Angular Momentum

in

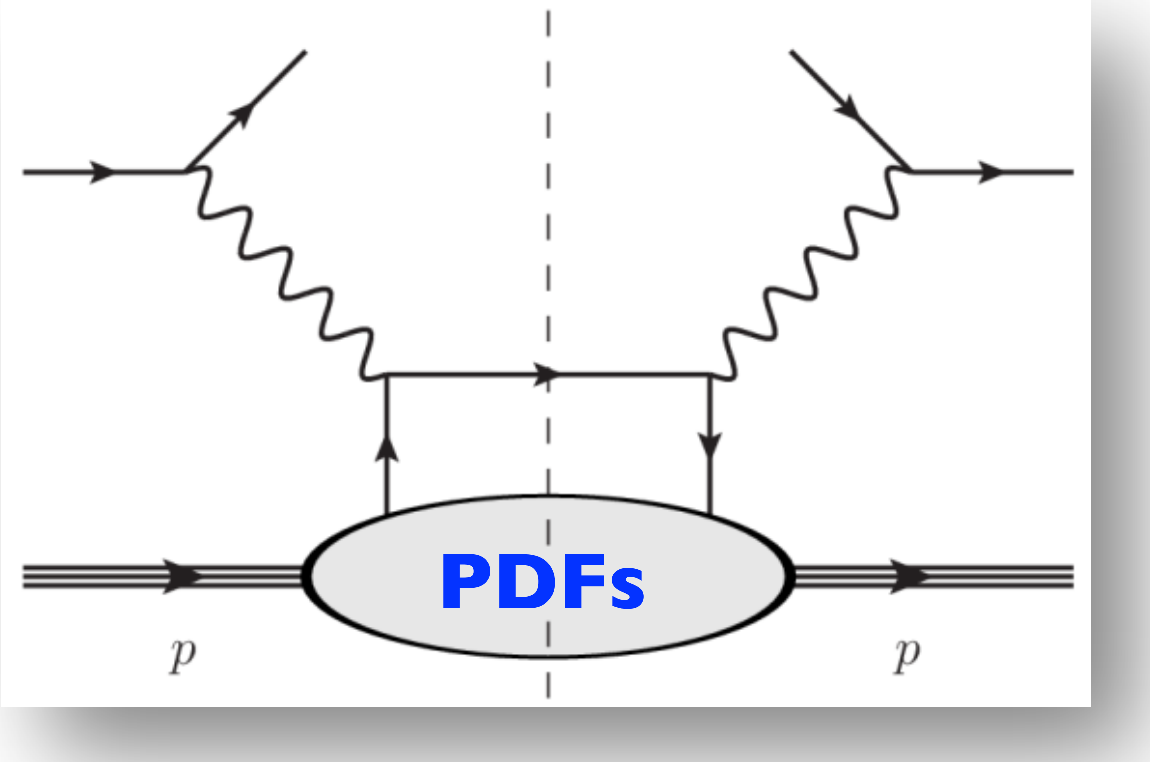
Phenomenological Models

Barbara Pasquini

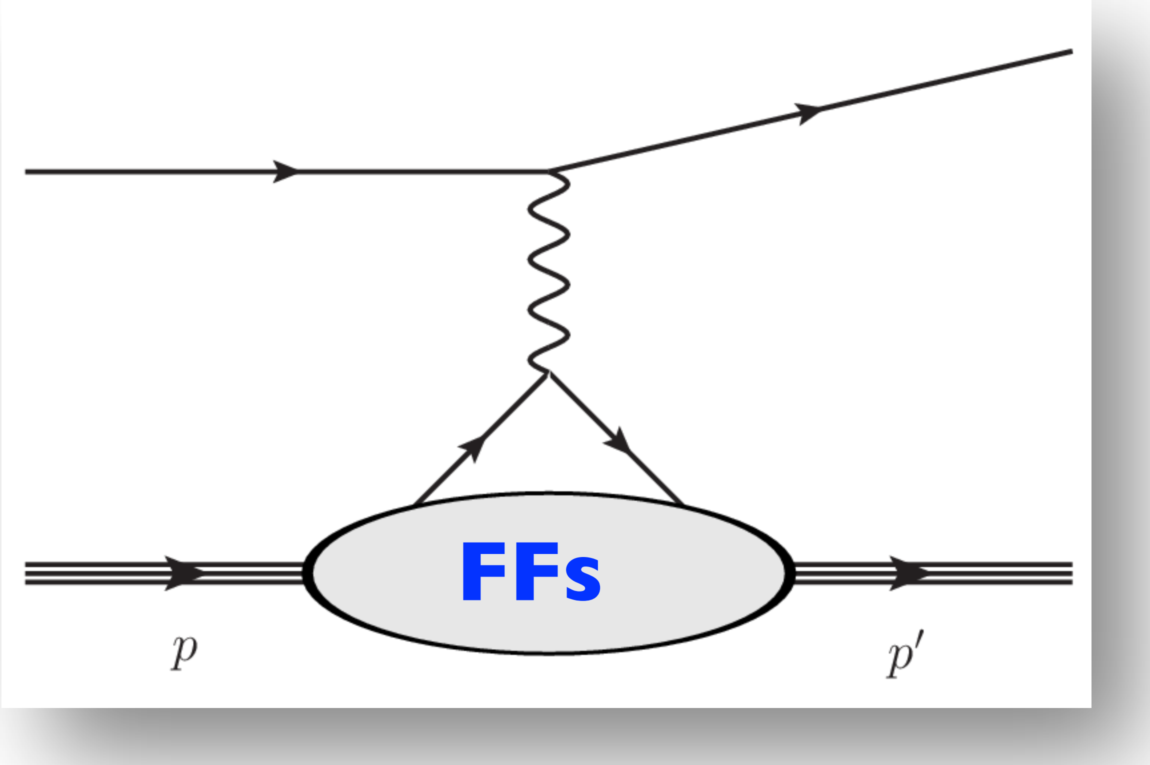
Università di Pavia & INFN

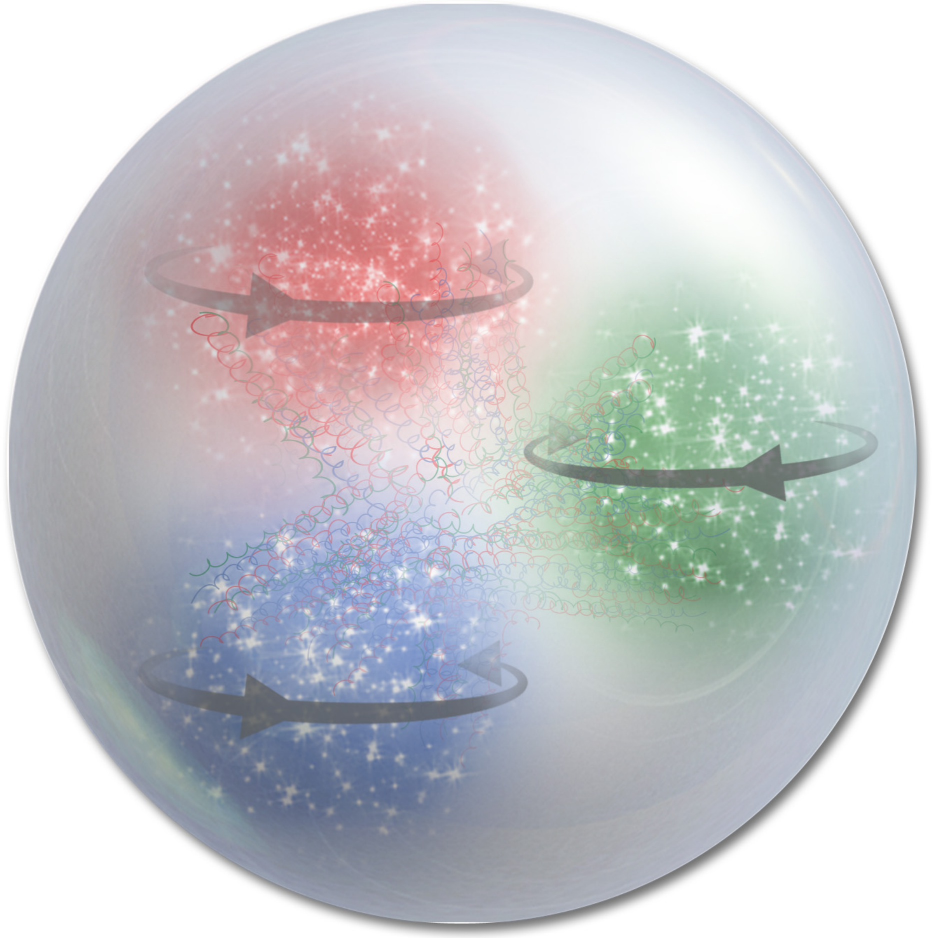
Goal: understanding the partonic structure of the nucleon

Deep Inelastic Scattering

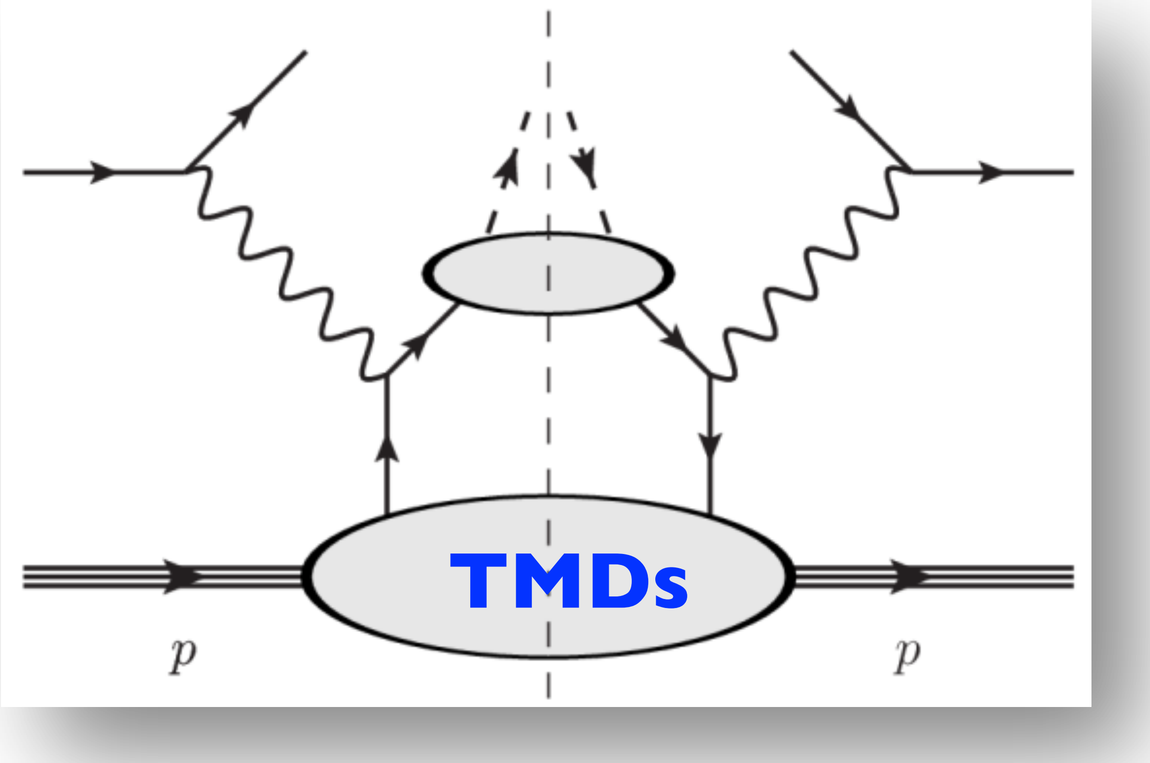


Elastic Scattering

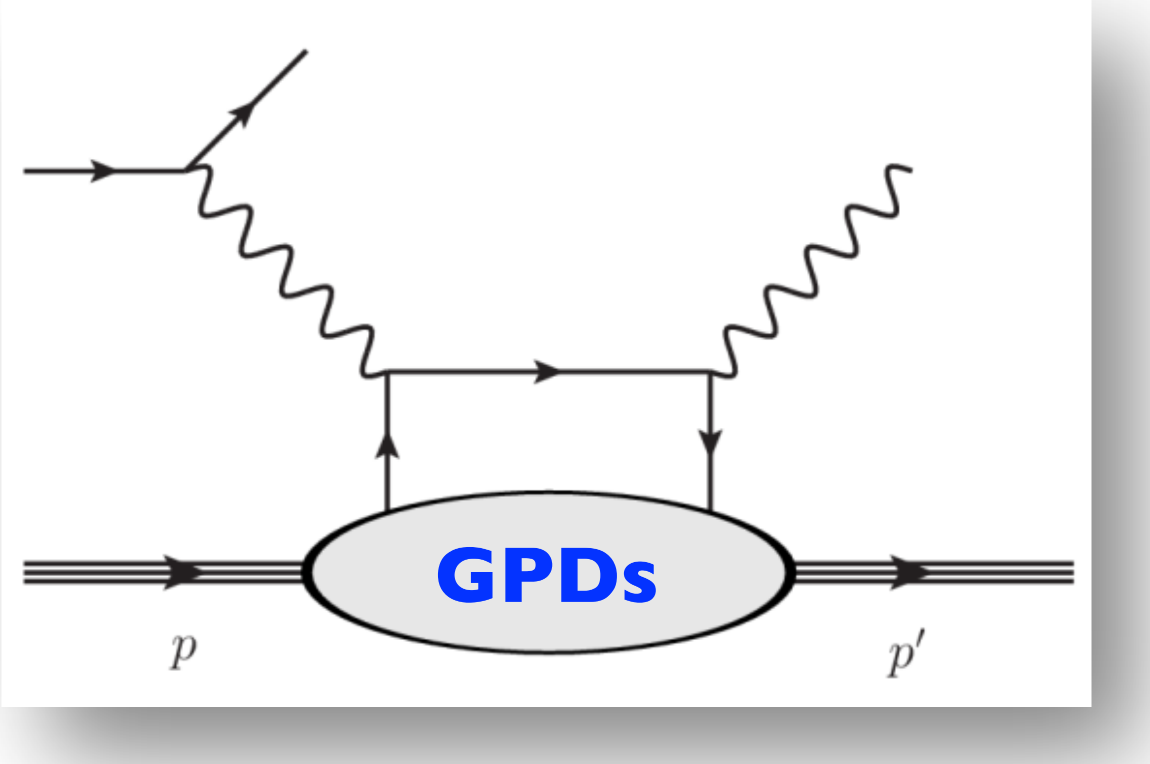




Semi-Inclusive
Deep Inelastic Scattering



Deeply Virtual Compton
Scattering



Momentum Space

Transverse Coordinate Space

Goal: understanding the partonic structure of the nucleon

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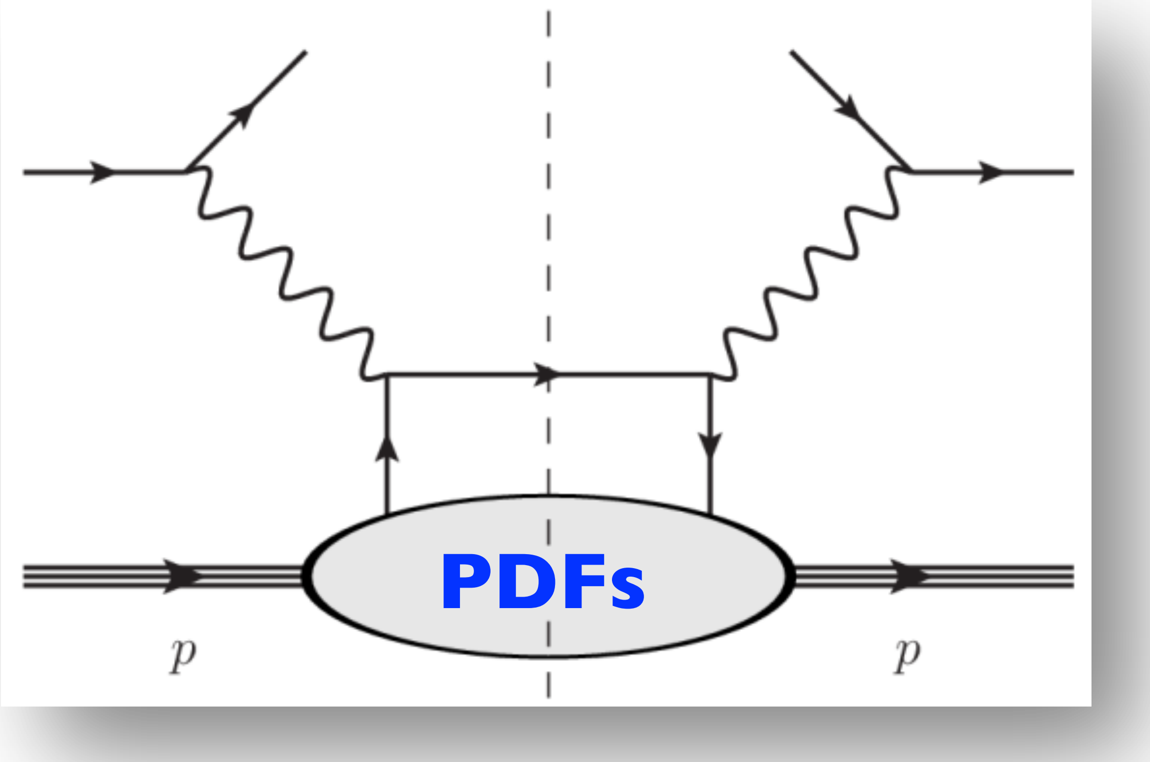
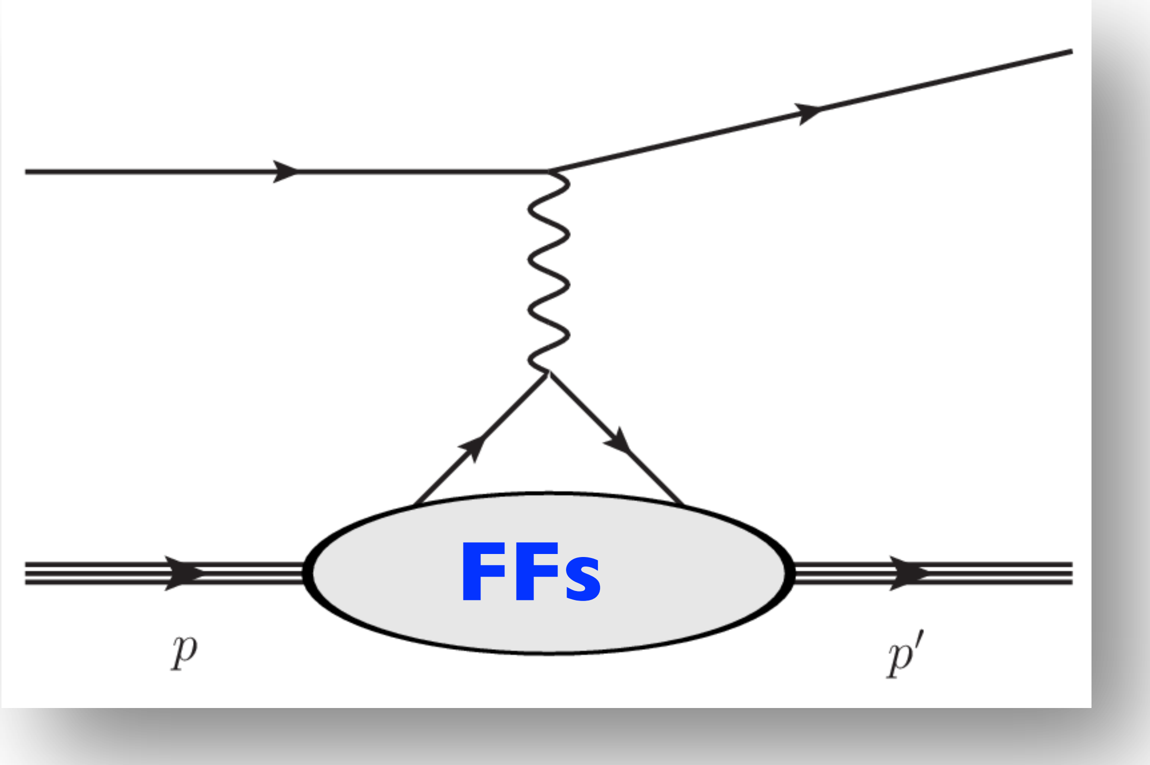


Elastic Scattering



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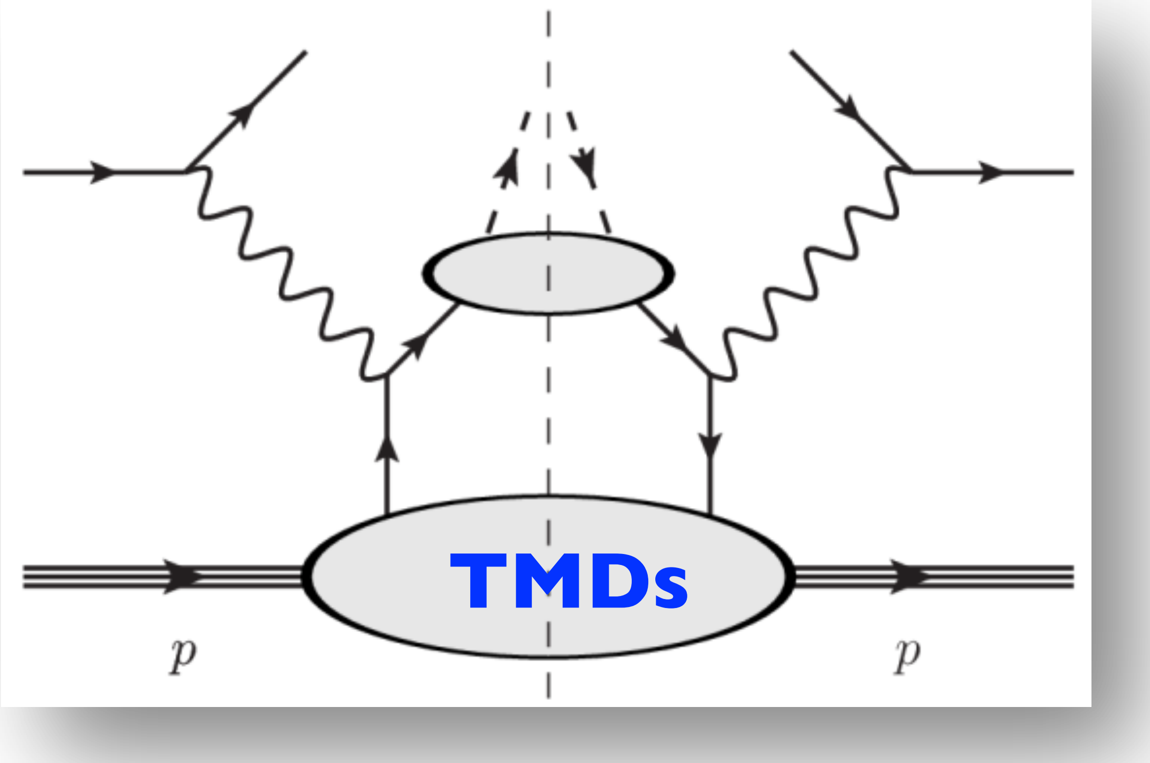

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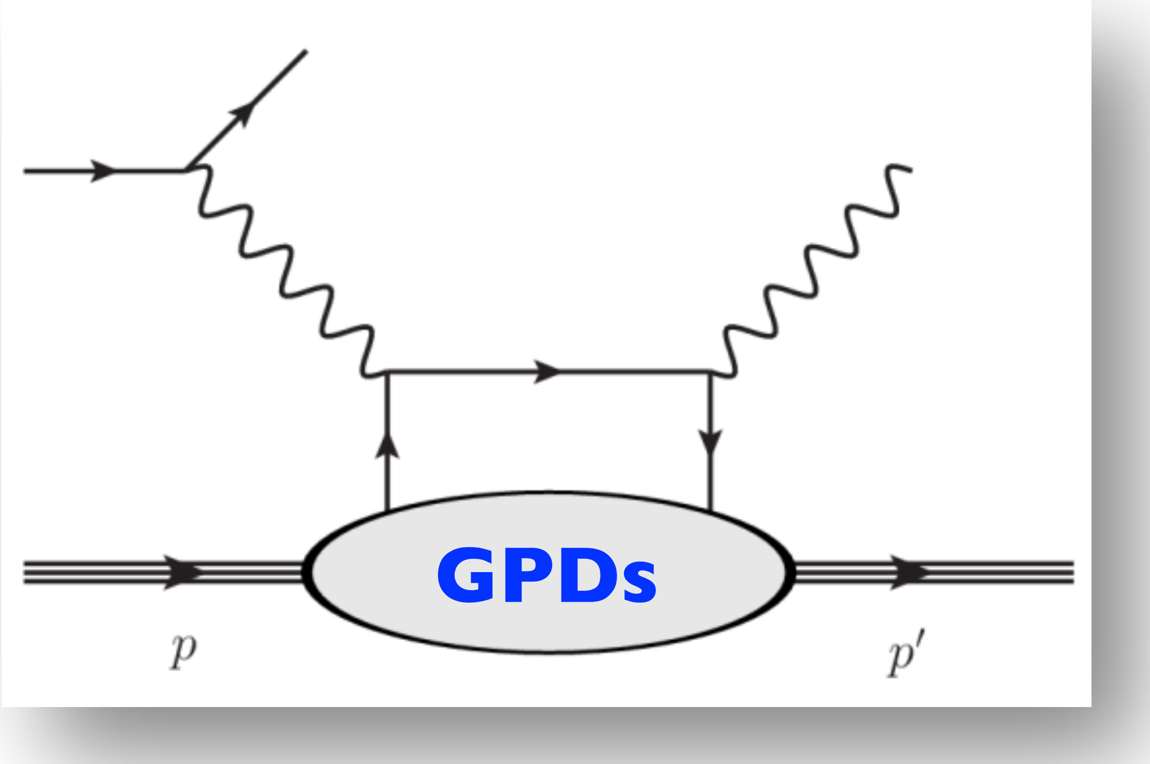


Complete Proton Tomography
in 3+2 D
from phase-space distributions

GTMDs ⟷ Wigner distr.

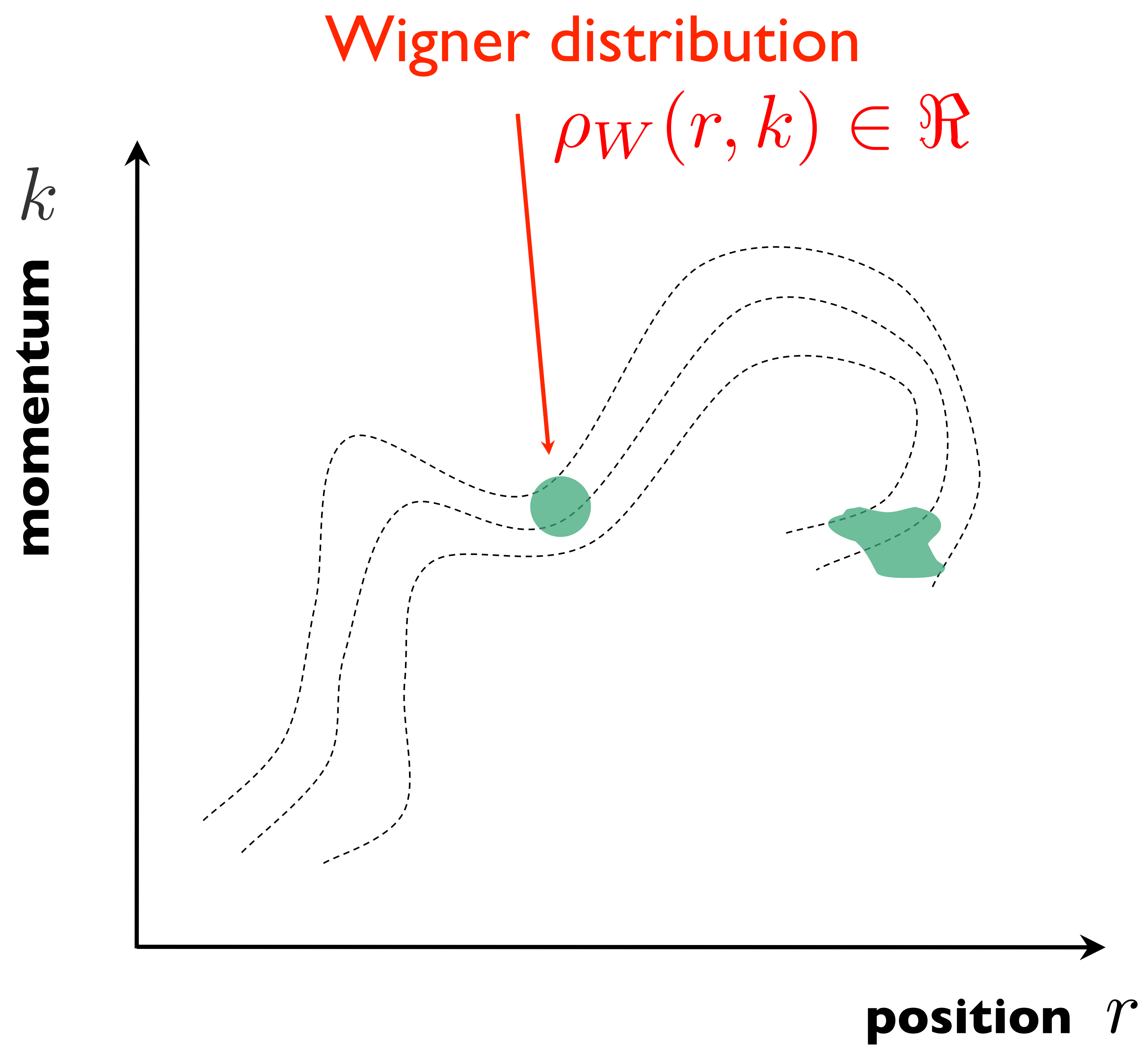
Momentum Space

Transverse Coordinate Space

Phase-space distribution in QM

Quantum Mechanics

[Wigner (1932)]
[Moyal (1949)]



$$\rho_W(r,k) = \int \frac{\mathrm{d}z}{2\pi} e^{-ikz} \psi^*(r-\frac{z}{2})\psi(r+\frac{z}{2})$$
$$= \int \frac{\mathrm{d}\Delta}{2\pi} e^{-i\Delta r} \phi^*(k+\frac{\Delta}{2})\phi(k-\frac{\Delta}{2})$$

