# An Overview of $B \rightarrow K^{(*)} \mu^{+} \mu^{-}$: What shall we expect? 

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## Paris

June 2, 2014

## PLAN of the TALK

- Theoretical and experimental overview of the 4 -body decay $B \rightarrow K^{*}(\rightarrow K \pi) I^{+} I^{-}$mode in terms of an optimal basis of CP conserving observables: $P_{i}^{\left({ }^{\prime}\right)}$. Implications of BR of $B \rightarrow K^{*} \mu^{+} \mu^{-}$and $B \rightarrow K \mu^{+} \mu^{-}$on the analysis.
- Possible explanations for the observed tensions with the SM.
- A consistency test on data on $B \rightarrow K^{*} \mu^{+} \mu^{-}$for future analysis. A new relation between the zero of $A_{F B}$ and the anomaly in $P_{5}^{\prime}$.


## Future? ...

The angular distribution $\overline{\mathbf{B}}_{\mathbf{d}} \rightarrow \overline{\mathbf{K}}^{* \mathbf{0}}\left(\rightarrow \mathbf{K}^{-} \pi^{+}\right) \mathbf{I}^{+} \mathbf{I}^{-}$with the $K^{* 0}$ on the mass shell. It is described by $\mathbf{s}=\mathbf{q}^{\mathbf{2}}$ and three angles $\theta_{\ell}, \theta_{\mathrm{K}}$ and $\phi$

$$
\frac{d^{4} \Gamma\left(\bar{B}_{d}\right)}{d q^{2} d \cos \theta_{\ell} d \cos \theta_{K} d \phi}=\frac{9}{32 \pi} \mathbf{J}\left(\mathbf{q}^{2}, \theta_{\ell}, \theta_{K}, \phi\right)=\frac{9}{32 \pi} \sum_{i} J_{i}\left(q^{2}\right) f_{i}\left(\theta_{\ell}, \theta_{K}, \phi\right)
$$


$\theta_{\ell}$ : Angle of emission between $\bar{K}^{* 0}$ and $\mu^{-}$in di-lepton rest frame.
$\theta_{\mathrm{K}}$ : Angle of emission between $\bar{K}^{* 0}$ and $K^{-}$in di-meson rest frame.
$\phi$ : Angle between the two planes.
$\mathbf{q}^{\mathbf{2}}$ : dilepton invariant mass square.

Notice LHCb uses $\theta_{\ell}^{L H C b}=\pi-\theta_{\ell}^{u s}$

- large recoil for $K^{*}: E_{K^{*}} \gg \Lambda_{Q C D}$ or $4 m_{\ell}^{2} \leq q^{2}<9 \mathrm{GeV}^{2}$

Three regions in $q^{2}$ :

- resonance region ( $q^{2}=m_{J / \Psi}^{2}, \ldots$ ) betwen $9<q^{2}<14 \mathrm{GeV}^{2}$.
- low-recoil for $K^{*}: E_{K^{*}} \sim \Lambda_{Q C D}$ or $14<q^{2} \leq\left(m_{B}-m_{K^{*}}\right)^{2}$.

1 Use $J_{k} \rightarrow \operatorname{Re}\left(A_{i}^{L} A_{j}^{L} \pm A_{i}^{R} A_{j}^{R}\right)$ or $\operatorname{Im}\left(A_{i}^{L} A_{j}^{L} \pm A_{i}^{R} A_{j}^{R}\right)$

$$
\text { However } J_{k} \propto \xi_{\perp, \|}^{2} \Rightarrow \text { Theory uncertainty very large. }
$$

2 Use $S_{i} \rightarrow\left(J_{i}+\bar{J}_{i}\right) /(d \Gamma+d \bar{\Gamma})$ and $A_{i} \rightarrow\left(J_{i}-\bar{J}_{i}\right) /(d \Gamma+d \bar{\Gamma})$
However: - Sensitivity at LO to soft form factors $\xi_{\perp, \|} \Downarrow$

- Scheme dependence when SFF are used $\Downarrow$ but not if pc are included $\Uparrow$
- Dependence on problematic form factor $A_{0}$ albeit $m_{\ell}$ suppressed $\Downarrow$
- $m_{\ell}$ dependence on $\beta_{\ell}$ but also $m_{\ell}$ terms. $\Uparrow \Downarrow$

3 Use $P_{k}^{(\prime)} \rightarrow \operatorname{Re} \tilde{A}_{i} \tilde{A}_{j} / \sqrt{\left|\tilde{A}_{i}\right|^{2}\left|\tilde{A}_{j}\right|^{2}}$ (or Im) with $\tilde{A}_{i} \rightarrow A_{i}+\bar{A}_{i}$

$$
P_{k}^{C P} \rightarrow \operatorname{Re} \hat{A}_{i} \hat{A}_{j} / \sqrt{\left|\tilde{A}_{i}\right|^{2}\left|\tilde{A}_{j}\right|^{2}} \text { (or Im) with } \hat{A}_{i} \rightarrow A_{i}-\bar{A}_{i}
$$

However: - Sensitivity only at NLO to soft form factors $\xi_{\perp, \|} \Uparrow$

- Marginal scheme dependence when SFF are used $\Uparrow$ with or without pc.
- No dependence on problematic form factor $A_{0} \Uparrow$
- Some observables identical for $m_{\ell}=e, \mu, \tau \Rightarrow$ excellent cross check . $\Uparrow$
$P_{i}, P_{i}^{\prime}$ defines an Optimal Basis of observables, a compromise between:
- I. Excellent experimental accessibility and simplicity of the fit.
- II. Reduced FF dependence (in the large-recoil region: $0.1 \leq q^{2} \leq 8 \mathrm{GeV}^{2}$ ).

Our proposal for CP-conserving basis:

$$
\left\{\frac{\mathbf{d} \boldsymbol{\Gamma}}{\mathbf{d q}}, \mathbf{A}_{\mathbf{F B}}, \mathbf{P}_{\mathbf{1}}, \mathbf{P}_{\mathbf{2}}, \mathbf{P}_{\mathbf{3}}, \mathbf{P}_{4}^{\prime}, \mathbf{P}_{\mathbf{5}}^{\prime}, \mathbf{P}_{6}^{\prime}\right\} \text { or } \mathbf{P}_{\mathbf{3}} \leftrightarrow \mathbf{P}_{8}^{\prime} \text { and } \mathbf{A}_{\mathrm{FB}} \leftrightarrow \mathbf{F}_{\mathrm{L}}
$$

where $P_{1}=A_{T}^{2}$ [Kruger, J.M'05],

```
\[
P_{2}=\frac{1}{2} A_{T}^{\mathrm{re}}, P_{3}=-\frac{1}{2} A_{T}^{\mathrm{im}} \text { [Becirevic, Schneider'12] }
\]
\[
P_{4,5,6}^{\prime} \text { [Descotes, JM, Ramon, Virto'13]). }
\]
```

The corresponding CP-violating basis $\left(J_{i}+\bar{J}_{i} \rightarrow J_{i}-\bar{J}_{i}\right.$ in numerators $)$ :

$$
\left\{\mathbf{A}_{\mathrm{CP}}, \mathbf{A}_{\mathrm{FB}}^{\mathrm{CP}}, \mathbf{P}_{1}^{\mathrm{CP}}, \mathbf{P}_{2}^{\mathrm{CP}}, \mathbf{P}_{3}^{\mathrm{CP}}, \mathbf{P}_{4}^{\prime \mathrm{CP}}, \mathbf{P}_{5}^{\prime \mathrm{CP}}, \mathbf{P}_{6}^{\prime \mathrm{CP}}\right\} \text { or } \mathbf{P}_{3}^{\mathrm{CP}} \leftrightarrow \mathbf{P}_{8}^{\prime \mathrm{CP}} \text { and } \mathbf{A}_{\mathrm{FB}}^{\mathrm{CP}} \leftrightarrow \mathbf{F}_{\mathrm{L}}^{\mathrm{CP}}
$$

