$b \to s\,\ell^+\ell^-$ form factors from lattice QCD

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1. Local and nonlocal matrix elements
2. Summary of unquenched lattice calculations
3. $B \to K\ell^+\ell^-$
4. $B \to K^*\mu^+\mu^-$, $B_s \to \phi\,\mu^+\mu^-$
5. $\Lambda_b \to \Lambda\,\mu^+\mu^-$

Local and nonlocal matrix elements

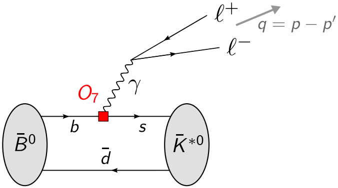
Decay amplitude:

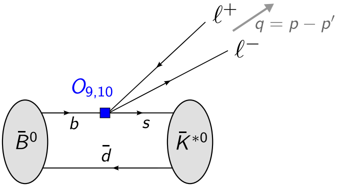
$$\mathcal{M} = \frac{G_F\,\alpha}{\sqrt{2}\pi} V_{tb} V_{ts}^* \Big[(\mathcal{A}_\mu + \mathcal{T}_\mu) \bar{u}_\ell \gamma^\mu v_\ell + \mathcal{B}_\mu \bar{u}_\ell \gamma^\mu \gamma_5 v_\ell \Big],$$

- Local:

$$\mathcal{A}_\mu = -\frac{2m_b}{q^2}\, q^\nu\, C_7\, \langle K^* |\, \bar{s}\, i\sigma_{\mu\nu} \tfrac{1+\gamma_5}{2}\, b\, | B \rangle + C_9\, \langle K^* |\, \bar{s} \gamma_\mu \tfrac{1-\gamma_5}{2}\, b\, | B \rangle$$

$$\mathcal{B}_\mu = C_{10}\, \langle K^* |\, \bar{s} \gamma_\mu \tfrac{1-\gamma_5}{2}\, b\, | B \rangle$$





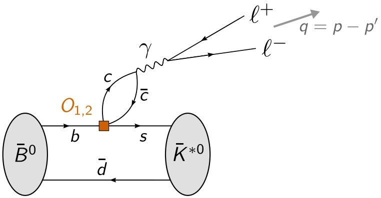
Local and nonlocal matrix elements

Decay amplitude:

$$\mathcal{M} = \frac{G_F\,\alpha}{\sqrt{2}\pi} V_{tb} V_{ts}^* \Big[(\mathcal{A}_\mu + \mathcal{T}_\mu) \bar{u}_\ell \gamma^\mu v_\ell + \mathcal{B}_\mu \bar{u}_\ell \gamma^\mu \gamma_5 v_\ell \Big],$$

- Nonlocal:

$$\mathcal{T}_\mu = \frac{-16 i \pi^2}{q^2} \sum_{i=1\ldots6;8} C_i \int \mathrm{d}^4 x\ \ e^{iq\cdot x} \, \langle K^* |\, \mathrm{T}\ O_i(0)\, j_\mu(x)\, |B\rangle$$



Local and nonlocal matrix elements

At high q^2: OPE in $1/q^2$ [Grinstein and Pirjol, hep-ph/0404250]

$$
\begin{aligned}
\mathcal{T}_\mu \quad &= \quad -T_7(q^2)\ \frac{2m_b}{q^2}\, q^\nu \langle K^* |\, \bar{s}\, i\sigma_{\mu\nu} \tfrac{1+\gamma_5}{2}\, b\, | B \rangle \\
&\quad + T_9(q^2)\ \langle K^* | \bar{s} \gamma_\mu \tfrac{1-\gamma_5}{2}\, b | B \rangle \\
&\quad + \mathcal{O}(\Lambda^2/m_b^2,\ m_c^4/q^4,\ \alpha_s \Lambda/m_b),
\end{aligned}
$$

where $T_7(q^2)$ and $T_9(q^2)$ are computed perturbatively

Summary of unquenched lattice calculations

	$B \to K\ell^+\ell^-$ arXiv:1306.2384, arXiv:1306.0434	$B \to K^*\mu^+\mu^-$, $B_s \to \phi\,\mu^+\mu^-$ arXiv:1310.3722, arXiv:1310.3887	$\Lambda_b \to \Lambda\,\mu^+\mu^-$ arXiv:1212.4827
u, d, s-quark action	HISQ/AsqTad	AsqTad	Domain Wall
b-quark action	NRQCD	NRQCD	static
Chiral extrapolation	HMχPT	linear	linear
Continuum extrapolation	✓	×	✓
Extrap. to low q^2	z-expansion	(high q^2 only)	dipole model
Treatment of $O_{1\ldots6}, O_8$	1-loop	2-loop	1-loop
Strong decay of final hadron	N/A	Narrow-width approx.	N/A

$B \to K\ell^+\ell^-$

Bouchard, Lepage, Monahan, Na, Shigemitsu (HPQCD), arXiv:1306.0434, arXiv:1306.2384

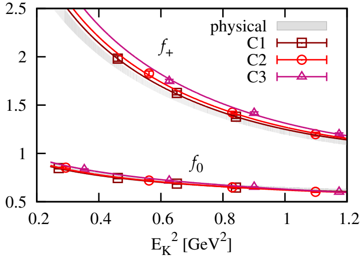
Lattice parameters:

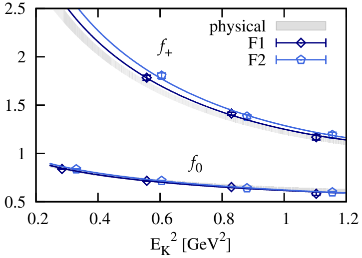
Set	$L^3 \times T$	a (fm)	m_π (MeV)
C1	$24^3 \times 64$	≈ 0.12	≈ 260
C2	$20^3 \times 64$	≈ 0.12	≈ 370
C3	$20^3 \times 64$	≈ 0.12	≈ 520
F1	$28^3 \times 96$	≈ 0.09	≈ 340
F2	$28^3 \times 96$	≈ 0.09	≈ 480

$B \to K\ell^+\ell^-$

Bouchard, Lepage, Monahan, Na, Shigemitsu (HPQCD), arXiv:1306.0434, arXiv:1306.2384

Data analysis step 1: chiral/continuum extrapolation using HMχPT including a^2 terms and finite-V corrections





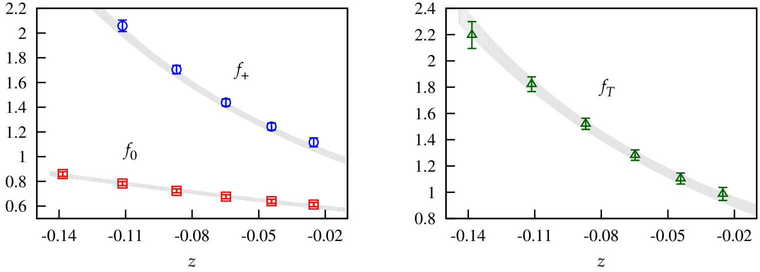
To estimate uncertainties, add NNLO terms with size limited by Bayesian constraints; also try removing highest E_K

+ 4% from missing higher-order terms in matching to $\overline{\text{MS}}$ scheme (done at one loop)

