Lattice determination of the B/B_s decay constant

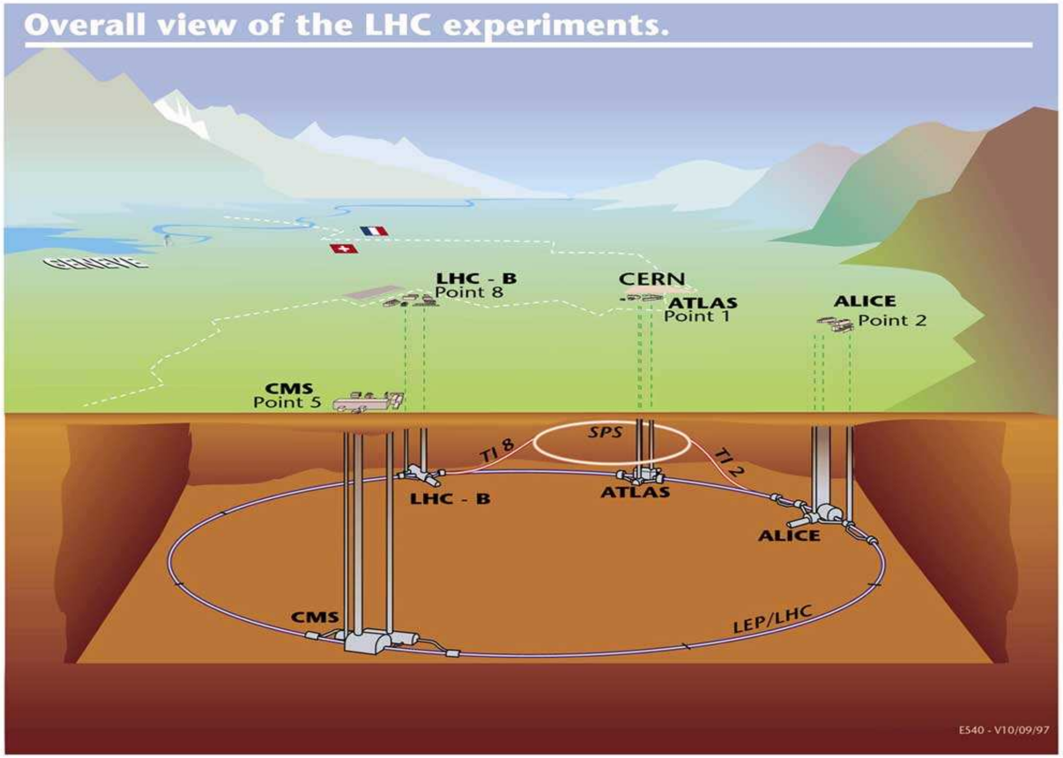
Benoît Blossier

LPT Orsay

Flavour of new physics in $b \to s$ transitions, Paris, 2 – 3 June 2014

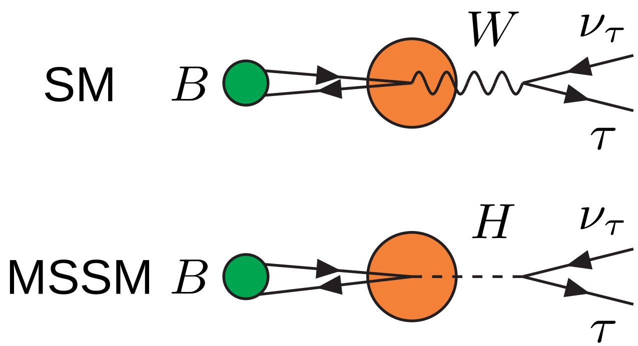
- Generalities
- Heavy quark on the lattice
- Collection of lattice results and averages

Generalities



LHC is working very well, a lot of forthcoming data will be analysed to hopefully give an answer to important questions (hierarchy problem, ...). Lattice QCD is a powerful tool to bring theoretical ingredients that are necessary as soon as bound states of quarks and gluons are involved in processes under study.

Leptonic decay (FCCC process)