## Analysis of new LHCb data On

$$
B \rightarrow K^{*} \mu^{+} \mu^{-}
$$

Present bins: $[0.1,2],[2,4.3],[4.3,8.68],[1,6],[14.18,16],[16,19] \mathrm{GeV}^{2}$.

| Observable | Experiment | SM prediction | Pull |  |
| :---: | :---: | :---: | :---: | :---: |
| $\left\langle P_{1}\right\rangle$ | $-0.19_{-0.35}^{+0.40}$ | $0.007_{-0.044}^{+0.043}$ | -0.5 | (large error bars) |
| $\left\langle P_{1}\right\rangle_{[2,4.3]}$ | $-0.29_{-0.46}^{+0.65}$ | $-0.051_{-0.046}^{+0.046}$ | -0.4 |  |
| $\left\langle P_{1}\right\rangle_{[4.3,8.68]}$ | $0.36_{-0.31}^{+0.30}$ | $-0.117_{-0.052}^{+0.056}$ | +1.5 | ver value of the second and |
| $\left\langle P_{1}\right\rangle_{[1,6]}$ | $0.15_{-0.41}^{+0.39}$ | $-0.055_{-0.043}^{+0.041}$ | +0.5 | ird bins of $A_{\text {FB }}$ is consist |
| $\left\langle P_{2}\right\rangle_{[0.1,2]}$ | $0.03_{-0.15}^{+0.14}$ | $0.172_{-0.02}^{+0.020}$ | -1.0 | $2.9 \sigma(1.7 \sigma)$ deviation in cond (third) bin of $P_{2}$. |
| $\left\langle P_{2}\right\rangle_{[2,4.3]}$ | $0.50_{-0.07}^{+0.00}$ | $0.234_{-0.086}^{+0.060}$ | +2.9 |  |
| $\left\langle P_{2}\right\rangle_{[4.3,8.68]}$ | $-0.25_{-0.08}^{+0.07}$ | $-0.407_{-0.037}^{+0.049}$ | +1.7 |  |
| $\left\langle P_{2}\right\rangle_{[1,6]}$ | $0.33_{-0.12}^{+0.11}$ | $0.084_{-0.078}^{+0.060}$ | +1.8 | $A_{\mathrm{FB}}$ (same as the zero of |
| $\left\langle A_{\text {FB }}\right\rangle_{[0.1,2]}$ | $-0.02_{-0.13}^{+0.13}$ | $-0.136_{-0.048}^{+0.051}$ | +0.8 | Both effects can be |
| $\left\langle A_{\text {FB }}\right\rangle_{[2,4.3]}$ | $-0.20_{-0.08}^{+0.08}$ | $-0.081_{-0.069}^{+0.055}$ | -1.1 | accommodated with $\mathcal{C}_{7}^{\mathbb{N P}}<0$ |
| $\left\langle A_{\text {FB }}\right\rangle_{[4.3,8.68]}$ | $0.16_{-0.05}^{+0.06}$ | $0.220_{-0.113}^{+0.138}$ | -0.5 | and/or $\mathcal{C}_{9}^{\mathrm{NP}}<0$. |
| $\left\langle A_{\text {FB }}\right\rangle_{[1,6]}$ | $-0.17_{-0.06}^{+0.06}$ | $-0.035_{-0.034}^{+0.037}$ | -2.0 |  |


| Observable | Experiment | SM prediction | Pull |
| :---: | :---: | :---: | :---: |
| $\left\langle P_{4}^{\prime}\right\rangle_{[0.1,2]}$ | $0.00_{-0.52}^{+0.52}$ | $-0.342_{-0.026}^{+0.031}$ | +0.7 |
| $\left\langle P_{4}^{\prime}\right\rangle_{[2,4.3]}$ | $0.74{ }_{-0.60}^{+0.54}$ | $0.569_{-0.063}^{+0.073}$ | +0.3 |
| $\left\langle P_{4}^{\prime}\right\rangle_{[4.3,8.68]}$ | $1.18{ }_{-0.32}^{+0.26}$ | $1.003_{-0.032}^{+0.028}$ | +0.6 |
| $\left\langle P_{4}^{\prime}\right\rangle_{[1,6]}$ | $0.58{ }_{-0.36}^{+0.32}$ | $0.555_{-0.058}^{+0.067}$ | +0.1 |
| $\left\langle P_{5}^{\prime}\right\rangle_{[0.1,2]}$ | $0.45{ }_{-0.24}^{+0.21}$ | $0.533_{-0.041}^{+0.033}$ | -0.4 |
| $\left\langle P_{5}^{\prime}\right\rangle_{[2,4.3]}$ | $0.29{ }_{-0.39}^{+0.40}$ | $-0.334_{-0.113}^{+0.097}$ | +1.6 |
| $\left\langle P_{5}^{\prime}\right\rangle_{[4.3,8.68]}$ | $-0.19_{-0.16}^{+0.16}$ | $-0.872_{-0.041}^{+0.053}$ | +4.0 |
| $\left\langle P_{5}^{\prime}\right\rangle_{[1,6]}$ | $0.21_{-0.21}^{+0.20}$ | $-0.349_{-0.100}^{+0.088}$ | +2.5 |
| $\left\langle P_{4}^{\prime}\right\rangle_{[14.18,16]}$ | $-0.18_{-0.70}^{+0.54}$ | $1.161_{-0.332}^{+0.190}$ | -2.1 |
| $\left\langle P_{4}^{\prime}\right\rangle_{[16,19]}$ | $0.70_{-0.52}^{+0.44}$ | $1.263_{-0.248}^{+0.119}$ | -1.1 |
| $\left\langle P_{5}^{\prime}\right\rangle_{[14.18,16]}$ | $-0.79_{-0.22}^{+0.27}$ | $-0.779_{-0.363}^{+0.328}$ | +0.0 |
| $\left\langle P_{5}^{\prime}\right\rangle_{[16,19]}$ | $-0.60{ }_{-0.18}^{+0.21}$ | $-0.601_{-0.367}^{+0.282}$ | +0.0 |

## Definition of the anomaly:

- $\mathbf{P}_{\mathbf{5}}^{\prime}$ : There is a striking $4.0 \sigma(1.6 \sigma)$ deviation in the third (second) bin of $P_{5}^{\prime}$.

Consistent with large negative contributions in $\mathcal{C}_{7}^{\mathrm{NP}}$ and/or $\mathcal{C}_{9}^{\mathrm{NP}}$.

- $\mathbf{P}_{4}^{\prime}$ : in agreement with the SM , but within large uncertainties, and it has future potential to determine the sign of $\mathcal{C}_{10}^{\mathrm{NP}}$.
- $\mathbf{P}_{6}^{\prime}$ and $\mathbf{P}_{8}^{\prime}$ : show small deviations with respect to the SM, but such effect would require complex phases in $\mathcal{C}_{9}^{\mathrm{NP}}$ and/or $\mathcal{C}_{10}^{\mathrm{NP}}$.

Us: $(-0.19-(-0.872)) / \sqrt{0.16^{2}+0.053^{2}}=4.05$ and Exp: $(-0.19-(-0.872+0.053)) / \sqrt{0.16^{2}+0.053^{2}}=3.73$

## Our SM predictions+LHCb data



Figure: Experimental measurements and SM predictions for some $B \rightarrow K^{*} \mu^{+} \mu^{-}$observables. The black crosses are the experimental LHCb data. The blue band corresponds to the SM predictions for the differential quantities, whereas the purple boxes indicate the corresponding binned observables.

Goal: Determine the Wilson coefficients $\mathcal{C}_{7,9,10}, \mathcal{C}_{7,9,10}^{\prime}: \mathcal{C}_{i}=\mathcal{C}_{i}^{S M}+\mathcal{C}_{i}^{N P}$
Standard $\chi^{2}$ frequentist approach: Asymmetric errors included, estimate theory uncertainties for each set of $\mathcal{C}_{i}^{N P}$ and all uncertainties are combined in quadrature.

