

Wigner Distributions and Orbital Angular Momentum of Quarks

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- Wigner distribution for the quarks
- Reduced wigner distributions in position and momentum space
- Model calculations for a dressed quark
- Quark orbital angular momentum
- Ongoing work on gluon Wigner distributions
- Summary and conclusions

Spin and Orbital Angular Momentum of Quarks and Gluons in the Nucleon, ECT Trento,
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Wigner distributions for quarks in the nucleon

- In quantum mechanics position and momentum operators do not commute and they cannot be measured simultaneously : one cannot define a joint position and momentum space distribution of quarks inside the nucleon
- Wigner distributions : not positive definite (no probabilistic interpretation); reduces to classical phase space distribution for $\hbar \rightarrow 0$
- For a 1-D quantum system with wave function $\psi(x)$, Wigner distribution can be defined as

$$W(x, p) = \int dy e^{ip \cdot y} \psi^*(x - y/2) \psi(x + y/2)$$

- Matrix elements of Wigner operator for a nucleon state can be interpreted as distributions of the partons in 6-D space (3 position and 3 momentum)

X. Ji, PRL (2003); Belitsky, Ji, Yuan, PRD (2004)

Wigner Distributions (contd.)

- 5-D Wigner distributions (by integrating one position variable) can be studied in infinite momentum frame or light-front formalism

Lorce, Pasquini (2011)

- Advantage : boost invariant definition of Wigner distributions
- Integrating over k_{\perp} Wigner distributions reduce to the Fourier transform of GPDs
- Integrating over transverse position b_{\perp} they reduce to TMD correlators
- Wigner distributions are related to GPCFs and GTMDs
- Contains informations on both GPDs and TMDs
- Are related to the orbital angular momentum carried by quarks in the nucleon
- Model studies : constituent quark model and chiral quark soliton model in the above reference
- We calculate the reduced Wigner distributions for a quark dressed with a gluon in light-front Hamiltonian perturbation theory : relativistic composite spin 1/2 state

Wigner Distributions : Definitions

Wigner distributions for the quarks are defined as

$$\rho^{[\Gamma]}(\vec{b}_\perp, \vec{k}_\perp, x, \sigma) = \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot b_\perp} W^{[\Gamma]}(\Delta_\perp, \vec{k}_\perp, x, \sigma);$$

Δ_\perp is momentum transfer of dressed quark in transverse direction and \vec{b}_\perp is impact parameter conjugate to Δ_\perp

$W^{[\Gamma]}$ is the quark-quark correlator given by

$$W^{[\Gamma]}(\vec{\Delta}_\perp, \vec{k}_\perp, x, \sigma) = \frac{1}{2} \int \frac{dz^- d^2 z_\perp}{(2\pi)^3} e^{i(xp^+ z^- / 2 - k_\perp \cdot z_\perp)} \left\langle p^+, \frac{\Delta_\perp}{2}, \sigma \left| \bar{\psi}\left(-\frac{z}{2}\right) \Omega \Gamma \psi\left(\frac{z}{2}\right) \right| p^+, -\frac{\Delta_\perp}{2}, \sigma \right\rangle \Big|_{z^+=0}.$$

Ω is the gauge link needed for color gauge invariance : we use light-front gauge and take the gauge link to be unity

Γ represents the Dirac matrix defining the types of quark densities

Wigner Distributions : Model Calculations

We calculate the above Wigner distributions in a quark state dressed with a gluon

The state can be expanded in Fock space in terms of multi-parton light-front wave functions (LFWFs)

$$\left| p^+, p_\perp, \sigma \right\rangle = \Phi^\sigma(p) b_\sigma^\dagger(p) |0\rangle + \sum_{\sigma_1 \sigma_2} \int [dp_1] \int [dp_2] \sqrt{16\pi^3 p^+} \delta^3(p - p_1 - p_2) \Phi_{\sigma_1 \sigma_2}^\sigma(p; p_1, p_2) b_{\sigma_1}^\dagger(p_1) a_{\sigma_2}^\dagger(p_2) |0\rangle;$$

Used light-front gauge, took gauge link as unity and used two-component formalism introduced in

Zhang, Harindranath, PRD (1993)

$\Phi^\sigma(p)$ gives the normalization of the wave function, $\Phi_{\sigma_1 \sigma_2}^\sigma(p; p_1, p_2)$ is the two-particle light-front wave function (LFWF), related to the boost invariant wave function

single particle sector contributes through the normalization of the state, which is important to get the complete contribution at $x = 1$

Wigner Distributions : Model Calculations

Two-particle LFWFs can be calculated

$$\Psi_{\sigma_1 \sigma_2}^{\sigma a}(x, q_{\perp}) = \frac{1}{\left[m^2 - \frac{m^2 + (q_{\perp})^2}{x} - \frac{(q_{\perp})^2}{1-x} \right]} \frac{g}{\sqrt{2}(2\pi)^3} T^a \chi_{\sigma_1}^{\dagger} \frac{1}{\sqrt{1-x}} \left[-2 \frac{q_{\perp}}{1-x} - \frac{(\sigma_{\perp} \cdot q_{\perp}) \sigma_{\perp}}{x} + \frac{im \sigma_{\perp} (1-x)}{x} \right] \chi_{\sigma}(\epsilon_{\perp \sigma_2})^*.$$

χ is the two-component spinor

The state above mimicks the bound state of a two particle system. Note that for a bound state, the bound state mass M should be less than the sum of the masses of the constituents

Using this, we can calculate the Wigner distributions for given helicity of the initial and final target state in terms of overlaps of LFWFs

We use the symbol $\rho_{\lambda \lambda'}$ for Wigner distributions, where $\lambda(\lambda')$ is longitudinal polarization of target state(quark)

$+e_z$ and $-e_z$ correspond to helicity up and down of the target state, respectively

Wigner Distributions : Definitions

Wigner distribution of unpolarized quarks in unpolarized target state is given by

$$\rho_{UU}(\vec{b}_\perp, \vec{k}_\perp, x) = \frac{1}{2} \left[\rho^{[\gamma^+]}(\vec{b}_\perp, \vec{k}_\perp, x, +\vec{e}_z) + \rho^{[\gamma^+]}(\vec{b}_\perp, \vec{k}_\perp, x, -\vec{e}_z) \right]$$

Distortion due to longitudinal polarization of the target state :

$$\rho_{LU}(\vec{b}_\perp, \vec{k}_\perp, x) = \frac{1}{2} \left[\rho^{[\gamma^+]}(\vec{b}_\perp, \vec{k}_\perp, x, +\vec{e}_z) - \rho^{[\gamma^+]}(\vec{b}_\perp, \vec{k}_\perp, x, -\vec{e}_z) \right]$$

Distortion due to the longitudinal polarization of quarks :

$$\rho_{UL}(\vec{b}_\perp, \vec{k}_\perp, x) = \frac{1}{2} \left[\rho^{[\gamma^+ \gamma_5]}(\vec{b}_\perp, \vec{k}_\perp, x, +\vec{e}_z) + \rho^{[\gamma^+ \gamma_5]}(\vec{b}_\perp, \vec{k}_\perp, x, -\vec{e}_z) \right]$$

Distortion due to the correlation between the longitudinal polarized target state and quarks :

$$\rho_{LL}(\vec{b}_\perp, \vec{k}_\perp, x) = \frac{1}{2} \left[\rho^{[\gamma^+ \gamma_5]}(\vec{b}_\perp, \vec{k}_\perp, x, +\vec{e}_z) - \rho^{[\gamma^+ \gamma_5]}(\vec{b}_\perp, \vec{k}_\perp, x, -\vec{e}_z) \right]$$

Connection with GTMDs

- All the Wigner distributions introduced are related to the Fourier transforms of GTMDs (Defs to be shown later)
- ρ_{UU} is related to F_{11} , ρ_{UL} is related to G_{11} , ρ_{LU} to F_{14} and ρ_{LL} to G_{14}
- On integration over k_{\perp} or b_{\perp} , ρ_{UL} and ρ_{LU} give zero : there is no GPD or TMD associated with these two. They give new information not contained in the GPDs and TMDs
- ρ_{UU} can be considered as the mother distribution for the GPD H and TMD f_1
- ρ_{LL} is the mother distribution for the GPD \tilde{H} and TMD g_{1L}
- k_{\perp} and b_{\perp} are not Fourier conjugates. k_{\perp} is the Fourier conjugate of z_{\perp} and b_{\perp} is the Fourier conjugate of Δ_{\perp} : relative average transverse momentum and relative average transverse position of the quark

Wigner Distributions : Model Calculations

In this work we restrict ourselves to the kinematic region $x < 1$, and in this case the contribution from $\Phi^\sigma(p)$ can be taken to be 1

Wigner distribution from the two particle sector in the Fock space expansion in our model is given by

$$W^{[\gamma^+] }(\Delta_\perp, k_\perp, x, \sigma) = \frac{1}{(2\pi)^3} \sum_{\sigma_1, \sigma_2} \Psi_{\sigma_1 \sigma_2}^{* \sigma a}(x, q'_\perp) \Psi_{\sigma_1 \sigma_2}^{\sigma a}(x, q_\perp),$$

$$W^{[\gamma^+ \gamma_5]}(\Delta_\perp, k_\perp, x, \sigma) = \frac{1}{(2\pi)^3} \sum_{\sigma_1, \sigma_2, \lambda_1} \Psi_{\lambda_1 \sigma_2}^{* \sigma a}(x, q'_\perp) \chi_{\lambda_1}^\dagger \sigma_3 \chi_{\sigma_1} \Psi_{\sigma_1 \sigma_2}^{\sigma a}(x, q_\perp);$$

where the Jacobi relation for the transverse momenta in the symmetric frame is given by

$$q'_\perp = k_\perp - \frac{\Delta_\perp}{2}(1-x) \text{ and } q_\perp = k_\perp + \frac{\Delta_\perp}{2}(1-x).$$

Wigner Distributions : Model Calculations

In our model $\rho_{LU} = \rho_{UL}$

$$\rho_{UU}^{[\gamma^+]}(b_{\perp}, k_{\perp}, x) = N \int d\Delta_x \int d\Delta_y \frac{\cos(\Delta_{\perp} \cdot b_{\perp})}{D(q_{\perp})D(q'_{\perp})} \left[I_1 + \frac{4m^2(1-x)}{x^2} \right];$$