$B \to K\ell^+\ell^-$

Bouchard, Lepage, Monahan, Na, Shigemitsu (HPQCD), arXiv:1306.0434, arXiv:1306.2384

Data analysis step 2: Generate synthetic data points from physical curves. Then extrapolate to low q^2 using z expansion



$$z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}},$$

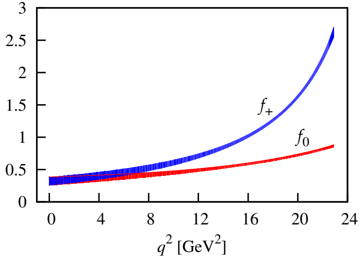
$$f_0(q^2) = \sum_{k=0}^{K} a_k^0\, z(q^2)^k,$$

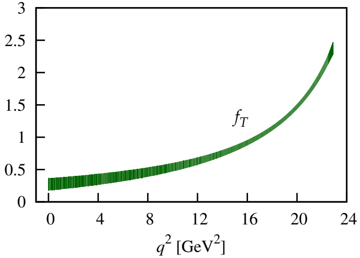
$$f_i(q^2) = \frac{1}{P_i(q^2)} \sum_{k=0}^{K-1} a_k^i \left[z(q^2)^k - (-1)^{k-K} \frac{k}{K} z(q^2)^K \right]$$

$B \to K\ell^+\ell^-$

Bouchard, Lepage, Monahan, Na, Shigemitsu (HPQCD), arXiv:1306.0434, arXiv:1306.2384

Data analysis step 2: Generate synthetic data points from physical curves.
Then extrapolate to low q^2 using z expansion





Main fits use $K = 3$. To estimate uncertainty, try $K = 2$ and $K = 4$. Also propagate uncertainties from changing the chiral/continuum fit functions.

$B \to K\ell^+\ell^-$

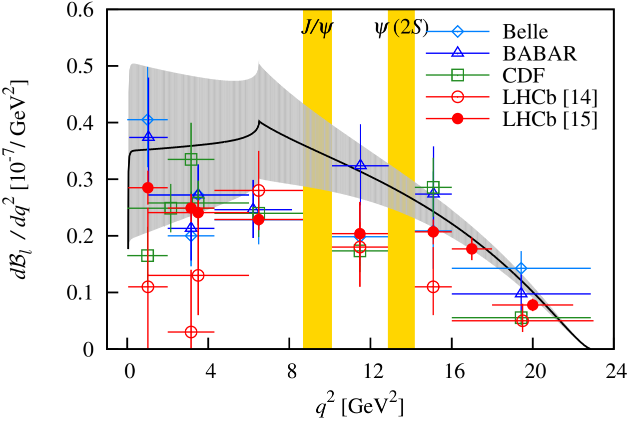
Bouchard, Lepage, Monahan, Na, Shigemitsu (HPQCD), arXiv:1306.0434, arXiv:1306.2384

Assumptions in calculation of $B \to K\ell^+\ell^-$ observables (decay rate, flat term in angular distribution):

- Nonlocal matrix elements are included only at one loop. No uncertainty is assigned to them!
- Uncertainty of NNLO Wilson coefficients (taken from [Altmannshofer et al., arXiv:0811.1214]): 2%
- $\alpha_{\text{e.m.}}$ is evaluated at $\mu = m_Z$, giving $\alpha_{\text{e.m.}} = 128.957(20)$. No uncertainty is assigned to this scale choice
- observables are isospin-averaged

$B \to K\ell^+\ell^-$

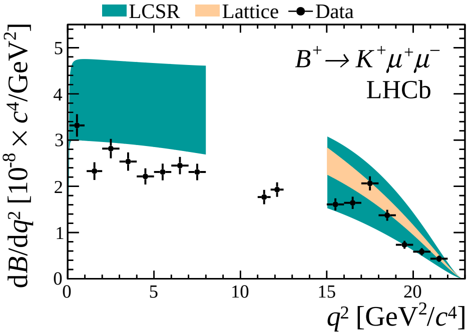
Bouchard, Lepage, Monahan, Na, Shigemitsu (HPQCD), arXiv:1306.0434, arXiv:1306.2384



$B \to K\ell^+\ell^-$

Bouchard, Lepage, Monahan, Na, Shigemitsu (HPQCD), arXiv:1306.0434, arXiv:1306.2384

New LHCb results with 3 fb^{-1} [arXiv:1403.8044]:



Large contribution from $\psi(4160)$ at high q^2

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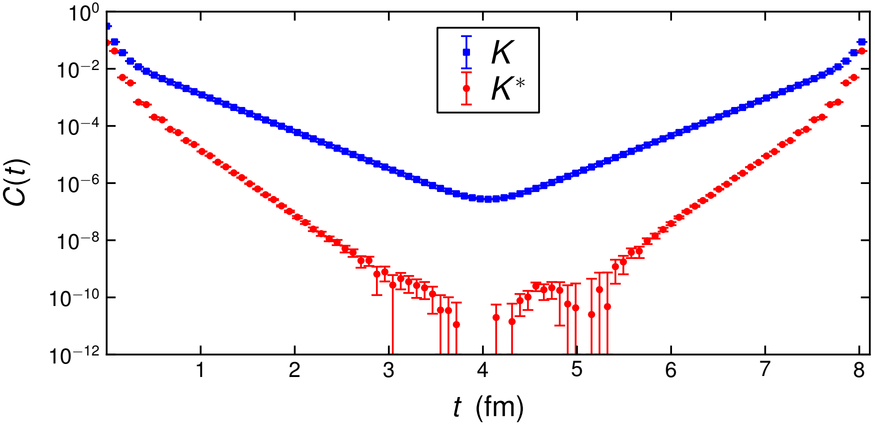
5. $\Lambda_b \to \Lambda \mu^+ \mu^-$

$B \to K^*\mu^+\mu^-$, $B_s \to \phi\,\mu^+\mu^-$ Horgan, Liu, Meinel, Wingate, arXiv:1310.3722, arXiv:1310.3887

Statistical uncertainties: K vs K^* two-point function
using equal number of gauge configurations



$B \to K^*\mu^+\mu^-$, $B_s \to \phi\,\mu^+\mu^-$ Horgan, Liu, Meinel, Wingate, arXiv:1310.3722, arXiv:1310.3887

$\longrightarrow$ **Statistical uncertainties will be dominant.**

We use fewer ensembles but higher statistics on each one:

Set	$L^3 \times T$	a (fm)	m_π (MeV)
c007	$24^3 \times 64$	0.1187(9)	313.4(2.4)
c02	$20^3 \times 64$	0.1183(9)	519.2(3.9)
f0062	$28^3 \times 96$	0.0846(6)	344.3(2.4)

$B \to K^* \mu^+ \mu^-$, $B_s \to \phi\, \mu^+ \mu^-$ Horgan, Liu, Meinel, Wingate, arXiv:1310.3722, arXiv:1310.3887

