# Missing top properties

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# Missing top properties?

Focus not on what you already know from TOP $\leq$ 2013 but on what you may not know [yet no claim this is all new and revolutionary, ok?]

The anomalous $A_{FB}$ at Tevatron — not settled despite claims — has fostered the study of top properties.

JAAS et al. RMP '14
not only $A_{FB}$

Discuss observables which are simple and interesting [sensitive to new physics] but have not been yet measured.

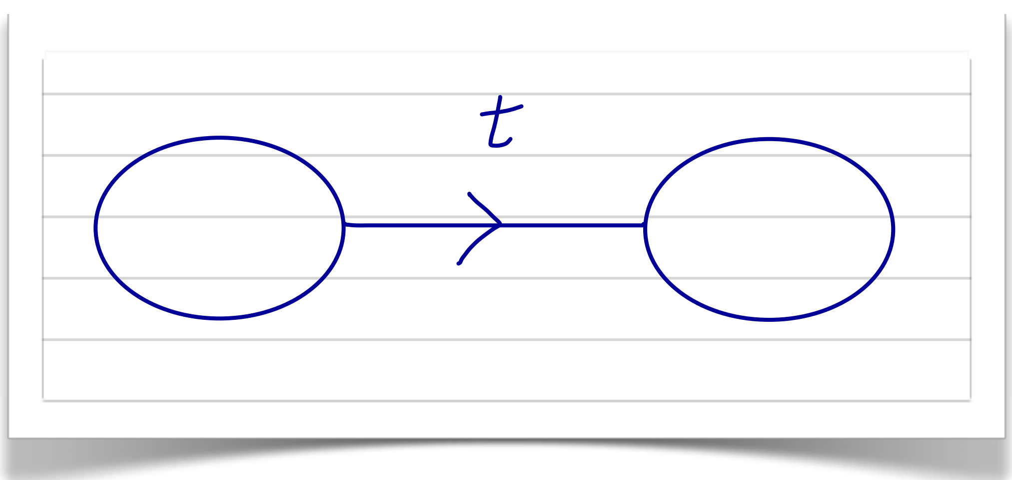
Outlook:

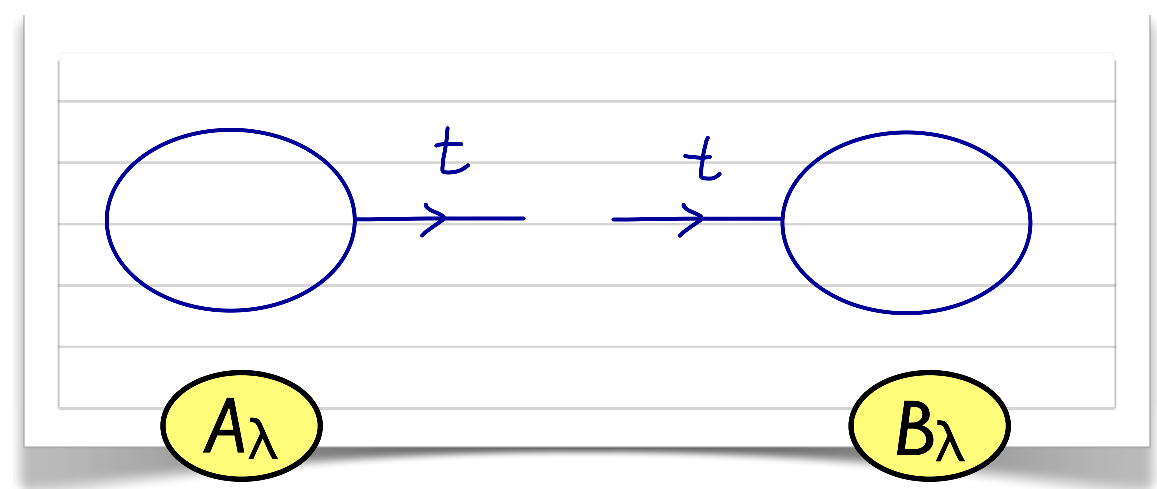
- framework
- top polarisation at Tevatron
- top polarisation at LHC

$A_{FB}$ / $A_C$ not explicitly covered but will always be around

# Production and decay of top quarks

The top is not stable but decays. The full matrix element contains a top propagator. Since the top is a narrow resonance [$\Gamma_t/m_t \ll 1$], the amplitude can decomposed into production $\times$ decay.




$$\frac{\not{p} + m_t}{p^2 - m_t^2 + i\Gamma_t m_t} \longrightarrow \frac{\pi}{\Gamma_t m_t}\delta(p^2 - m_t^2)\sum_\lambda u(p,\lambda)\bar{u}(p,\lambda)$$

Barger et al. '88

$$\Rightarrow \quad \mathcal{M} \propto \sum_\lambda A_\lambda \times B_\lambda$$

Therefore, the squared matrix element [and the differential cross section] can be written as

$$|\mathcal{M}|^2 \propto \sum_{\lambda\lambda'} A_\lambda B_\lambda A^*_{\lambda'} B^*_{\lambda'}$$

[also after integration in production phase space]

$$= \begin{pmatrix} \cdot & \cdot \end{pmatrix}_{\lambda'} \begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}_{\lambda'\lambda} \begin{pmatrix} \cdot \\ \cdot \end{pmatrix}_{\lambda}$$

$B^*_{\lambda'}$

$A^*_{\lambda'} A_\lambda$

spin density matrix

$B_\lambda$

$\langle O \rangle = \mathrm{tr}[\rho O]$

$$\rho = \frac{1}{2} \begin{pmatrix} 1 + P_z & P_x - iP_y \\ P_x + iP_y & 1 - P_z \end{pmatrix}$$

General form for a spin 1/2 particle, with $P_i = 2 \langle S_i \rangle$

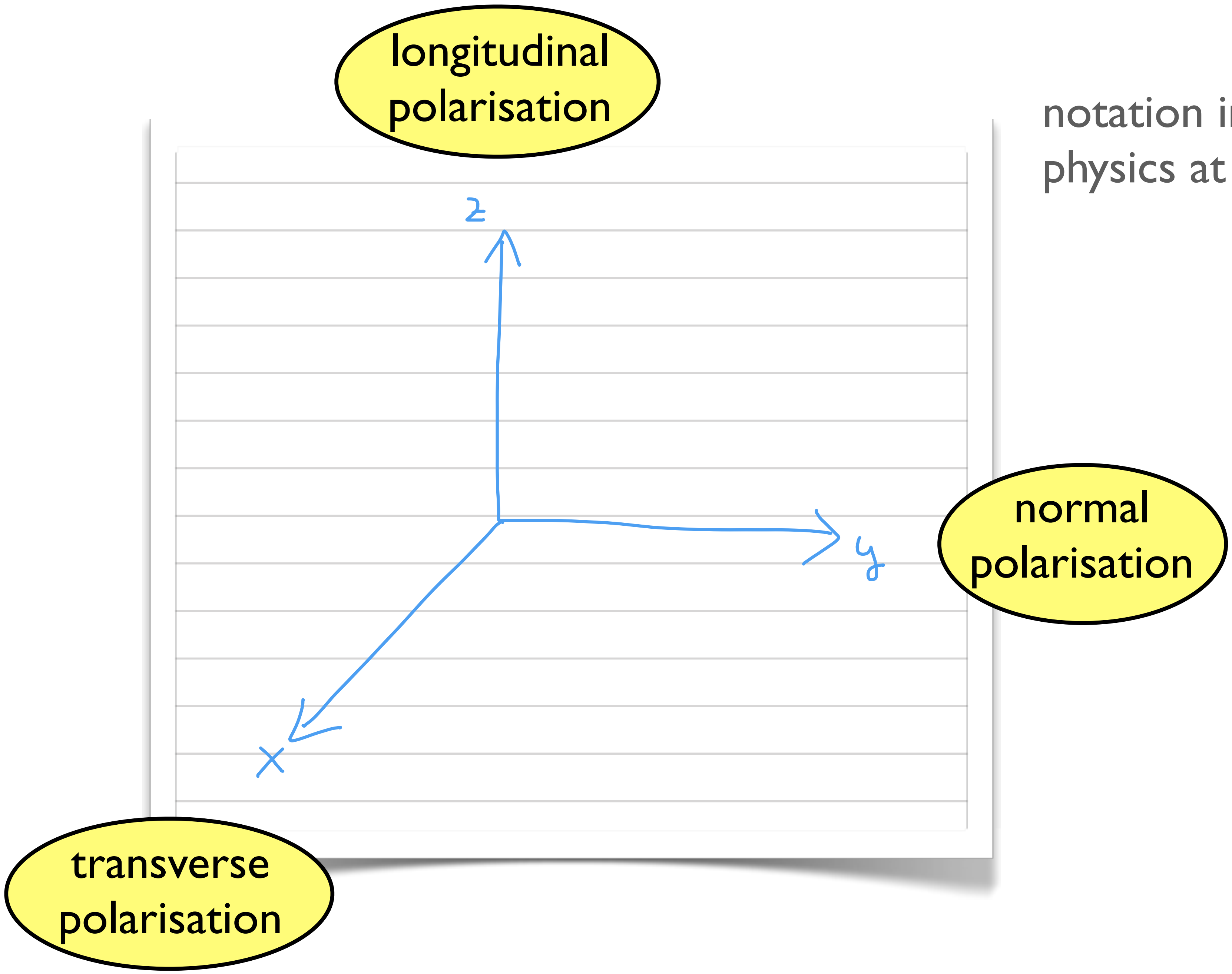
SU(2)

By introducing ρ we are “ignoring” on purpose the details of the top production process. This applies to tops produced singly, in pairs, from a black hole, etc.

Here, we are taking the z direction as the top momentum in the CM frame [helicity].

As it is clear, we can measure not only $\langle S_z \rangle$ but also $\langle S_x \rangle$ and $\langle S_y \rangle$.

Obviously, to do that we must further specify the reference system, not only the z direction.




notation inherited from τ physics at LEP

Similar thing already proposed and measured for $W$ decays. [ JAAS & Bernabéu '10 ] [ATLAS]

# Top ≃ tau! 16 years back, the same was done at LEP.

30 April 1998

PHYSICS LETTERS B

ELSEVIER

Physics Letters B 426 (1998) 207–216

## Measurement of the weak dipole moments of the $\tau$ lepton

L3 Collaboration

M. Acciarri [aa], O. Adriani [p], M. Aguilar-Benitez [z], S. Ahlen [k], J. Alcaraz [z], G. Alemanni [v], J. Allaby [q], A. Aloisio [ac], M.G. Alviggi [ac], G. Ambrosi [s], H. Anderhub [aw], V.P. Andreev [al], T. Angelescu [m], F. Anselmo [i], A. Arefiev [ab], T. Azemoon [c], T. Aziz [j], P. Bagnaia [ak], L. Baksay [ar], R.C. Ball [c], S. Banerjee [j], Sw. Banerjee [j], K. Banicz [at], A. Barczyk [aw,au], R. Barillère [q], L. Barone [ak], P. Bartalini [ah], A. Baschirotto [aa], M. Basile [i], R. Battiston [ah], A. Bay [v], F. Becattini [p], U. Becker [o], F. Behner [aw], J. Berdugo [z], P. Berges [o], B. Bertucci [ah], B.L. Betev [aw], S. Bhattacharya [j], M. Biasini [q], A. Biland [aw], G.M. Bilei [ah], J.J. Blaising [d], S.C. Blyth [ai], G.J. Bobbink [b], R. Bock [a], A. Böhm [a], L. Boldizsar [n], B. Borgia [ak], D. Bourilkov [aw], M. Bourquin [s], D. Boutigny [d], S. Braccini [s], J.G. Branson [an], V. Brigljevic [aw], I.C. Brock [ai], A. Buffini [p], A. Buijs [as], J.D. Burger [o], W.J. Burger [s], J. Busenitz [ar], X.D. Cai [o], M. Campanelli [aw], M. Capell [o], G. Cara Romeo [i], G. Carlino [ac], A.M. Cartacci [p], J. Casaus [z], G. Castellini [p], F. Cavallari [ak], N. Cavallo [ac], C. Cecchi [s], M. Cerrada [z], F. Cesaroni [w], M. Chamizo [z], Y.H. Chang [ay], U.K. Chaturvedi [r], S.V. Chekanov [ae], M. Chemarin [y], A. Chen [ay], G. Chen [g], G.M. Chen [g], H.F. Chen [t], H.S. Chen [g], M. Chen [o], G. Chiefari [ac], C.Y. Chien [e], L. Cifarelli [am], F. Cindolo [i], C. Civinini [p], I. Clare [o], R. Clare [o], H.O. Cohn [af], G. Coignet [d], A.P. Colijn [b], N. Colino [z], S. Costantini [h], F. Cotorobai [m], B. de la Cruz [z], A. Csilling [n], T.S. Dai [o], R. D'Alessandro [p], R. de Asmundis [ac], A. Degré [d], K. Deiters [au], P. Denes [aj], F. DeNotaristefani [ak], D. DiBitonto [ar], M. Diemoz [ak], D. van Dierendonck [b], F. Di Lodovico [aw], C. Dionisi [ak], M. Dittmar [aw], A. Dominguez [an], A. Doria [ac], M.T. Dova [r,1], E. Drago [ac], D. Duchesneau [d], P. Duinker [b], I. Duran [ao], S. Dutta [j], S. Easo [ah], Yu. Efremenko [af], H. El Mamouni [y], A. Engler [ai], F.J. Eppling [o], F.C. Erné [b], J.P. Ernenwein [y], P. Extermann [s], M. Fabre [au], R. Faccini [ak], S. Falciano [ak], A. Favara [p], J. Fay [y], O. Fedin [al], M. Felcini [aw], B. Fenyi [ar], T. Ferguson [ai],