We do three analysis: a) large-recoil data b) large+low-recoil data c) [1-6] bin Observables:

- $B \rightarrow K^{*} \mu^{+} \mu^{-}$: We take observables $P_{1}, P_{2}, P_{4}^{\prime}, P_{5}^{\prime}, P_{6}^{\prime}, P_{8}^{\prime}$ and $A_{F B}$ in the following binning: -large-recoil: $[0.1,2],[2,4.3],[4.3,8.68] \mathrm{GeV}^{2}$.
-low recoil: $[14.18,16]$, $[16,19] \mathrm{GeV}^{2}$ -wide large-recoil bin: $[1,6] \mathrm{GeV}^{2}$.
- Radiative and dileptonic $B$ decays: $\mathcal{B}\left(B \rightarrow X_{s} \gamma\right)_{E_{\gamma}>1.6 \mathrm{GeV}}, \mathcal{B}\left(B \rightarrow X_{s} \mu^{+} \mu^{-}\right)_{[1,6]}$ and $\mathcal{B}\left(B_{s} \rightarrow \mu^{+} \mu^{-}\right), A_{l}\left(B \rightarrow K^{*} \gamma\right)$ and the $B \rightarrow K^{*} \gamma$ time-dependent CP asymmetry $S_{K^{*} \gamma}$

In conclusion our pattern of [PRD88 (2013) 074002] obtained from an $\mathcal{H}_{\text {eff }}$ approach is $\mathrm{C}_{9}^{\mathrm{NP}} \sim[-1.6,-0.9]>3 \sigma, \mathrm{C}_{7}^{\text {NP }} \sim[-\mathbf{0 . 0 5},-0.01]$ at $2 \sigma, \mathrm{C}_{9}^{\prime} \sim \pm \delta(\mathrm{SM}) \quad \mathrm{C}_{10}, \mathrm{C}_{7,10}^{\prime} \sim \pm \epsilon(\mathrm{SM})$
where $\delta$ is small and compatible with zero (SM) and $\epsilon$ is smaller.
$\Rightarrow$ There is strong evidence for a $\mathcal{C}_{9}^{\mathrm{NP}}<\mathbf{0}$, preference for $\mathcal{C}_{7}^{\mathrm{NP}}<\mathbf{0}$ and no clear-cut evidence for $\mathcal{C}_{10,71,9,10}^{\mathrm{NP}} \neq 0$.

A simplified version is $C_{9}^{N P}=-1.2$ or -1.5 with $C_{9}^{S M}$ including all known em corrections.

## Best fit points we find in different scenarios:

Large recoil: $\mathrm{C}_{9}^{\mathrm{NP}}=-1.5, C_{7 \text { eff }}^{\mathrm{NP}}=-0.02$
Large recoil: $\mathrm{C}_{9}^{\mathrm{NP}}=-1.6, C_{7 \text { eff }}^{\mathrm{NP}}=-0.02, C_{10}^{\mathrm{NP}}>0, \mathbf{C}_{9 /}^{\mathrm{NP}}<\mathbf{0}, C_{7 \prime}^{\mathrm{NP}}>0, C_{10 \prime}^{\mathrm{NP}}<0$.
Large+Low: $\mathrm{C}_{9}^{\mathrm{NP}}=-1.2, C_{7 \text { eff }}^{\mathrm{NP}}=-0.03, C_{10}^{\mathrm{NP}}>0, \mathbf{C}_{9,}^{\mathrm{NP}}>\mathbf{0}, C_{7 \prime}^{\mathrm{NP}}<0, C_{10 \prime}^{\mathrm{NP}}<0$
$\rightarrow \mathrm{BR}$ are strongly dependent on FF choices (for lattice FF see S. Meinel's talk)
$\rightarrow$ We use them as cross check. $\rightarrow B \rightarrow K \mu \mu$ provides info on $C_{9}^{N P}+C_{9}^{\prime}$.

(left) $B R\left(B \rightarrow K^{*} \mu \mu\right)$ : blue curve is SM and red curve corresponds to $C_{9}^{N P}=-1.5$.

- $3 \mathrm{fb}^{-1}$ data on $B^{-} \rightarrow K^{-} \mu^{+} \mu^{-}$and $B^{0} \rightarrow K^{0} \mu^{+} \mu^{-}$exhibit a similar deficit w.r.t. SM at large-recoil.
- A $\mathbf{2}^{\text {nd }}$ proposed solution by Altmannshofer-Straub $-C_{9}^{N P} \sim C_{9}^{\prime} \sim 1$ constructed to reproduce the SM is in trouble. At large-recoil this $2^{\text {nd }}$ solution is in tension with the new $3 \mathrm{fb}^{-1}$ data of both modes.
- On the contrary this data is in very good agreement with our solution particularly at large but also at low-recoil. We use updated WC, full expressions for the BR including important quadratic terms.
[Hofer, Descotes, Mescia, JM, Virto]

1 Underestimated theory errors: Our plots include all known pieces in QCDF in SM.
There are three issues that can only be estimated:
i) Factorizable and non-factorizable power corrections (see J. Virto talk's).
ii) Charm-loop contributions.
iii) Quark-hadron duality violations at low-recoil.

We have verified that the "usual suspects" i) and ii) does not provide a successful explanation of the observed discrepancies at large-recoil. At low-recoil iii) will be important.

2 Experimental issue [statistical Fluctuation (we are still with $1 \mathrm{fb}^{-1}$ dataset), ...].
3 New Physics contribution mainly to the Wilson coefficient of the semileptonic operator $\mathrm{O}_{9}$. This is already discussed and it is the main conclusion of our analysis.

## OPTION 1:

## Theoretical uncertainties

There are basically two approaches:
$\Rightarrow$ Soft FF + NLO QCDF ( $\mathrm{O}\left(\alpha_{s}\right)$-factorizable+non-factorizable+weak annihilation) [BFS'01,'04]
[Descotes, Matias, Virto]

+ factorizable and non-factorizable power corrections [S. Descotes, L. Hofer, J. Matias, J. Virto]

$$
F=F^{\text {soft }}\left(\xi_{\perp}, \xi_{\|}\right)+\Delta F^{\alpha_{s}}+a_{F}+b_{F} \frac{q^{2}}{m_{B}^{2}}+c_{F} \frac{q^{4}}{m_{B}^{4}} \quad \text { and } \quad A_{i}=A_{i}^{0}+d_{F}+e_{F} \frac{q^{2}}{m_{B}^{2}}
$$

[Jaeger, Camalich]
Naive factorization (full form factors) + non-factorizable QCDF
(some weak annihilation neglected ) [Altmannshoffer, Ball, Barucha, Buras, Straub, Wick]

+ non-factorizable power corrections [S. Descotes, L. Hofer, J. Matias, J. Virto]
We have worked out the two options in three different schemes.
Conclusion: We find that the impact of power corrections is SUBSTANTIALLY smaller than claimed in previous works and far from explaining any anomaly.

A third complementary approach for a future analysis of data for a second round of LHCb data
$\Rightarrow$ Amplitude analysis (see Petridis's talk)

- Use symmetries to fix 4 real/imaginary parts of amplitudes to zero.
- Parametrize amplitudes by $A_{i}=\alpha_{i}+\beta_{i} q^{2}+\gamma_{i} / q^{2}$. How to provide this info to theorists?


## Result for $P_{5}^{\prime}$ (preliminary)

The impact of factorizable power corrections is indeed smaller than previous estimates.


We use KMPW Form Factors. Similar results using Ball-Zwicky.

## Our method: SFF from full FF

I. Fit $a_{F}, b_{F}, c_{F}$ to reproduce exactly full FF
II. Take $\Delta a_{F}=\Delta b_{F}=\Delta c_{F}=\mathcal{O}\left(\Lambda / m_{B}\right) \times F(0)$ corresponding to $>100 \%$ error w.r. to $a_{F}, b_{F}, c_{F}$.

