Form factors for $B \to V$ decays:

updated LCSR results and related fits

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Flavor of New Physics in $b \to s$ transitions

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Work in progress with David Straub and Roman Zwicky

Why calculate form factors for exclusive $B \to V$?

and why is an update required?

- Exclusive decays, to which the LHC is more sensitive (e.g. $B \to K^*$), require form factors: non-perturbative quantities
- Need accurate FFs to detect NP via $B \to K^*$ or $B_s \to \phi$ or measure $|V_{ub}|$ via $B \to \rho$, $B_s \to K^*$
- LCSR[1] at low q^2, Lattice[2] at high q^2
- Best coverage in q^2: fit to LCSR and Lattice using e.g. series expansion, coefficients satisfy dispersive bounds.[3]

Critical point: correlated errors between FFs not yet available...

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[1] see e.g. P. Ball and R. Zwicky, Phys. Rev. D **71** (2005) 014015 [arXiv:hep-ph/0406232] and Phys. Rev. D **71** (2005) 014029 [arXiv:hep-ph/0412079]

[2] see e.g. A. Al-Haydari *et al.* [QCDSF Collaboration], Eur. Phys. J. A **43**, 107 (2010) [arXiv:0903.1664 [hep-lat]]

[3] AB, T. Feldmann, M. Wick, JHEP **1009** (2010) 090 [arXiv:1004.3249 [hep-ph]]

Form Factor Definitions

The standard basis

$V(q^2)$, $A_{0-3}(q^2)$, $T_{1-3}(q^2)$ conventionally defined: (note $A_0(0) = A_3(0)$, $T_1(0) = T_2(0)$)

$$
\begin{aligned}
\langle V(k,\varepsilon)|\bar{q}\gamma_\mu b|\bar{B}(p)\rangle &= i\epsilon_{\mu\nu\rho\sigma}\,\varepsilon^{*\nu}(k)\,p^\rho k^\sigma\,\frac{2V(q^2)}{m_B+m_V}\,,\\
\langle V(k,\varepsilon)|\bar{q}\gamma_\mu\gamma_5 b|\bar{B}(p)\rangle &= -\varepsilon^*_\mu(k)\,(m_B+m_V)\,A_1(q^2) + (p+k)_\mu\,(\varepsilon^*(k)\cdot q)\,\frac{A_2(q^2)}{m_B+m_V}\\
&\quad + q_\mu\,(\varepsilon^*(k)\cdot q)\,\frac{2m_V}{q^2}\left(A_3(q^2)-A_0(q^2)\right)\,,\\
\langle V(k,\varepsilon)|\bar{q}\sigma_{\mu\nu}q^\nu b|\bar{B}(p)\rangle &= i\epsilon_{\mu\nu\rho\sigma}\,\varepsilon^{*\nu}\,p^\rho k^\sigma\,2T_1(q^2)\,,\\
\langle V(k,\varepsilon)|\bar{q}\sigma_{\mu\nu}q^\nu\gamma_5 b|\bar{B}(p)\rangle &= T_2(q^2)\left(\varepsilon^*_\mu(k)\,(m_B^2-m_V^2) - (\varepsilon^*(k)\cdot q)\,(p+k)_\mu\right)\\
&\quad + T_3(q^2)(\varepsilon^*(k)\cdot q)\left(q_\mu - \frac{q^2}{m_B^2-m_V^2}\,(2p-q)_\mu\right)\,,
\end{aligned}
$$

Due to EOM for quarks, $A_3(q^2) = \frac{m_B+m_V}{2m_V}\,A_1(q^2) - \frac{m_B-m_V}{2m_V}\,A_2(q^2)\,,$

$\Rightarrow$ Only 7 independent FFs

Definition of Helicity Amplitudes

Vector current

$$\mathcal{B}_{V,\sigma}(q^2) = \sqrt{\frac{q^2}{\lambda}} \sum_{\varepsilon(k)} \varepsilon_\sigma^{*\mu}(q) \, \langle V(k,\varepsilon(k))| \bar{q}\, \gamma_\mu (1-\gamma^5)\, b |\bar{B}(p)\rangle$$

with

$$\mathcal{B}_{V,0}(q^2) = \frac{(m_B+m_V)^2\,(m_B^2-m_V^2-q^2)\,A_1(q^2) - \lambda\, A_2(q^2)}{2m_V\sqrt{\lambda}\,(m_B+m_V)}\,,$$

$$\mathcal{B}_{V,t}(q^2) = A_0(q^2)\,,$$

$$\mathcal{B}_{V,1}(q^2) \equiv -\frac{\mathcal{B}_{V,-}-\mathcal{B}_{V,+}}{\sqrt{2}} = \frac{\sqrt{2\,q^2}}{m_B+m_V}\,V(q^2)\,,$$

$$\mathcal{B}_{V,2}(q^2) \equiv -\frac{\mathcal{B}_{V,-}+\mathcal{B}_{V,+}}{\sqrt{2}} = \frac{\sqrt{2\,q^2}\,(m_B+m_V)}{\sqrt{\lambda}}\,A_1(q^2)\,.$$

