# The Top Mass: Interpretation and Theoretical Uncertainties

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#### **Motivation**





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#### **CMS Preliminary** CMS 2010, dilepton 175.5 ± 4.6 ± 4.6 GeV JHEP 07 (2011) 049, 36 pb<sup>-1</sup> (value ± stat ± syst) CMS 2010, lepton+jets 173.1 ± 2.1 ± 2.6 GeV PAS TOP-10-009, 36 pb<sup>-1</sup> (value ± stat ± syst) CMS 2011, dilepton 172.5 ± 0.4 ± 1.4 GeV EPJC 72 (2012) 2202, 5.0 fb<sup>-1</sup> (value ± stat ± syst) CMS 2011, lepton+jets 173.5 ± 0.4 ± 1.0 GeV JHEP 12 (2012) 105, 5.0 fb<sup>-1</sup> (value ± stat ± syst) CMS 2011, all-hadronic 173.5 ± 0.7 ± 1.2 GeV arXiv:1307.4617, 3.5 fb<sup>-1</sup> (value ± stat ± syst) CMS 2012, lepton+jets 172.0 ± 0.2 ± 0.8 GeV PAS TOP-14-001, 19.7 fb<sup>-1</sup> (value ± stat ± syst) **CMS** combination $172.2 \pm 0.1 \pm 0.7 \text{ GeV}$ (value ± stat ± syst) March 2014 **Tevatron combination** $173.2 \pm 0.6 \pm 0.8 \text{ GeV}$ Phys. Rev. D86 (2012) 092003 (value ± stat ± syst) World combination 2014 $173.3 \pm 0.3 \pm 0.7 \text{ GeV}$ ATLAS, CDF, CMS, D0 (value ± stat ± syst) 165 170 175 180 m, [GeV]



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# Outline

<u>**Part 1:**</u>  $\rightarrow$  Theoretical considerations on  $m_t^{MC}$ 

 $\rightarrow$  "What is the physics MC mass ?"  $\leftarrow$ 

• How  $m_t^{
m MC}$  is related to field theoretic masses.

- **<u>Part 2:</u>**  $\rightarrow$  Method to determine  $m_t^{MC}$ 
  - Variable Flavor Number Scheme for final state jets.
     Full massive event shape distribution
  - First encouraging preliminary results



#### **QCD** Parameters

**QCD Lagrangian:** 
$$\mathcal{L}_{QCD} = \mathcal{L}_{classic} + \mathcal{L}_{gauge-fix} + \mathcal{L}_{ghost}$$

$$\mathcal{L}_{\text{classic}} = -\frac{1}{4} F^A_{\alpha\beta} F^{\alpha\beta}_A + \sum_{\text{flavors } q} \bar{q}_\alpha (i D - m_q)_{\alpha\beta} q_b$$
$$D^\mu = \partial^\mu + i g T^C A^{\mu C}$$

Formally  $m_{\rm top}$  and  $\alpha_s$  are couplings of the Lagrangian.

$$\begin{array}{ll} m^0_{\mathrm{top}} \,, \ \alpha^0_s & \to \mathrm{bare} \ \mathrm{UV}\mathrm{-divergent} \\ & \to \mathrm{field} \ \mathrm{theoretically} \ \mathrm{unique} \\ & \to \mathrm{pure} \ \mathrm{UV}\mathrm{-object} - \mathrm{NO} \ \mathrm{IR} \ \mathrm{dependence} \end{array}$$

$$m^R_{\mathrm{top}} \,, \ \alpha^R_s & \to \mathrm{renormalized} \ \mathrm{UV}\mathrm{-finite} \end{array}$$

- $\rightarrow$  renormalization scheme dependent
- $\rightarrow$  regularization scheme dependent





