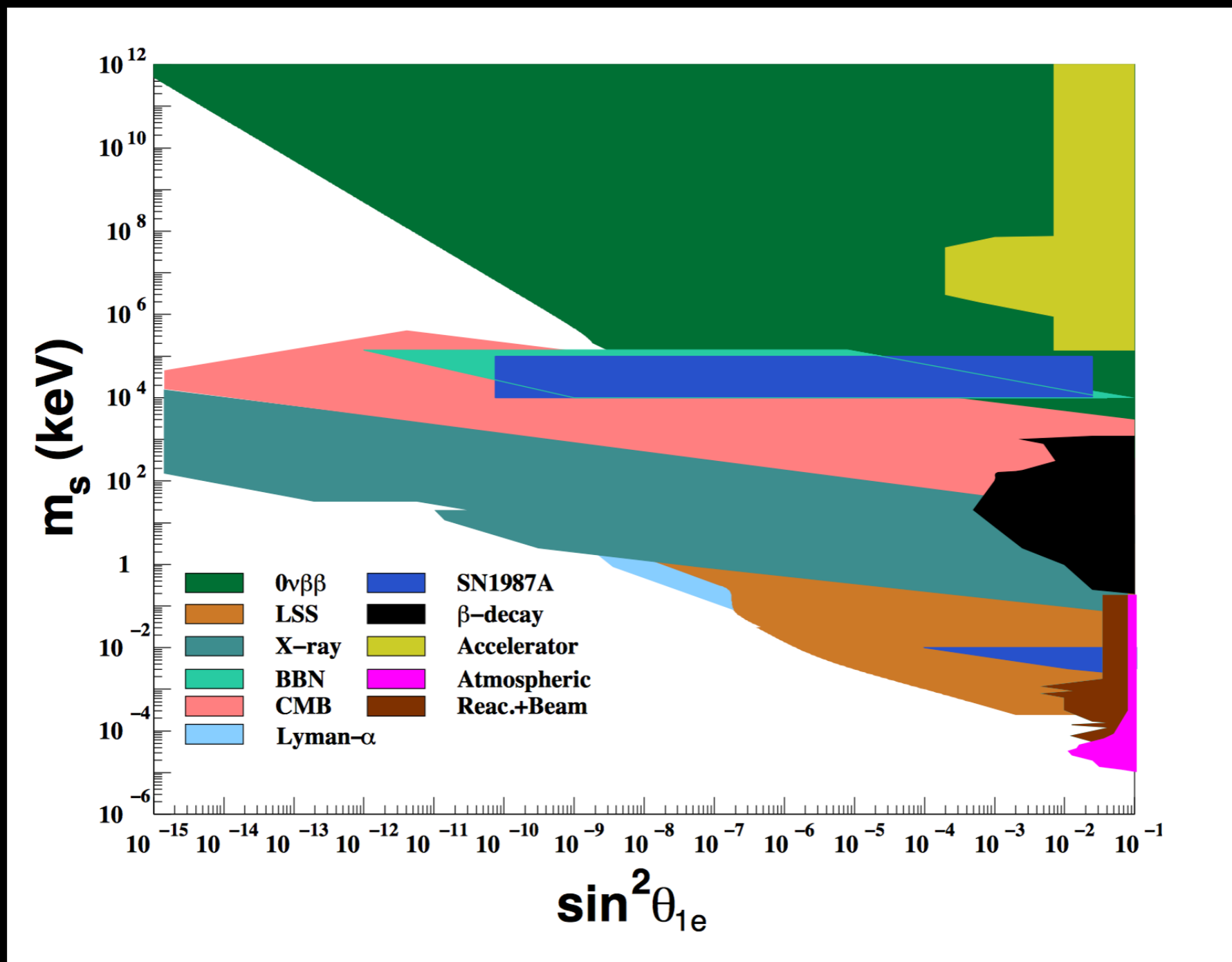
6. Laboratory bounds: direct searches for sterile neutrinos (Atre et al. 2009, Kusenko et al. 2009) e.g. $\pi^\pm \rightarrow \mu^\pm \nu_s$, the lepton spectrum would show a monochromatic line.

7. Lepton flavor violation ($\mu \rightarrow e \gamma$) (Ilakovac and Pilaftsis, 1995, Deppisch and Valle, 2005) $Br(\mu \rightarrow e \gamma)_{MEG} = 0.57 \times 10^{-12}$

9. Neutrinoless double beta decay (Blennow et al. 2010, Lopez-Pavon et al. 2013, Abada et al. 2014) $m_\nu^{\beta\beta} = \sum_i U_{ei}^2 m_i \leq (140 - 700) meV$ (EXO-200, KamLAND-Zen, GERDA, CUORICINO)