Position-space density

$$|\psi(r)|^2 = \int \mathrm{d}k\, \rho_W(r,k)$$

Momentum-space density

$$|\phi(k)|^2 = 2\pi \int \mathrm{d}r\, \rho_W(r,k)$$

Quantum average

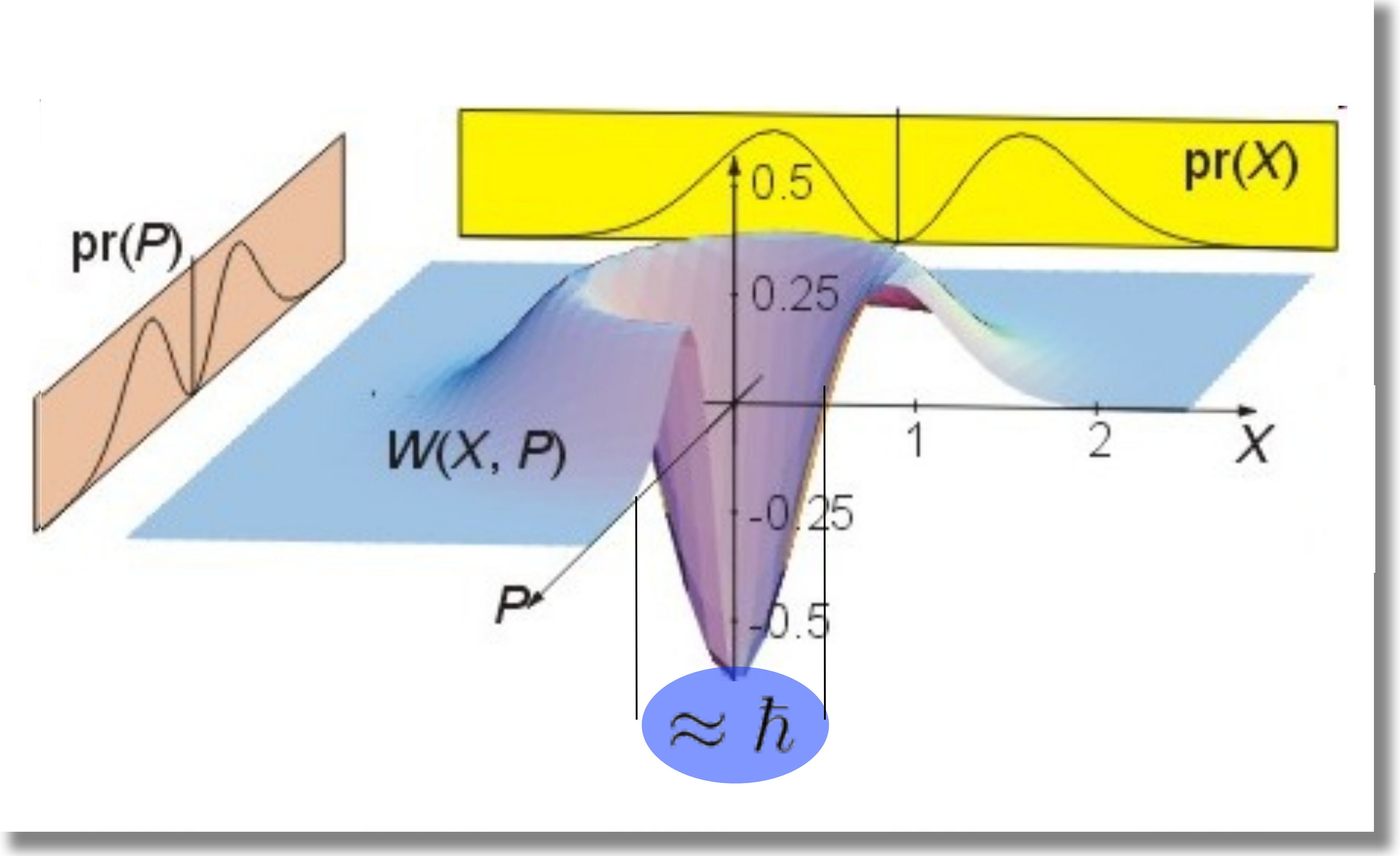
$$\langle \hat{O} \rangle = \int \mathrm{d}r\, \mathrm{d}k\, O(r,k)\, \rho_W(r,k)$$

Phase-space distribution

Wigner distribution

Numerous applications in

- Nuclear physics
- Quantum chemistry
- Quantum molecular dynamics
- Quantum information
- Quantum optics
- Classical optics
- Signal analysis
- Image processing
- Heavy ion collisions
- …



[Antonov et al. (1980-1989)]

Heisenberg's uncertainty relation

→ Quasi-probabilistic interpretation $\xrightarrow{\hbar \to 0}$ classical density

Wigner Distributions in QFT

Dirac matrix

$\sim$ quark polarization

Quark Wigner operator

$$\widehat{W}^{[\Gamma]}(\vec{r},k)=\int\frac{\mathrm{d}^4z}{(2\pi)^4}e^{ik\cdot z}\bar{\psi}(\vec{r}-\frac{z}{2})\,\Gamma\,\mathcal{W}\,\psi(\vec{r}+\frac{z}{2})$$

Wilson line

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Fixed light-front time

$$z^+=0 \quad \Longleftrightarrow \quad \int \mathrm{d}k^-$$

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Wigner distributions
in the Breit frame

$$\rho^{[\Gamma]}_{\Lambda'\Lambda}(\vec{r},k^+,\vec{k}_\perp) = \frac{1}{2}\int \frac{\mathrm{d}^3\vec{\Delta}}{(2\pi)^3} e^{-i\vec{\Delta}\cdot\vec{r}}\langle\frac{\vec{\Delta}}{2},\Lambda'|\widehat{W}^{[\Gamma]}(0,k^+,\vec{k}_\perp)|-\frac{\vec{\Delta}}{2},\Lambda\rangle$$

3+3 D

no semi-classical interpretation

[Ji (2003)]
[Belitsky, Ji, Yuan (2004)]

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3+3 D

no semi-classical interpretation

Wigner distributions in the Drell-Yan frame

$(\Delta^+=0)$

$$\rho^{[\Gamma]}_{\Lambda'\Lambda}(\vec{b}_\perp,k^+,\vec{k}_\perp)=\frac{1}{2}\int\frac{\mathrm{d}^2\vec{\Delta}_\perp}{(2\pi)^2}e^{-i\vec{\Delta}_\perp\cdot\vec{b}_\perp}\langle p^+,\frac{\vec{\Delta}_\perp}{2},\Lambda'|\widehat{W}^{[\Gamma]}(0,k^+,\vec{k}_\perp)|p^+,-\frac{\vec{\Delta}_\perp}{2},\Lambda\rangle$$

[Lorcè, BP (2011)]
Lorcè, BP, Xiong, Yuan (2012)]

2+3 D

semi-classical interpretation

Wigner Distributions in QFT

Quark Wigner operator

Dirac matrix
$\sim$ quark polarization

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2+3 D

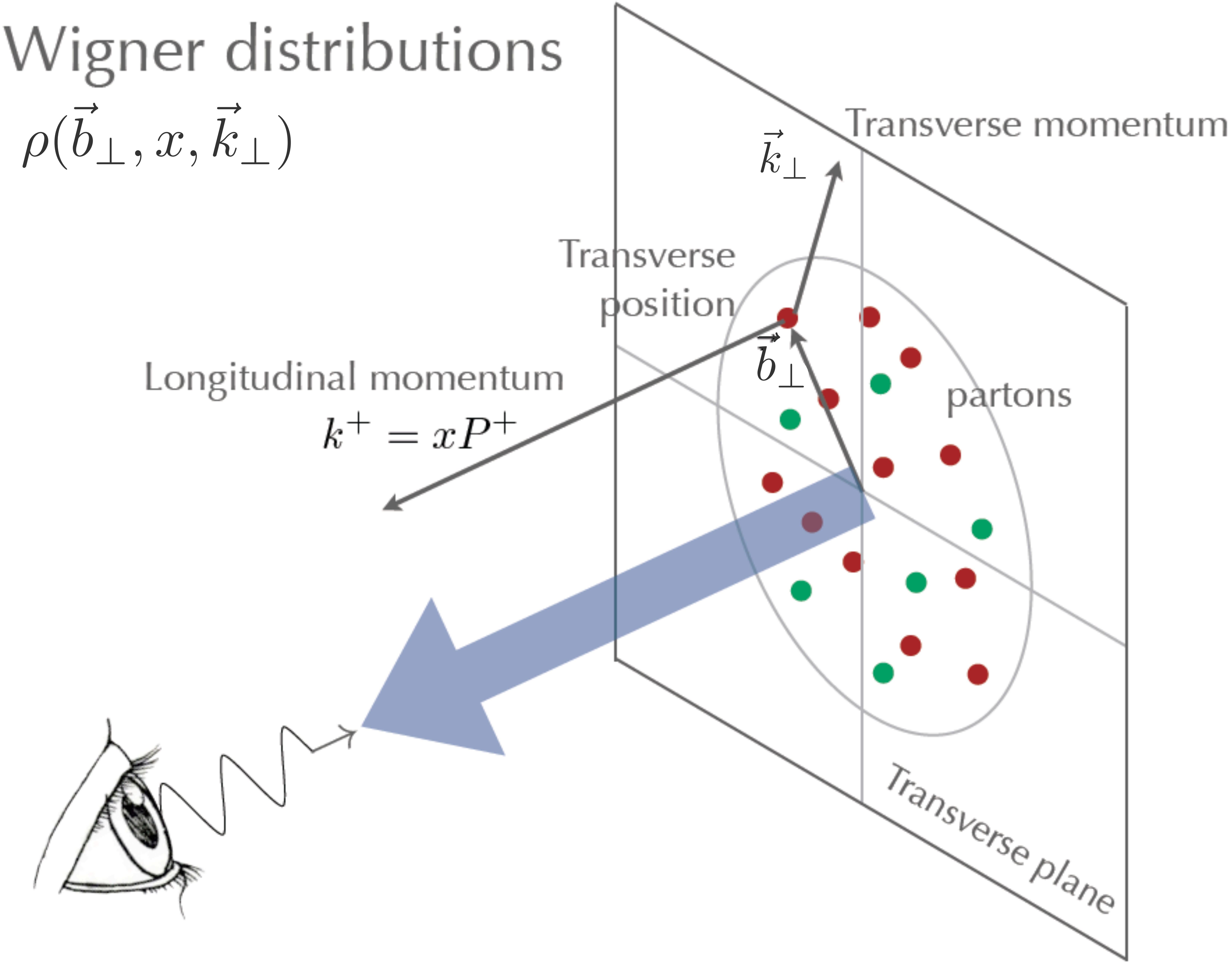
semi-classical interpretation

Generalized Transverse Momentum Dependent

[Meissner, Metz, Schlegel (2007)]

Wigner distributions

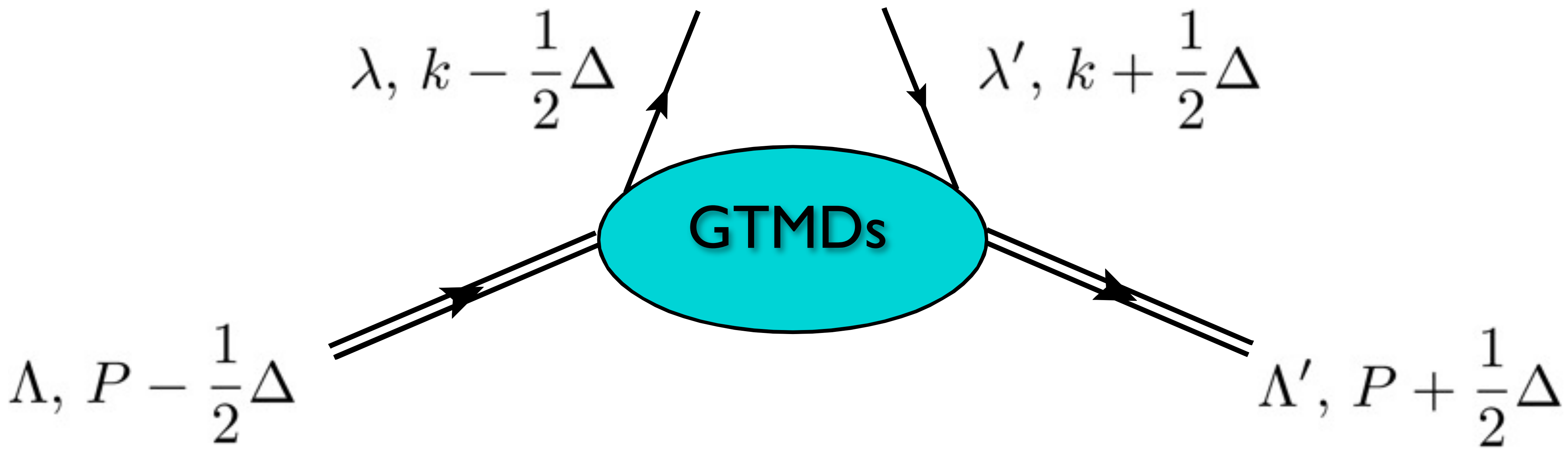
$\rho(\vec{b}_\perp, x, \vec{k}_\perp)$



Generalized TMDs and Wigner Distributions

[Meißner, Metz, Schlegel (2009)]

[Gluon GTMDs: Lorcé, BP, (2014)]



Quark polarization

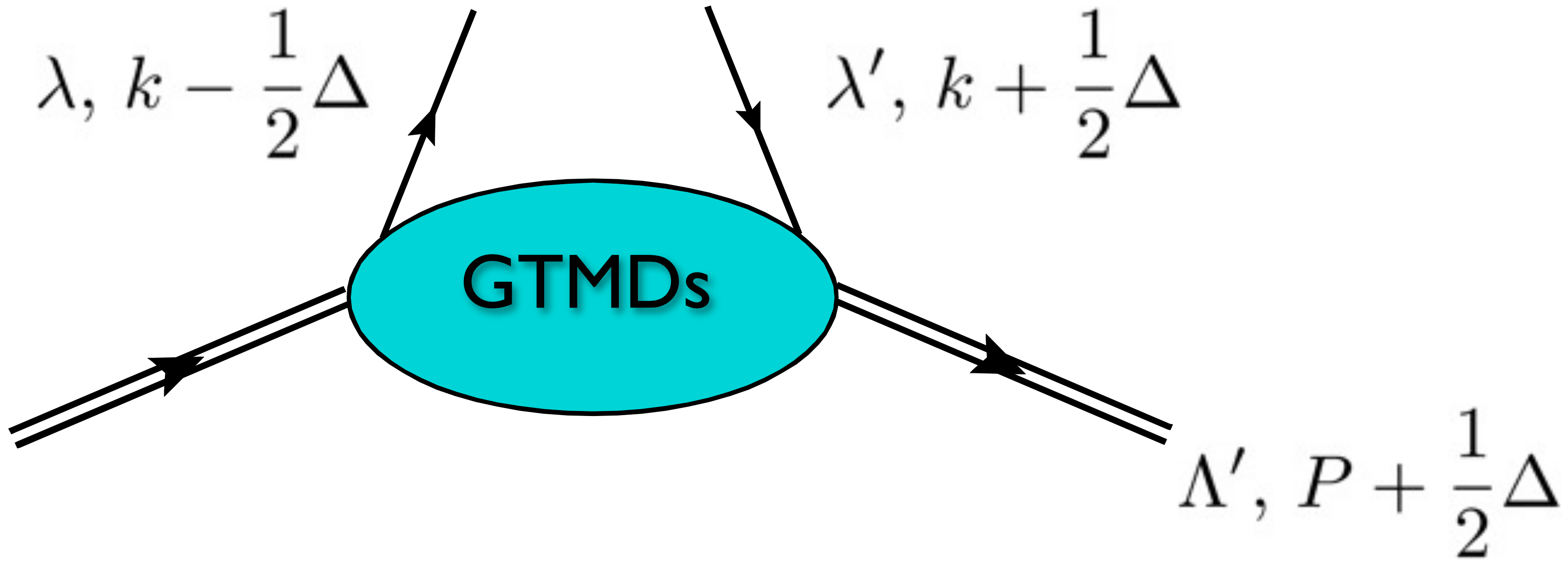
$$W^{[\Gamma]}_{\Lambda'\Lambda} = \frac{1}{2}\int \frac{\mathrm{d}z^-\mathrm{d}^2z_\perp}{(2\pi)^3}\langle P',\Lambda'|\bar{\psi}^q_{\lambda'}(-\frac{z}{2})\,\Gamma\,\mathcal{W}(-\frac{z}{2},\frac{z}{2}|n)\psi^q_\lambda(\frac{z}{2})|P,\Lambda\rangle e^{i(xP^+z^- - \vec{k}_\perp\cdot\vec{z}_\perp)}$$

Nucleon polarization

4 X 4 =16 polarizations ⟷ 16 complex GTMDs (at twist-2)

$$W^{\Gamma}_{\Lambda',\Lambda}(x,\xi,\vec{k}_\perp,\vec{\Delta}_\perp)$$

Generalized TMDs and Wigner Distributions



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Nucleon polarization

4 X 4 =16 polarizations ↔ 16 complex GTMDs (at twist-2)

$$W^{\Gamma}_{\Lambda',\Lambda}(x,\xi,\vec{k}_\perp,\vec{\Delta}_\perp)$$

x: average fraction of quark longitudinal momentum

ξ: fraction of longitudinal momentum transfer

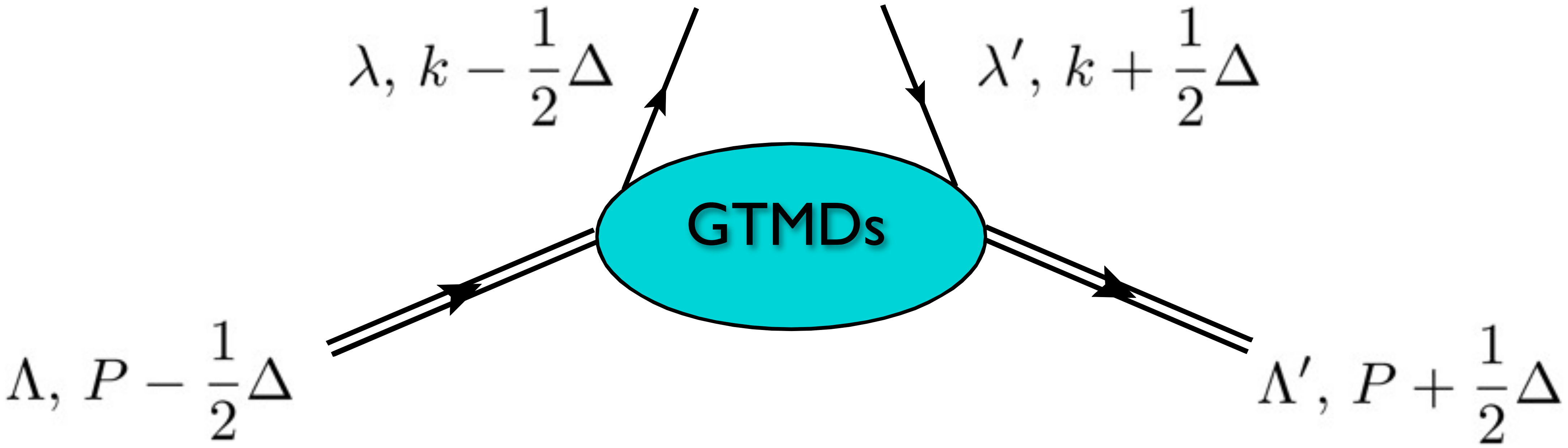
$\vec{k}_\perp$: average quark transverse momentum

$\vec{\Delta}_\perp$: nucleon transverse-momentum transfer

Generalized TMDs and Wigner Distributions

[Meißner, Metz, Schlegel (2009)]

[Gluon GTMDs: Lorcé, BP, (2014)]



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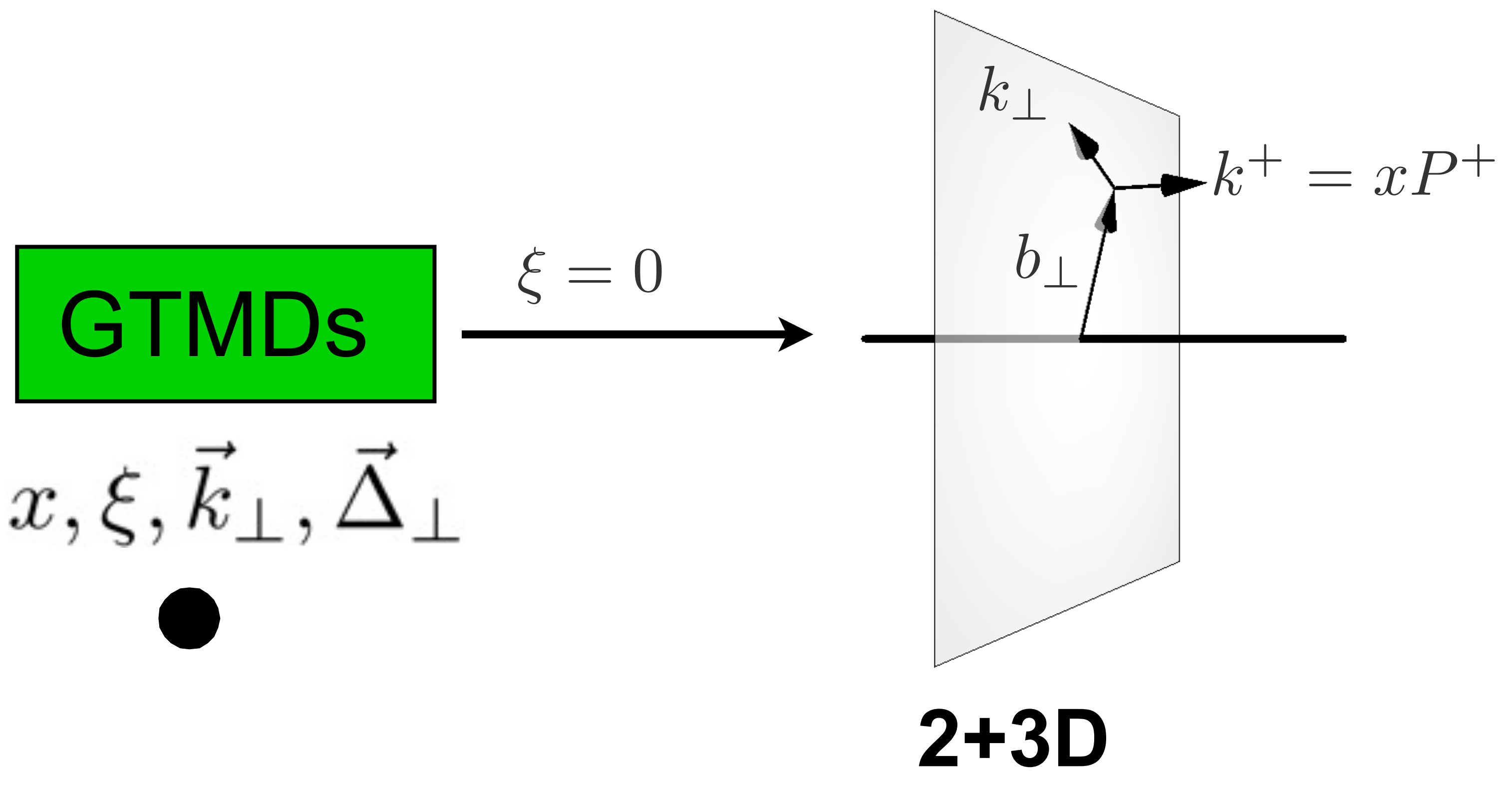
Nucleon polarization

4 X 4 =16 polarizations ⟷ 16 complex GTMDs (at twist-2)

$$W^{\Gamma}_{\Lambda',\Lambda}(x,\xi,\vec{k}_\perp,\vec{\Delta}_\perp)$$

$\downarrow$ Fourier transform $\vec{\Delta}_\perp \leftrightarrow \vec{b}_\perp$

$\tilde{W}^{\Gamma}_{\Lambda',\Lambda}(x,\xi,\vec{k}_\perp,\vec{b}_\perp)$ 16 real Wigner distributions



GTMDs

$x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp$

●

$\xi = 0$ →

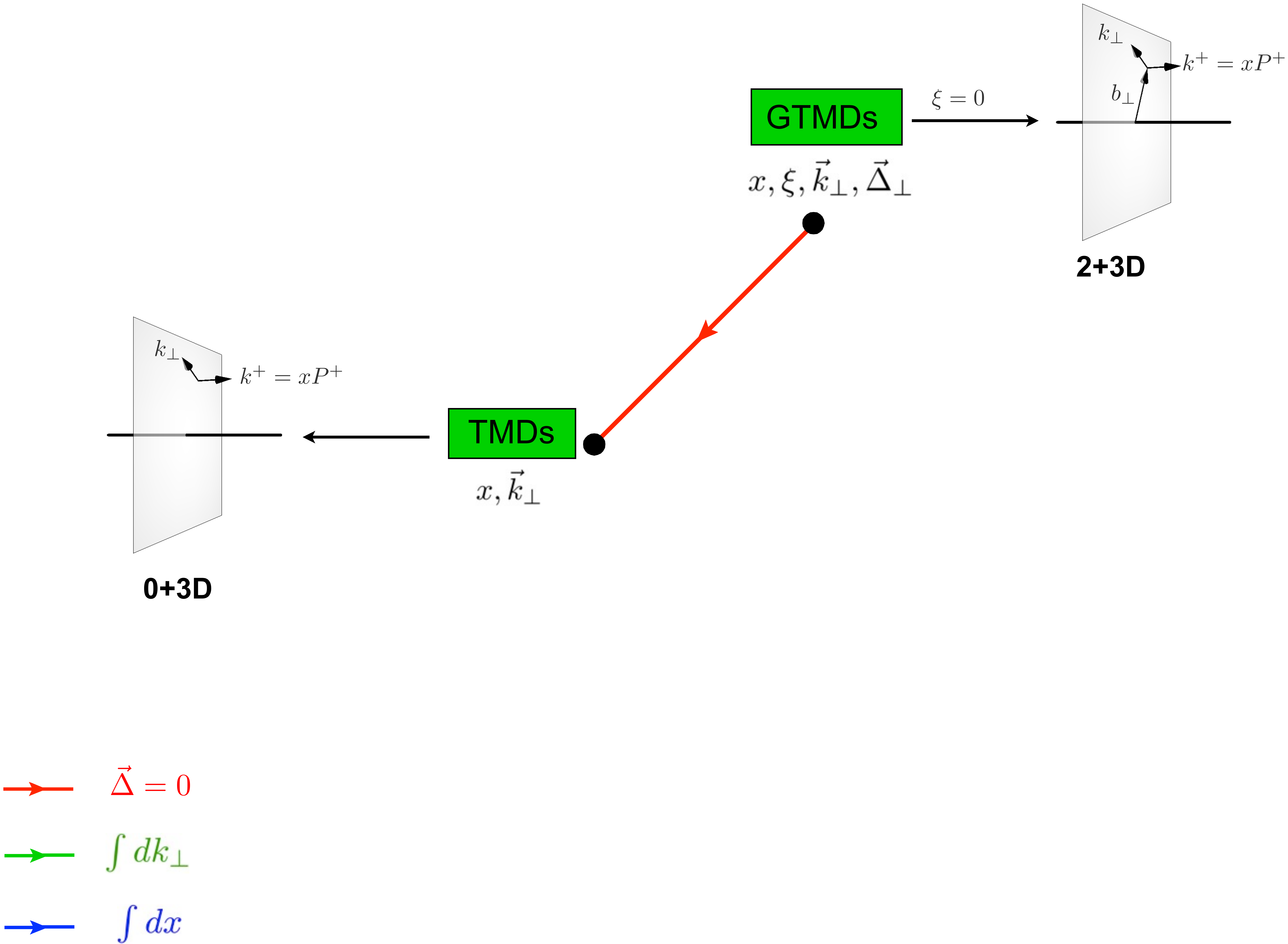
2+3D

→ $\vec{\Delta} = 0$

→ $\int dk_\perp$

→ $\int dx$

[Lorcé, BP, Vanderhaeghen, JHEP05 (2011)]



GTMDs

$x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp$

$\xi = 0$

2+3D

TMDs

$x, \vec{k}_\perp$

0+3D

$\vec{\Delta} = 0$

$\int dk_\perp$

$\int dx$

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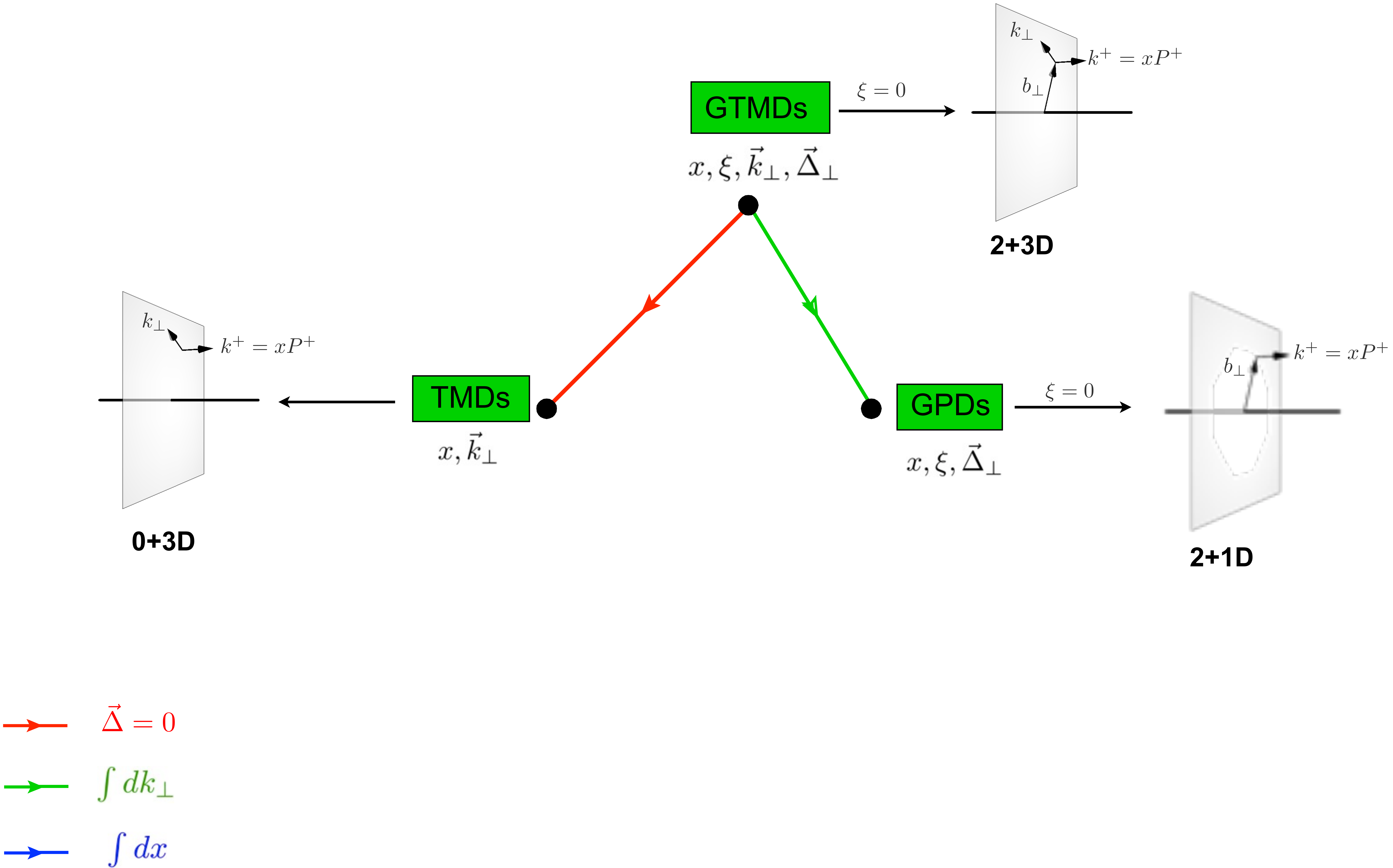


