Summary : meaning some points that caught my eye

Bakker and **Lorcê** were given the tasks of explaining Light Front Dynamics and introducing the whole subject of the Angular Momentum Controversy.
These tasks ere carried admirably!

SEATTLE 2011:

Aim: Workshop will resolve controversy about AM of quarks and gluons

Result: Opened Pandora's box. Even greater controversy. Masses of papers.

TRENTO 2014:

Cedric Lorcé and I feel there has been a convergence.
Less controversy, more agreement.

Wakamatsu: very clear explanation of beautiful developments of gauge invariant versions based on splitting

$$A^{\mu} = A^{\mu}_{\text{phys}} + A^{\mu}_{\text{pure}} \tag{1}$$

which is an extension of the very old idea

$$\boldsymbol{A} = \boldsymbol{A}_{\perp} + \boldsymbol{A}_{\parallel}. \tag{2}$$

However:there is an infinite number of ways to do this.

Main points

1)Out of the infinite number of possibilities there are two fundamental versions of momentum and angular momentum: CANONICAL and KINETIC.

Canonicals generate translations and rotations at equal time, but are **NOT** gauge invariant.

Kinetics are **NOT** generators, but are gauge invariant.

Both reflect some aspects of internal structure of the nucleon.

Major motivation for the above developments was the belief:

THE OPERATOR CORRESPONDING TO A MEASURABLE QUANTITY MUST BE GAUGE INVARIANT

QCD: Wakamatsu's comment:

3. “Canonical” or “Mechanical” decomposition ?

Historically, it was a common belief that the canonical OAM appearing in the Jaffe-Manohar decomposition would not correspond to observables, because they are not gauge-invariant quantities.

This nebulous impression did not change even after a gauge-invariant version of the Jaffe-Manohar decomposition a la Bashinsky and Jaffe appeared.

However, the situation has changed drastically after Lorcé and Pasquini showed that the canonical quark OAM can be related to a certain moment of a quark distribution function in a phase space, called the Wigner distribution.

$$\rho^q(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp ; \mathcal{W}) \;=\; \int \frac{d^2 \boldsymbol{\Delta}_\perp}{(2\,\pi)^2}\, e^{-\,i\,\boldsymbol{\Delta}_\perp \cdot \boldsymbol{b}_\perp}\, \frac{1}{2} \int \frac{dz^-\, d^2 z_\perp}{(2\,\pi)^3}\, e^{i\,(x\,\bar{P}^+\, z^- \,-\, \boldsymbol{k}_\perp \cdot \boldsymbol{z}_\perp)}$$
$$\times\; \langle P'^+, \frac{\boldsymbol{\Delta}_\perp}{2}, S\,|\, \bar{\psi}\left(-\frac{z}{2}\right) \gamma^+\, \mathcal{W}\, \psi\left(\frac{z}{2}\right) |\, P^+, -\frac{\boldsymbol{\Delta}_\perp}{2}, S \rangle \,|_{z^+ = 0}$$

$x \;=\; k^+ \,/\, \bar{P}^+,$

$\boldsymbol{k}_\perp$: `transverse momentum`

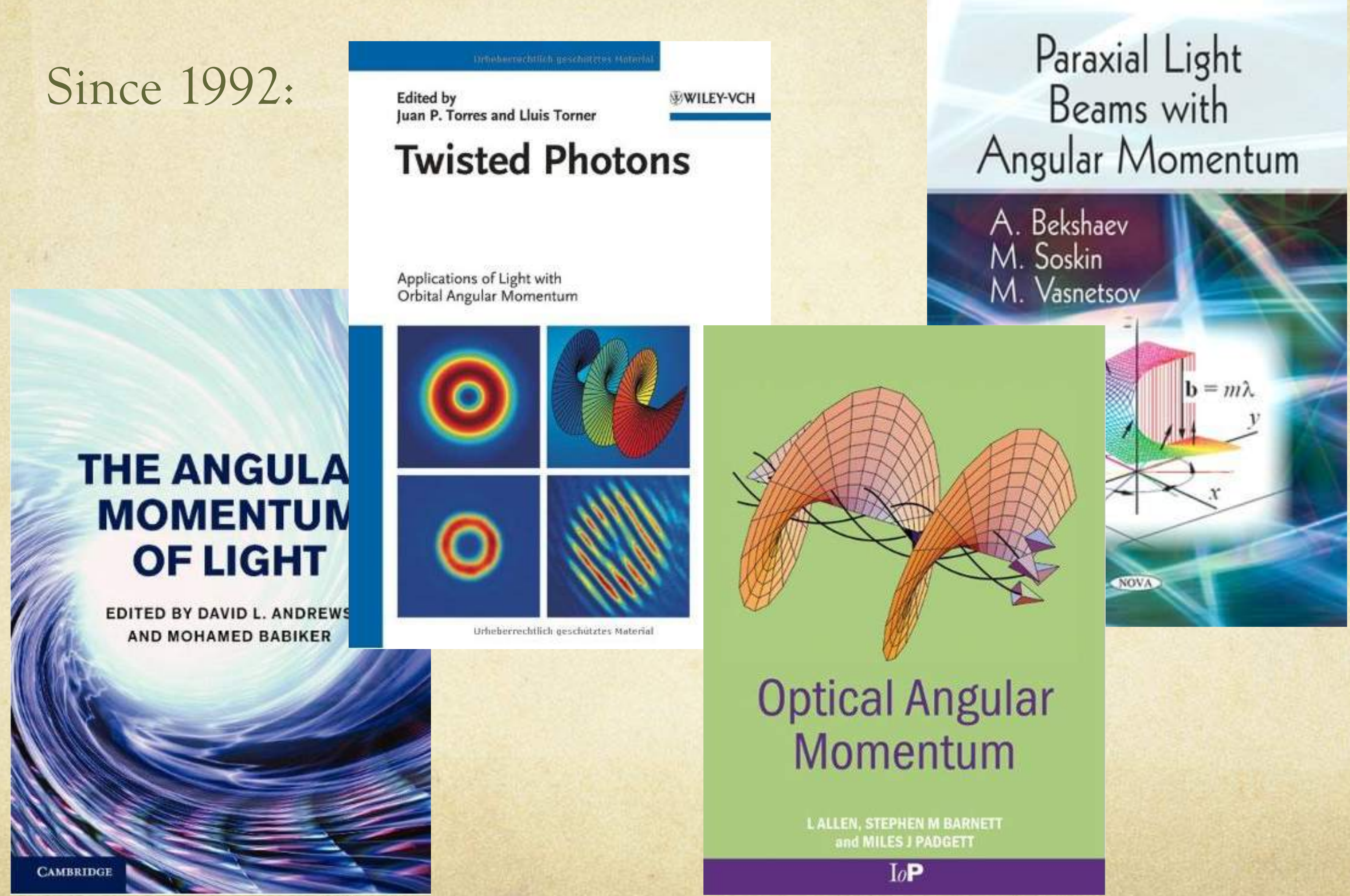
$\mathcal{W}$: `gauge-link`,

$\boldsymbol{b}_\perp$: `impact parameter`

Bliokh's talk provided a severe shock!
In Quantum Optics they have, for decades, been happily MEASURING quantities that most of us considered not measurable, because not represented by a gauge-invariant operator.

SAM and OAM in paraxial beams

Since 1992:



Urheberrechtlich geschütztes Material

Edited by
Juan P. Torres and Lluis Torner

WILEY-VCH

Twisted Photons

Applications of Light with
Orbital Angular Momentum

Urheberrechtlich geschütztes Material

Paraxial Light
Beams with
Angular Momentum

A. Bekshaev
M. Soskin
M. Vasnetsov

NOVA

THE ANGULA
MOMENTUM
OF LIGHT

EDITED BY DAVID L. ANDREWS
AND MOHAMED BABIKER

CAMBRIDGE

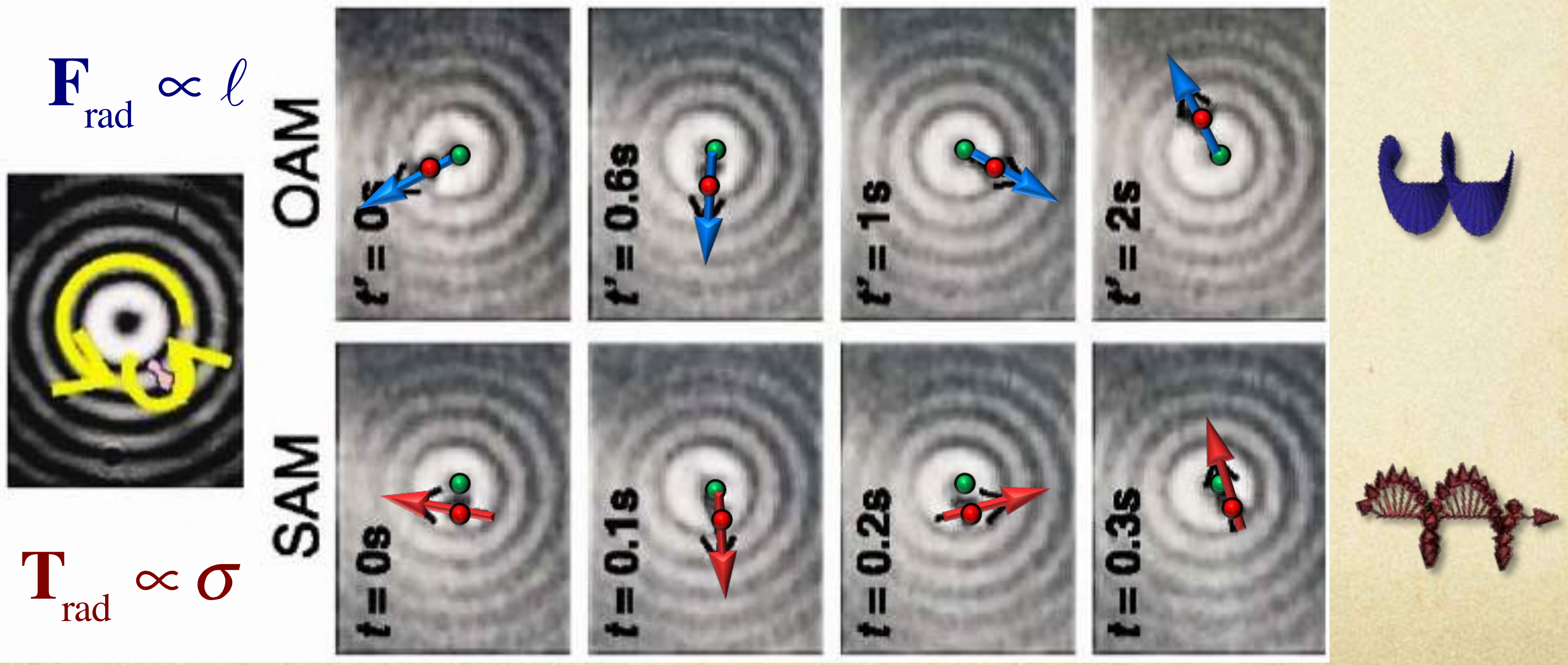
Optical Angular
Momentum

L ALLEN, STEPHEN M BARNETT
and MILES J PADGETT

IoP

Observations of the SAM and OAM

We clearly see different manifestations of the SAM and OAM in paraxial optical beams via spinning and orbital motion of a probe particle, determined by the helicity and vortex quantum numbers:



$\mathbf{F}_{\mathrm{rad}} \propto \ell$

$\mathbf{T}_{\mathrm{rad}} \propto \sigma$

O’Neil *et al.*, PRL (2002); Garces-Chavez *et al.*, PRL (2003)

Essential point:

You **can** measure the value of a **gauge-non-invariant** quantity expressed in some particular gauge.
AND that might be interesting.

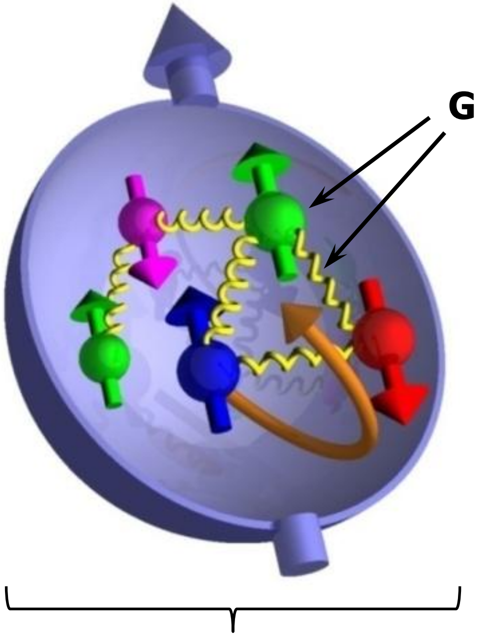
Moreover, post-Seattle developments tell us that we can write

$$\left(O_{\text{non-GI}}\right)\Big|_{\text{gauge X}} = \left(O'_{\text{GI}}\right) \qquad O' = \text{GIE of}\, O \qquad (3)$$

Another argument why you should not be unhappy working in a particular gauge:

Back to basics

Gauge theory



Gauge non-invariant

Gauge invariant

[…] in QCD we should make clear what a quark or gluon parton is in an interacting theory. The subtlety here is in the issue of gauge invariance: a pure quark field in one gauge is a superposition of quarks and gluons in another. Different ways of gluon field gauge fixing predetermine different decompositions of the coupled quark-gluon fields into quark and gluon degrees of freedom.

[Bashinsky, Jaffe (1998)]

A choice of gauge is a choice of basis

Why use notation J_{kin} ?