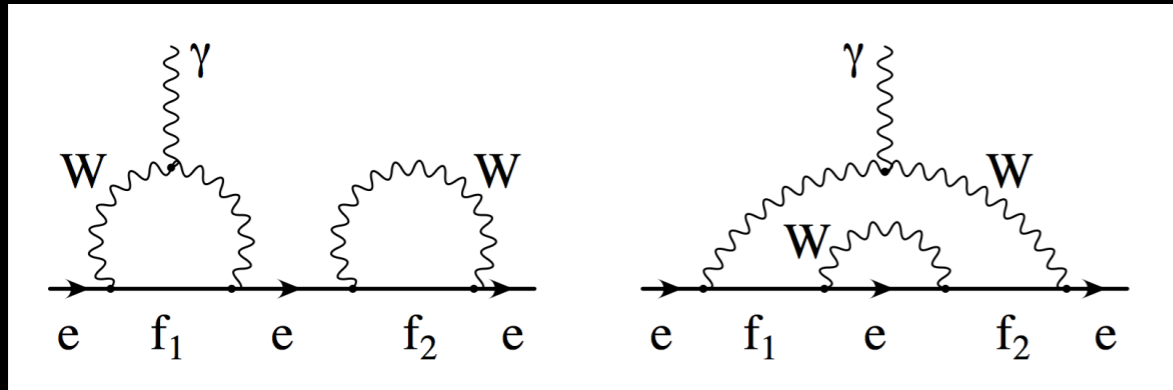
10. Cosmological bounds on sterile neutrinos (Smirnov et al. 2006, Kusenko 2009, Gelmini 2010) Large scale structure, Lyman- α , BBN, CMB, X-ray constraints (from $\nu_i \rightarrow \nu_j \gamma$), SN1987a

Cosmological bounds



(Kusenko 2009)

Electric dipole moments



the CP-odd part of a diagram with the selection of eigenstates f_1 and f_2 is always opposite to the CP-odd part of the diagram with interchanged flavors. As a result of this antisymmetrization, in the expansion over small q the first class of diagrams, turns out to be proportional to the cube of the photon momentum $O(q^3)$, while $G_2(0)$ vanishes identically

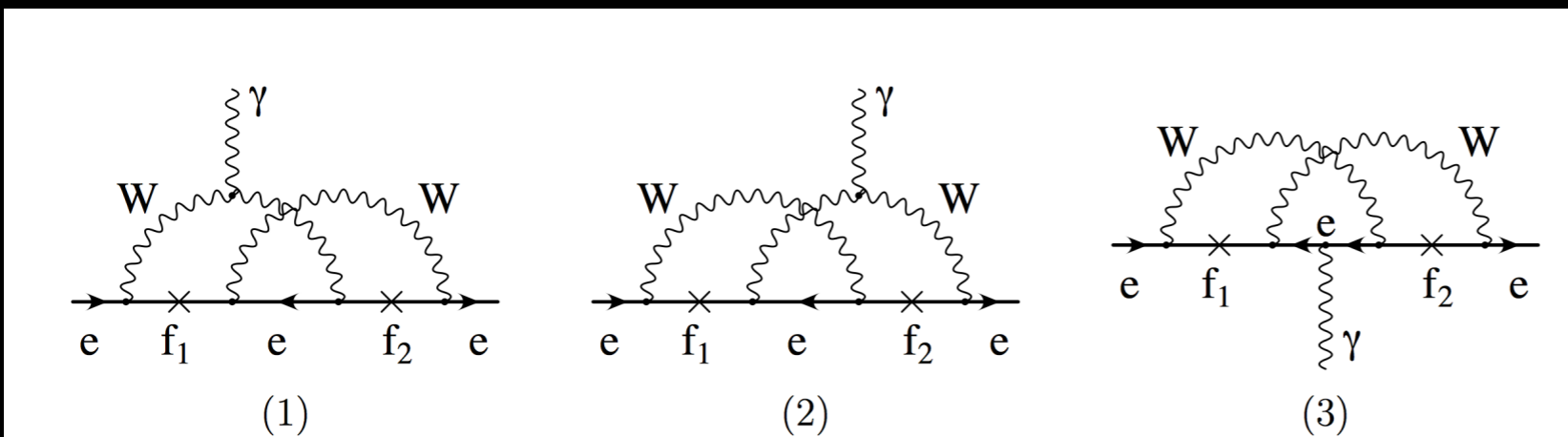
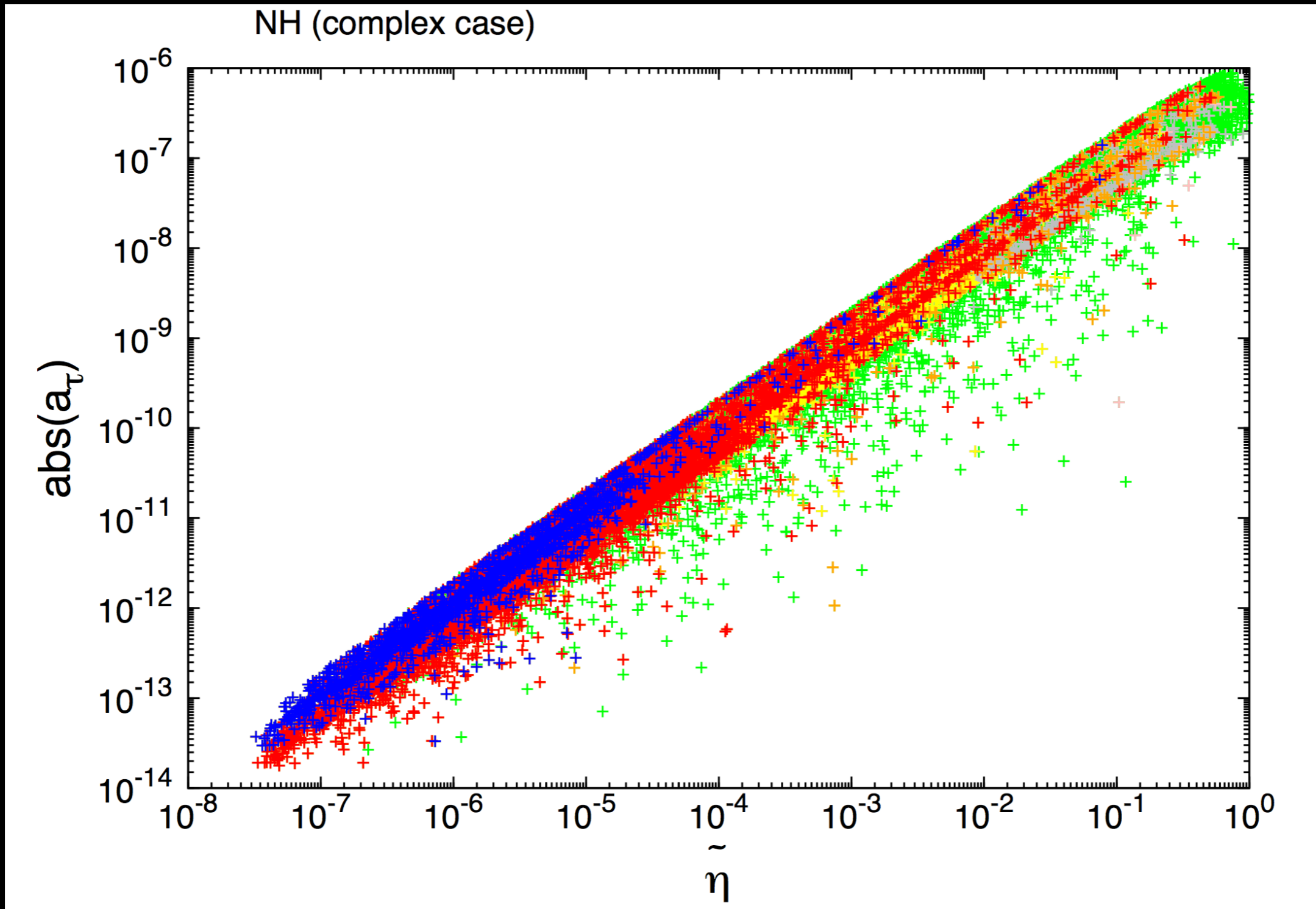