## Main improvements:

- Understand importance of choosing the right scheme to
- maximize the dependence on known quantities $\xi_{\perp, \|}$
- minimizes the dependence on unknown ones (correlation between errors of PC).
- Use of all correlations among $a_{i}, b_{i}, c_{i}$ :
- depend on scheme choice
- exact relations of FF at $q^{2}=0$.
- In JC approach:
- two different $q^{2}$ dependence for soft FF used at the same time

$$
\xi_{\perp}\left(q^{2}\right)=\xi_{\perp}(0) /\left(1-\frac{q^{2}}{m_{b}^{2}}\right)^{2} \text { and } \xi_{\perp}\left(q^{2}\right)=T_{1}\left(q^{2}\right)
$$

- averaging different FF parametrization break correlations.
- "unlucky" choice of scheme that maximizes error


- Charm loop: Insertion of 4-quark operators $\left(\mathcal{O}_{1,2}^{c}\right)$ or penguin operators $\left(\mathcal{O}_{3-6}\right)$ are important and influence the extraction of $C_{9}$. Perturbative contribution is absorbed in $C_{9}^{\text {eff }}$. Long distance:
- Partly cancelled experimentally by removing charmonium resonances.
- We followed LCSR computation and prescription from KMPW to recast the effect inside $C_{9}$ as an effective contribution (different for each amplitude). See plots above taken from KMPW.


## Result:

- An increase of charm mass, for instance, from 1.27 to 1.4 GeV shifts $C_{9}^{N P}$ by +0.3 in the third bin, reducing its negative value.
- On the contrary we checked explicitly that this long distance charm contributions obtained by KMPW will tend to slightly enhance the negativity of $C_{9}^{N P} \Rightarrow \mathrm{It}$ increases the anomaly in $P_{5}^{\prime}$.
- It corresponds to large $\sqrt{q^{2}} \sim \mathcal{O}\left(m_{b}\right)$ above $\psi^{\prime}$ mass, i.e., $E_{K}$ is around GeV or below.
- Methodology:
- Operator Product Expansion in $E_{K} / \sqrt{q^{2}}$ or $\Lambda_{Q C D} / \sqrt{q^{2}}$
- Form factors in this region:
$\rightarrow$ Extrapolation of LCSR FF above $14 \mathrm{GeV}^{2}$ [Bobeth, van Dyk, Hiller]
$\rightarrow$ Lattice form factors [Bouchard et al,. S. Meinel et al.]
- Main problem:

Existence of $c \bar{c}$ resonances observed in this region for the related mode $B^{-} \rightarrow K^{-} \mu^{+} \mu^{-}$ (clearly seen $\psi(4160)$ ), but many more expected to be seen from BESS-II data.


One would expect to observe also a peaking structure at the low-recoil of $B \rightarrow K^{*} \mu^{+} \mu^{-}$
$\Rightarrow$ Quark-hadron duality-violations. How to estimate quark-hadron duality violations:

- First estimate from [Grinstein-Pirjol] combining OPE/HQET gives violation around 5\%
- Analysis from [Beylich, Buchalla,Feldmann] estimated to 2\%. BUT:
- Simple toy model estimate based on Shifman's model: OPE $+\Delta_{2}$ (oscillating function, exponentially suppressed: duality violating term). Right: simple model based on BES data summing pairs of close resonances, excluding first narrow resonance at $\psi(3770)\left(q^{2}=14.2 \mathrm{GeV}^{2}\right)$.
Assumption: Neglect interactions between $c \bar{c}$ and $B \rightarrow K$ system.


Still open questions:

1. Integrate all bin (starting from 15 and not $14.18 \mathrm{GeV}^{2}$ to avoid $\psi(3770)$ problem) and then assign a duality-violation: $2 \%$ ?, $5 \%$ ?, $10 \%$ ?
2. Focus near the endpoint where resonances are exponentially suppressed?
3. Results of bin-by-bin at low-recoil are unclear if no model included.

Interesting experiment by S. Meinel (remove 1st low-recoil bin and repeat the analysis) in a lattice FF analysis of $B \rightarrow K^{*} \mu \mu$ and $B_{s} \rightarrow \phi \mu \mu$ at high- $q^{2}$ [hep-ph 1310.3887].

Result:

- $C_{9}^{N P}$ remained negative, while $C_{9}^{\prime}$ became compatible with zero:

$$
\mathrm{C}_{9}^{\mathrm{NP}}=-1.0 \pm 0.6, \quad \mathrm{C}_{9}^{\prime}=1.2 \pm 1.0 \quad \rightarrow \quad \mathrm{C}_{9}^{N P}=-0.9 \pm 0.7, \quad \mathrm{C}_{9}^{\prime}=0.4 \pm 0.7
$$

in agreement with our pattern.

This might help in removing the second argument in favor of a $C_{9}^{\prime}$ positive and large as claimed by AS:

- large-recoil has a strong preference for $C_{9}^{\prime}<0$.
- low-recoil shows a preference for $C_{9}^{\prime}>0$.
$\Rightarrow$ It is possible that the reason of the low-recoil preference for $C_{9}^{\prime}>0$ might be a problem
(experimental or presence of a resonance or its tail) in the first low recoil bin.
Don't forget the strange behavior of this first low-recoil bin of $P_{4}^{\prime}!!!$
Suggestion to LHCb: Move the first low recoil bin to the right (like in $B^{-} \rightarrow K^{-} \mu^{+} \mu^{-}$):

at $15 \mathrm{GeV}^{2}$ or above

OPTION 2:

## Experimental issue

The 4-body angular distribution ( $m_{\ell}=0$, no scalars) exhibits 4 symmetries: transformations of the transversity amplitudes ( 2 phases, 2 rotations) that leaves the distribution invariant

Symmetries $\Rightarrow$ the minimal \# observables for each scenario, AND how many $J_{i}$ are independent:

$$
n_{o b s}=2 n_{A}-n_{S}=n_{c}-n_{d}
$$

| Case | $n_{c}$ Coefficients | $n_{A}$ Amplitudes | $n_{S}$ Symmetries | $n_{\text {obs }}$ Observables | $n_{d}$ relations |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $m_{\ell}=0, A_{S}=0$ | 11 | 6 | 4 | 8 | $3 \Leftarrow$ |
| $m_{\ell}=0$ | 11 | 7 | 5 | 9 | 2 |
| $m_{\ell}>0, A_{S}=0$ | 11 | 7 | 4 | 10 | 1 |
| $m_{\ell}>0$ | 12 | 8 | 4 | 12 | 0 |

$J_{1 s}=3 J_{2 s}, J_{1 c}=-J_{2 c}$ and a third non-trivial consistency relation:

$$
\begin{aligned}
J_{2 c}= & -2 \frac{\left(2 J_{2 s}+J_{3}\right)\left(4 J_{4}^{2}+\beta_{\ell}^{2} J_{7}^{2}\right)+\left(2 J_{2 s}-J_{3}\right)\left(\beta_{\ell}^{2} J_{5}^{2}+4 J_{8}^{2}\right)}{16 J_{2 s}^{2}-\left(4 J_{3}^{2}+\beta_{\ell}^{2} J_{6 s}^{2}+4 J_{9}^{2}\right)} \\
& +4 \frac{\beta_{\ell}^{2} J_{6 s}\left(J_{4} J_{5}+J_{7} J_{8}\right)+J_{9}\left(\beta_{\ell}^{2} J_{5} J_{7}-4 J_{4} J_{8}\right)}{16 J_{2 s}^{2}-\left(4 J_{3}^{2}+\beta_{\ell}^{2} J_{6 s}^{2}+4 J_{9}^{2}\right)},
\end{aligned}
$$

Identical equation can be written in terms of the $\overline{J_{i}}$.