# **Strong Coupling**





# Heavy Quark Mass

$$+ \underbrace{\leq} \underbrace{\sum}_{\Sigma} \underbrace{\leq} = p - m^{0} - \Sigma(p, m^{0}, \mu)$$
$$\Sigma(m^{0}, m^{0}, \mu) = m^{0} \left[ \frac{\alpha_{s}}{\pi \epsilon} + \dots \right] + \underbrace{\sum}_{n} \underbrace{(m^{0}, m^{0}, \mu)}_{n}$$
$$\underbrace{MS \text{ scheme:}}_{MS \text{ scheme:}} m^{0} = \overline{m}(\mu) \left[ 1 - \frac{\alpha_{s}}{\pi \epsilon} + \dots \right]$$
$$\stackrel{\text{. Very energetic processes (E>m)}_{n} \underbrace{Total cross sections}_{n} \underbrace{Off-shell massive quarks}_{n} \underbrace{Off-shell massive quarks}$$

→ Separation: self energy corrections ↔ inter quark/gluon interactions for all momenta

ions

→ Has perturbative instabilities due to sensitivity to momenta < 1 GeV ( $\Lambda_{QCD}$ )



## **Heavy Quark Mass**

$$+ \underbrace{\sum \sum \sum }_{\substack{m \in \mathbb{Z}^{n} \\ \sum (m^{0}, m^{0}, \mu) = m^{0} - \sum(p, m^{0}, \mu)}_{\sum(m^{0}, m^{0}, \mu) = m^{0} \left[\frac{\alpha_{s}}{\pi \epsilon} + \dots\right] + \underbrace{\sum \min(m^{0}, m^{0}, \mu)}_{\sum(m^{0}, m^{0}, \mu) = m^{0} = \overline{m}(\mu) \left[1 - \frac{\alpha_{s}}{\pi \epsilon} + \dots\right]}$$

$$\frac{\text{MS scheme:}}{\text{m}^{0} = m^{\text{pole}} \left[1 - \frac{\alpha_{s}}{\pi \epsilon} + \dots\right] - \sum \min(m^{\text{pole}}, m^{\text{pole}}, \mu)$$

$$\frac{\text{MSR scheme:}}{m^{\text{MSR}}(R) = m^{\text{pole}} - \sum \min(R, R, \mu)}$$

$$\text{Jain, AH, Scimemi, Stewart (2008)}$$

 $\rightarrow$  Interpolates between MS and pole mass scheme

$$m_t^{\mathrm{MSR}}(R=0) = m^{\mathrm{pole}}$$

$$m_t^{\text{MSR}}(R = \overline{m}(\overline{m})) = \overline{m}(\overline{m})$$

- $\rightarrow$  Stable in perturbation theory.



# **Masses Loop-Theorists Like to use**









- parton shower evolution
- nonperturbative gluon splitting
- colour singlets
- colourless clusters
- cluster fission
- cluster  $\rightarrow$  hadrons
- hadronic decays

#### Monte-Carlo QCD Computer:

- Computes all inter-quark/gluon
   and radiation processes
- Computes hadronization of partons
- Electroweak radiation effects
- Does NOT calculate self-energy processes
- Value of MC mass parameter is intrinsically related to the interquark/gluon radiation contained in the MC

MC top mass does NOT depend on the observable since the MC calculates always the same way !

MC top mass is unique for each MC.









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- Computes all inter-quark/gluon and radiation processes
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Inter-quark/gluon radiation/ Parton shower cut-off at  $\Lambda_s=1~{
m GeV}$ 

Hadronization model below.

Shower, shower cut, model details affect the value of top mass.











# Lessons on the MC top mass (for a perfect MC)

The interpretation (and value) of the top mass parameter in each MC generator is unique and should be observable-independent.

The value measured for the top mass depends on details of the MC, so in principle different MC have different top mass values.

The MC top mass parameter has features similar to a Top meson mass, and the way how to extract a field theoretical mass (in a suitable scheme) is analogous to methods in B physics

Without further knowledge there is an uncertainty of order  $\lesssim 1 GeV$  one has to add when translating the MC top mass to a suitable suitable field theoretical top mass ( $m_t^{MSR}(R=1-3 GeV)$ )



# **MSR Mass Definition**





# **Theory Tools to Measure the MC mass**

#### <u>Part 2</u>

The relation between MC mass and field theoretical mass can be made more precise by measuring the MC mass using a hadron level QCD prediction of a mass-dependent observable.