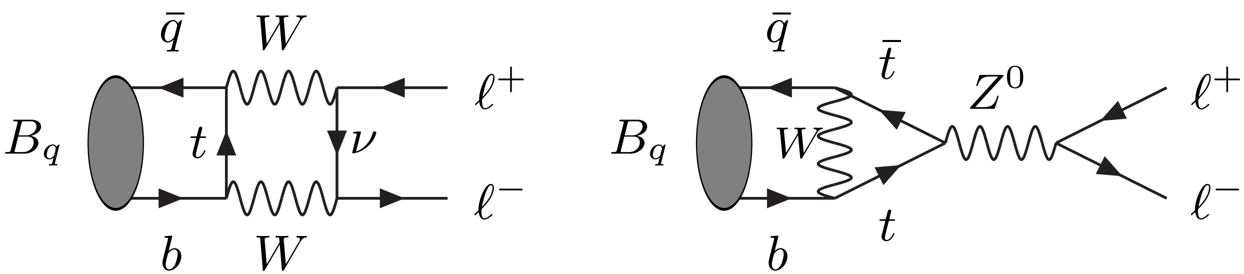


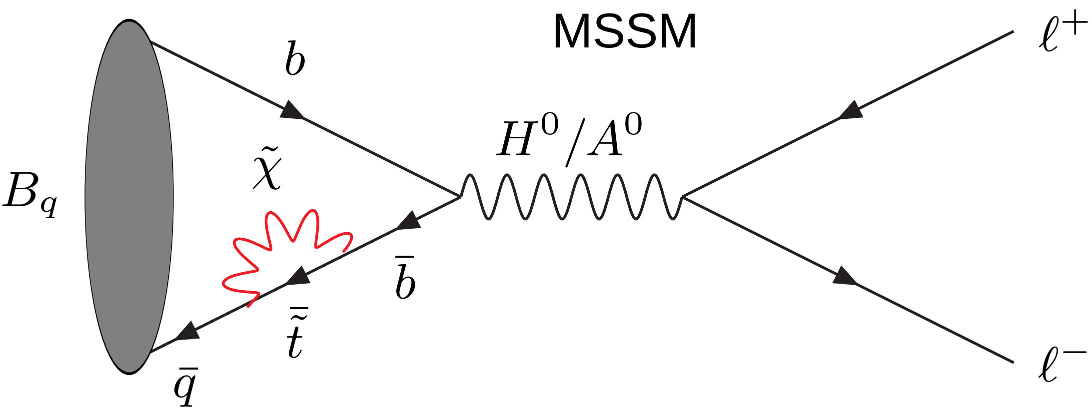
$$\Gamma(B^- \to \tau\nu_\tau) = \frac{G_F^2 |V_{ub}|^2}{8\pi} f_B^2 m_\tau^2 m_B \times \left(1 - \frac{m_\tau^2}{m_B^2}\right)^2 \left|1 + \frac{m_B^2}{m_b m_\tau} C_{\rm NP}^\tau\right|^2$$

$\delta(f_B^2) \sim 10$ %

Leptonic decay (FCNC process)

SM





$$\Gamma(B_q \to \ell^+\ell^-) = \frac{G_F^2}{\pi} Y\left(\frac{\alpha}{4\pi \sin^2 \Theta_W}\right)^2 \times m_{B_q} f_{B_q}^2 |V_{tb}^* V_{tq}|^2 m_\ell^2 \sqrt{1 - 4\frac{m_\ell^2}{m_{B_q}^2}}$$

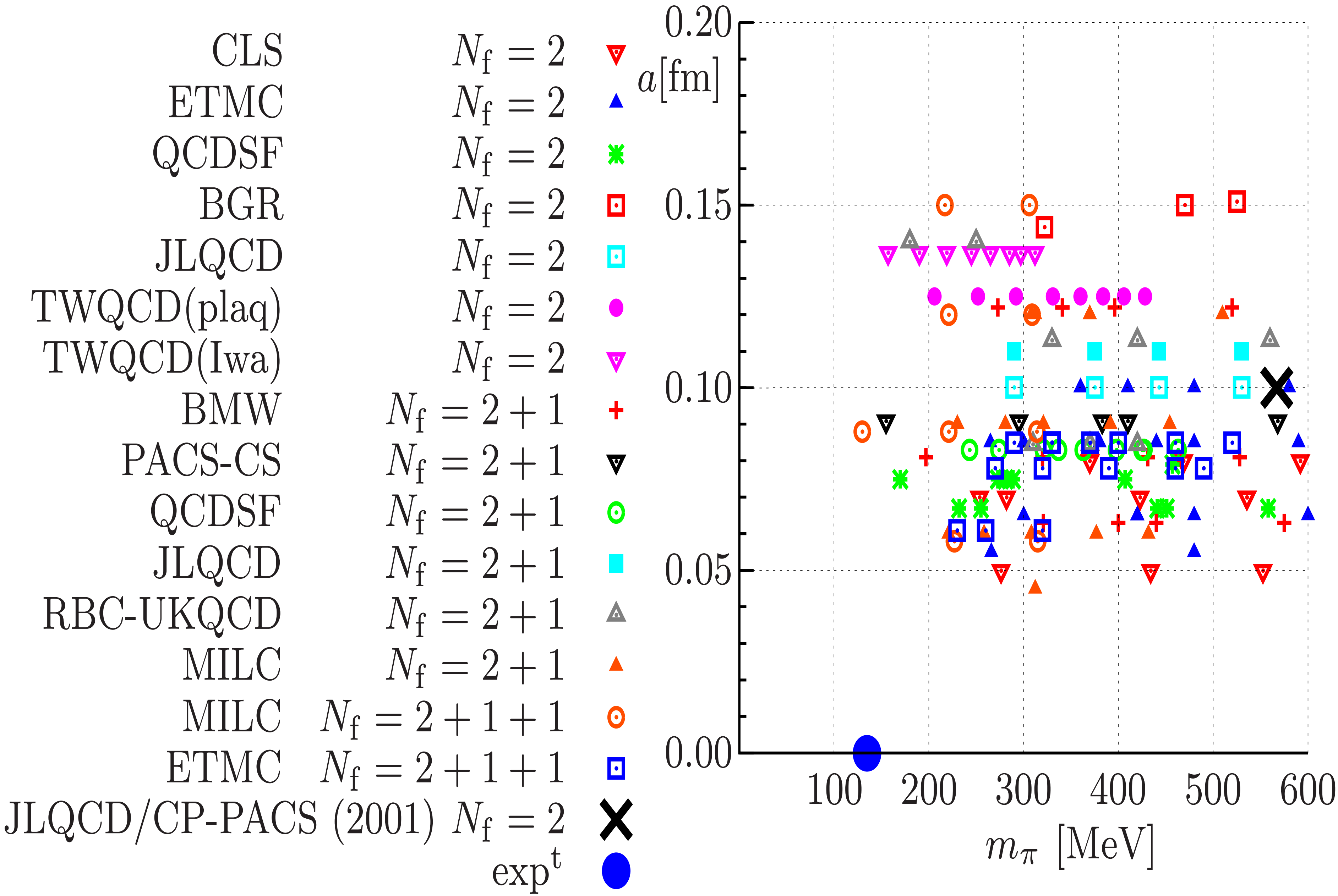
$$\frac{\Gamma(B_s \to \mu^+\mu^-)}{\Gamma(B_d \to \mu^+\mu^-)} \propto \left(\frac{f_{B_s}}{f_{B_d}}\right)^2$$