[1] Also supported by CONICET and Universidad Nacional de La Plata, CC 67, 1900 La Plata, Argentina.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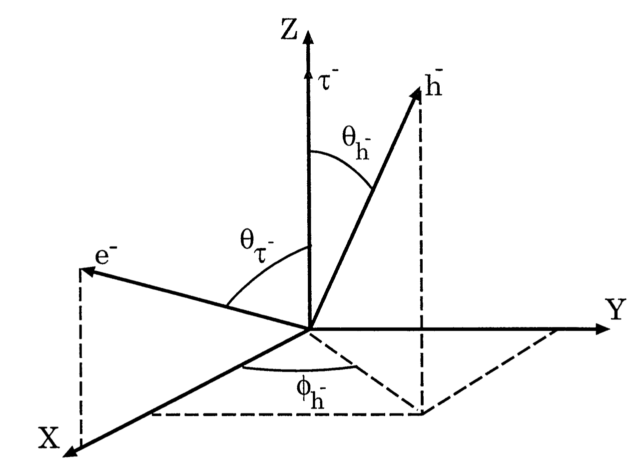
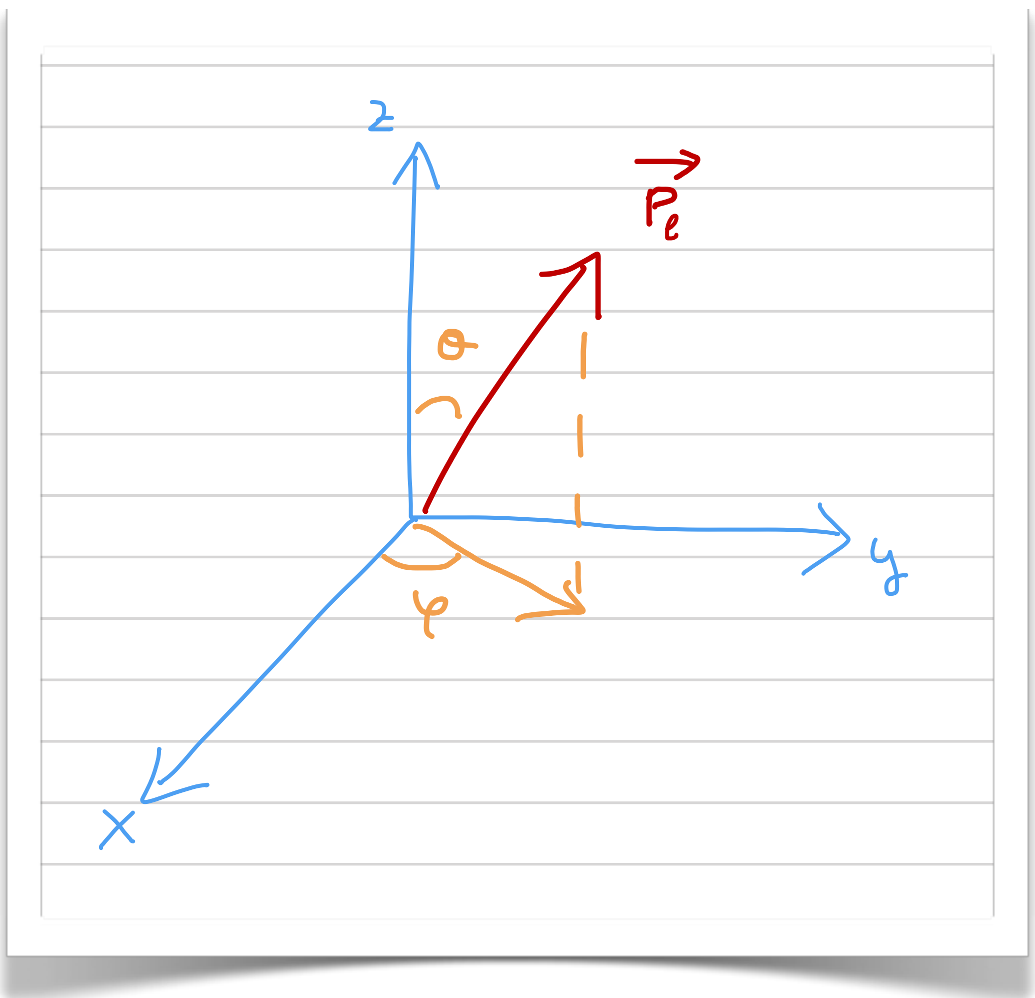



Fig. 1. Reference system used in this analysis. The $z$ axis points in the $\tau$ flight direction and the $x$ axis is fixed by the plane containing the $\tau$ and the electron flight directions.

# Measuring top polarisation

As always, one can use the charged lepton momentum in the top quark rest frame.




ignoring anomalous *Wtb* couplings that correspond to Cen's talk

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta_\ell} = \frac{1}{2}(1 + P_z\cos\theta_\ell)$$

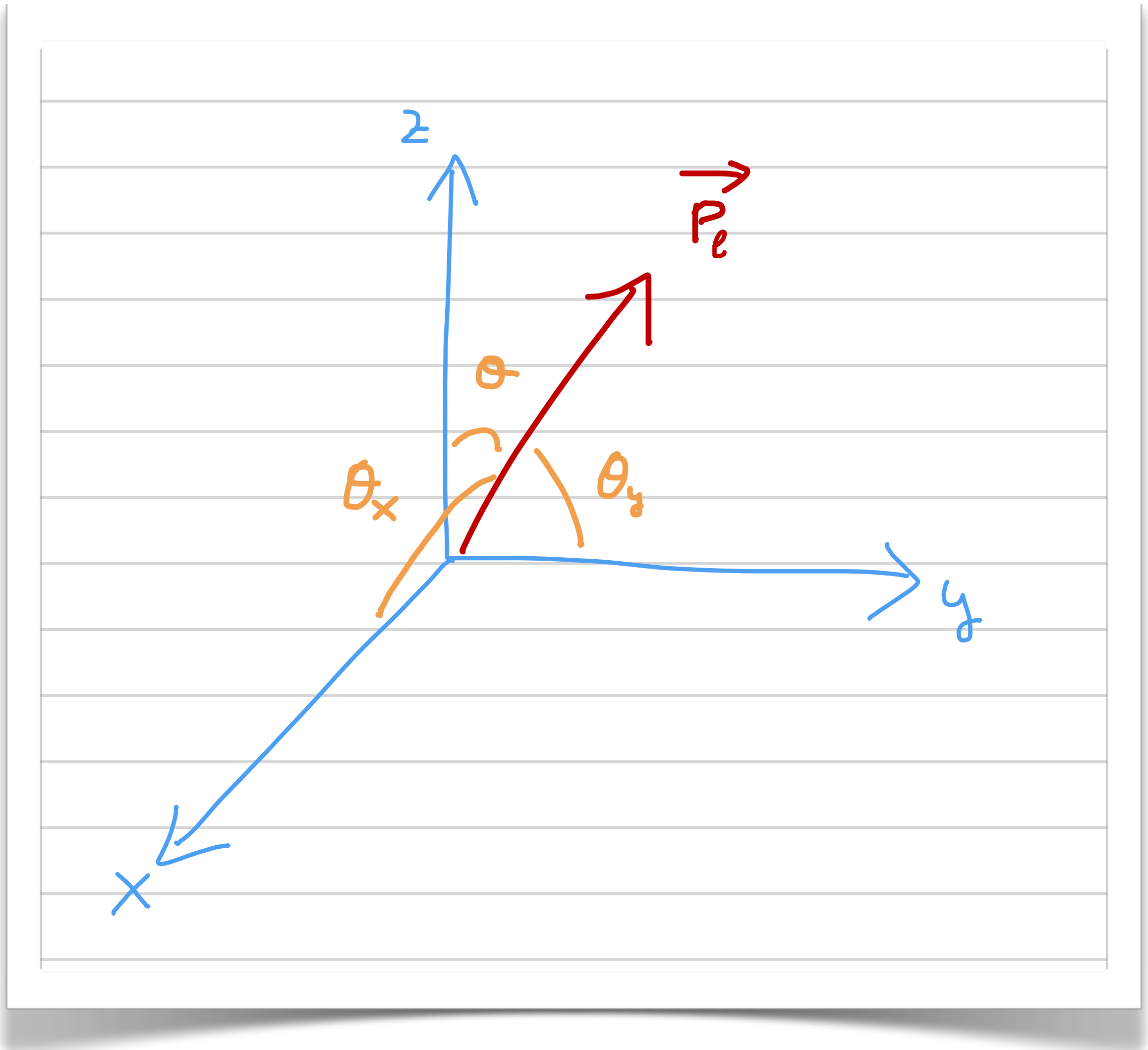
more general

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta_\ell d\varphi_\ell} = \frac{1}{4\pi}(1 + P_z\cos\theta_\ell + P_x\sin\theta_\ell\cos\varphi_\ell + P_y\sin\theta_\ell\sin\varphi_\ell)$$

Godbole et al. '06

for example

Alternatively,




$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta_x} = \frac{1}{2}(1 + P_x\cos\theta_x)$$

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta_y} = \frac{1}{2}(1 + P_y\cos\theta_y)$$

Baumgart & Tweedie '13

for example

It may be more convenient experimentally to reduce 2D to 1D measurements and clearly no information is lost.

# Warning!

We have been speaking about expectation values of spin operators.

Nowhere have I mentioned such thing as that top quarks are produced with a definite helicity [spin].

Saying that a top quark is produced with a definite helicity is in general incorrect: the differential cross section does <u>not</u> factorise into production × decay of helicity states.

$$\begin{pmatrix} \cdot & \cdot \end{pmatrix}_{\lambda'} \begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}_{\lambda'\lambda} \begin{pmatrix} \cdot \\ \cdot \end{pmatrix}_{\lambda} \neq \begin{pmatrix} \cdot & \cdot \end{pmatrix}_{\lambda'} \begin{pmatrix} \cdot & 0 \\ 0 & \cdot \end{pmatrix}_{\lambda'\lambda} \begin{pmatrix} \cdot \\ \cdot \end{pmatrix}_{\lambda}$$

We are not performing a Stern-Gerlach-like experiment on top quarks to force them into helicity eigenstates [before they decay].

However, we often see mentions to top spin, for example:

The angles $\theta_X, \theta_{\bar{X}'}$ are measured using as spin axis the parent top (anti)quark momentum in the $t\bar{t}$ CM system. The factor

$$C \equiv \frac{\sigma(t_R\bar{t}_R)+\sigma(t_L\bar{t}_L)-\sigma(t_R\bar{t}_L)-\sigma(t_L\bar{t}_R)}{\sigma(t_R\bar{t}_R)+\sigma(t_L\bar{t}_L)+\sigma(t_R\bar{t}_L)+\sigma(t_L\bar{t}_R)} \qquad (18)$$

is the relative number of like helicity minus opposite helicity $t\bar{t}$ pairs, and measures the spin correlation between the

Paper #1

$$C \cancel{A} = \frac{N_{\rm like} - N_{\rm unlike}}{N_{\rm like} + N_{\rm unlike}} = \frac{N(\uparrow\uparrow) + N(\downarrow\downarrow) - N(\uparrow\downarrow) - N(\downarrow\uparrow)}{N(\uparrow\uparrow) + N(\downarrow\downarrow) + N(\uparrow\downarrow) + N(\downarrow\uparrow)}, \qquad (3)$$

where $N_{\rm like} = N(\uparrow\uparrow) + N(\downarrow\downarrow)$ is the number of events where the top quark and top antiquark spins are parallel, and $N_{\rm unlike} = N(\uparrow\downarrow) + N(\downarrow\uparrow)$ is the number of events where they are anti-parallel. The strength of the spin correlation is defined by

Paper #2

Given the previous remarks, several questions arise:

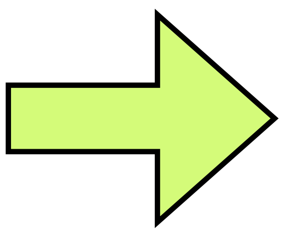
- Speaking about "parallel spins" or "like helicity" makes sense at all? Or should I withdraw my paper from EPJC?
- If it does in some sense, are we measuring what it is written?
- Ok, imagine we are, then why?

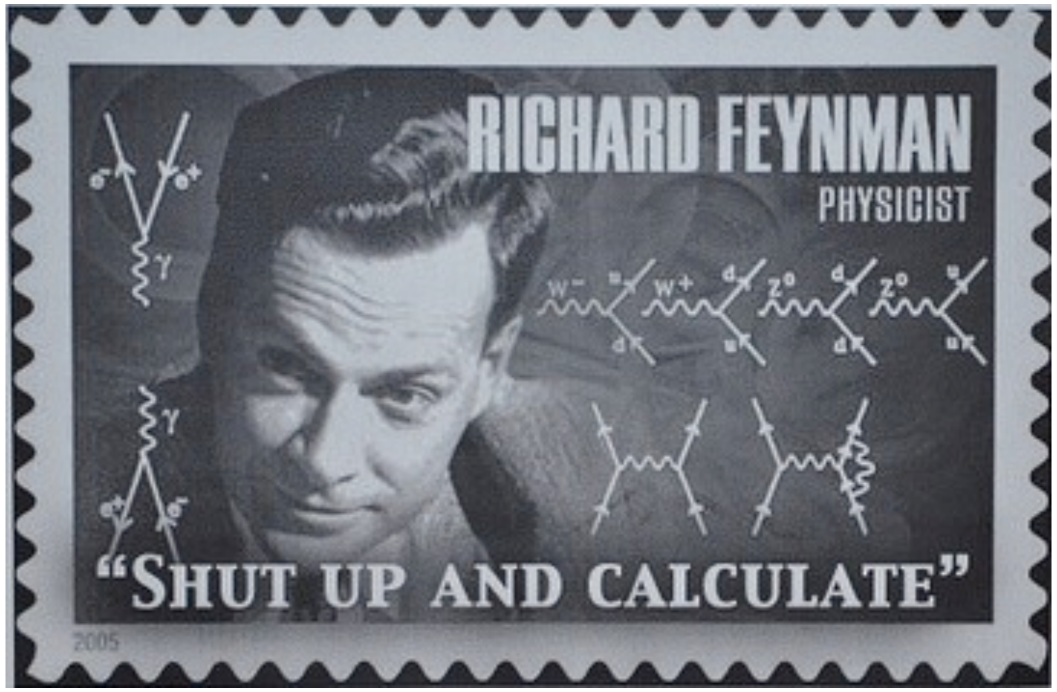
Notice that the same questions / concerns apply to $W$ helicity fractions. The $W$ bosons are not produced on shell and their spin is not measured.