#### Need:

- Accurate analytic QCD predictions beyond LL/LO with full control over the quark mass dependence
- Theoretical description at the hadron level for comparison with MC at the hadron level
- Implementation of massive quarks into the SCET framework
- VFNS for final state jets (with massive quarks)\*

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* In collaboration with: P. Pietrulewicz, V. Mateu, I. Jemos, S. Gritschacher
arXiv:1302.4743 (PRD 88, 034021 (2013))
arXiv:1309.6251 (PRD 89, 014035 (2013))
arXiv:1405.4860 (PRD ..)
More to come ...
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#### **Theory Tools to Measure the MC mass**

Observable: Thust in e+e-

$$\tau = 1 - \max_{\vec{n}} \frac{\sum_{i} |\vec{n} \cdot \vec{p_i}|}{Q}$$
$$\tau \stackrel{\tau \to 0}{\approx} \frac{M_1^2 + M_2^2}{Q^2}$$

Invariant mass distribution in the resonance region !









## **Factorization for Massless Quarks**





# **VFN Scheme for Final State Jets**

- $\rightarrow$  consider: dijet in e<sup>+</sup>e<sup>-</sup> annihilation, n<sub>l</sub> light quarks  $\oplus$  one massive quark
- $\rightarrow$  obvious: (n<sub>1</sub>+1)-evolution for  $\mu \gtrsim m$  and (n<sub>1</sub>)-evolution for  $\mu \leq m$
- $\rightarrow$  obvious: different EFT scenarios w.r. to mass vs. Q J S scales

 $\mu_H \sim Q$ Q $\mu_J \sim Q \sqrt{\tau}$  $n_l + 1$ m  $\mu_S \sim Q \tau$  $n_l$  $Q\Lambda_{QCD}$  $\tau$  $\Lambda_{QCD}$ 0.1 0.3 0.0 0.2 0.4 05

"profile functions"

- $\rightarrow$  Deal with collinear and soft "mass modes"
- ightarrow Additional power counting parameter  $\lambda_m = m/Q$

mode	${\pmb  ho}^\mu = (+,-,\perp)$	p <sup>2</sup>
<i>n</i> -coll MM	$Q(\lambda_m^2, 1, \lambda_m)$	$m^2$
soft MM	$Q(\lambda_m, \lambda_m, \lambda_m)$	$m^2$

#### Aims:

- Full mass dependence (little room for any strong hierarchies): decoupling, massless limit
- Smooth connections between different EFTs
- Determination of flavor matching for current-, jet- and soft-evolution
- Reconcile problem of SCET<sub>2</sub>-type rapidity divergences



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arks  $\oplus$  one massive quark evolution for  $\mu \le m$ is vs. Q – J – S scales

# **Fully Massive Thrust**



universität wien

# **Counting Rules**

NLL NNLL NNNLL  $\ln \frac{d\sigma}{dy} = (\alpha_s \ln)^k \ln + (\alpha_s \ln)^k + \alpha_s (\alpha_s \ln)^k + \alpha_s^2 (\alpha_s \ln)^k + \dots$ 

						Classic Counting
standard		$\operatorname{cusp}$	non-cusp	$\operatorname{matching}$	alphas	
counting	LL	1	_	tree	1	LLA
	$\mathbf{NLL}$	2	1	$\operatorname{tree}$	2	NLLA
	NNLL	3	2	1	3	NNLLA + LLO
	$ m N^3LL$	$4^{pade}$	3	(2)	4	NNNLLA + NLO
	-LL'	1	_	tree	1	LLA
primed	$\mathrm{NLL}'$	2	1	1	2	NLLA + LLO
counting	NNLL'	3	2	2	3	NNLLA + NLO
emphasizes fixed order	$N^{3}LL'$	$4^{pade}$	3	3	4	NNNLLA + NNLC

Theory error from Padé estimate of  $\Gamma_3^{\text{cusp}}$ 





# **VFNS for Inclusive Hadron Collisions**

 $Q^2 = -q^2$ 

e.g. Deep Inelastic Scattering:

$$\frac{d\sigma(e^-p \to e^- + X)}{dQ \, dx}$$

- $\rightarrow$  consider all quarks as as light (m<sub>q</sub> <  $\Lambda$ )
- $\rightarrow$  quark number operators with an anomalous dimension between proton states  $\rightarrow\,$  DGLAP equations
- $\rightarrow$  Hadronic tensor:

$$W_{\mu\nu}(Q,x) \sim \sum_{\text{partons a}} f_a(\mu) \otimes w_{\mu\nu}(Q,x,\mu)$$