Gauge invariant version of J due to Belinfante

$$\begin{aligned} \boldsymbol{J}_{\text{Bel}} &= \underbrace{\int \mathrm{d}^3x\, \overline{\psi} \left[\boldsymbol{x} \times \tfrac{1}{2} (\gamma^0\, i\boldsymbol{D} + \boldsymbol{\gamma}\, iD^0) \right] \psi}_{\boldsymbol{J}^e_{\text{Bel}}} \\ &+ \underbrace{\int \mathrm{d}^3x\, \boldsymbol{x} \times (\boldsymbol{E} \times \boldsymbol{B})}_{\boldsymbol{J}^\gamma_{\text{Bel}}} \end{aligned} \tag{4}$$

No electron spin! In Ji's 1991 paper

$$\begin{aligned} \boldsymbol{J}_{\text{Ji?}} &= \underbrace{\int \mathrm{d}^3x\, \psi^\dagger \tfrac{1}{2} \boldsymbol{\Sigma} \psi}_{\boldsymbol{S}^e_{\text{Ji}}} + \underbrace{\int \mathrm{d}^3x\, \psi^\dagger (\boldsymbol{x} \times i\boldsymbol{D}) \psi}_{\boldsymbol{L}^e_{\text{Ji}}} \\ &+ \underbrace{\int \mathrm{d}^3x\, \boldsymbol{x} \times (\boldsymbol{E} \times \boldsymbol{B})}_{\boldsymbol{J}^\gamma_{\text{Ji}}} . \end{aligned} \tag{5}$$

Why not label "Ji"?
Ji stated that he had found this expression somewhere, but could not remember where.

We suggest "**kinetic**" because $\boldsymbol{p}_{\text{kin}}$ points in direction of motion of a classical particle.

e.g. charge moving in helix in given uniform magnetic field $\boldsymbol{B}$:
$\boldsymbol{p}_{\text{kin}}$ follows motion.

Care needed in labelling what you are talking about

Nucleon moving along OZ.
Canonical case: e.g.

$$\ell^q_{\text{can},z} \equiv \langle\langle P, S_L | L^q_{\text{can},\,z} | P, S_L \rangle\rangle \tag{6}$$

$$J^q_{\text{can},z} \equiv \langle\langle P, S_L | J^q_{\text{can},\,z} | P, S_L \rangle\rangle, \tag{7}$$

Kinetic case: e.g.

$$\ell^q_{\text{kin},z} \equiv \langle\langle P, S_L | L^q_{\text{kin},z} | P, S_L \rangle\rangle \tag{8}$$

$$J^q_{\text{kin},z} \equiv \langle\langle P, S_L | J^q_{\text{kin},z} | P, S_L \rangle\rangle. \tag{9}$$

es

One more subtlety

Should state somewhere whether you are using

INSTANT form or LIGHT FRONT form dynamics

Burkardt: very nice interpretation of the difference $\ell^q_{\text{can},z} - \ell^q_{\text{kin},z}$.
= average change in OAM due to torque caused by final state interactions as the quark leaves the target.

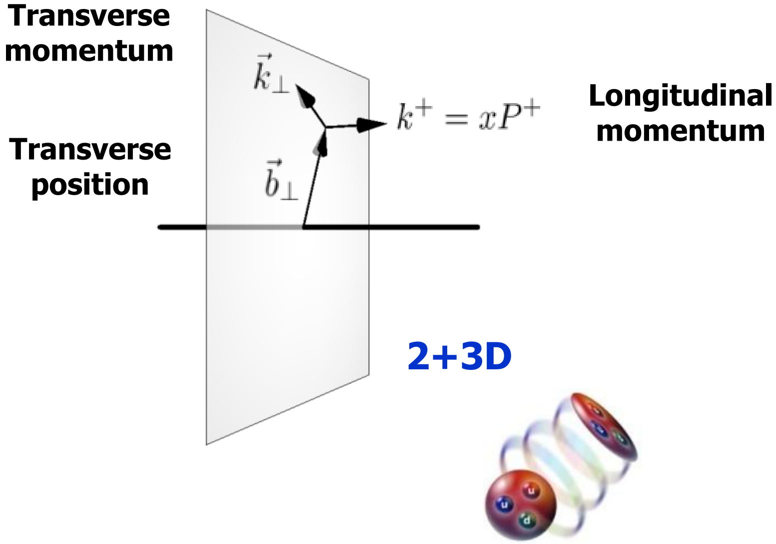
Key question: how to measure $\ell^q_{\mathrm{can},z}$ and $\ell^q_{\mathrm{kin},z}$

Pasquini (1) Approach via GMTDs and then (2) models of the GTMDs.
Courtoy Criticizes the starting point ie the approach via GTMDs
Liuti Agrees with Courtoy
Lorcé and **Pasquini** disagree

Partonic interpretation

Phase-space «density»

$$\rho^{\mathcal{O}}_{\Lambda'\Lambda}(P,k,\vec{b}_\perp;\mathcal{W})=\int\frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2}\,e^{-i\vec{\Delta}_\perp\cdot\vec{b}_\perp}\,W^{\mathcal{O}}_{\Lambda'\Lambda}(P,k,\Delta;\mathcal{W})\big|_{\Delta^+=0}$$



Transverse momentum

Transverse position

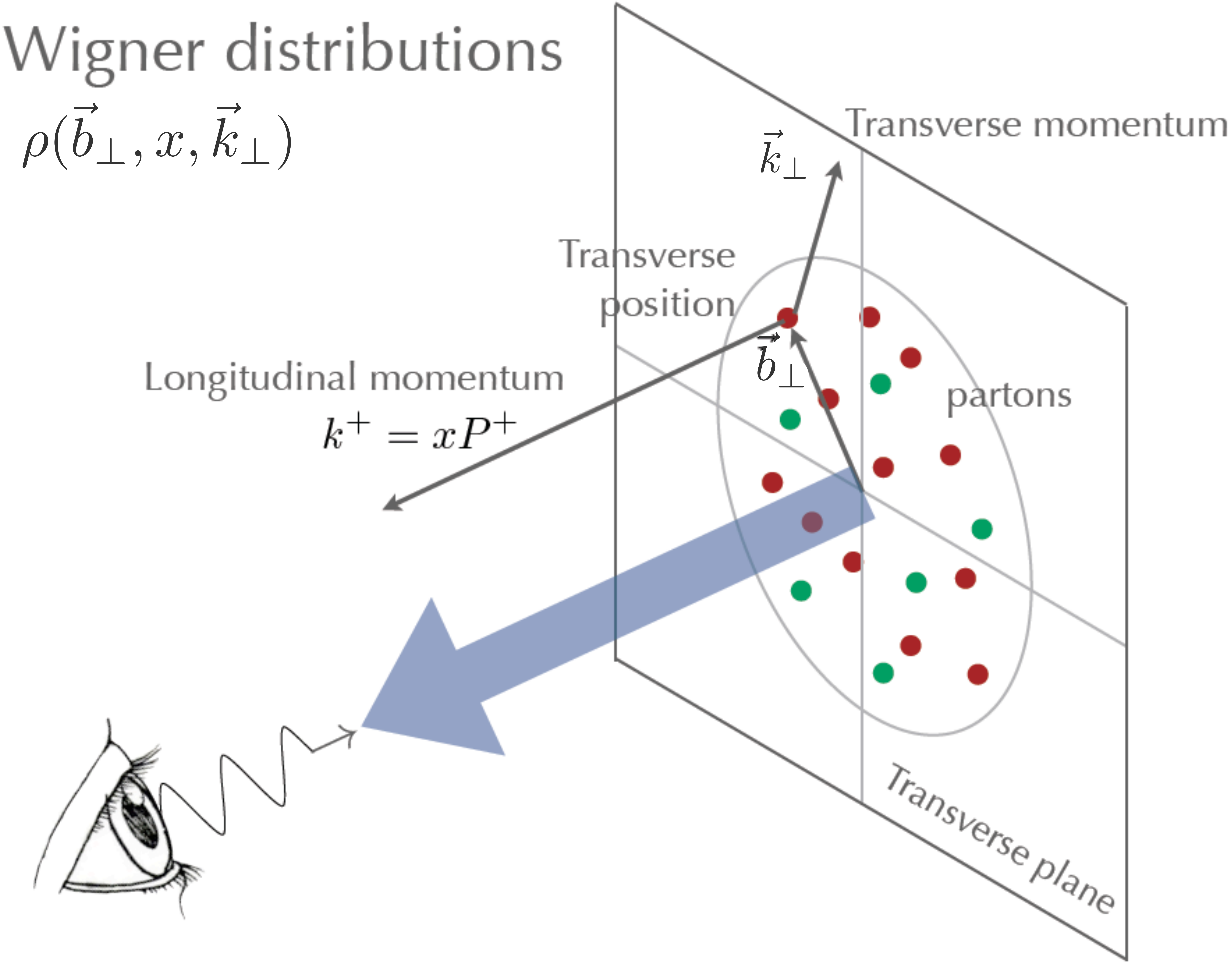
Longitudinal momentum

2+3D

[Ji (2003)]
[Belitsky, Ji, Yuan (2004)]
[C.L., Pasquini (2011)]

Wigner distributions

$\rho(\vec{b}_\perp, x, \vec{k}_\perp)$



Results of model calculations

Light Front Constituent Quark Model (LFCQM): similar to **Ma**
Light Front Chiral Quark Soliton Model (LFχQSM)

Model	LFCQM			LFχQSM		
q	u	d	Total	u	d	Total
$\ell^q_{\mathrm{kin},z}$	0.071	0.055	0.126	−0.008	0.077	0.069
$\ell^q_{\mathrm{can},z}$	0.131	−0.005	0.126	0.073	−0.004	0.069
$\mathcal{L}^q_{\mathrm{can},z}$	0.169	−0.042	0.126	0.093	−0.023	0.069

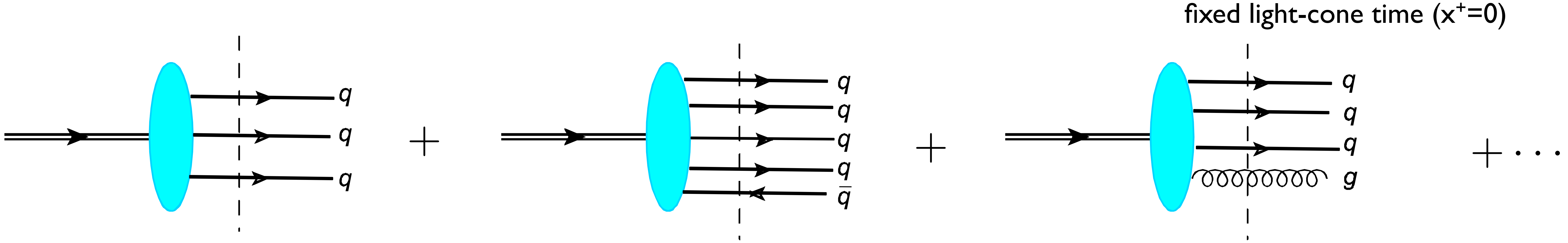
A puzzle

Lorcéand Pasquini use only one term in the Fock expansion:

Light-Front Wave Functions (LFWFs)

- ✦ Fock expansion of Nucleon state:

$$|N\rangle = \Psi_{3q}|qqq\rangle + \Psi_{3q\,q\bar{q}}|3q\,q\bar{q}\rangle + \Psi_{3q\,g}|qqqg\rangle + \cdots$$



In Fock states the quanta are **FREE** particles, so would expect no difference between Canonical and Kinetic OAM for **quarks**

Bacchetta was pessimistic about ever being able to test the Jaffe-Manohar sum rule. The least tractable part is the gluon spin term $\Delta G(x)$. The error corridor is large, especially at small x, leading to large errors in the integral.

I think the same problem arises in every relation or sum rule in QCD e.g. the Bjorken sum rule and we simply have to try to continuously improve the data.

Bacchetta also discussed the lensing formula relating TMDs to GPDs:

2

Sivers function and dynamical AM

$$f_{1T}^{\perp(0)a}(x;Q_L^2) = -L(x)\,E^a(x,0,0;Q_L^2)$$

Sivers TMD

Lensing function

TMDs

GPDs

Unfortunately there seems to be no way to establish such a relation beyond models.

Liu presented beautiful results of a quenched lattice calculation of kinetic angular momentum.

Most noteworthy points are:

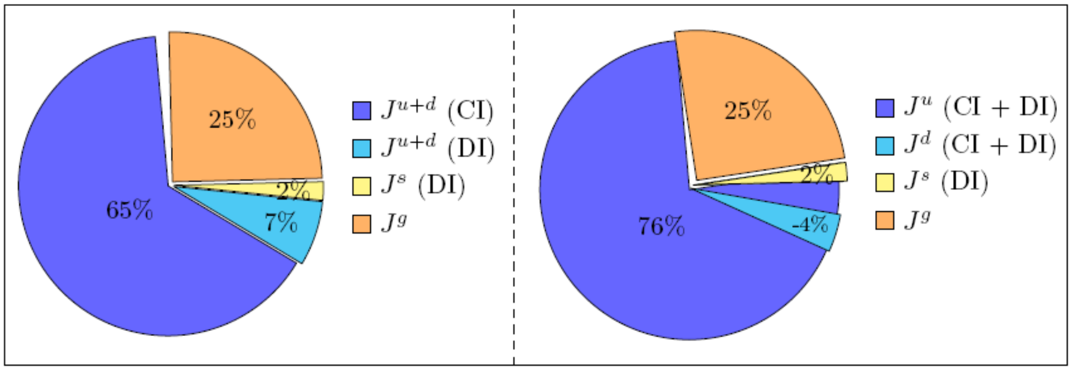
1) There is a considerable contribution from Disconnected Insertions (DI).

