

Dark energy : a revival of the quantum vacuum scenario¹

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1. A&A 554, A60 (2013)

- Visibility for OCEVU research.



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COSMOLOGY

The accelerating universe: spacetime structure and matter structures

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SCOPE OF THE CONFERENCE

Dark energy cosmology, the study of the causes and effects of the accelerated expansion of the universe, is the natural crossroad where fundamental physics, astrophysics and particle physics meet. This interdisciplinary character has enormously accelerated the progress of researches in this field, and exciting challenges and breakthroughs are expected in the next decade.

Revealing the finest details of cosmic acceleration, unveiling the effects of dark energy on structure formation and evolution processes, and advancing dark energy studies to the next level of complexity, critically depend on the



Stephan's Quintet in X-ray and Optical

TOPICS

- Dark Energy - Dark Gravity
- Inflation
- Primordial Non-Gaussianity
- Cosmological parameters
- Cosmic Microwave Background
- Large Scale Structure of the Universe
- Galaxy formation and evolution
- Galaxy clustering
- Gravitational Lensing
- Redshift surveys

Dark energy status

What tell us observations ?

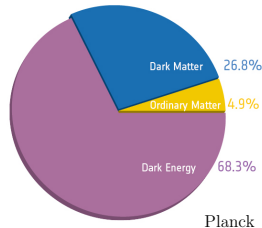
Einstein equation with cosmological constant

$$R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R + \Lambda g^{\mu\nu} = -\frac{8\pi G}{c^4}T^{\mu\nu}$$

- Λ a positive constant with dimension $[\Lambda] = \text{L}^{-2}$.

- Equivalently, $T_{\Lambda}^{\mu\nu} = \frac{\Lambda c^4}{8\pi G}g^{\mu\nu}$

$$\rightarrow \rho_{\Lambda} \equiv \frac{\Lambda c^4}{8\pi G} = 0,7\rho_c \sim 4 \text{ keV}/\text{cm}^3.$$



Which conclusion can be drawn ?

- Λ is « theoretically well-defined, observationally acceptable, phenomenologically simple and economical » (J.P. Uzan).

\rightarrow But not determined by the theory : **a new fundamental constant ?**

Macroscopic versus microscopic scales

Estimate orders $\rightarrow \Lambda$ defines a cosmological scale

- $\ell_\Lambda \equiv \Lambda^{-1/2} \sim$ size of Einstein universe
- $\tau_\Lambda \equiv \ell_\Lambda/c \sim H_0^{-1}$

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Introducing \hbar

- New length scale : $\rho_\Lambda \equiv \frac{\hbar c}{\ell_\Lambda'^4} \Rightarrow \ell_\Lambda' \simeq 83 \mu\text{m}$
 - $\rightarrow \ell_\Lambda' \sim \sqrt{\ell_\Lambda \ell_P}$ (geometric mean).
 - \rightarrow No modification of the gravitational force below this scale.

PRL 98, 021101 (2007)

PHYSICAL REVIEW LETTERS

week ending
12 JANUARY 2007



Tests of the Gravitational Inverse-Square Law below the Dark-Energy Length Scale

D. J. Kapner,* T. S. Cook, E. G. Adelberger, J. H. Gundlach, B. R. Heckel, C. D. Hoyle, and H. E. Swanson

- $E_\Lambda = \rho_\Lambda \ell_\Lambda'^3 \sim 2 \text{ meV}$.
 - \rightarrow No new physics expected at this energy scale.

Connection with vacuum energy

Some historical perspectives

The early times

- 1916 : Nernst cosmic quantum-ether from the zero point energies.
- 1920s : Lenz (and later Pauli). « If one allows waves of the shortest observed wavelengths of $\lambda \sim 2 \times 10^{-11}$ cm (as in radioactive γ -rays) - and if this radiation, converted to material density ($u/c^2 \sim 10^6$), contributed to the curvature of the world - one would obtain a vacuum energy density of such a value that the world would not reach even to the moon. »

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Conclusion so far : « vacuum catastrophe »

The vacuum energy as a sum of zero point energy $\frac{1}{2} \hbar \omega$ is much too large to be compatible with the observed cosmological constant.

- 120 orders of magnitude if Planck scale cutoff, 60 orders of magnitude for a cutoff at 1 TeV.
- Unknown mechanism to « degravitate » the zero point energies.

Connection with vacuum energy

$\langle T^{\mu\nu} \rangle_{\text{vac}} = 0$ for massless field in isotropic $(D + 1)$ -dimensional space

Three qualitative arguments :

- ① Equation of state of radiation $p = \rho/D$ while Lorentz invariance of vacuum implies $p = -\rho$
→ « Naive » solution $p = \rho = 0$.