 $\rightarrow$  µ-dependence with DGLAP equations for (light) parton distribution functions

$$\frac{\partial}{\partial \ln Q^2} \begin{pmatrix} q_i(x, Q^2) \\ g(x, Q^2) \end{pmatrix} = \frac{\alpha_s(Q^2)}{2\pi} \sum_j \int_x^1 \frac{d\xi}{\xi} \\
\times \begin{pmatrix} P_{q_i q_j} \left(\frac{x}{\xi}, \alpha_s(Q^2)\right) & P_{q_i g} \left(\frac{x}{\xi}, \alpha_s(Q^2)\right) \\
P_{g q_j} \left(\frac{x}{\xi}, \alpha_s(Q^2)\right) & P_{g g} \left(\frac{x}{\xi}, \alpha_s(Q^2)\right) \end{pmatrix} \begin{pmatrix} q_j(\xi, Q^2) \\ g(\xi, Q^2) \end{pmatrix},$$
(11)

$$\frac{d\alpha_s(Q)}{d\ln Q^2} = -\beta_0 \,\frac{\alpha_s^2(Q)}{(4\pi)} + \dots \qquad \beta_0 = 11 - \frac{2}{3}n_{\text{light}}$$



Q

Λ

m<sub>light</sub>

# **VFNS for Inclusive Hadron Collisions**

 $\frac{d\sigma(e^-p \to e^- + X)}{dQ \, dx}$ 

- e.g. Deep Inelastic Scattering:
  - → realistic case: massive quarks with Q > m > Λ (charm, bottom [top])
  - $\rightarrow$  Hadronic tensor:

$$W_{\mu\nu}(m,Q,x) \sim \sum_{a=q,g,Q} f_a^{(n_l+1)}(\mu) \otimes w_{\mu\nu}(m,Q,x,\mu) \overset{\checkmark}{\underset{P}{\longrightarrow}}$$

#### VFNS for pdf evolution:

- DGLAP evolution for  $n_1$  flavors for  $\mu \leq m$  (only light quarks)
- DGLAP evolution for  $n_i$ +1 flavors for  $\mu \ge m$  (light quarks + massive quark)
- Flavor matching for  $\alpha_s$  and the pdfs at  $\mu_m \sim m$

$$f_{q,g,Q}^{(n_l+1)}(\mu_m) = \sum_{a=q,g} F_{q,g,Q|a}(m,\mu_m) \otimes f_a^{(n_l)}(\mu_m)$$

- $\rightarrow$  hard coefficient  $w_{\mu\nu}(m,Q,x)$  approaches massless  $w_{\mu\nu}(Q,x)$  for  $m{\rightarrow}0$
- $\rightarrow$  calculations of w<sub>µv</sub>(m,Q,x) involves subtraction of pdf IR mass singularities
- $\rightarrow$  full dependence on m/Q without any large logarithms

Q

m

Λ

m<sub>light</sub>

### **VFN Scheme: Secondary Massive Quarks**

 $\bigotimes_{p'}^{p} m \bigotimes_{p'}^{p} m$ 



- Provided results for factors with complete mass dependence at O(as^2) [NNNLL/NNLL']
- Flavor threshold correction factors at O(as^2)
- Reconcile problem of SCET<sub>2</sub>-type rapidity divergences
- Establish consistency conditions of flavor threshold matching factors (e.g. universality between thrust and DIS@ large x
- Simple implementation rules related to modified renormalization conditions
- Removal of O(Λ<sub>QCD</sub>) renormalon effects concerning mass and soft effects