LHCb + CMS: $\mathcal{B}(B_s \to \mu^+\mu^-) = 2.9 \pm 0.7 \times 10^{-9}$

LHCb: $\mathcal{B}(B_d \to \mu^+\mu^-) < 7.4 \times 10^{-10}$ @ 95% CL

Lattice simulations set up

Nowadays, simulations are quite close to the physical point.



Flavour Lattice Averaging Group (FLAG) [http://itpwiki.unibe.ch/flag/]

The lattice community is doing an effort in providing to phenomenologists a collection of useful results after a careful survey of the world-wide work.

Quantities under study:

– u, d and s quark masses
– V_{ud} and V_{us}
– Low Energy Constants
– Kaon mixing bag parameter B_K
– Strong coupling constant α_s
– $B_{(s)}$ and $D_{(s)}$ meson decay constants
– B mixing bag parameter B_B
– form factors of $B_{(s)}$ and D semileptonic decays

A lot of technicalities and issues about systematics, difficult to present outside our community in a pedagogical way, are thus often hidden. FLAG is performing global averages of results, after a selection according to several quality criteria:

– continuum limit extrapolation

★ 3 or more lattice spacings, $a^2_{\rm max}/a^2_{\rm min} \geq 2$, $D(a_{\rm min}) \leq 2\%$, $\delta(a_{\rm min}) \leq 1$
○ 2 or more lattice spacings, $a^2_{\rm max}/a^2_{\rm min} \geq 1.4$, $D(a_{\rm min}) \leq 10\%$, $\delta(a_{\rm min}) \leq 2$
■ otherwise

$$D(a) = \frac{Q(a)-Q(0)}{Q(a)} \quad \delta(a) = \frac{Q(a)-Q(0)}{\sigma_Q^{\rm cont}}$$

– renormalization and matching:

★ absolutely renormalized or non-pertubative
○ 1-loop perturbation theory or higher with an estimate of truncation error
■ otherwise

Flavour Lattice Averaging Group (FLAG) [http://itpwiki.unibe.ch/flag/]

The lattice community is doing an effort in providing to phenomenologists a collection of useful results after a careful survey of the world-wide work.

Quantities under study:

- u, d and s quark masses
- V_{ud} and V_{us}
- Low Energy Constants
- Kaon mixing bag parameter B_K
- Strong coupling constant α_s
- $B_{(s)}$ and $D_{(s)}$ meson decay constants
- B mixing bag parameter B_B
- form factors of $B_{(s)}$ and D semileptonic decays

A lot of technicalities and issues about systematics, difficult to present outside our community in a pedagogical way, are thus often hidden. FLAG is performing global averages of results, after a selection according to several quality criteria:

– finite-volume

★ $m_\pi L \gtrsim 3.7$ or 2 volumes at fixed parameters of the simulation

○ $m_\pi L \gtrsim 3$

■ otherwise

– chiral extrapolation

★ $m_{\pi\,\mathrm{min}} \lesssim 200$ MeV

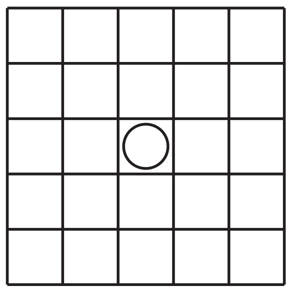
○ 200 MeV $\lesssim m_{\pi\,\mathrm{min}} \lesssim 400$ MeV

■ otherwise

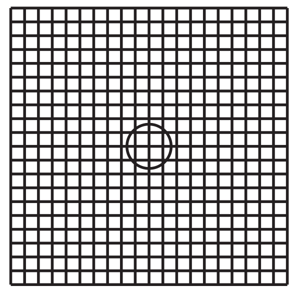
Results with tiny errors must be taken with care, unfortunately they sometimes dominate too much the averages.

Heavy quark on the lattice

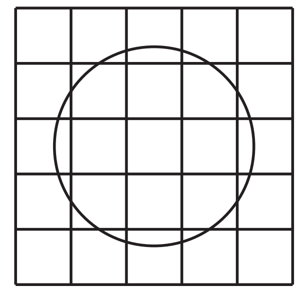
Issue for B-physics on the lattice: systematics coming from large discretisation effects ($\Lambda_{\mathrm{Compt}} \sim 1/m_Q$).