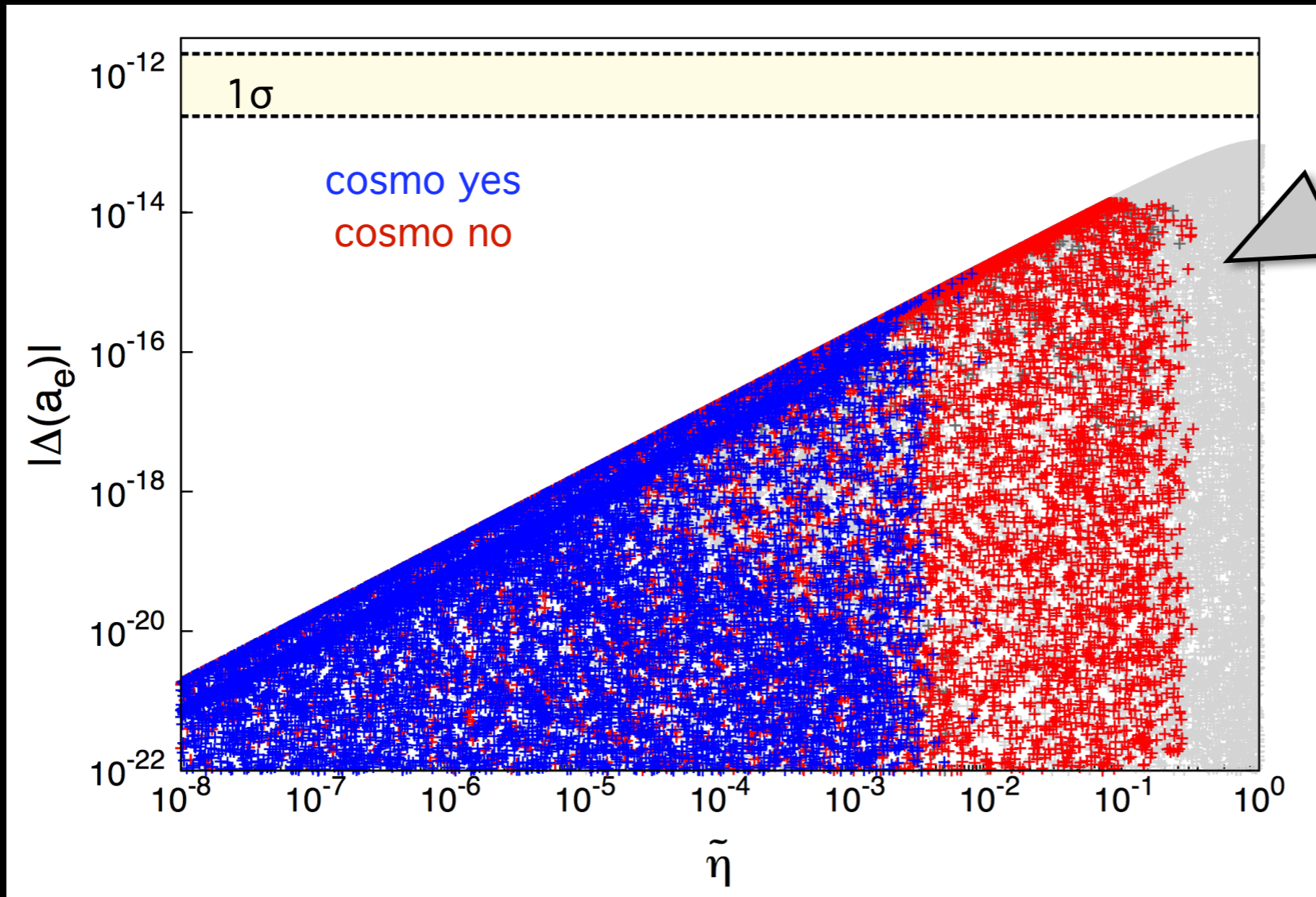
Figure 2: Contributions to the electron EDM in a model with Majorana masses of neutrinos. $f_{1,2}$ denote all possible neutrinos (see text). Crosses denote insertions of lepton-number violating mass parameters. Note that the direction of the internal electron line is opposite to the external ones.