GTMDs

$x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp$

$\xi = 0$

2+3D

TMDs

$x, \vec{k}_\perp$

0+3D

GPDs

$x, \xi, \vec{\Delta}_\perp$

$\xi = 0$

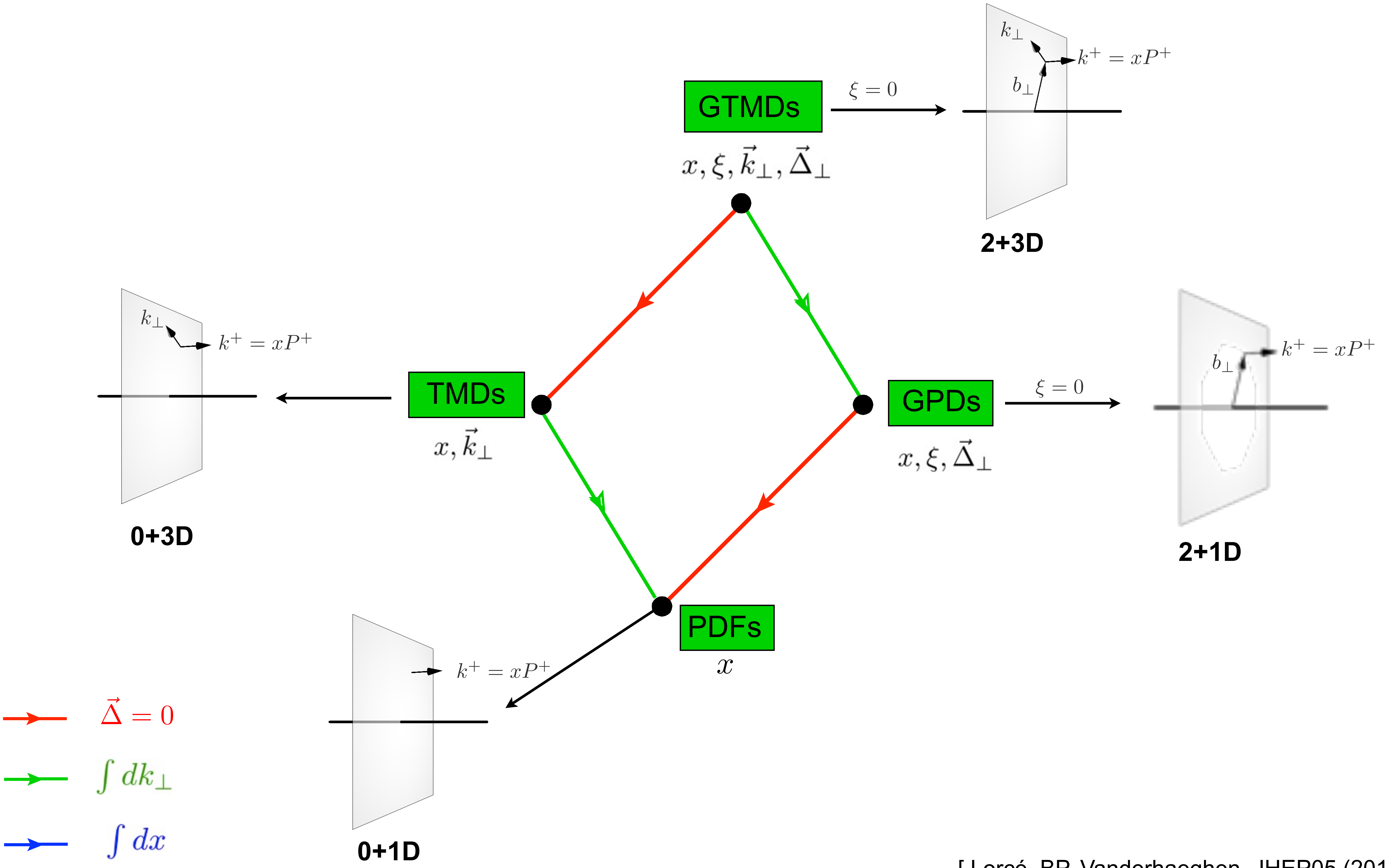
2+1D

$\vec{\Delta} = 0$

$\int dk_\perp$

$\int dx$

[Lorcé, BP, Vanderhaeghen, JHEP05 (2011)]



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2+3D

TMDs

$x, \vec{k}_\perp$

0+3D

GPDs

$x, \xi, \vec{\Delta}_\perp$

$\xi = 0$

2+1D

PDFs

x

0+1D

$\vec{\Delta} = 0$

$\int dk_\perp$

$\int dx$

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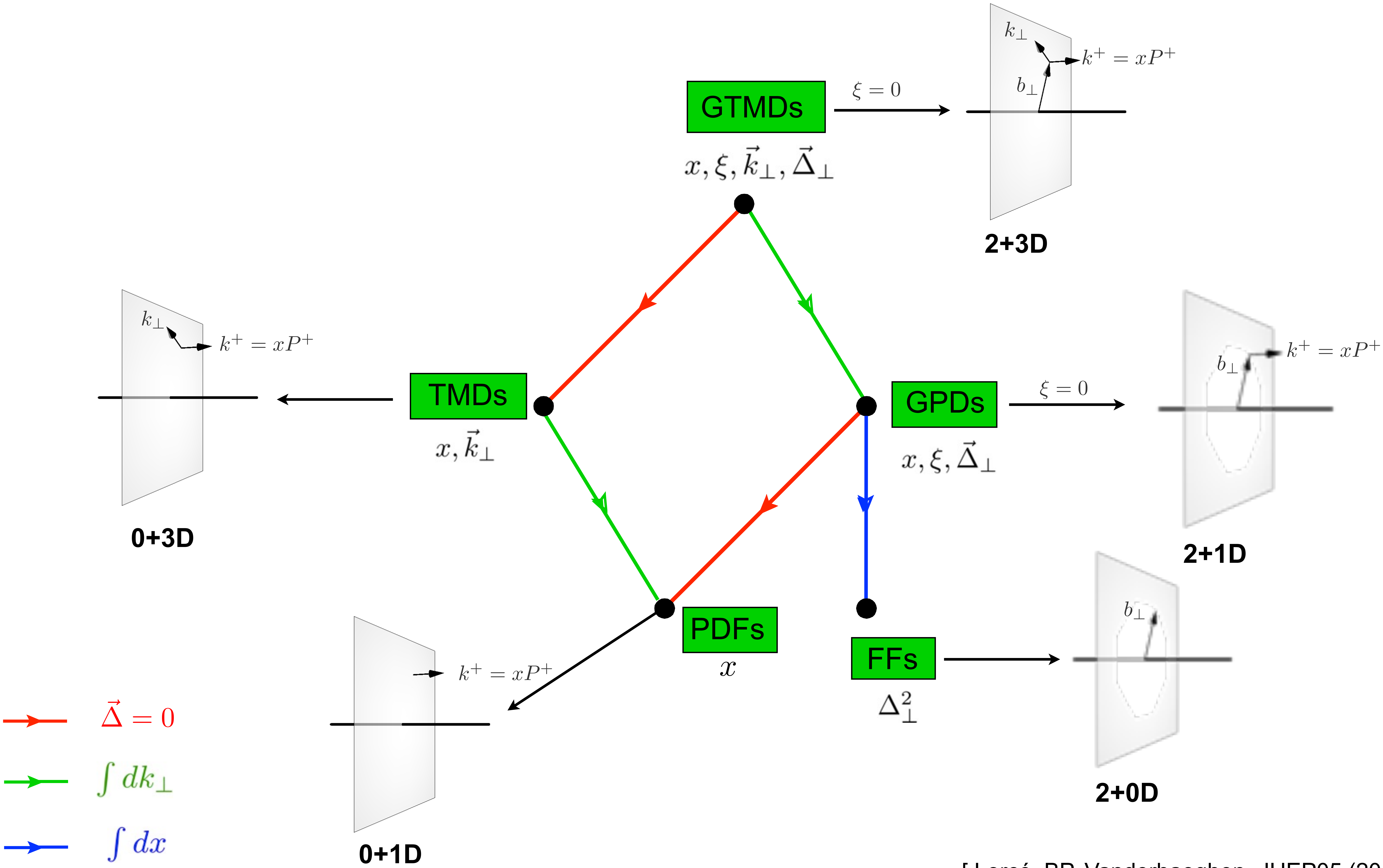


GTMDs

$x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp$

$\xi = 0$

$k_\perp$

$k^+ = xP^+$

$b_\perp$

2+3D

TMDs

$x, \vec{k}_\perp$

$k_\perp$

$k^+ = xP^+$

0+3D

GPDs

$x, \xi, \vec{\Delta}_\perp$

$\xi = 0$

$b_\perp$

$k^+ = xP^+$

2+1D

PDFs

x

$k^+ = xP^+$

0+1D

FFs

$\Delta_\perp^2$

$b_\perp$

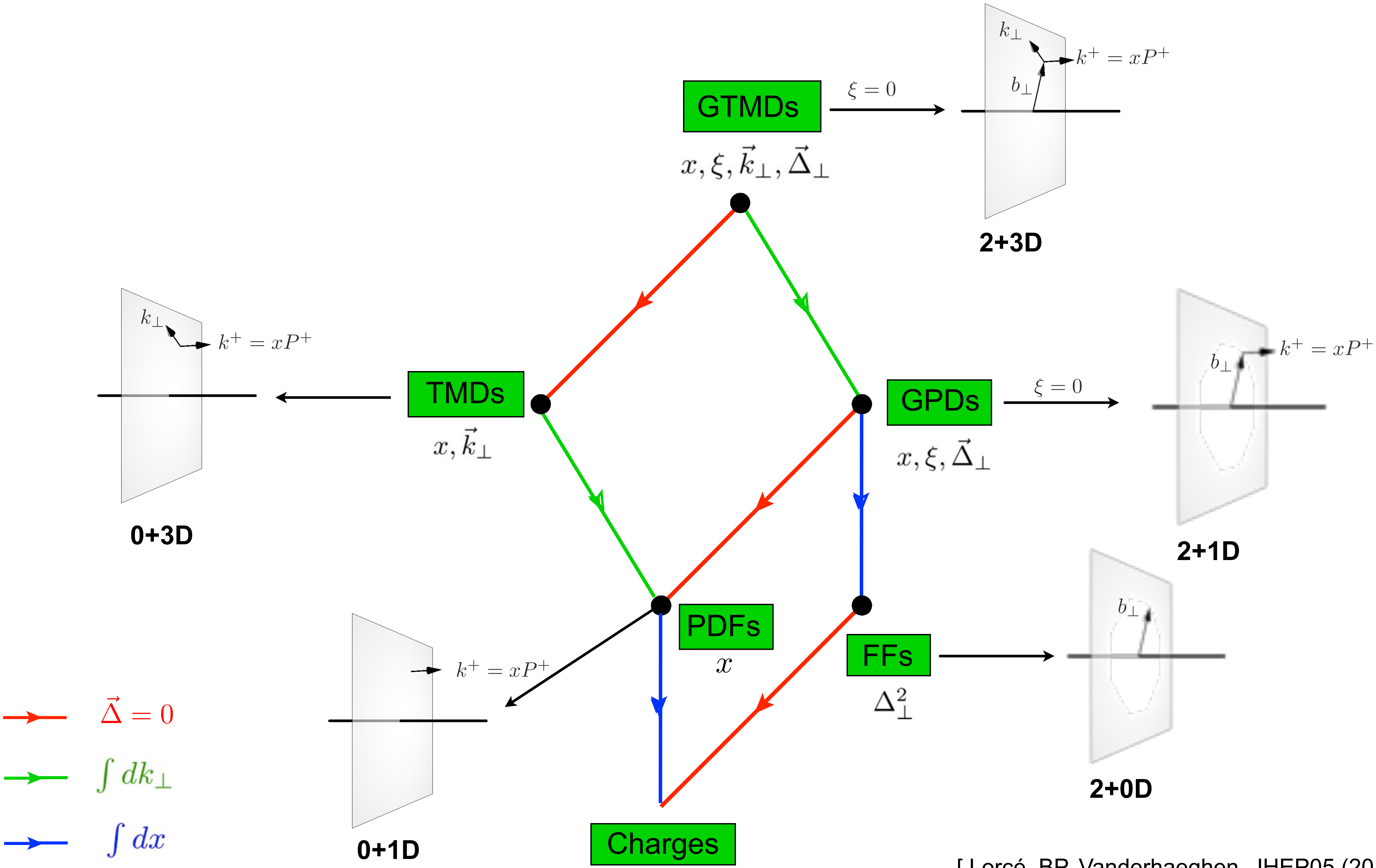
2+0D

$\vec{\Delta} = 0$

$\int dk_\perp$

$\int dx$

[Lorcé, BP, Vanderhaeghen, JHEP05 (2011)]



GTMDs

$x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp$

$\xi = 0$

$k_\perp$ $b_\perp$ $k^+ = xP^+$

2+3D

TMDs

$x, \vec{k}_\perp$

$k_\perp$ $k^+ = xP^+$

0+3D

GPDs

$x, \xi, \vec{\Delta}_\perp$

$\xi = 0$

$b_\perp$ $k^+ = xP^+$

2+1D

PDFs

x

$k^+ = xP^+$

0+1D

FFs

$\Delta_\perp^2$

$b_\perp$

2+0D

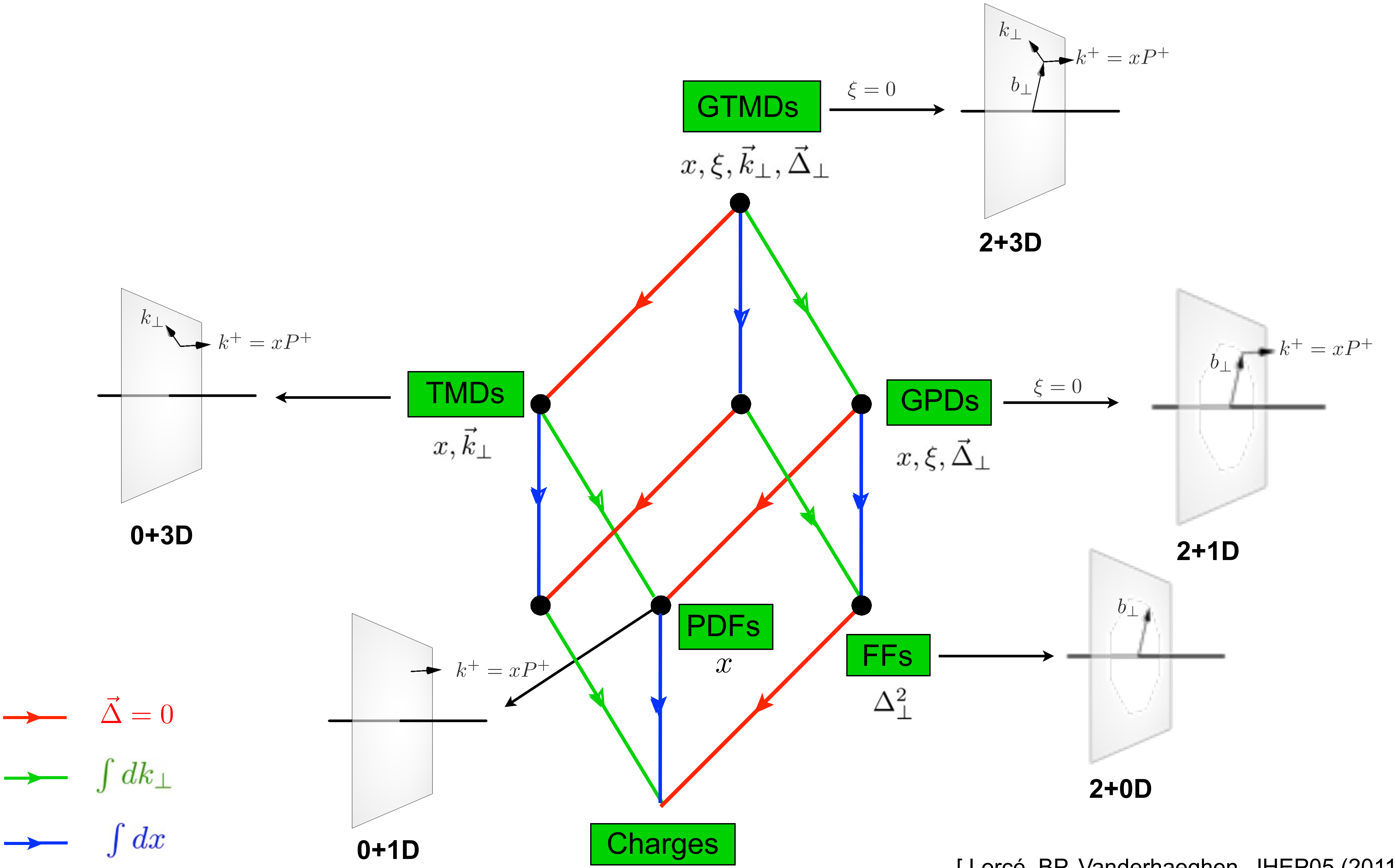
Charges

$\vec{\Delta} = 0$

$\int dk_\perp$

$\int dx$

[Lorcé, BP, Vanderhaeghen, JHEP05 (2011)]



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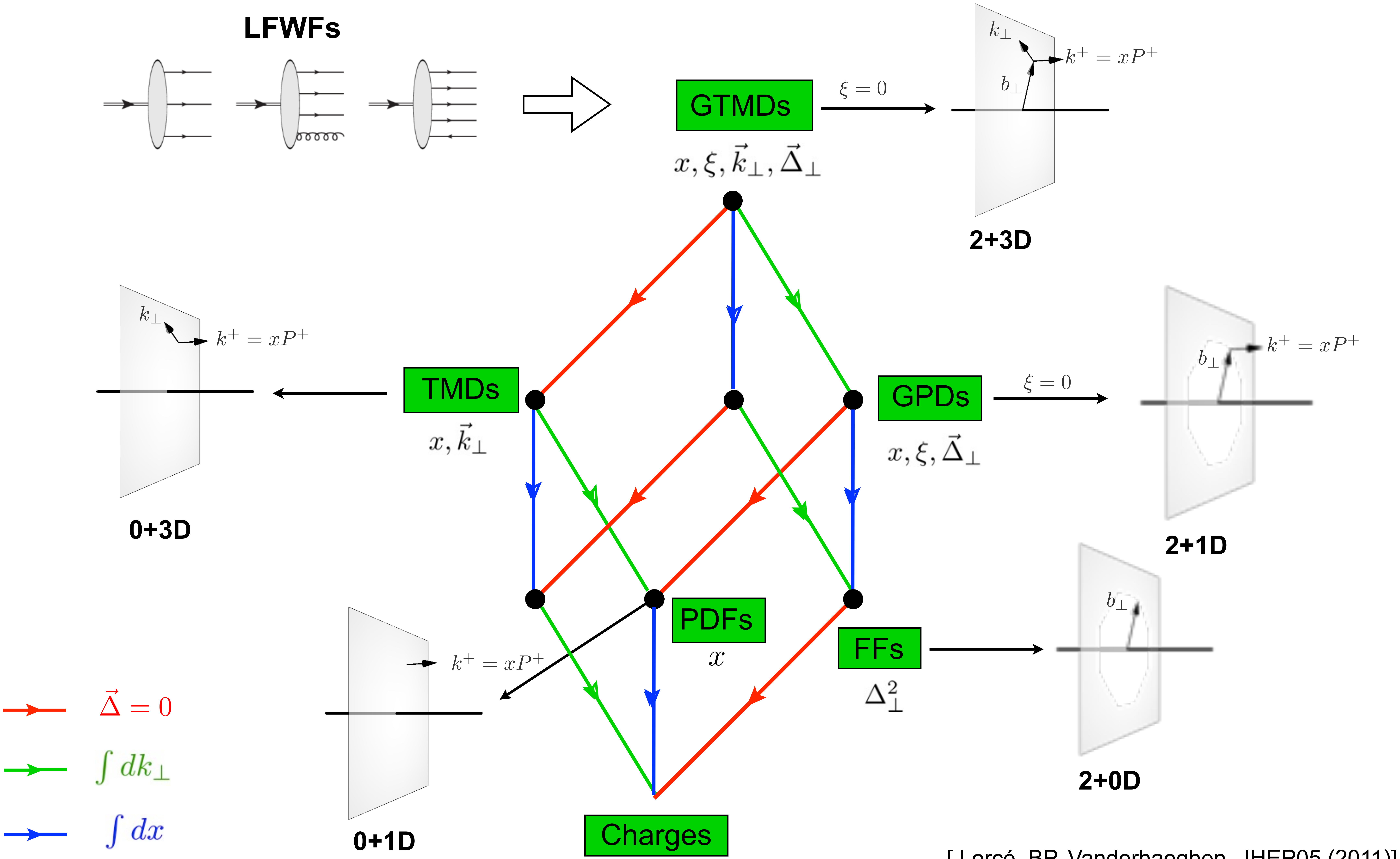


LFWFs

GTMDs

$x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp$

$\xi = 0$

2+3D

TMDs

$x, \vec{k}_\perp$

0+3D

GPDs

$x, \xi, \vec{\Delta}_\perp$

$\xi = 0$

2+1D

PDFs

x

0+1D

FFs

$\Delta_\perp^2$

2+0D

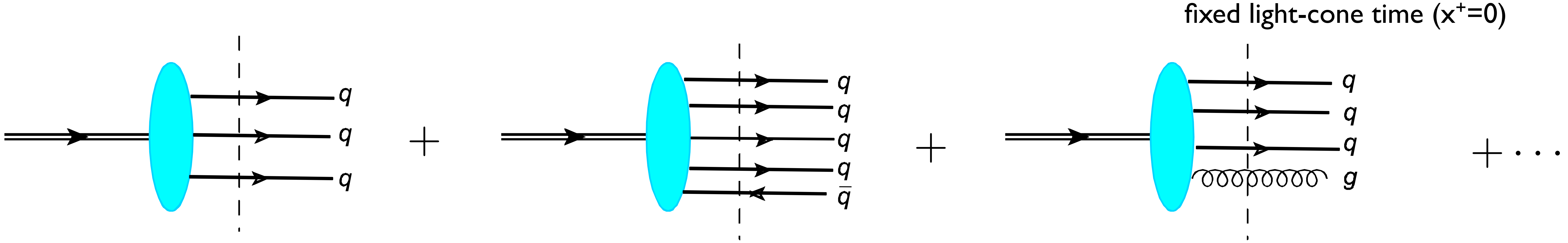
Charges

[Lorcé, BP, Vanderhaeghen, JHEP05 (2011)]

Light-Front Wave Functions (LFWFs)

- ✦ Fock expansion of Nucleon state:

$$|N\rangle = \Psi_{3q}|qqq\rangle + \Psi_{3q\,q\bar{q}}|3q\,q\bar{q}\rangle + \Psi_{3q\,g}|qqqg\rangle + \cdots$$



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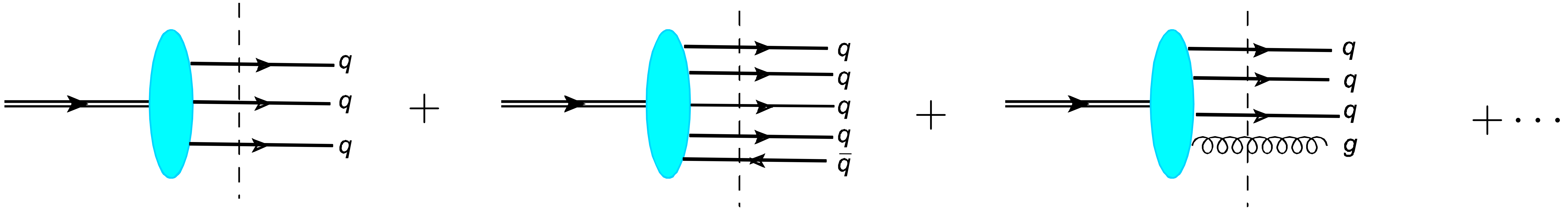
fixed light-cone time ($x^+=0$)



- Probability to find N partons in the nucleon $\rho^{\Lambda}_{N,\beta} = \int [dx]_N [d^2k_\perp]_N |\Psi^{\Lambda}_{\lambda_1\cdots\lambda_N}|^2$

normalization $\sum_{N,\beta} \rho^{\Lambda}_{N,\beta} = 1$

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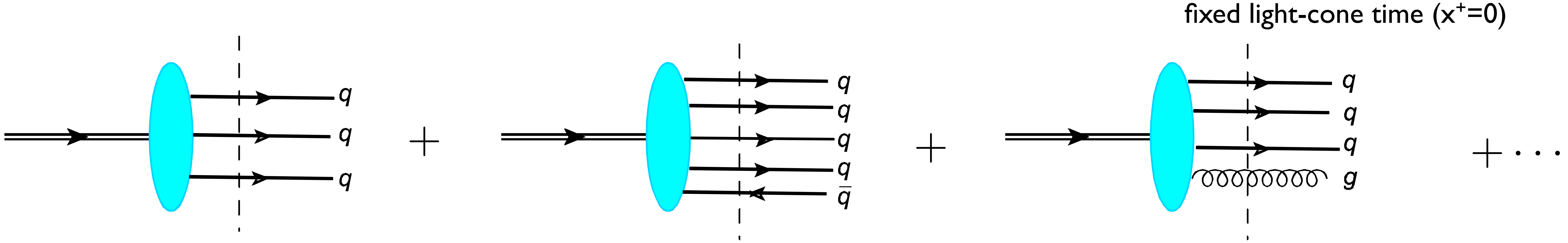

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✦ Eigenstates of momentum $P^+ = \sum_{i=1}^{N} k_i^+ \qquad \vec{P}_\perp = \sum_{i=1}^{N} \vec{k}_{i\,\perp} = \vec{0}_\perp$

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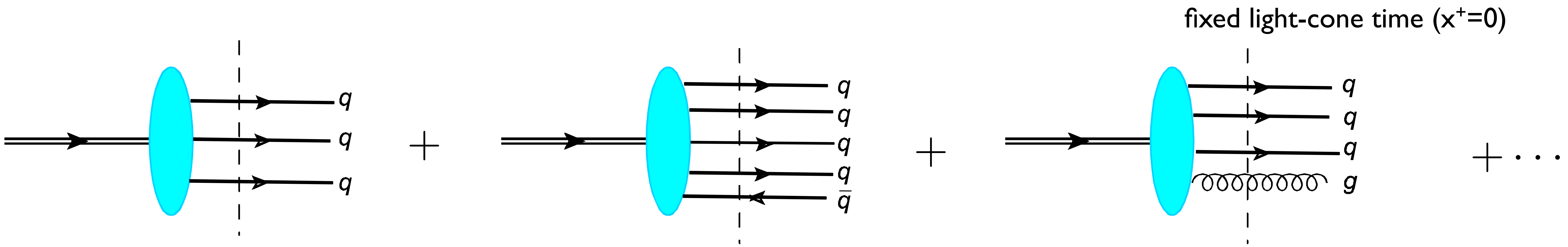

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✦ Eigenstates of parton light-front helicity $\hat{S}_{iz}\,\Psi^{\Lambda}_{\lambda_1\cdots\lambda_N} = \lambda_i\,\Psi^{\Lambda}_{\lambda_1\lambda_2\cdots\lambda_N}$

$$\boxed{\Lambda = \sum_{i=1}^{N} \lambda_i + l_z}$$

✦ Eigenstates of total orbital angular momentum $\hat{L}_z\,\Psi^{\Lambda}_{\lambda_1\cdots\lambda_N} = l_z\,\Psi^{\Lambda}_{\lambda_1\lambda_2\cdots\lambda_N}$

⚠ $A^+ = 0$ gauge

Light-Front Wave Functions (LFWFs)

✦ Fock expansion of Nucleon state:

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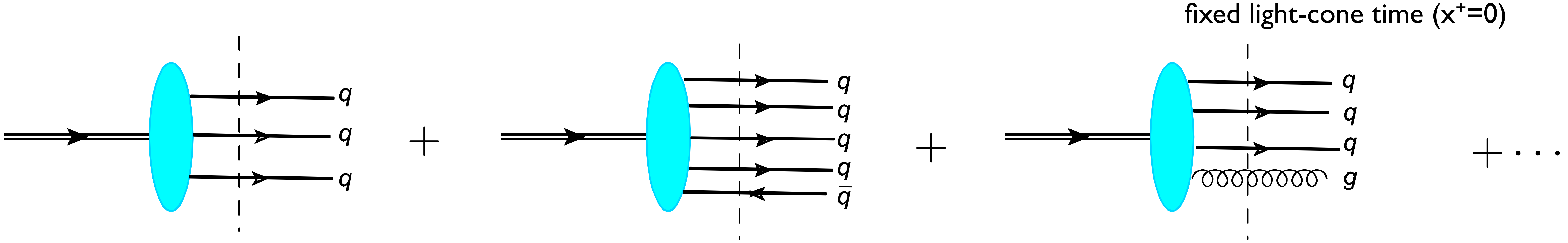

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✦ Eigenstates of momentum $P^+ = \sum_{i=1}^{N} k_i^+$ $\vec{P}_\perp = \sum_{i=1}^{N} \vec{k}_{i\,\perp} = \vec{0}_\perp$

✦ Eigenstates of parton light-front helicity $\hat{S}_{iz}\,\Psi^{\Lambda}_{\lambda_1\cdots\lambda_N} = \lambda_i\,\Psi^{\Lambda}_{\lambda_1\lambda_2\cdots\lambda_N}$

$$\boxed{\Lambda = \sum_{i=1}^{N} \lambda_i\, + l_z}$$

✦ Eigenstates of total orbital angular momentum $\hat{L}_z\,\Psi^{\Lambda}_{\lambda_1\cdots\lambda_N} = l_z\,\Psi^{\Lambda}_{\lambda_1\lambda_2\cdots\lambda_N}$

⚠ $A^+ = 0$ gauge

total helicity

$$s_z = \langle \hat{S}_z \rangle = \sum_{N,\beta} \sum_{i=1}^{N} \lambda_i\, \rho^{\Lambda}_{N,\beta}$$