### **VFN Scheme: Secondary Massive Quarks**





# **Rapidity Logarithms**



$$L_M = \ln\left(\frac{m^2}{\mu_m^2}\right)$$



# **VFN Scheme: Primary Massive Quarks**





# **MC vs. SCET: Primary Bottom Production**

#### Preliminary !!

Denahdi, AHH, Mateu

Compare MC with SCET (pQCD, summation, hadronization effects) @ NNLL for Thrust

- Take central values for  $\alpha_s$  and  $\Omega_1$  from our earlier NNLL thrust analysis for data on all-flavor production (=massless quarks)  $\alpha_s(M_Z) = 0.1192 \pm 0.006$  $\Omega_1 = 0.276 \pm 0.155$
- Compare with Pythia (m<sub>b</sub><sup>Pythia</sup>=4.8 GeV) for consistency and mass sensitivity
- Which mass does m<sub>b</sub><sup>Pythia</sup>=4.8 GeV correspond to for a field theoretic bottom mass?

order	$\overline{\Omega}_1 \ (\overline{\mathrm{MS}})$	$\Omega_1$ (R-gap)	order	$lpha_s(m_Z)~({ m with}~ar\Omega_1^{\overline{ m MS}})$	$lpha_s(m_Z) \; ( ext{with} \; \Omega_1^{ ext{Rgap}})$
$\mathbf{NLL}'$	$0.264 \pm 0.213$	$0.293 \pm 0.203$	$\mathrm{NLL}'$	$0.1203 \pm 0.0079$	$0.1191 \pm 0.0089$
NNLL	$0.256 \pm 0.197$	$0.276 \pm 0.155$	NNLL	$0.1222 \pm 0.0097$	$0.1192 \pm 0.0060$
$\mathrm{NNLL}'$	$0.283 \pm 0.097$	$0.316\pm0.072$	NNLL'	$0.1161 \pm 0.0038$	$0.1143 \pm 0.0022$
$N^{3}LL$	$0.274 \pm 0.098$	$0.313 \pm 0.071$	$N^{3}LL$	$0.1165 \pm 0.0046$	$0.1143 \pm 0.0022$
$N^{3}LL'$ (full)	$0.252\pm0.069$	$0.323 \pm 0.045$	$N^{3}LL'$ (full)	$0.1146 \pm 0.0021$	$0.1135 \pm 0.0009$
$\mathrm{N}^{3}\mathrm{LL'}_{(\mathrm{QCD}+m_{b})}$	$0.238 \pm 0.070$	$0.310\pm0.049$	$\mathrm{N}^{3}\mathrm{LL'}_{(\mathrm{QCD}+m_b)}$	$0.1153 \pm 0.0022$	$0.1141 \pm 0.0009$
$\rm N^3 L L'_{(pure QCD)}$	$0.254 \pm 0.070$	$0.332 \pm 0.045$	$ m N^3LL'_{(pure QCD)}$	$0.1152 \pm 0.0021$	$0.1140 \pm 0.0008$

Abbate, Fickinger, AHH, Mateu, Stewart 2010



### **MC vs. SCET: Primary Bottom Production**





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### **MC vs. SCET: Primary Bottom Production**

#### Preliminary !! (No fit yet)

 $\overline{m}_b(\overline{m}_b) = 3.7, 4.2, 4.7 \text{ GeV}$   $\alpha_s(M_Z) = 0.1192$   $\Omega_1 = 0.276 \text{ GeV}$ 



# Conclusions

- → The MC top mass parameter has the status of a hadronic parameter and is therefore not a field theoretic mass definition
- → The issue is becomes relevant when uncertainties in the MC top mass are becoming smaller than 1 GeV.
- → Ignoring the issue means that there is a conceptual uncertainty of about 1 GeV one needs to account for when relating the MC mass to a field theory mass.
- $\rightarrow$  Suitable field theory mass definition in this context: e.g. MSR mass (R=1-3 GeV)
- → It is possible to relate the MS top mass to a field theoretic mass by fits of QCD calculations at the hadron level to MC output for very mass sensitive quantities.
- → When one does that there are still theoretical uncertainties (in the QCD predictions used for the fit) one has to account for.
- $\rightarrow$  Fully massive thrust using a VFNS for final state inclusive jets.
- $\rightarrow$  Upcoming:
- C parameter, heavy jet mass, inv. mass distr. @ NNLL
- NNNLL for e+e-  $\rightarrow$  Need: 2-loop massiv quark jet function
- DIS for massive quarks @ large x
- pp  $\rightarrow$  tt+X (2-jettiness) @ NLL  $\rightarrow$  NNLL possible, NNNLL need NNLO full. Diff.