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- 3 Dimensional regularization ($D = N + \epsilon$ with $\epsilon \rightarrow 0$)

$$p = -\rho = \frac{m^{D+1} \Gamma\left(-\frac{D+1}{2}\right)}{\mu^\epsilon 2^{D+2} \pi^{\frac{D+1}{2}}}$$

→ $m = 0 \Rightarrow p = \rho = 0$.

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→ how to generate a non zero energy density from a massless field?
(the gravitational field).

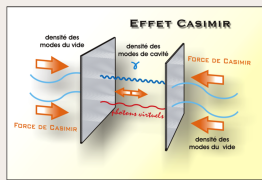
Analogy with the EM Casimir effect

Energy momentum tensor inside the plates

- Point-splitting regularization.

$$\langle T^{\mu\nu} \rangle = \frac{\pi^2}{720a^4} \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -3 \end{pmatrix}$$

$$\langle T^{\mu\nu} \rangle = \rho_{\text{cas}}(g^{\mu\nu} - 4\hat{z}^\mu \hat{z}^\nu) \neq \rho_{\text{cas}} g^{\mu\nu}$$



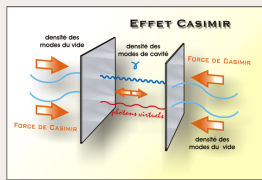
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→ Extra spatial dimension + boundary conditions allow a Lorentz invariant equation of state in a subspacetime with a massless field.

Kaluza-Klein picture

- Gravity and EM emerges from a 4+1 spacetime with one extra compact dimension (circle of radius R)

$$f(x, y + 2\pi R) = f(x, y)$$

Fourier series :

$$g_{AB}(x, y) = \sum_{n=-\infty}^{\infty} g_{AB}^{(n)}(x) e^{iny/R}$$

- $g_{\mu\nu}^{(0)}$ is GR (massless spin 2), $g_{\mu y}^{(0)} \equiv A_{\mu}$ is EM (massless spin 1) and $g_{yy}^{(0)}$ is a dilaton (massless scalar field).
- Modes with $n \neq 0$ are new massive fields of masses $m_n = n/R$ (KK tower).

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Limitation

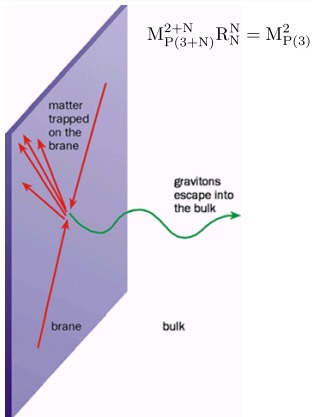
- Massive modes are not observed experimentally
 - the size of the extra dimension should be very small if universal.

$$R \lesssim 10^{-19} \text{ m}$$

Extra compact dimensions

Braneworld paradigm

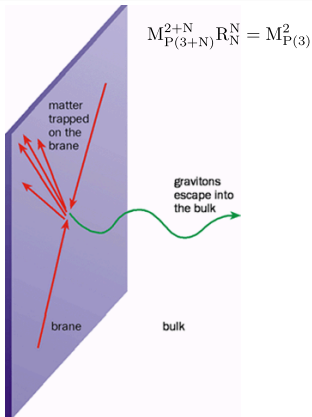
- Standard model fields are confined in a 3+1 brane. Only gravity propagates in the N extra dimensions.
 - ADD model (flat bulk).
 - Proposed to solve the hierarchy problem ($M_{P(3+N)} = 1 \text{ TeV}$).



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N	R_N (μm)	Constraints (μm)
1	2.6×10^{19}	< 44 (ISL) < 44 (NS)
2	2.2×10^3	< 30 (ISL) < 0.00016 (astr)
3	9.7×10^{-3}	$< 2.6 \times 10^{-3}$ (astr) $< 10^{-3}$ (LHC)
4	2.0×10^{-5}	$< 3.4 \times 10^{-7}$ (NS)
5	5.0×10^{-7}	$< 1.0 \times 10^{-7}$ (NS)
6	4.3×10^{-8}	$< 4.4 \times 10^{-8}$ (NS)
7	7.3×10^{-9}	$< 2.4 \times 10^{-8}$ (NS)

(from J. Beringer et al. (Particle Data Group), Phys. Rev. D 86 (2012), 010001)

Gravitational Casimir effect

Quantum effects of compactification of one extra dimension

Sum over the Kaluza Klein tower

- 5 polarization states in 4+1 gravity.

$$\rho = \frac{5\hbar c}{2\pi R} \sum_{n=-\infty}^{\infty} \int \frac{d^3k}{(2\pi)^3} \frac{1}{2} \sqrt{k^2 + m_n^2} \quad , \quad m_n = \frac{n}{R}$$

→ The presence of the KK tower allows a non-zero result.