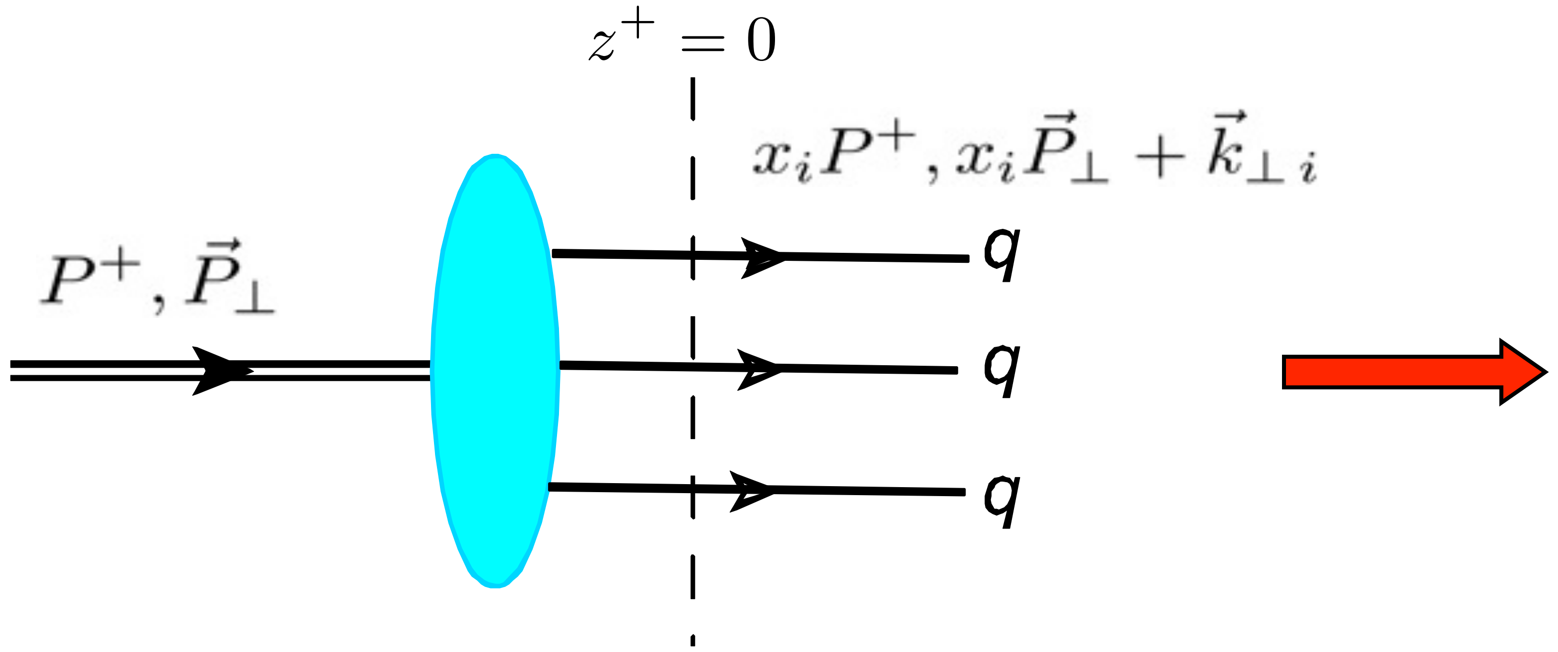
total OAM

$$\ell_z = \langle \hat{L}_z \rangle = \sum_{N,\beta} \sum_{i=1}^{N} l_z\, \rho^{\Lambda}_{N,\beta}$$

nucleon helicity

$$\Lambda = s_z + \ell_z$$

LFWF Overlap Representation



$\Psi^{\Lambda; q_1 q_2 q_3}_{\lambda_1 \lambda_2 \lambda_3}(x_i, \vec{k}_{\perp,i})$

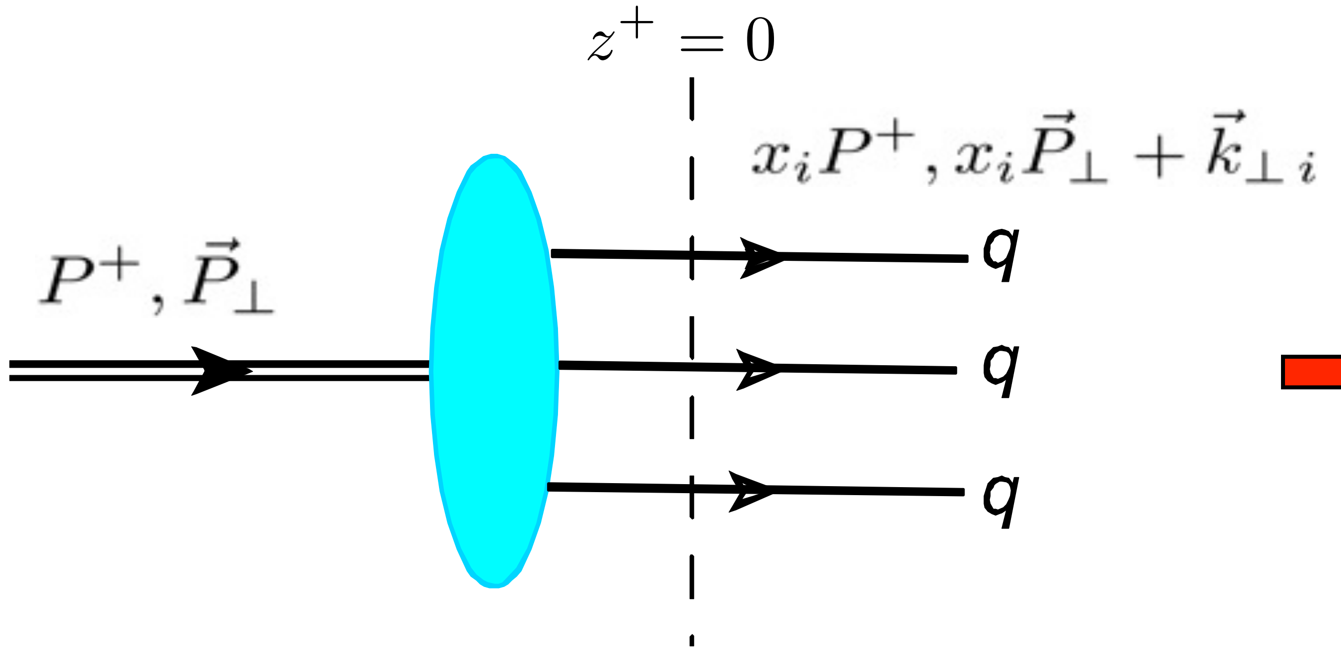
invariant under boost, independent of Pμ

internal variables: $\sum_{i=1}^{3} x_i = 1, \sum_{i=1}^{3} \vec{k}_{\perp\, i} = \vec{0}_\perp$

[Brodsky, Pauli, Pinsky, ’98]

[Lorcé, BP, Vanderhaeghen, JHEP05 (2011)]

LFWF Overlap Representation



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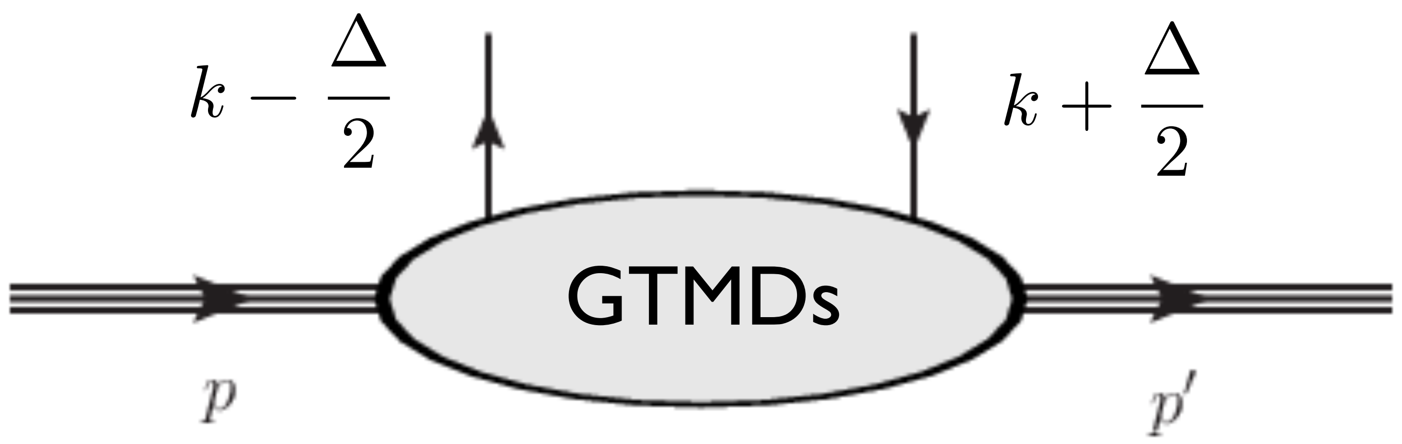
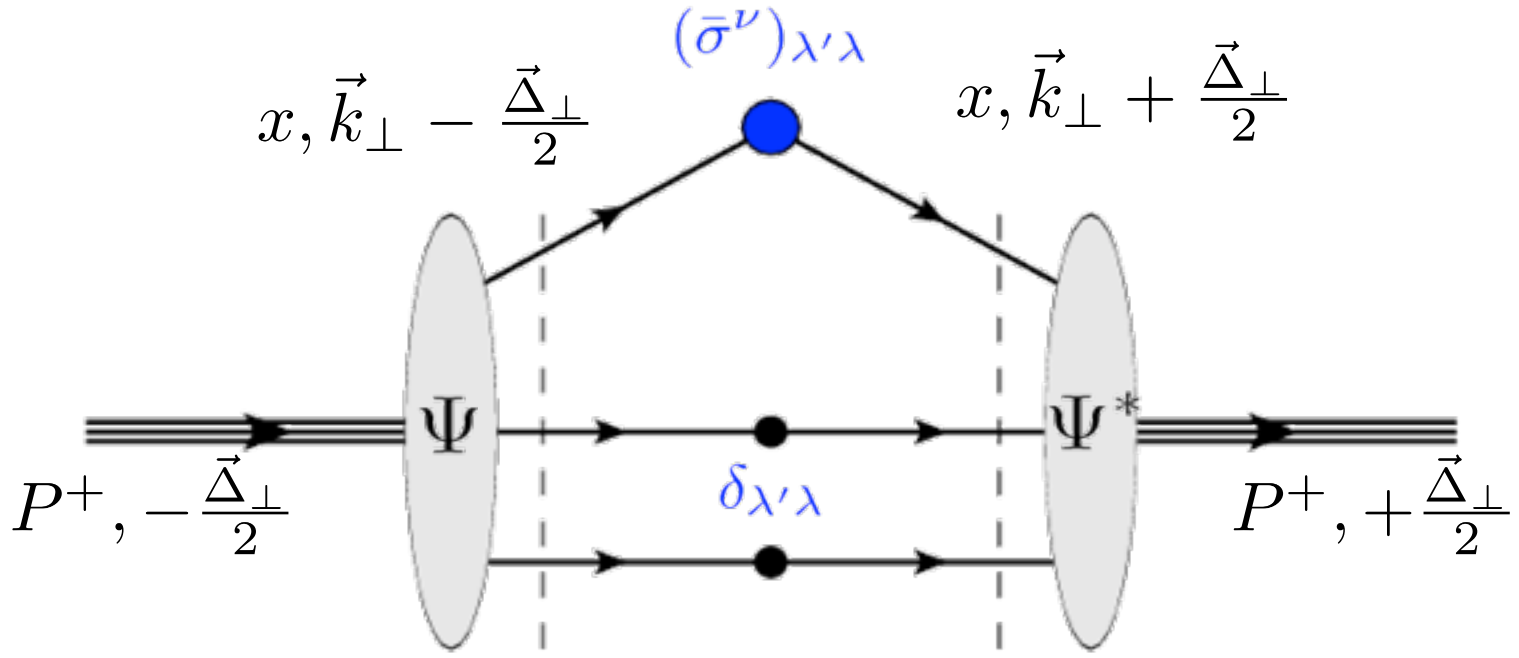
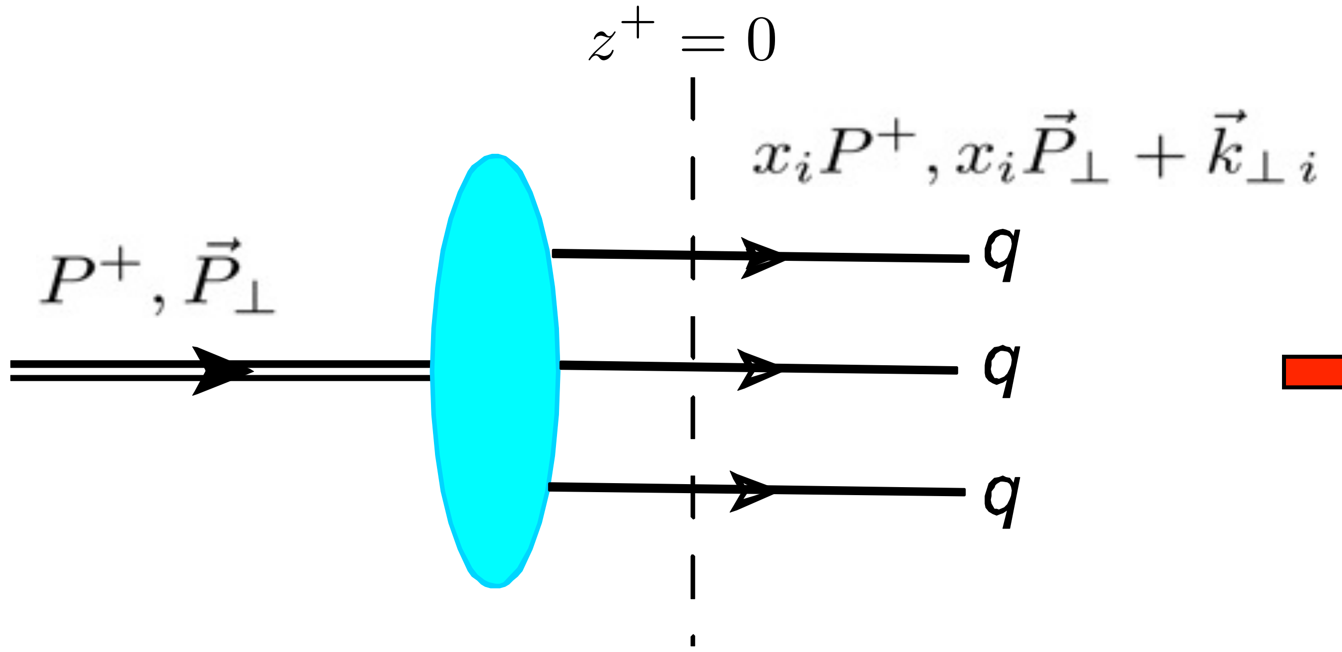


(Δ^+ =0)



[Lorcé, BP, Vanderhaeghen, JHEP05 (2011)]

LFWF Overlap Representation



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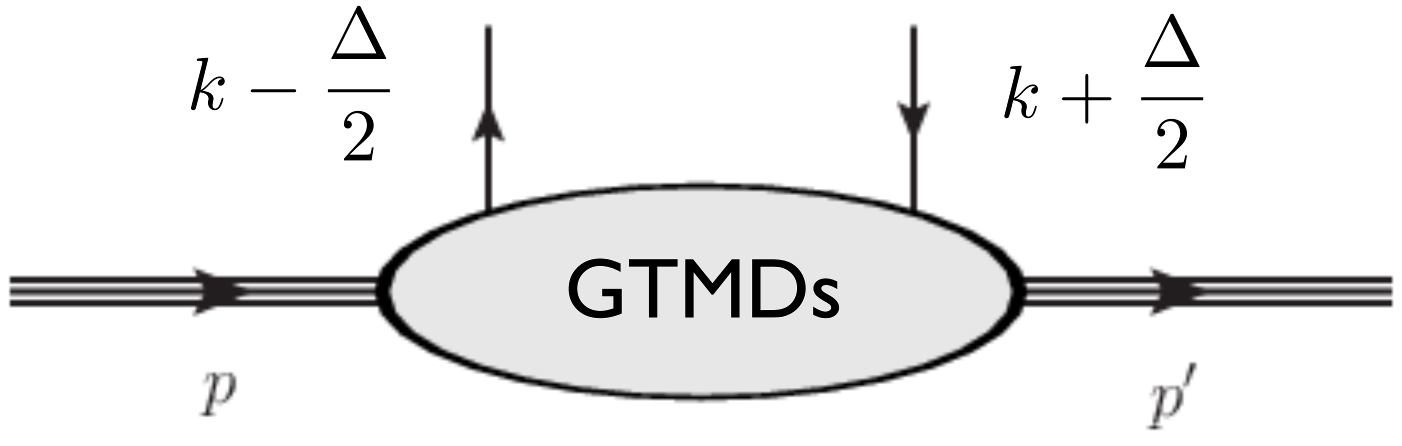
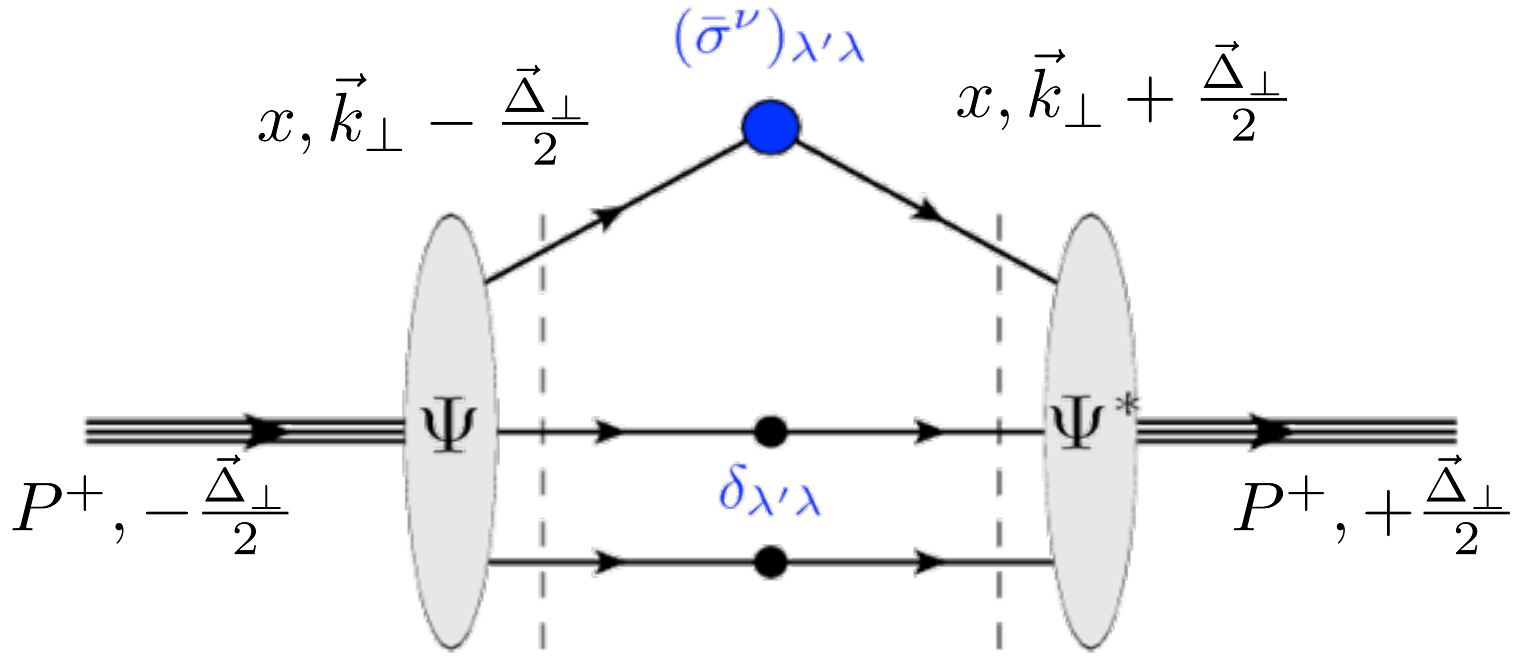


(Δ^+ =0)



$$\Psi^{\Lambda;q_1q_2q_3}_{\lambda_1\lambda_2\lambda_3}(x_i,\vec{k}_{\perp,i})=\sum_{s_i}\phi(x_i,\vec{k}_{\perp,i})\,\Phi^{\Lambda;q_1q_2q_3}_{s_1s_2s_3}\prod_i D^{1/2*}_{s_i\lambda_i}(R_{cf})$$

momentum wf

spin-flavor wf

rotation from canonical spin to light-front spin

General formalism valid for

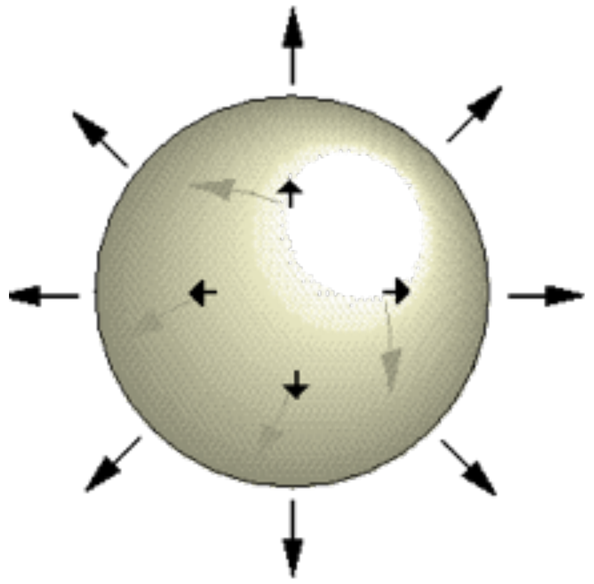
Bag Model, LFχQSM, LFCQM, Quark-Diquark, Covariant Parton Models

[Lorcé, BP, Vanderhaeghen, JHEP05 (2011)]

Common assumptions :

- ➢ No gluons
- ➢ Independent quarks
- ➢ Spherical symmetry in the nucleon rest frame

spherical symmetry
in the rest frame



the quark distribution does not depend on the
direction of polarization

rest frame

$|\vec{0}, \sigma\rangle$

zero OAM

Light-front boost

$\longrightarrow$

infinite-momentum frame

$|\vec{k}, \lambda\rangle_{LC}$

NON-zero OAM

[Lorce',BP, 2011]

Light-Cone Helicity and Canonical Spin

$$q_\lambda^{LC}(k) = D_{\lambda s}^{(1/2)*}\, q_s^{C}(k)$$

LC helicity

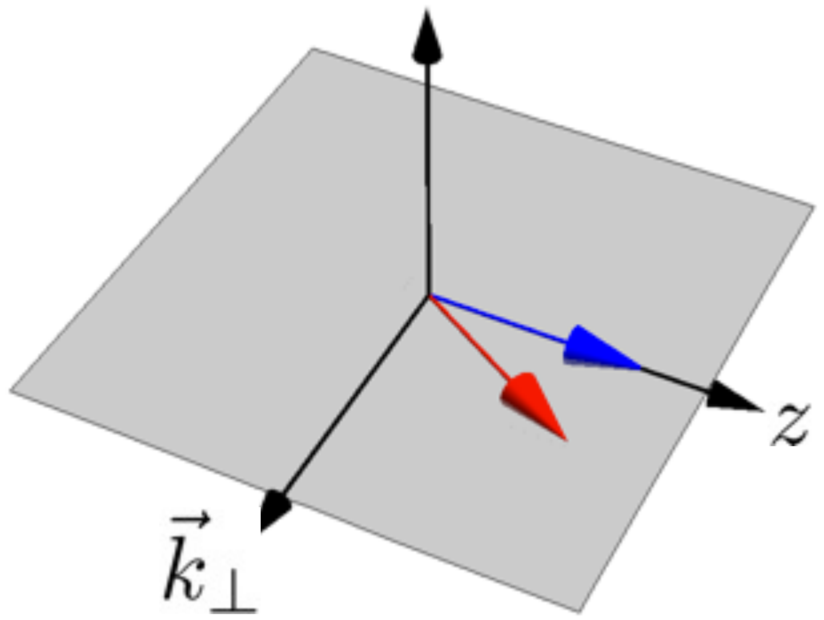
canonical spin

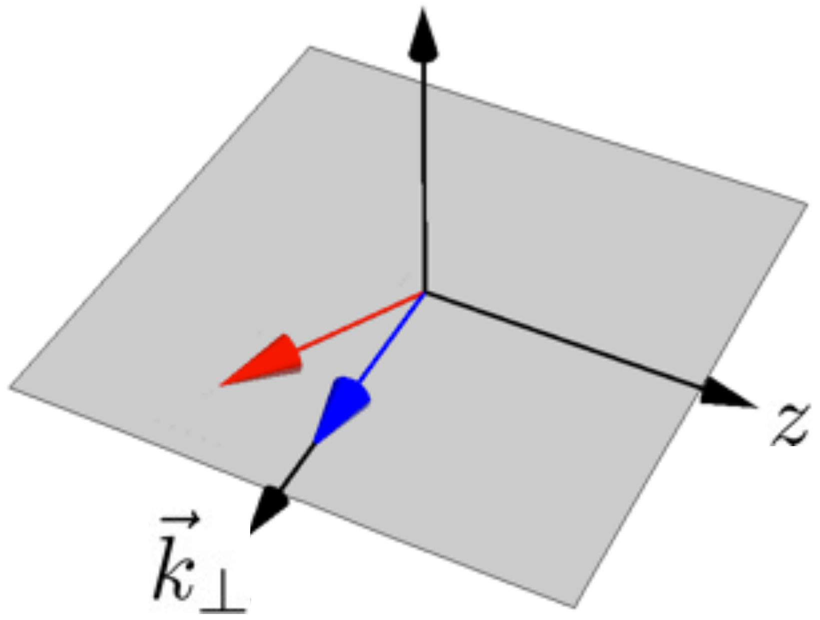
$$D_{\lambda s}^{(1/2)*}(k) = \frac{1}{|\vec{K}|}\begin{pmatrix} K_z & K_L \\ -K_R & K_z \end{pmatrix}$$

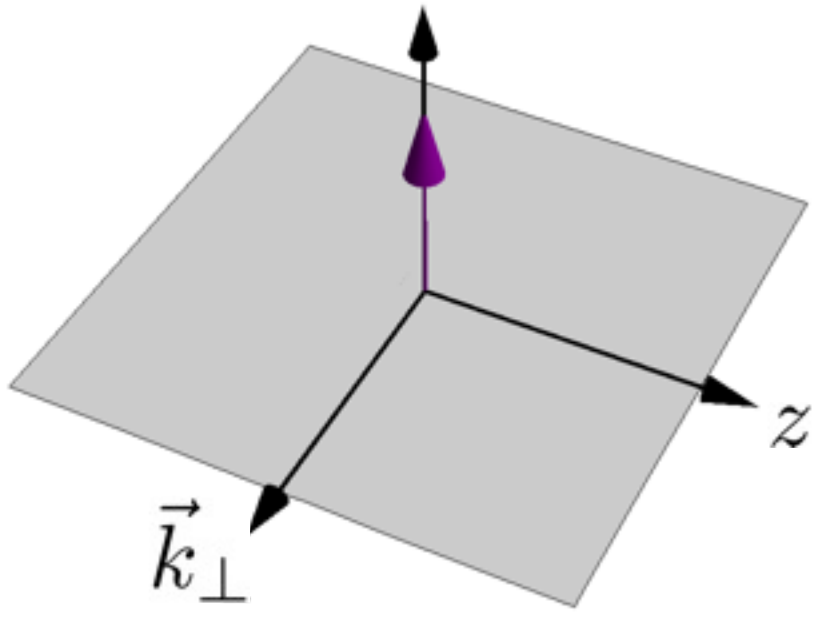
rotation around an axis orthogonal to z and k

$$K_L,\ K_R \to 0$$

in the limit of $k_\perp \to 0$







Light-Cone CQM

$$K_z = m + x\mathcal{M}_0$$

$$\vec{K}_\perp = \vec{k}_\perp$$

$$k_z = x\mathcal{M}_0 - \sqrt{\vec{k}^2 + m^2}$$

(Melosh rotation)

Chiral Quark-Soliton Model

$$K_z = h(|\vec{k}|) + \frac{k_z}{|\vec{k}|}\, j(|\vec{k}|)$$

$$\vec{K}_\perp = \frac{\vec{k}_\perp}{|\vec{k}|}\, j(|\vec{k}|)$$

$$k_z = x\mathcal{M}_N - E_{\text{lev}}$$

Bag Model

$$K_z = t_0(|\vec{k}|) + \frac{k_z}{|\vec{k}|}\, t_1(|\vec{k}|)$$

$$\vec{K}_\perp = \frac{\vec{k}_\perp}{|\vec{k}|}\, t_1(|\vec{k}|)$$

$$k_z = x\mathcal{M}_N - \omega/R_0$$

Light-Front Constituent Quark Model

Light-Front Constituent Quark Model

- **momentum-space wf**

  [Schlumpf, Ph.D. Thesis, hep-ph/921155]

$$\Psi(k_i) = \frac{N}{(M_0^2 + \beta^2)^\gamma}$$

N : normalization constant

$$M_0 = \sum_i^3 \sqrt{m_i^2 + \vec{k}_i^2}$$

β, γ parameters fitted to anomalous magnetic moments of the nucleon

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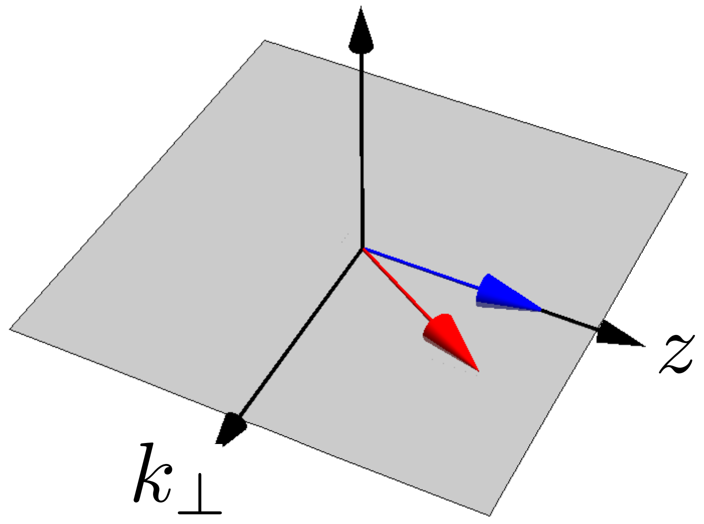
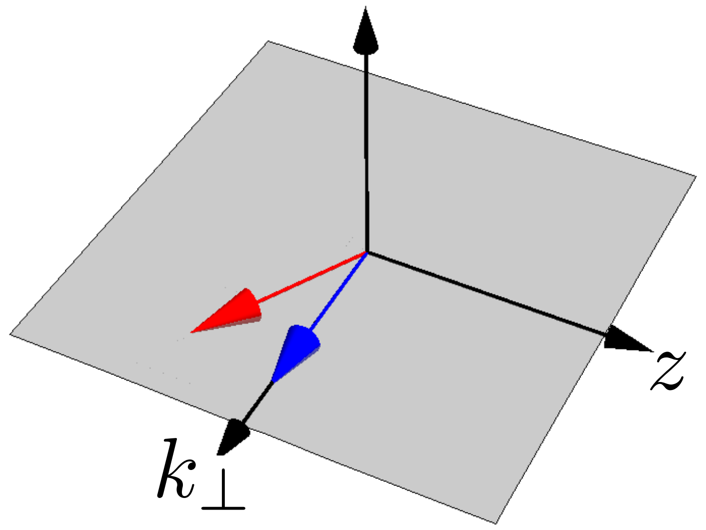
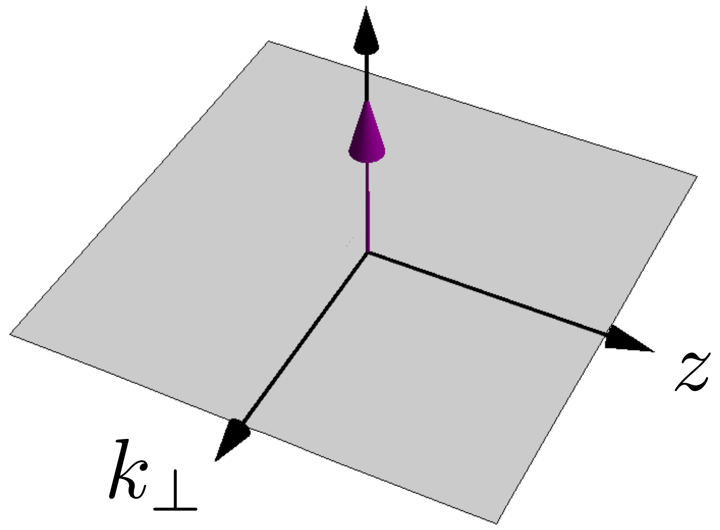
- **spin-structure:**

$$q_\lambda^{LC}(k) = D_{\lambda s}^{(1/2)*}\, q_s^C(k) \qquad D_{\lambda s}^{(1/2)*}(k) = \frac{1}{|\vec{K}|}\begin{pmatrix} K_z & K_L \\ -K_R & K_z \end{pmatrix}$$