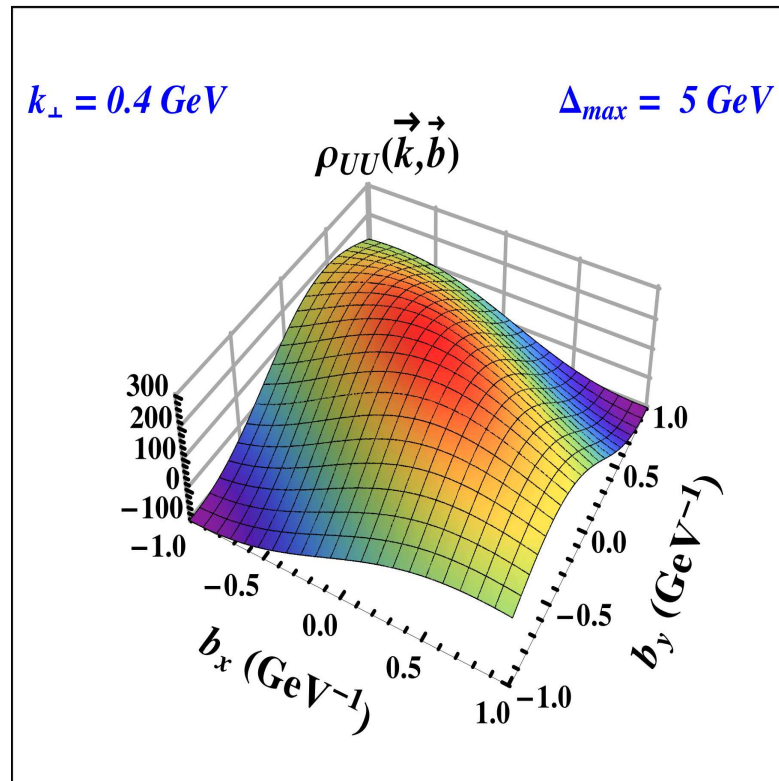
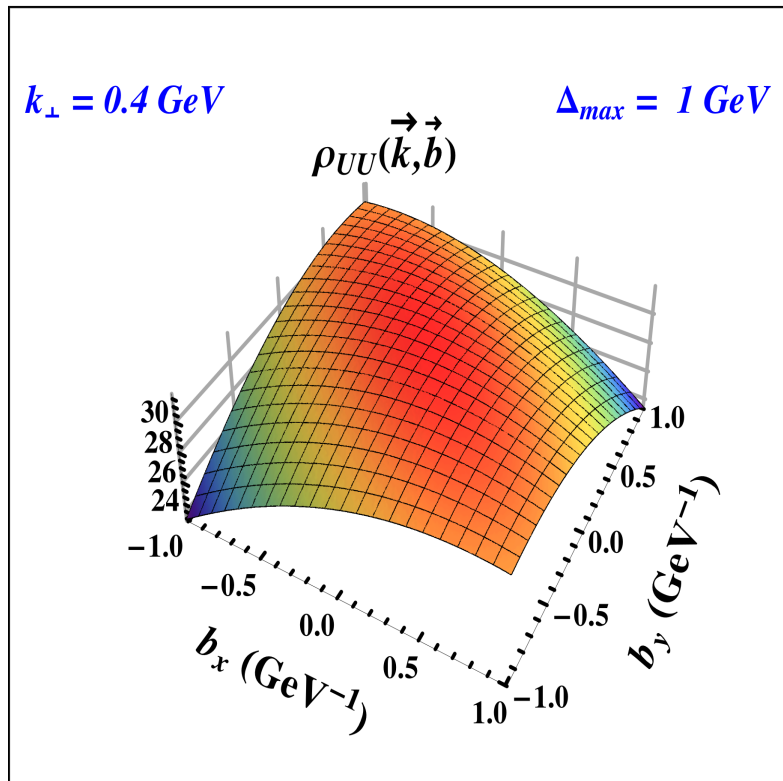
$$\rho_{LU}^{[\gamma^+]}(b_{\perp}, k_{\perp}, x) = N \int d\Delta_x \int d\Delta_y \frac{\sin(\Delta_{\perp} \cdot b_{\perp})}{D(q_{\perp})D(q'_{\perp})} \left[4(k_x \Delta_y - k_y \Delta_x) \frac{(1+x)}{x^2(1-x)} \right];$$

$$\rho_{LL}^{[\gamma^+ \gamma^5]}(b_{\perp}, k_{\perp}, x) = N \int d\Delta_x \int d\Delta_y \frac{\cos(\Delta_{\perp} \cdot b_{\perp})}{D(q_{\perp})D(q'_{\perp})} \left[I_1 - \frac{4m^2(1-x)}{x^2} \right];$$

$$D(k_{\perp}) = \left(m^2 - \frac{m^2 + (k_{\perp})^2}{x} - \frac{(k_{\perp})^2}{1-x} \right) \quad I_1 = 4 \left((k_{\perp})^2 - \frac{\Delta_{\perp}^2 (1-x)^2}{4} \right) \frac{(1+x^2)}{x^2(1-x)^3}.$$

N is a normalization constant

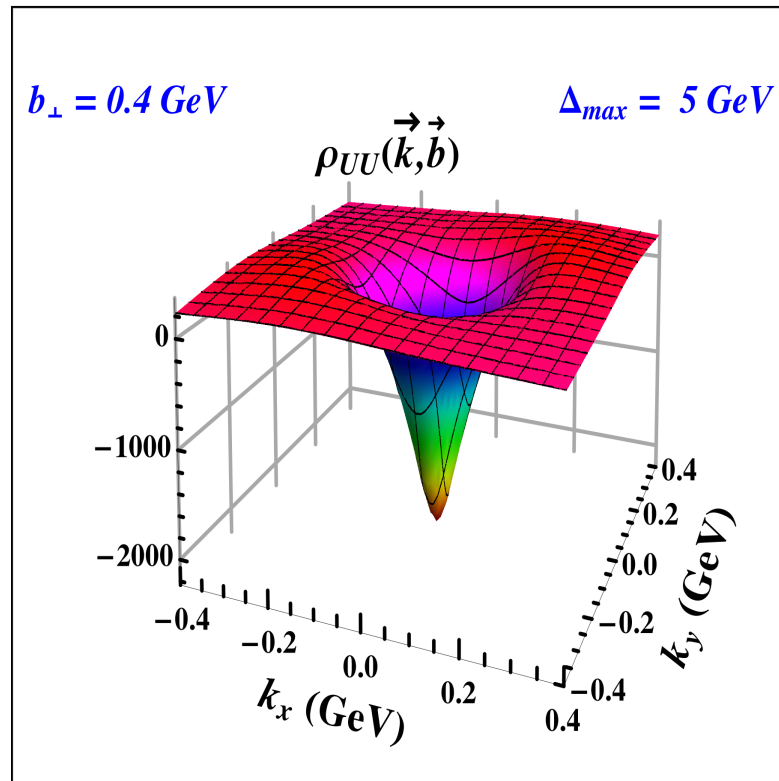
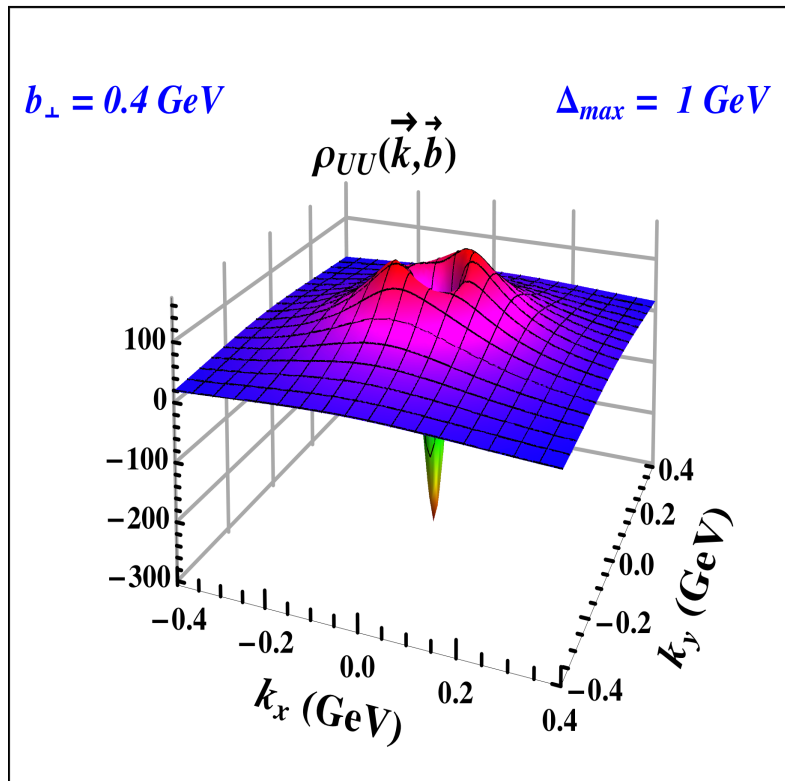
Wigner Distributions : Results



AM, Nair, Ojha, PRD 90 (2014) 014024

- Integrated over x (all plots)
- Used an upper limit on Δ_{\perp} integration, k_{\perp} in y direction
- Asymmetry related to OAM (no confining potential)