Previously

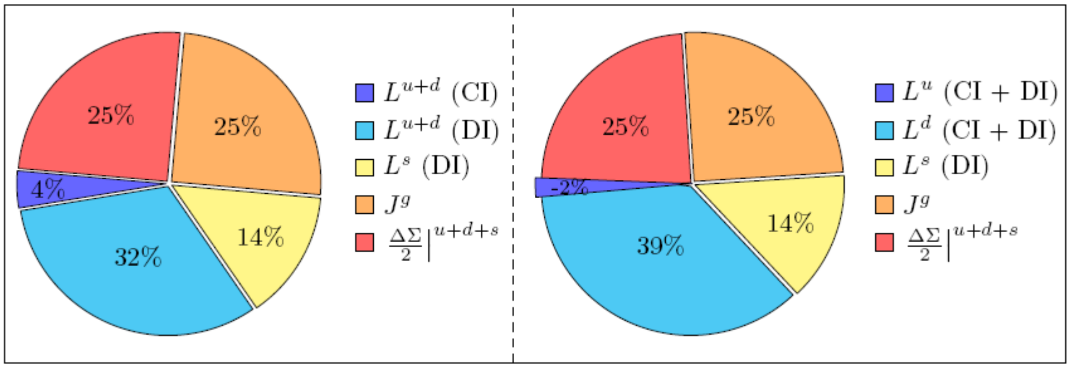
$$L_u \approx -L_d \tag{1}$$

so that the total quark orbital contribution was zero. Now DI shifts them upwards, but does not alter the surprising result that L_u is negative.

2) For the first time it was possible to measure J_G which is far from negligible in value.



G. 9. Preliminary results for the quark and gluon contributions to the angular momentum, presented at Lattice 2013 ourtesy of Keh-Fei Liu.



lieved to alter the results by just a few percent.

	CI(u)	CI(d)	CI($u+d$)	DI(u/d)	DI(s)	Glue
$\langle x\rangle$	0.428(40)	0.156(20)	0.586(45)	0.038(7)	0.024(6)	0.313(56)
$2J$	0.726(128)	-0.072(82)	0.651(51)	0.036(7)	0.023(7)	0.254(76)
a_0	0.91(11)	-0.30(12)	0.61(8)	-0.12(1)	-0.12(1)	–
$2L$	-0.18(18)	0.23(14)	0.04(10)	0.16(2)	0.14(2)	–

Interesting to compare with results presented by textbfKroll, based on the Ji relation between J_{kin} and GPDs.
The GPDs are parametrized so as to respect key properties and are fitted to EM form factor data.
This means that only valence quark information can be obtained.

Application: Angular momenta of partons

$$J^q = \frac{1}{2}\Big[q_{20} + e^q_{20}\Big] \qquad J^g = \frac{1}{2}\Big[g_{20} + e^g_{20}\Big] \qquad (\xi = t = 0)$$

q_{20}, g_{20} from ABM11 (NLO) PDFs

$e^{q_v}_{20}$ from form factor analysis Diehl-K. (13)

$e^s_{20} \approx 0 \cdots - 0.026$ from A_{UT} in DVMP GK(09) and DVCS KMS(13)

and saturation of pos. bound (flavor symm. sea assumed for E)

e^g_{20} from sum rule for e_{20}

$$J^{u+\bar{u}} = 0.261\,;\, J^{d+\bar{d}} = 0.035\,;\, J^{s+\bar{s}} = \quad 0.017\,;\, J^g = 0.188 \quad (e^s_{20} = 0)$$

$$= 0.235\,; \qquad = 0.009\,; \qquad = -0.009\,; \qquad = 0.266 \quad (e^s_{20} = -0.026)$$

J^i quoted at scale $2\,\mathrm{GeV}$ $\quad \sum J^i = 1/2$ spin of the proton

need better determ. of E^s (smaller errors of A_{UT} in DVCS)

Strange and Gluon are in fair agreement

$u + \bar{u}$ of Kroll is considerably smaller

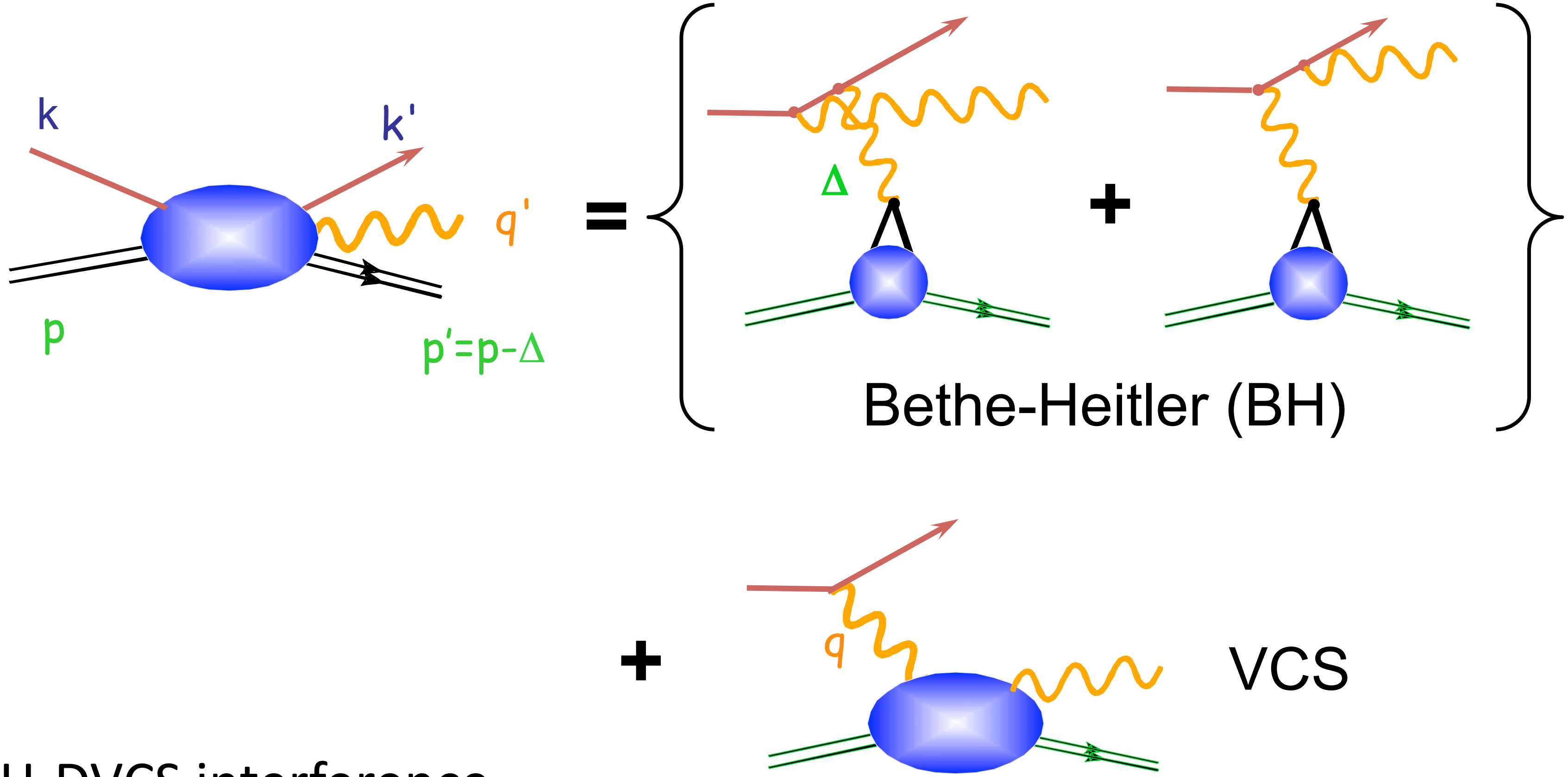
Sign of small contribution $d + \bar{d}$ is different.

Several experimental talks: the field is full of promise

Hyde: GPDs at Jefferson lab

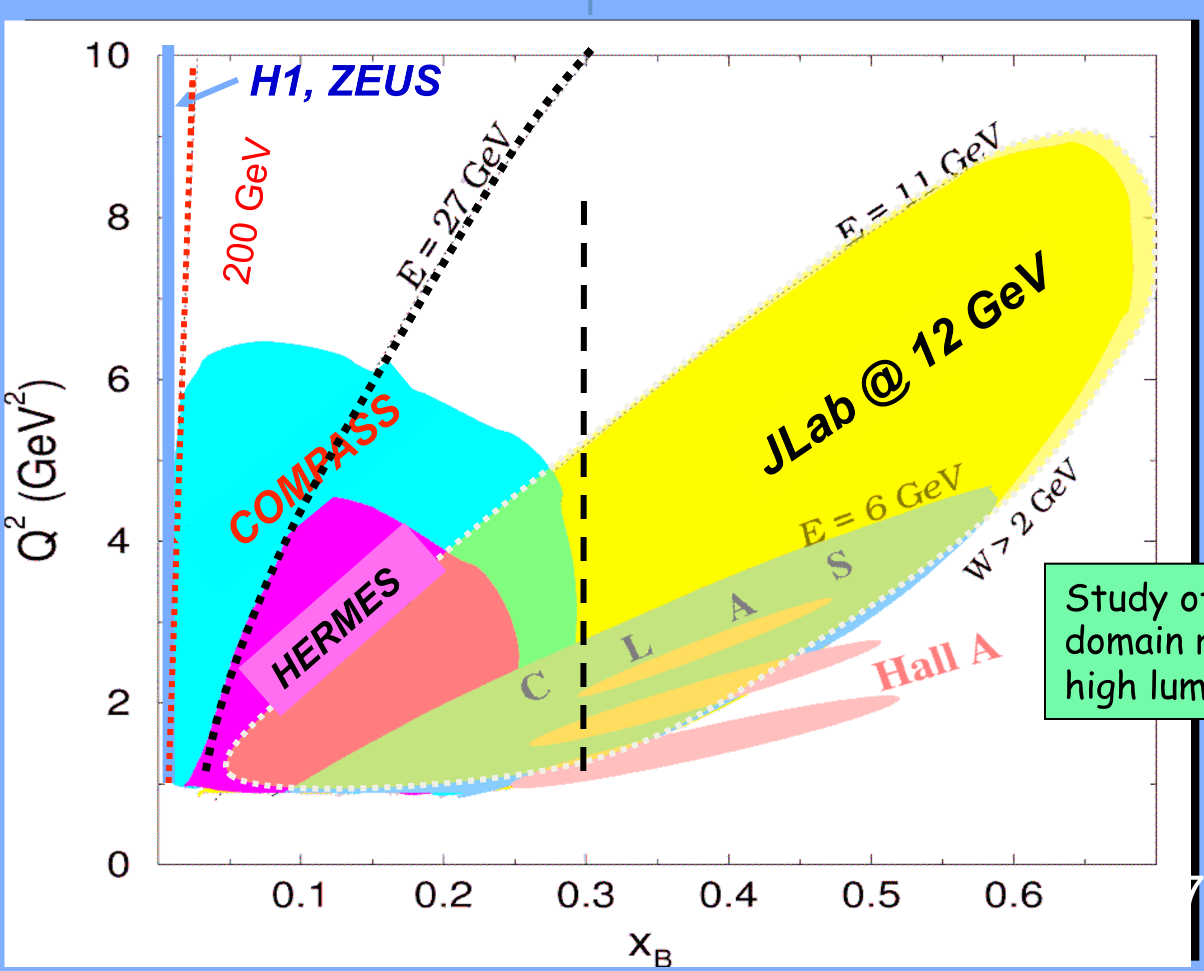
Bethe-Heitler (BH) and Virtual Compton Scattering (VCS)

$e\,p \rightarrow e\,p\,\gamma$



- BH-DVCS interference
  - Access to DVCS amplitude, linear in GPDs

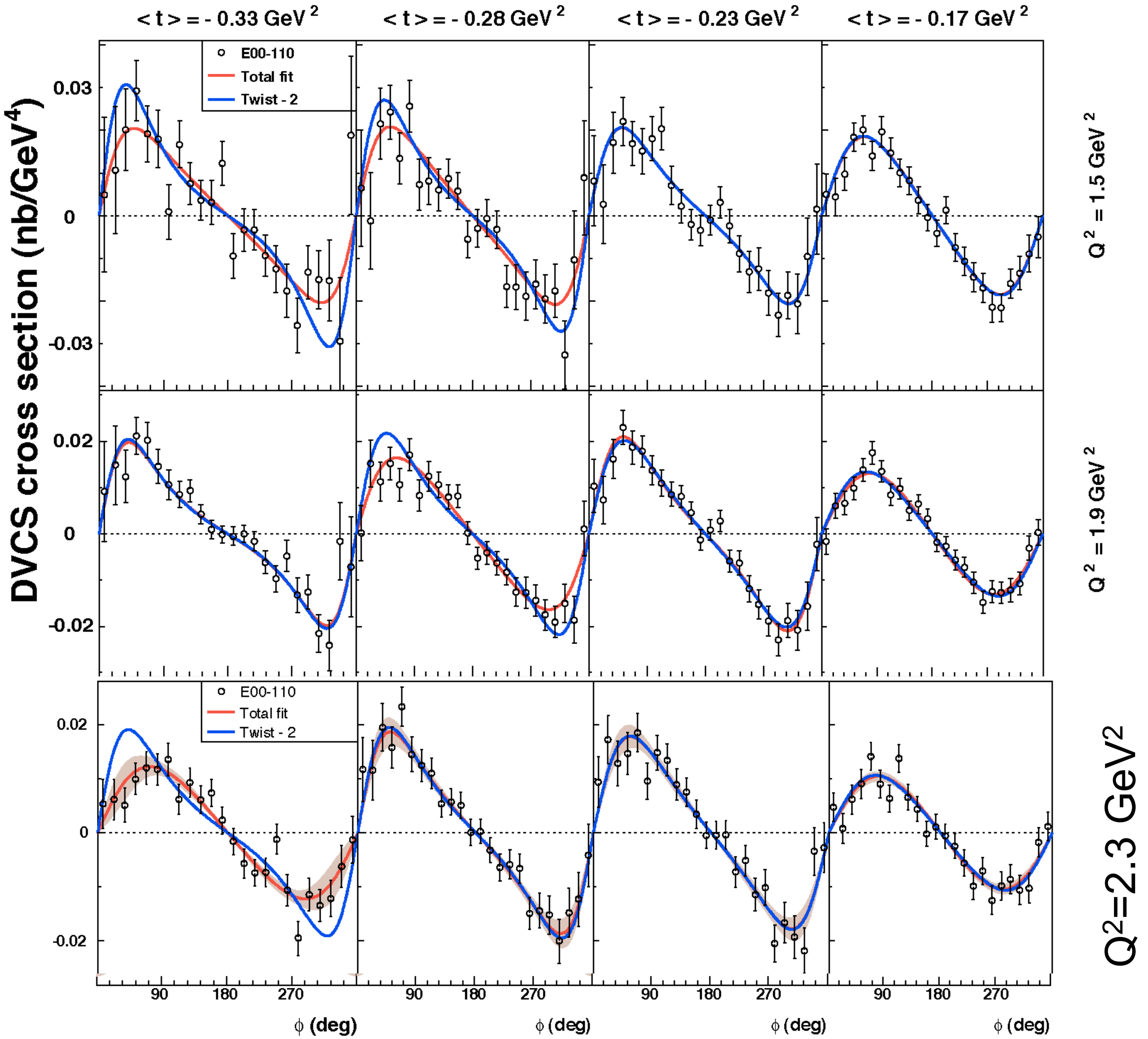
Deeply Virtual Exclusive Processes - Kinematic Coverage