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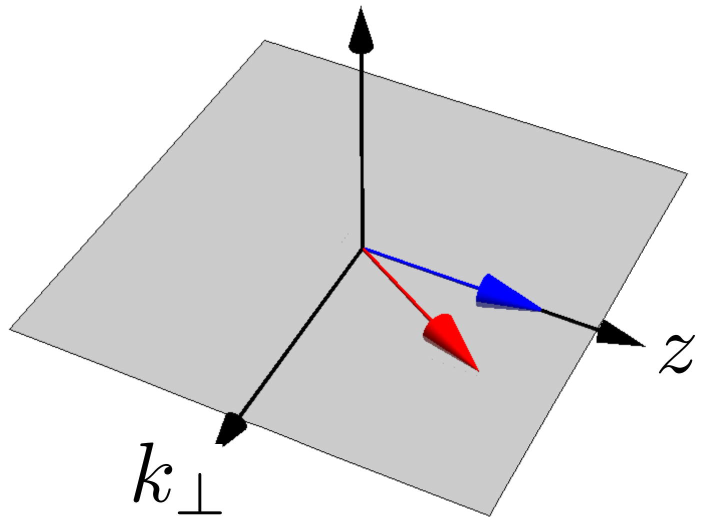
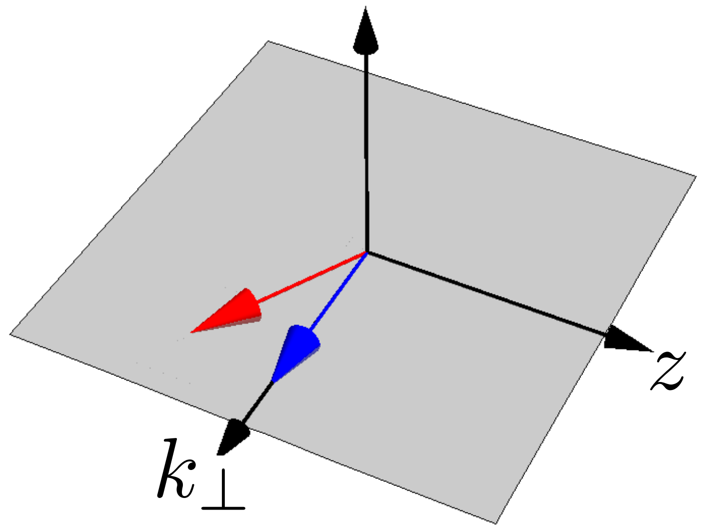
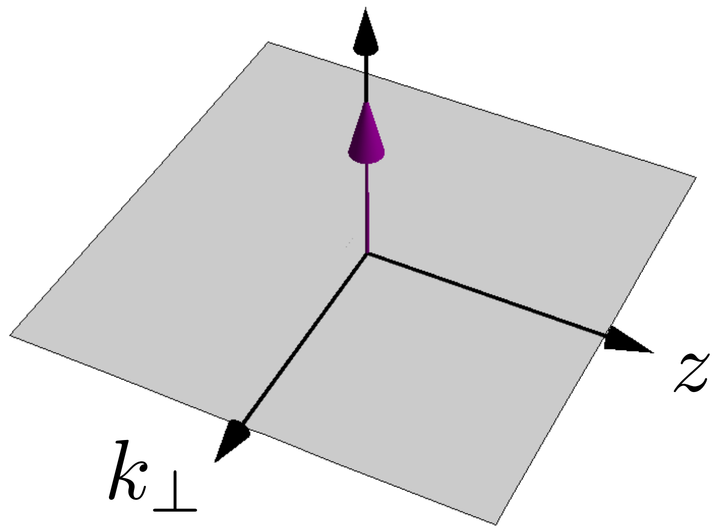
free quarks $\longrightarrow$ $K_z = m + x\mathcal{M}_0 \qquad \vec{K}_\perp = \vec{k}_\perp$ (Melosh rotation)







Light-Front Constituent Quark Model

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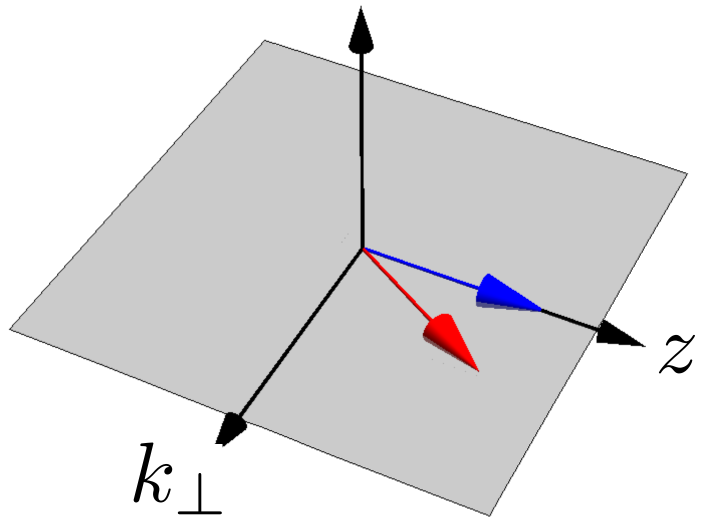

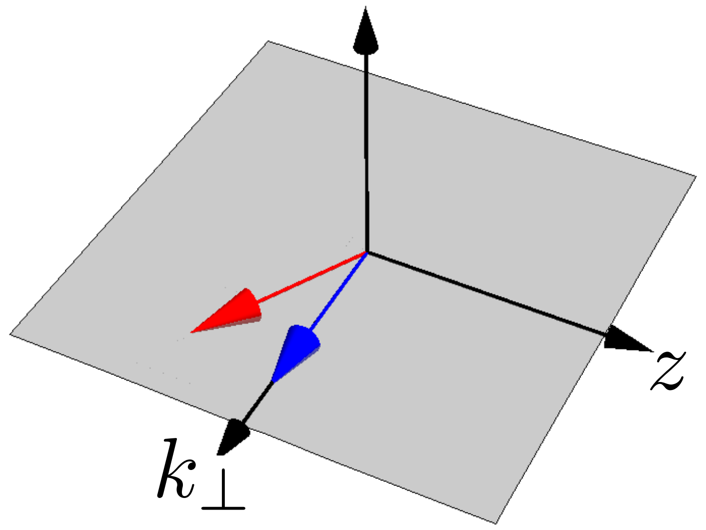

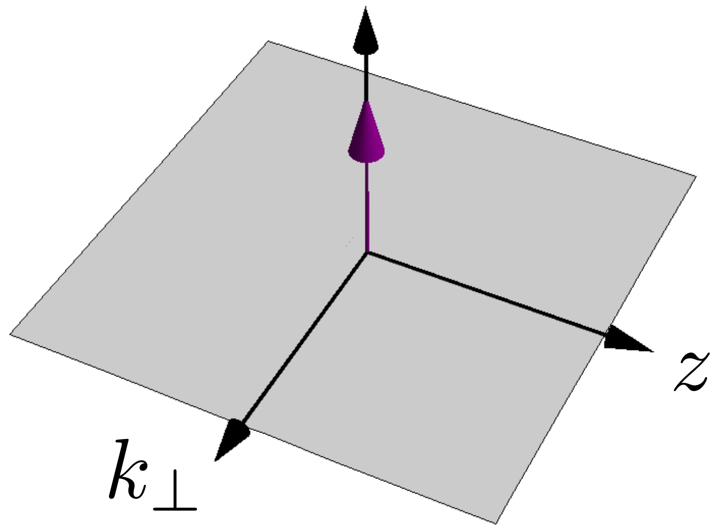

- SU(6) symmetry

Light-Front Constituent Quark Model

- **momentum-space wf**

  [Schlumpf, Ph.D. Thesis, hep-ph/921155]

$$\Psi(k_i) = \frac{N}{(M_0^2 + \beta^2)^\gamma} \qquad M_0 = \sum_i^3 \sqrt{m_i^2 + \vec{k}_i^2}$$

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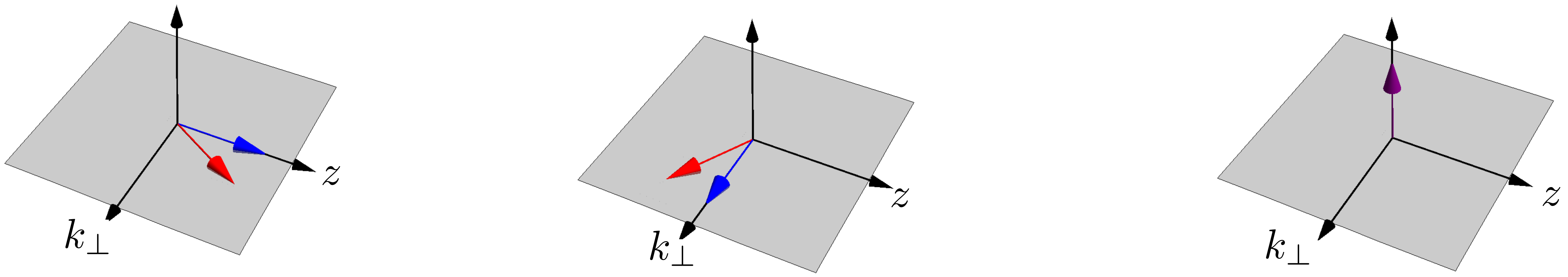


- **SU(6) symmetry**

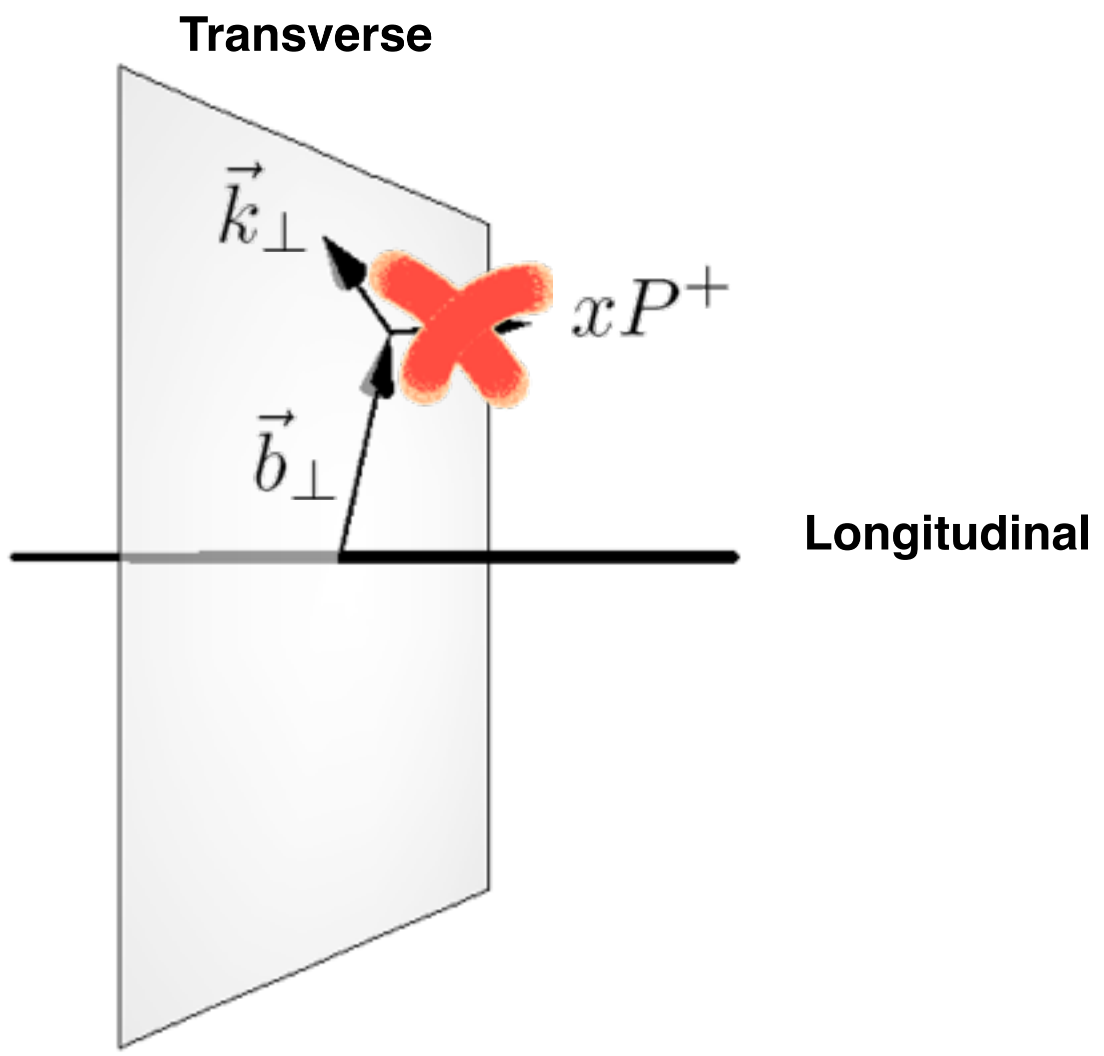
Applications of the model to:

GPDs and Form Factors: BP, Boffi, Traini (2003)-(2005);

TMDs: BP, Cazzaniga, Boffi (2008); BP, Yuan (2010); Lorcè, BP, Vanderhaeghen (2011)

Azimuthal Asymmetries: Schweitzer, BP, Boffi, Efremov (2009)

Quark Wigner Distributions



$$\rho(\vec{k}_\perp, \vec{b}_\perp) = \int \mathrm{d}x \, \rho(x, \vec{k}_\perp, \vec{b}_\perp)$$ **2+2D**

at fixed $\vec{k}_\perp$ → two-dimensional distributions in impact-parameter space

★ Twist-2 ~ LO in P$^+$ $\boxed{\Gamma_{\mathrm{twist}-2} = \gamma^+, \, \gamma^+\gamma_5, \, i\sigma^{j+}\gamma_5}$

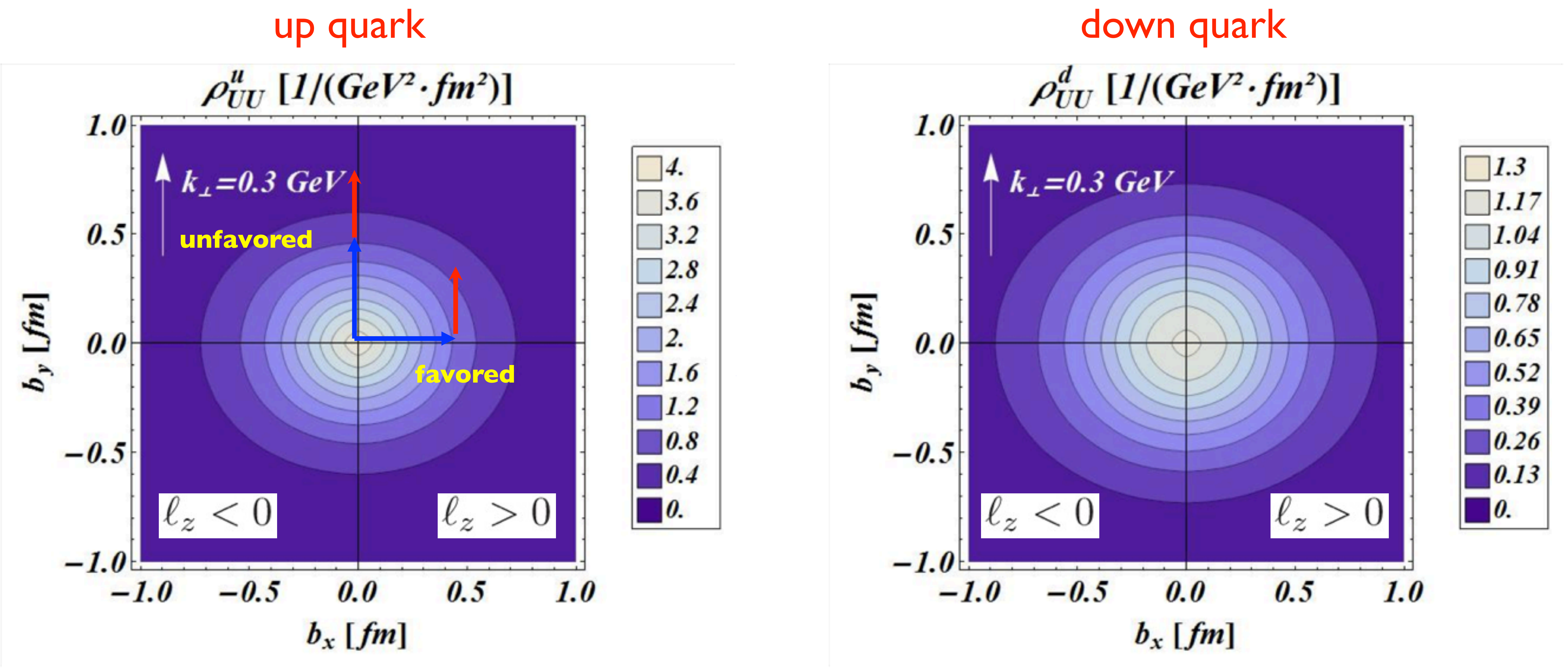
quark polarization → **U** **L** **T**

★ Nucleon polarization: **U** **L** **T**

→ 16 independent Wigner distributions

Unpol. quarks in Unpol. Proton

fixed $\vec{k}_\perp$: ↑



Distortion due to correlations between $\vec{k}_\perp$ and $\vec{b}_\perp$

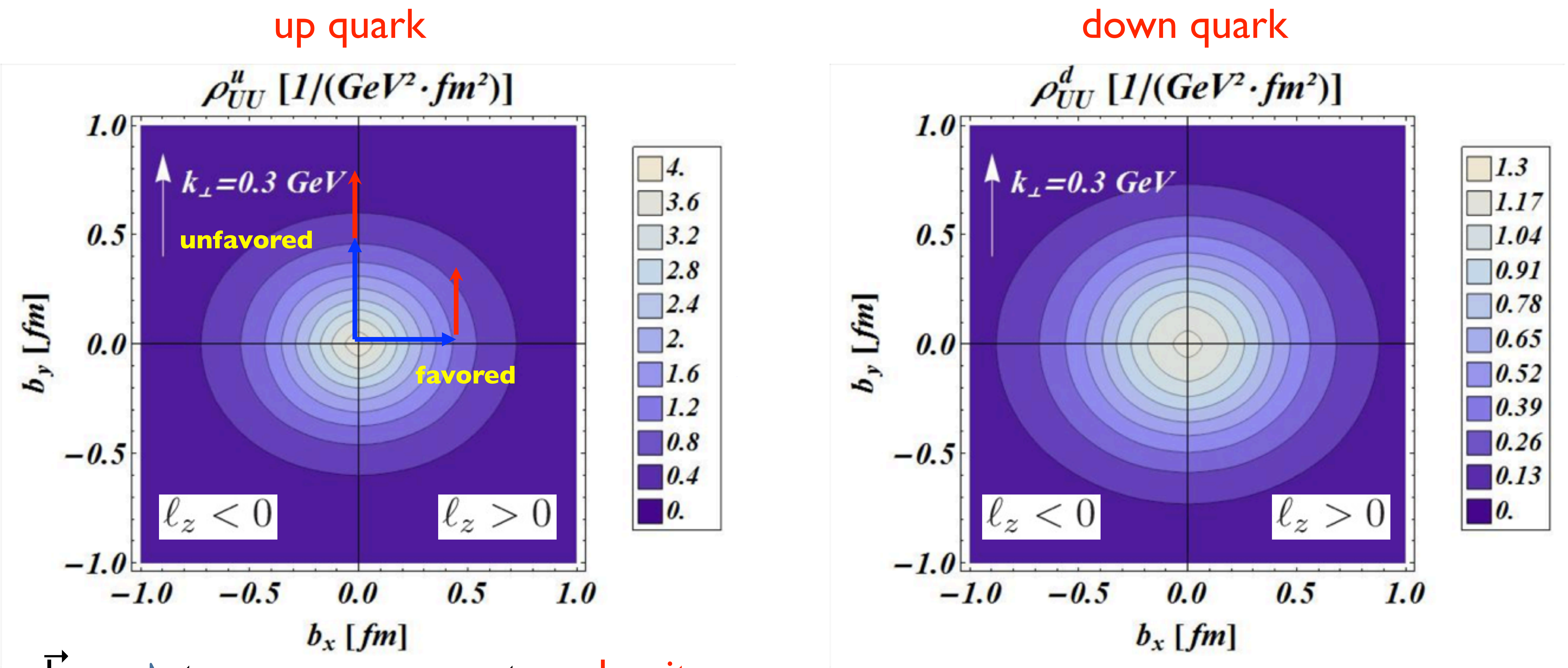
→ absent in GPD and TMD!

Left-right symmetry → no net quark OAM

[Lorcé, Pasquini (2011)]

Unpol. quarks in Unpol. Proton

fixed $\vec{k}_\perp$: ↑



✦ integrating over $\vec{b}_\perp$ ➡ transverse-momentum density

$$f_1^q(k_\perp^2) = \int \mathrm{d}x\, f_1^q(x, k_\perp^2)$$

✦ integrating over $\vec{k}_\perp$ ➡ charge density in the transverse plane $\vec{b}_\perp$

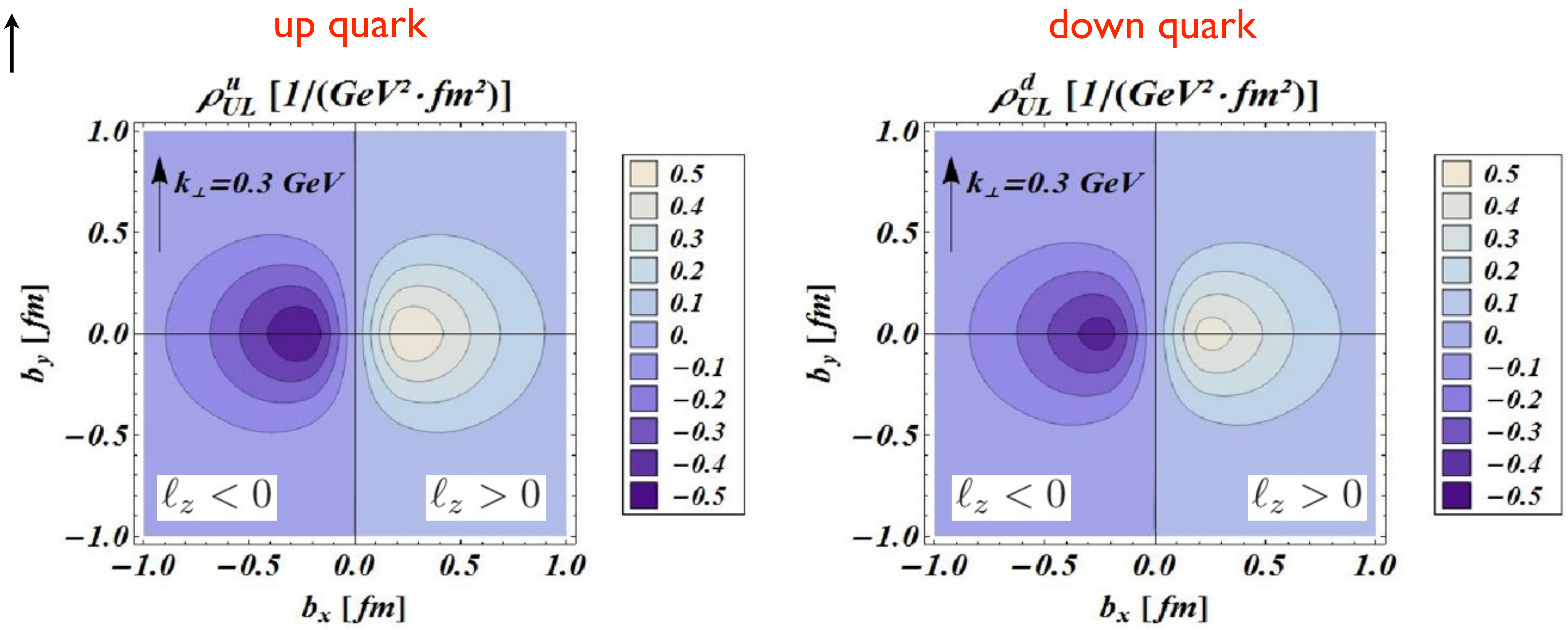
Monopole Distributions

$$\rho^q(b_\perp^2) = e^q \int \mathrm{d}^2\Delta_\perp e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp} F_1^q(\Delta_\perp^2)$$

[Miller (2007); Burkardt (2007)]

Long. pol. quark in Unpol. Proton

fixed $\vec{k}_\perp \uparrow$



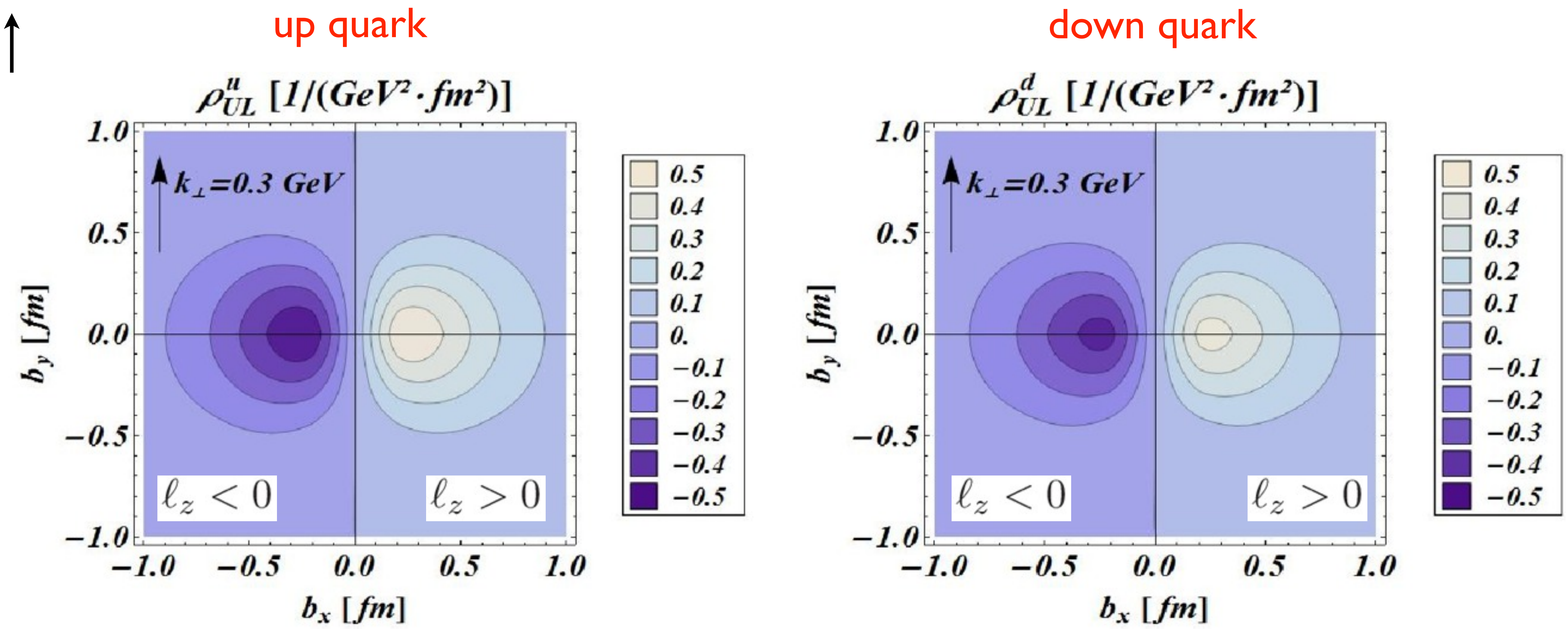
✦ projection to GPD and TMD is vanishing

➡ unique information on OAM from Wigner distributions

[Lorcé, Pasquini (2011)]
[Lorcé, (2014)]