Cut-off Effects



cut-off effects



cut-off effects

Several strategies are proposed in the literature to deal with those cut-off effects:

- Use NRQCD to describe the heavy quark [P. Lepage and B. Thacker, '91]; though, no continuum limit when the theory is regularised on the lattice
- Define an action with counterterms that are tuned to get $\mathcal{O}(a)$, $\mathcal{O}(am_Q)$ and $\mathcal{O}(\alpha_s(am_Q)^n)$ improvements [A El Khadra *et al*, '96; N. Christ *et al*, '06]
- Computation within Heavy Quark Effective Theory, the effective couplings are determined non perturbatively by imposing matching conditions between QCD and HQET [J. Heitger and R. Sommer, '03]
- Computation within QCD: use of the HQET scaling laws to interpolate easily a quantity between the charm region and the (exactly known) infinite heavy mass limit [B. B. *et al*, '09]

Extraction of $f_{B_{(s)}}$ with $N_{\mathrm{f}} = 2+1$ RHQ [RBC/UKQCD: N. Christ *et al*, '14]

Purpose: define a lattice action for heavy quarks such that the improvement of the hadron spectrum is realised at $\mathcal{O}(a)$, $\mathcal{O}(a\vec{p})$ and at all orders of (am_0), $am_0 \sim 1$.

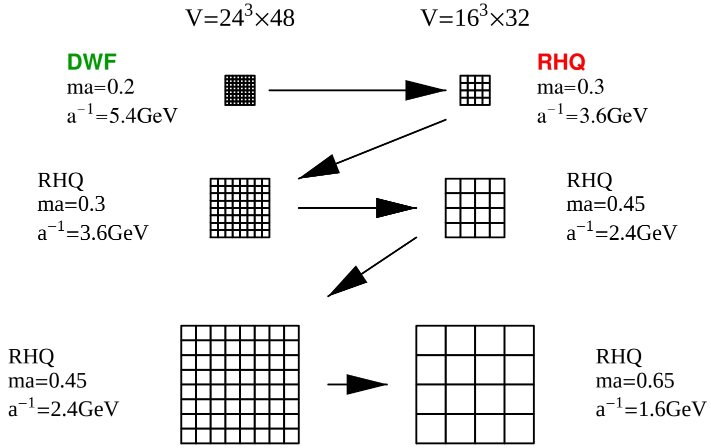
Kinematics: $|\vec{p}| \sim \Lambda_{QCD}$ (hl mesons), $|\vec{p}| \sim \alpha_s m_Q$ (hh mesons).

Necessity to break the axis symmetry because $p_0 \gg \Lambda_{QCD}$.

Only 3 parameters are required in the effective action; the improvement of matrix elements needs the introduction of further counter-terms to the operators and interpolating fields:

$$\begin{aligned} S_{\mathrm{lat}} &= \sum_{n',n} \overline{\psi}_{n'} \Big(\gamma^0 D^0 + \zeta \vec{\gamma} \cdot \vec{D} + m_0 - \frac{r_t}{2} (D^0)^2 - \frac{r_s}{2} \vec{D}^2 \\ &\quad + \sum_{i,j} \frac{i}{4} c_B \sigma_{ij} F_{ij} + \sum_i \frac{i}{2} c_E \sigma_{0i} F_{0i} \Big)_{n',n} \psi_n \qquad (1) \\ \Psi &= z_q^{-1/2} (1 + \delta a \vec{\gamma} \cdot \vec{\partial}) \psi \end{aligned}$$

It has been shown that one can fix $r_t = 0$, $r_s = 0$ and $c_E = c_B \equiv c_P$.



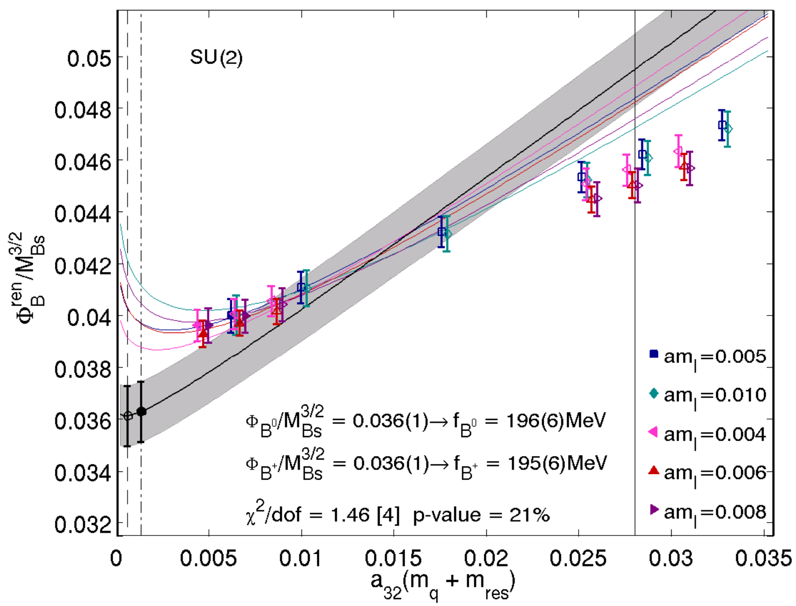
Several applications of step scaling and matching are performed.