Study of high x_B domain requires high luminosity

Hall A Helicity Dependent Cross Sections E00-110

PRL**97**:262002 (2006)
C. MUNOZ CAMACHO, et al.,



Twist-2(GPD)+…

Twist-3(qGq)+…

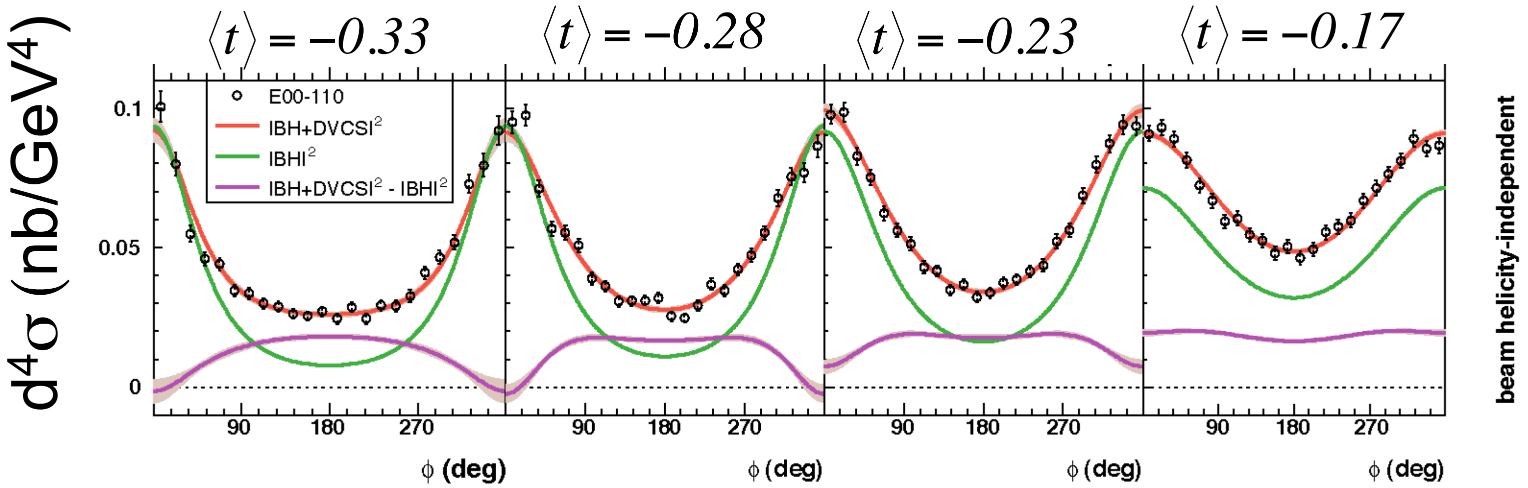
$$\sum h d\sigma(h) = \frac{\boxed{s_1}\sin(\phi_{\gamma\gamma})\boxed{\Gamma_{s1}} + \boxed{s_2}\sin(2\phi_{\gamma\gamma})\boxed{\Gamma_{s2}}}{P_1(\phi_{\gamma\gamma})P_1(\phi_{\gamma\gamma})}$$

$\Gamma_{s1,2}$ = kinematic factors

Beam helicity-independent cross sections at $Q^2=2.3$ GeV^2, $x_B=0.36$

- Contribution of Re[DVCS*BH] + $|\text{DVCS}|^2$ large.
- Measurements at multiple incident energies to separate these two terms and isolate Twist 2 from Twist-3 contributions

PRL**97**:262002 (2006) *C. MUNOZ CAMACHO, et al.,*

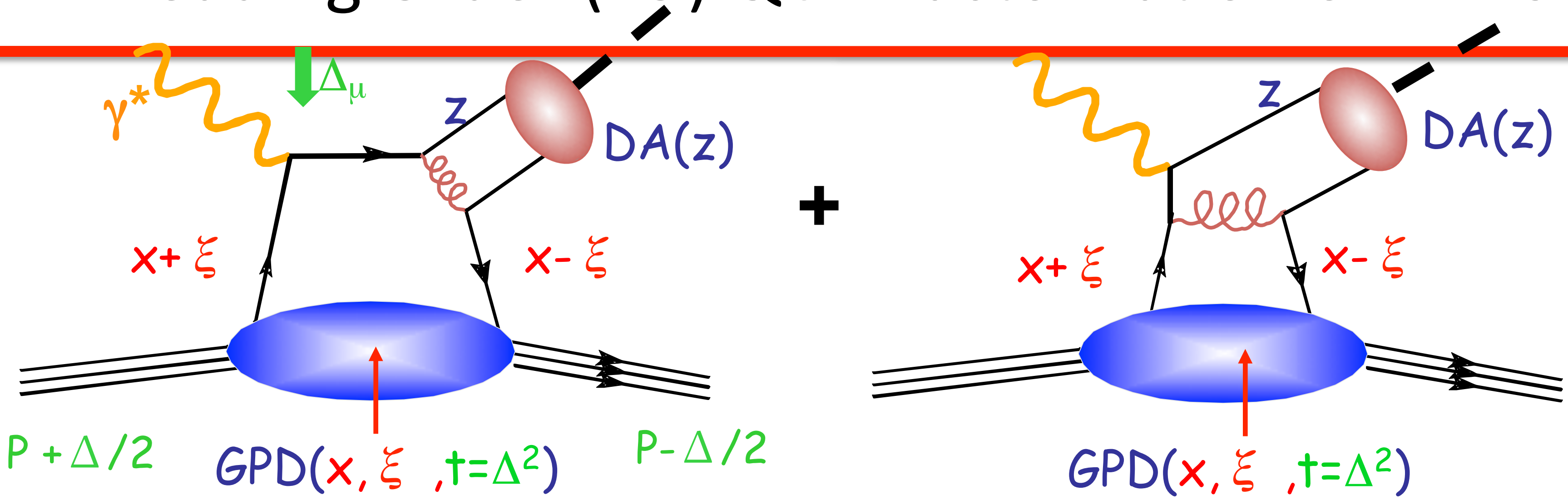


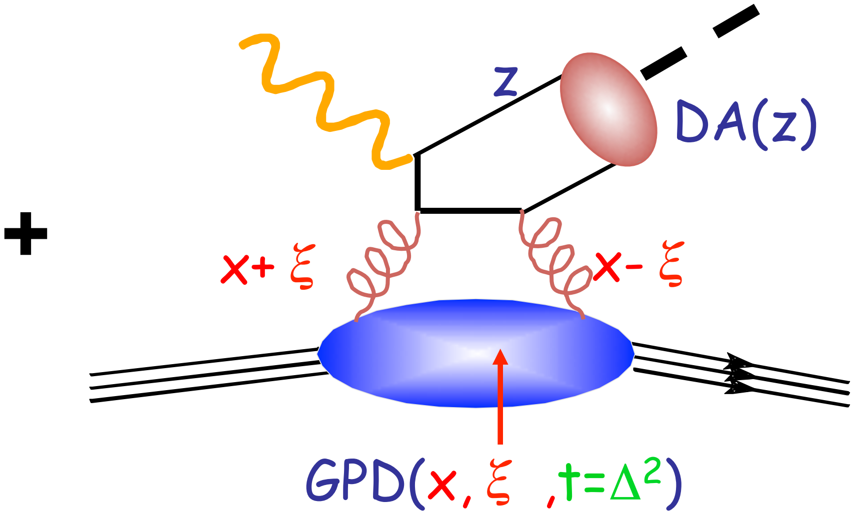
$$\boxed{d\sigma} = \boxed{d\sigma(|BH|^2)} + \boxed{2\,\mathrm{Re}[DVCS^* BH] + |DVCS|^2}$$

$$= d\sigma(|BH|^2) + \boxed{\frac{c_0\Gamma_0 + c_1\cos(\phi_{\gamma\gamma})\Gamma_1 + c_2\cos(2\phi_{\gamma\gamma})\Gamma_2 + \ldots}{P_1(\phi_{\gamma\gamma})P_1(\phi_{\gamma\gamma})}}$$

$$\left.\begin{array}{l} \boxed{c_{0,1}(t)} \approx \mathrm{Re}[C^I(GPD)] \pm C^{DVCS}\left(GPD^2\right)\ldots + \mathrm{Re}\left[\Delta C^I(GPD)\right] \\ \boxed{c_2(t)} = \mathrm{Twist}-3 = (qGq) \end{array}\right\}$$

Leading Order (LO) QCD Factorization of DVES





Gluon and quark GPDs enter to same order in α_S.

SCHC: $\sigma_L \sim [Q^2]^{-3}$ $\sigma_T \sim [Q^2]^{-4}$

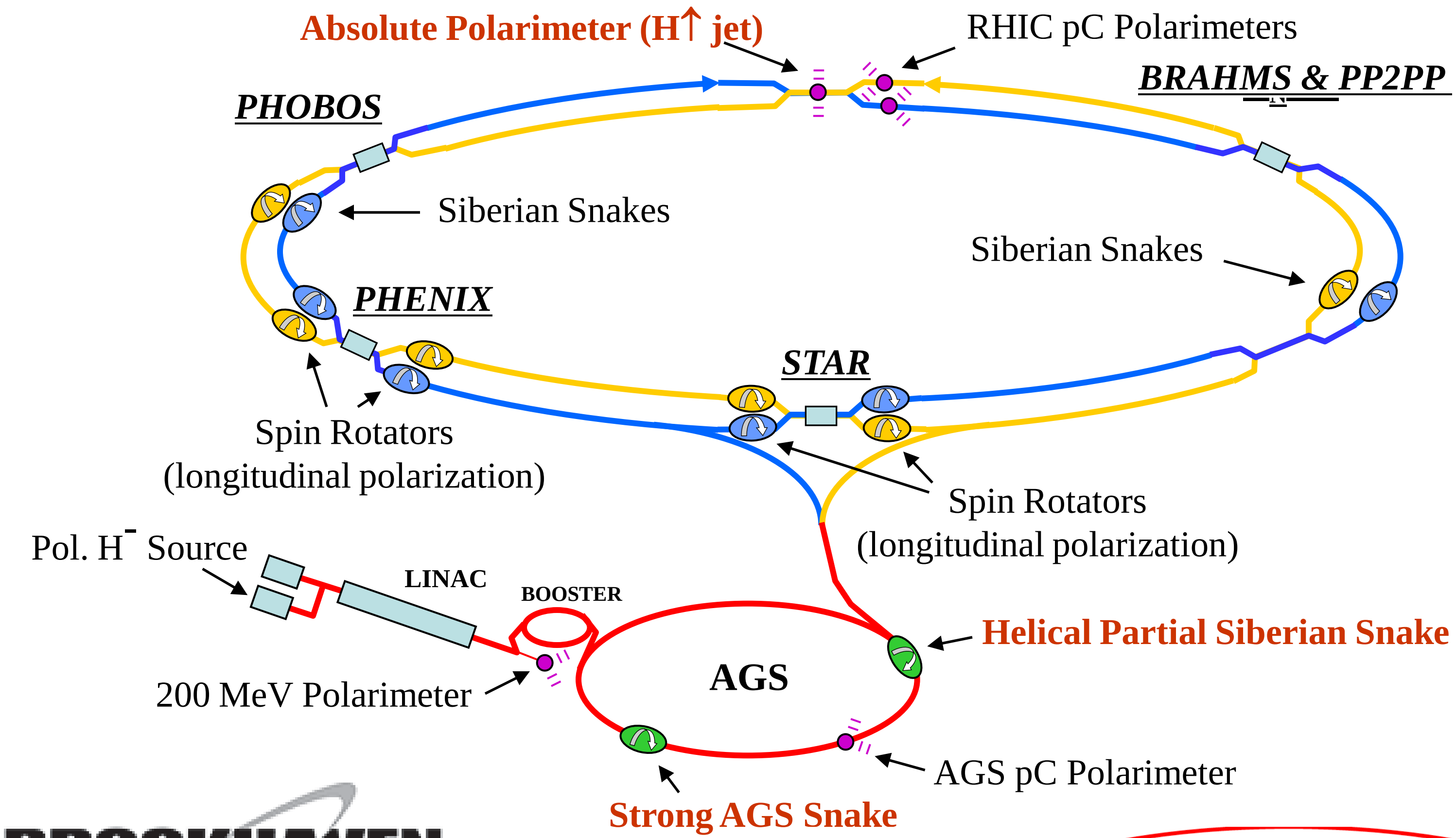
Spin/Flavor selectivity

[Diffractive channels only]

Bland: TMDs from hadron-hadron collisions: RHIC

Schematic of Measurement Apparatus

RHIC for Spin



Absolute Polarimeter (H↑ jet)

RHIC pC Polarimeters

BRAHMS & PP2PP

PHOBOS

Siberian Snakes

Siberian Snakes

PHENIX

STAR

Spin Rotators
(longitudinal polarization)

Spin Rotators
(longitudinal polarization)