We fit the lattice data using

$$F(q^2) = P_F(q^2) \left[\left(1 + c_{01} \frac{m_\pi^2 - m_{\pi,\mathrm{phys}}^2}{(4\pi f)^2} + c_{01s} \frac{m_{\eta_s}^2 - m_{\eta_s,\mathrm{phys}}^2}{(4\pi f)^2} \right) a_0 + a_1\, z(q^2, t_0) \right],$$

where

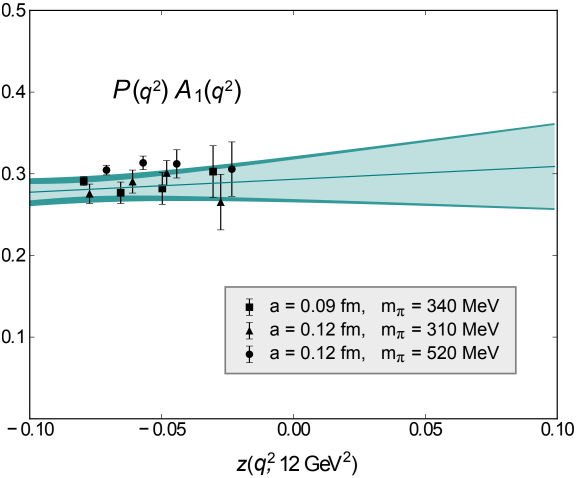
$$P_F(q^2) = \frac{1}{1 - q^2/m_{\mathrm{pole(F)}}^2},$$

$$z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}, \qquad t_+ = (m_B + m_{K^*})^2, \qquad t_0 = 12\ \mathrm{GeV}$$

- There is no χPT for vector mesons $\to$ we assume linear $m_{u,d}$ dependence
- Because of the larger statistical uncertainties, we cannot resolve a nonzero lattice-spacing dependence of the form factors
- To describe the lattice data region, first order in z expansion is sufficient. We do not use the form factors at low q^2

$B \to K^* \mu^+ \mu^-$, $B_s \to \phi\, \mu^+ \mu^-$ Horgan, Liu, Meinel, Wingate, arXiv:1310.3722, arXiv:1310.3887

For example A_1:



$B \to K^* \mu^+ \mu^-$, $B_s \to \phi \, \mu^+ \mu^-$ Horgan, Liu, Meinel, Wingate, arXiv:1310.3722, arXiv:1310.3887

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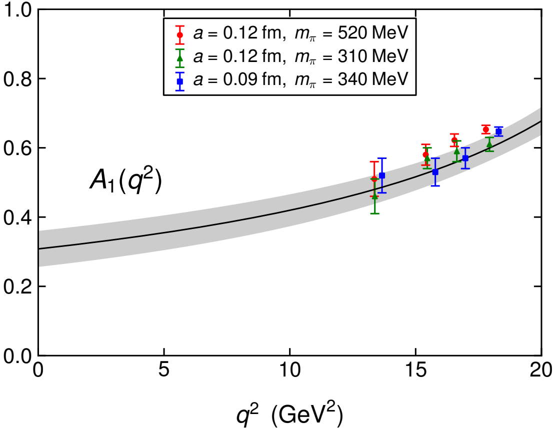


$B \to K^*\mu^+\mu^-$, $B_s \to \phi\,\mu^+\mu^-$ Horgan, Liu, Meinel, Wingate, arXiv:1310.3722, arXiv:1310.3887

Estimates of systematic uncertainties in form factors:

- Missing $\mathcal{O}(\alpha_s^2)$ terms in $b \to s$ current: 4%
- Missing $\mathcal{O}(\alpha_s\,\Lambda/m_b)$ terms in $b \to s$ current: 2%
- Missing $\mathcal{O}(\Lambda^2/m_b^2)$ terms in $b \to s$ current: 1%

No estimates of discretization errors and chiral-extrapolation errors, but the results show no significant dependence on lattice spacing, and only mild quark-mass dependence.

No estimates of q^2-extrapolation errors – that's ok because we only use our form factors in the region where we have lattice data

$B \to K^* \mu^+ \mu^-$, $B_s \to \phi\, \mu^+ \mu^-$ Horgan, Liu, Meinel, Wingate, arXiv:1310.3722, arXiv:1310.3887

Assumptions in calculation of $B \to K^* \mu^+ \mu^-$, $B_s \to \phi\, \mu^+ \mu^-$ observables:

- Nonlocal matrix elements are included using two-loop OPE. Included an extra 5% systematic uncertainty in vector amplitude $(\mathcal{A}_\mu + \mathcal{T}_\mu)$ due to truncation of OPE / duality violations
- Uncertainty of NNLO Wilson coefficients (taken from [Altmannshofer et al., arXiv:0811.1214]): 2%
- $\alpha_{\mathrm{e.m.}}$ is evaluated at $\mu = m_b$, which minimizes higher-order electroweak corrections (at least for inclusive decay [Bobeth et al., hep-ph/0312090])
- Narrow-width approximation (next slide)

$B \to K^*\mu^+\mu^-$, $B_s \to \phi\,\mu^+\mu^-$ Horgan, Liu, Meinel, Wingate, arXiv:1310.3722, arXiv:1310.3887

Narrow-width approximation [Krüger et al., hep-ph/9907386]:

$$\langle K^-(p_K)\pi^+(p_\pi)|J^\mu|\bar{B}(p)\rangle \approx -D_{K^*}(p'^2)\left[p_K^\nu - p_\pi^\nu - \frac{M_K^2 - M_\pi^2}{p'^2}(p_K^\nu + p_\pi^\nu)\right] A_{\nu\mu}$$

where $D_{K^*}(p'^2)$ is defined through

$$|D_{K^*}(p'^2)|^2 = \frac{48\pi^2}{\beta^3 M_{K^*}^2}\delta(k^2 - M_{K^*}^2)$$

and $A_{\nu\mu}$ is defined through

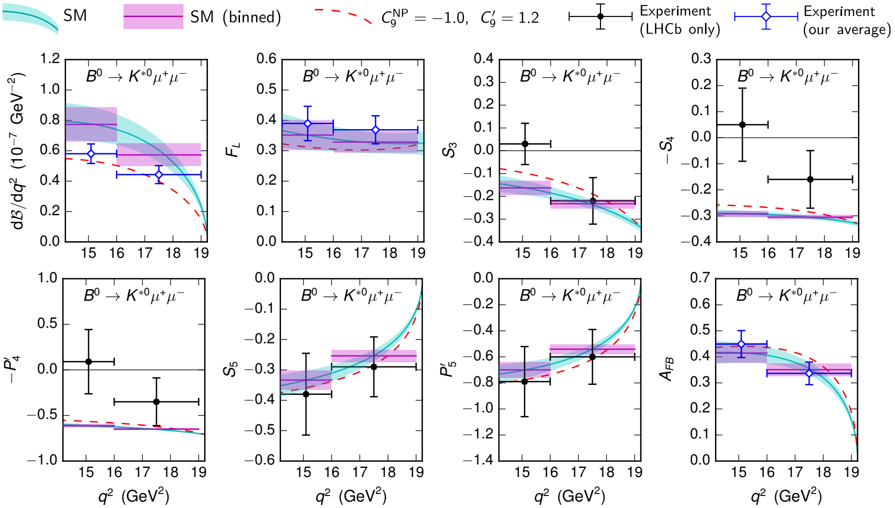
$$\langle\bar{K}^*(p',\varepsilon)|\,J^\mu|\bar{B}(p)\rangle = \varepsilon^{*\nu} A_{\nu\mu}.$$

Corresponds to pure P-wave decay $K^* \to K\,\pi$.