## Does it make sense?

Of course, one can always calculate *C* pretending that the top spin is measured. As if tops were electrons.

just use your Feynman rules for tops as external particles






Then, the problem comes to whether the experimental measurement with decaying tops whose spins are not measured, <u>correspond to</u> this *C*.

# Are we measuring what is written? Why?

The measurement corresponds to the theoretical calculation for on-shell stable tops when you measure $C$ using

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos(\theta_+)\; d\cos(\theta_-)} = \frac{1}{4}\left(1 + \overset{C}{\cancel{A}}\alpha_+\alpha_-\cos(\theta_+)\cos(\theta_-)\right)\,, \tag{2}$$

because the off-diagonal matrix elements [interference $A_\lambda A^*_{\lambda'}$] do not contribute to this distribution

the off-diagonal density matrix terms cancel when integrating over lepton azimuthal angles [slide 7]

and for <u>this observable</u> measured with <u>this distribution</u> the production $\times$ decay interpretation of tops with definite spin is correct.

[sighs of relief here]

# Moral from all this

- One has to be especially careful when studying spin.
- Being overconfident is dangerous. Spins of particles can be considered *classically* only under certain conditions.
- For example, they cannot when computing $A^l_{FB}$. JAAS '14
- The language used may hide all this for brevity, but one should be aware of all the caveats. Especially when attempting to do something new.

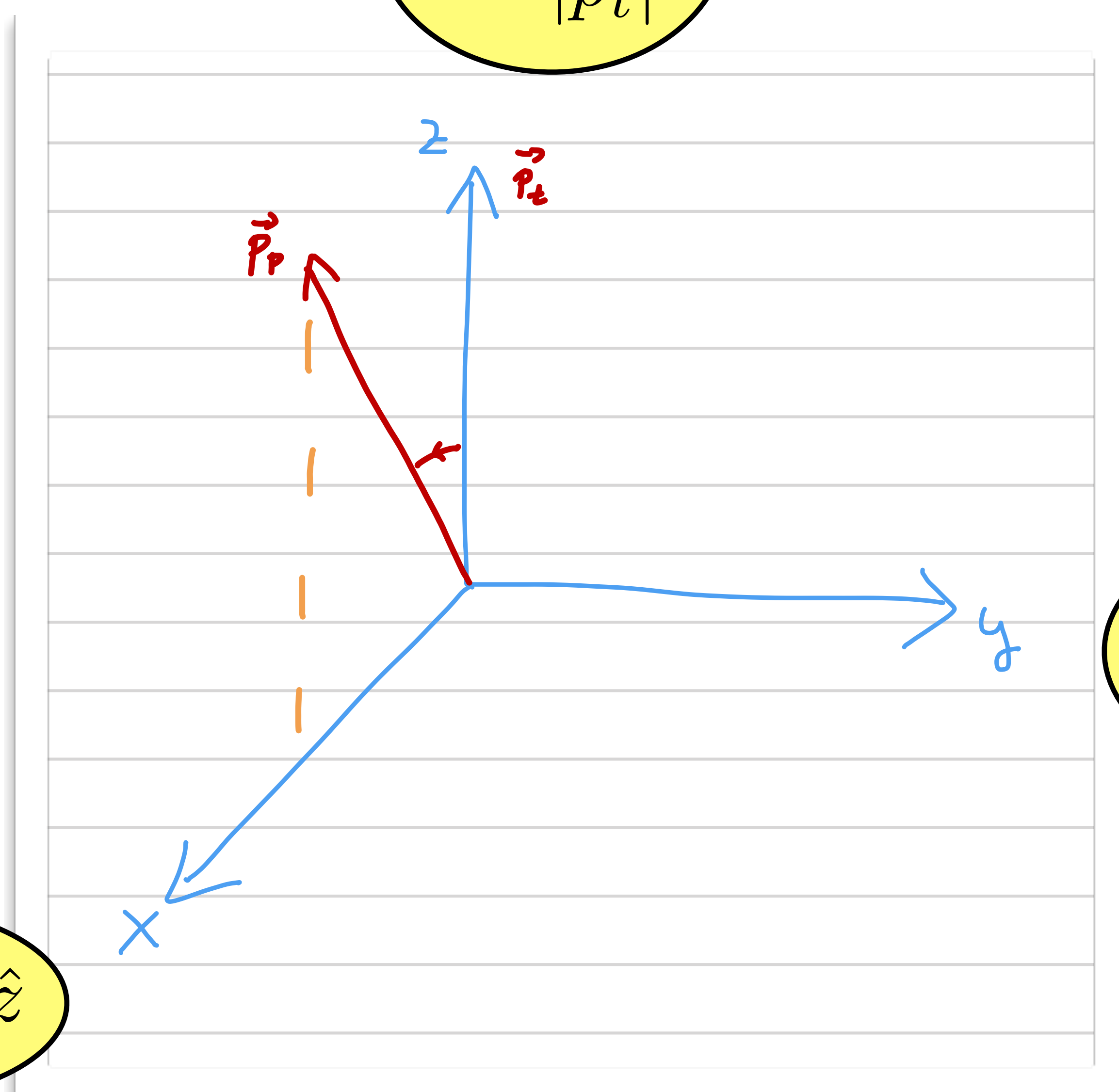
once this is remarked we can proceed with

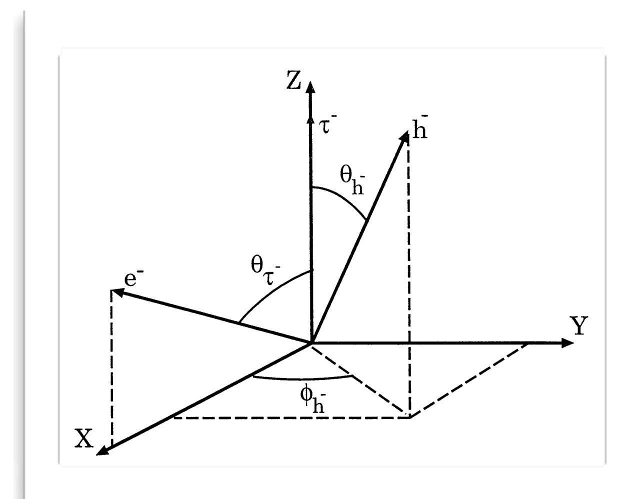
"missing top properties"

# Top pair production at Tevatron

The *x* direction can be taken in the plane spanned by the top quark momentum and the proton, in CM frame. The *y* direction is perpendicular to that plane.

$$\hat{z} = \frac{\vec{p}_t}{|\vec{p}_t|}$$



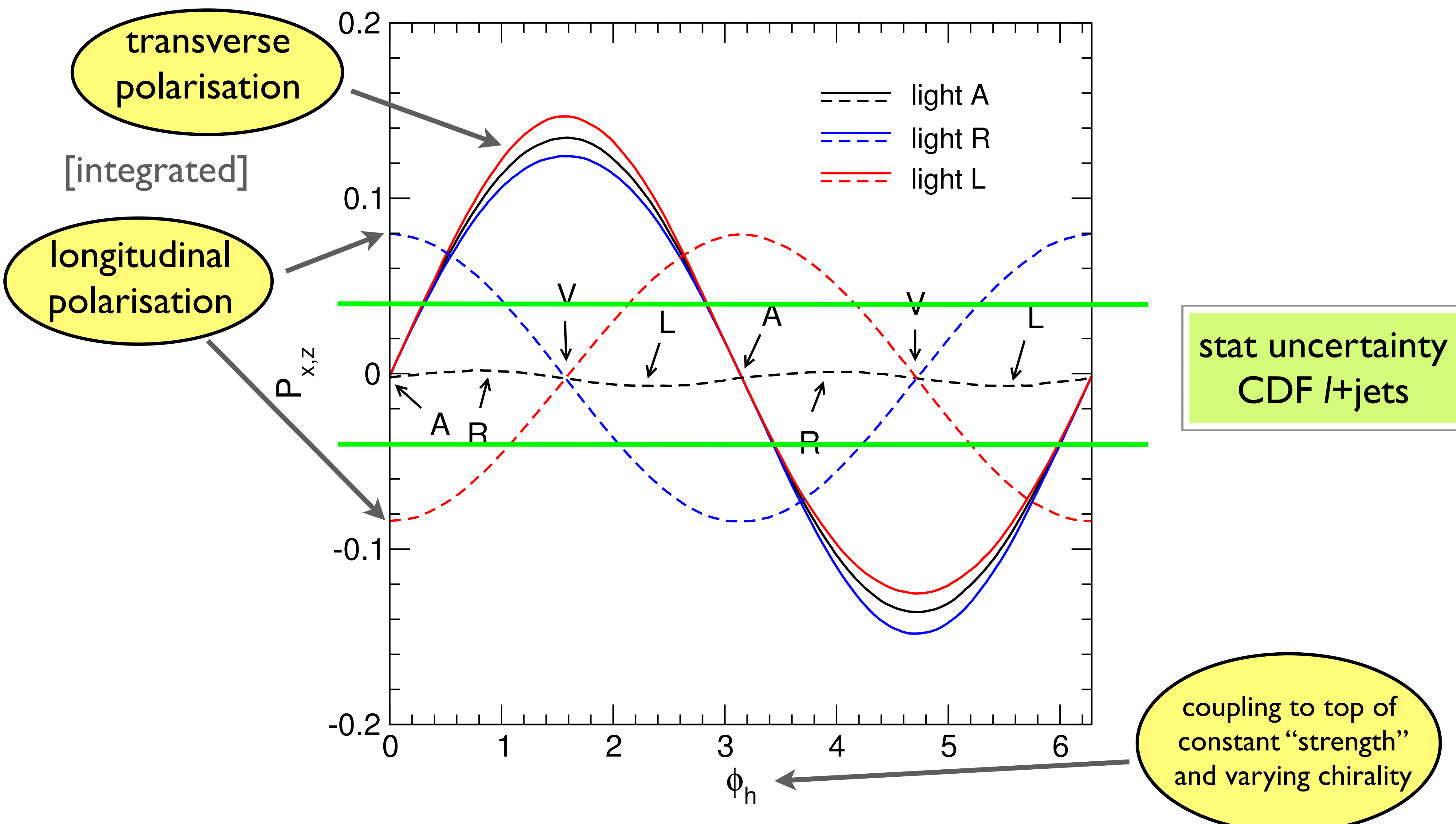




$$\hat{y} = \frac{\vec{p}_t \times \vec{p}_p}{|\vec{p}_t \times \vec{p}_p|}$$

$$\hat{x} = \hat{y} \times \hat{z}$$

The transverse and normal polarisations provide independent probes for new physics.

Example: $P_x$ and $P_z$ for new colour octet $M$ = 250 GeV with *reasonable* couplings to generate a FB asymmetry at Tevatron. JAAS '14



A normal polarisation $P_y$ requires some nontrivial complex phase in the amplitude.

$$s_t \cdot p_t = 0$$

always!

$$\left.\begin{aligned} s_t \cdot p_p &= 0 \\ s_t \cdot p_{\bar{t}} &= 0 \end{aligned}\right\}$$

$s_t$ normal to production plane

$\mathrm{tr}\, \gamma^5 \gamma_\mu \gamma_\nu \gamma_\rho \gamma_\sigma$

only $i\varepsilon_{\mu\nu\rho\sigma} s_t^\mu p_t^\nu p_p^\rho p_{\bar{t}}^\sigma$ terms nonzero in $|\mathcal{M}|^2$ [V−A interference]

and Re [*i*ε … ] = 0 unless there are nontrivial complex phases in the amplitude [interference]:

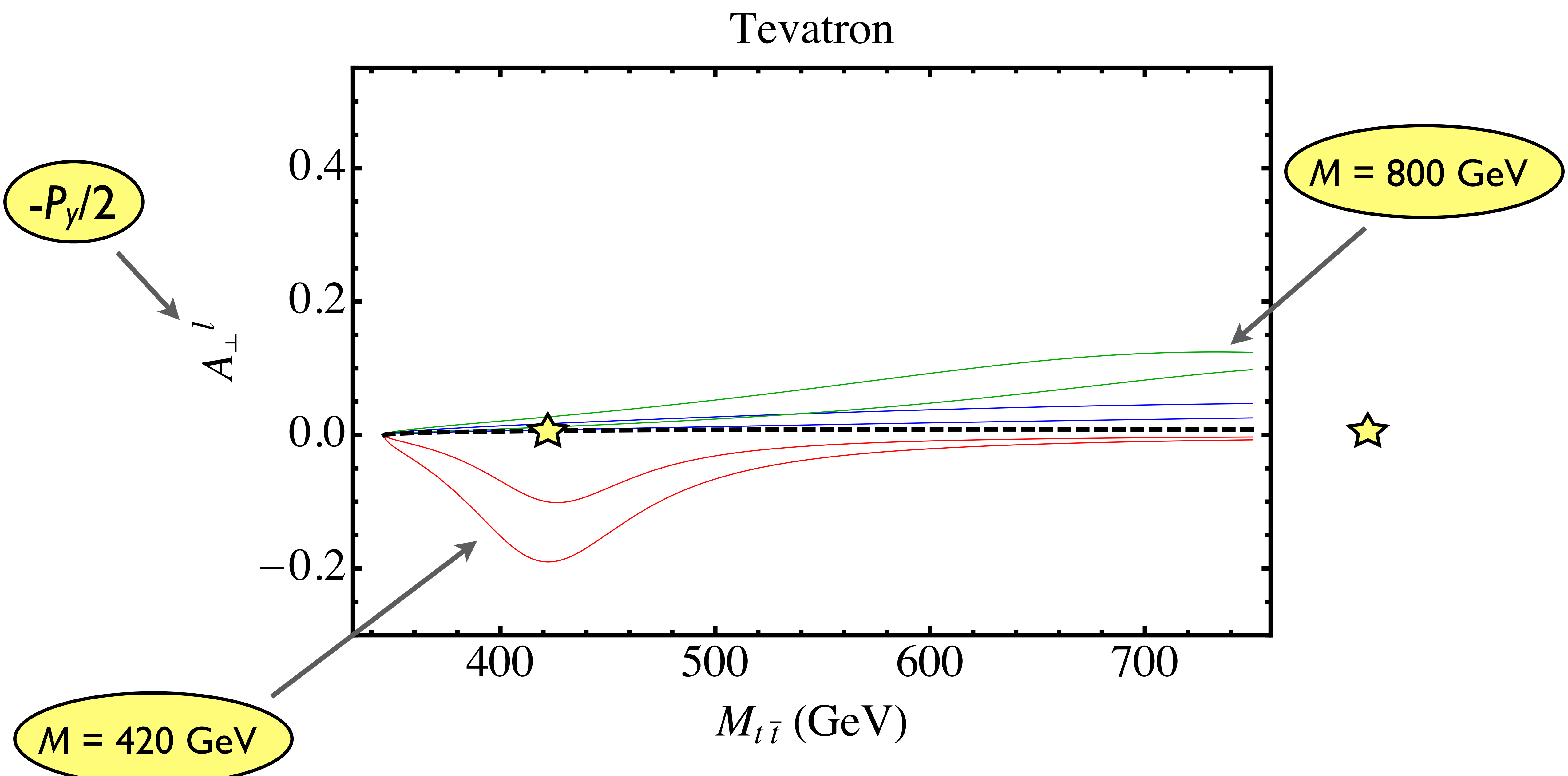
- complex anomalous couplings $V_i V^*_j$ JAAS & Santos '14 JAAS & Bernabéu '10
- large particle widths Baumgart & Tweedie '13