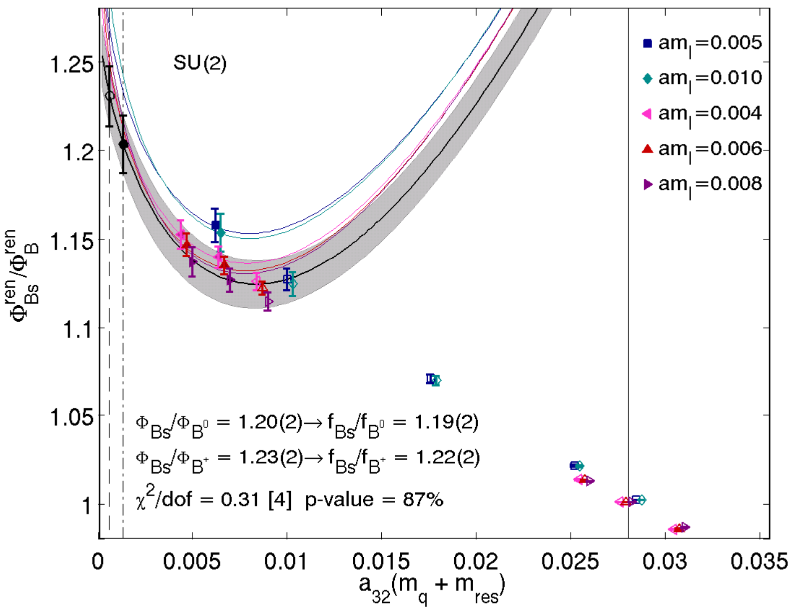
The matching at the smallest lattice spacing is done with Domain Wall Fermions

$\Longrightarrow$ discretisation effects are $\mathcal{O}(am)^2$

Parameters of the RHQ action are finally interpolated to m_b.

One performs a combined extrapolation to the chiral and continuum limit of f_B and f_{B_s}/f_B.





$f_{B+} = 195.4(15.8)$ MeV, $f_{B^0} = 196.2(15.7)$ MeV, $f_{B_s} = 235.4(12.2)$ MeV

$f_{B_s}/f_{B+} = 1.220(82)$, $f_{B_s}/f_{B^0} = 1.193(59)$

Extraction of $f_{B_{(s)}}$ in $N_f = 2$ HQET [ALPHA: B. B. *et al*, '14]

$$\mathcal{L}^{\mathrm{HQET},1/\mathrm{m}} = \bar\psi_h D_0 \psi_h + m_{\mathrm{bare}} \bar\psi_h \psi_h - \omega_{\mathrm{kin}} \bar\psi_h \mathbf{D}^2 \psi_h - \omega_{\mathrm{spin}} \bar\psi_h \sigma \cdot \mathbf{B} \psi_h$$

$$A_0^{\mathrm{HQET},1/\mathrm{m}} = Z_A^{\mathrm{HQET}} [\bar\psi_l \gamma_0 \gamma^5 \psi_h + c_A^{(1)} \bar\psi_l \frac{1}{2} \gamma^5 \gamma_i (\nabla_i - \overleftarrow{\nabla}_i) \psi_h + c_A^{(2)} \partial_i [\bar\psi_l \gamma_i \gamma^5 \psi_h]$$

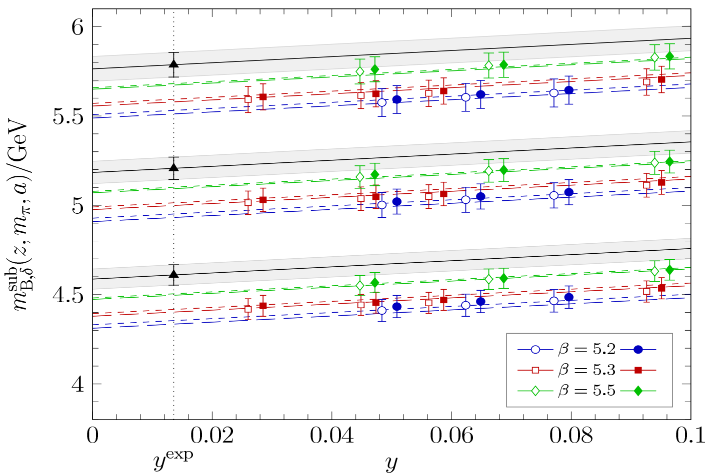
The effective couplings are determined non perturbatively by imposing matching conditions between QCD and HQET. Hadronic matrix elements are extracted with a particular care to excited states.