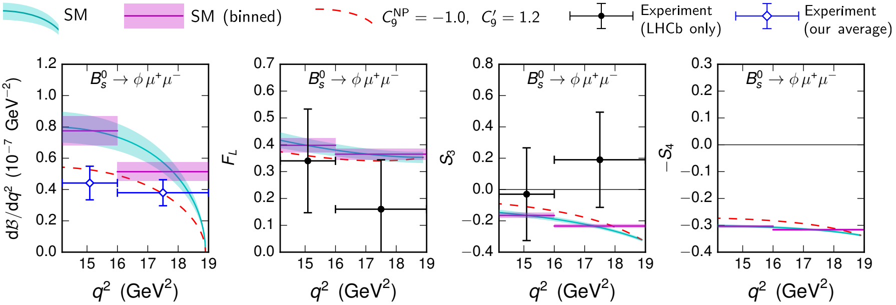
S-wave pollution is expected to be negligible at high q^2
[Bećirević and Tayduganov, arXiv:1207.4004]

$B \to K^* \mu^+ \mu^-$, $B_s \to \phi\, \mu^+ \mu^-$

Horgan, Liu, Meinel, Wingate, arXiv:1310.3722, arXiv:1310.3887



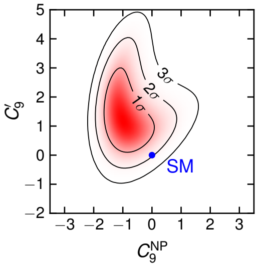
$B \to K^*\mu^+\mu^-$, $B_s \to \phi\,\mu^+\mu^-$ Horgan, Liu, Meinel, Wingate, arXiv:1310.3722, arXiv:1310.3887



$B \to K^* \mu^+ \mu^-$, $B_s \to \phi\, \mu^+ \mu^-$ Horgan, Liu, Meinel, Wingate, arXiv:1310.3722, arXiv:1310.3887

Fit Wilson coefficients of

$$O_9 = e^2/(16\pi^2)\, \bar{s}\gamma_\mu b_L \;\; \bar{\ell}\gamma_\mu \ell, \qquad O_9' = e^2/(16\pi^2)\, \bar{s}\gamma_\mu b_R \;\; \bar{\ell}\gamma_\mu \ell$$



- This fit (high q^2, lattice QCD):

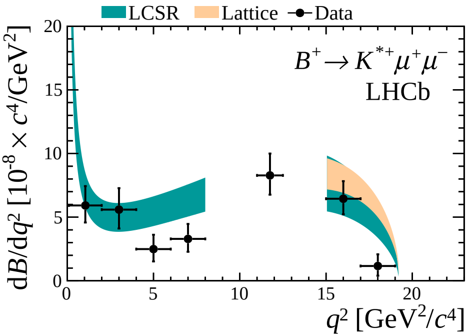
$$C_9^{\rm NP} = -1.0 \pm 0.5, \qquad C_9' = 1.2 \pm 1.0$$

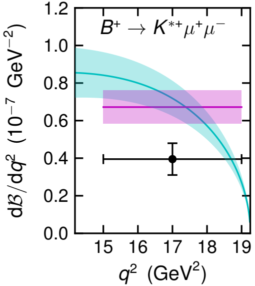
- Altmannshofer and Straub (dominated by low q^2, without $B_s \to \phi\mu^+\mu^-$) [arXiv:1308.1501]:

$$C_9^{\rm NP} = -1.0 \pm 0.3, \qquad C_9' = 1.0 \pm 0.5$$

$B \to K^* \mu^+ \mu^-$, $B_s \to \phi\, \mu^+ \mu^-$ Horgan, Liu, Meinel, Wingate, arXiv:1310.3722, arXiv:1310.3887

LHCb results for $B^+ \to K^{*+} \mu^+ \mu^-$ with 3 fb^{-1} [arXiv:1403.8044]:





$\Lambda_b \to \Lambda\, \mu^+ \mu^-$ (static b)

Detmold, Lin, Meinel, Wingate, arXiv:1212.4827

- Leading-order (static) heavy-quark effective theory for b quark. Only 2 form factors in this limit:

$$\langle \Lambda |\, \bar{s}\, \Gamma Q \,| \Lambda_Q \rangle \;=\; \bar{u}_\Lambda \;[F_1 + \not{v}\, F_2]\; \Gamma\; u_{\Lambda_Q}$$

  We use the combinations

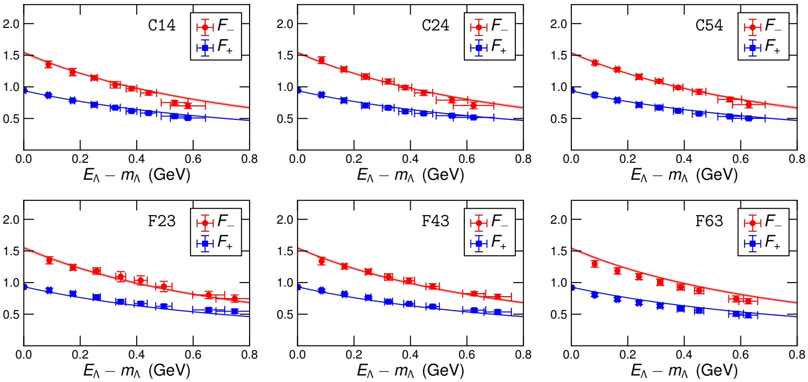
$$F_+ = F_1 + F_2, \qquad F_- = F_1 - F_2$$

- Domain-wall action for u, d, s quarks

Set	$L^3 \times T$	$am_{u,d}^{(\mathrm{sea})}$	$am_s^{(\mathrm{sea})}$	a (fm)	$am_{u,d}^{(\mathrm{val})}$	$am_s^{(\mathrm{val})}$	$m_\pi^{(\mathrm{vv})}$ (MeV)
C14	$24^3 \times 64$	0.005	0.04	0.1119(17)	0.001	0.04	245(4)
C24	$24^3 \times 64$	0.005	0.04	0.1119(17)	0.002	0.04	270(4)
C54	$24^3 \times 64$	0.005	0.04	0.1119(17)	0.005	0.04	336(5)
C53	$24^3 \times 64$	0.005	0.04	0.1119(17)	0.005	0.03	336(5)
F23	$32^3 \times 64$	0.004	0.03	0.0849(12)	0.002	0.03	227(3)
F43	$32^3 \times 64$	0.004	0.03	0.0849(12)	0.004	0.03	295(4)
F63	$32^3 \times 64$	0.006	0.03	0.0848(17)	0.006	0.03	352(7)

$\Lambda_b \to \Lambda\,\mu^+\mu^-$ (static b)

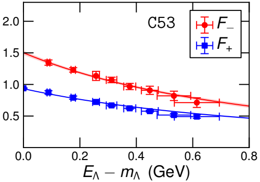
Detmold, Lin, Meinel, Wingate, arXiv:1212.4827



Dipole fit model:

$$F_\pm = \frac{Y_\pm}{(\widetilde{X}_\pm + E_\Lambda - m_\Lambda)^2}\left[1 + d_\pm (aE_\Lambda)^2\right],$$