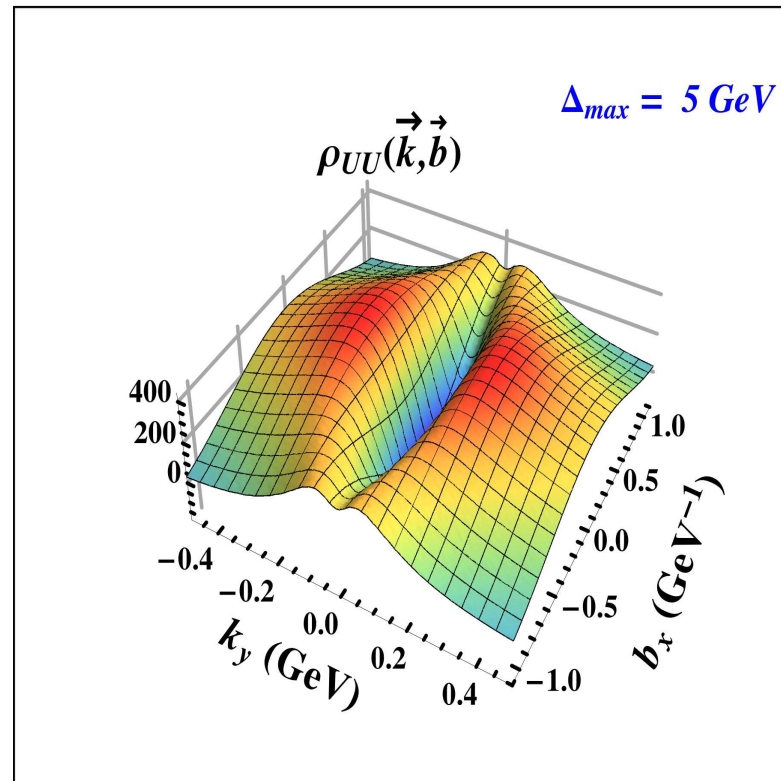
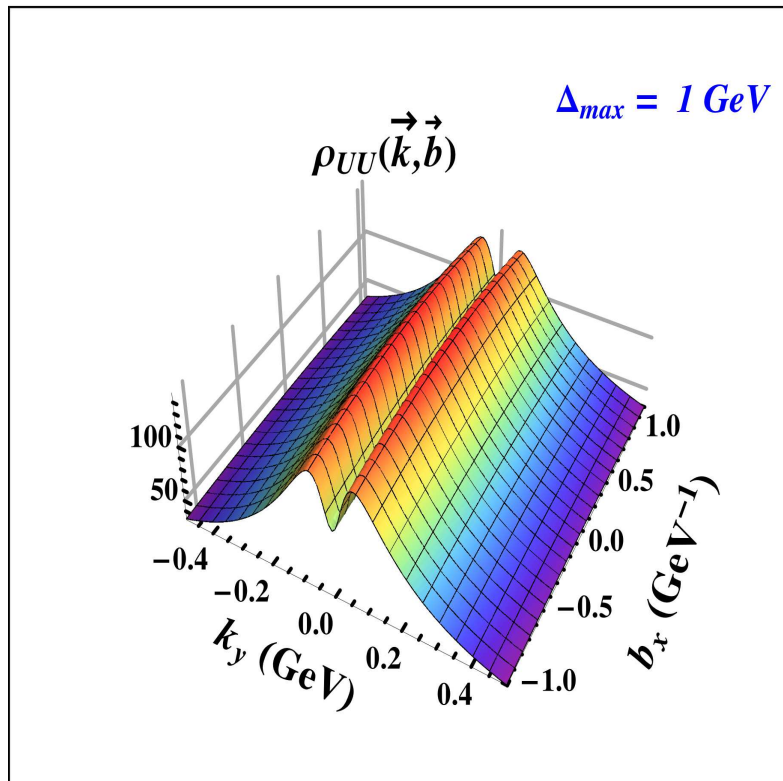
Wigner Distributions : Results



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- For all plots took $m = 0.33 \text{ GeV}$
- Plots in k_{\perp} space
- b in y direction

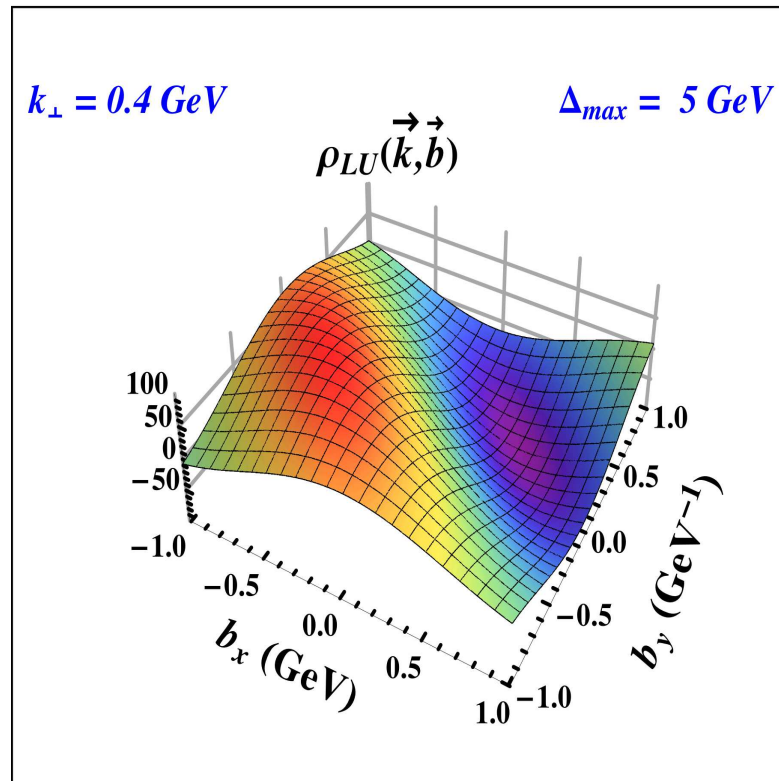
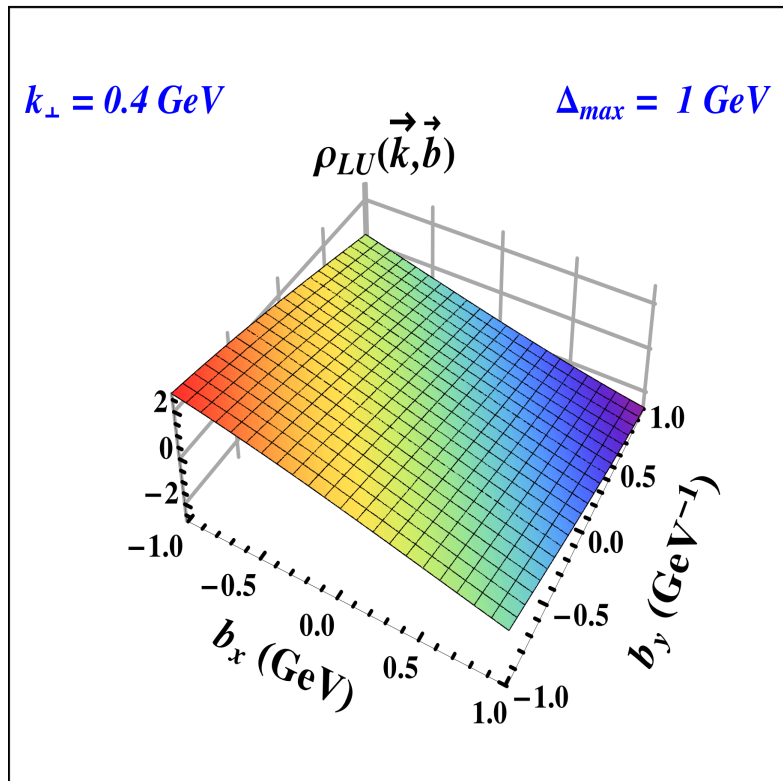
Wigner Distributions : Results



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- Plots in mixed space : probabilistic interpretation
- Minima at $b_x = 0$ and $k_y = 0$: minima is observed for all b_x values for $k_y = 0$
- Probability of finding a quark with fixed k_y and b_x first increases away from $k_y = 0$ and then decreases

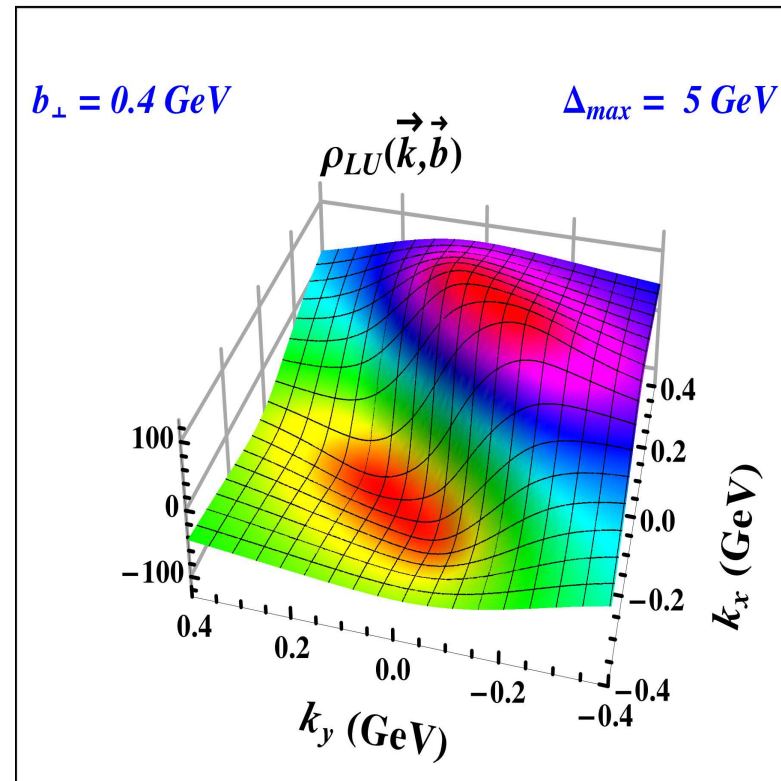
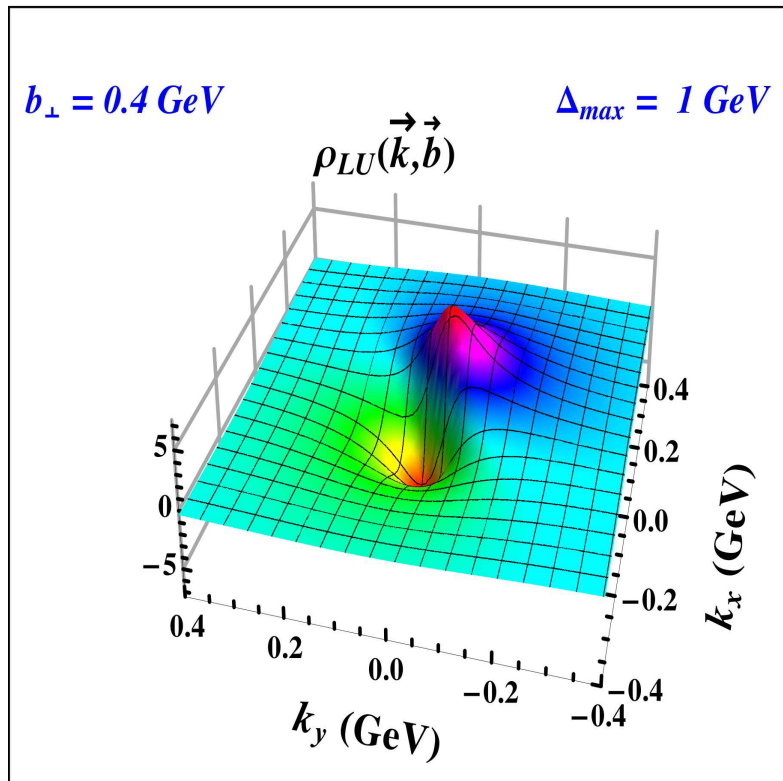
Wigner Distributions : Results