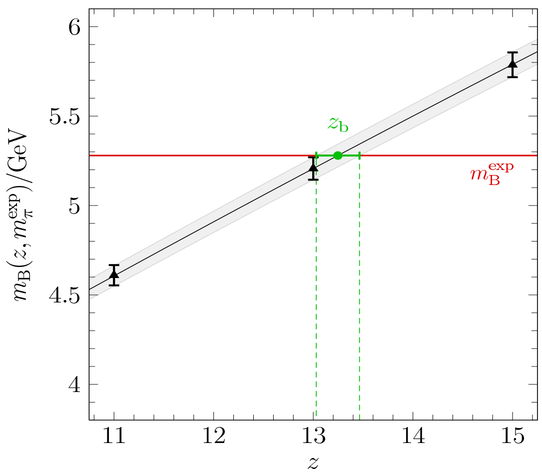
$$m_B = m_{\mathrm{bare}} + E^{\mathrm{stat}} + \omega_{\mathrm{kin}} E^{\mathrm{kin}} + \omega_{\mathrm{spin}} E^{\mathrm{spin}}$$

$$f_B \sqrt{m_B/2} = Z_A^{\mathrm{HQET}} (1 + b_A^{\mathrm{stat}} m_q) p^{\mathrm{stat}} \left(1 + \omega_{\mathrm{kin}} p^{\mathrm{kin}} + \omega_{\mathrm{spin}} p^{\mathrm{spin}} + c_A^{(1)} p^{\mathrm{A}^{(1)}}\right)$$

Chiral and continuum limit extrapolations of m_B are performed to get m_b^{RGI}. Several heavy quark masses m_h are considered on the QCD side of the whole program $\Longrightarrow$ effective couplings $\omega(m_h)$ and meson masses $m_B(m_h)$.

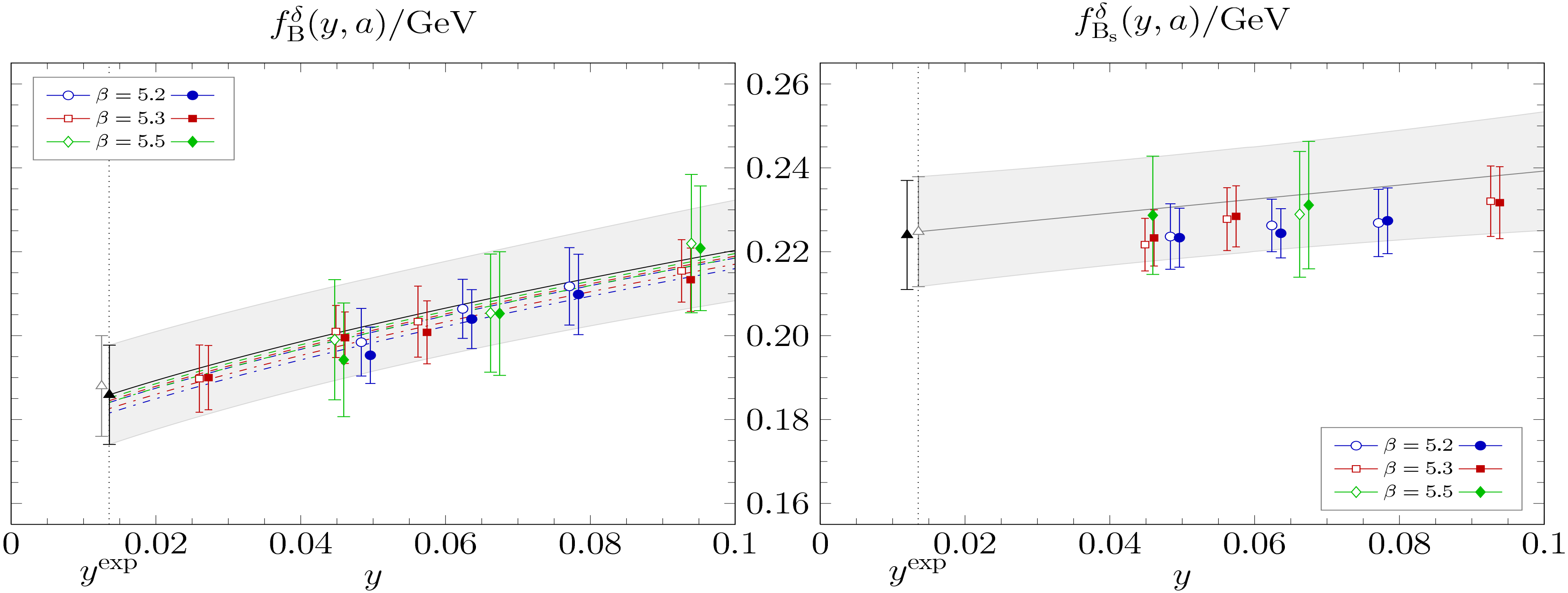
$$y = \frac{m_\pi^2}{8\pi f_\pi^2} \qquad\qquad z = L_1 m_h^{\mathrm{RGI}}$$





Having interpolated the HQET coupling at m_b, it is then straightforward to perform a combined chiral and continuum extrapolation of $f_{B_{d,s}}\sqrt{m_{B_{d,s}}/2}$..

$$y = \frac{m_\pi^2}{8\pi f_\pi^2}$$



$f_B = 186(13)(2)_\chi$ MeV, $f_{B_s}/f_B = 1.203(62)(19)_\chi$, $f_{B_s} = 224(14)(2)_\chi$ MeV

$f_B^{\mathrm{stat}} = 190(5)(2)_\chi$ MeV, $(f_{B_s}/f_B)^{\mathrm{stat}} = 1.189(24)(30)_\chi$, $f_{B_s}^{\mathrm{stat}} = 226(6)(9)_\chi$ MeV

Extraction of $f_{B_{(s)}}$ in $\mathrm{N_f} = 2$ TmQCD [ETMC: N. Carrasco *et al*, '13]