Pol. H^- Source

LINAC

BOOSTER

AGS

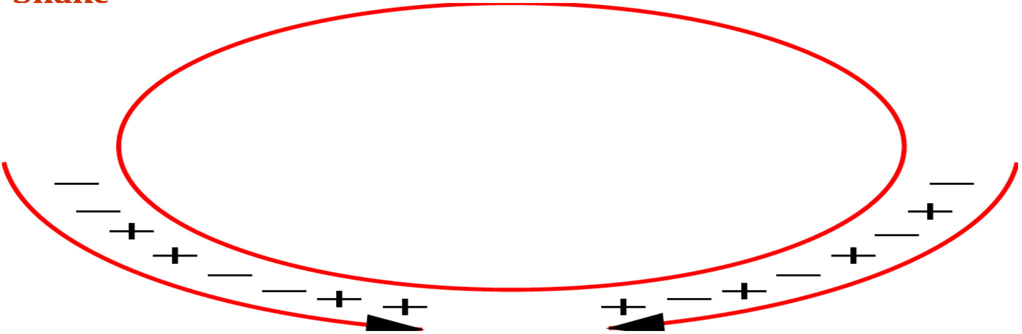
Helical Partial Siberian Snake

200 MeV Polarimeter

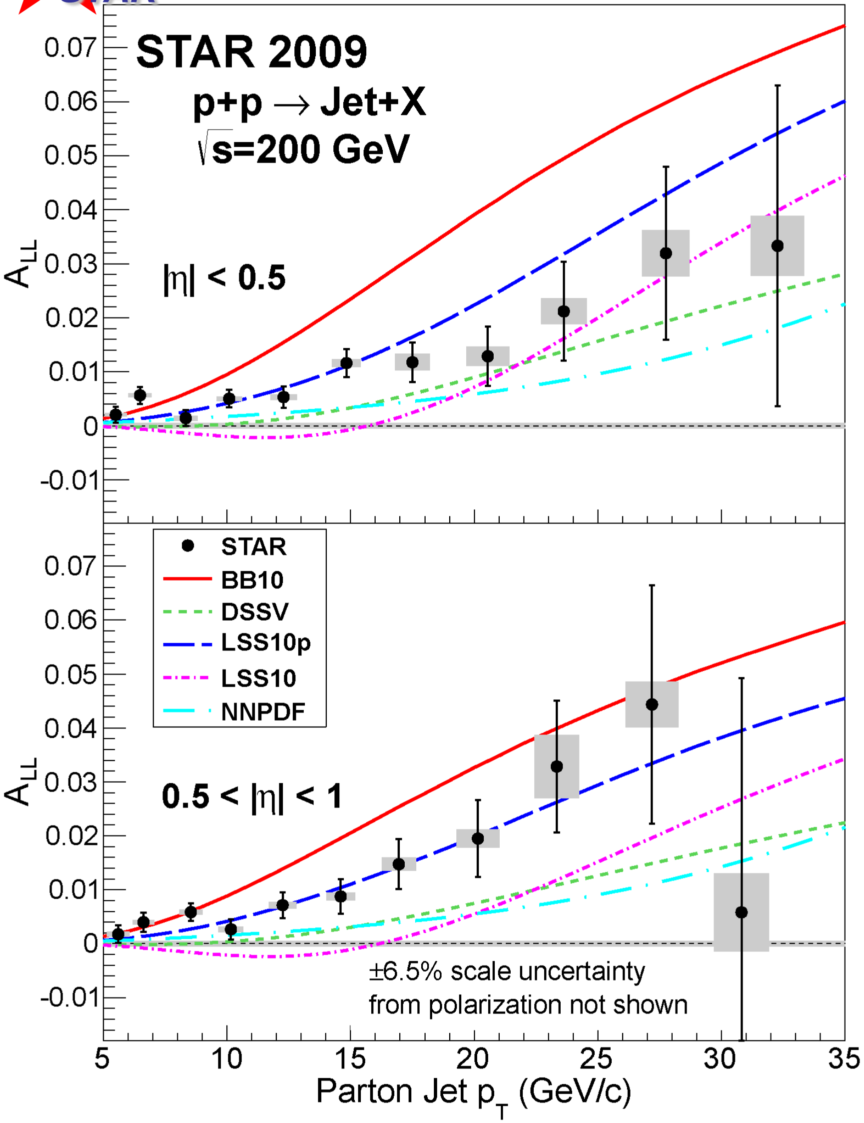
AGS pC Polarimeter

Strong AGS Snake

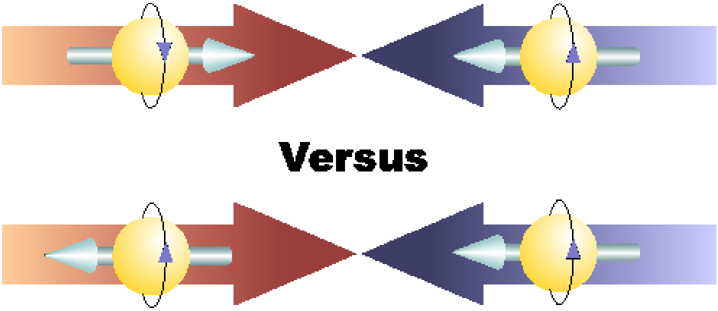
BROOKHAVEN
NATIONAL LABORATORY



Measured Quantity



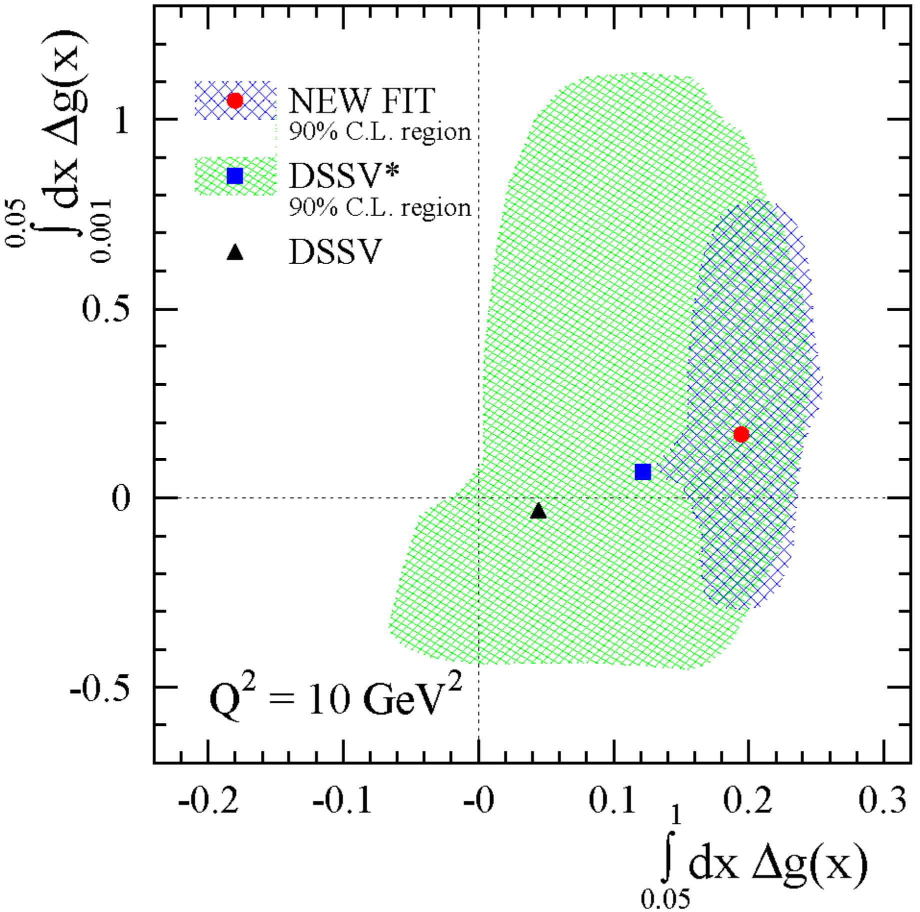
arXiv:1405.5134



$$A_{LL} = \frac{\sigma_{++} - \sigma_{+-}}{\sigma_{++} + \sigma_{+-}}$$

- Helicity asymmetry for inclusive jet production is measured as a function of p_T.
- Measurements are sensitive to $<x>\sim 2p_T/\sqrt{s}$

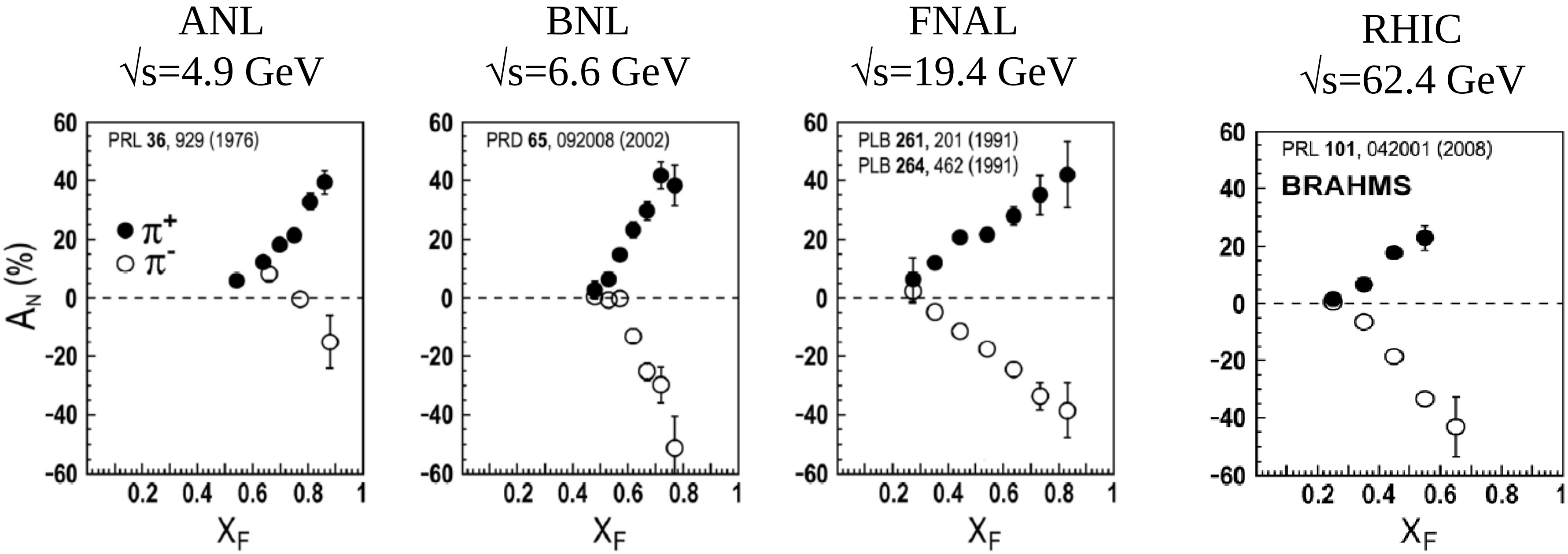
Implications of Measurement



Evidence for polarization of gluons from global NLO fit to preliminary version of inclusive jet data from STAR, neutral pion data from PHENIX and polarized deep inelastic scattering

de Florian, Sassot, Stratmann, Vogelsang
PRL 113 (2014) 012001 / arXiv:1404.4293

Forward Pion Transverse SSA Versus √s



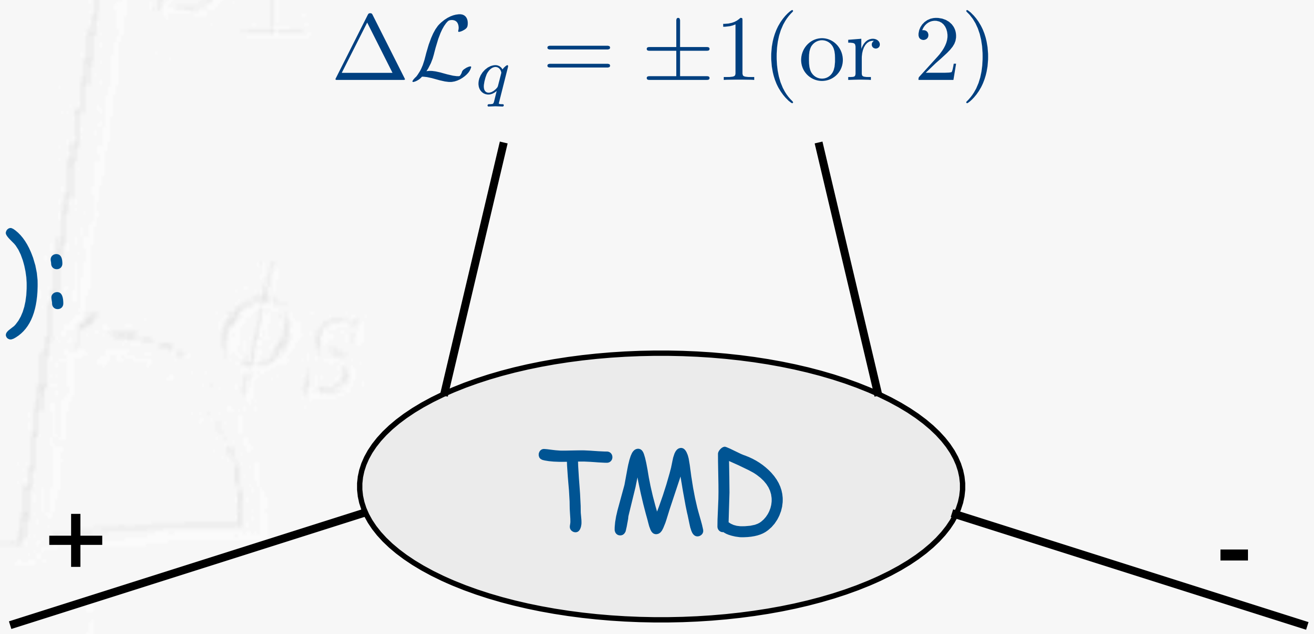
Aidala, Bass, Hasch, Mallot RMP 85 (2013) 655 / arXiv:1209.2803