Long. pol. quark in Unpol. Proton

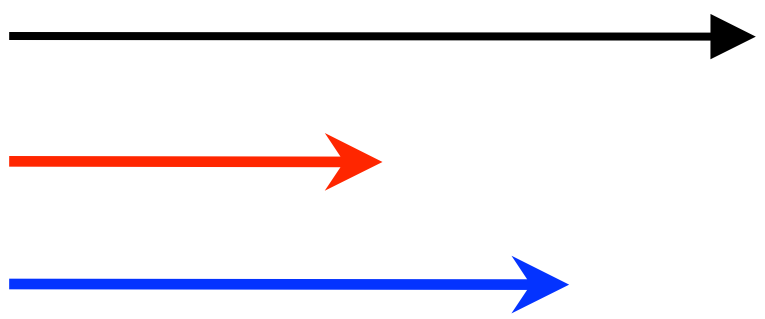
fixed $\vec{k}_\perp \uparrow$



correlation between quark spin and quark OAM

$$\boxed{C^q_z = \int \mathrm{d}x\, \mathrm{d}\vec{k}_\perp\, \mathrm{d}\vec{b}_\perp \left(\vec{b}_\perp \times \vec{k}_\perp\right)\, \rho^q_{UL}(x, \vec{k}_\perp, \vec{b}_\perp)}$$

	u-quark	d-quark
C^q_z	0.23	0.19



Quark spin

u-quark OAM

d-quark OAM

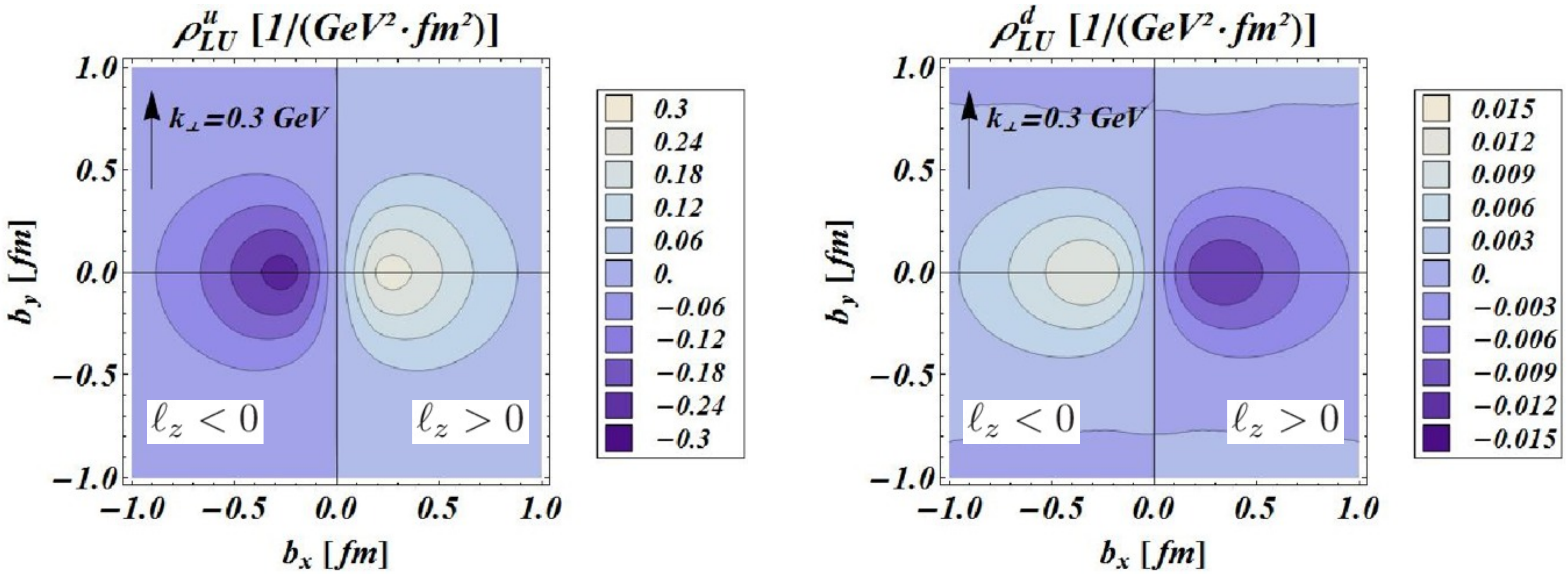
[Lorcé, Pasquini (2011)]
[Lorcé, (2014)]

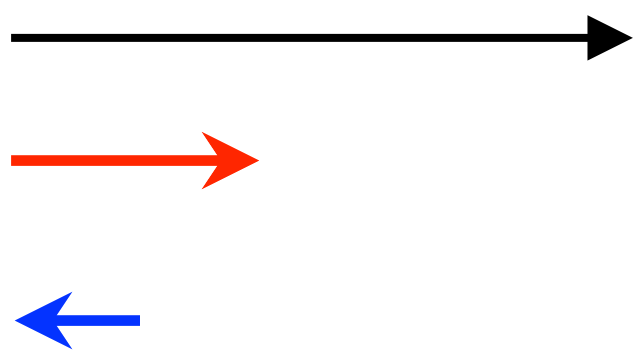
Unpol. quark in Long. pol. Proton

fixed $\vec{k}_\perp \uparrow$

up quark

down quark





Proton spin

u-quark OAM

d-quark OAM

★ projection to GPD and TMD is vanishing

➡ unique information on OAM from Wigner distributions

[Lorcé, Pasquini (2011)]

Quark Orbital Angular Momentum

$$\ell_z^q = \int \mathrm{d}x\, \mathrm{d}^2\vec{k}_\perp \mathrm{d}^2\vec{b}_\perp (\vec{b}_\perp \times \vec{k}_\perp) \rho^q_{LU}(\vec{b}_\perp, \vec{k}_\perp, x) = -\int \mathrm{d}x \mathrm{d}^2\vec{k}_\perp \frac{\vec{k}_\perp^2}{M^2} F^q_{14}(x, 0, \vec{k}_\perp, \vec{0}_\perp)$$

⬇

Wigner distribution
for Unpolarized quark in a Longitudinally pol. nucleon

[Lorcé, BP (11)
Hatta (12)
Ji, Xiong, Yuan (12)
Kanazawa et al., (2014)]

Quark Orbital Angular Momentum

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$$= \int \mathrm{d}^2\vec{b}_\perp \vec{b}_\perp \times \langle \vec{k}^q_\perp \rangle \longrightarrow \langle \vec{k}^q_\perp \rangle = \int \mathrm{d}x\, \mathrm{d}\vec{k}_\perp\, \vec{k}_\perp \rho^q_{LU}(\vec{b}_\perp, \vec{k}_\perp, x)$$

[Lorcé, BP (11)
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[Lorcé, BP (11)
Hatta (12)
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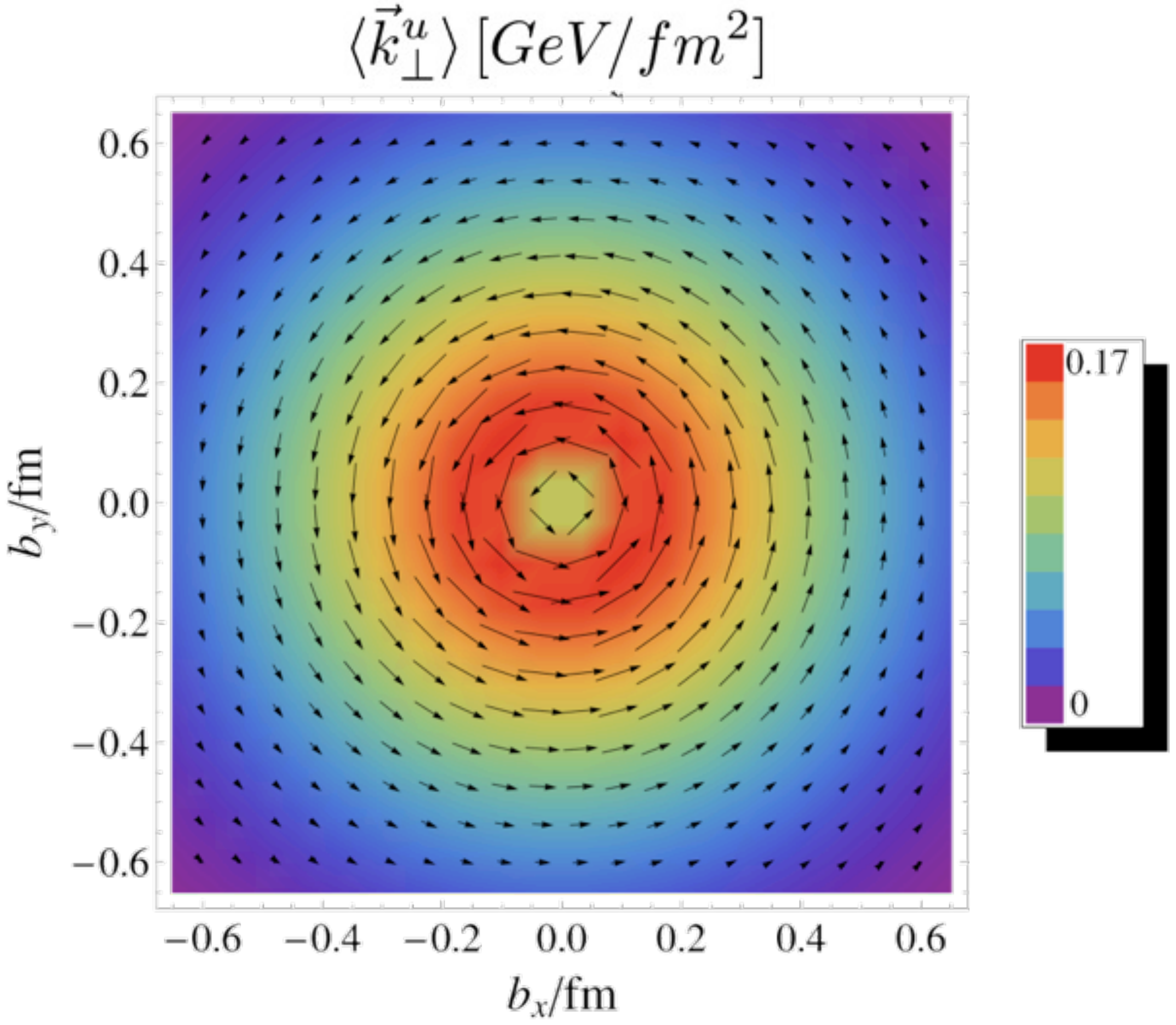
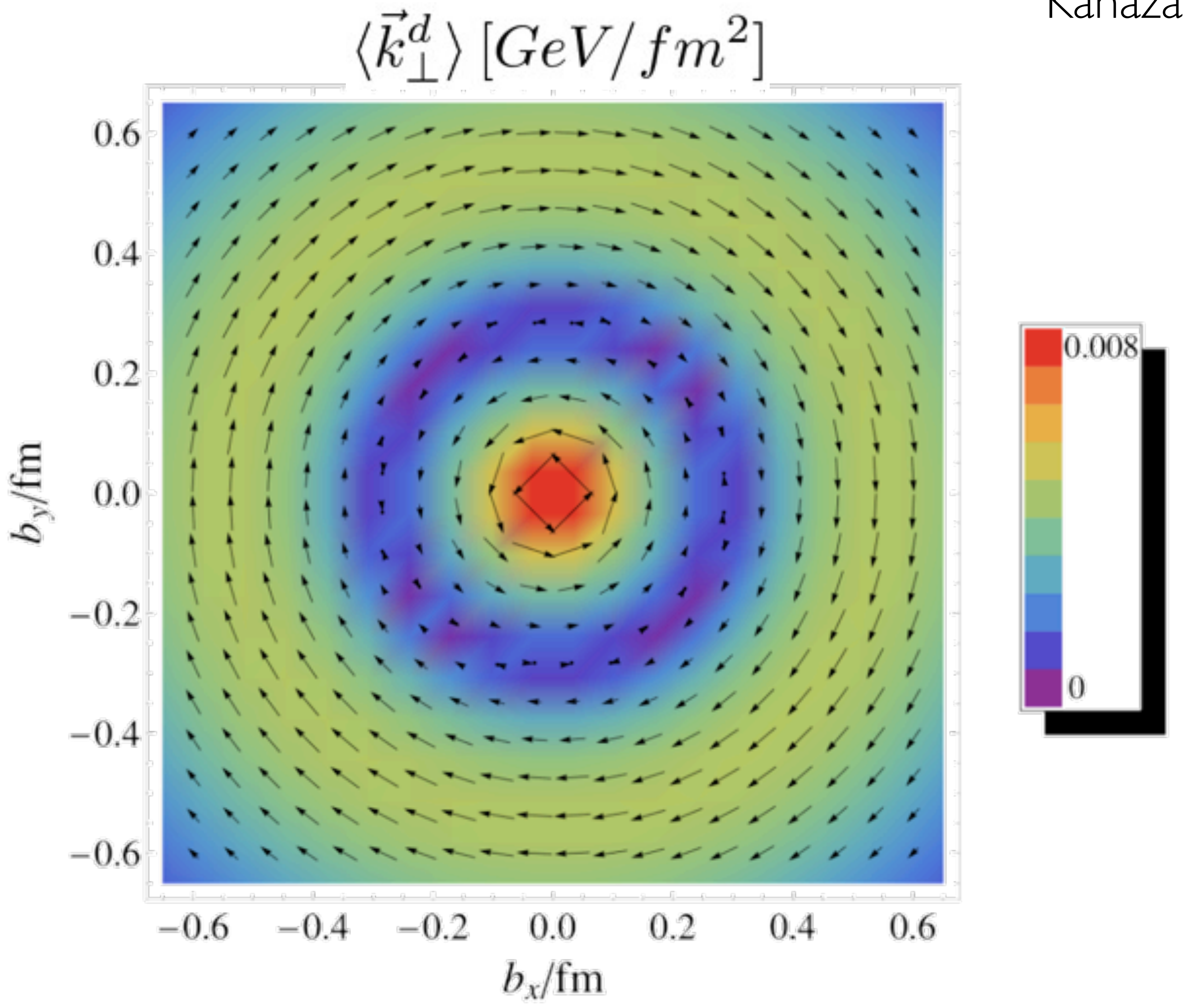
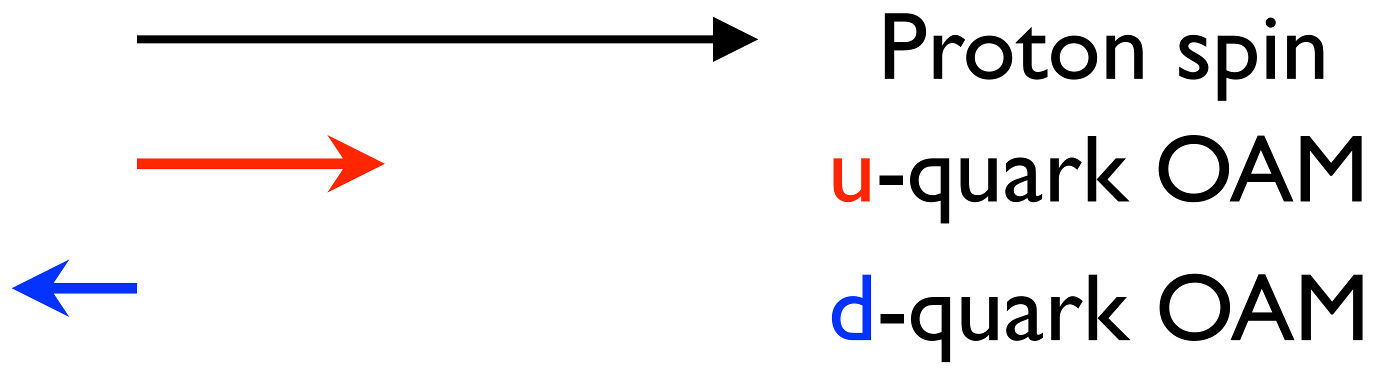





Proton spin

u-quark OAM

d-quark OAM

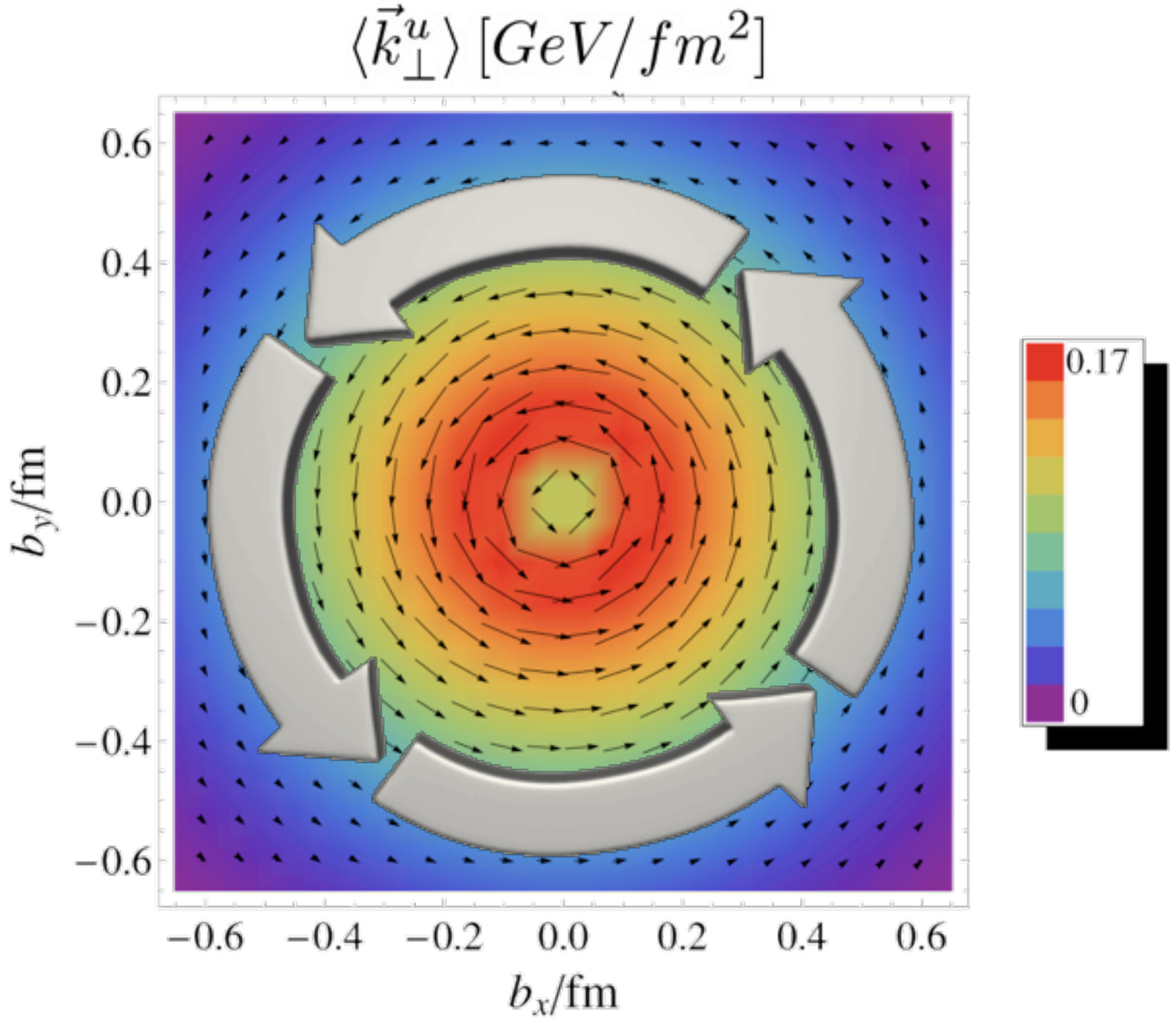
Results in a light-front constituent quark model:
Lorcé, BP, Xiong, Yuan, PRD85 (2012)

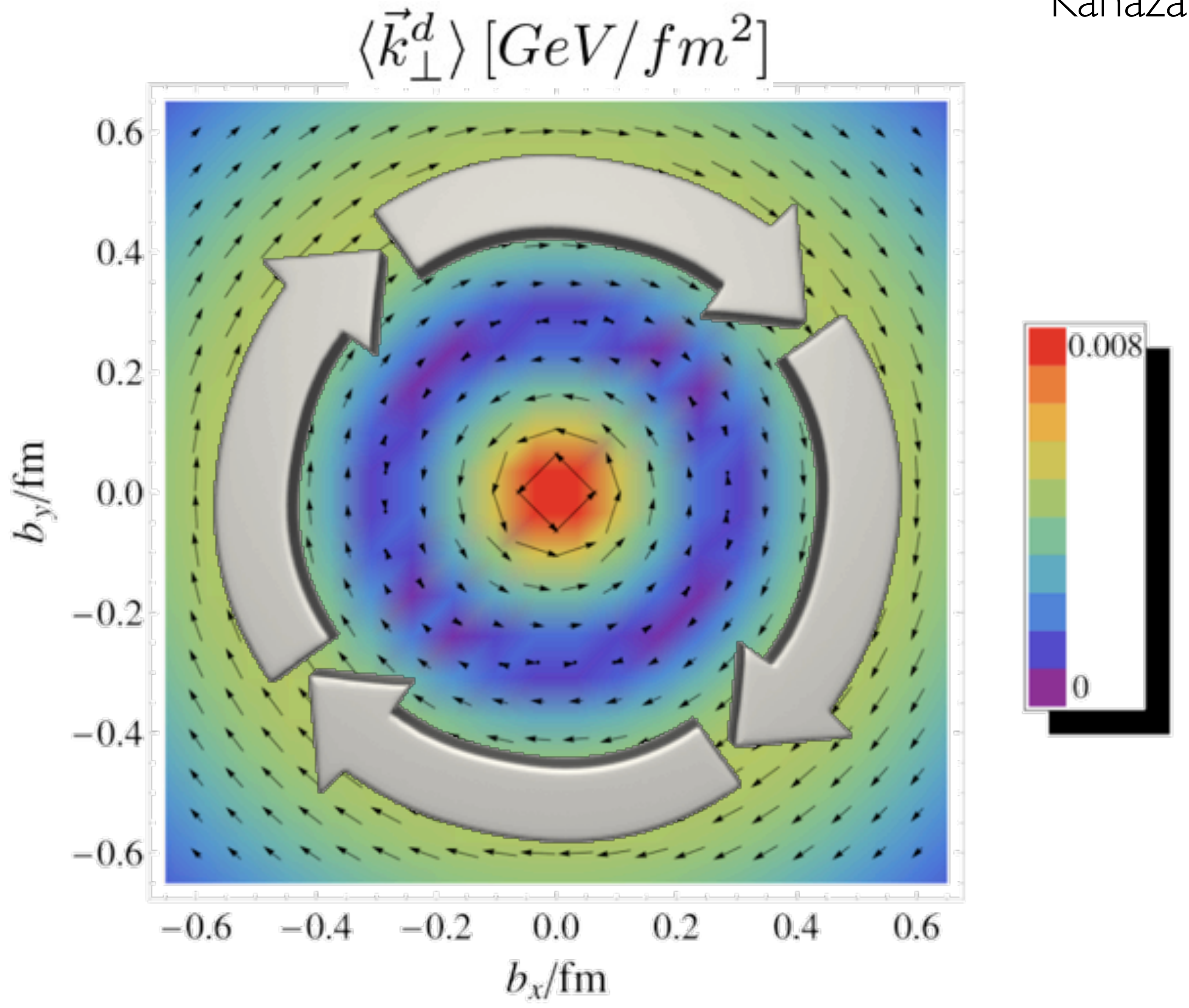
Quark Orbital Angular Momentum

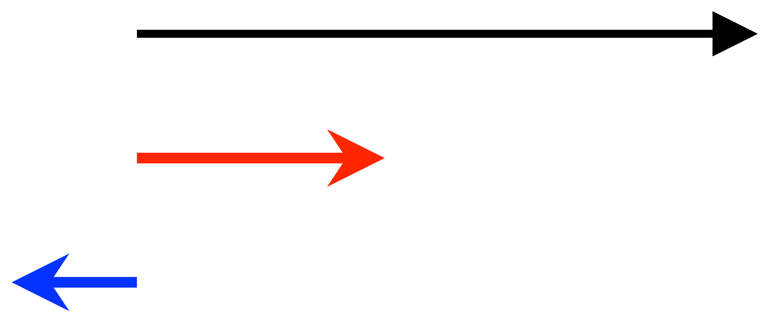
$$\ell_z^q = \int \mathrm{d}x\, \mathrm{d}^2\vec{k}_\perp \mathrm{d}^2\vec{b}_\perp (\vec{b}_\perp \times \vec{k}_\perp)\rho^q_{LU}(\vec{b}_\perp, \vec{k}_\perp, x) = -\int \mathrm{d}x \mathrm{d}^2\vec{k}_\perp \frac{\vec{k}^2_\perp}{M^2} F^q_{14}(x, 0, \vec{k}_\perp, \vec{0}_\perp)$$

$$= \int \mathrm{d}^2\vec{b}_\perp \vec{b}_\perp \times \langle \vec{k}^q_\perp \rangle \longrightarrow \quad \langle \vec{k}^q_\perp \rangle = \int \mathrm{d}x\, \mathrm{d}\vec{k}_\perp\, \vec{k}_\perp \rho^q_{LU}(\vec{b}_\perp, \vec{k}_\perp, x)$$

[Lorcé, BP (11)
Hatta (12)
Ji, Xiong, Yuan (12)
Kanazawa et al., (2014)]







Proton spin

u-quark OAM

d-quark OAM

Results in a light-front constituent quark model:
Lorcé, BP, Xiong, Yuan, PRD85 (2012)

$$\boxed{\ell_z^q = \int \mathrm{d}x\mathrm{d}^2\vec{k}_\perp \mathrm{d}^2\vec{b}_\perp (\vec{b}_\perp \times \vec{k}_\perp)\rho_{LU}(\vec{b}_\perp, \vec{k}_\perp, x)}$$

[Lorcé, BP (2011)]
[Lorcé, BP, Xiong, Yuan(2011)]

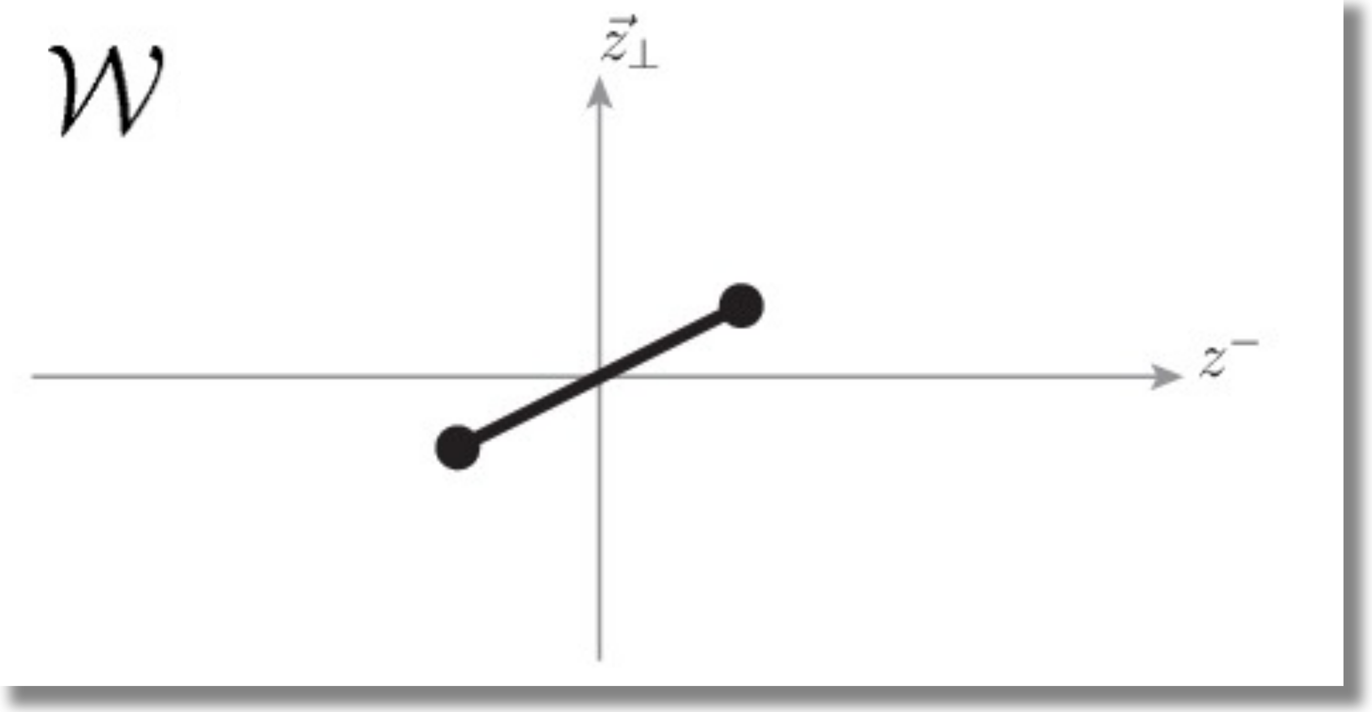
Light-cone gauge $A^+ = 0$
not gauge invariant, but with simple partonic interpretation

Gauge-invariant extension

$$\rho_{LU} \to \rho_{LU}^{\mathcal{W}} \longrightarrow \text{Wilson line}$$

Kinetic

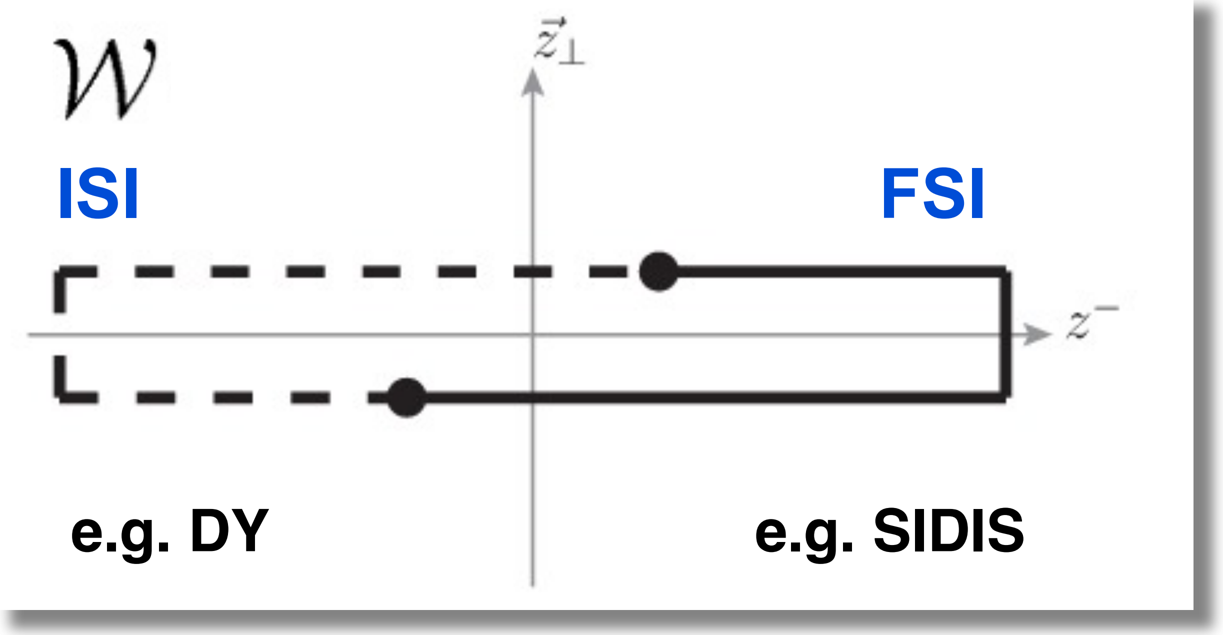
[Ji (1997)]



[Ji, Xiong, Yuan (2012)]
[Burkardt (2012)]