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- Distortion of the Wigner distribution of unpolarized quarks due to the longitudinal polarization of the dressed quark.
- ρ_{LU} is the same as ρ_{UL} in this model
- k_{\perp} in y direction; dipole structure similar to other models

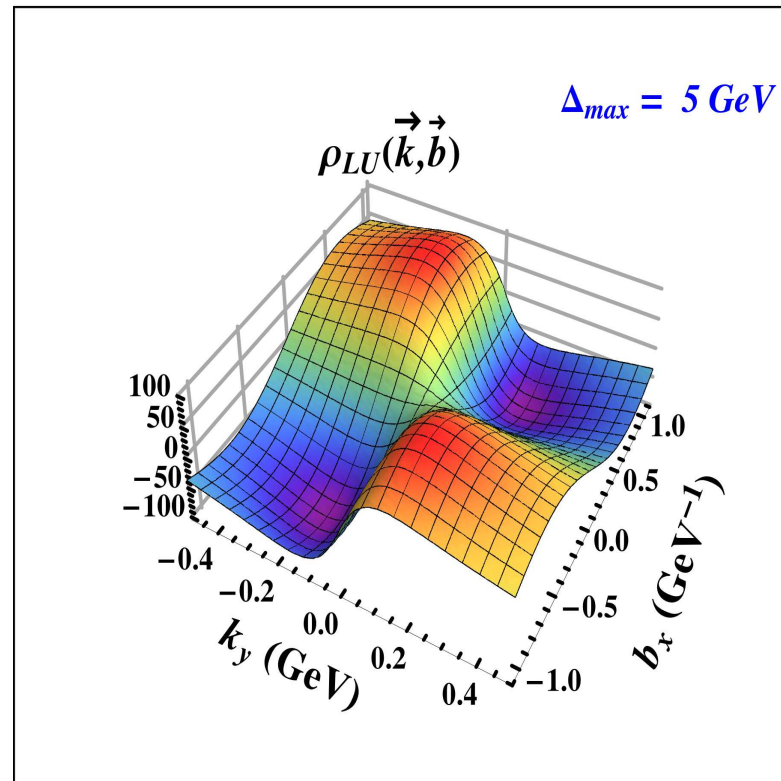
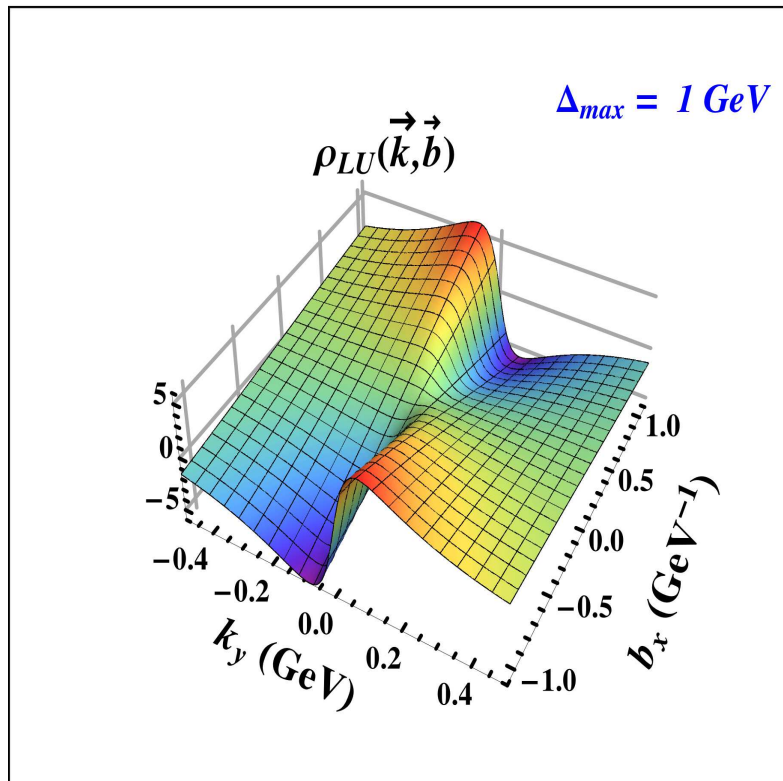
Wigner Distributions : Results



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- b in y direction
- Dipole structure

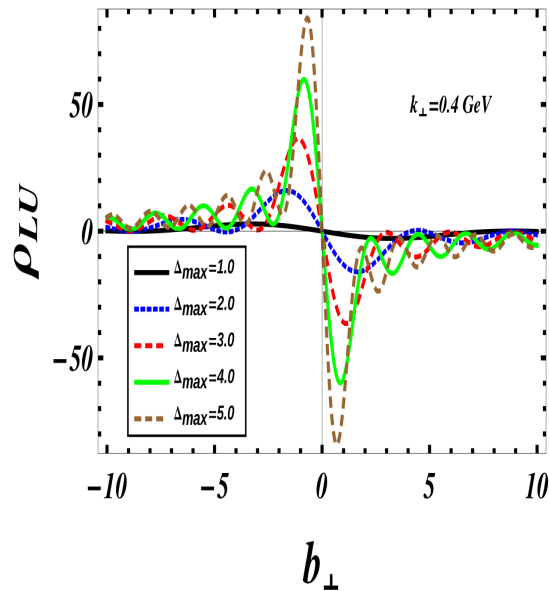
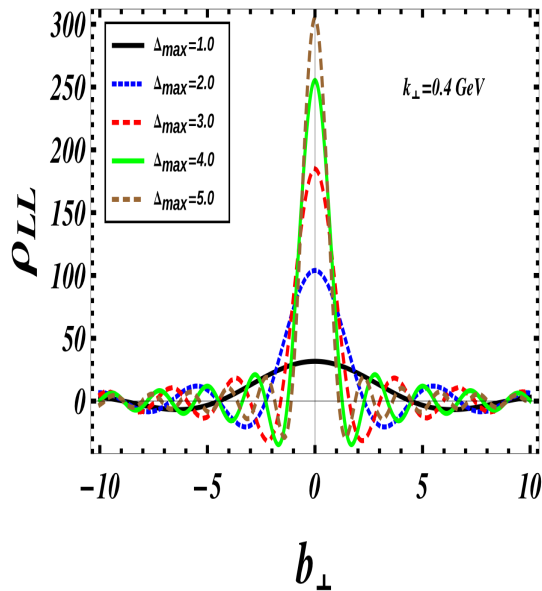
Wigner Distributions : Results



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- Quadrupole structure in the mixed space like other models
- Peaks increase in magnitude with increasing Δ_{\perp}^{max}

Wigner Distributions : Results



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- Correlations between the unpolarized quarks inside the unpolarized target as well as the distortions due to the longitudinal polarization of the quarks in the longitudinally polarized dressed quark target are large in the close vicinity of $b_{\perp} = 0$ for fixed k_{\perp} . If the allowed transverse momentum transfer is higher, these correlations move closer to the origin
- The distortions of the Wigner functions due to the longitudinal polarization of the quark in an unpolarized target changes sign for negative b_{\perp} , these distortions are related to the OAM of the quark. Such distortions are also more concentrated near the origin in b space as the transverse momentum transfer is higher

Orbital Angular Momentum of Quarks

- EMC experiment (1989) showed that the quark intrinsic spin contribution was much less than expected
- A substantial part of the spin of the nucleon comes from quark and gluon OAM
- Issue of gauge invariance and experimental measurability of the OAM contribution complicates the issue of a full understanding of such contributions
- Theoretically there exist mainly two definitions of OAM : one obtained from the sum rules of GPDs and the other, canonical OAM distribution in the light cone gauge
- These two different distributions are projections of Wigner distributions with different choice of gauge links and they are related by a gauge dependent potential term

Hatta, PLB (2012), Lorce, PLB (2013), Burkardt, PRD (2013).

Orbital Angular Momentum of Quarks

Quark-quark correlator defining the Wigner distributions can be parameterized in terms of generalized transverse momentum dependent parton distributions (GTMDs)

$$W_{\lambda,\lambda'}^{[\gamma^+]} = \frac{1}{2M} \bar{u}(p', \lambda') \left[F_{1,1} - \frac{i\sigma^{i+} k_{i\perp}}{P^+} F_{1,2} - \frac{i\sigma^{i+} \Delta_{i\perp}}{P^+} F_{1,3} + \frac{i\sigma^{ij} k_{i\perp} \Delta_{j\perp}}{M^2} F_{1,4} \right] u(p, \lambda);$$

$$W_{\lambda,\lambda'}^{[\gamma^+ \gamma_5]} = \frac{\bar{u}(p', \lambda')}{2M} \left[\frac{-i\epsilon_{\perp}^{ij} k_{i\perp} \Delta_{j\perp}}{M^2} G_{1,1} - \frac{i\sigma^{i+} \gamma_5 k_{i\perp}}{P^+} G_{1,2} - \frac{i\sigma^{i+} \gamma_5 \Delta_{i\perp}}{P^+} G_{1,3} + i\sigma^{+-} \gamma_5 G_{1,4} \right] u(p, \lambda).$$

For the dressed quark, the GTMDs can be calculated analytically

Confirm in our model calculation that the GTMDs F_{14} and G_{11} exist and are non-zero; conclusion agrees with

Kanazawa, Lorce, Metz, Pasquini, Schlegel, PRD (2014); arXiv:1403.5226[hep-ph]