(Archambault et al. 2004)

ISS a_τ



Effective "3+1": a_e



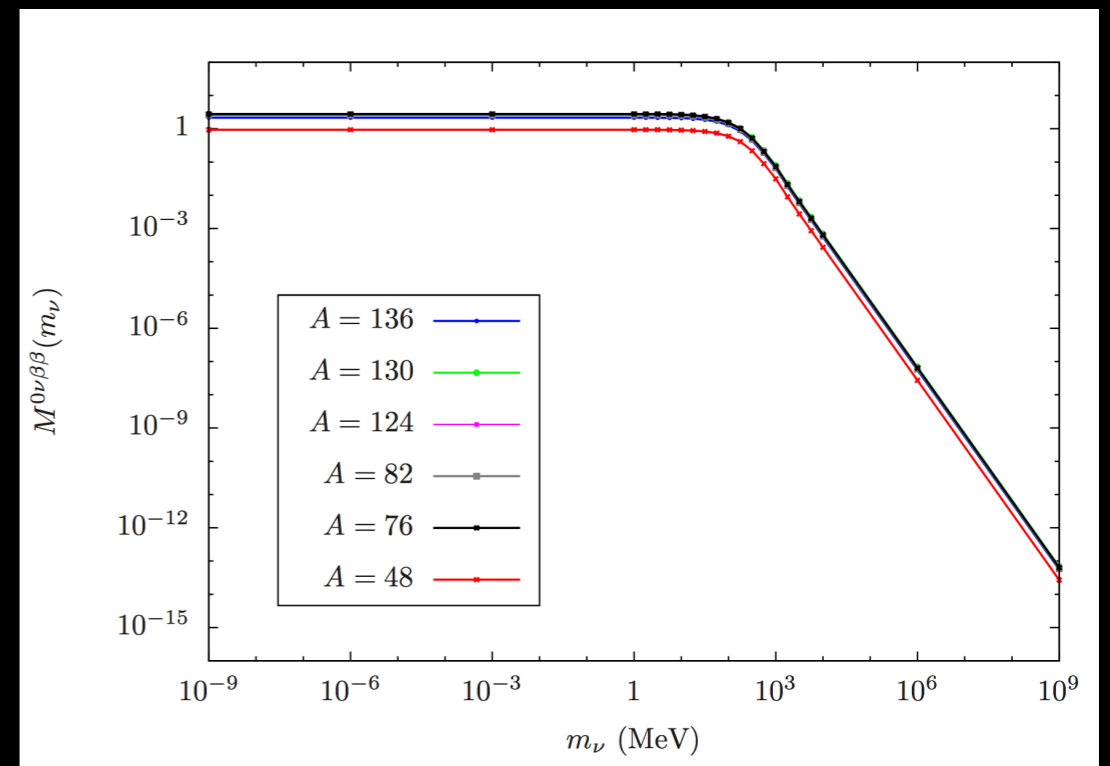
mainly excluded by
 ν oscillation data
 and lab bounds

$\tilde{\eta} = 1 - \det(\tilde{U}_{\text{PMNS}})$
 measures the deviation from
 unitarity.