not my business today

# Example: wide colour octet $M$ = 420 (800) GeV and *reasonable* couplings

Baumgart & Tweedie '13

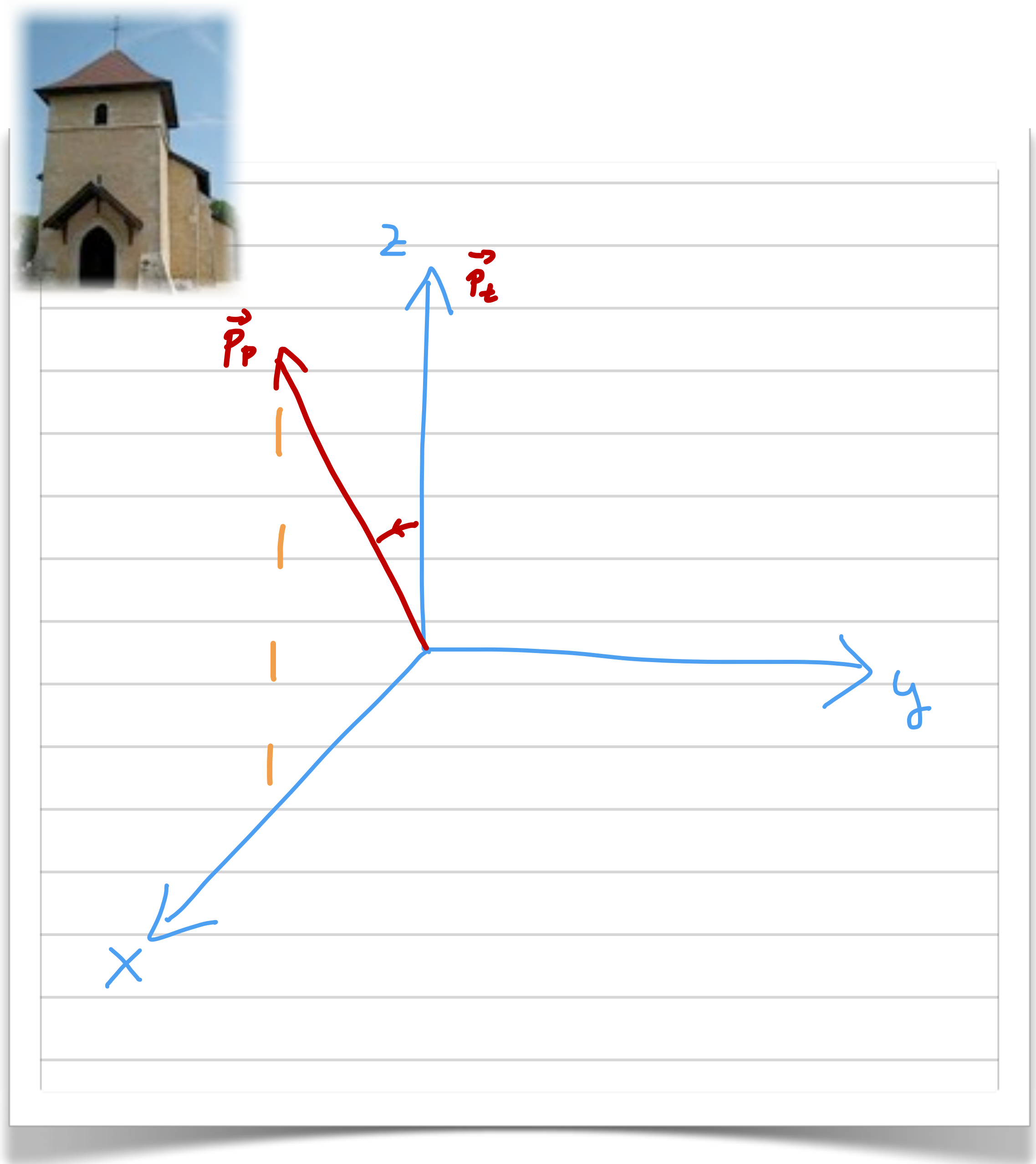
masses chosen to have $P_y$ from width



# Top pair production at LHC

At the LHC we have two protons, we need to choose between them to build our reference system.

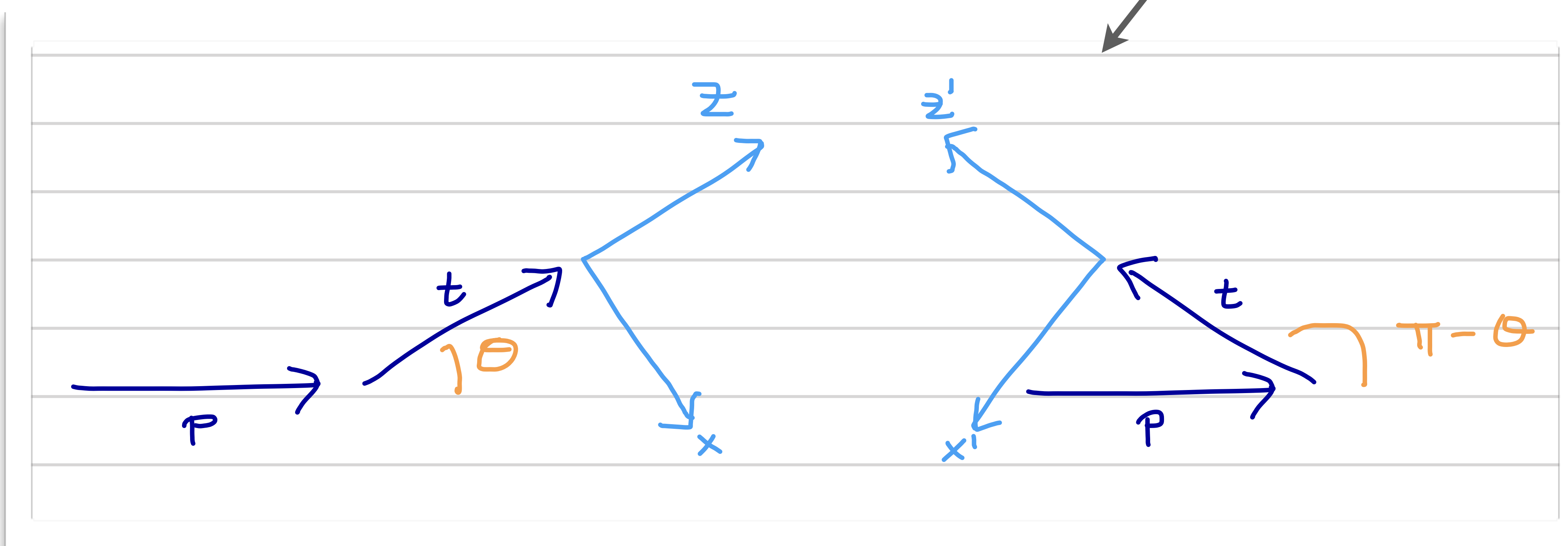
Let us, for example, choose the proton in direction Saint-Genis.



Since the interactions mediating $q\bar{q} \to t\bar{t}$ do not really care where Saint-Genis is, we have [differentially]

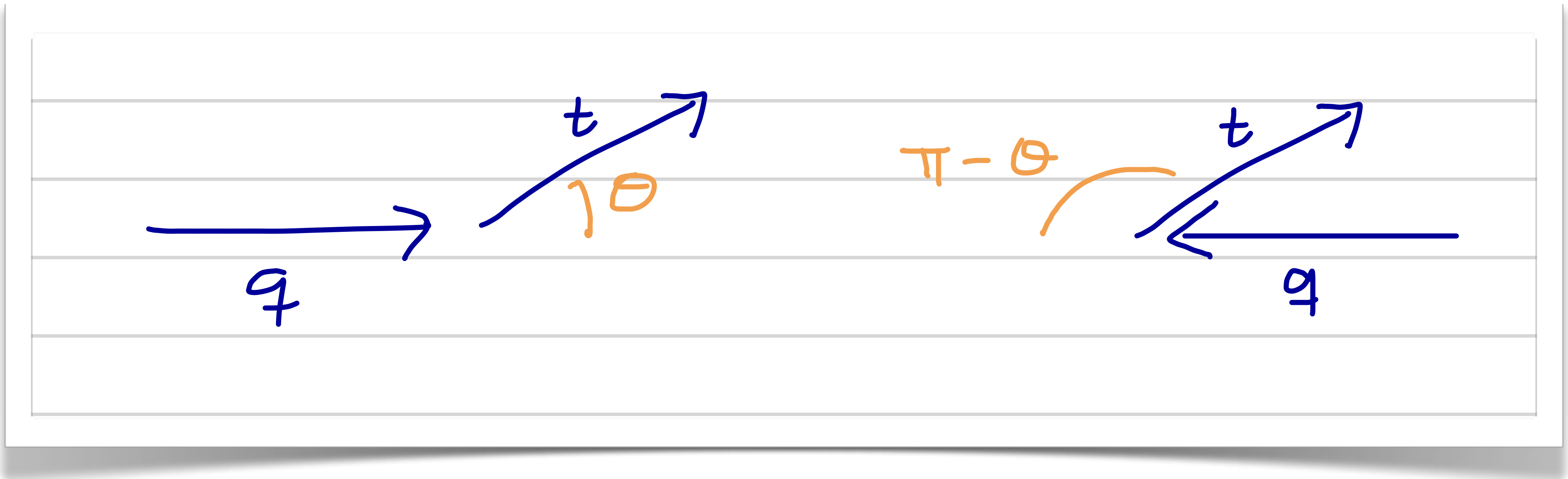
$$
\begin{aligned}
P_x(\theta) &= -P_{x'}(\pi - \theta) \\
P_y(\theta) &= -P_{y'}(\pi - \theta) \\
P_z(\theta) &= P_{z'}(\pi - \theta)
\end{aligned}
$$

note that "longitudinal" and "transverse" depend on θ!




so that $P_x$ and $P_y$ vanish after integration over θ.

This is of course because the quark (antiquark) can come from either proton with equal probability.




Good exercise for students: derive the relations between $P(\theta)$ and $P(\pi-\theta)$ using this fact and the symmetries of the problem.

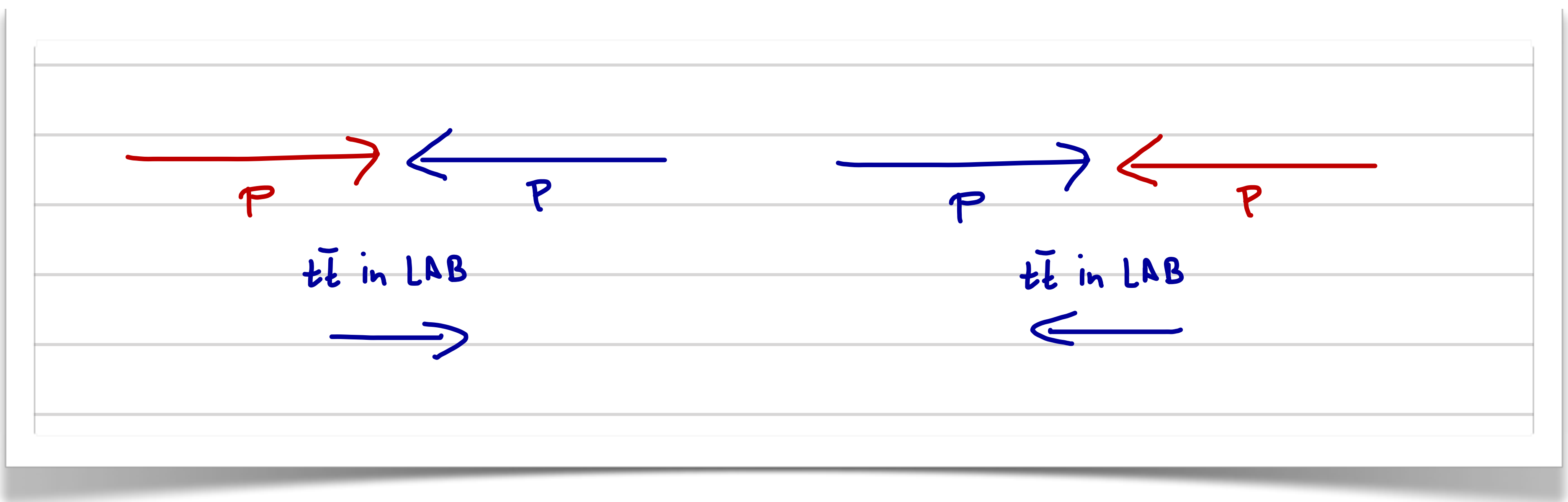
Possible solutions to yield non-zero $P_x$ and $P_y$ :