Canonical

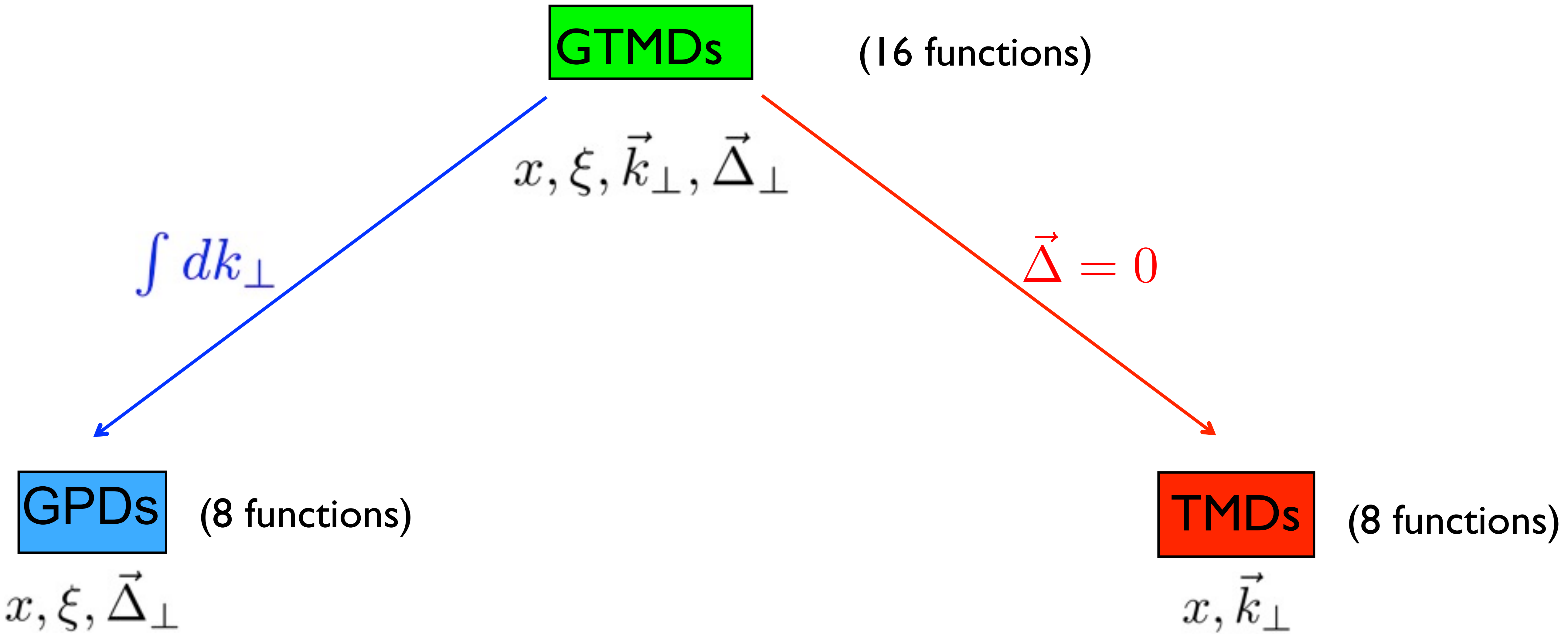
[Jaffe, Manohar (1990)]



$$\ell_z^{\mathrm{DY}} = \ell_z^{\mathrm{SIDIS}}$$

[Hatta (2012)]

⟶ see talk of Matthias Burkardt


GTMDs (16 functions)

$x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp$

$\int dk_\perp$

$\vec{\Delta} = 0$

GPDs (8 functions)

$x, \xi, \vec{\Delta}_\perp$

TMDs (8 functions)

$x, \vec{k}_\perp$

Quark polarization

Nucleon polarization

	U	T	L
U	H	$\mathcal{E}_T$	
T	E	$H_T, \tilde{H}_T$	$\tilde{E}$
L		$\tilde{E}_T$	$\tilde{H}$

Quark polarization

Nucleon polarization

	U	T	L
U	f_1	$h_1^\perp$	
T	$f_{1T}^\perp$	$h_1, \ h_{1T}^\perp$	g_{1T}
L		$h_{1L}^\perp$	g_{1L}

✦ almost all distributions (in red) vanish if there is no quark orbital angular momentum

✦ quark GPDs (at ξ=0) and TMDs given by the same overlap of LFWFs but in different kinematics
⇒ each distribution contains unique information

⇒ no model-independent relations between GPDs and TMDs

Quark OAM from Pretzelosity

"pretzelosity"

model-dependent relation

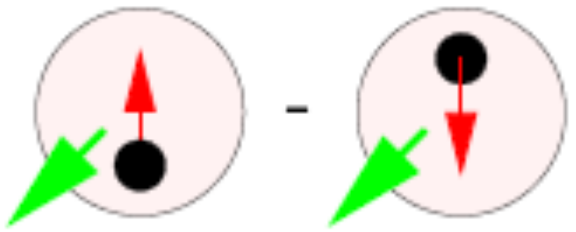
$$\mathcal{L}_z = -\int \mathrm{d}x \mathrm{d}^2\vec{k}_\perp \frac{k_\perp^2}{2M^2}\, h_{1T}^{\perp}(x, k_\perp^2)$$

first derived in LF-diquark model and bag model

[She, Zhu, Ma, 2009; Avakian, Efremov, Schweitzer, Yuan, 2010]

Quark OAM from Pretzelosity

$h_{1T}^{\perp} =$



“pretzelosity”

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first derived in LF-diquark model and bag model

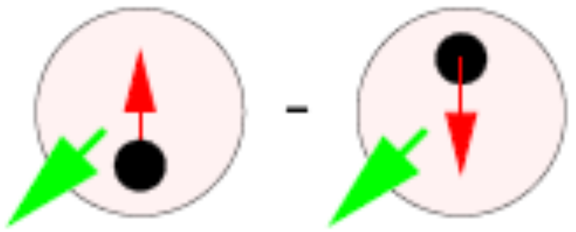
[She, Zhu, Ma, 2009; Avakian, Efremov, Schweitzer, Yuan, 2010]

$\mathcal{L}_z$	$h_{1T}^{\perp}$
chiral even and charge even	chiral odd and charge odd
$\Delta L_z = 0$	$\lvert \Delta L_z \rvert = 2$

no operator identity
relation at level of matrix elements of operators

Quark OAM from Pretzelosity

$h_{1T}^{\perp} =$



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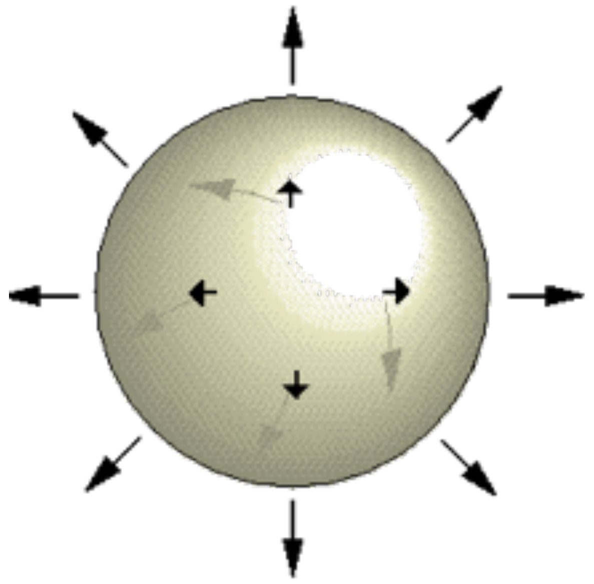
no operator identity
relation at level of matrix elements of operators

$\Downarrow$



valid in all quark models with spherical symmetry in the rest frame

[Lorcé, BP, PLB (2012)]

Quark spin and OAM

GTMDs

Quark spin (from DIS)

$$S_z^q = \frac{1}{2}\int \mathrm{d}x\,\mathrm{d}^2k_\perp\, G_{14}^q(x,0,\vec{k}_\perp,\vec{0}_\perp)$$

polarized PDF
inclusive DIS

$$\ell_z^q = -\int \mathrm{d}x\,\mathrm{d}^2k_\perp\, \frac{\vec{k}_\perp^2}{M^2}\, F_{14}^q(x,0,\vec{k}_\perp,\vec{0}_\perp)$$

[Lorcé, BP (2011)]
[Hatta (2011)]
[Lorce',BP, et al. (2012)]

$$\ell_{iz}^{\text{int}} = \vec{b}_{i\perp} \times \vec{k}_{i\perp}$$

TMDs

Quark spin (from DIS)

$$S_z^q = \frac{1}{2}\int \mathrm{d}x\,\mathrm{d}^2k_\perp\, g_{1L}^q(x,\vec{k}_\perp)$$

polarized PDF
inclusive DIS

$$\mathcal{L}_z^q(x,\vec{k}_\perp) = -\frac{\vec{k}_\perp^2}{2M^2}\, h_{1T}^{\perp q}(x,\vec{k}_\perp^2)$$

[Burkardt (2007)]
[Efremov et al. (2008,2010)]
[She, Zhu, Ma (2009)]
[Avakian et al. (2010)]
[Lorcé, BP (2011)]

- **Model-dependent**
- **Not intrinsic!**

⚠

$$\mathcal{L}_{iz} = \vec{r}_{i\perp} \times \vec{k}_{i\perp}$$

GPDs

Quark spin (from DIS)

$$S_z^q = \frac{1}{2}\int \mathrm{d}x\, \tilde{H}^q(x,0,0)$$

polarized PDF
inclusive DIS

Ji's relation

$$J^q = \tfrac{1}{2}\int \mathrm{d}x\, x\,[H^q(x,0,0) + E^q(x,0,0)]$$

$$L^q = J^q - S_z^q$$

[Ji (1997)]

Twist-3

$$L_z^q = -\int \mathrm{d}x\, x\, G_2^q(x,0,0)$$

Pure twist-3!

[Penttinen et al. (2000)]

for a review: Lorcé, Leader, Phys.Rep. 541 (2014)

LCWF overlap representation

$$\ell_z^{N\beta,q} = -\frac{i}{2}\int [\mathrm{d}x]_N\,[\mathrm{d}^2k_\perp]_N \sum_{i=1}^{N} \delta_{qq_i} \sum_{n=1}^{N} \underline{(\delta_{ni}-x_n)} \left[\Psi^{*\uparrow}_{N\beta}\left(\underline{\vec{k}_i}\times \overleftrightarrow{\nabla}_{k_n}\right)_z \Psi^{\uparrow}_{N\beta}\right]$$

GTMDs
Jaffe-Manohar

$$\mathcal{L}_z^{N\beta,q} = -\frac{i}{2}\int [\mathrm{d}x]_N\,[\mathrm{d}^2k_\perp]_N \sum_{i=1}^{N} \delta_{qq_i} \left[\Psi^{*\uparrow}_{N\beta}\left(\vec{k}_i\times \overleftrightarrow{\nabla}_{k_i}\right)_z \Psi^{\uparrow}_{N\beta}\right]$$

TMD

$$L_z^{N\beta,q} = \frac{1}{2}\int [\mathrm{d}x]_N\,[\mathrm{d}^2k_\perp]_N \sum_{i=1}^{N} \delta_{qq_i} \left\{(x_i-\lambda_i)|\Psi^{\uparrow}_{N\beta}|^2 + M\underline{x_i\sum_{n=1}^{N}(\delta_{ni}-x_n)}\left[\Psi^{*\uparrow}_{N\beta}\,\frac{\overleftrightarrow{\partial}}{\partial k_n^x}\,\Psi^{\downarrow}_{N\beta}\right]\right\}$$

GPDs
Ji sum rule

LCWF overlap representation

$$\ell_z^{N\beta,q} = -\frac{i}{2}\int [\mathrm{d}x]_N\, [\mathrm{d}^2k_\perp]_N \sum_{i=1}^{N} \delta_{qq_i} \sum_{n=1}^{N} \underline{(\delta_{ni}-x_n)} \left[\Psi^{*\uparrow}_{N\beta}\left(\underline{\vec{k}_i}\times \overleftrightarrow{\nabla}_{k_n}\right)_z \Psi^{\uparrow}_{N\beta}\right]$$

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GPDs
Ji sum rule

⟶ sum over all parton contributions

Conservation of transverse momentum:

$$\sum_{i=1}^{N}\vec{k}_{i\perp}\,(\delta_{ni}-x_n) = \vec{k}_{n\perp} - x_n \underbrace{\sum_{i=1}^{N}\vec{k}_{i\perp}}_{0} = \vec{k}_{n\perp}$$

LCWF overlap representation

$$\ell_z^{N\beta,q} = -\frac{i}{2}\int [\mathrm{d}x]_N\,[\mathrm{d}^2k_\perp]_N \sum_{i=1}^{N}\delta_{qq_i}\sum_{n=1}^{N}\underline{(\delta_{ni}-x_n)}\left[\Psi^{*\uparrow}_{N\beta}\left(\underline{\vec{k}_i}\times\overleftrightarrow{\nabla}_{k_n}\right)_z\Psi^{\uparrow}_{N\beta}\right]$$

GTMDs
Jaffe-Manohar

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TMD

$$L_z^{N\beta,q} = \frac{1}{2}\int [\mathrm{d}x]_N\,[\mathrm{d}^2k_\perp]_N \sum_{i=1}^{N}\delta_{qq_i}\left\{(x_i-\lambda_i)|\Psi^{\uparrow}_{N\beta}|^2+\underline{Mx_i\sum_{n=1}^{N}(\delta_{ni}-x_n)}\left[\Psi^{*\uparrow}_{N\beta}\,\frac{\overleftrightarrow{\partial}}{\partial k_n^x}\,\Psi^{\downarrow}_{N\beta}\right]\right\}$$

GPDs
Ji sum rule

→ sum over all parton contributions

Conservation of transverse momentum:

$$\sum_{i=1}^{N}\vec{k}_{i\perp}\,(\delta_{ni}-x_n)=\vec{k}_{n\perp}-x_n\underset{\substack{\downarrow\\ 0}}{\sum_{i=1}^{N}\vec{k}_{i\perp}}=\vec{k}_{n\perp}$$

Conservation of longitudinal momentum

$$\sum_{i=1}^{N}x_i\,(\delta_{ni}-x_n)=x_n\left(1-\underset{\substack{\downarrow\\ 1}}{\sum_{i=1}^{N}x_i}\right)=0$$

LCWF overlap representation

$$\ell_z^{N\beta,q} = -\frac{i}{2}\int [\mathrm{d}x]_N\,[\mathrm{d}^2k_\perp]_N \sum_{i=1}^{N}\delta_{qq_i}\sum_{n=1}^{N}\underline{(\delta_{ni}-x_n)}\left[\Psi^{*\uparrow}_{N\beta}\left(\underline{\vec{k}_i}\times\overleftrightarrow{\nabla}_{k_n}\right)_z\Psi^{\uparrow}_{N\beta}\right]$$

GTMDs Jaffe-Manohar

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TMD

$$L_z^{N\beta,q} = \frac{1}{2}\int [\mathrm{d}x]_N\,[\mathrm{d}^2k_\perp]_N \sum_{i=1}^{N}\delta_{qq_i}\left\{(x_i-\lambda_i)|\Psi^{\uparrow}_{N\beta}|^2 + \underline{Mx_i\sum_{n=1}^{N}(\delta_{ni}-x_n)}\left[\Psi^{*\uparrow}_{N\beta}\,\frac{\overleftrightarrow{\partial}}{\partial k_n^x}\,\Psi^{\downarrow}_{N\beta}\right]\right\}$$

GPDs Ji sum rule

⟶ sum over all parton contributions

Conservation of transverse momentum:

$$\sum_{i=1}^{N}\vec{k}_{i\perp}(\delta_{ni}-x_n) = \vec{k}_{n\perp} - x_n\underset{\substack{\downarrow\\ 0}}{\sum_{i=1}^{N}\vec{k}_{i\perp}} = \vec{k}_{n\perp}$$

Conservation of longitudinal momentum

$$\sum_{i=1}^{N}x_i(\delta_{ni}-x_n) = x_n\left(1-\underset{\substack{\downarrow\\ 1}}{\sum_{i=1}^{N}x_i}\right) = 0$$

LCWFs are eigenstates of total OAM

$$-i\sum_{n=1}^{N}\left(\vec{k}_n\times\vec{\nabla}_{k_n}\right)_z\Psi^{\Lambda}_{\lambda_1\cdots\lambda_N} = l_z\,\Psi^{\Lambda}_{\lambda_1\cdots\lambda_N}$$

$$l_z = \left(\Lambda-\sum_{n=1}^{N}\lambda_n\right)/2$$

For **total** OAM

$$\boxed{\ell_z^N = \mathcal{L}_z^N = L_z^N = \sum_{\lambda_1\cdots\lambda_N} l_z\int[\mathrm{d}x]_N\,[\mathrm{d}^2k_\perp]_N\left|\Psi^{\uparrow}_{\lambda_1\cdots\lambda_N}\right|^2} = \sum_{l_z} l_z\;{}^{l_z}\langle P,\uparrow|P,\uparrow\rangle^{l_z}$$

OAM and origin dependence

OAM from Pretzelosity

$$\mathcal{L}_{iz} = \vec{r}_{i\perp} \times \vec{k}_{i\perp}$$

"naive" OAM

Depends on proton position

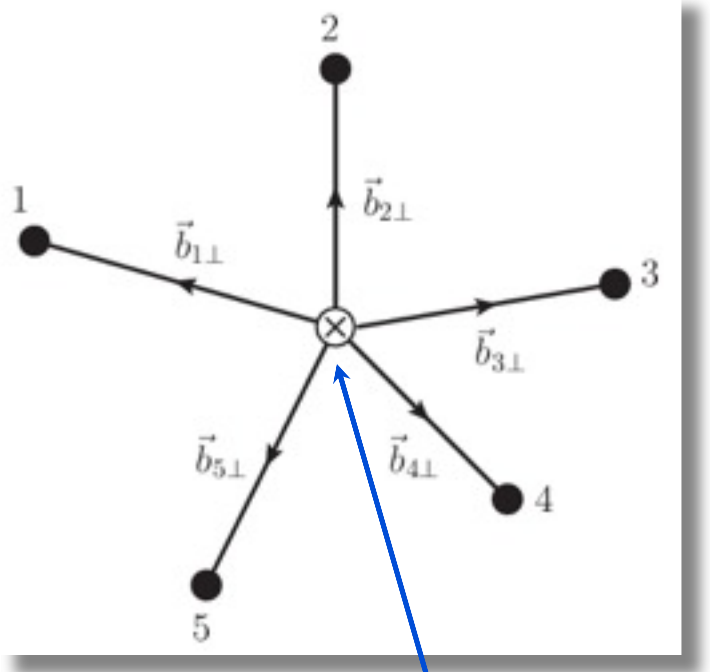
Momentum conservation

$$\boxed{\sum_{i=1}^{N} \vec{k}_{i\perp} = \vec{0}_\perp}$$

OAM from GTMDs

$$\ell_{iz}^{\text{int}} = \vec{b}_{i\perp} \times \vec{k}_{i\perp}$$

intrinsic OAM



Transverse center of momentum

$$\vec{R}_\perp = \sum_{i=1}^{N} x_i\, \vec{r}_{i\perp}$$

equivalence for TOTAL OAM

Model	LCCQM			χQSM		
q	u	d	Total	u	d	Total
ℓ_z^q	0.131	−0.005	0.126	0.073	−0.004	0.069
L_z^q	0.071	0.055	0.126	−0.008	0.077	0.069
$\mathcal{L}_z^q$	0.169	−0.042	0.126	0.093	−0.023	0.069

$$\mathcal{L}_{iz} \neq \ell_{iz}^{\text{int}} \xrightarrow{\text{momentum conservation}} \mathcal{L}_z = l_z^{\text{int}}$$

$$\underbrace{\sum_{i=1}^{N} \vec{b}_{i\perp} \times \vec{k}_{i\perp}}_{\text{Intrinsic}} = \sum_{i=1}^{N} \left(\vec{r}_{i\perp} - \vec{R}_\perp\right) \times \vec{k}_{i\perp} = \underbrace{\sum_{i=1}^{N} \vec{r}_{i\perp} \times \vec{k}_{i\perp}}_{\text{Naive}}$$

OAM and origin dependence

OAM from Pretzelosity

$$\mathcal{L}_{iz} = \vec{r}_{i\perp} \times \vec{k}_{i\perp}$$

"naive" OAM

Depends on proton position

Momentum conservation

$$\boxed{\sum_{i=1}^{N} \vec{k}_{i\perp} = \vec{0}_\perp}$$

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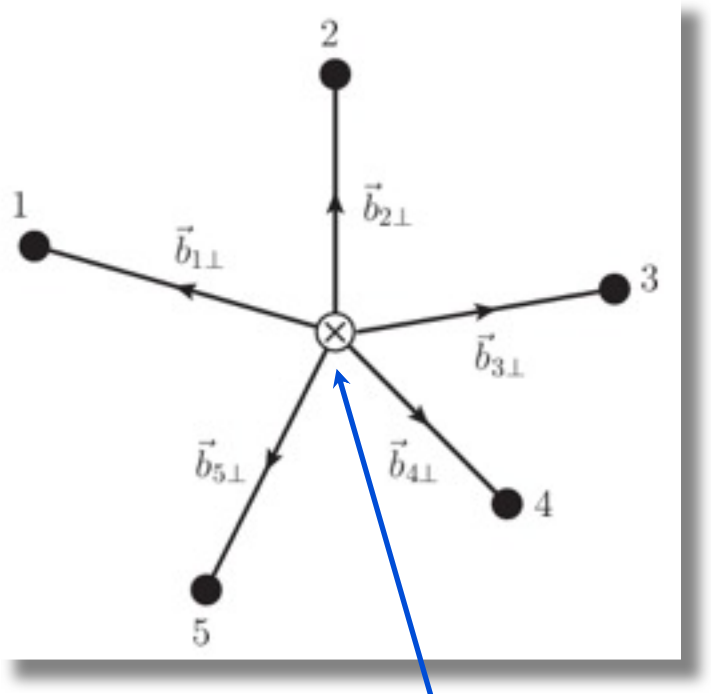
intrinsic OAM



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$$\sum_{i=1}^{N} \vec{b}_{i\perp} \times \vec{k}_{i\perp} = \sum_{i=1}^{N} \left(\vec{r}_{i\perp} - \vec{R}_\perp \right) \times \vec{k}_{i\perp} = \sum_{i=1}^{N} \vec{r}_{i\perp} \times \vec{k}_{i\perp}$$

Intrinsic

Naive

OAM and origin dependence

OAM from Pretzelosity

$$\mathcal{L}_{iz} = \vec{r}_{i\perp} \times \vec{k}_{i\perp}$$

“naive” OAM

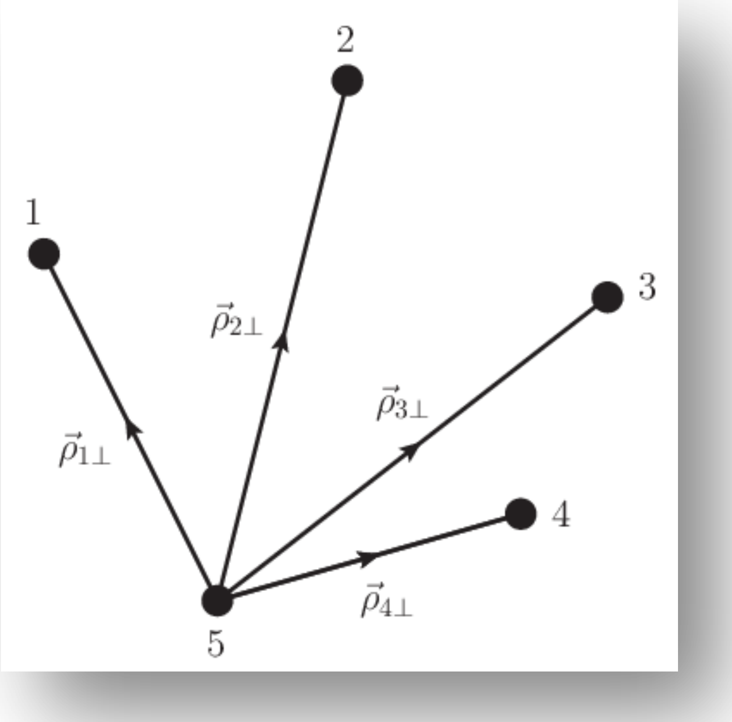
⚠ **Depends on proton position**

Momentum conservation

$$\boxed{\sum_{i=1}^{N} \vec{k}_{i\perp} = \vec{0}_\perp}$$

Relative OAM

$$\ell_{iz}^{\mathrm{rel}} = \vec{\rho}_{i\perp} \times \vec{k}_{i\perp}$$

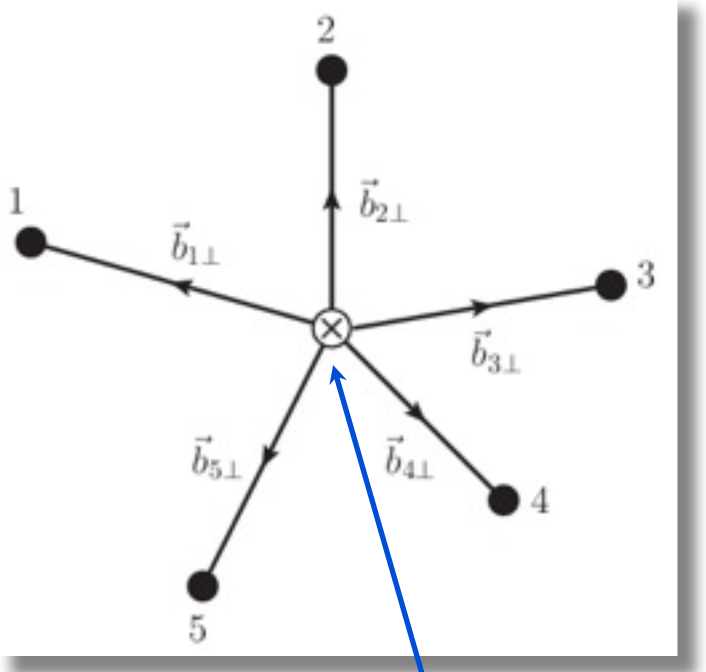


physical interpretation?

OAM from GTMDs

$$\ell_{iz}^{\mathrm{int}} = \vec{b}_{i\perp} \times \vec{k}_{i\perp}$$

intrinsic OAM



Transverse center of momentum

$$\vec{R}_\perp = \sum_{i=1}^{N} x_i\, \vec{r}_{i\perp}$$

OAM and origin dependence

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"naive" OAM

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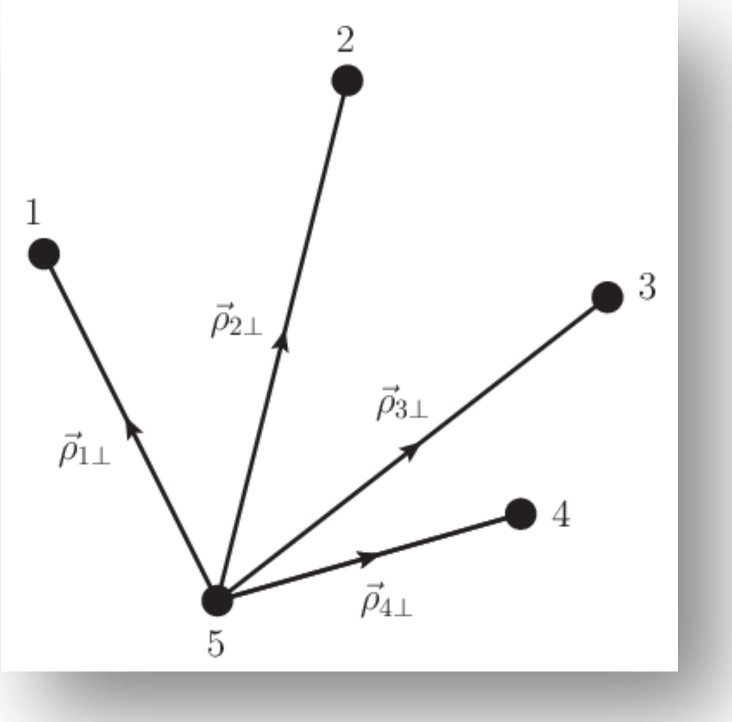

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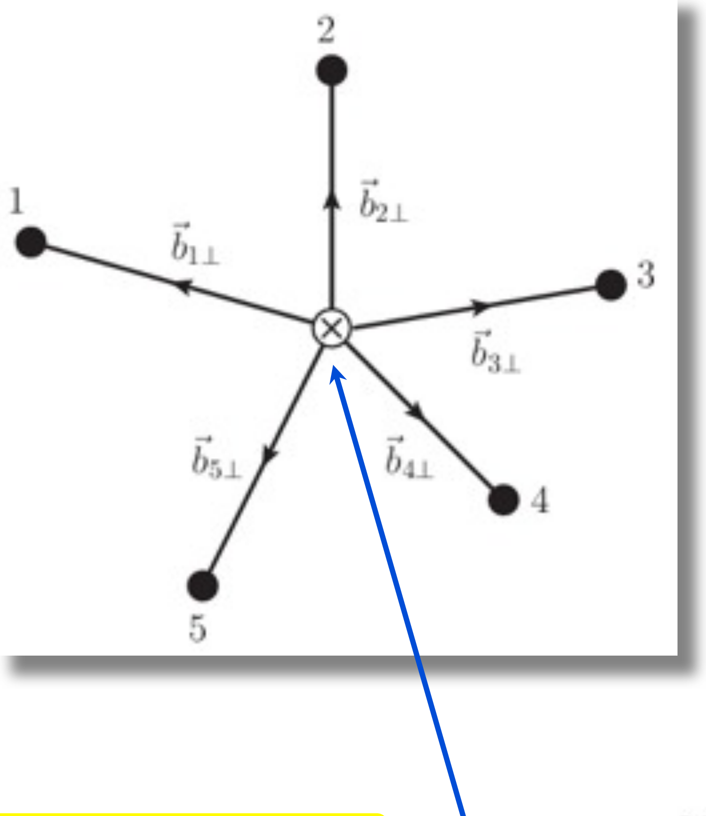