Quark GTMDs

$$F_{11} = - \frac{N \left[4k_{\perp}^2 (1 + x^2) + (x - 1)^2 (4m^2 (x - 1)^2 - (1 + x^2) \Delta_{\perp}^2) \right]}{D(q_{\perp}) D(q'_{\perp}) 2x^2 (x - 1)^3};$$

$$F_{12} = \frac{2Nm^2 \Delta_{\perp}^2}{D(q_{\perp}) D(q'_{\perp}) x (k_y \Delta_x - k_x \Delta_y)};$$

$$F_{13} = \frac{N}{D(q_{\perp}) D(q'_{\perp}) 4x (k_y \Delta_x - k_x \Delta_y)} \left[8m^2 (k_{\perp} \Delta_{\perp}) \right. \\ \left. - \frac{(k_y \Delta_x - k_x \Delta_y) (4k_{\perp}^2 (1 + x^2) + (x - 1)^2 (4m^2 (x - 1)^2 - (1 + x^2) \Delta_{\perp}^2))}{x (x - 1)^3} \right];$$

$$F_{14} = \frac{2Nm^2 (1 + x)}{D(q_{\perp}) D(q'_{\perp}) x^2 (1 - x)}.$$

Quark GTMDs

$$G_{11} = -\frac{2Nm^2(1+x)}{D(q_{\perp})D(q'_{\perp})x^2(x-1)};$$

$$G_{12} = \frac{-N}{D(q_{\perp})D(q'_{\perp})x(x-1)} \left[4m^2 \frac{k_{\perp} \cdot \Delta_{\perp}}{(k_y \Delta_x - k_x \Delta_y)} - \frac{(1+x)\Delta_{\perp}^2}{x} \right];$$

$$G_{13} = \frac{N \left[(1+x) \left(\Delta_y^2 - \Delta_x^2 + \Delta_x \Delta_y (k_y^2 - k_x^2) \right) + 4xm^2 k_{\perp}^2 \right]}{D(q_{\perp})D(q'_{\perp})x^2(x-1)(k_y \Delta_x - k_x \Delta_y)};$$

$$G_{14} = \frac{N \left[-4k_{\perp}^2(1+x^2) + (x-1)^2 \left(4m^2(x-1)^2 - (1+x^2)\Delta_{\perp}^2 \right) \right]}{D(q_{\perp})D(q'_{\perp})2x^2(x-1)^3};$$

Quark Orbital Angular Momentum

$$L_z^q = \frac{1}{2} \int dx \{ x [H^q(x, 0, 0) + E^q(x, 0, 0)] - \tilde{H}^q(x, 0, 0) \}$$

X. Ji, PRL (1997)

GPDs in the above equation are defined at $\xi = 0$ or when the momentum transfer is purely in the transverse direction

The GPDs in the above expression are related to the GTMDs by the following relations :

$$H(x, 0, t) = \int d^2 k_\perp F_{11}$$

$$E(x, 0, t) = \int d^2 k_\perp \left[-F_{11} + 2 \left(\frac{k_\perp \cdot \Delta_\perp}{\Delta_\perp^2} F_{12} + F_{13} \right) \right]$$

$$\tilde{H}(x, 0, t) = \int d^2 k_\perp G_{14}$$

Meissner, Metz, Schlegel, JHEP (2009)

Quark Orbital Angular Momentum

Using the GTMDs calculated we have the following final expression for the kinetic orbital angular momentum of quarks in the dressed quark model:

$$L_z^q = \frac{N}{2} \int dx \left\{ -f(x)I_1 + 4m^2(1-x)^2 I_2 \right\};$$

$$I_1 = \int \frac{d^2 k_{\perp}}{m^2(1-x)^2 + (k_{\perp})^2} = \pi \log \left[\frac{Q^2 + m^2(1-x)^2}{\mu^2 + m^2(1-x)^2} \right];$$

$$I_2 = \int \frac{d^2 k_{\perp}}{\left(m^2(1-x)^2 + (k_{\perp})^2 \right)^2} = \frac{\pi}{(m^2(1-x)^2)}; f(x) = 2(1+x^2)$$

Q and μ are the upper and lower limits of the integration respectively, Q is the large scale

μ can be safely taken to be zero provided the quark mass is non-zero

Quark Orbital Angular Momentum

$F_{1,4}$ is not reducible to any GPDs or transverse-momentum dependent parton distributions (TMDs) in any limit: related to the canonical OAM as

$$l_z^q = - \int dx d^2 k_\perp \frac{k_\perp^2}{m^2} F_{14}.$$

Canonical quark OAM in the dressed quark model becomes

$$l_z^q = -2N \int dx (1 - x^2) \left[I_1 - m^2 (x - 1)^2 I_2 \right].$$

Agrees with

Harindranath and Kundu (1999); Hikmat and Burkardt(2012),
Kanazawa, Lorce, Metz, Pasquini, Schlegel, arXiv:1403.5226[hep-ph]

Gauge link does not contribute to these GTMDs and the result is independent of the choice of the gauge link

Correlation Between Quark Spin and OAM

Correlation between the quark spin and its OAM is given by

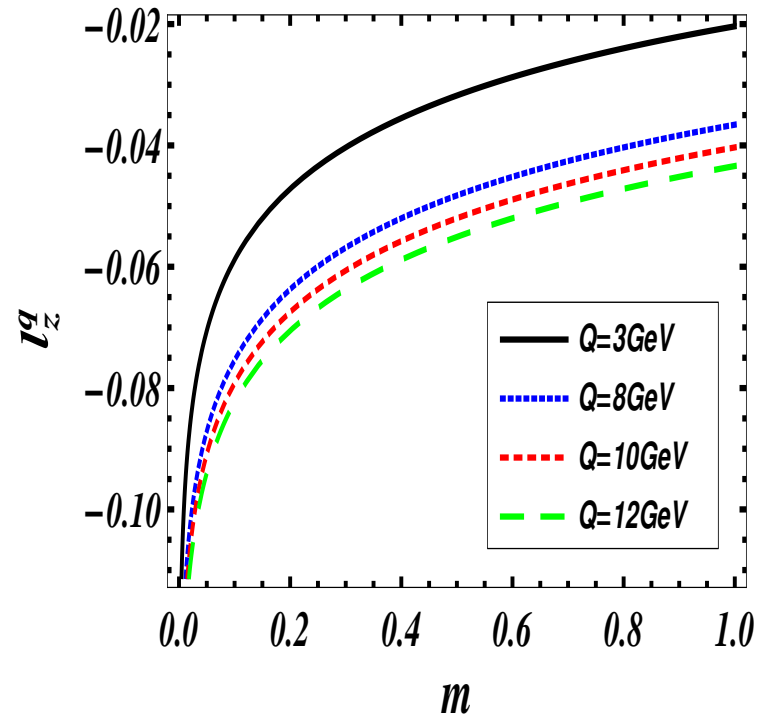
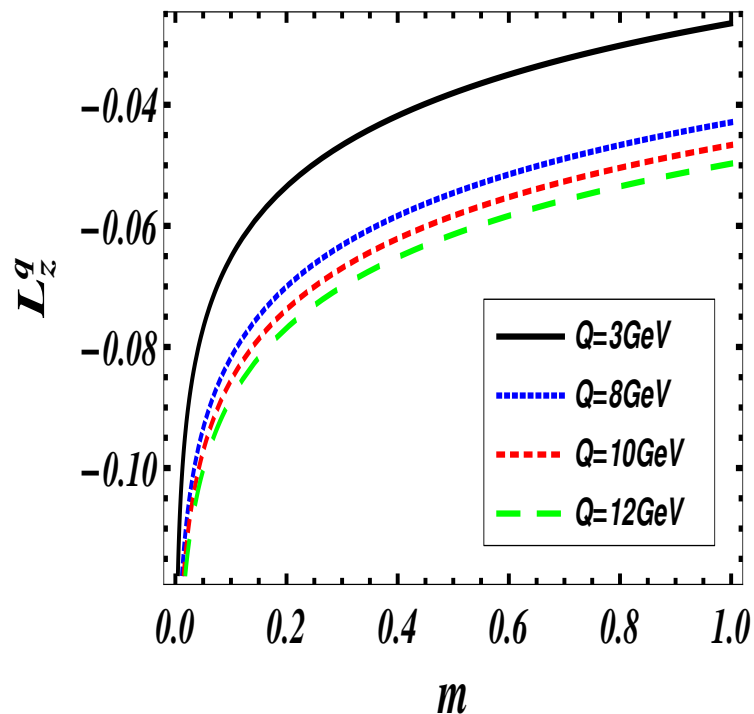
$$C_z^q = \int dx d^2 k_{\perp} \frac{k_{\perp}^2}{m^2} G_{11}.$$

As in our model $F_{14} = -G_{11}$, the above correlation is the same as the canonical OAM

Spin-orbit correlation for the quark in the dressed quark is negative

This is opposite to what is observed in chiral quark-soliton model and constituent quark model, namely here the quark spin is anti-aligned with its OAM, unlike the other two models where there is no gluon

Orbital Angular Momentum : Results



AM, Nair, Ojha, PRD 90 (2014) 014024

Both OAM decrease in magnitude with increasing mass of the quark

In models without any gluonic degrees of freedom, the contribution to the OAM from different quark flavors were found to be different, but the sum over all flavors were equal for the two definitions of OAM

Here the magnitude of the two OAM differs : effect of gluonic degree of freedom

In this model gluon intrinsic helicity contribution to the helicity sum rule cancels the contribution from the quark and gluon OAM and the helicity sum rule is satisfied

Helicity Sum Rule

In light-front quantization, with $A^+ = 0$ gauge, $\mathcal{J}^3 = J^3$ (the naive canonical form independent of interactions) at the operator level, provided the fields vanish at the boundary

$$J_{f(o)}^3 = \int dx^- d^2 x^\perp \psi^{+\dagger} i(x^1 \partial^2 - x^2 \partial^1) \psi^+,$$

$$J_{f(i)}^3 = \frac{1}{2} \int dx^- d^2 x^\perp \psi^{+\dagger} \Sigma^3 \psi^+,$$

$$J_{g(o)}^3 = \frac{1}{2} \int dx^- d^2 x^\perp \left\{ x^1 [\partial^+ A^1 \partial^2 A^1 + \partial^+ A^2 \partial^2 A^2] - x^2 [\partial^+ A^1 \partial^1 A^1 + \partial^+ A^2 \partial^1 A^2] \right\},$$

$$J_{g(i)}^3 = \frac{1}{2} \int dx^- d^2 x^\perp [A^1 \partial^+ A^2 - A^2 \partial^+ A^1]$$

Helicity sum rule for the fermion target is given by

$$\frac{1}{\mathcal{N}} \langle PS | \left[J_{q(i)}^3 + J_{q(o)}^3 + J_{g(i)}^3 + J_{g(o)}^3 \right] | PS \rangle = \pm \frac{1}{2}$$

