## $B_{s} \rightarrow \mu \mu$ within (mostly) the $S M$

Diego Guadagnoli<br>LAPTh Annecy

## Outline

$\square$ Why $\mathrm{B}_{\mathrm{s}} \rightarrow \mu \mu$
$\square \quad \mathrm{B}_{\mathrm{s}} \rightarrow \mu \mu$ in the SM: structure and theory errors

V $\quad \mathrm{B}_{\mathrm{s}} \rightarrow \mu \mu$ beyond the SM : possible directions

## $B R\left[B_{s} \rightarrow \mu^{+} \mu^{-}\right]$: a hard probe of scalar-fermion interactions

$\square$ Model-independent approach: effective operators

Beyond the SM,
a total of 6 operators can contribute:
(One may write also two tensor operators, but their matrix elements vanish for this process.)

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& \begin{array}{c}
\text { the } \mathbf{g}^{2} \text { dependence } \\
\text { cancels out }
\end{array}
\end{aligned}
$$

So this process is a genuine probe of Yukawa interactions i.e. of the scalar-fermion sector

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\begin{gathered}
\mathrm{B}_{\mathrm{s}} \rightarrow \mu \mu \\
\text { within the } \mathrm{SM}
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## The $B_{s} \rightarrow \mu \mu$ decay within the $S M$ : structure

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hadronic matrix element

Recall: the final state is purely leptonic


The only non-null matrix elem' is:

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\langle 0| \bar{b} \gamma^{\alpha} \gamma_{5} s\left|B_{s}(p)\right\rangle=-i f_{B_{s}} p^{\alpha}
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- Easy to understand: = take the $B$ momentum $p$
$=$ contract $p$ with the lepton current, using $p=p\left(\mu^{+}\right)+p\left(\mu^{-}\right)$
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chiral suppression
- Masses' \& couplings' dependence of the BR =

$\mathrm{BR}\left[\mathrm{B}_{\mathrm{s}} \rightarrow \mu \mu\right]$ error: parametric
$\boxed{\square}$ The main sources of error within the BR formula are:

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Thus, one can write the following phenomenological expression for the BR

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B R\left[B_{s} \rightarrow \mu^{+} \mu^{-}\right]=3.23 \cdot 10^{-9} \cdot\left(\frac{\tau_{B_{s}}}{1.466 \mathrm{ps}}\right) \cdot\left(\frac{\operatorname{Re}\left(V_{t b}^{*} V_{t s}\right)}{4.05 \cdot 10^{-2}}\right)^{2} \cdot\left(\frac{f_{B_{s}}}{227 \mathrm{MeV}}\right)^{2} \cdot\left(\frac{M_{t}}{173.2 \mathrm{GeV}}\right)^{3.07}
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This average is however dominated by one determination (HPQCD collab.), that has about half the error of the other ones.

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This issue is still debatable to some extent (or at least it would be so in case of a SM vs. exp discrepancy)
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\begin{gathered}
\frac{\mathrm{BR}_{\mathrm{th}}}{1-y_{s}}=\mathrm{BR}_{\mathrm{exp}} \\
\text { with } y_{s}=\Delta \Gamma_{s} /\left(2 \Gamma_{s}\right) \simeq 0.088
\end{gathered}
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## $\mathrm{BR}\left[\mathrm{B}_{\mathrm{s}} \rightarrow \mu \mu\right]$ systematics: the initial state oscillates

 Descotes, Matias, Virto, PRD 12;$\square$ The $B_{s} \rightarrow \mu \mu$ rate is measured as follows:



See:

- LHCb 1212.4140
- latest HFAG average: 1207.1158

■ Intuitive picture of this correction

Recall: $\quad \mathrm{BR}_{\mathrm{th}} \propto \frac{1}{\Gamma_{s}}$

Then one finds:

$$
\frac{1}{\Gamma_{s}} \times \frac{1}{1-\Delta \Gamma_{s} /\left(2 \Gamma_{s}\right)}=\frac{1}{\Gamma_{s}} \frac{\Gamma_{s}}{\Gamma_{\text {long }}}
$$

Namely the 1/(1-ys) factor just "renormalizes" $B R_{t h}$ to the width of the long-lived $B_{s}$ eigenstate

## $B R\left[B_{s} \rightarrow \mu \mu\right]$ error: systematics

Initial-state effect

- Effect of $\mathrm{B}_{\mathrm{s}}-\overline{\mathrm{B}}_{\mathrm{s}}$ oscillations: $\quad B R_{\text {exp }}=B R_{\text {th }} \frac{1}{1-\Delta \Gamma_{s} / 2 \Gamma_{s}}=B R_{\mathrm{th}} \times 1.09$
D. Guadagnoli, $B_{s} \rightarrow \mu \mu$ : theory


## $\mathrm{BR}\left[\mathrm{B}_{\mathrm{s}} \rightarrow \mu \mu\right]$ error: systematics

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- Effect of $B_{s}-B_{s}$ oscillations:

De Bruyn et al., PRL 12 \& PRD 12

$$
B R_{\mathrm{exp}}=B R_{\mathrm{th}} \frac{1}{1-\Delta \Gamma_{s} / 2 \Gamma_{s}}=B R_{\mathrm{th}} \times 1.09
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- Effect of soft undetected photons in the final state:

$$
B R_{\text {exp }}=B R_{\mathrm{th}} \times 0.89
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Buras, Girrbach, DG, Isidori, EPJC 13

- Implied systematic error comparable to $f_{B s}$ error

Albeit impact arguably small ( $\sim$ O(1\%))
in appropriate scheme
[see Buras, Girrbach, DG, Isidori, EPJC 13]

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- Incomplete knowledge of NLO EW corrections:
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- Final answer: full calculation
- NLO EW: Bobeth et al., 1311.1348, PRD14
- SM pred.: Bobeth et al., 1311.0903, PRL14
- See also NNLO QCD: Hermann et al., 1311.1347, JHEP13


## $\mathrm{BR}\left[\mathrm{B}_{\mathrm{s}} \rightarrow \mu \mu\right]$ error: systematics

## Initial-state effect <br>  effect

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De Bruyn et al., PRL 12 \& PRD 12

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Taken into account by exp

Buras, Girrbach, DG, Isidori, EPJC 13

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All in all, theory (SM) ready to match expected experimental accuracy
D. Guadagnoli, $B_{s} \rightarrow \mu \mu$ : theory

## Some considerations on

## $B_{s} \rightarrow \mu \mu$ beyond the SM

## How to probe scalar operators and their phases thru $B_{s} \rightarrow \mu \mu$

$\sqrt{\square}$ Back to the initial-state systematic effect. For general new physics, the correction factor becomes

$$
\mathrm{BR}_{\mathrm{th}} \cdot\left(\frac{1+A_{\Delta \Gamma} y_{s}}{1-y_{s}^{2}}\right)=\mathrm{BR}_{\mathrm{exp}}
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$$
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$$

where

$$
A_{\Delta \Gamma}=\frac{|P|^{2} \cos \left(2 \Phi_{P}\right)-|S|^{2} \cos \left(2 \Phi_{S}\right)}{|P|^{2}+|S|^{2}}
$$

normalized Wilson coeff
for $\mathrm{O}_{\mathrm{A}}$ (= SM operator)
and $\mathrm{O}_{\mathrm{P}}$ (and primed counterparts)

$$
P_{S M}=1
$$

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$$
\begin{aligned}
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& \text { eff } \\
& \text { r) } \quad \begin{array}{l}
\text { normalized Wilson coeff } \\
\text { for } \mathrm{O}_{\mathrm{s}} \\
\text { (and primed counterpart) } \\
\text { rennerparts }^{S_{\mathrm{SM}}=0}
\end{array}
\end{aligned}
$$

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$$

where

## NP phases from $P$ and $S$

$$
A_{\Delta \Gamma}=
$$

$$
P_{\mathrm{sm}}=1
$$



- this NP could involve CPV or not
( The crucial point is that $A_{\Delta r}$ can be extracted from

$$
\begin{gathered}
\substack{B_{s} \rightarrow \mu \mu \\
\text { effective } \\
\text { lifetime }}
\end{gathered} \tau_{\mu \mu} \equiv \frac{\int t d t\left(\Gamma\left(B_{s}(t) \rightarrow \mu \mu\right)+\Gamma\left(\bar{B}_{s}(t) \rightarrow \mu \mu\right)\right)}{\int d t\left(\Gamma\left(B_{s}(t) \rightarrow \mu \mu\right)+\Gamma\left(\bar{B}_{s}(t) \rightarrow \mu \mu\right)\right)}
$$

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$$

$$
=\text { known function of } A_{\Delta r}, \underbrace{\tau_{\mathrm{Bs}} \text { and } \mathrm{y}_{\mathrm{s}}}
$$

## Scalar operators and their phases thru $B_{s} \rightarrow \mu \mu$

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$$

$$
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$$

$$
\text { (After having measured } \left.A_{\Delta \Gamma} \neq 1\right)
$$

- To clarify whether it will be new CPV or not will call for measuring the time-dependent CP asymmetry
- This quantity requires tagging \& time-dependence measurements in an ultra-rare decay


## Conclusions

$\square B_{s} \rightarrow \mu \mu$ SM prediction

- Systematics is under control - within below O(1\%)
- Parametrics soon (or already?) dominated by CKM error
- Outlook: The SM error is, and will remain, negligible w.r.t. exp error


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$\nabla B_{s} \rightarrow \mu \mu$ beyond the SM
- Exquisite probe of the Yukawa sectorScalar operators $\mathrm{O}_{s, \mathrm{P}}{ }^{(\text {(') }}$ and their phases


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Scalar operators $O_{s, P}{ }^{\left({ }^{\prime}\right)}$ and their phases

- Excellent probe of anomalous Z-to-quark couplings


Vector operators $\mathrm{O}_{A}{ }^{(\prime)}$

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Dos
Constraining power superior to Z-peak observables measured at LEP (within reasonable flavor frameworks such as MFV or partial compositeness)