This equation can be expressed in terms of $P_{i}$ and $P_{i}^{C P}$ observables to get:

$$
\bar{P}_{2}=+\frac{1}{2 \bar{k}_{1}}\left[\left(\bar{P}_{4}^{\prime} \bar{P}_{5}^{\prime}+\delta_{1}\right)+\frac{1}{\beta} \sqrt{\left(-1+\bar{P}_{1}+\bar{P}_{4}^{\prime 2}\right)\left(-1-\bar{P}_{1}+\beta^{2} \bar{P}_{5}^{\prime 2}\right)+\delta_{2}+\delta_{3} \bar{P}_{1}+\delta_{4} \bar{P}_{1}^{2}}\right]
$$

where

$$
\bar{P}_{i}=P_{i}+P_{i}^{C P} \quad \beta=\sqrt{1-4 m_{\ell}^{2} / s}
$$

The sign in front of the square root is taken "+" everywhere by comparison with exact result in SM, at low-recoil both solutions ( + and -) converge. (Plot with $\delta_{i} \rightarrow 0$ )


REMARK:

- This is an exact equation valid for any $q^{2}$ (low, large) and obtained from symmetries.
- It involves $6 P_{i}$ of the basis plus one redundant.

An identical equation can be written in terms of $\hat{P}_{i}=P_{i}-P_{i}^{C P}$, substituting $\bar{P}_{i} \rightarrow \hat{P}_{i}$ everywhere. More importantly all terms inside the $\delta_{i}$ are strongly suppressed (by small strong and weak phases):

$$
\delta_{i} \sim \mathcal{O}\left(\left(\operatorname{Im} A_{i}\right)^{2}, 1-\bar{k}_{1}\right) \quad \text { and } \quad \bar{k}_{1}=1+F_{L}^{C P} / F_{L}
$$

Hypothesis: No New Physics in weak phases entering Wilson coefficients and not scalars/tensors. Both hypothesis can be tested, measuring $P_{i}^{C P}$ and $S_{1}$.

To an excellent approximation we have:
$P_{2}=\frac{1}{2}\left[P_{4}^{\prime} P_{5}^{\prime}+\frac{1}{\beta} \sqrt{\left(-1+P_{1}+P_{4}^{\prime 2}\right)\left(-1-P_{1}+\beta^{2} P_{5}^{\prime 2}\right)}\right]$
This equation can be used in binned form if:

- Observables are nearly constant inside the bin
- Or the size of the bin is very small.

We correct for this by $\left\langle P_{2}\right\rangle \rightarrow\left\langle P_{2}\right\rangle+\Delta_{\text {exact-relation }}^{\mathrm{NP}}$ where $\Delta_{\text {exact-relation }}^{\mathrm{NP}}$ is order $10^{-2}$ except for [0.1-2] bin and [1-6] bin.


Figure: Green: SM exact, dashed inside approximation, Red: NP $C_{9}^{N P}=-1.5$ exact, dashed inside approximation

The striking consequence of this equation is that it allows you to use data to predict the impact of the anomaly in $P_{5}^{\prime}$ in a completely different observable: $P_{2}$

Imposing that the square root is well defined one finds:

$$
P_{5}^{\prime 2}-1 \leq P_{1} \leq 1-P_{4}^{\prime 2}
$$

- Indeed this is an exact bound that could be alternatively obtained from

$$
\left|P_{4}\right|=\left|P_{4}^{\prime}\right| / \sqrt{1-P_{1}} \leq 1 \quad \text { and } \quad\left|P_{5}\right|=\left|P_{5}^{\prime}\right| / \sqrt{1+P_{1}} \leq 1
$$

$\left|P_{4,5}\right| \leq 1$ comes from the geometrical interpretation of those observables in terms of $n_{i}$.



- The new upper bound is very stringent for the [4.3,8.68] bin, cutting most of the space for a positive $P_{1}: P_{1}^{[4.3,8.68]}<0.33$
- The lower bound is particularly relevant for the $[16,19]$ bin of $P_{1}: P_{1}^{[16,19]}>-0.68$.

Implication II: At the position of the zero $q_{0}^{2}$ of $P_{2}$ (same as $A_{F B}$ ) the following relation holds:

$$
\left.\left[P_{4}^{2}+P_{5}^{2}\right]\right|_{q^{2}=q_{0}^{2}}=1 \quad \text { or }\left.\quad\left[P_{4}^{\prime 2}+P_{5}^{\prime 2}\right]\right|_{q^{2}=q_{0}^{2}}=1-\eta\left(q_{0}^{2}\right)
$$

where

$$
\eta\left(q_{0}^{2}\right)=P_{1}^{2}+\left.P_{1}\left(P_{4}^{\prime 2}-P_{5}^{\prime 2}\right)\right|_{q^{2}=q_{0}^{2}}
$$

SM Zero of $A_{F B}: q_{0}^{2 S M}=3.95 \pm 0.38$ (our), $3.90 \pm 0.12$ (Buras'08), $2.9 \pm 0.3$ (LO-Khodj.'10) $\mathrm{GeV}^{2}$
Experimental LHCb data: $q_{0}^{2 L H C b}=4.9 \pm 0.9 \mathrm{GeV}^{2}$


If a future precise measurement of the zero confirms $q_{0}^{2 e x p} \sim 4.9 \mathrm{GeV}^{2}$ and $P_{4}^{\prime} \sim 1$ and $P_{1} \geq 0$ at this point (as present data suggests) THEN

$$
\begin{gathered}
P_{1}\left(q_{0}^{2}\right) \leq 1-P_{4}^{\prime 2} \sim 0 \\
\eta\left(q_{0}^{2}\right) \sim 0 \text { and } P_{5}^{\prime}\left(q_{0}^{2}\right) \sim 0
\end{gathered}
$$

(notice that in $\operatorname{SM} P_{5}^{\prime}\left(q_{0}^{2}\right)=-0.75$ )
A precise measurement of $q_{0}^{2}$ (zero of $A_{F B}$ ) outside the $S M$ region would serve as
an indirect confirmation of the anomaly

Implication III: We can establish a new relation between the anomaly bin in $P_{5}^{\prime}$ and $P_{2}$ :

$$
\left\langle P_{2}\right\rangle=\frac{1}{2}\left[\left\langle P_{4}^{\prime}\right\rangle\left\langle P_{5}^{\prime}\right\rangle+\sqrt{\left(-1+\left\langle P_{1}\right\rangle+\left\langle P_{4}^{\prime}\right\rangle^{2}\right)\left(-1-\left\langle P_{1}\right\rangle+\left\langle P_{5}^{\prime}\right\rangle^{2}\right)}\right]+\Delta_{\text {exact }}^{b i n}
$$

where $\Delta_{\text {exact }}^{\text {bin }}=-0.04$ for NP best fit point at 2 nd and 3 rd bin, while $\Delta_{\text {exact }}^{\text {bin }}=-0.01$ for $1 \mathrm{GeV}^{2}$ size.


GRAY band: SM prediction.
BLUE cross: Measured value of $P_{2}$
RED rectangle: $C_{9}^{N P}=-1.5 \mathrm{NP}$ solution.
Green cross is $\left\langle P_{2}\right\rangle$ obtained from combining data of $\left\langle P_{4,5}^{\prime}\right\rangle$, $\left\langle P_{1}\right\rangle$, considering asymmetric errors and bound on $P_{1}$

- Bin [2,4.3]: LHCb data: $+\mathbf{0 . 5 0}_{-0.07}^{+0}$, Relation: $+0.46_{-0.19}^{+0}$
$\underline{0.2 \sigma}$ from relation (green cross) to measured $P_{2}$ (blue)
- $\operatorname{Bin}[4.3,8.68]$ : LHCb data: $-\mathbf{0 . 2 5}_{-0.08}^{+0.07}$, Relation: $+0.10_{-0.13}^{+0.13}$
$\underline{\mathbf{2 . 4} \sigma}$ from relation (green cross) to measured $P_{2}$ (blue), $1.9 \sigma$ from relation to NP best fit point (red box), $3.6 \sigma$ from relation to SM.