- Dimensional regularization

$$\rho = -\frac{15\hbar c \zeta_R(5)}{128\pi^7 R^5} \quad \Rightarrow \quad \rho_{\text{brane}} = 2\pi R \rho = -2.5 \times 10^{-4} \frac{\hbar c}{R^4}$$

→ In accordance with Appelquist^a.

^a. T. Appelquist and A. Chodos, Phys. Rev. D **28** (1983), 772; D. Rohrlich, Phys. Rev. D **29** (1984), 330

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Conclusion

- « Wrong » sign for the energy density ($\rho < 0$).

→ Something is missing?

Gravitational Casimir effect

Generalization to $M^4 \times S^N$

Generalization to N extra dimension (N odd)

$$\rho = \kappa_N \frac{\hbar c}{a^4}$$

$$a_N = \left(\frac{\kappa_N \hbar c}{\rho \Lambda} \right)^{1/4}$$

N	κ_N	a_N (μm)	Constraints (μm)
1	-2.5×10^{-4}	(10.5)	< 44 (ISL) < 44 (NS)
3	1.1×10^{-3}	15.0	$< 2.6 \times 10^{-6}$ (NS) $< 10^{-3}$ (LHC)
5	1.2×10^{-2}	27.2	$< 1.0 \times 10^{-7}$ (NS)
7	3.6×10^{-2}	36.1	$< 2.4 \times 10^{-8}$ (NS)
9	7.4×10^{-2}	43.2	
11	1.2×10^{-1}	48.8	
13	1.6×10^{-1}	52.7	
15	1.9×10^{-1}	54.8	
17	1.8×10^{-1}	54.1	
19	1.1×10^{-1}	47.8	
21	-4.9×10^{-2}	(39.1)	

→ Positive value of ρ are obtained for N odd from $N = 3$ to $N = 19^a$.

^a. P. Candelas and S. Weinberg, Nucl. Phys. B **237** (1984), 397; K.A. Milton, The Casimir effect (2001), ISBN 981-02-4397-9

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- The end of the story ?

One proposition

Adding a cosmological argument

Assumptions

- 1 Only modes with wavelength λ lower than the Hubble radius $\sim ct$ should contribute to the Casimir energy.
- 2 As long as the Hubble radius (cosmological horizon) is lower than the radius of the extra dimension, the energy density stays equal to zero.

Dark Energy as a « temporal » Casimir effect

- Modes with wavelength larger than R appear after the horizon crosses the fifth dimension (equivalent to have an UV cutoff $1/R$ in momentum space) and give a positive contribution.

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Conclusion

$$\rho_{\text{brane}} = \frac{5\hbar c}{16\pi^2 R^4} > 0$$

- Compatible with a cosmological constant if $R \simeq 35 \mu\text{m}!$

Modification of the gravitational law

Point-point interaction

Modified gravitational potential

- Obtained as a sum of Yukawa potential for each massive KK modes.

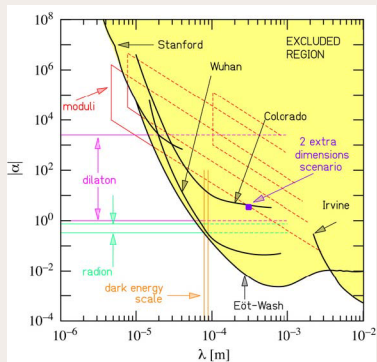
$$V(r) = -\frac{GM}{r} \coth\left(\frac{r}{2R}\right)$$

- Two limiting cases

$$\rightarrow r \ll R, V(r) = -\frac{2RG_3M}{r^2}$$

$$\rightarrow r \gg R,$$

$$V(r) = -\frac{G_3M}{r} \left(1 + 2e^{-r/R}\right)$$



(From Adelberger et al., progr. part. Nucl. Phys. 62 (2009), 102)

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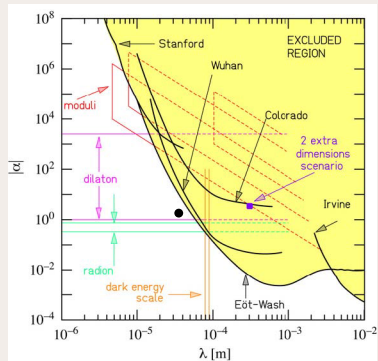
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Summary

- Reconsider the connection between cosmological constant and vacuum energy from the gravitational field.
- Usual computation leads to a wrong sign, which could be inverted with thanks to the horizon.
- The prediction is testable in a very near future.