- Include sign(cos θ) in the definition of observables. In other words: integral in *forward* − integral in *backward*

Bernreuther, Brandenburg & Uwer '95 … Bernreuther & SI '13

- Select among protons based on the momentum of the top pair in the LAB frame [try to guess the quark direction]

Baumgart & Tweedie '13; JAAS '14



# From Tevatron to LHC

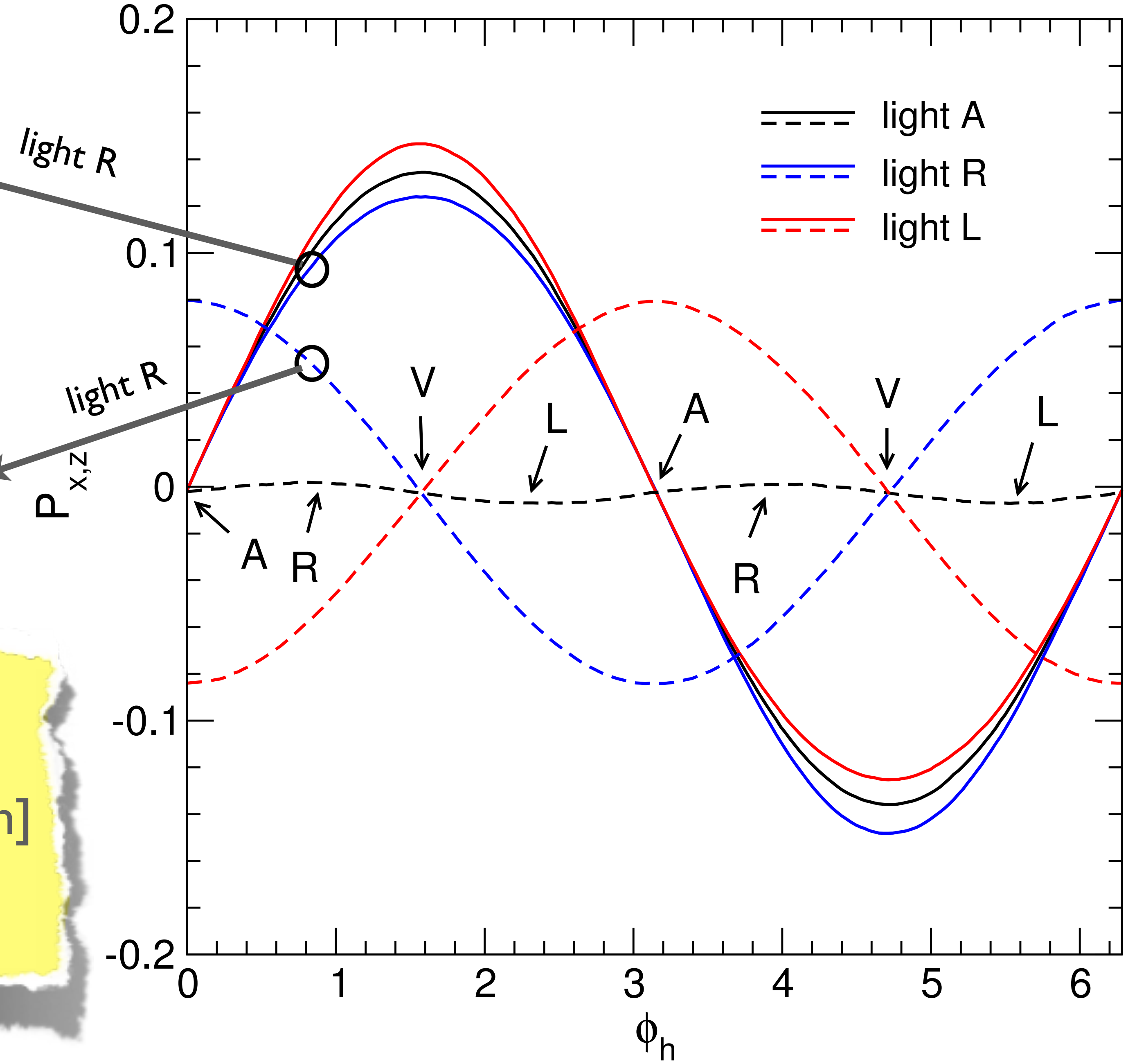
LHC 8

[1] $P_x = 0.0021$
[2] $P_x = 0.0106$ [0.0186]
[3] $P_x = 0.0212$

LHC 8

$P_z = 0.0126$

Tevatron



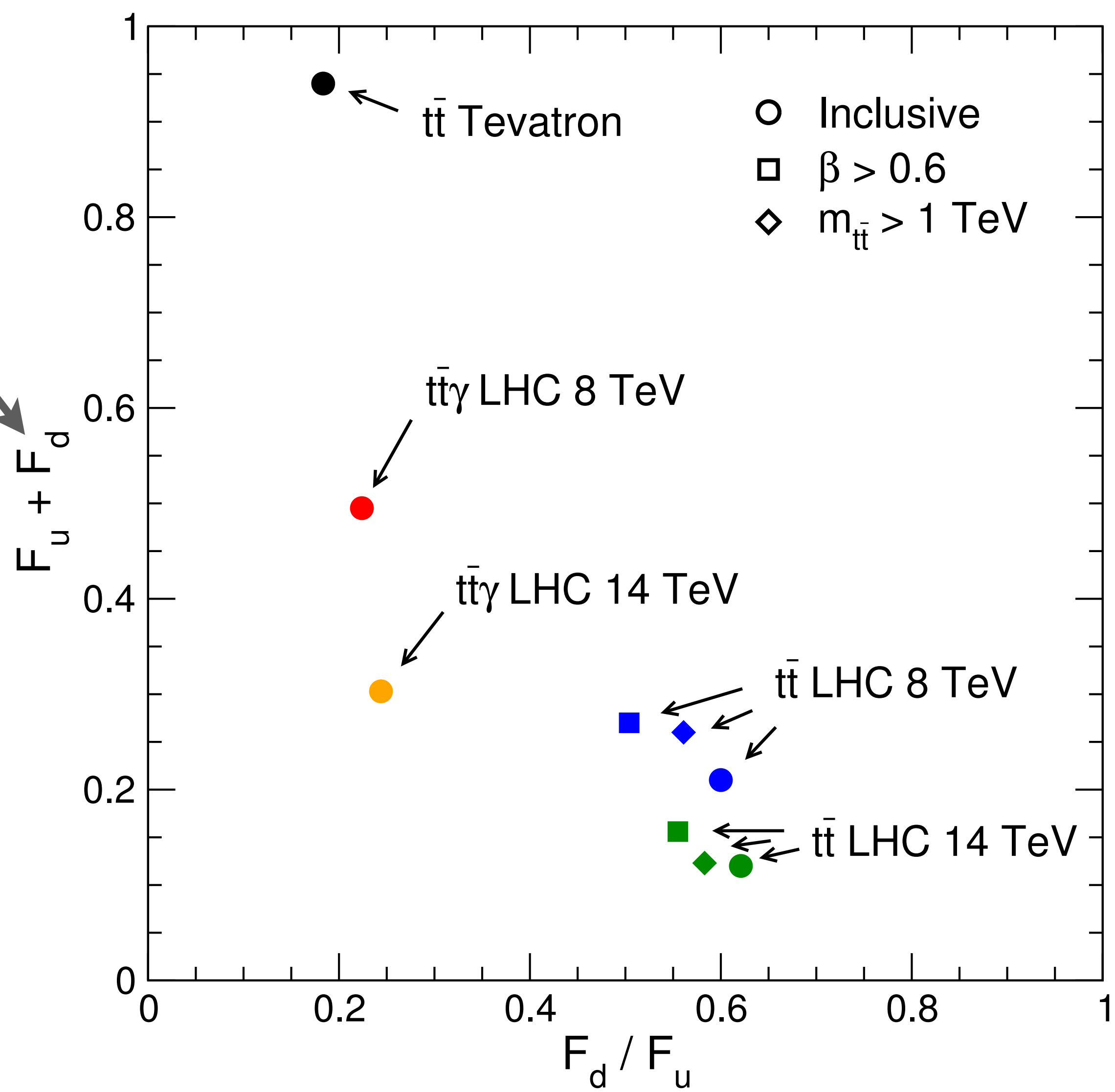

[1] include sign(cos θ)
[2] select proton by $p_z$ [true proton]
[3] select proton by $p_z$ and $\beta > 0.6$

Main penalty: large gg fraction

# Photon handle for polarisation?

Already proposed for charge asymmetry $A_C$ JAAS et al. '14

total fraction of qq




relative importance of uu vs dd

$t\bar{t}$ 7 TeV

$\Delta P_z$ (sys) = 0.037

$t\bar{t}\gamma$ 8 TeV

$\Delta P_z$ (stat) = 0.09 same sensitivity

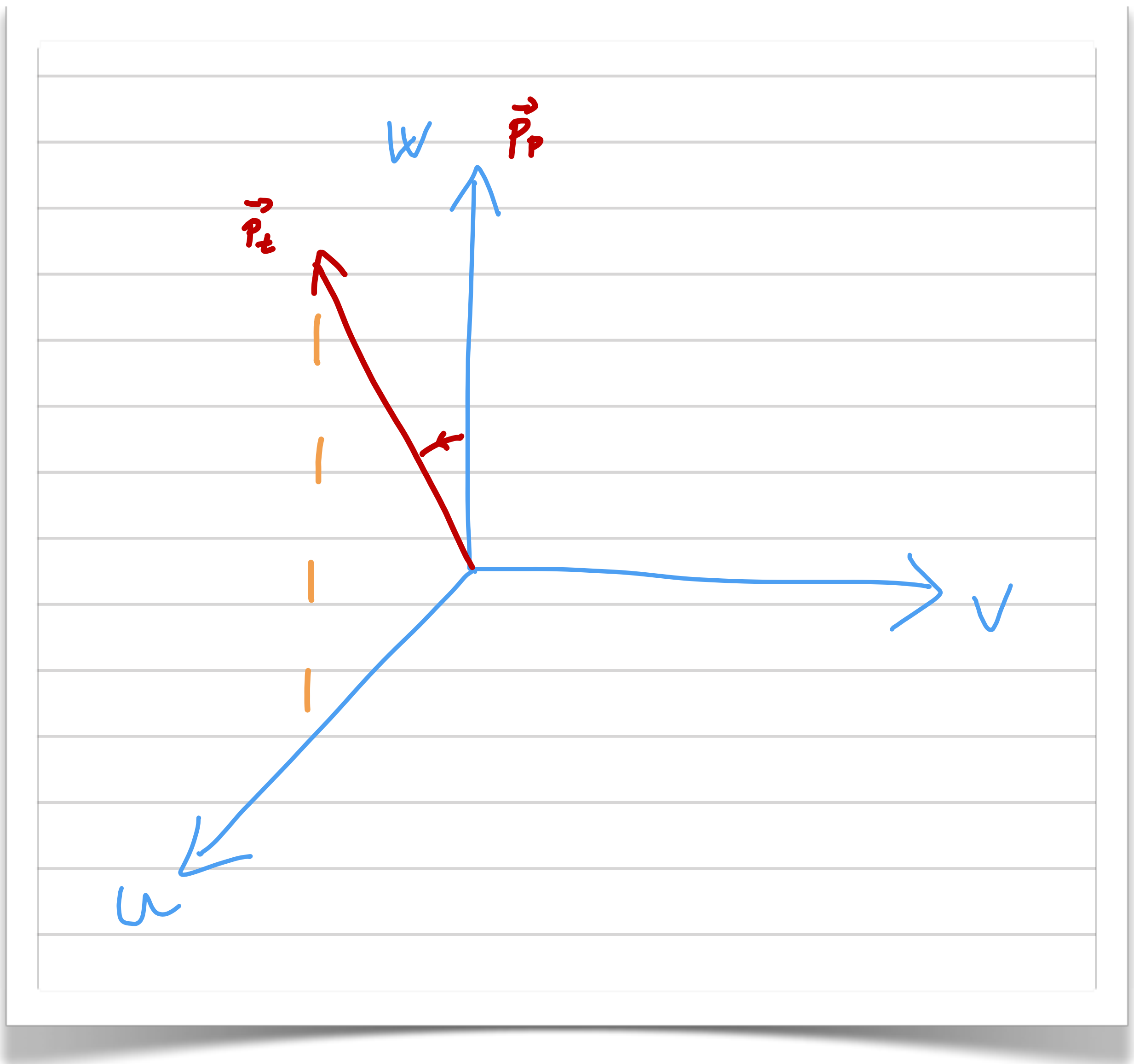
$t\bar{t}\gamma$ 14 TeV 100 $\mathrm{fb}^{-1}$