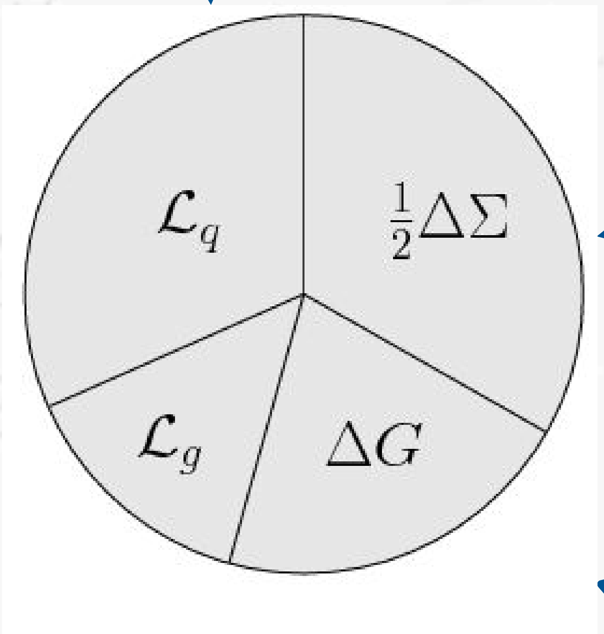
Forward pion analyzing power in p+p collisions exhibits similar x_F dependence over a broad range of √s

Schnell: TMDs via DIS and SIDIS
HERMES, COMPASS and JLab 6 GeV
Sivers, Collins, Boer-Mulders etc

Access to angular momentum in (SI)DIS

TMDs? (SIDIS):





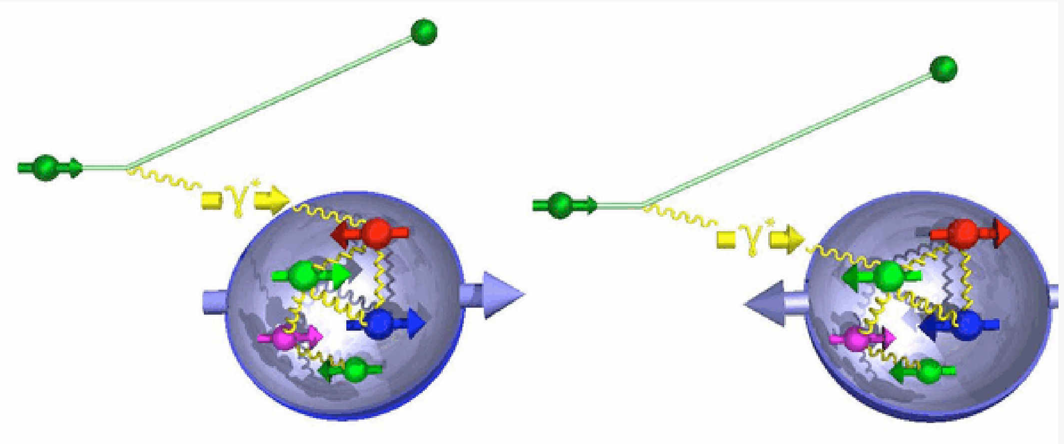
'pizza quattro stagioni'

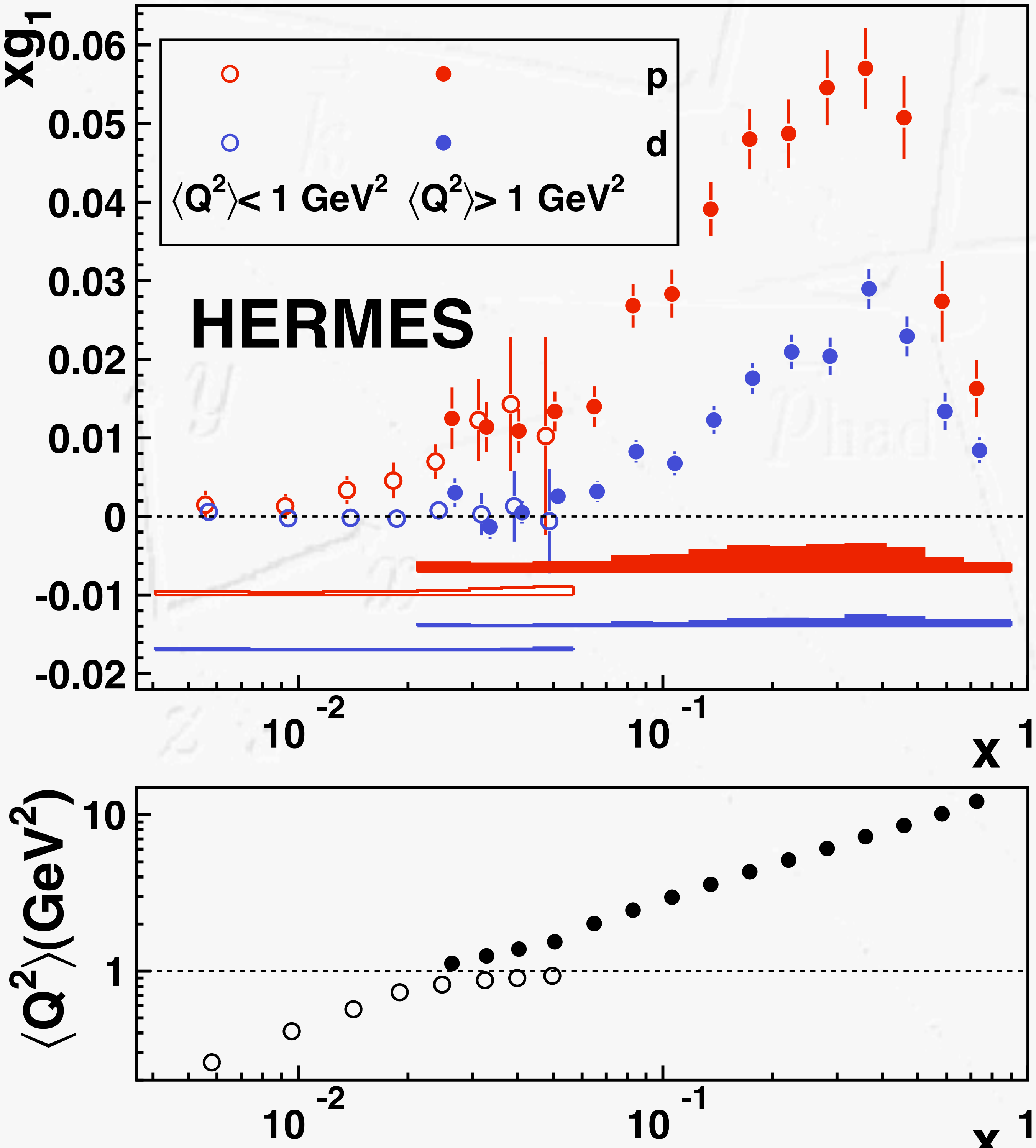
[M. Burkardt]

(semi-)inclusive longitudinal double-spin asymmetries

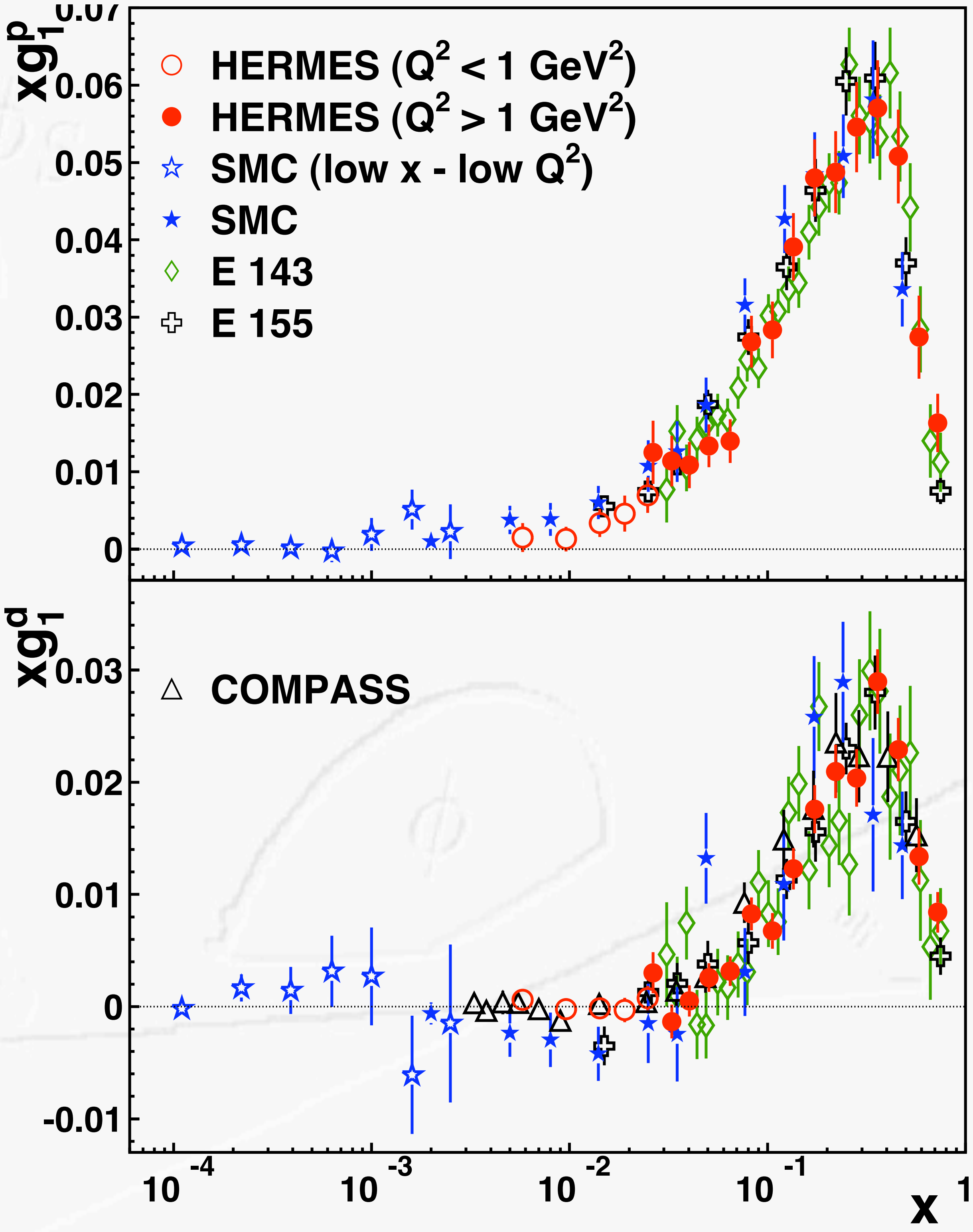
evolution

Polarized SF g_1





A. Airapetian et al., PRD 75 (2007) 012007



Polarized structure function g_1

Saturation

➠ **close to full integral?**

$$\Delta\Sigma \overset{\overline{\mathrm{MS}}}{=} \frac{1}{\Delta C_{\mathrm{S}}}\left[\frac{9\Gamma_1^d}{1-\frac{3}{2}\omega_D} - \frac{1}{4}a_8\Delta C_{\mathrm{NS}}\right]$$