## The first low-recoil bin $[14.18,16]$ can also be tested using this equation

LHCb data on $P_{2}$ in this bin gives: $-\mathbf{0 . 5 0}_{-\mathbf{0 . 0 0}}^{+\mathbf{0 . 0 3}}$
LHCb data on $P_{4}^{\prime}, P_{1}, P_{5}^{\prime}$ implies that $P_{2}$ should be: $+\mathbf{0 . 5 0} 0_{-0.27}^{+\mathbf{0}}$ (if + ) or $-\mathbf{0 . 5 0} 0_{+0}^{+0.33}$ (if -)

- This shows a discrepancy of $3.7 \sigma$ if + solution is taken
- Or agreement if - solution is chosen

However both solutions + and - should give same result at low-recoil

Conclusion: The measurement of this first low recoil bin is probably exhibiting a statistical fluctuation or signaling a problem at low recoil (a large strong phase driven by resonances?)

## ALTERNATIVELY Full fit of the angular distribution with a small dataset

Under the assumption of real Wilson coefficients one has

- Free parameters $F_{L}, P_{1}, P_{4,5}^{\prime}$.
- $P_{2}$ is a function of the other observables and $P_{6,8}^{\prime}$ are set to zero.


Figure: Residual distribution of $P_{5}^{\prime}$ when fitting with 100 events. The fit of a gaussian distribution is superimosed.

We find testing this fit for values around the measured values: convergence and unbiased pulls with as little as 50 events per bin. Gaussian pulls are obtained with only 100 events.

This opens the possibility to perform a full angular fit analysis with small bins in $q^{2}$

The main hypothesis (real WC) can be tested measuring $P_{i}^{C P}$.

Implication VI: Some of the endpoint symmetries [Zwicky et al] obtained automatically:

$$
P_{1}=-1 \rightarrow P_{5}^{\prime}=0=P_{2}
$$

## Decision tree

A discrepancy between a measurement of $P_{2}$ and the value obtained from data on $P_{1}, P_{4,5}^{\prime}$ should be interpreted according to one of the following options:

- At low-q ${ }^{2}$ (large-recoil region):

I presence of scalar or tensors. Improbable given constraints on $C_{S}$ from $B_{s} \rightarrow \mu^{+} \mu^{-}$(limit on $C_{S}$ ) and strong double suppression $m_{\ell}^{2} / q^{2}$ of this term in $J_{5}$. Alternatively, one can construct dedicated observables for scalars ( $S_{1}$ or tensor observables) to test directly this possibility. New Physics

II presence of new Physics contributions in weak phases. New Physics
III experimental effects (statistical fluctuation, uncontrolled systematic error, ...)
II and III can be distinguished by studying the pattern of breaking of the equation in all bins:

- an experimental effect would be most likely localized in particular bins
- new weak phases would produce a consistent pattern of breaking of the relation and can be observed in $A_{i}^{C P}, P_{i}^{C P}$
- At high-q ${ }^{2}$ (low-recoil region): previous possibilities

IV An extra possibility is opened: a new large strong phase brought by a resonance. This case would be recognized by a local breaking of the relation on the bin where the resonance sits.

- Disentangling II from IV same procedure as before.
- Disentangling III from IV requires a dedicated experimental analysis of the angles and $q^{2}$ in that region allowing for the presence of resonances should be performed


## Conclusions

- The analysis of LHCb data on the 4-body angular distribution of $B \rightarrow K^{*}(\rightarrow K \pi) \mu^{+} \mu^{-}$using clean $P_{i}^{(\prime)}, A_{F B}+$ radiative observables gives the pattern:

$$
\mathrm{C}_{9}^{\mathrm{NP}} \sim[-1.6,-0.9], \quad \mathrm{C}_{7}^{\mathrm{NP}} \sim[-0.05,-0.01], \quad \mathrm{C}_{9}^{\prime} \sim \pm \delta \quad \mathrm{C}_{10}, \mathrm{C}_{7,10}^{\prime} \sim \pm \epsilon
$$

where $\delta$ is small and $\epsilon$ is smaller. Consistent with new $3 \mathrm{fb}^{-1}$ data on $B \rightarrow K \mu \mu$.

- Large-recoil: 0.1-8.6 GeV². We have shown that the 'usual suspects' does not help in explaining the pattern of deviations in front of a NP explanation:
- Charm loops: The first results from this kind of contributions [Khodjamirian et al.'10] show that they add a positive contribution to $C_{9}$, enlarging the size of the discrepancy of data with SM prediction.
- Factorizable Power Corrections: A careful implementation of correlations between PC + the freedom to choose an appropriate scheme to define soft FF shows that PC are substantially smaller than previously claimed.
$\Rightarrow$ Naive statement "It is QCD" is in tension with our detailed analysis.
- Low-recoil: 15-19.22 $\mathbf{G e V}^{2}$ : The difficulty to establish the size of quark-hadron duality violations in this region $(2 \% ?, 5 \%$ ?, ...) complicates the analysis. Different possibilities: i) model resonances, ii) integrate over the whole $q^{2}$ region and assign an error,iii) take only the bin near the endpoint...
- We have established a new connection between the zero of $A_{F B}$ and the anomaly in $P_{5}^{\prime}$ and a full set of consistency tests that experimentalists can use to check the consistency of future data on $B \rightarrow K^{*} \mu^{+} \mu^{-}$.


## The best possible scenario to move from evidence to discovery:

- $P_{2}$ :
- bin $[2,4.3]$ keep the $2.9 \sigma$ discrepancy
- bin $[4,3,8.68]$ should increase the discrepancy w.r.t. the SM. $\Uparrow$
- $P_{5}^{\prime}$ :
- bin $[2,4.3]$ should increase the significance of the discrepancy with SM $\Downarrow$
- bin $[4.3,8.68]$ should decrease the discrepancy possibly reducing a bit the significance. $\Downarrow$

The best strategy would be to split bin $[4.3,8.68]$ into 2 or 3 bins and each one exhibiting a consistent pattern.



## Back-up slides

The coefficients $\mathbf{J}_{\mathbf{i}}$ of the distribution can be reexpressed now in terms of this basis of clean observables:

Correspondence $\mathbf{J}_{\mathbf{i}} \leftrightarrow \mathbf{P}_{\mathbf{i}}^{(\prime)}$ :
BROWN: LO FF-dependent observables ( $F_{L}$ Longitudinal Polarization Fraction of $K^{*}$ )
RED: LO FF-independent observables at large-recoil (defined from these eqs.)

Here for simplicity $\left(m_{\ell}=0\right)$. See [J.M'12] for $m_{\ell} \neq 0$.