No relevant contribution
 $\Delta(a_e)$: no new constraint on the
 model

Nuclear matrix element dependence on the neutrino mass

There are two distinct regions where the behaviour of the NME as a function of the neutrino mass changes from almost constant up to $m_i \approx 100$ MeV to decreasing quadratically as the neutrino mass increases beyond 100 MeV. The neutrino can be characterized as light if $m_i^2 \ll |p^2|$ or heavy if $m_i^2 \gg |p^2|$, which would mean that the neutrino propagator in the NME would be dominated by $|p^2|$ or m_i^2 , respectively, where p is the momentum exchanged in the process.



$$m_{\text{eff}}^{\nu e} \simeq \sum_{i=1}^7 U_{e,i}^2 p^2 \frac{m_i}{p^2 - m_i^2} \simeq \left(\sum_{i=1}^3 U_{e,i}^2 m_{\nu_i} \right) + p^2 \left(-U_{e,4}^2 \frac{|m_4|}{p^2 - m_4^2} + U_{e,5}^2 \frac{|m_5|}{p^2 - m_5^2} - U_{e,6}^2 \frac{|m_6|}{p^2 - m_6^2} + U_{e,7}^2 \frac{|m_7|}{p^2 - m_7^2} \right)$$

(Fernandez-Martinez et al, 2010, Abada and Lucente, 2014)

Current bounds on effective neutrino masses from total lepton number violating processes

Flavors	Exp. technique	Exp. bound	Mass bound (eV)
(e, e)	$\beta\beta 0\nu$	$T_{1/2}(^{76}\text{Ge} \rightarrow ^{76}\text{Se} + 2e^-) > 1.9 \times 10^{25} \text{ yr}$	$ m_{ee} < 3.6 \times 10^{-1}$
(e, μ)	$\mu^- \rightarrow e^+$ conversion	$\Gamma(\text{Ti} + \mu^- \rightarrow e^+ + \text{Ca}_{\text{gs}}) / \Gamma(\text{Ti} + \mu^- \text{ capture}) < 1.7 \times 10^{-12}$	$ m_{e\mu} < 1.7 \times 10^7$
(e, τ)	Rare τ decays	$\Gamma(\tau^- \rightarrow e^+ \pi^- \pi^-) / \Gamma_{\text{tot}} < 8.8 \times 10^{-8}$	$ m_{e\tau} < 2.6 \times 10^{12}$
(μ, μ)	Rare kaon decays	$\Gamma(K^+ \rightarrow \pi^- \mu^+ \mu^+) / \Gamma_{\text{tot}} < 1.1 \times 10^{-9}$	$ m_{\mu\mu} < 2.9 \times 10^8$
(μ, τ)	Rare τ decays	$\Gamma(\tau^- \rightarrow \mu^+ \pi^- \pi^-) / \Gamma_{\text{tot}} < 3.7 \times 10^{-8}$	$ m_{e\tau} < 2.1 \times 10^{12}$
(τ, τ)	none	none	none

(Gómez-Cadenas et al. 2012)

Neutrinoless double beta decay

Isotope	Experiment	$T_{1/2}^{0\nu\beta\beta}$ [yr]	$\langle m_{\beta\beta} \rangle$ [meV]
^{136}Xe	EXO-200	$>1.6 \cdot 10^{25}$	$<140\text{--}380$
^{136}Xe	KamLAND-Zen	$>1.9 \cdot 10^{25}$	$<120\text{--}250$
^{76}Ge	GERDA phase I	$>2.1 \cdot 10^{25}$	$<200\text{--}400$
^{130}Te	CUORICINO	$>2.8 \cdot 10^{24}$	$<300\text{--}700$

Future sensitivities

Isotope	Experiment	$T_{1/2}^{0\nu\beta\beta}$ sensitivity [yr]	$\langle m_{\beta\beta} \rangle$ sensitivity [meV]
^{136}Xe	EXO-200 (4 yr)	$5.5 \cdot 10^{25}$	75–200
^{136}Xe	nEXO (5 yr)	$3 \cdot 10^{27}$	12–29
^{136}Xe	nEXO (5 yr + 5 yr w/ Ba tagging)	$2.1 \cdot 10^{28}$	5–11
^{136}Xe	KamLAND-Zen (300 kg, 3 yr)	$2 \cdot 10^{26}$	45–110
^{136}Xe	KamLAND2-Zen (1 ton, post 2016)	IH	IH
^{76}Ge	GERDA phase II	$2 \cdot 10^{26}$	90–290
^{130}Te	CUORE-0 (2 yr)	$5.9 \cdot 10^{24}$	204–533
^{130}Te	CUORE (5 yr)	$9.5 \cdot 10^{25}$	51–133
^{130}Te	SNO+	$4 \cdot 10^{25}$	70–140

(Tosi - EXO. 2014)

Moreover...