$\Delta P_z$ (stat) = 0.02 1.5 × sensitivity

# Azimuthal distributions

Godbole et al. '10

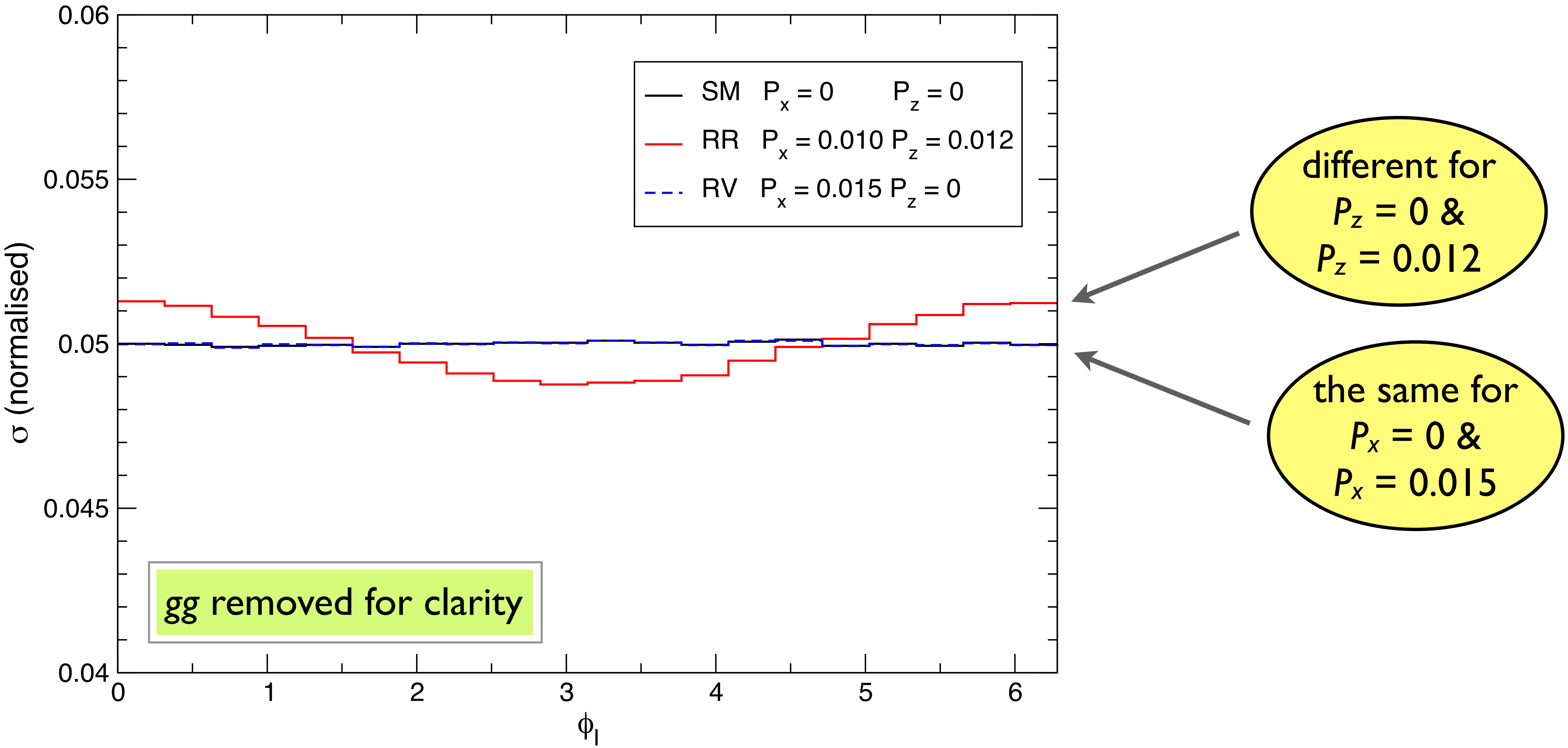
A different reference system [call it ($u$,$v$,$w$) ] is chosen by with the $w$ axis in the direction of one of the protons [fixed].




The azimuthal distribution of the charged lepton in this reference system depends on $P_u$ and $P_v$.

What about $P_x$, $P_y$, $P_z$?

# Azimuthal distributions for the colour octet benchmark



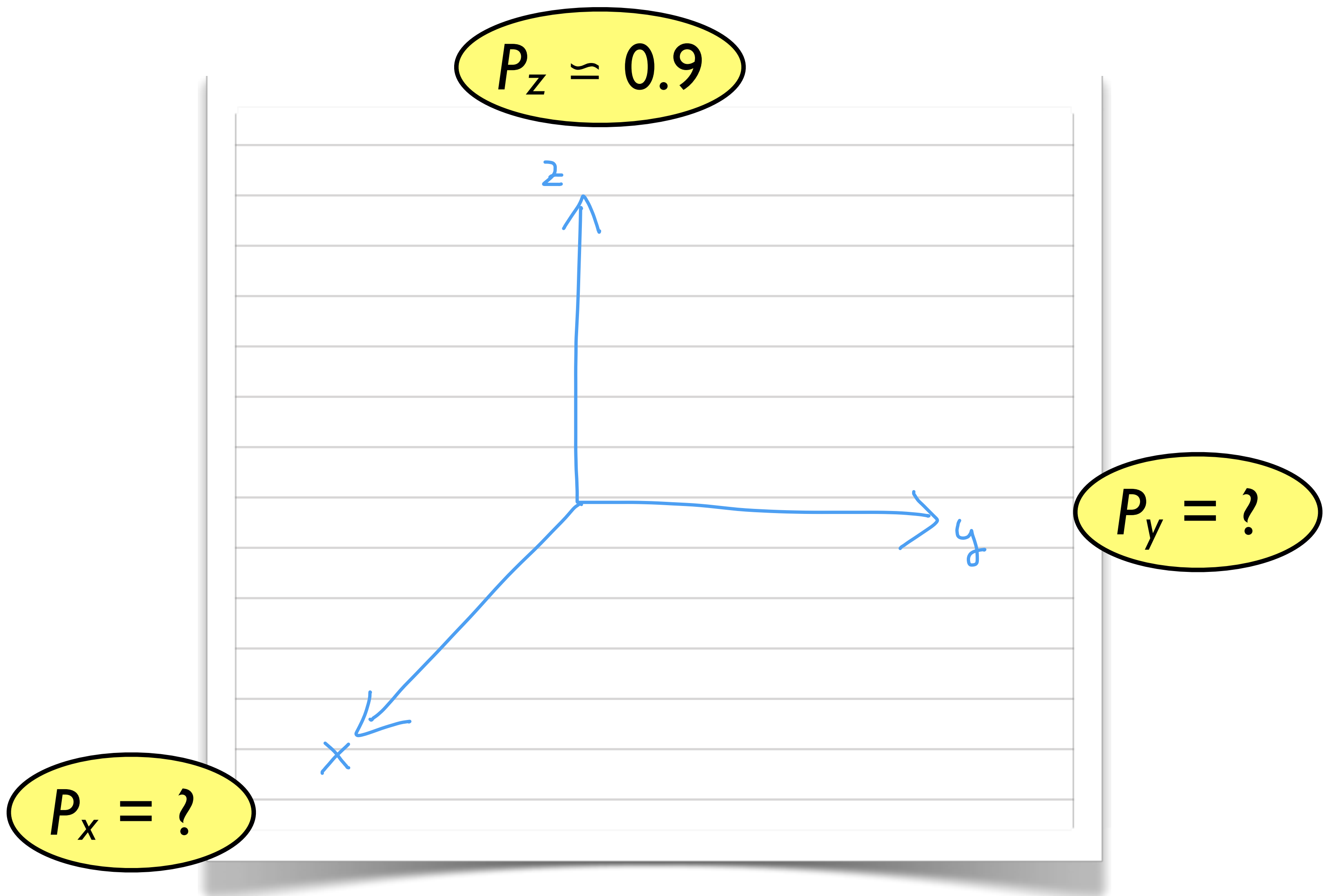

these azimuthal distributions, sensitive to $P_u$ and $P_v$, are in fact only sensitive to $P_z$ and <u>not</u> to $P_x$ nor $P_y$.

[this was clear from the beginning since we don't distinguish protons]

# Single top production at LHC

Since long, we know that in the *t*-channel process the tops have a large polarisation in the spector quark direction. Mahlon & Parke '00

But what about other directions?

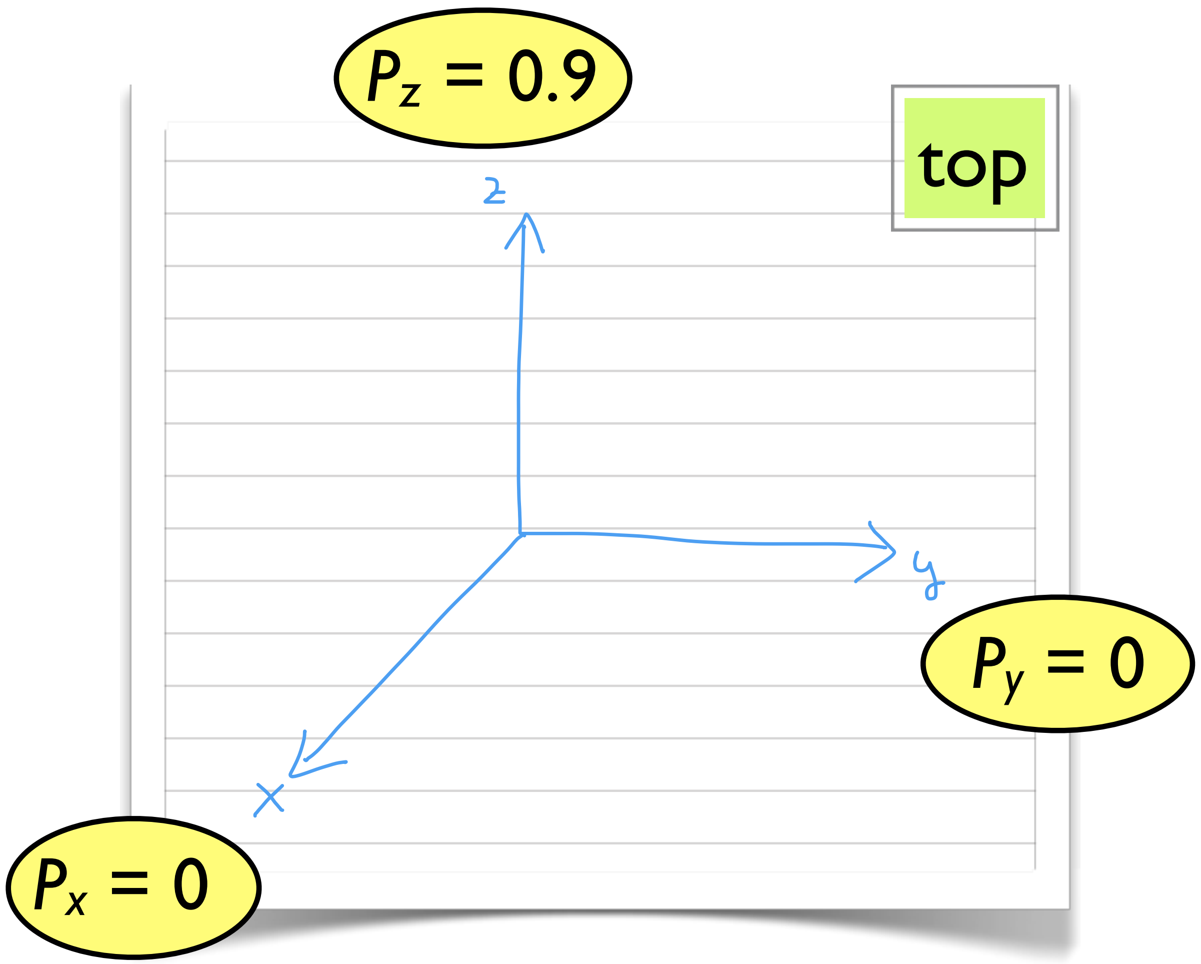


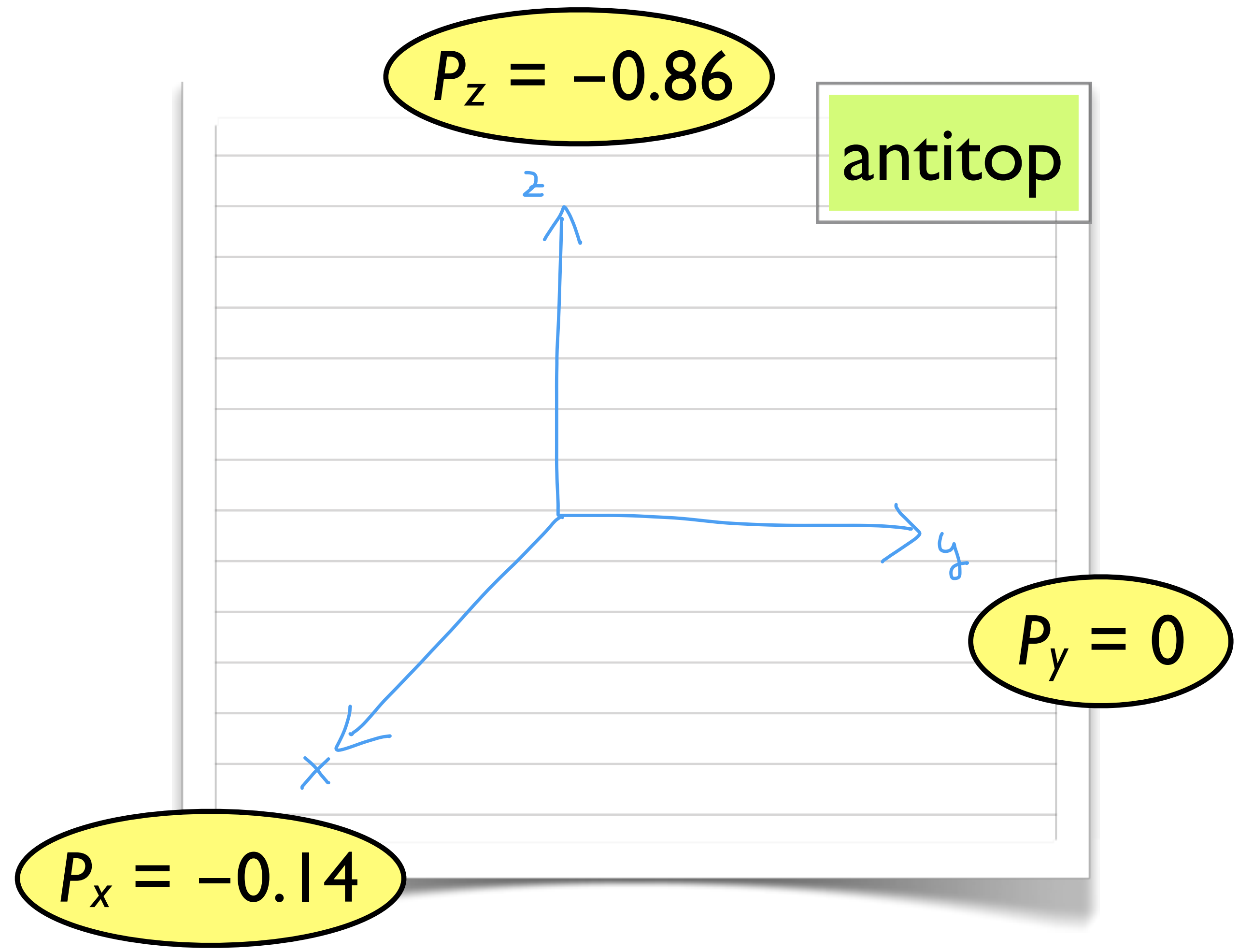

because tr[$\rho^2$]≤1

Of course, $P_x$ and $P_y$ cannot be very large since $P_x^2 + P_y^2 + P_z^2 \leq 1$