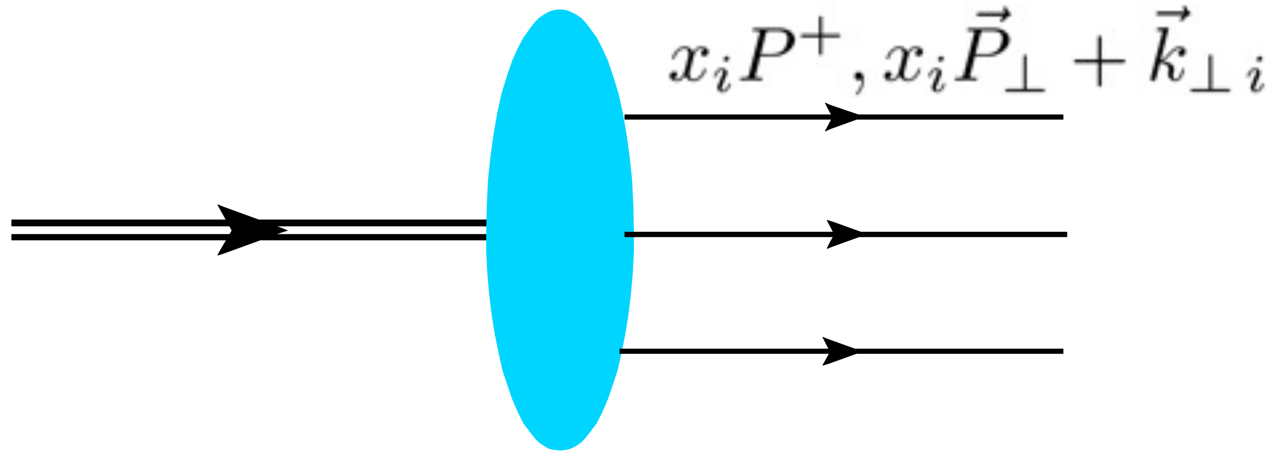
Transverse center of momentum

$$\vec{R}_{\perp} = \sum_{i=1}^{N} x_i \, \vec{r}_{i\perp}$$

$$\mathcal{L}_{iz} \neq \ell_{iz}^{\text{int}} \neq \ell_{iz}^{\text{rel}} \quad \xrightarrow{\text{momentum conservation}} \quad \mathcal{L}_z = l_z^{\text{int}} = l_z^{\text{rel}}$$

$$\underbrace{\sum_{i=1}^{N} \vec{b}_{i\perp} \times \vec{k}_{i\perp}}_{\text{Intrinsic}} = \sum_{i=1}^{N} \left(\vec{r}_{i\perp} - \vec{R}_{\perp}\right) \times \vec{k}_{i\perp} = \underbrace{\sum_{i=1}^{N} \vec{r}_{i\perp} \times \vec{k}_{i\perp}}_{\text{Naive}} = \sum_{i=1}^{N-1} \vec{r}_{i\perp} \times \vec{k}_{i\perp} - \vec{r}_{N\perp} \times \sum_{i=1}^{N-1} \vec{k}_{i\perp} = \underbrace{\sum_{1=1}^{N-1} \vec{\rho}_{i\perp} \times \vec{k}_{i\perp}}_{\text{Relative}}$$

Quark OAM: Partial-Wave Decomposition



$$| P, \Lambda\rangle = \int \mathrm{d}[1]\mathrm{d}[2]\mathrm{d}[3]\ \Psi^{\Lambda}_{\lambda_1\lambda_2\lambda_3}(x_i, \vec{k}_{\perp,i})\ \frac{\varepsilon^{ijk}}{\sqrt{6}}\, u^\dagger_{i\lambda_1}(1)u^\dagger_{j\lambda_2}(2)d^\dagger_{k\lambda_3}(3)|\, 0\rangle$$

$\downarrow$

LCWF: eigenstate of OAM
(gauge A^+=0 ⟶ Jaffe-Manohar)

J^q_z	$\rightarrow$	$(\uparrow\uparrow\uparrow)_{LF} = \frac{3}{2}$	$(\uparrow\uparrow\downarrow)_{LF} = \frac{1}{2}$	$(\uparrow\downarrow\downarrow)_{LF} = -\frac{1}{2}$	$(\downarrow\downarrow\downarrow)_{LF} = -\frac{3}{2}$
$L^q_z = \frac{1}{2} - J^q_z$	$\rightarrow$	$L^q_z = -1$	$L^q_z = 0$	$L^q_z = 1$	$L^q_z = 2$

${}^{L_z}\langle P, \uparrow | P, \uparrow\rangle^{L_z}$:probability to find the proton in a state with eigenvalue of OAM L_z

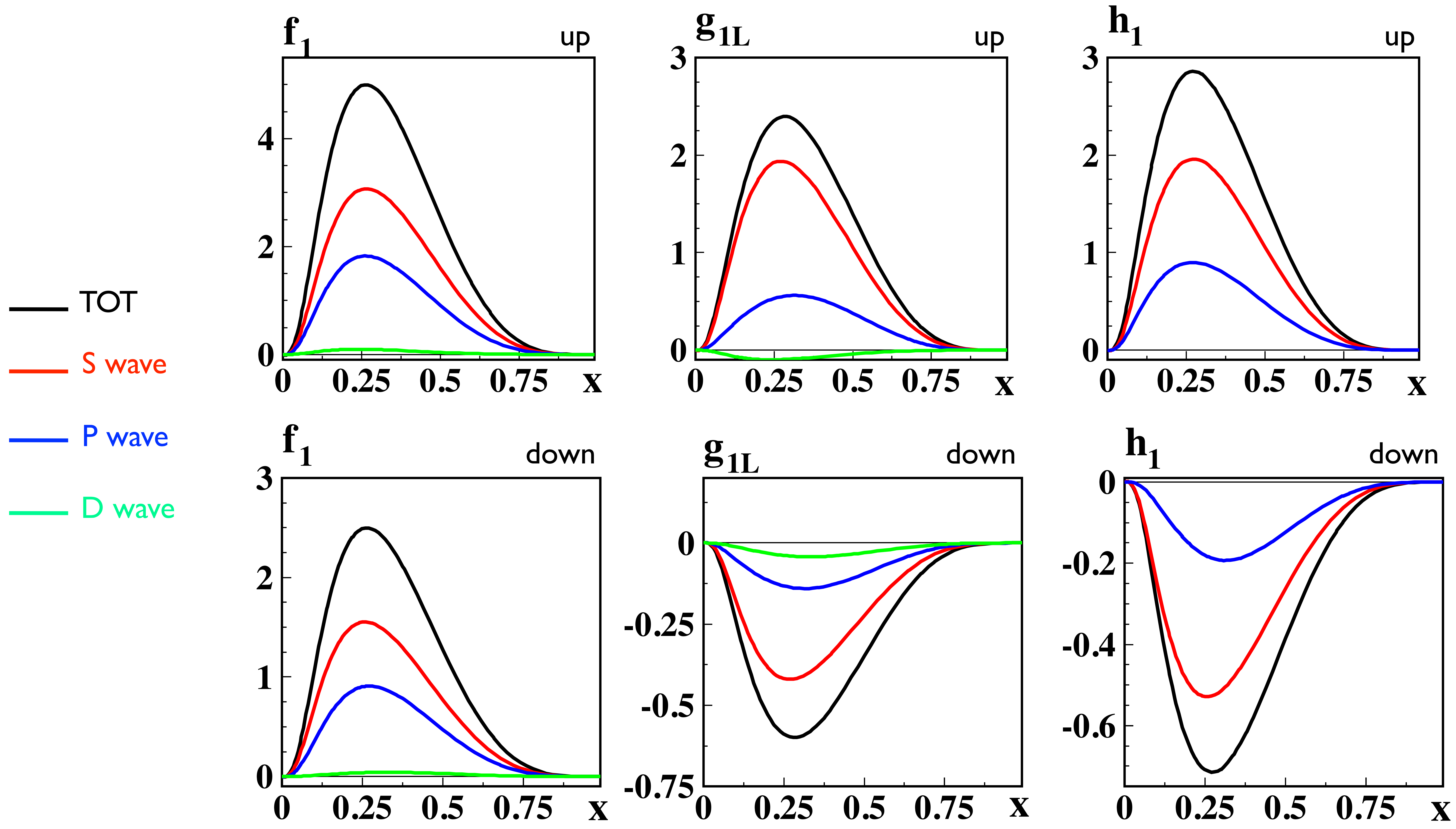
$\Downarrow$

$$\ell_z = \sum_{L_z} L_z\ {}^{L_z}\langle P, \uparrow | P, \uparrow\rangle^{L_z}$$

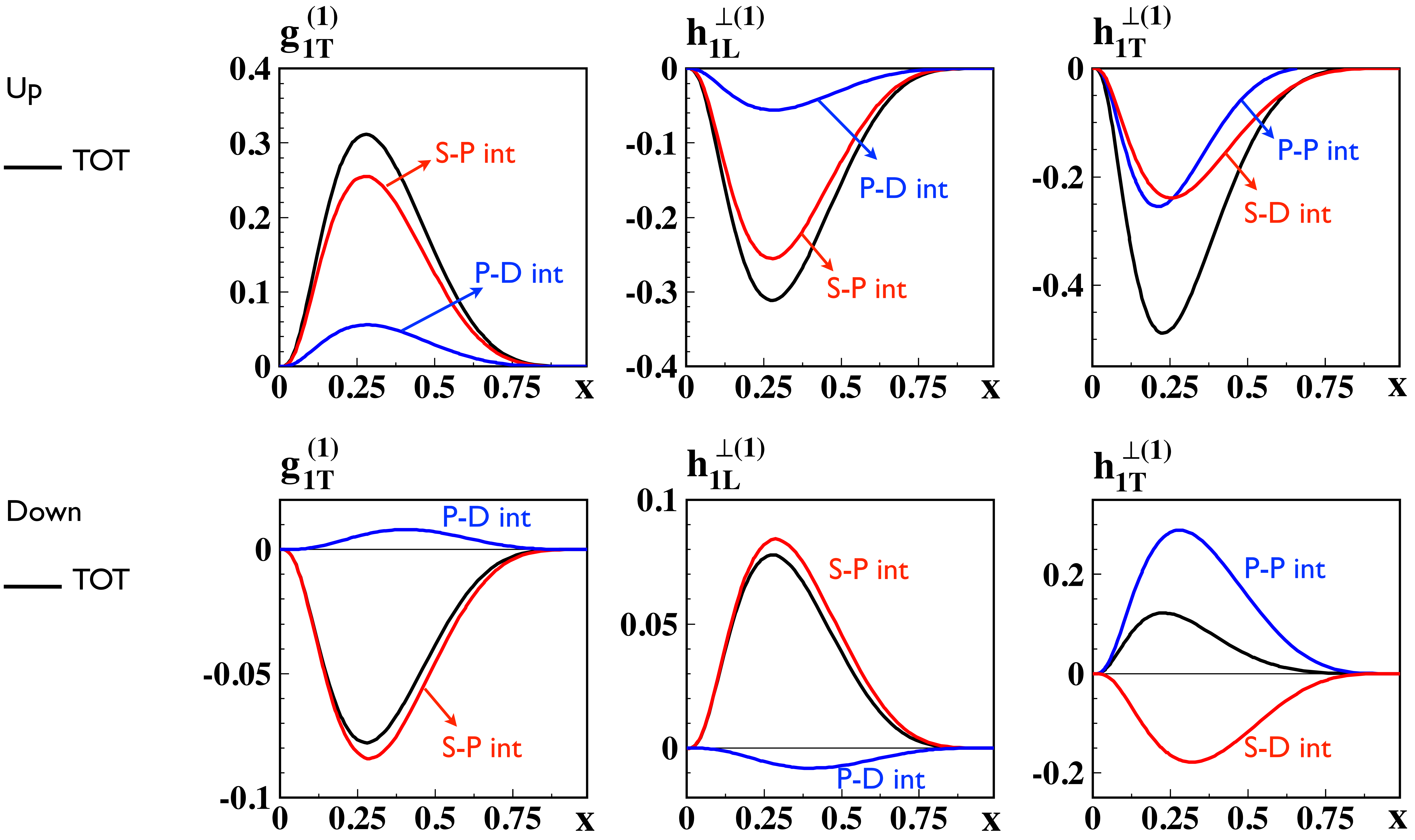
$\downarrow$

squared of
partial-wave amplitude

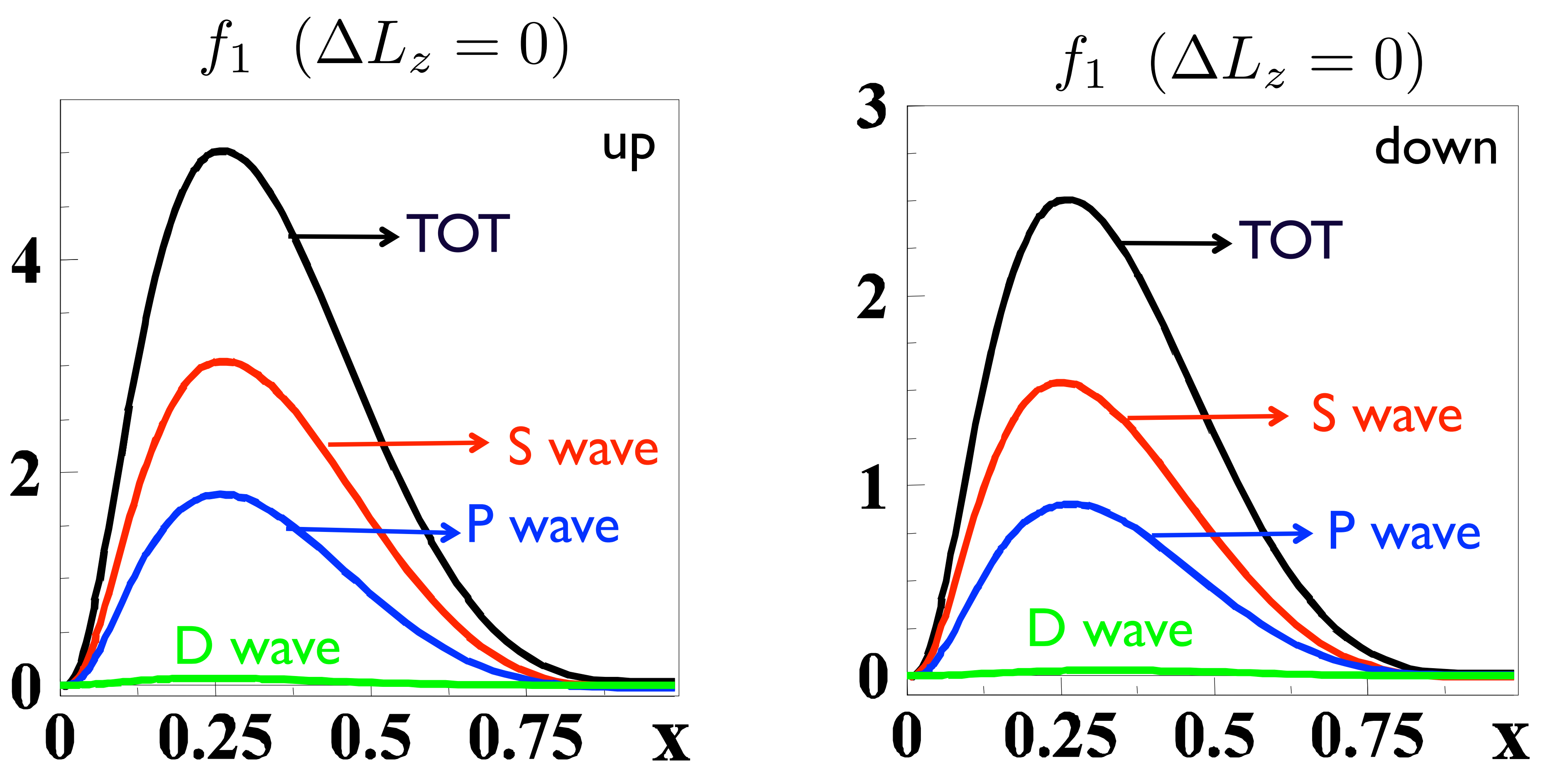
OAM content of TMDs

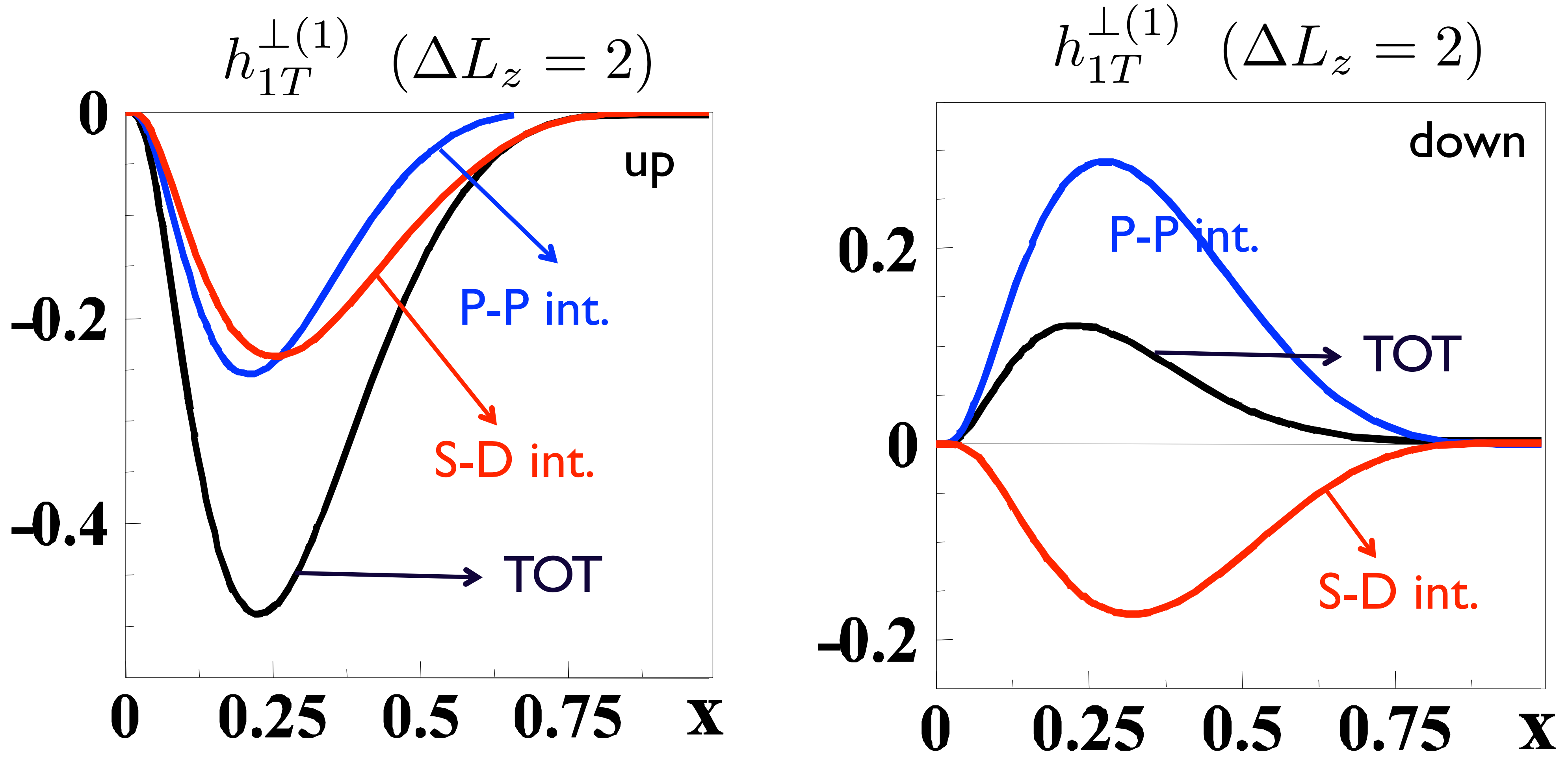


OAM content of TMDs



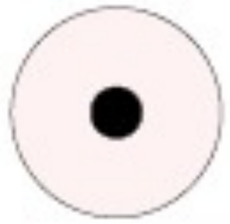
✦ Orbital angular momentum content of TMDs (light-front constituent quark model)



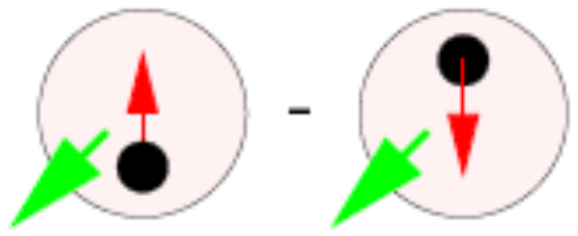


“pretzelosity”

f_1 =



$h_{1T}^{\perp}$ =



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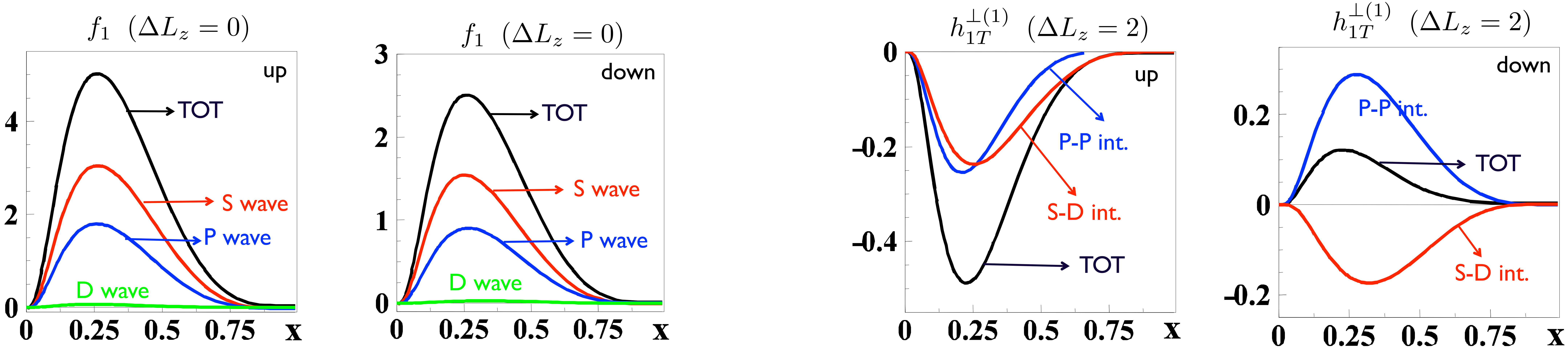
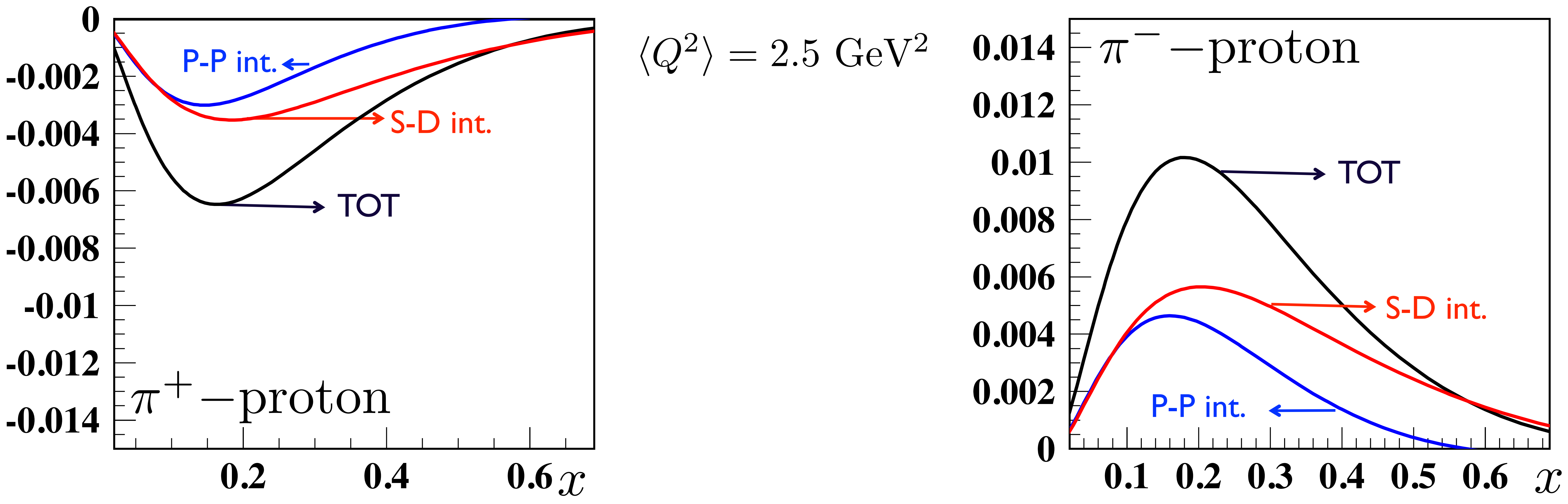


✦ Effects on SIDIS observables

$$A_{UT}^{\sin(3\phi-\phi_S)} \sim \frac{h_{1T}^{\perp} \otimes H_1}{f_1 \otimes D_1}$$



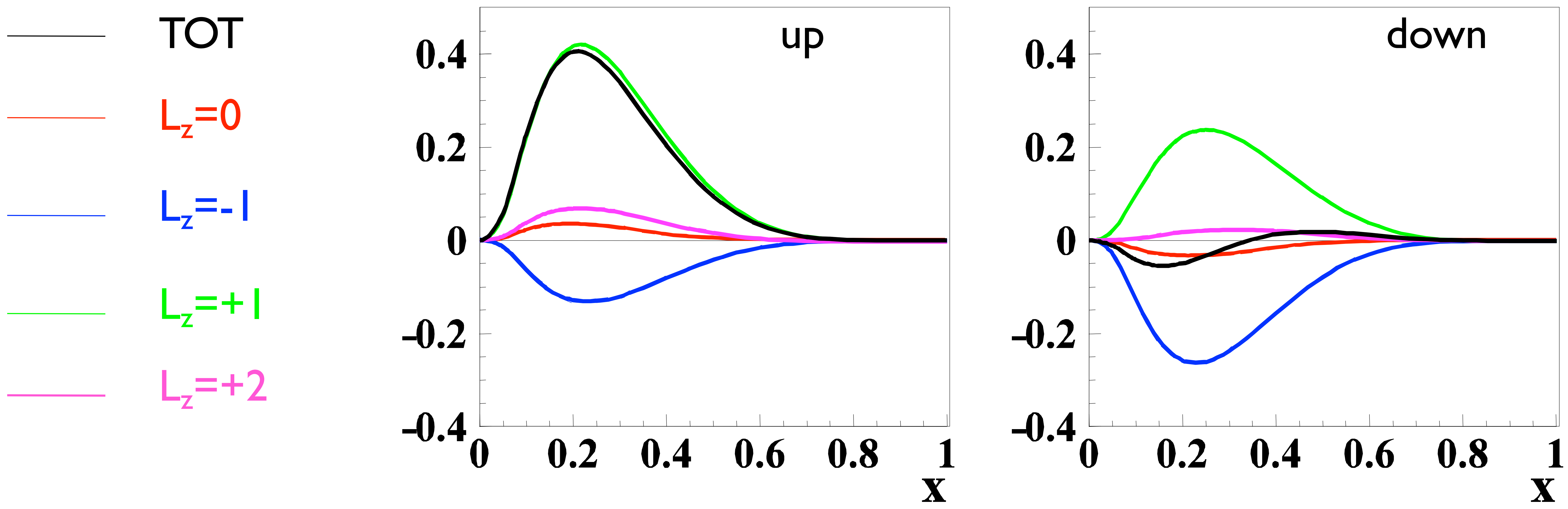
Boffi, Efremov, BP, Schweitzer, PRD79(2009)

Quark OAM: Partial-Wave Decomposition

$$\ell_z = \sum_{L_z} L_z \; {}^{L_z}\langle P,\uparrow|P,\uparrow\rangle^{L_z}$$

OAM	L_z=0	L_z=-1	L_z=+1	L_z=+2	TOT
UP	0.013	-0.046	0.139	0.025	**0.131**
DOWN	-0.013	-0.090	0.087	0.011	**-0.005**
UP+DOWN	0	-0.136	0.226	0.036	**0.126**
<P" |P">	0.62	0.136	0.226	0.018	**1**

distribution in x of OAM



[Lorce,B.P., Xiong,Yuan, 2012]

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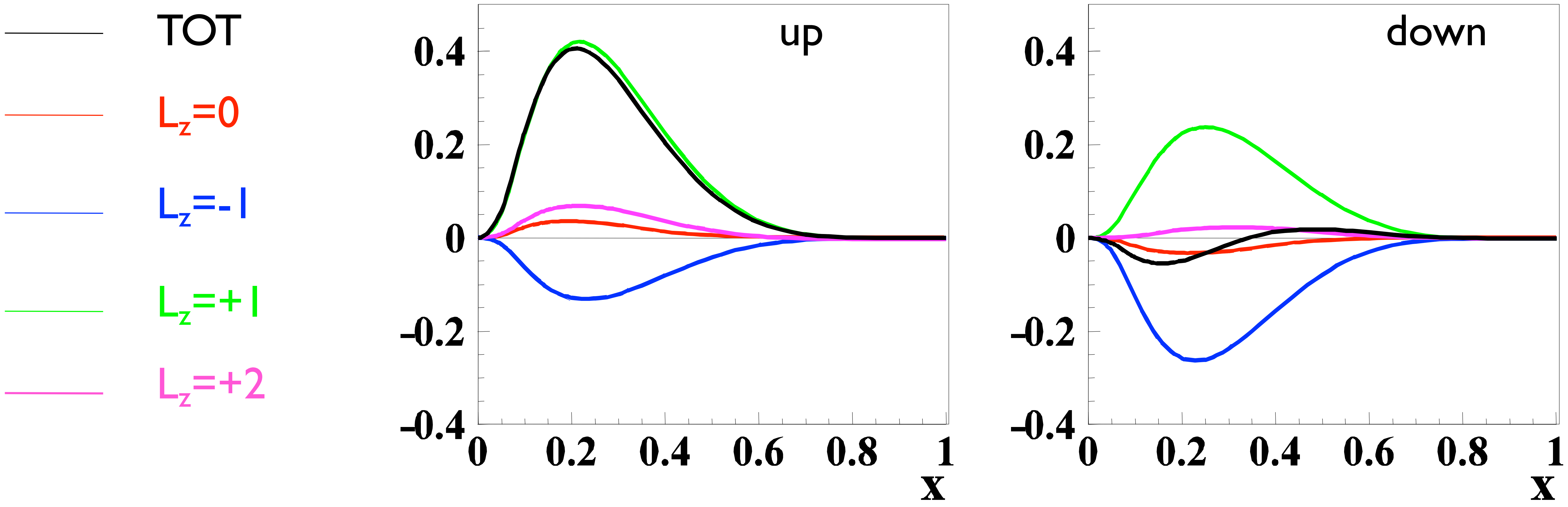

Open question: scale dependence of the different partial-wave contribution

[Lorce,B.P., Xiong,Yuan, 2012]

Summary

- GTMDs and Wigner Distributions
  - the most complete information on partonic structure of the nucleon
  - not yet directly related to physical observables, but no general principle forbids observability of GTMDs
- Results for Wigner distributions in the transverse plane
  - non-trivial correlations between $\vec{b}_\perp$ and $\vec{k}_\perp$ due to orbital angular momentum
- Orbital Angular Momentum from phase-space average with Wigner distributions
- GPDs and TMDs probe the same overlap of 3-quark LCWF in different kinematics
  - no model-independent relations between GPDs and TMDs
  - give complementary information useful to reconstruct the nucleon wf
- No direct connection between TMDs and OAM ➡ need to use model-inspired connections
  - use LCWF (eigenstate of quark OAM) to quantify amount of OAM in different observables
  - model relation between pretzelosity and OAM