Helicity Sum Rule

For a dressed quark target having helicity $+\frac{1}{2}$ we get

$$\begin{aligned} \frac{1}{\mathcal{N}} \langle P, \uparrow | J_{f(i)}^3 | P, \uparrow \rangle_q &= \int dx \left[\frac{1}{2} \delta(1-x) + \frac{\alpha}{2\pi} C_f \ln \frac{Q^2}{\mu^2} \left[\frac{1+x^2}{(1-x)_+} + \frac{3}{2} \delta(1-x) \right] \right] \\ &= \frac{1}{2} \end{aligned}$$

$$\frac{1}{\mathcal{N}} \langle P, \uparrow | J_{f(o)}^3 | P, \uparrow \rangle_q = -\frac{\alpha}{2\pi} C_f \ln \frac{Q^2}{\mu^2} \int dx (1-x)(1+x)$$

$$\frac{1}{\mathcal{N}} \langle P, \uparrow | J_{g(i)}^3 | P, \uparrow \rangle_q = \frac{\alpha}{2\pi} C_f \ln \frac{Q^2}{\mu^2} \int dx (1+x)$$

$$\frac{1}{\mathcal{N}} \langle P, \uparrow | J_{g(o)}^3 | P, \uparrow \rangle_q = -\frac{\alpha}{2\pi} C_f \ln \frac{Q^2}{\mu^2} \int dx x(1+x).$$

Harindranath and Kundu, PRD,(1999)

Adding all the contributions, we get

$$\frac{1}{\mathcal{N}} \langle P, \uparrow | J_{f(i)}^3 + J_{f(o)}^3 + J_{g(i)}^3 + J_{g(o)}^3 | P, \uparrow \rangle_q = \frac{1}{2}$$

Wigner Distributions for the Gluon

Wigner distribution for gluons can be defined as

$$xW^g(x, k_{\perp}, b_{\perp}) = \int \frac{d^2 q_{\perp}}{(2\pi)^2} e^{-iq_{\perp} \cdot b_{\perp}} \int \frac{dz^- d^2 z_{\perp}}{(2\pi)^3} e^{ik \cdot z} \left\langle p^+, \frac{\Delta_{\perp}}{2}, S \left| F^{+i} \left(-\frac{z}{2} \right) F^{+i} \left(\frac{z}{2} \right) \right| p^+, -\frac{\Delta_{\perp}}{2}, S \right\rangle \Big|_{z^+=0}$$

Ji, Xiong, Yuan (2013)

Operator contains gauge links : we chose the light front gauge and took the gauge link to be unity

We calculate the above for a quark dressed with a gluon

Contribution comes from the two-particle light-front wave function

We define the unpolarized gluon Wigner distribution as

$$W^{UU} = W^{\uparrow\uparrow}(x, k_{\perp}, b_{\perp}) + W^{\downarrow\downarrow}(x, k_{\perp}, b_{\perp})$$

\uparrow and \downarrow correspond to the helicity of the target state

Gluon Wigner Distributions: Results

Wigner distribution corresponding to the distortion due to longitudinal polarization of the target as

$$W^{LU} = W^{\uparrow\uparrow}(x, k_{\perp}, b_{\perp}) - W^{\downarrow\downarrow}(x, k_{\perp}, b_{\perp})$$

We calculate the Wigner distributions for the gluon in the same model : results are given by

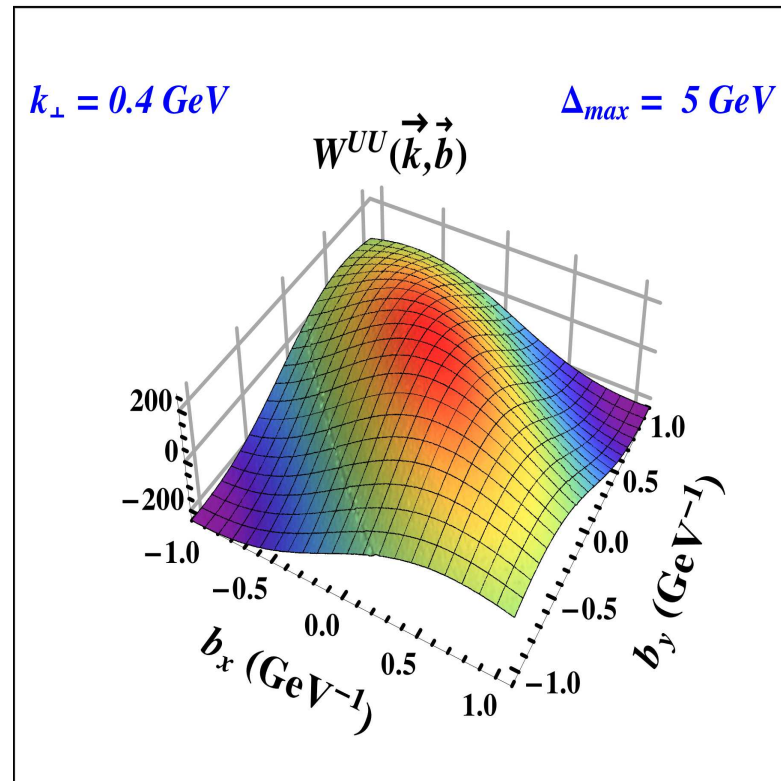
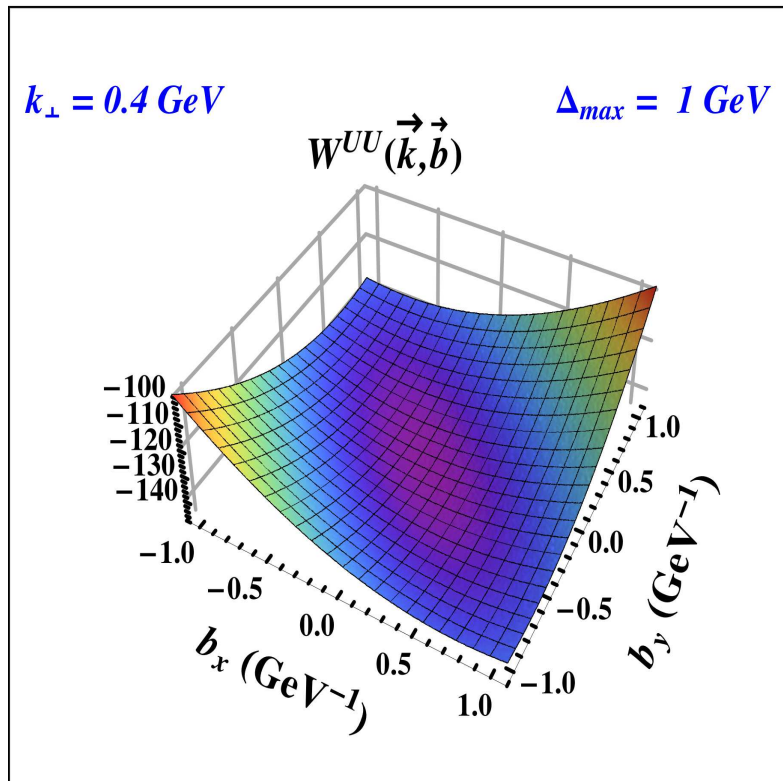
$$W^{UU}(x, k_{\perp}, b_{\perp}) = N \int d\Delta_x \int d\Delta_y \frac{\cos(\Delta_{\perp} \cdot b_{\perp})}{D(q_{\perp})D(q'_{\perp})} \left[\frac{-4 \left((q_{\perp} q'_{\perp})(x^2 - 2x + 2) + m^2 x^4 \right)}{x^3 (x - 1)^2} \right];$$

$$W^{LU}(x, k_{\perp}, b_{\perp}) = N \int d\Delta_x \int d\Delta_y \frac{\sin(\Delta_{\perp} \cdot b_{\perp})}{D(q_{\perp})D(q'_{\perp})} \left[\frac{4(2 - x)(q_2 q'_1 - q_1 q'_2)}{x^2 (x - 1)^2} \right];$$

where A_x, A_y are x, y component of A_{\perp} and N is a normalization constant

$$D(k_{\perp}) = \left(m^2 - \frac{m^2 + (k_{\perp})^2}{1 - x} - \frac{(k_{\perp})^2}{x} \right)$$

Gluon Wigner Distributions : Results



AM, Nair, Ojha, ongoing work

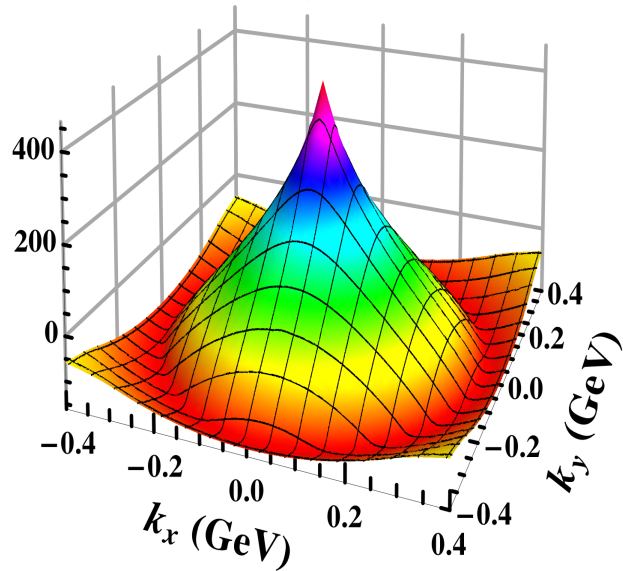
For all plots we took $m = 0.33 \text{ GeV}$
 Δ_{max} is the upper limit of Δ_{\perp} integration