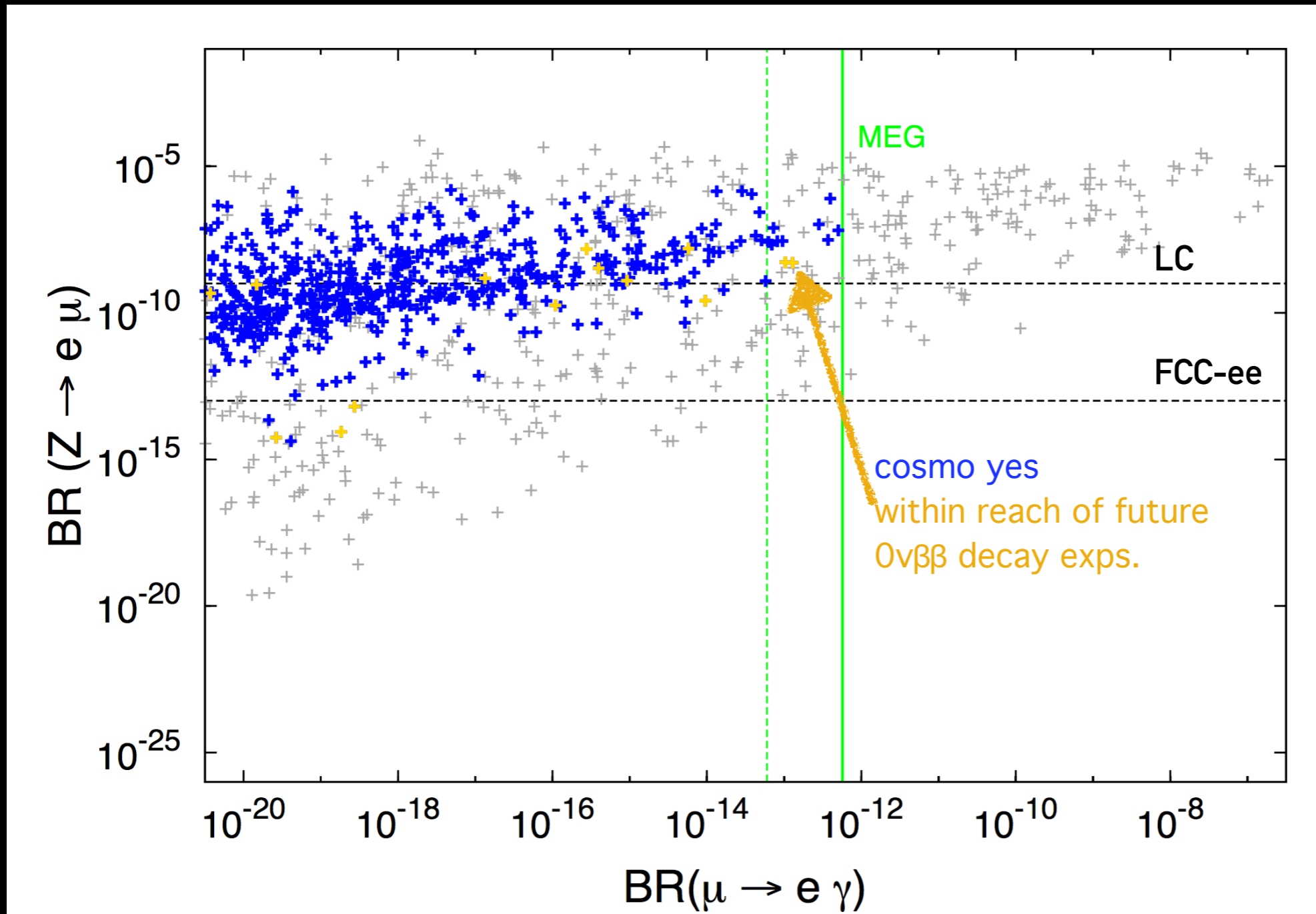
No dependence of the leptons anomalous magnetic moments on the phases (nor Majorana nor Dirac).

We calculated also a_τ , which is of the order of $10^{-14} - 10^{-6}$, while the experimental precision is too low to be compared.