$$
\begin{aligned}
& \left(\mathbf{J}_{2 \mathrm{~s}}+\bar{J}_{2 \mathrm{~s}}\right)=\frac{1}{4} \mathrm{~F}_{\mathrm{T}} \frac{\mathrm{~d} \bar{\Gamma}+\mathrm{d} \bar{\Gamma}}{d q^{2}} \quad\left(\mathrm{~J}_{2 \mathrm{c}}+\overline{\mathbf{J}}_{2 \mathrm{c}}\right)=-\mathrm{F}_{\mathrm{L}} \frac{\mathrm{~d} \boldsymbol{\Gamma}+\mathrm{d} \bar{\Gamma}}{d q^{2}} \\
& \mathrm{~J}_{3}+\bar{J}_{3}=\frac{1}{2} \mathrm{P}_{1} \mathrm{~F}_{\mathrm{T}} \frac{\mathrm{~d} \mathrm{\Gamma}+\mathrm{d} \bar{\Gamma}}{\mathrm{dq}{ }^{2}} \quad \mathrm{~J}_{3}-\bar{J}_{3}=\frac{1}{2} \mathrm{P}_{1}^{\mathrm{CP}} \mathrm{~F}_{\mathrm{T}} \frac{\mathrm{~d} \mathrm{\Gamma}+\mathrm{d} \bar{\Gamma}}{\mathrm{dq} q^{2}} \\
& \mathbf{J}_{6 \mathrm{~s}}+\overline{\mathbf{J}}_{6 \mathrm{~s}}=2 \mathrm{P}_{2} \mathrm{~F}_{\mathrm{T}} \frac{\mathrm{~d} \boldsymbol{\Gamma}+\mathrm{d} \bar{\Gamma}}{\mathrm{dq}}{ }^{2} \quad \quad \mathrm{~J}_{6 \mathrm{~s}}-\overline{\mathbf{J}}_{6 \mathrm{~s}}=2 \mathrm{P}_{2}^{\mathrm{CP}} \mathrm{~F}_{\mathrm{T}} \frac{\mathrm{~d} \boldsymbol{\Gamma}+\mathrm{d} \overline{\boldsymbol{\Gamma}}}{\mathrm{dq}} \\
& \mathrm{~J}_{9}+\overline{\mathrm{J}}_{9}=-\mathrm{P}_{3} \mathrm{~F}_{\mathrm{T}} \frac{\mathrm{~d} \mathrm{\Gamma}+\mathrm{d} \bar{\Gamma}}{\mathrm{dq} q^{2}} \quad \mathrm{~J}_{9}-\overline{\mathrm{J}}_{9}=-\mathrm{P}_{3}^{\mathrm{CP}} \mathrm{~F}_{\mathrm{T}} \frac{\mathrm{~d} \mathrm{\Gamma}+\mathrm{d} \bar{\Gamma}}{\mathrm{dq}^{2}} \\
& \mathrm{~J}_{4}+\bar{J}_{4}=\frac{1}{2} \mathrm{P}_{4}^{\prime} \sqrt{\mathrm{F}_{\mathrm{T}} \mathrm{~F}_{\mathrm{L}}} \frac{\mathrm{~d} \mathrm{\Gamma}+\mathrm{d} \bar{\Gamma}}{d q^{2}} \quad \mathrm{~J}_{4}-\bar{J}_{4}=\frac{1}{2} P_{4}^{\prime C P} \sqrt{\mathrm{~F}_{\mathrm{T}} \mathrm{~F}_{\mathrm{L}}} \frac{\mathrm{~d} \Gamma+\mathrm{d} \bar{\Gamma}}{d q^{2}} \\
& \mathrm{~J}_{5}+\overline{\mathrm{J}}_{5}=\mathrm{P}_{5}^{\prime} \sqrt{\mathrm{F}_{\mathrm{T}} \mathrm{~F}_{\mathrm{L}}} \frac{\mathrm{~d} \bar{\Gamma}+\mathrm{d} \bar{\Gamma}}{\mathrm{dq}} \\
& \mathrm{~J}_{5}-\bar{J}_{5}=\mathrm{P}_{5}^{\prime \mathrm{CP}} \sqrt{\mathrm{~F}_{\mathrm{T}} \mathrm{~F}_{\mathrm{L}}} \frac{\mathrm{~d} \boldsymbol{\Gamma}+\mathrm{d} \overline{\boldsymbol{\Gamma}}}{\mathrm{dq}^{2}} \\
& J_{7}+\bar{J}_{7}=-P_{6}^{\prime} \sqrt{F_{\mathrm{T}} F_{\mathrm{L}}} \frac{d \bar{\Gamma}+d \bar{\Gamma}}{d q^{2}} \quad J_{7}-\bar{J}_{7}=-P_{6}^{\prime C P} \sqrt{F_{\mathrm{T}} F_{\mathrm{L}}} \frac{d \Gamma+d \bar{\Gamma}}{d q^{2}}
\end{aligned}
$$



It is not surprising that the second bin in $P_{2}$ fits perfectly, while the third bin in $P_{2}$ goes on the right direction but does not fit perfectly.

Reason It is very difficult to get excellent agreement with the third bin of $P_{5}^{\prime}$ inside a global fit.

-     - (magenta, green, red) $C_{9}^{\prime} \leq 0$
- (brown) $C_{9}^{\prime}>0$
- Our large recoil best fit point gives $\left\langle P_{5}^{\prime}\right\rangle_{[4.3,8.68]}=-0.49$ and reduces tension with data at $1.8 \sigma$ (from $4 \sigma$ in SM ): $C_{9}^{\prime}<0$ is strongly favored by this bin.
- The best fit point with $C_{9}^{N P}=-1.5$ gives $\left\langle P_{5}^{\prime}\right\rangle_{[4.3,8.68]}=-0.61$.
- Any analysis with $C_{9}^{\prime}>0$ provides a much worst disagreement with data in this bin.

Most plausible scenario: Third bin in $P_{5}^{\prime}$ will go down (reducing distance with SM) while third bin in $P_{2}$ might go up (enlarging distance with SM ): Global picture much more consistent.


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If we neglect scalars/tensors the 4-body angular distribution can be written in terms of 3 vectors:

$$
\mathbf{n}_{\|}=\binom{A_{\|}^{L}}{A_{\|}^{R *}}, \quad \mathbf{n}_{\perp}=\binom{A_{\perp}^{L}}{-A_{\perp}^{R *}}, \quad \mathbf{n}_{0}=\binom{A_{0}^{L}}{A_{0}^{R *}} .
$$

All the coefficients $\boldsymbol{J}_{\mathbf{i}}$ can be expressed in terms of the products $\mathbf{n}_{\mathbf{i}}^{\dagger} \boldsymbol{n}_{\mathbf{j}}$ (example):

$$
J_{3}=\frac{1}{2}\left(\left|n_{\perp}\right|^{2}-\left|n_{\|}\right|^{2}\right), \quad J_{4}=\frac{1}{\sqrt{2}} \operatorname{Re}\left(n_{0}^{\dagger} n_{\|}\right), \quad J_{5}=\sqrt{2} \operatorname{Re}\left(n_{0}^{\dagger} n_{\perp}\right), \quad J_{9}=-\operatorname{lm}\left(n_{\perp}^{\dagger} n_{\|}\right)
$$

The angular distribution is invariant under a unitary transformation $n_{i} \rightarrow U n_{i}$

$$
n_{i}^{\prime}=U n_{i}=\left[\begin{array}{ll}
e^{i \phi_{\mathrm{L}}} & 0 \\
0 & e^{-i \phi_{\mathrm{R}}}
\end{array}\right]\left[\begin{array}{rr}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{array}\right]\left[\begin{array}{rr}
\cosh i \tilde{\theta} & -\sinh i \tilde{\theta} \\
-\sinh i \tilde{\theta} & \cosh i \tilde{\theta}
\end{array}\right] n_{i} .
$$

$U$ defines the four symmetries of the massless angular distribution:

- two global phase transformations ( $\phi_{\mathrm{L}}$ and $\phi_{\mathrm{R}}$ ),
- a rotation $\theta$ among the real and imaginary components of the amplitudes independently
- another rotation $\tilde{\theta}$ that mixes real and imaginary components of the transversity amplitudes.
- Another possible source of uncertainty is the S -wave contribution coming from $B \rightarrow K_{0}^{*} I^{+} I^{-}$. [Becirevic, Tayduganov '13], [Blake et al.'13]
- We assume that both $P$ and $S$ waves are described by $q^{2}$-dependent FF $\times$ a Breit-Wigner function.
- The distinct angular dependence of the S -wave terms in folded distributions allow to disentangle the signal of the P -wave from the S -wave: $P_{i}^{(1)}$ can be disentangled from $S$-wave pollution [JM'12].