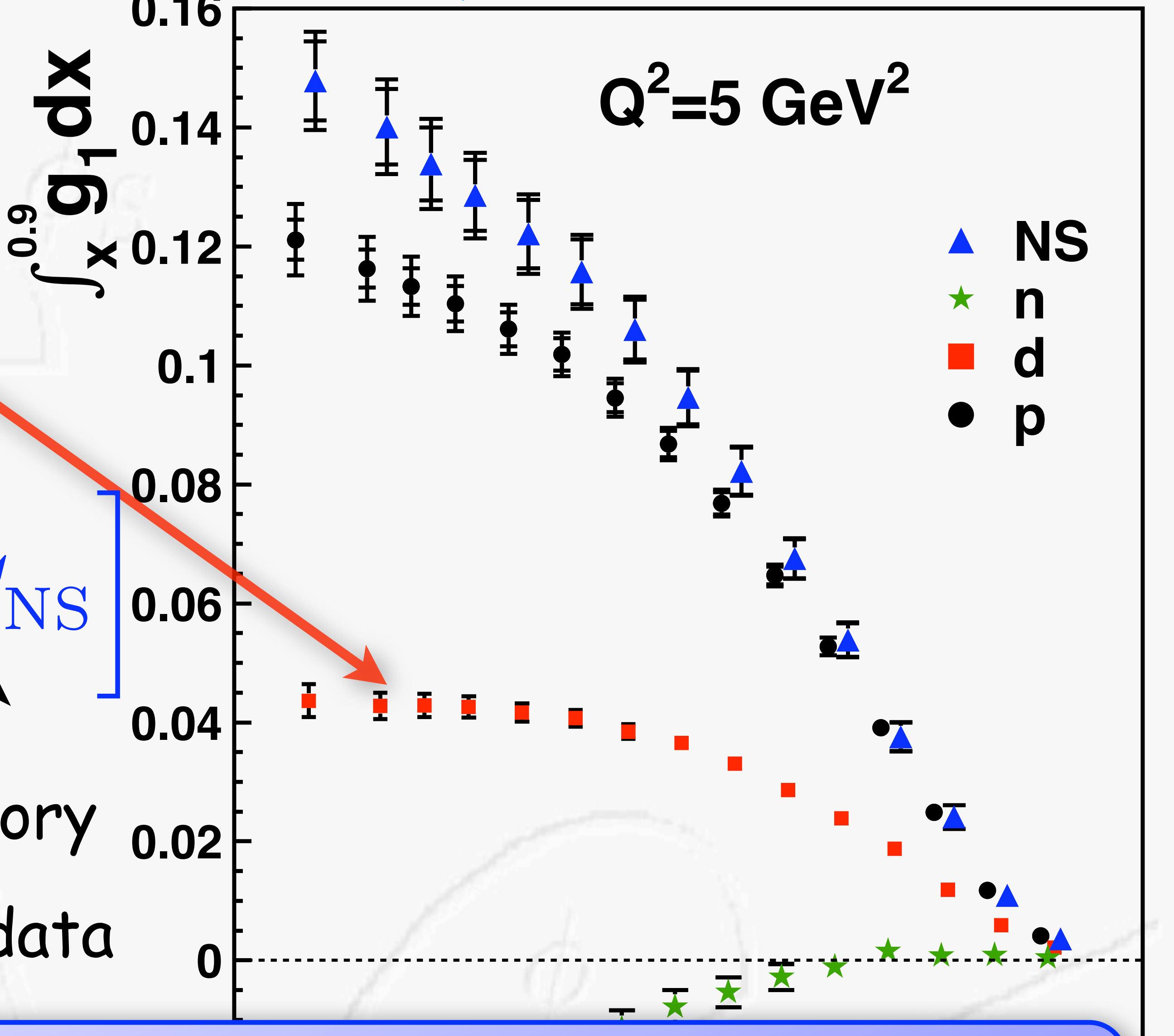
↑ theory

↑ 0.05±0.05

↑ hyperon-decay data

↑ theory

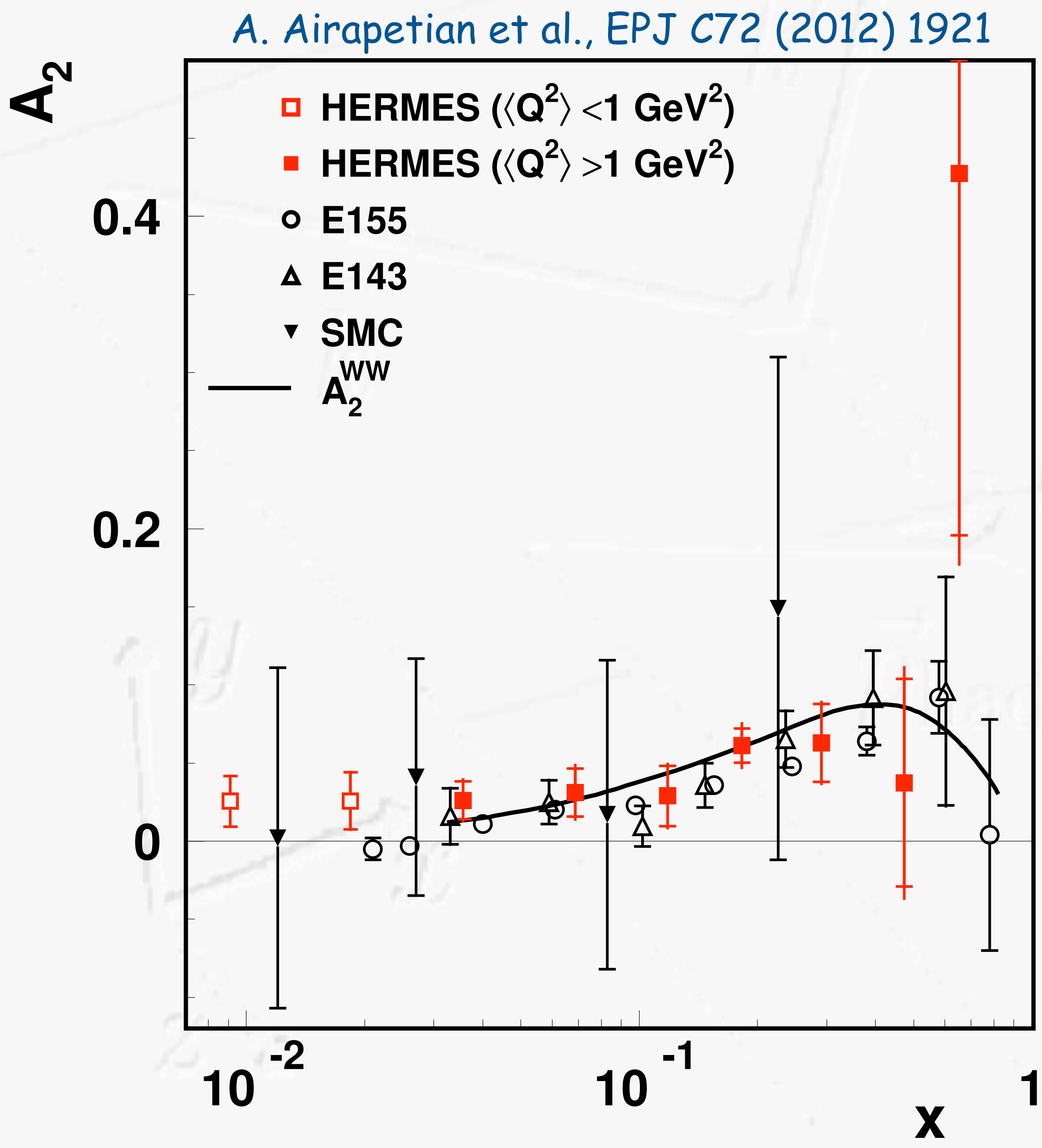
A. Airapetian et al., PRD 75 (2007) 012007

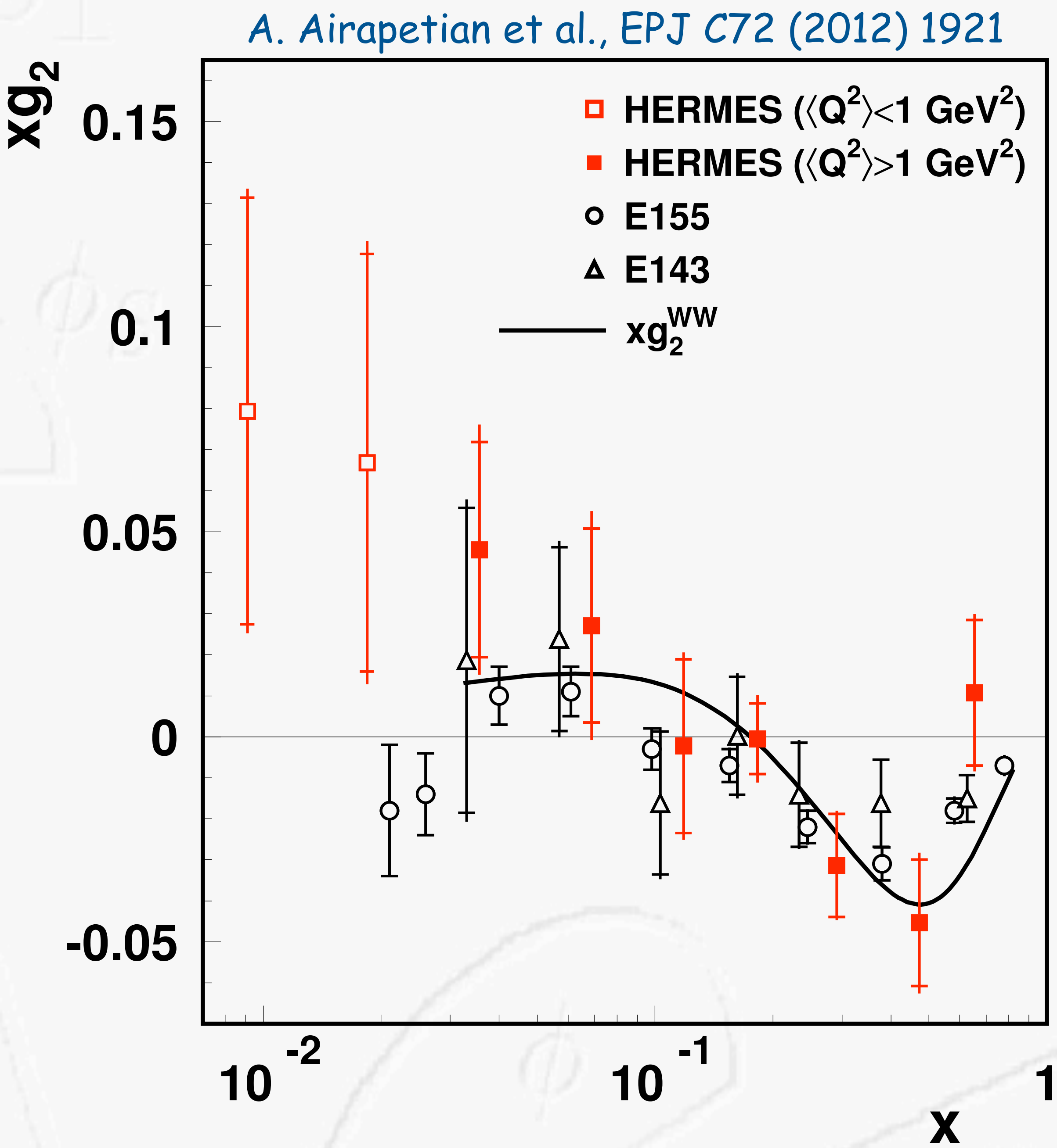


$$\Delta\Sigma \overset{\overline{\mathrm{MS}}}{=} 0.330 \pm 0.011_{\mathrm{theory}} \pm 0.025_{\mathrm{exp}} \pm 0.028_{\mathrm{evol}}$$

$$\Delta\Sigma \overset{\overline{\mathrm{MS}}}{=} 0.35 \pm 0.03_{\mathrm{stat}} \pm 0.05_{\mathrm{sys}}$$ COMPASS

Results on A_2 and xg_2


A. Airapetian et al., EPJ C72 (2012) 1921

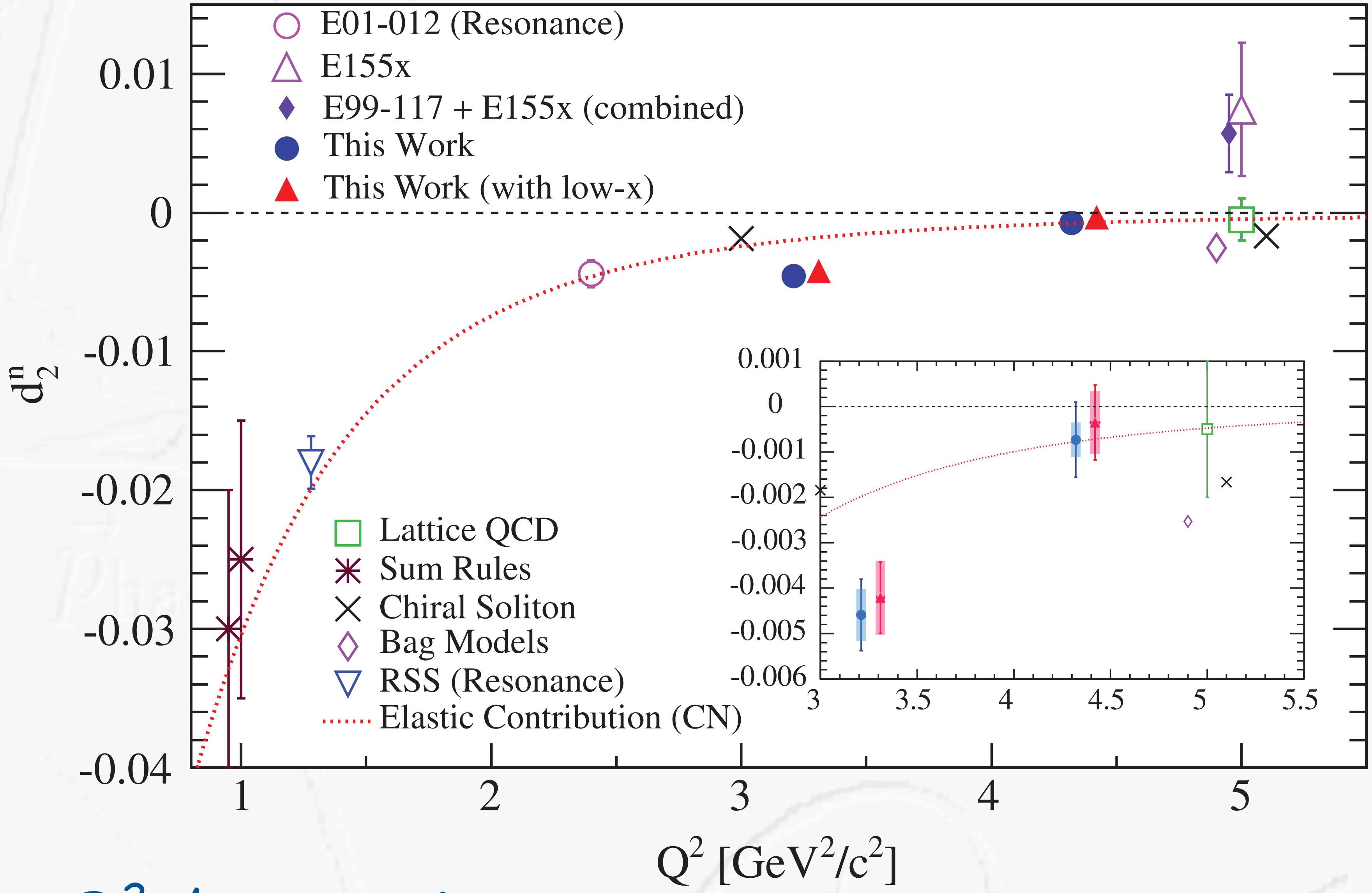

A. Airapetian et al., EPJ C72 (2012) 1921

$$\int_{0.023}^{0.9} g_2(x,Q^2)\,dx = \begin{cases} \mathbf{0.006 \pm 0.024_{\mathrm{stat}} \pm 0.017_{\mathrm{syst}}} & \text{HERMES} \\ \mathbf{-0.042 \pm 0.008} & \text{E143+E155} \end{cases}$$

$$\mathbf{d_2(Q^2)} \equiv 3\int_0^1 x^2\,\bar{g}_2(x,Q^2)\,dx = \begin{cases} \mathbf{0.0148 \pm 0.0096_{\mathrm{stat}} \pm 0.0048_{\mathrm{syst}}} & \text{HERMES} \\ \mathbf{0.0032 \pm 0.0017} & \text{E143+E155} \end{cases}$$

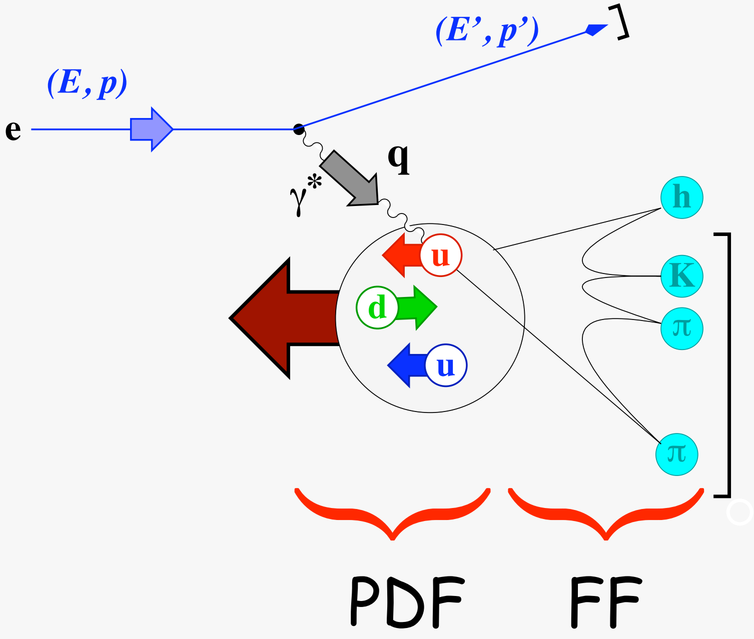
… the neutron case

[M. Posik et al., PRL 113, 022002 (2014)]



- sizable in the lower-Q^2 / -x region
- opposite sign compared to proton case (as expected, e.g., by M. Burkardt, PRD 88, 114502 (2013) due to "instantaneous transverse color force")

Probing TMDs in semi-inclusive DIS



quark pol.

nucleon pol.