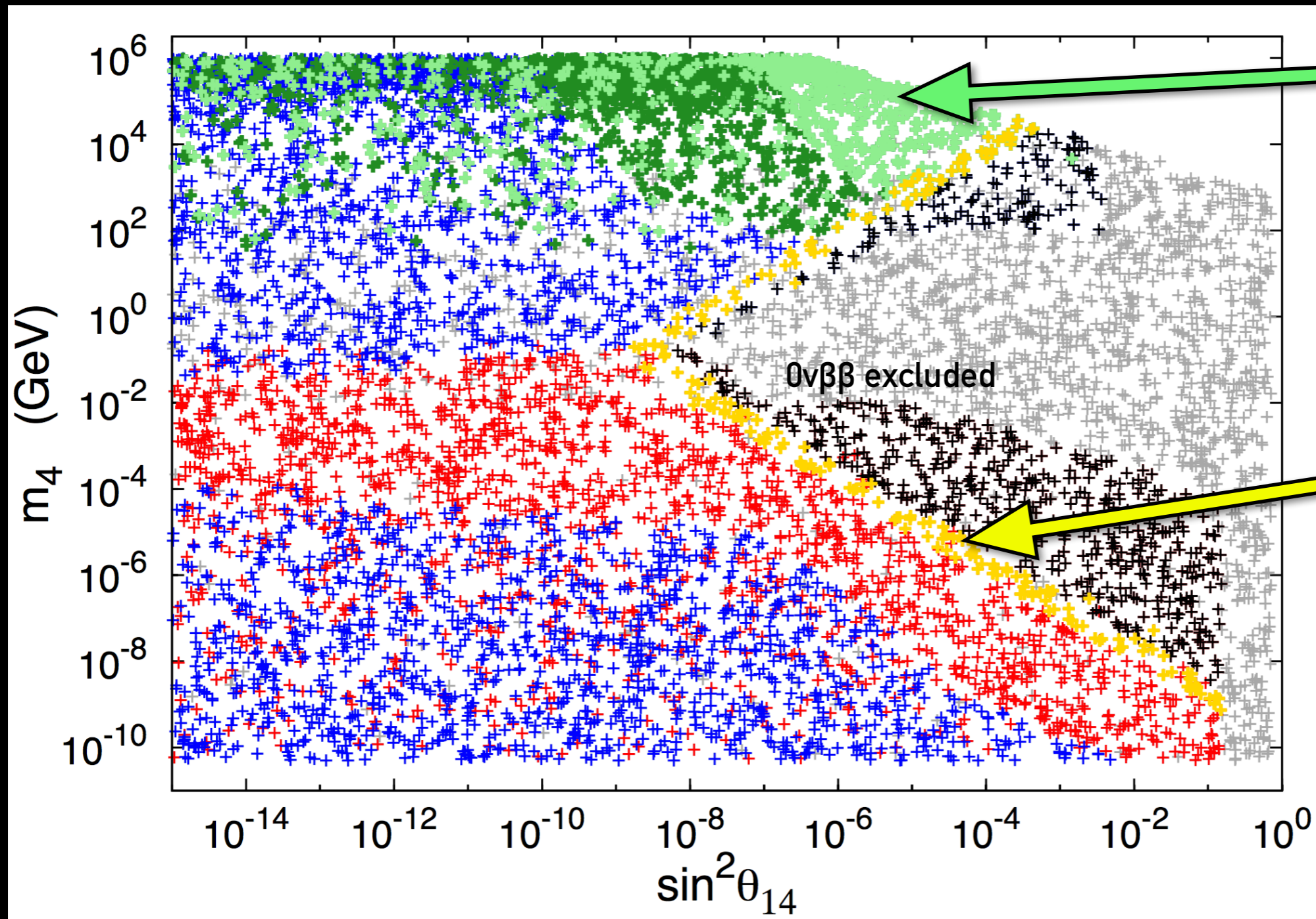
The Lepton Collider is designed to provide e^+e^- collisions in the beam energy range of 40 to 175 GeV. The main centre-of-mass operating points with large physics interest are 91 GeV (Z-pole), 160 GeV (W pair production threshold), 240 GeV (Higgs resonance) and 350 GeV ($t\bar{t}$ threshold). The machine would have a circumference of the order of 80 to 100 km in order to limit the synchrotron radiation power.

As explained in the text, the TLEP luminosity at each interaction point would increase significantly if fewer interaction points were considered. The possible TLEP energy upgrade up to 500 GeV, represented by a dashed line, is briefly discussed in section 5.