Definition of Helicity Amplitudes

Tensor current

$$\mathcal{B}_{T,\sigma}(q^2) = \sqrt{\frac{1}{\lambda}} \sum_{\varepsilon(k)} \varepsilon^{*\mu}_{\sigma}(q) \, \langle V(k,\varepsilon(k))|\bar{q}\,\sigma_{\mu\alpha} q^{\alpha} (1+\gamma^5)\, b|\bar{B}(p)\rangle$$

$$\mathcal{B}_{T,0}(q^2) = \frac{\sqrt{q^2}\,(m_B^2 + 3m_V^2 - q^2)}{2m_V\sqrt{\lambda}}\, T_2(q^2) - \frac{\sqrt{q^2\,\lambda}}{2m_V\,(m_B^2 - m_V^2)}\, T_3(q^2)$$

$$\mathcal{B}_{T,1}(q^2) = -\frac{\mathcal{B}_{V,-} - \mathcal{B}_{V,+}}{\sqrt{2}} = \sqrt{2}\, T_1(q^2)\,,$$

$$\mathcal{B}_{T,2}(q^2) = -\frac{\mathcal{B}_{V,-} + \mathcal{B}_{V,+}}{\sqrt{2}} = \frac{\sqrt{2}\,(m_B^2 - m_V^2)}{\sqrt{\lambda}}\, T_2(q^2)\,.$$

Calculating the FFs at low q^2?

Non-perturbative techniques

- Appropriate method for low q^2 is LCSR
- However, using full form factors from LCSR correlations in errors between form factors not available.
- Many people resorting to using soft form factors with corrections in order to include correlations[4]
- This can be improved if correlated FFs available: **Our Aim**
- Can fit LCSR and Lattice: Results valid in both low and high q^2 regimes

Burning question: how does one calculate FFs in LCSR?[5]

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[4] e.g. S. Descotes-Genon, T. Hurth, J. Matias and J. Virto, JHEP **1305** (2013) 137 [arXiv:1303.5794 [hep-ph]], S. Jaeger and J. Martin Camalich, JHEP **1305** (2013) 043 [arXiv:1212.2263 [hep-ph]].

[5] P. Ball and R. Zwicky, hep-ph/0406232, Phys. Rev. D **71**, 014015 (2005), P. Ball and R. Zwicky, hep-ph/0412079, Phys. Rev. D **71**, 014029 (2005)

What is LCSR?

taking the example of f_+ for $B \to \pi$

On one hand....

In physical region, correlator dominated by B pole:

$$\Pi_\mu = i\, m_b \int d^D x e^{-i\, p_B \cdot x} \langle \pi(p) | T\{\bar{u}(0)\gamma_\mu b(0) \bar{b}(x) i\gamma_5 d(x)\} | 0 \rangle,$$
$$= (p_B + p)_\mu \Pi_+(p_B^2, q^2) + (p_B - p)_\mu \Pi_-(p_B^2, q^2).$$

into

$B \to \pi$ transition $(f_+(q^2))$

$\langle \pi(p) | \bar{u}\gamma_\mu b | B(p_B) \rangle = (p_B + p)_\mu f_+(q^2) + (p_B - p)_\mu f_-(q^2)$

B meson decay (f_B)

$m_b \langle 0 | \bar{d} i\gamma_5 b | B \rangle = m_B^2 f_B$

$$\Pi_+(p_B^2, q^2) = f_B m_B^2 \frac{f_+(q^2)}{m_B^2 - p_B^2} + \int_{s > m_B^2} ds \frac{\rho_{\rm had}}{s - p_B^2},$$

($\rho_{\rm had}$ is spectral density of the higher-mass hadronic states)

What is LCSR?

on the other hand..