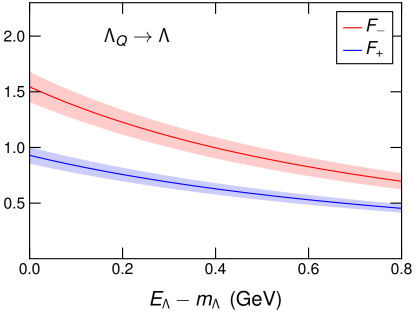
$$\widetilde{X}_\pm = X_\pm + c_{l,\pm}\left[(m_\pi)^2 - (m_\pi^{\rm phys})^2\right] + c_{s,\pm}\left[(m_{\eta_s})^2 - (m_{\eta_s}^{\rm phys})^2\right]$$



$\Lambda_b \to \Lambda\, \mu^+ \mu^-$ (static b)

Detmold, Lin, Meinel, Wingate, arXiv:1212.4827

$\Lambda_Q \to \Lambda$ form factors at $m_\pi = m_\pi^{\rm phys}$, $m_{\eta_s} = m_{\eta_s}^{\rm phys}$, $a = 0$:



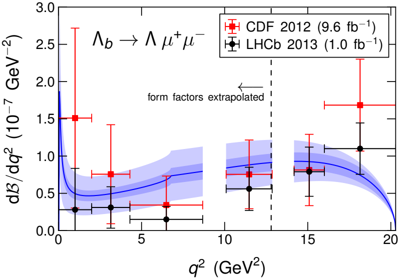
Error bands shown here include the following estimates of systematic uncertainties:

- renormalization: $\sim$6%
- finite volume: $\sim$3%
- chiral extrapolation: $\sim$3%
- continuum extrapolation: $\sim$2%

$\Lambda_b \to \Lambda\, \mu^+ \mu^-$ (static b)

Detmold, Lin, Meinel, Wingate, arXiv:1212.4827

$\Lambda_b \to \Lambda\mu^+\mu^-$ differential branching fraction



- Inner error band from statistical and systematic uncertainty in F_+, F_-
- Outer error band includes $\dfrac{\sqrt{|\mathbf{p}'|^2 + \Lambda_{\rm QCD}^2}}{m_b}$ uncertainty from static approximation
- no uncertainty estimates for: Wilson coefficients, nonlocal matrix elements one-loop only, extrapolation to low q^2

$\Lambda_b \to \Lambda\,\mu^+\mu^-$ (relativistic b)

Meinel, arXiv:1401.2685 and work in progress

- for b quark, replace static action by "relativistic" heavy quark action
  [RBC/UKQCD Collaboration, arXiv:1206.2554]

- "mostly nonperturbative" renormalization of heavy-light currents

- compute full set of 10 relativistic form factors

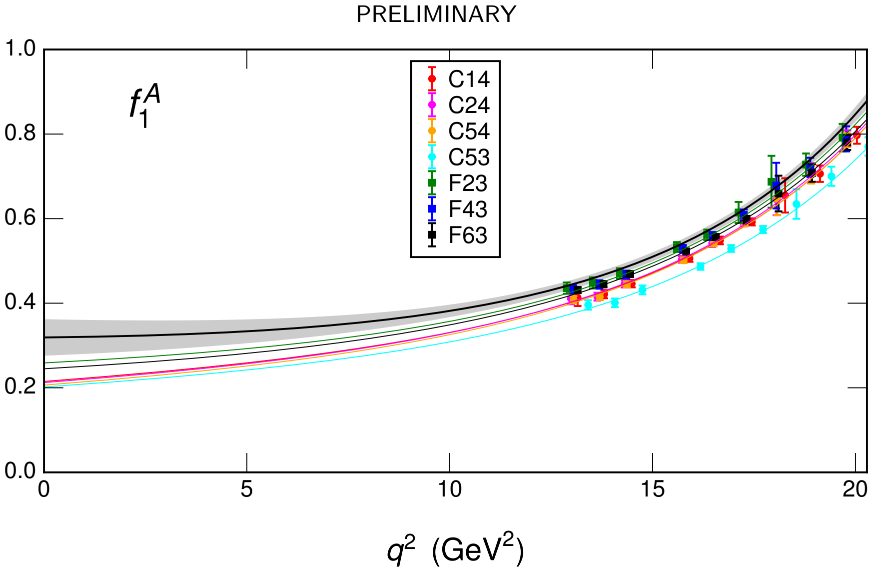
$$
\begin{aligned}
\langle\Lambda|\,\bar{s}\,\gamma^\mu\,b\,|\Lambda_b\rangle &= \bar{u}_\Lambda\Big[f_1^V\,\gamma^\mu - f_2^V\,i\sigma^{\mu\nu}q_\nu/m_{\Lambda_b} + f_3^V\,q^\mu/m_{\Lambda_b}\Big]u_{\Lambda_b},\\
\langle\Lambda|\,\bar{s}\,\gamma^\mu\gamma_5\,b\,|\Lambda_b\rangle &= \bar{u}_\Lambda\Big[f_1^A\,\gamma^\mu - f_2^A\,i\sigma^{\mu\nu}q_\nu/m_{\Lambda_b} + f_3^A\,q^\mu/m_{\Lambda_b}\Big]\gamma_5\,u_{\Lambda_b},\\
\langle\Lambda|\,\bar{s}\,i\sigma^{\mu\nu}q_\nu\,b\,|\Lambda_b\rangle &= \bar{u}_\Lambda\Big[f_1^{TV}\,(\gamma^\mu q^2 - q^\mu\not{q})/m_{\Lambda_b} - f_2^{TV}\,i\sigma^{\mu\nu}q_\nu\Big]u_{\Lambda_b},\\
\langle\Lambda|\,\bar{s}\,i\sigma^{\mu\nu}q_\nu\,\gamma_5\,b\,|\Lambda_b\rangle &= \bar{u}_\Lambda\Big[f_1^{TA}\,(\gamma^\mu q^2 - q^\mu\not{q})/m_{\Lambda_b} - f_2^{TA}\,i\sigma^{\mu\nu}q_\nu\Big]\gamma_5\,u_{\Lambda_b}
\end{aligned}
$$

- will publish updated predictions for decay rate and **angular observables** of $\Lambda_b \to \Lambda(\to p\,\pi^-)\mu^+\mu^-$

$\Lambda_b \to \Lambda\,\mu^+\mu^-$ (relativistic b)

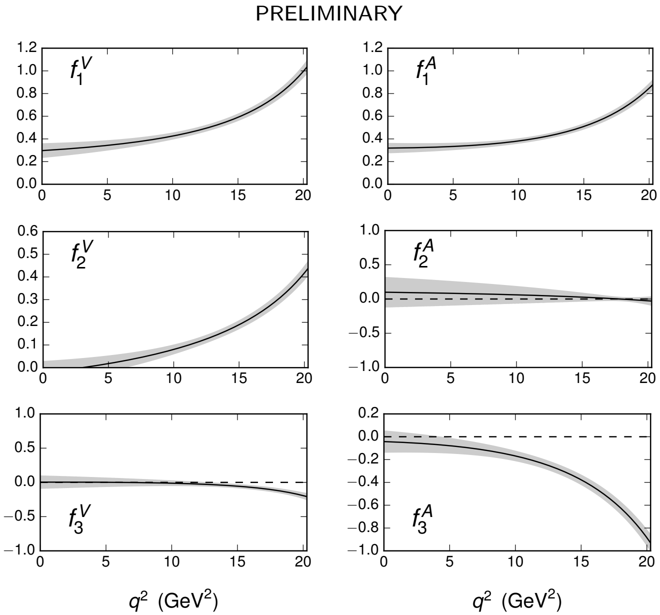
Meinel, arXiv:1401.2685 and work in progress