A techniques to interpolate in the m_b region results obtained around m_c, using scaling laws in the heavy quark limit, has been developed with great success.

$$q(x, \lambda, \hat{m}_l) = \lambda^{\alpha} \frac{A_{hl}(1/x, \hat{m}_l)}{A_{hl}(1/\lambda x, \hat{m}_l)} \frac{\mathcal{Z}(\ln x\lambda)}{\mathcal{Z}(\ln x)} \left[\frac{\rho(\ln x)}{\rho(\ln \lambda x)} \right]^{\alpha}$$

$\lambda = \frac{x^{(n-1)}}{x^n} > 1$ is the heavy mass step, $x = 1/\hat{m}_h$, $\hat{m}_{l(h)}$ are renormalised light (heavy) quark masses, $\rho(\ln \hat{m}_h)\hat{m}_h = m_h^{\rm pole}$, $A^{\rm QCD} = \mathcal{Z} A^{\rm HQET}$

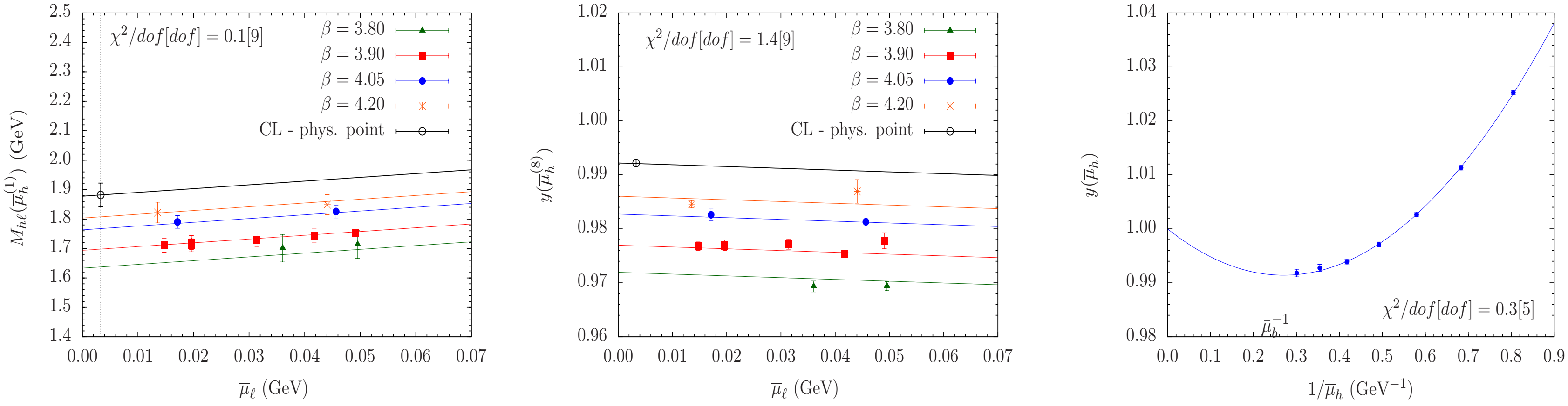
$$\lim|_{x\to 0}[\rho(\ln x)/x]^{\alpha} A_{hl}(1/x)/\mathcal{Z}(\ln x) = \mathrm{C_{ste}} \quad q(\Phi) = \lim|_{a\to 0} q^L(\Phi, a) \quad \hat{m}_b \sim \lambda^K \hat{m}_c$$

$$q_p^{(2)} q_p^{(3)} \cdots q_p^{(K+1)} = \lambda^{K\alpha} \frac{A_{hl}(\hat{m}_h^{(1)})}{A_{hl}(\hat{m}_h^{(K+1)})} \left\{ \frac{\mathcal{Z}(\ln \hat{m}_h^{(1)})}{\mathcal{Z}(\ln \hat{m}_h^{(K+1)})} \left[\frac{\rho(\ln \hat{m}_h^{(K+1)})}{\rho(\ln \hat{m}_h^{(1)})} \right]^{\alpha} \right\}_p$$