In Euclidean region ($p_B^2 - m_B^2$ is large and negative): light-cone expand about $x^2 = 0$[6]

$$\Pi_+(p_B^2, q^2) = \sum_n \int du\, \mathcal{T}_+{}^{(n)}(u, p_B^2, q^2, \mu^2)\phi^{(n)}(u, \mu^2) = \int ds \frac{\rho_{\rm LC}}{s - p_B^2},$$

- $\mathcal{T}_+^{(n)}(u, \mu^2)$: perturbatively calculable hard kernels
- $\phi^{(n)}(u, \mu^2)$: non-perturbative LCDAs, twist n
- $\langle \pi(p)|\bar{u}(0)\gamma_\mu\gamma_5\, d(x)|0\rangle = -if_\pi p_\mu \int_0^1 du\, e^{i\bar{u}p\cdot x}\phi(u, \mu^2) + \ldots,$
- $\phi(u, \mu^2) = 6u(1-u)\sum_{n=0}^{\infty} a_n(\mu^2) C_n^{3/2}(2u-1)$

[6] Factorisation theorem not proven to all orders, verified at given order by cancellation of IR and soft divergences

What is LCSR?

....which leads to the sum rule

Above the continuum threshold s_0, a continuum of states contribute and approximation of quark-hadron duality is thought to be reasonable, such that

$$\rho_{\rm had} = \rho_{\rm LC}\,\Theta(s - s_0).$$

Subtracting from both sides, and Borel transforming (M^2=Borel parameter):

Sum rule for $f_+(q^2)$

$$f_+(q^2) = \frac{1}{f_B m_B^2} \int_{m_b^2}^{s_0} ds\, \rho_{\rm LC}\, e^{-(s-m_B^2)/M^2},$$

Parameters and uncertainties

Choosing s_0 and $M2$

We carefully choose the sum rules parameters using the following:

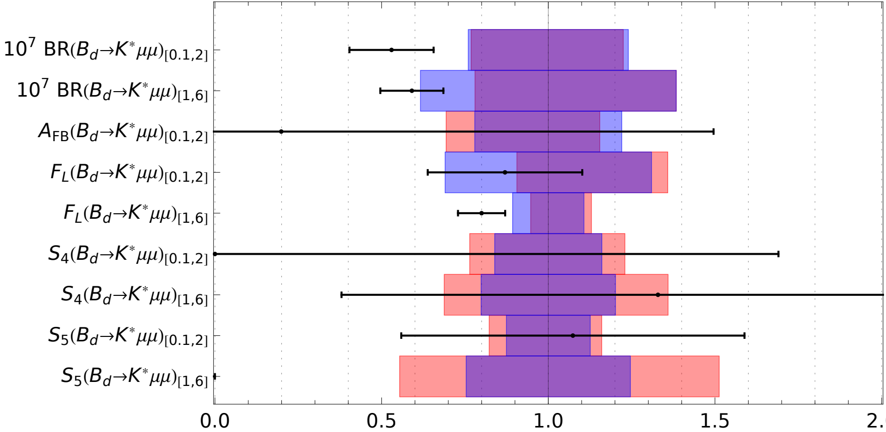
- SR depends little on, but is clear extremum as fn of s_0, M^2;
- SR for m_B, (differentiate SR by M^{-2}), fulfilled to 0.1%;
- the continuum contribution is under control, i.e. integral of the spectral density between s_0 and ∞ should be $\sim$25-30% of the B contribution, between m_b^2 and s_0;

Dominant uncertainties arise due to varying the following:

- the continuum threshold s_0 by $\pm 0.5\,\mathrm{GeV}^2$ and the Borel parameter M_2 by $\pm 1.2\,\mathrm{GeV}^2$;
- the condensates $\langle \bar{q}q\rangle = (-0.24 \pm 0.01)^3 \mathrm{GeV}^3$, $\frac{\langle \bar{q}\sigma gGq\rangle}{\langle \bar{q}q\rangle} = (0.8 \pm 0.2)$
- the twist-3 parameter η_3 by $\pm 50\%$;
- the factorisation scale in the range $\mu/2$ to 2μ.

Results (Preliminary)

Do the correlations affect observables?[7]



[7] Thanks to David Straub for preparing this at short notice

Summary

and Outlook

Correlated errors for $B \to K^*$ form factors:

- Prevent community from resorting to soft form factors
- Include the factorizable $\mathcal{O}(1/m_B)$ corrections

Thanks for listening![8]

[8] and to: the organisers for the great conference;David Straub for some great plots; and Flip Tanedo for letting me use his beamer theme

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- Latest input parameters, $B \to V$ form factors
- Full correlated errors and fit with Lattice using various parameterizations

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Things for the future:

- Full paper to appear in the coming months....

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Sum rules for m_B

$$m_B^2 = \int_{m_b^2}^{s_0} ds\, s\, \rho^{\mathrm{tot}}(s) / \int_{m_b^2}^{s_0} ds\, \rho^{\mathrm{tot}}(s).$$