Chiral/continuum extrapolation using modified z expansion (to order z^2)



$\Lambda_b \to \Lambda\, \mu^+\mu^-$ (relativistic b)

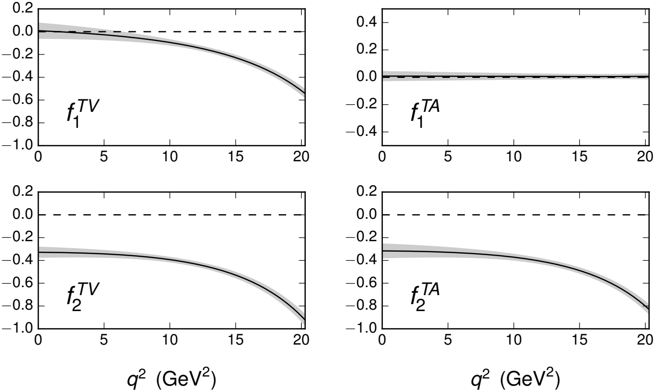
Meinel, arXiv:1401.2685 and work in progress



$\Lambda_b \to \Lambda\,\mu^+\mu^-$ (relativistic b)

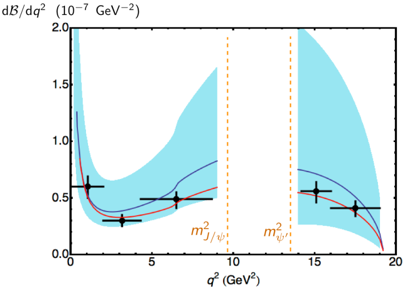
Meinel, arXiv:1401.2685 and work in progress

PRELIMINARY



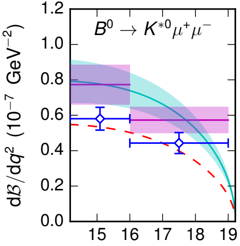
Conclusions

Lattice QCD has significantly improved the precision at high q^2.



[Descotes-Genon, Matias, Virto, arXiv:1311.3876]

$\rightarrow$



[Horgan, Liu, Meinel, Wingate, arXiv:1310.3887]

More work is needed for the nonlocal matrix elements of $O_{1,\ldots,6;8}$.

Extra slides

- Extracting matrix elements from correlation functions
- K^* and ρ resonances on the lattice: Lüscher method
- Modelling the effects of the $\psi(3770)$ and $\psi(4160)$

Matrix elements from correlation functions

Interpolating fields; $b \to s$ current:

$$\begin{aligned} \Phi^{(B)} &= \bar{d}\,\gamma_5\, b \\ \Phi_\mu^{(K^*)} &= \bar{d}\,\gamma_\mu\, s \\ J_\Gamma &= Z_\Gamma\, \bar{s}\,\Gamma\, b \end{aligned}$$

Three-point and two-point functions:

$$\begin{aligned} C_\mu^{(B\to K^*)} &= \sum_{\mathbf{y}}\sum_{\mathbf{z}} e^{-i\mathbf{p}'\cdot(\mathbf{x}-\mathbf{y})} e^{-i\mathbf{p}\cdot(\mathbf{y}-\mathbf{z})} \left\langle \Phi_\mu^{(K^*)}(x)\;\; J_\Gamma(y)\;\; \Phi^{(B)\dagger}(z) \right\rangle_{U,\psi,\bar{\psi}} \\ C^{(B)} &= \sum_{\mathbf{x}} e^{-i\mathbf{p}'\cdot(\mathbf{x}-\mathbf{z})} \left\langle \Phi^{(B)}(x)\;\; \Phi^{(B)\dagger}(z) \right\rangle_{U,\psi,\bar{\psi}} \\ C_{\mu\nu}^{(K^*)} &= \sum_{\mathbf{x}} e^{-i\mathbf{p}\cdot(\mathbf{x}-\mathbf{z})} \left\langle \Phi_\mu^{(K^*)}(x)\;\; \Phi_\nu^{(K^*)\dagger}(z) \right\rangle_{U,\psi,\bar{\psi}} \end{aligned}$$

$$\left\langle O \right\rangle_{U,\psi,\bar{\psi}} \equiv \frac{1}{\mathcal{Z}} \int \mathcal{D}U\,\mathcal{D}\psi\,\mathcal{D}\bar{\psi}\;\; O\;\; e^{-(S_G + S_F)}$$

Matrix elements from correlation functions

After performing the path integral over fermions:

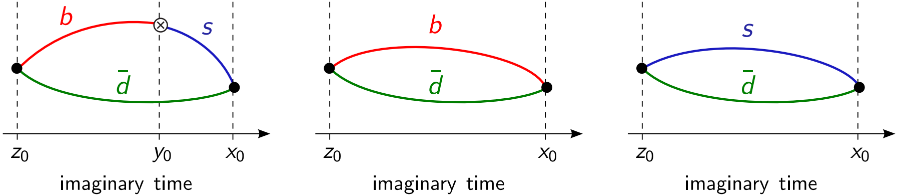
$$C_{\mu}^{(B\to K^*)} \;=\; \sum_{\mathbf{y}}\sum_{\mathbf{z}} e^{-i\mathbf{p}'\cdot(\mathbf{x}-\mathbf{y})}e^{-i\mathbf{p}\cdot(\mathbf{y}-\mathbf{z})}\left\langle \mathrm{Tr}\left[\gamma_\mu G_s(x,y)\,\Gamma_J\, G_b(y,z)\,\gamma_5\, G_d(z,x)\right]\right\rangle$$

$$C^{(B)} \;=\; \sum_{\mathbf{x}} e^{-i\mathbf{p}'\cdot(\mathbf{x}-\mathbf{z})}\left\langle \mathrm{Tr}\left[\gamma_5\, G_d(x,z)\,\gamma_5\, G_b(z,x)\right]\right\rangle$$

$$C_{\mu\nu}^{(K^*)} \;=\; \sum_{\mathbf{x}} e^{-i\mathbf{p}\cdot(\mathbf{x}-\mathbf{z})}\left\langle \mathrm{Tr}\left[\gamma_\mu\, G_d(x,z)\,\gamma_\nu\, G_s(z,x)\right]\right\rangle$$

where

$$\left\langle O\right\rangle \equiv \frac{1}{\mathcal{Z}}\int \mathcal{D}U\;\; O\;\det[D_F]\;\, e^{-S_G} \qquad \text{(done numerically)}$$



Matrix elements from correlation functions

Spectral decomposition for large $|x_0 - y_0|$ and $|y_0 - z_0|$:

$$C_\mu^{(B\to K^*)} = \frac{1}{2E_{K^*}}\frac{1}{2E_B}\sum_{K^*\ \mathrm{spin}} \langle 0|\Phi_\mu^{(K^*)}|K^*\rangle\langle K^*|\bar{s}\,\Gamma\, b|B\rangle\langle B|\Phi^{(B)\dagger}|0\rangle \times\ e^{-E_{K^*}|x_0-y_0|}\ e^{-E_B|y_0-z_0|}$$

$$C^{(B)} = \frac{1}{2E_B}\ \langle 0|\Phi^{(B)}|B\rangle\ \langle B|\Phi^{(B)\dagger}|0\rangle\ e^{-E_B|x_0-y_0|}$$

$$C_{\mu\nu}^{(K^*)} = \frac{1}{2E_{K^*}}\sum_{K^*\ \mathrm{spin}} \langle 0|\Phi_\mu^{(K^*)}|K^*\rangle\ \langle K^*|\Phi_\nu^{(K^*)\dagger}|0\rangle\ e^{-E_{K^*}|x_0-y_0|}$$