Gluon Wigner Distributions : Results

$b_{\perp} = 0.4 \text{ GeV}$

$\Delta_{max} = 1 \text{ GeV}$

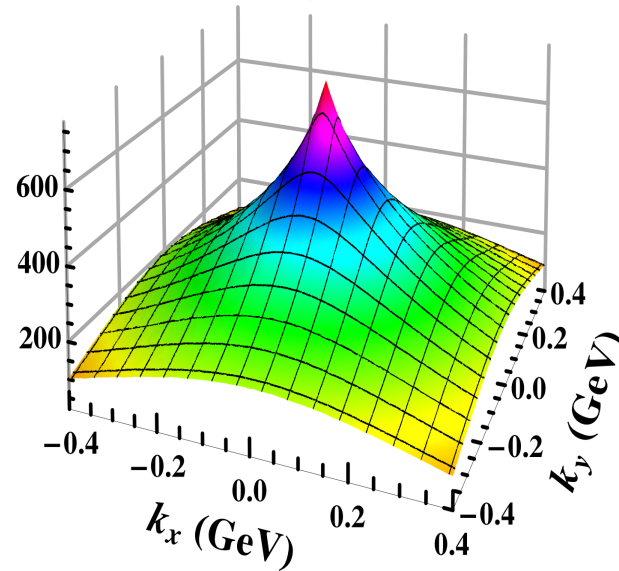
$W^{UU}(\vec{k}, \vec{b})$



$b_{\perp} = 0.4 \text{ GeV}$

$\Delta_{max} = 5 \text{ GeV}$

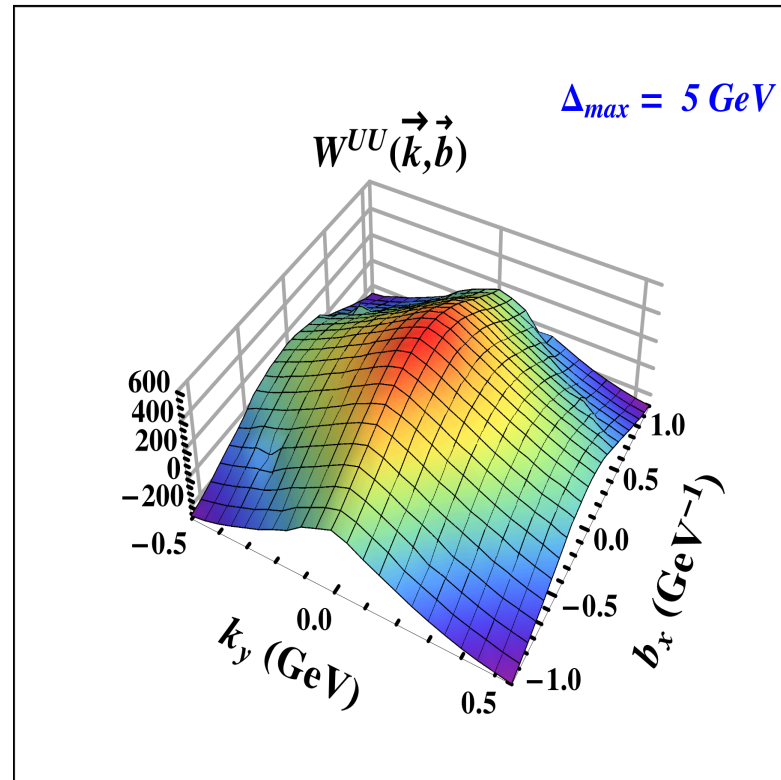
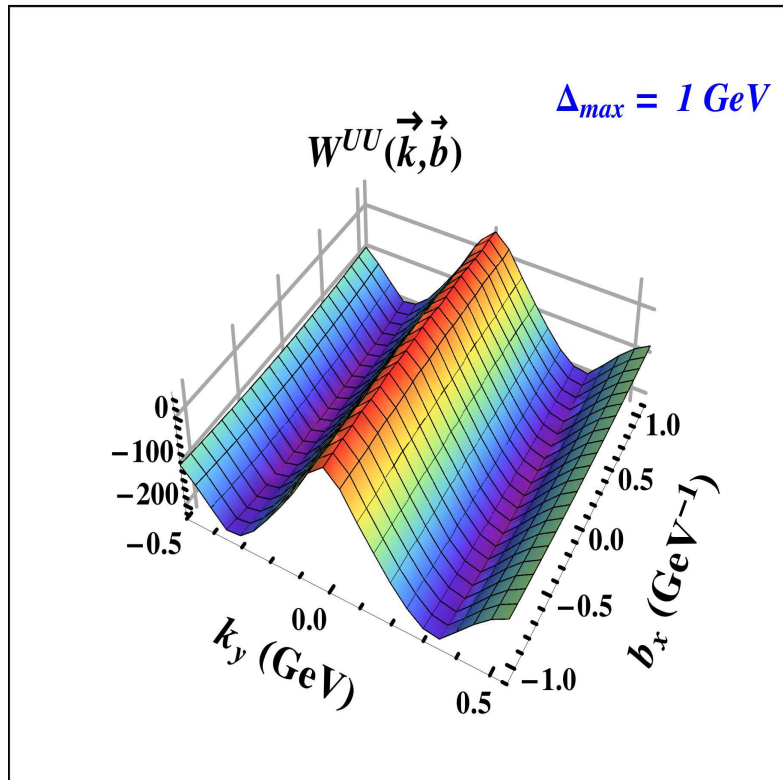
$W^{UU}(\vec{k}, \vec{b})$



AM, Nair, Ojha, ongoing work

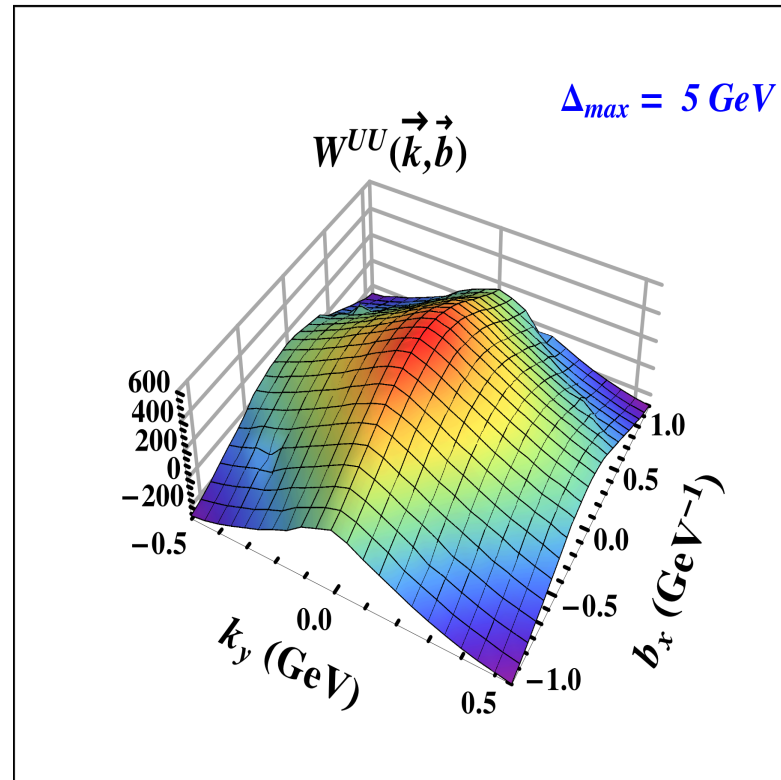
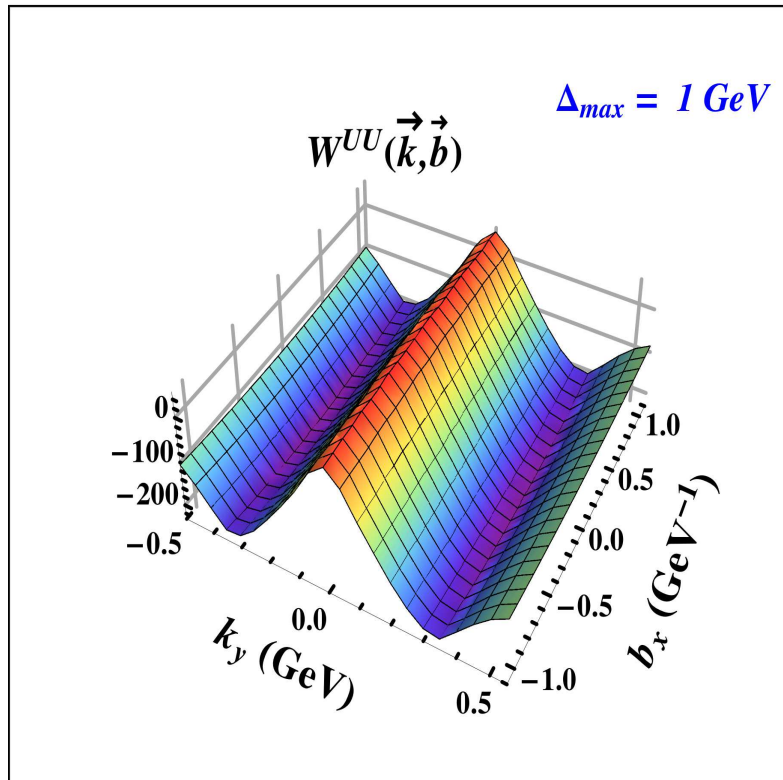
Integrated over x (all plots)

Gluon Wigner Distributions : Results



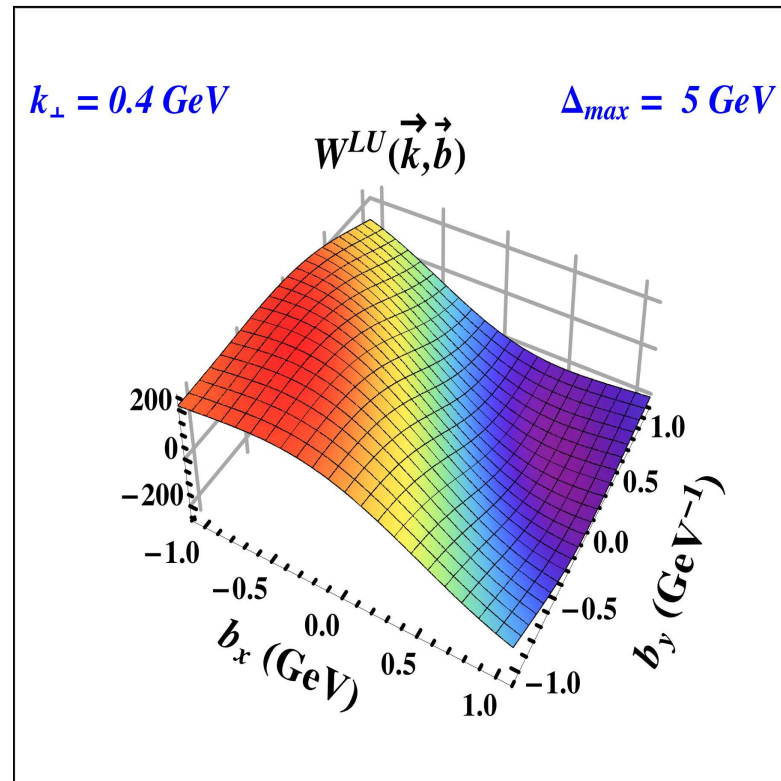
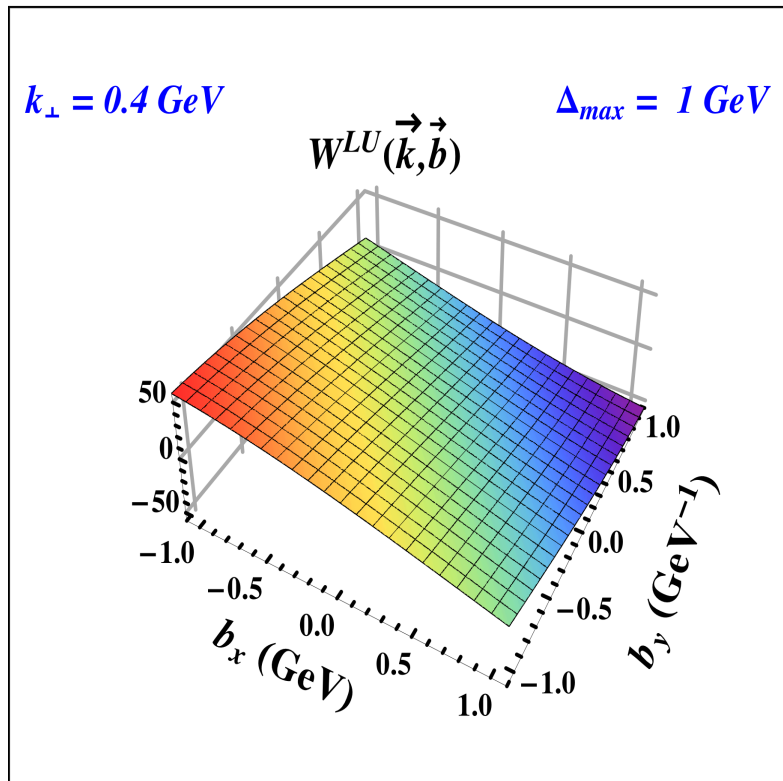
AM, Nair, Ojha, ongoing work