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

in SIDIS*) couple PDFs to:

Collins FF: $H_1^{\perp,q\to h}$

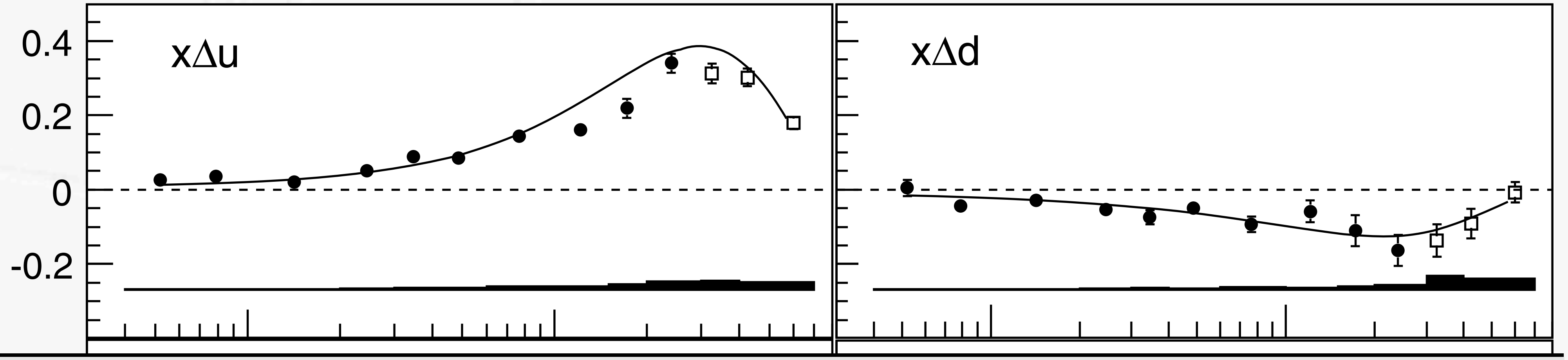
ordinary FF: $D_1^{q\to h}$

*) semi-inclusive DIS with unpolarized final state

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$

Helicity density

[M. Alekseev et al., PLB 693 (2010) 227]



	Extrapolation	DSSV
Δu	$0.71 \pm 0.02 \pm 0.03$	$0.71 \pm 0.02 \pm 0.03$
Δd	$-0.34 \pm 0.04 \pm 0.03$	$-0.35 \pm 0.04 \pm 0.03$
$\Delta \bar{u}$	$0.02 \pm 0.02 \pm 0.01$	$0.03 \pm 0.02 \pm 0.01$
$\Delta \bar{d}$	$-0.05 \pm 0.03 \pm 0.02$	$-0.07 \pm 0.03 \pm 0.02$
$\Delta s (\Delta \bar{s})$	$-0.01 \pm 0.01 \pm 0.01$	$-0.05 \pm 0.01 \pm 0.01$
Δu_v	$0.68 \pm 0.03 \pm 0.03$	$0.68 \pm 0.03 \pm 0.03$
Δd_v	$-0.29 \pm 0.06 \pm 0.03$	$-0.28 \pm 0.06 \pm 0.03$
$\Delta \Sigma$	$0.32 \pm 0.03 \pm 0.03$	$0.22 \pm 0.03 \pm 0.03$

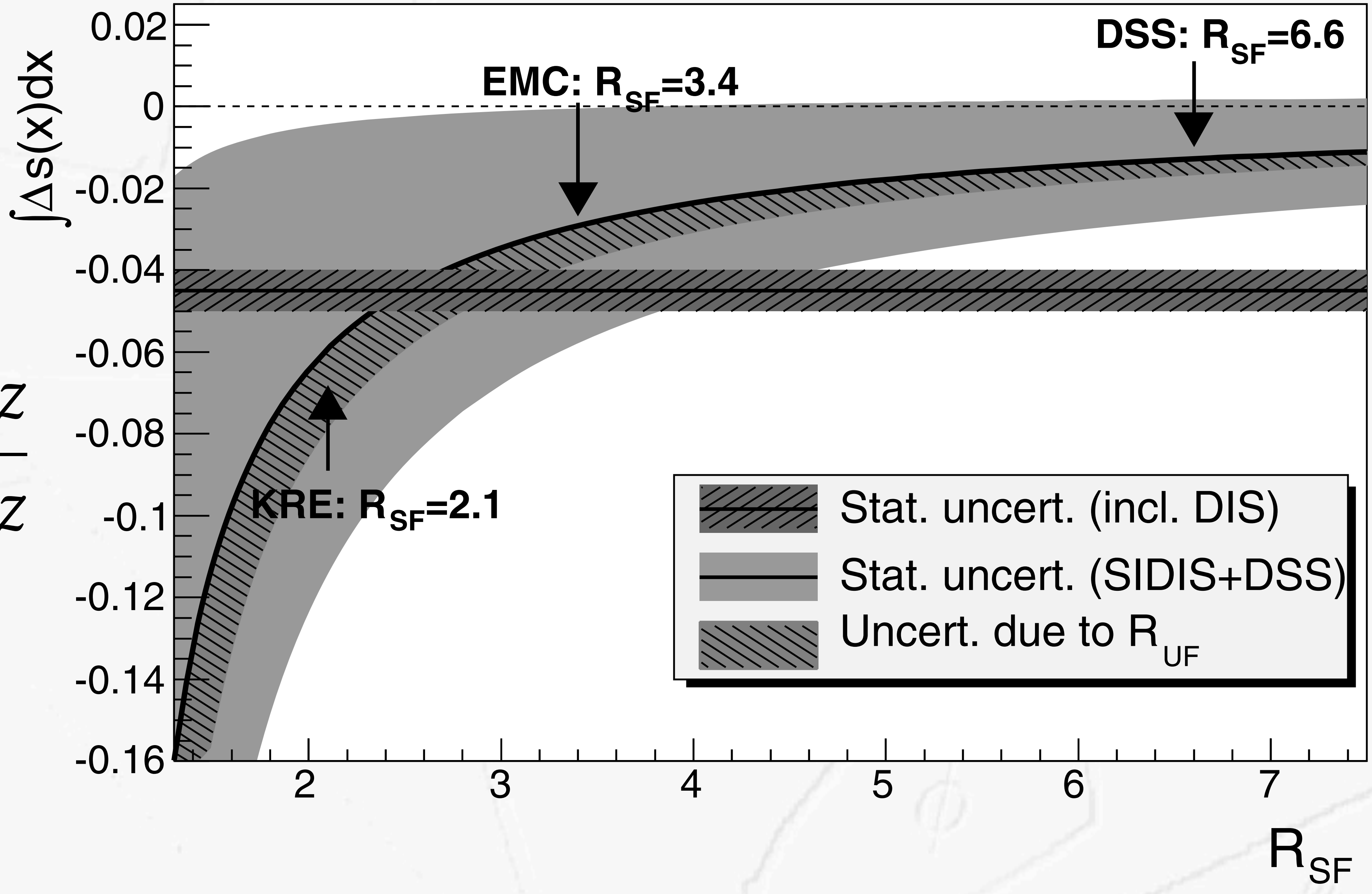
flavor separation of quark-helicity distribution using DIS data only

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$

Helicity density

[M. Alekseev et al., PLB 680 (2009) 217]

$$R_{\rm SF} = \frac{\int D_{\bar{s}}^{K^+}(z)\,dz}{\int D_u^{K^+}(z)\,dz}$$



caveat: potentially large dependences on knowledge of FFs!

Summary

- first round of SIDIS measurements coming to an end
- current knowledge on quark- and gluon-spin contribution to nucleon spin leaves room for orbital angular momentum
- transversity is non-zero and quite sizable
  - can be measured, e.g., via Collins effect or s-p interference in 2-hadron fragmentation
- Sivers and Boer-Mulders effects are also non-zero
  - Sivers: opposite sign for up and down quarks in line with their contributions to the nucleon's anomalous magnetic moment
- so far no sign of a non-zero pretzelosity distribution
- first evidences for non-vanishing worm-gear functions
- precision measurements at ongoing and future SIDIS facilities needed to fully map TMD landscape

Wang: Very exciting news. Looks realistically possible that China will build an EIC several years before a US machine.

Several interesting theory papers:

Gamberg: proposes a different and perhaps more systematic method of deconvoluting theoretical expressions into factorized form

Zhou: small x behaviour of SSA, based on ODDERON exchange.

Recall

$$A_{pp} = A_{\bar{p}p} = s\,\mathrm{log}(s)^2 \qquad \text{Pomeron} \qquad (1)$$

In principle can have

$$A_{pp} - A_{\bar{p}p} \nrightarrow 0 \quad \text{as} \rightarrow 0 \qquad \text{Odderon} \qquad (2)$$

Schlegel: SSA in SIDIS

Very interesting study of 2-photon effects. Motivated by remarkable JLab EM form factor discovery. Interesting use of $q\gamma q$ correlator. Recall qGq correlator is really

$$\langle g\bar{\psi}G\psi\rangle \tag{1}$$

and not considered $O(g)$.

Similarly $q\gamma q$ correlator is

$$\langle e\bar{\psi}F\psi\rangle \tag{2}$$

and not considered $O(e)$.

Some food for thought: theoretical of course

1) Group theory commutation relations like

$$[L_i, L_j] = i\epsilon_{ijk}L_k \tag{1}$$

don't hold if $m = 0$.
Quite different in case $m = 0$! PHYSICS! Can never establish absolutely that $m_\gamma = 0$.

2) How to prove that an OPERATOR is G. Invt.? Defining theory by fixing Lagrangian we use Classical variables and demand G. Inv. under

$$A^{\mu} \to A^{\mu} + \partial^{\mu}\Lambda(x) \tag{2}$$

$$\Lambda(x) = \text{classical, ordinary function} \tag{3}$$

Chen: If you TEST G.Inv. of an OPERATOR this way you get contradictions.

$$L^i_{can}(q) = \int d^3x\, \psi^\dagger [\boldsymbol{x} \times \frac{1}{i}\, \boldsymbol{\nabla}]^i\, \psi \qquad (1)$$

under a gauge transformation

$$\psi \to \psi' = U\psi \qquad U = e^{i\omega(x)} \qquad \omega = \omega_\alpha\, t^\alpha \qquad (2)$$

Now assuming that the Belinfante version of $\boldsymbol{J}$ generates the standard transformations, and bearing in mind that it is G. Invt., so that if expressed in terms of ψ' it will generate the usual transformations on functions of ψ', we have

$$\delta L^i_{can}(q) = i \int d^3x\, \omega_\alpha\, [\, J^i_{Bel}\, ,\, \psi'^\dagger\, t^\alpha\, \psi'\,]. \qquad (5)$$

Therefore, for an eigenstate of angular momentum

$\langle\, j_z \,|\, L^i_{can}(q)\, |\, j_z\, \rangle$ is gauge invariant.

Questioning the path-integral proof of gauge-invariant matrix element for gauge-dependent operators

Explicit counter example by perturbative calculation

P. Hoodbhoy, X. Ji, W. Lu, PRD 59:074010 (1999);

P. Hoodbhoy, X. Ji, PRD 60, 114042 (1999).

$$\Delta g(Q^2) = \left\langle p^\mu \frac{1}{2} \right| \int d^3\vec{x} (\vec{E}\times\vec{A})^z \left| p^\mu \frac{1}{2} \right\rangle$$

$$\text{light-cone } A^+ = 0 \quad \Delta g = \frac{3}{2} C_F \frac{\alpha_s}{2\pi} \ln\left(\frac{Q^2}{\mu^2}\right),$$

$$\text{covariant gauge} \quad \Delta g = C_F \frac{\alpha_s}{2\pi} \ln\left(\frac{Q^2}{\mu^2}\right).$$

3) Can we understand the surprising result that in Fock space models, where the **quarks** are free, one finds

$$L^q_{\text{kin}} \neq L^q_{\text{can}}\,? \tag{1}$$

4)Although I now consider Canonical and Kinetic versions of AM to both be of importance, I feel that AM operators ought to be the **GENERATORS** of **ROTATIONS** (at least at equal times). This means the CANONICAL version.

Lastly

As an organiser I am not allowed to say the Workshop was a success.
But if the **speakers** have to apologise in order to continue with their talks, I think that is a good sign!