The modified distribution including the S-wave:

$$
\begin{gathered}
\frac{1}{\Gamma_{\text {full }}^{\prime}} \frac{d^{4} \Gamma}{\boldsymbol{C}^{2} d \cos \theta_{K} d \cos \theta_{l} d \phi}=P d f_{K^{*}}\left(1-\mathbf{F}_{\mathrm{S}}\right)+\frac{1}{\Gamma_{\text {full }}^{\prime}} \mathbf{W}_{\mathbf{S}} \\
\frac{\mathbf{W}_{\mathrm{S}}}{\Gamma_{\text {full }}^{\prime}}= \\
\\
\frac{3}{16 \pi}\left[\mathbf{F}_{\mathrm{S}} \sin ^{2} \theta_{\ell}+\mathbf{A}_{\mathbf{S}} \sin ^{2} \theta_{\ell} \cos \theta_{K}+\mathbf{A}_{\mathrm{S}}^{4} \sin \theta_{K} \sin 2 \theta_{\ell} \cos \phi\right. \\
\\
\left.+\mathbf{A}_{\mathrm{S}}^{5} \sin \theta_{K} \sin \theta_{\ell} \cos \phi+\mathbf{A}_{\mathrm{S}}^{7} \sin \theta_{K} \sin \theta_{\ell} \sin \phi+\mathbf{A}_{\mathrm{S}}^{8} \sin \theta_{K} \sin 2 \theta_{\ell} \sin \phi\right]
\end{gathered}
$$

$\Gamma_{\text {full }}^{\prime}=\Gamma_{K^{*}}^{\prime}+\Gamma_{S}^{\prime}$ and the longitudinal polarization fraction associated to $\Gamma_{S}^{\prime}$ is

$$
\mathrm{F}_{\mathrm{S}}=\frac{\Gamma_{S}^{\prime}}{\Gamma_{\text {full }}^{\prime}} \quad \text { and } \quad 1-\mathrm{F}_{\mathrm{S}}=\frac{\Gamma_{K^{*}}^{\prime}}{\Gamma_{\text {full }}^{\prime}}
$$

We can get bounds on the size of the S-wave polluting terms from Cauchy-Schwartz

$$
\begin{gathered}
\mathbf{A}_{\mathbf{S}}=2 \sqrt{3} \frac{1}{\Gamma_{\text {full }}^{\prime}} \int \operatorname{Re}\left[\left(A_{0}^{\prime L} A_{0}^{L *}+A_{0}^{\prime R} A_{0}^{R *}\right) B W_{K_{0}^{*}}\left(m_{K \pi}^{2}\right) B W_{K^{*}}^{\dagger}\left(m_{K \pi}^{2}\right)\right] d m_{K \pi}^{2} \\
\left|\mathbf{A}_{\mathbf{S}}\right| \leq 2 \sqrt{3} \frac{1}{\Gamma_{\text {full }}^{\prime}} \times \sqrt{\left[\left|A_{0}^{\prime} L\right|^{2}+\left|A_{0}^{\prime} R\right|^{2}\right]\left[\left|A_{0}^{L}\right|^{2}+\left|A_{0}^{R}\right|^{2}\right]} \mathbf{Z}=2 \sqrt{3} \sqrt{\mathbf{F}_{\mathbf{S}}\left(1-\mathbf{F}_{\mathrm{S}}\right) \mathbf{F}_{\mathbf{L}}} \mathbf{Z} / \sqrt{\mathbf{X Y}}
\end{gathered}
$$

$\mathbf{X}, \mathbf{Y}, \mathbf{Z}$ collect the Breit-Wigner.
[S.Descotes, T, Hurth, JM, J. Virto 1303.5794]

|  | Large <br> recoil <br> $\infty$ | Low recoil <br> $\infty$ Range <br> Range | Large Recoil <br> Finite <br> Range | Low Recoil <br> Finite <br> Range |
| :---: | :---: | :---: | :---: | :---: |
| $\left\|A_{S}\right\|$ | 0.33 | 0.25 | 0.67 | 0.49 |
| $\left\|A_{S}^{4}\right\|$ | 0.05 | 0.10 | 0.11 | 0.19 |
| $\left\|A_{S}^{5}\right\|$ | 0.11 | 0.11 | 0.22 | 0.23 |
| $\left\|A_{S}^{7}\right\|$ | 0.11 | 0.19 | 0.22 | 0.38 |
| $\left\|A_{S}^{8}\right\|$ | 0.05 | 0.06 | 0.11 | 0.11 |

Table: Illustrative values of the size of the bounds for the choices of $F_{S}, F_{L}, P_{1}$ and $\mathbf{F}=\mathbf{Z} / \sqrt{\mathbf{X Y}}$

- Large-recoil: $F_{S} \sim 7 \%\left(\right.$ like $\left.B^{0} \rightarrow J / \psi K^{+} \pi^{-}\right), F_{L} \sim 0.7$ and $P_{1} \sim 0$
- Low-recoil: $F_{S} \sim 7 \%, F_{L} \sim 0.38$ and $P_{1} \sim-0.48$.

This may help in estimating the systematics associated to S-wave.

## $P_{1}$ and $P_{2}$ observables function of $A_{\perp}$ and $A_{\|}$amplitudes

- $\mathbf{P}_{1}$ : Proportional to $\left|A_{\perp}\right|^{2}-\left|A_{\|}\right|^{2}$
- Test the LH structure of SM and/or existence of RH currents that breaks $A_{\perp} \sim-A_{\|}$
- $\mathbf{P}_{2}$ : Proportional to $\operatorname{Re}\left(A_{i} A_{j}\right)$
- Zero of $P_{2}$ at the same position as the zero of $A_{F B}$
- $P_{2}$ is the clean version of $A_{F B}$. Their different normalizations offer different sensitivities.


- $P_{3}$ and $P_{6,8}^{\prime}$ are proportional to $\operatorname{Im} A_{i} A_{j}$ and small if there are no large phases. All are $<0.1$.
- $P_{i}^{C P}$ are all negligibly small if there is no New Physics in weak phases.
$P_{4}^{\prime}$ and $P_{5}^{\prime}$ observables function of $A_{\perp, \|}$ and also $A_{0}$ amplitudes
- $\mathbf{P}_{4,5}^{\prime}$ : Proportional to $\operatorname{Re}\left(A_{i} A_{j}\right)$
- $\left|P_{4,5}\right| \leq 1$ but $\left|P_{4,5}^{\prime}\right|$ can be $>1$.


In the large-recoil limit

$$
\begin{aligned}
A_{\perp, \|}^{L} & \propto\left[\mathcal{C}_{9}^{\mathrm{eff}}-\mathcal{C}_{10}+\frac{2 \hat{m}_{b}}{\hat{s}} \mathcal{C}_{7}^{\mathrm{eff}}\right] \xi_{\perp}\left(E_{K^{*}}\right) \quad A_{\perp, \|}^{R} \propto\left[\mathcal{C}_{9}^{\mathrm{eff}}+\mathcal{C}_{10}+\frac{2 \hat{m}_{b}}{\hat{s}} \mathcal{C}_{7}^{\mathrm{eff}}\right] \xi_{\perp}\left(E_{K^{*}}\right) \\
A_{0}^{L} & \propto\left[\mathcal{C}_{9}^{\mathrm{eff}}-\mathcal{C}_{10}+2 \hat{m}_{b} \mathcal{C}_{7}^{\mathrm{eff}}\right] \xi_{\|}\left(E_{K^{*}}\right) \quad A_{0}^{R} \propto\left[\mathcal{C}_{9}^{\mathrm{eff}}+\mathcal{C}_{10}+2 \hat{m}_{b} \mathcal{C}_{7}^{\mathrm{eff}}\right] \xi_{\| \|}\left(E_{K^{*}}\right)
\end{aligned}
$$

- In the SM $C_{9}^{S M} \sim-C_{10}^{S M}$, this cancellation strongly suppresses $A_{\perp, \|}^{R}$ above $4 \mathrm{Gev}^{2}: A_{\perp, \|}^{L} \gg A_{\perp, \|}^{R}$. This makes $P_{4} \rightarrow 1$ and $P_{5} \rightarrow-1$ for $q^{2} \rightarrow 8 \mathrm{GeV}^{2}$ quite fast BUT the fact that $\left|A_{\|}\right|>\left|A_{\perp}\right|$ and that $P_{4}^{\prime} \propto A_{0}^{L *} A_{\|}^{L}+A_{0}^{R} A_{\|}^{R *}$ and $P_{5}^{\prime} \propto A_{0}^{L *} A_{\perp}^{L}-A_{0}^{R} A_{\perp}^{R *}$ makes less efficient the convergence in the case of $P_{5}^{\prime}$.
- In presence of New Physics affecting only $C_{9}$ the cancellation $C_{9} \sim-C_{10}$ is less efective, consequently $A_{\perp, \|}^{R}$ is less suppressed and one should expect to see the effect of $C_{9} \neq C_{9}^{S M}$ in $P_{5}^{\prime}$.