Gluon Wigner Distributions : Results



AM, Nair, Ojha, ongoing work

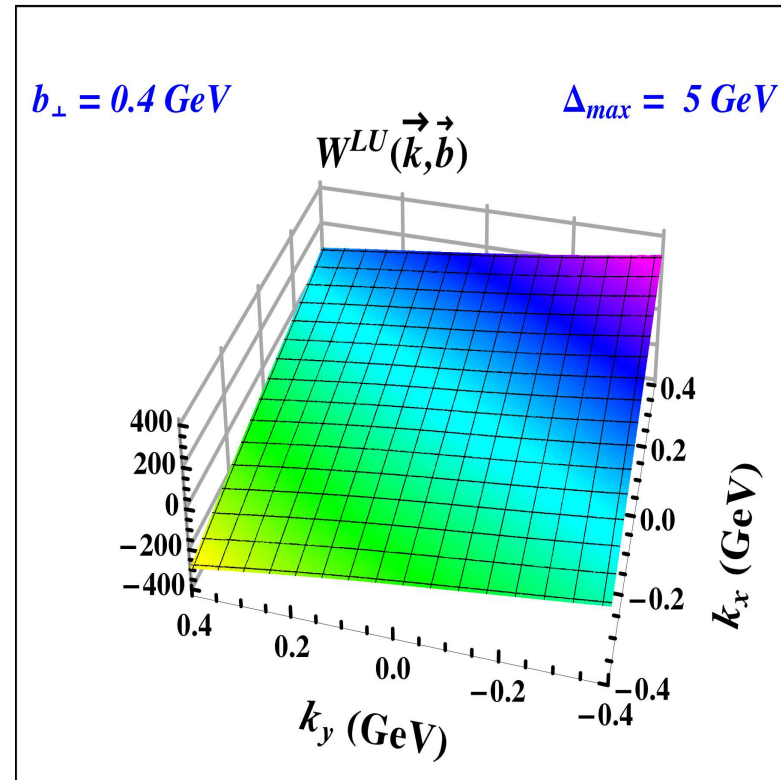
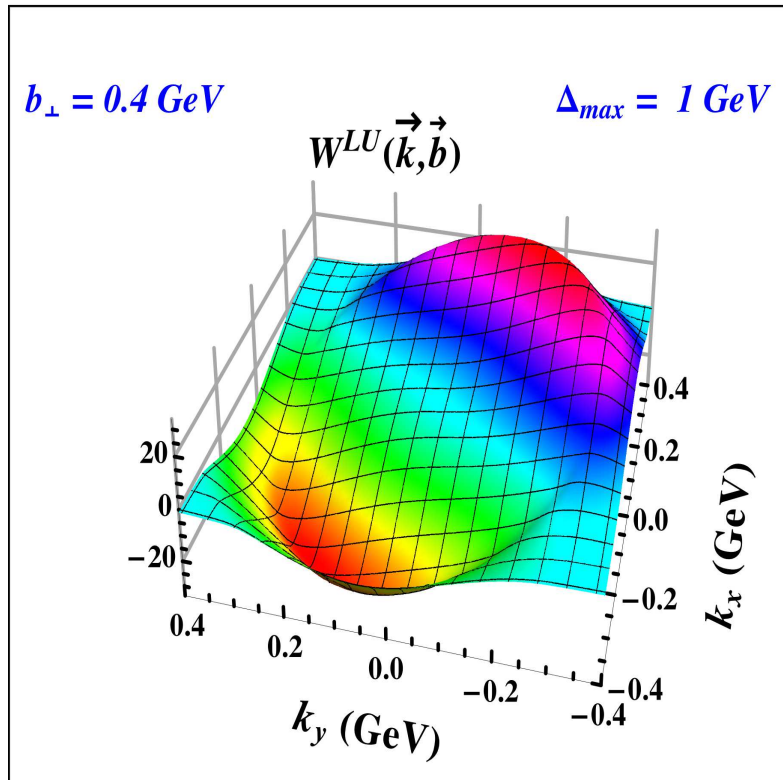
Gluon Wigner Distributions : Results



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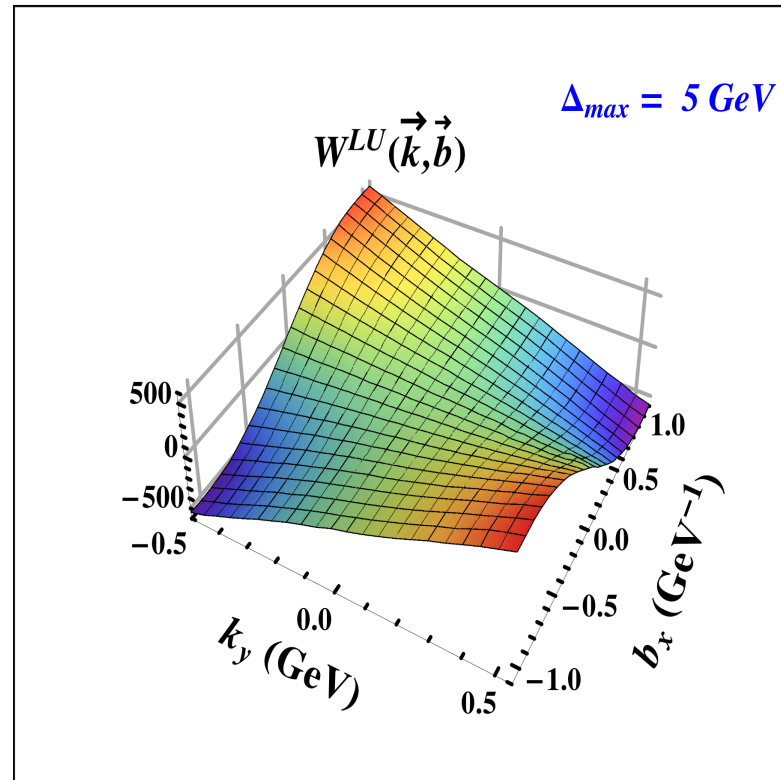
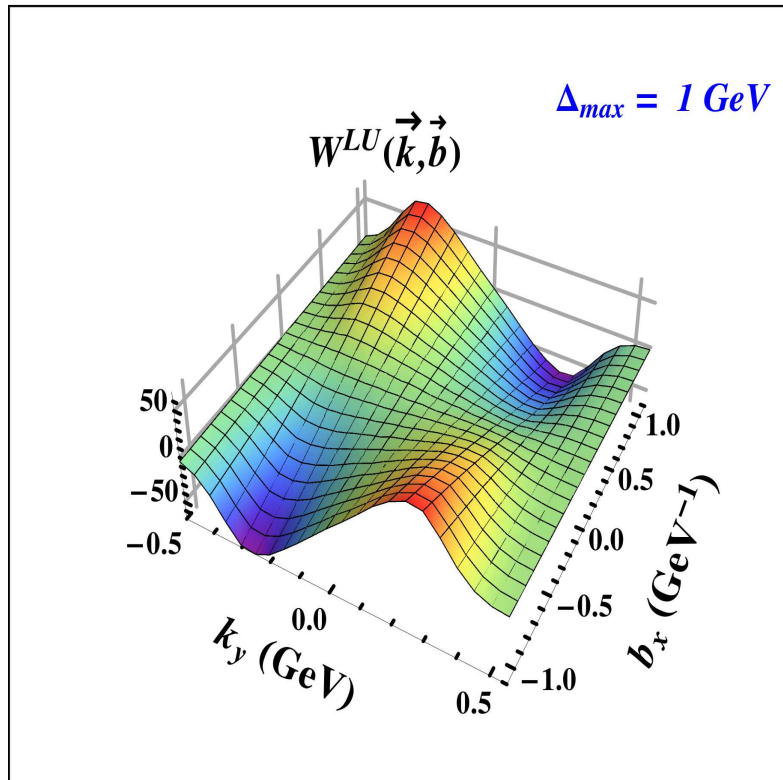
The Wigner distribution for the gluon can be related to gluon GTMDs : and gluon OAM is related to W^{LU}

Gluon Wigner Distributions : Results



AM, Nair, Ojha, ongoing work

Gluon Wigner Distributions : Results



AM, Nair, Ojha, ongoing work

Summary and Conclusions

- We calculated the Wigner distributions for a quark state dressed with a gluon using the overlap representation in terms of the LFWFs
- This is a simple composite relativistic spin-1/2 system which has a gluonic degree of freedom : used light-front wave functions
- Although the Wigner distributions in quantum mechanics are not measurable and do not have probabilistic interpretation, after integrating out some of the variables a probabilistic interpretation is possible to obtain
- We calculated the Wigner distributions both for unpolarized and longitudinally polarized target state and showed the correlations in transverse momentum and position space
- We also calculated the kinetic quark OAM using the GPD sum rule and the canonical OAM in light-front gauge and showed that these are different in magnitude
- Presented Wigner distributions for the gluon
- Further study would be to calculate the GTMDs for the gluons and include transverse polarization