$B \to K^*$ form factor definitions

$$
\begin{aligned}
\langle K^*(p',\varepsilon)|\bar{s}\gamma^\mu b|B(p)\rangle &= \frac{2iV}{M_B+M_{K^*}}\,\epsilon^{\mu\nu\rho\sigma}\varepsilon^*_\nu\,p'_\rho p_\sigma\,,\\
\langle K^*(p',\varepsilon)|\bar{s}\gamma^\mu\gamma_5 b|B(p)\rangle &= 2M_{K^*}A_0\,\frac{\varepsilon^*\cdot q}{q^2}\,q^\mu\\
&\quad +(M_B+M_{K^*})\,A_1\left[\varepsilon^{*\mu}-\frac{\varepsilon^*\cdot q}{q^2}\,q^\mu\right]\\
&\quad -A_2\,\frac{\varepsilon^*\cdot q}{M_B+M_{K^*}}\left[p^\mu+p'^\mu-\frac{M_B^2-M_{K^*}^2}{q^2}\,q^\mu\right],\\
q^\nu\langle K^*(p',\varepsilon)|\bar{s}\sigma_{\mu\nu}b|B(p)\rangle &= 4\,T_1\epsilon_{\mu\rho\kappa\sigma}\varepsilon^{*\rho}p^\kappa p'^\sigma\,,\\
q^\nu\langle K^*(p',\varepsilon)|\bar{s}\sigma_{\mu\nu}\gamma_5 b|B(p)\rangle &= -2i\,T_2\left[\varepsilon^*_\mu(M_B^2-M_{K^*}^2)-(\varepsilon^*\cdot q)(p+p')_\mu\right]\\
&\quad -2i\,T_3(\varepsilon^*\cdot q)\left[q_\mu-\frac{q^2}{M_B^2-M_{K^*}^2}(p+p')_\mu\right].
\end{aligned}
$$

Predicted L-dependence of spectrum

For single ρ-like resonance at zero momentum:

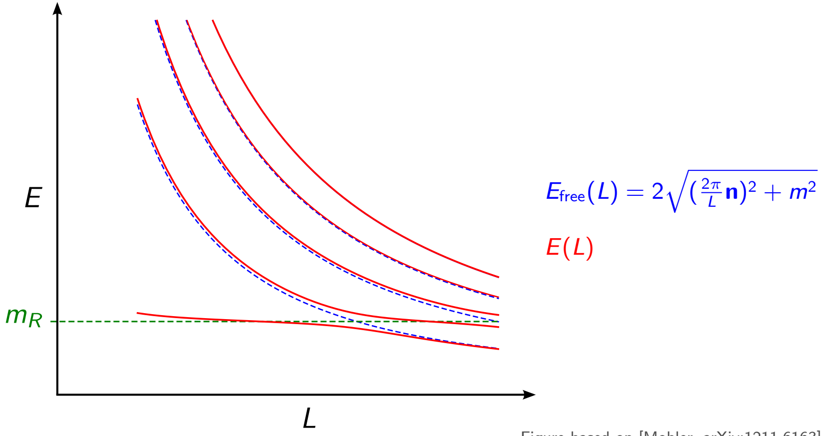
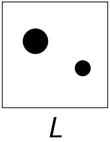


Figure based on [Mohler, arXiv:1211.6163]

Lüscher formula

Elastic scattering phase shifts from finite-volume energy shifts:



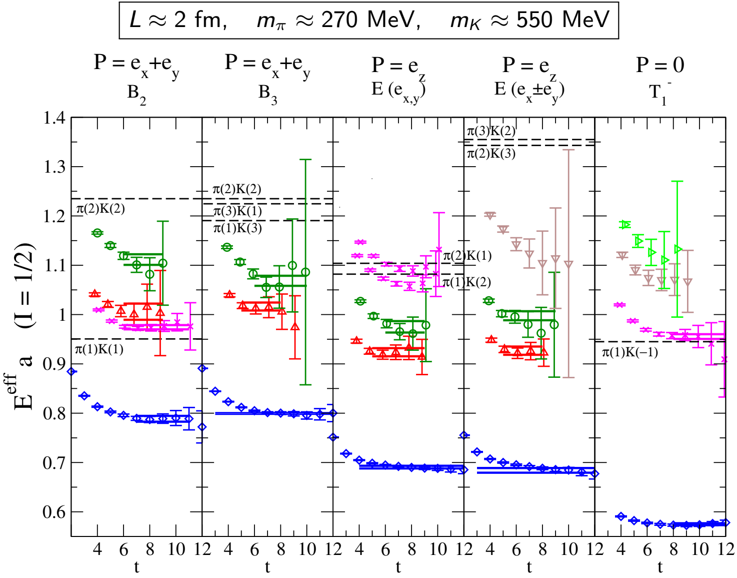
Define $p_*(L)$ through

$$E(L) = \sqrt{p_*^2 + m_\pi^2} + \sqrt{p_*^2 + m_K^2}$$

Lüscher's formula relates p^* to the scattering phase shift:

$$\delta(p^*) = f_{\text{Luscher}}(p^*, L)$$

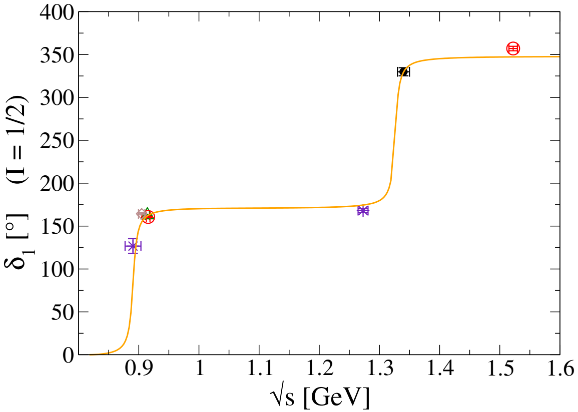
$I = 1/2$ P-wave $K\,\pi$ scattering in lattice QCD



[Prelovsek, Leskovec, Lang, Mohler, arXiv:1307.0736]

$I = 1/2$ P-wave $K\,\pi$ scattering in lattice QCD

$L \approx 2$ fm, $m_\pi \approx 270$ MeV, $m_K \approx 550$ MeV

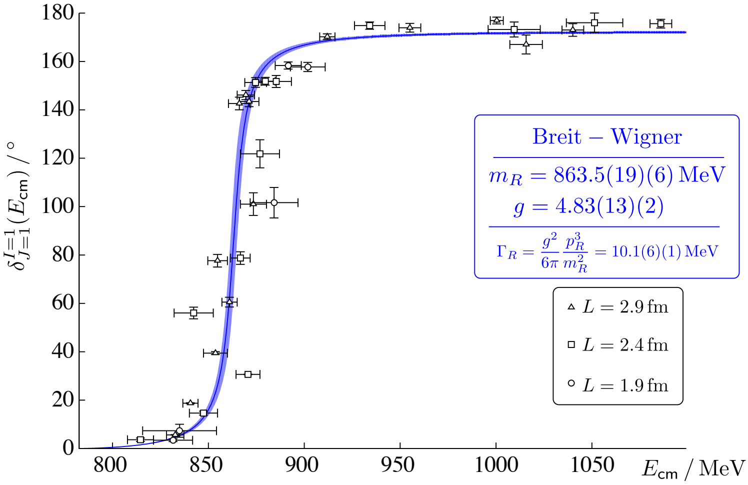


$\Rightarrow g_{K^* K\pi} = 5.7 \pm 1.6$ (experiment : 5.72 ± 0.06)

[Prelovsek, Leskovec, Lang, Mohler, arXiv:1307.0736]

$I = 1$ P-wave $\pi\,\pi$ scattering in lattice QCD

$m_\pi \approx 400$ MeV



[Dudek, Edwards, Thomas, arXiv:1212.0830]

Matrix elements: resonance or two-hadron?