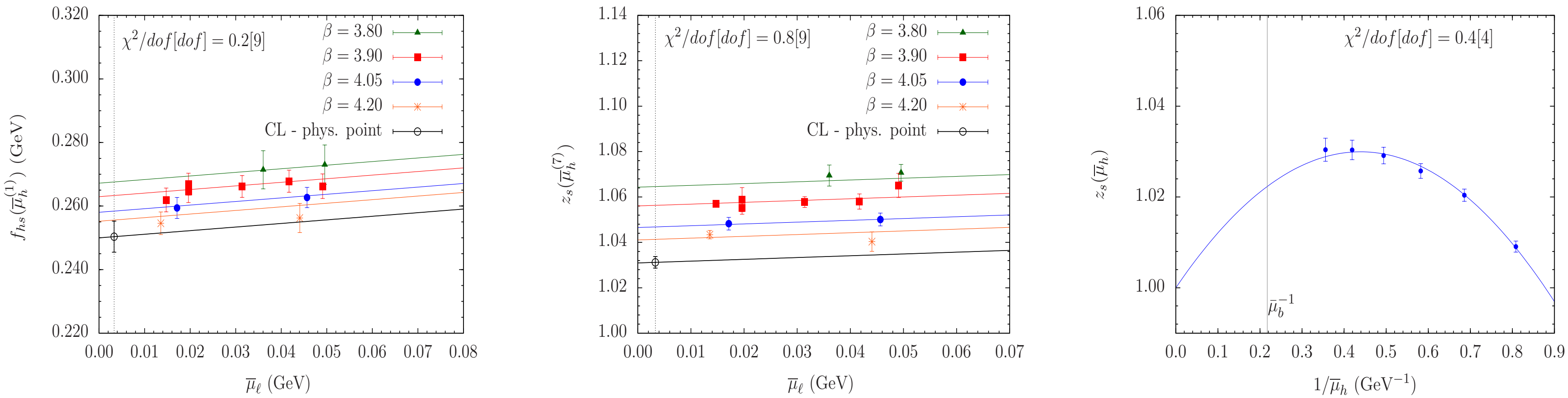
One has to determine K, λ and interpolate lattice data q^L to a sequence of "reference masses" $\hat{m}_h^i = \lambda^i \hat{m}_h^{(1)}$ by a smooth function, then perform a combined fit of $q^L(\hat{m}_h^{(i)})$ to extrapolate to the continuum limit; $A_{hl}(\hat{m}_b) = A_{hl}(\hat{m}_h^{(1)}) \times \Pi_i q(\hat{m}_h^{(i)})$.

$$y = \lambda^{-1} \frac{M_{hl}(1/\lambda x, \hat{m}_l)}{M_{hl}(1/x, \hat{m}_l)} \left[\frac{\rho(\ln x)}{\rho(\ln \lambda x)} \right]^{-1} \qquad z = \lambda^{1/2} \frac{f_{hl}(1/x, \hat{m}_l)}{f_{hl}(1/\lambda x, \hat{m}_l)} \frac{Z_{\text{stat}}(\ln x\lambda)}{Z_{\text{stat}}(\ln x)} \left[\frac{\rho(\ln x)}{\rho(\ln \lambda x)} \right]^{1/2}$$

The first task is to get the b quark mass, through m_{hl}.



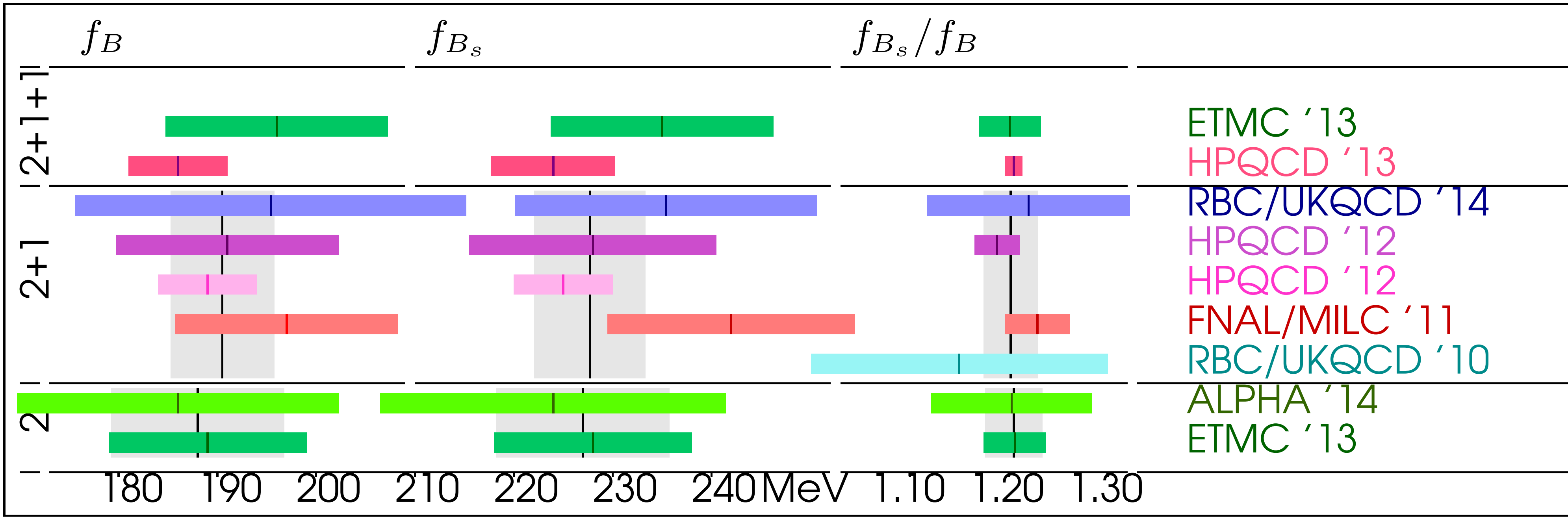
The second step is the extraction of the decay constants f_B and f_{B_s}.



$f_B = 189(8)$ MeV, $f_{B_s} = 228(8)$ MeV, $f_{B_s}/f_B = 1.206(24)$

Collection of lattice results and averages

[N. Christ *et al*, '14]



$$\delta f_B \sim 5\%, \quad \delta f_{B_s} \sim 3.5\%, \quad \delta |f_{B_s}/f_B - 1| \sim 10\%$$