Effective "3+1": $Z \rightarrow e^\pm \mu^\mp$ vs $\mu \rightarrow e \gamma$



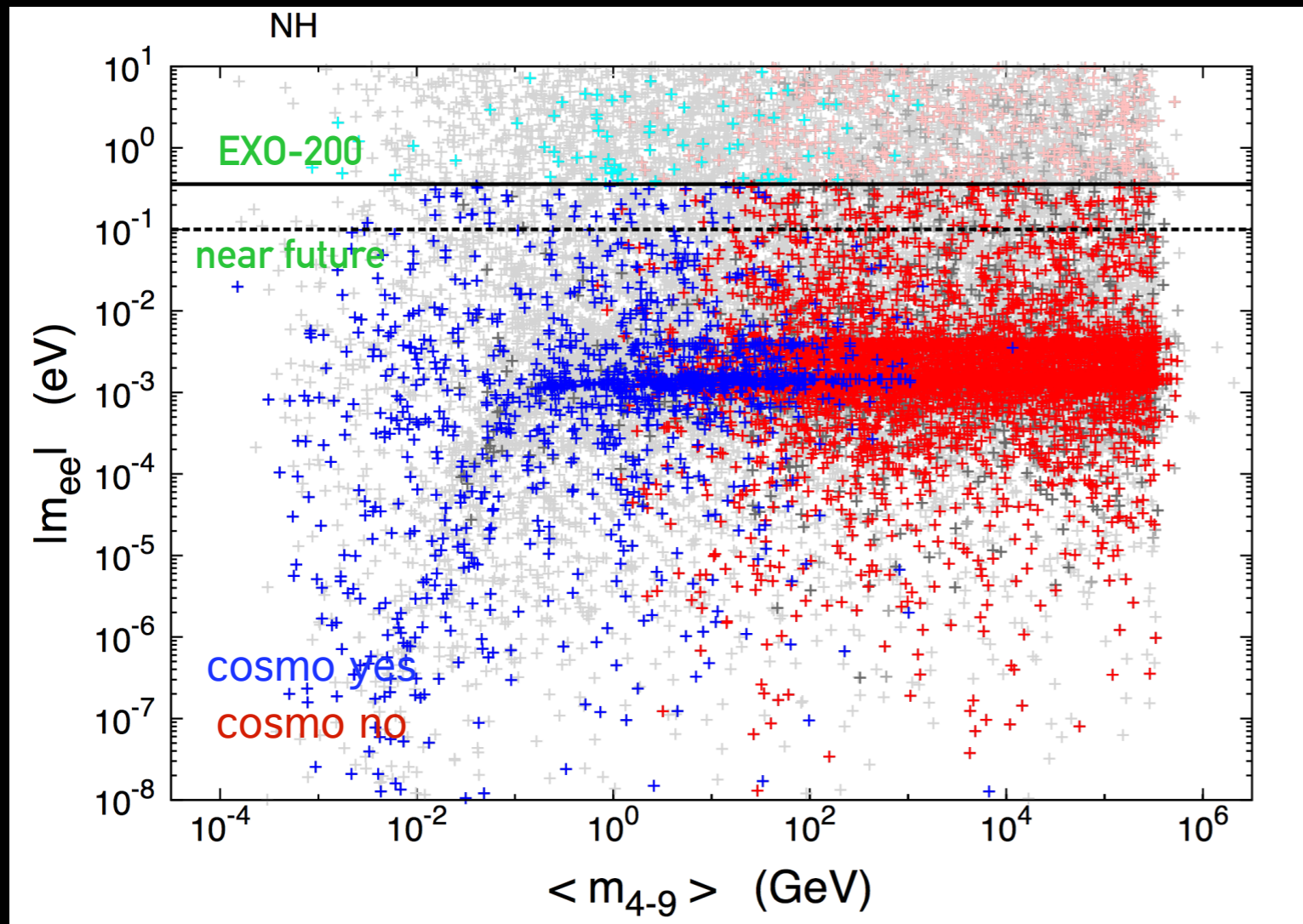
Effective "3+1": summary plot



$BR(Z \rightarrow e^+ \mu^-) > 10^{-9}$
 $BR(Z \rightarrow e^+ \mu^-) > 10^{-13}$

solutions within
reach of near future
 $0\nu\beta\beta$ experiments

ISS: $0\nu\beta\beta$ decay



p : momentum exchanged in the process

$m_s \ll |p|$: in this regime the effective mass goes to zero

$$m_{\text{eff}}^{\nu_e} = p^2 \sum_{i=1}^7 U_{e,i}^2 \frac{m_i}{p^2 - m_i^2} \simeq \sum_{i=1}^7 U_{e,i}^2 m_i$$

$m_s \approx |p|$: the contribution of the pseudo-Dirac states becomes more important, and can induce sizeable effects to m_{ee}

$m_s \gg |p|$: in this regime the heavy states decouple, and the contributions to m_{ee} only arise from the 3 light neutrino states.

$$m_{\nu}^{\beta\beta} = \sum_i U_{ei}^2 p^2 \frac{m_i}{p^2 - m_i^2}$$

- $0\nu\beta\beta$ decay excludes some solutions
- points within the reach of actual and near-future experiments

Conclusions

Measurements of the **electron and muon anomalous magnetic moments ($g-2$)** have recently reached an extraordinary precision. The discrepancy between the theoretical and the measured values of the muon $g-2$ could unveil NP signals.

We have considered **two extensions of the SM (ISS and $3+1$)** which add to the particle content of the SM one or more sterile neutrinos.

We have investigated the **contribution of the sterile states** to the anomalous magnetic moment of the leptons in these two classes of models and discussed them taking into account a number of **experimental and theoretical constraints**.

Even if the scale of such NP is low, its **contribution** to the anomalous magnetic moment of the leptons **is generically smaller** than the errors in theoretical calculation. However, **for large η** (deviation from unitarity) we can get solutions within 3σ of the expectation.

The **largest mixing angles (active-sterile)** which would give a sizeable contribution to the muon $g-2$ are indeed **strongly constrained** by other EW observables, e.g. $0\nu\beta\beta$.