Two possibilities:

- Aim to compute
$$\langle K(p_K)\pi(p_\pi)|\bar{s}\Gamma b|B(p_B)\rangle$$
on the lattice, using a generalization of the Lellouch-Lüscher approach
[hep-lat/0003023]

- Aim to compute "resonance matrix elements", which are defined in the continuum through analytic continuation of the $B \to K^*$ three-point function to complex momentum
$$p_{K^*}^2 = m_{K^*}^2 - \Gamma_{K^*}^2/4 - i\, m_{K^*}\Gamma_{K^*}$$
[Bernard, Hoja, Meißner, Rusetsky, arXiv:1205.4642]

Modelling the effects of the $\psi(3770)$ and $\psi(4160)$, part I

Replace

$$T_9(q^2) \quad \longrightarrow \quad T_9(q^2) \; + \; \frac{(R_1 + i I_1) m_1^2}{q^2 - m_1^2 + i m_1 \Gamma_1} + \frac{(R_2 + i I_2) m_2^2}{q^2 - m_2^2 + i m_2 \Gamma_2}$$

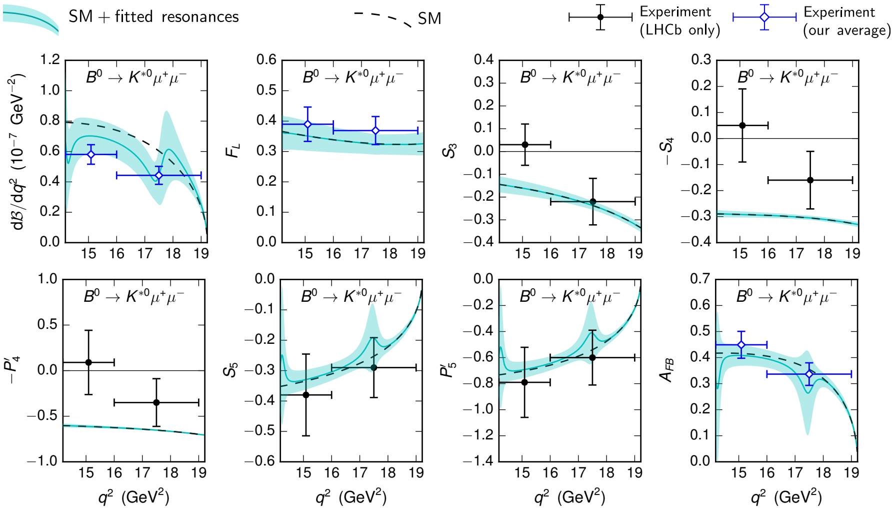
with

$$\begin{aligned} m_1 &= 3772 \text{ MeV}, \\ \Gamma_1 &= 30 \text{ MeV}, \\ m_2 &= 4190 \text{ MeV}, \\ \Gamma_2 &= 72 \text{ MeV}. \end{aligned}$$

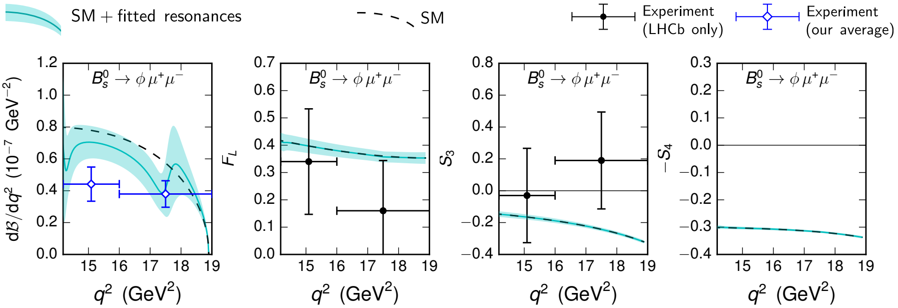
Set $C_9^{\rm NP} = 0$, $C_9' = 0$ and fit R_1, I_1, R_2, I_2 to the experimental data

(assuming that these couplings are the same
for $B \to K^* \mu^+ \mu^-$ and $B_s \to \phi \mu^+ \mu^-$).

Modelling the effects of the $\psi(3770)$ and $\psi(4160)$, part I



Modelling the effects of the $\psi(3770)$ and $\psi(4160)$, part I



Modelling the effects of the $\psi(3770)$ and $\psi(4160)$, part I

The fit gives

$$\begin{aligned} R_1 &= -0.008 \pm 0.020 \\ I_1 &= -0.013 \pm 0.022 \\ R_2 &= 0.034 \pm 0.036 \\ I_2 &= -0.024 \pm 0.032 \end{aligned}$$

and has

$$\Delta\chi^2 = 2.3$$

For comparison, the fit of C_9^{NP} and C_9' (without $c\bar{c}$ resonances) achieves

$$\Delta\chi^2 = 5.7$$

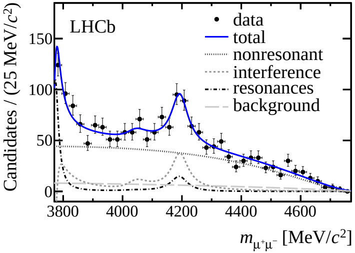
Modelling the effects of the $\psi(3770)$ and $\psi(4160)$, part II

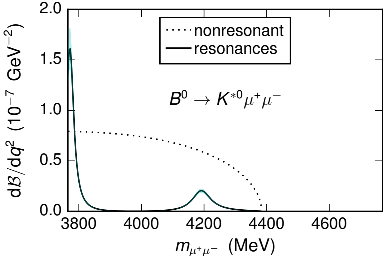
Alternative study: set

$$\begin{aligned} R_1 &= 0 \pm 0.045, \\ I_1 &= 0 \pm 0.045, \\ R_2 &= 0 \pm 0.04, \\ I_2 &= 0 \pm 0.04 \end{aligned}$$

(treated as independent parameters for $B \to K^* \mu^+ \mu^-$ and $B_s \to \phi \mu^+ \mu^-$) and perform fit of C_9^{NP} and C_9'.

The above limits on the magnitudes are chosen to match LHCb's fit in $B^+ \to K^+ \mu^+ \mu^-$:





Modelling the effects of the $\psi(3770)$ and $\psi(4160)$, part II