# Transverse and normal polarisation in single top $t$-channel

JAAS & Santos '14

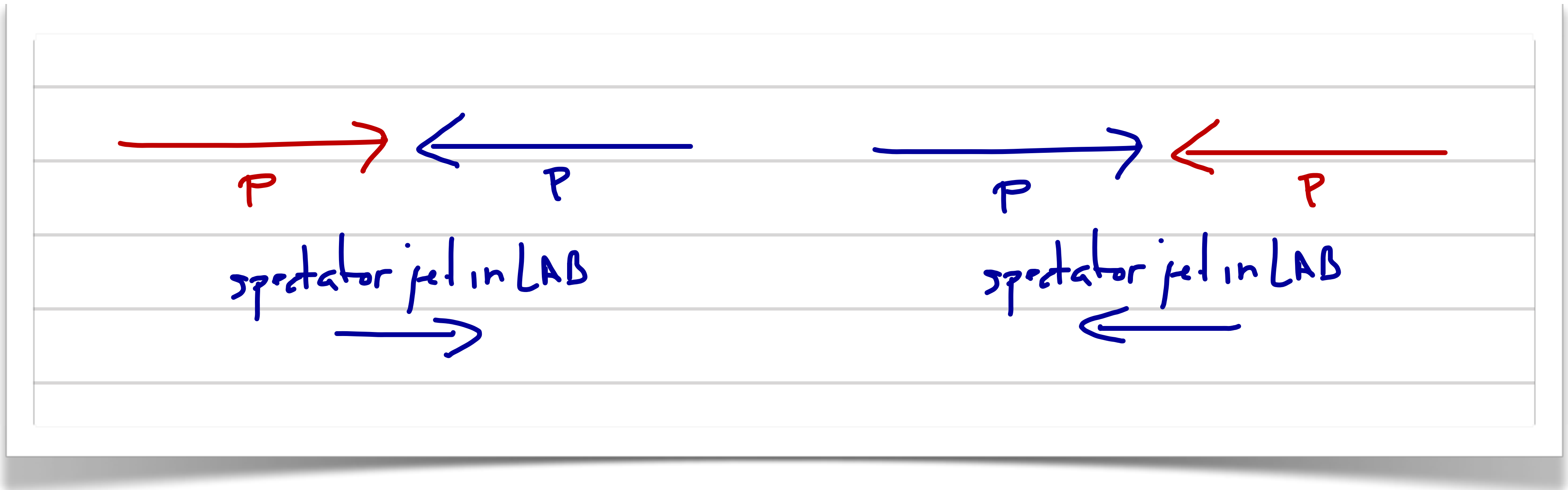



If the polarisation is so small, why should it be interesting? Because this can easily change with anomalous $Wtb$ couplings!

again, not my talk!

And how to choose among the two protons? Obvious: follow the jet.




Correct id of the initial quark [parton level]

95% for $t$

90% for $\bar{t}$

# Unconclusions

- Oh! But the polarisations you showed are so small…

  The benchmark points were chosen just for illustration. $P_x$ can be as large as the $P_z$ that you have measured. It can be increased with cuts, etc.

- Maybe the experimental systematics on these observables are large…

  These observables are not radically different from the ones already measured, I expect similar uncertainties in the measurement.

- But these predictions change at NLO…

- Does the polarisation depend a lot on the PDFs?

- …

# Conclusions

- We have been discussing new observables that might be measured

  ⇨ at least 4 interesting theses, 2 ATLAS + 2 CMS

- They provide independent unexplored information on top production

  ⇨ go for it now to have the measurement first!

- And this is just the beginning, because we have considered only polarisation, not correlations. Baumgart & Tweedie '11

- I'm around and happy to discuss physics — also about the asymmetry.

# Extra Slides

## Why is production $\times$ decay correct? More details, please.

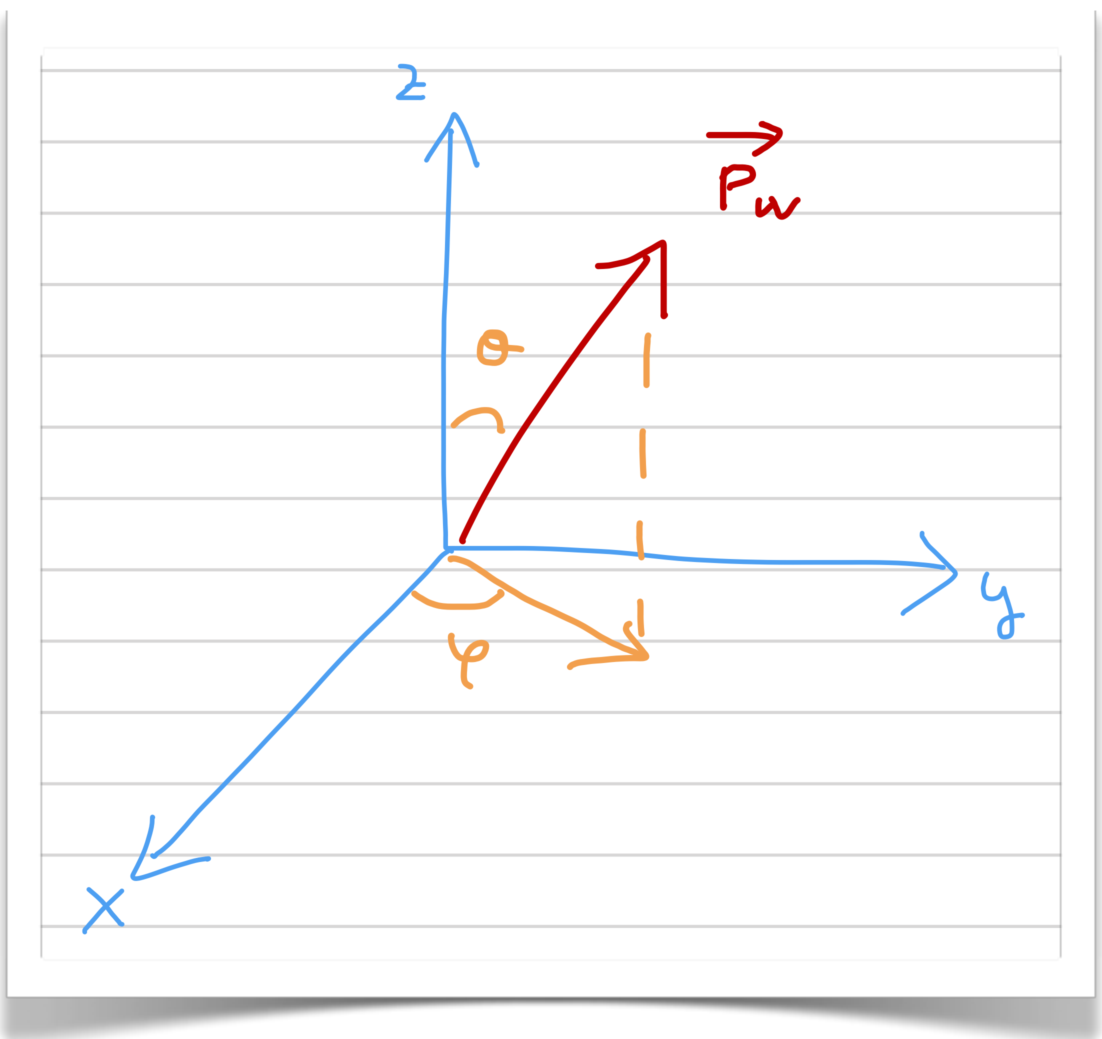
Let us describe the decay of an ensemble of top quarks in a $S_z$ eigenstate using the helicity formalism. Jacob & Wick '59

1/2 1 1/2

$$t \to W^+ b$$

$$A_{M\lambda_1\lambda_2} = a_{\lambda_1\lambda_2} D^{\frac{1}{2}*}_{M\Lambda}(\phi, \theta, 0)$$

$$D^j_{m'm}(\alpha, \beta, \gamma) \equiv \langle jm' | \, e^{-i\alpha J_z} e^{-i\beta J_y} e^{-i\gamma J_z} \, | jm \rangle$$




$M$ = top $S_z$ eigenvalue

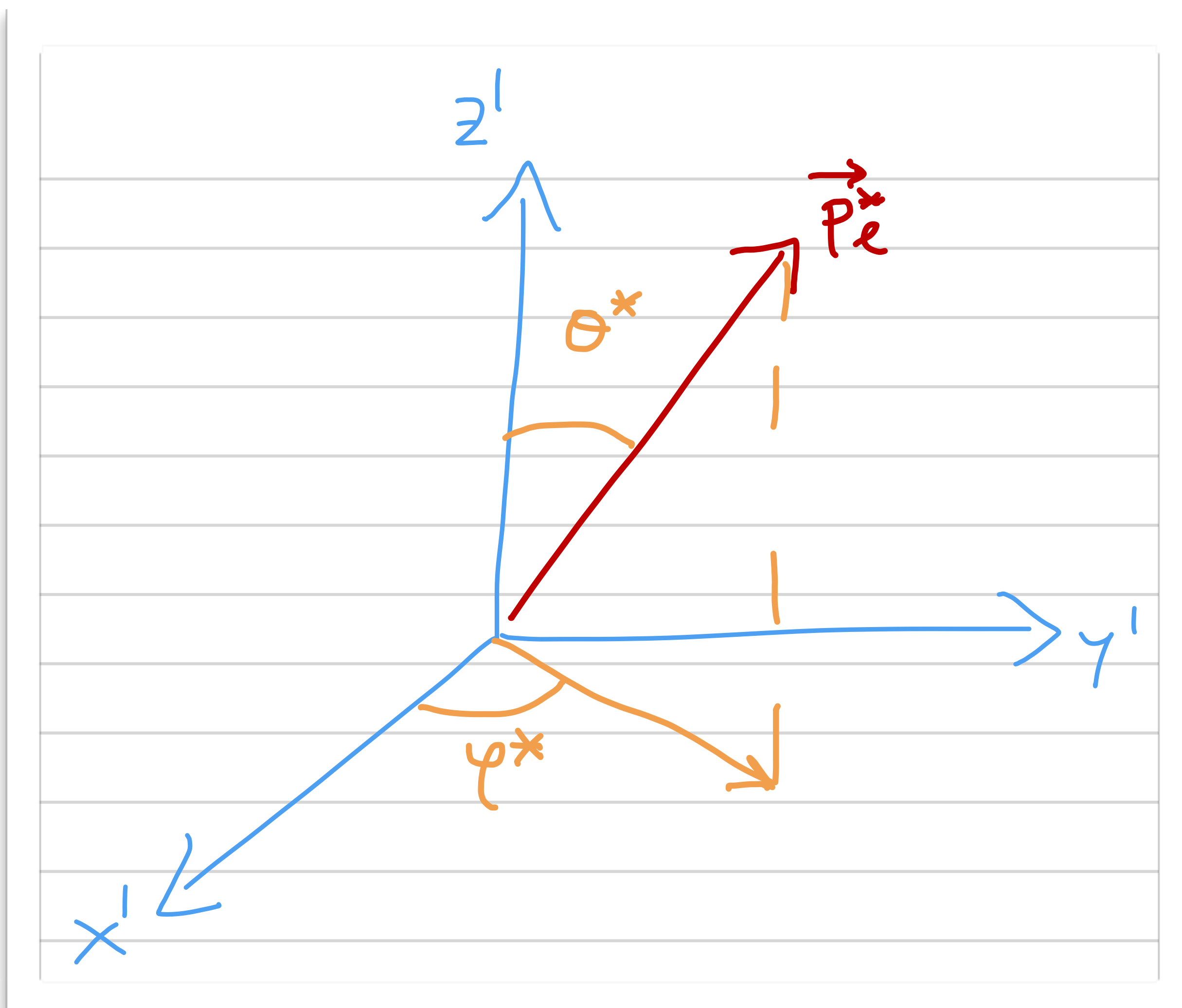
$\lambda_1$ = $W$ helicity $\lambda_2$ = $b$ helicity

$\Lambda = \lambda_1 - \lambda_2$

$\theta$, $\varphi$: spherical coordinates of the $W$ 3-momentum in this reference system. The $b$ quark moves in the opposite direction.

The [leptonic] decay of the $W$ can be described in a similar fashion introducing a $(x', y', z')$ coordinate system in the $W$ rest frame

$$\overset{1}{W^+} \to \overset{1/2}{\ell^+}\,\overset{1/2}{\nu} \quad \Rightarrow \quad A_{m\lambda_3\lambda_4} = b_{\lambda_3\lambda_4} D^{1*}_{m\lambda}(\phi^*, \theta^*, 0)$$




$m$ = W $S_z$ eigenvalue

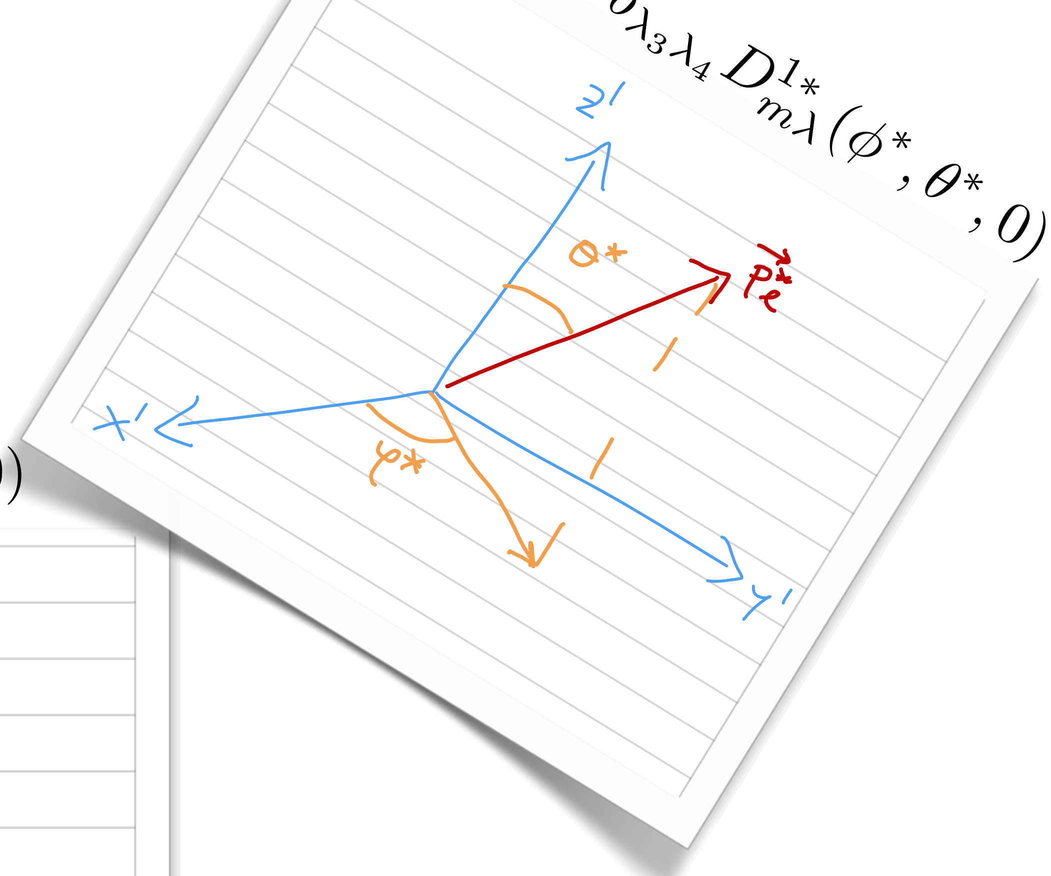
$\lambda_3$ = $l^+$ helicity  $\lambda_4$ = ν helicity

$\lambda = \lambda_3 - \lambda_4$

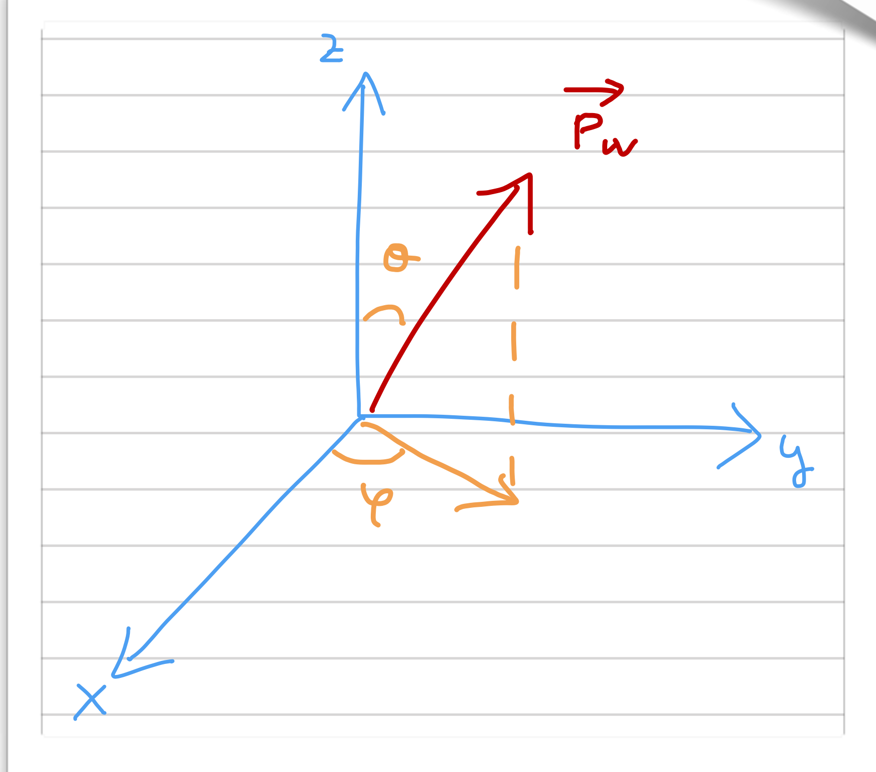
$\theta^*, \varphi^*$: spherical coordinates of the $l^+$ 3-momentum in this reference system.

Now, the decay chain can be connected by choosing $z'$ precisely in the direction of $\vec{p}_W$ , so that $m = \lambda_1$

$A_{m\lambda_3\lambda_4} = b_{\lambda_3\lambda_4} D^{1*}_{m\lambda}(\phi^*, \theta^*, 0)$




$A_{M\lambda_1\lambda_2} = a_{\lambda_1\lambda_2} D^{\frac{1}{2}*}_{M\Lambda}(\phi, \theta, 0)$




narrow width approximation for *W*

$$A_{M\lambda_2\lambda_3\lambda_4} = \sum_{\lambda_1} a_{\lambda_1\lambda_2} b_{\lambda_3\lambda_4} D^{\frac{1}{2}*}_{M\Lambda}(\phi, \theta, 0) D^{1*}_{\lambda_1\lambda}(\phi^*, \theta^*, 0)$$

Then, the differential decay width looks as terrible as

global phase space factor

*b* helicities summed at cross section level

$$\frac{d\Gamma}{d\phi\, d\cos\theta\, d\phi^* d\cos\theta^*} = C \sum_{MM'\lambda_1\lambda_1'\lambda_2} \rho_{MM'} a_{\lambda_1\lambda_2} a^*_{\lambda_1'\lambda_2} |b_{\lambda_3\lambda_4}|^2$$

$$\times D^{\frac{1}{2}*}_{M\lambda}(\phi,\theta,0) D^{\frac{1}{2}}_{M'\lambda'}(\phi,\theta,0)$$

$$\times D^{1*}_{\lambda_1\lambda}(\phi^*,\theta^*,0) D^{1}_{\lambda_1'\lambda}(\phi^*,\theta^*,0)$$

[we have assumed that W decay is SM-like to simplify the expression]

Notice we have not forgotten our quantum-mechanics course:

- we are summing over top $S_z$ [$M$,$M'$] at the amplitude level
- we are summing over possible helicities of intermediate $W$ [$\lambda_l$,$\lambda_l'$] at the amplitude level

The integration over azimuthal angles is easy since $J_z|jm\rangle = m|jm\rangle$

$$D^j_{m'm}(\alpha,\beta,\gamma) = \langle jm'|e^{-i\alpha J_z}e^{-i\beta J_y}e^{-i\gamma J_z}|jm\rangle = e^{-i\alpha m'}e^{-i\gamma m}\langle jm'|e^{-i\beta J_y}|jm\rangle$$
$$\equiv e^{-i\alpha m'}e^{-i\gamma m}d^j_{m'm}(\beta)$$

$$\Rightarrow \quad \int d\phi = 2\pi\delta_{MM'} \qquad \int d\phi^* = 2\pi\delta_{\lambda_1\lambda_1'}$$

kills off-diagonal density matrix contributions

kills interference of different *W* polarisations

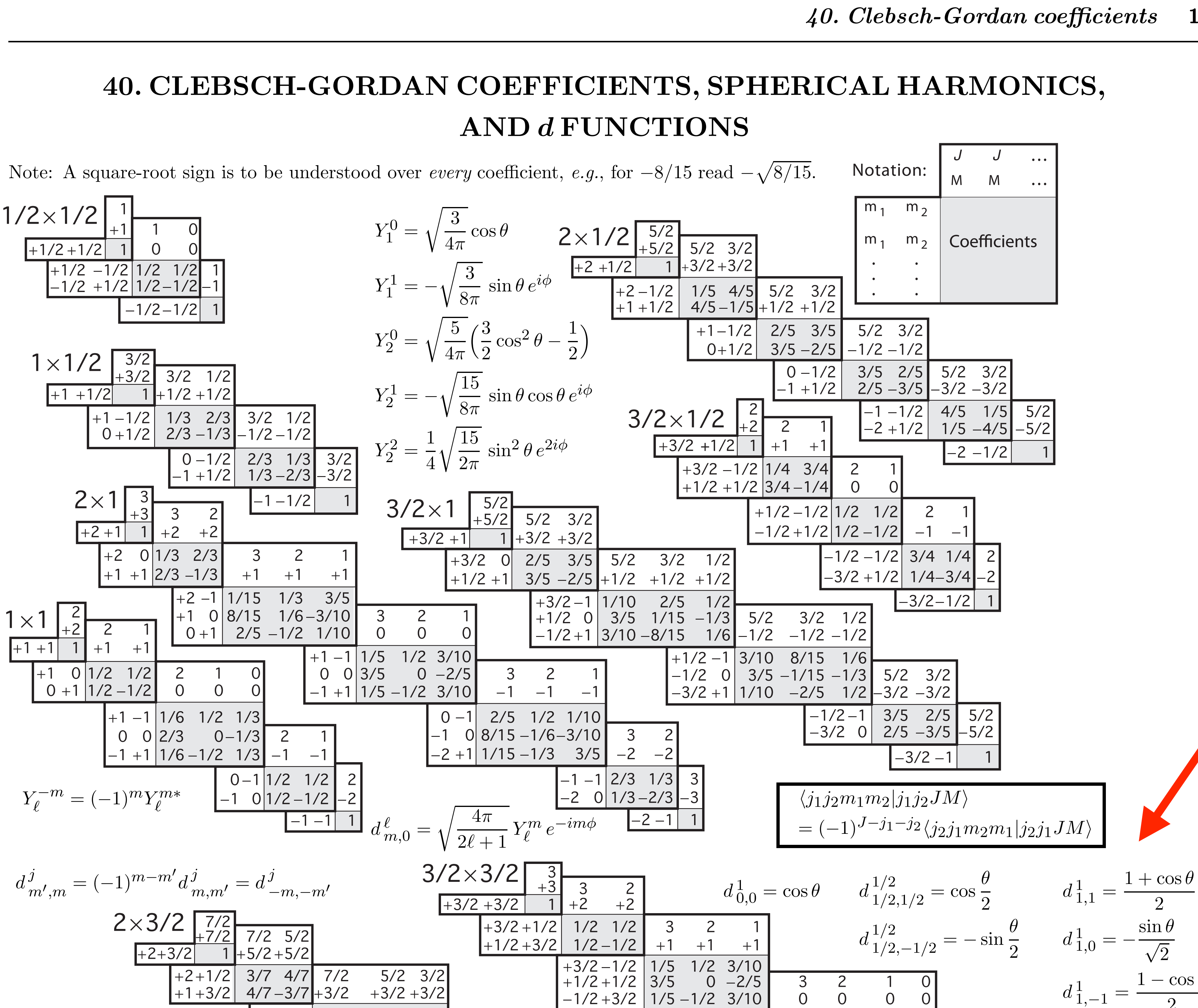
By integrating over $\phi, \phi^*$ we have erased all quantum interference effects!

And the result is

$$\frac{d\Gamma}{d\cos\theta\, d\cos\theta^*} = 4\pi^2 C|b_{\lambda_3\lambda_4}|^2 \sum_{M\lambda_1\lambda_2} \rho_{MM}|a_{\lambda_1\lambda_2}|^2 \left[d^{\frac{1}{2}}_{M\lambda}(\theta)d^1_{\lambda_1\lambda}(\theta^*)\right]^2$$

In the case of *C* it is more complicated but essentially equivalent: integrating over $\varphi_l$ eliminates dependence on $P_x$ and $P_y$

# Nice! Where can I get these *d*'s?





## 40. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND *d* FUNCTIONS

Note: A square-root sign is to be understood over *every* coefficient, *e.g.*, for $-8/15$ read $-\sqrt{8/15}$.

Notation: $J$ $J$ … / $M$ $M$ … ; $m_1$ $m_2$ / $m_1$ $m_2$ … Coefficients

1/2×1/2 · 1×1/2 · 2×1 · 1×1 · 2×1/2 · 3/2×1/2 · 3/2×1 · 3/2×3/2 · 2×3/2

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}}\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$$

$$Y_2^2 = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\phi}$$

$$Y_\ell^{-m} = (-1)^m Y_\ell^{m*}$$

$$d^\ell_{m,0} = \sqrt{\frac{4\pi}{2\ell+1}}\, Y_\ell^m\, e^{-im\phi}$$

$$\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M\rangle = (-1)^{J-j_1-j_2}\langle j_2 j_1 m_2 m_1 | j_2 j_1 J M\rangle$$

$$d^j_{m',m} = (-1)^{m-m'} d^j_{m,m'} = d^j_{-m,-m'}$$

$$d^1_{0,0} = \cos\theta \qquad d^{1/2}_{1/2,1/2} = \cos\frac{\theta}{2} \qquad d^{1/2}_{1/2,-1/2} = -\sin\frac{\theta}{2}$$

$$d^1_{1,1} = \frac{1+\cos\theta}{2} \qquad d^1_{1,0} = -\frac{\sin\theta}{\sqrt{2}} \qquad d^1_{1,-1} = \frac{1-\cos\theta}{2}$$

here!

# Warning!

Observables involving top decay products in general do depend on the interference. Example: $A^l_{FB}$ at Tevatron [interference missed in Berger et al. '12 '13]

continuous variation of the chirality of the octet coupling to top

A → R → V → L → …

$A^l_{FB}$ vs $A_{FB}$ for a new colour octet