The quark Orbital Angular Momentum with experimental prospect

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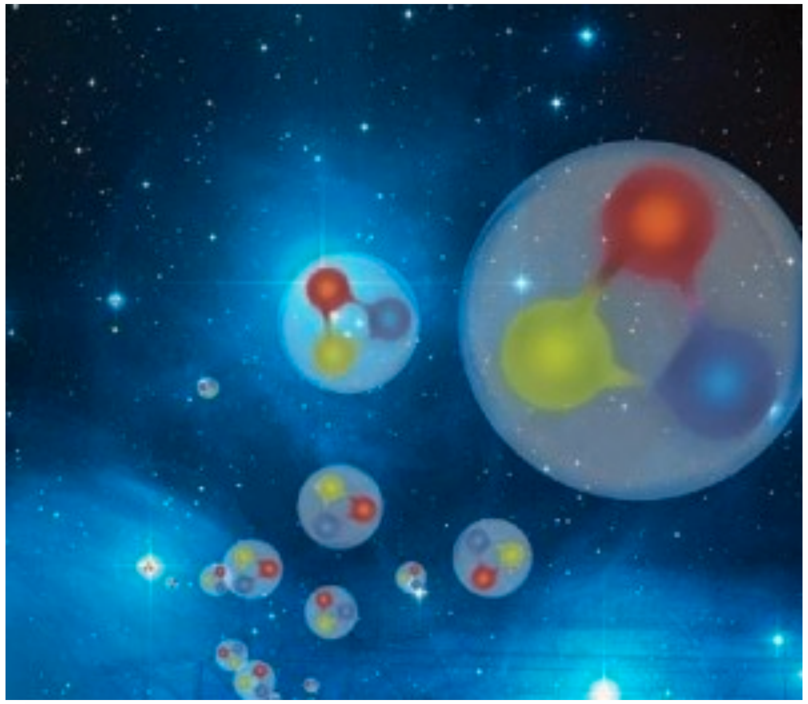
Interactions Fondamentales en Physique et Astrophysique

e-JLab12

Outline



- Where is the Orbital Angular Momentum?
  - ...theoretically
    - helicity amplitudes
    - models
  - ...experimentally

Based on Phys.Lett. B731 (2014) 141-147

with
Gary Goldstein
Osvaldo González Hernández
Simonetta Liuti
Abha Rajan

Spin crisis

$\Delta\Sigma \sim 30\%$ **≠100%**

... the rest must be in gluon and Orbital Angular Momentum

Spin crisis

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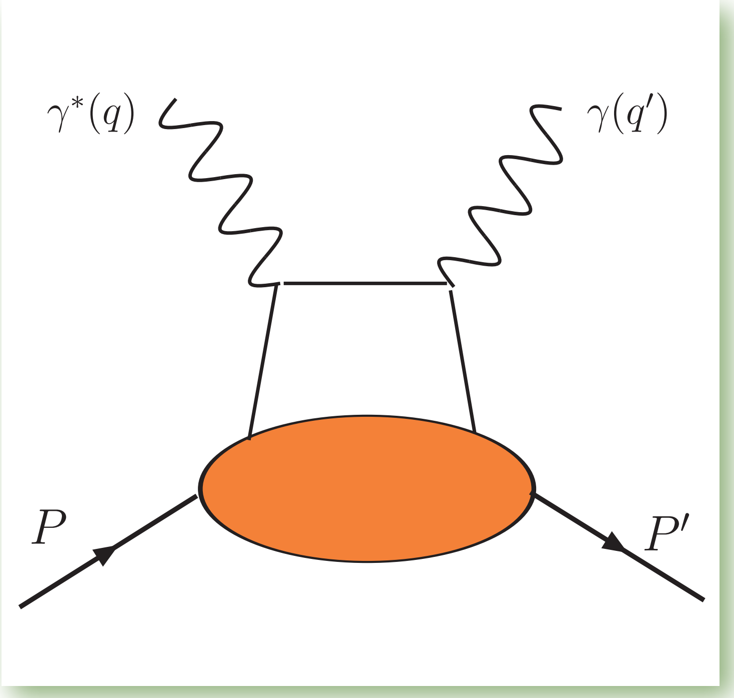
- Transverse spin?
  - higher-twist: g_T
  - role of $k_\perp$ highlighted long ago (e.g. **Jackson, Ross & Roberts, PLB226**)
  - formalized by **Mulders & Tangerman, NPB461** → Transverse Momentum Distributions

- Nucleon spin decomposition
  - **Ji PRL78**: related fo Form Factors of non-forward matrix elements
  - off-forward PDFs → Generalized Parton Distributions

Generalized Functions

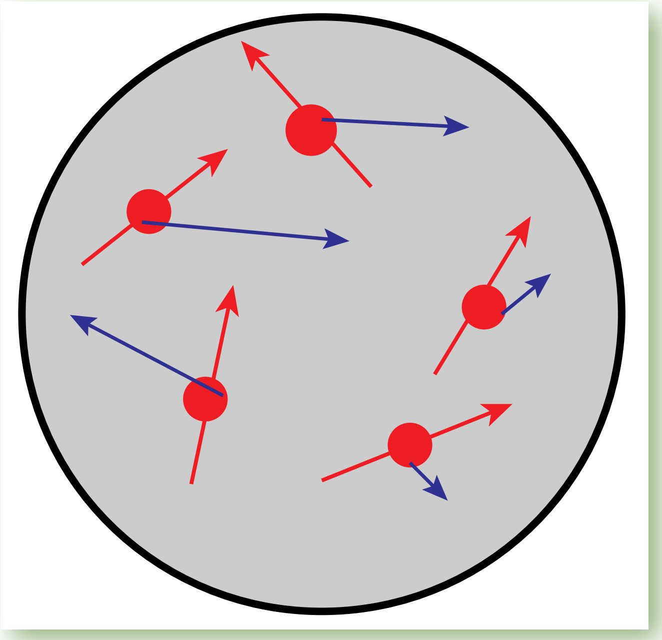
Momentum transfer b/w initial and final state



- Generalized Parton Distributions

$f(x) \rightarrow f(x, (P'-P)^2, n.(P'-P))$

Intrinsic quark transverse motion



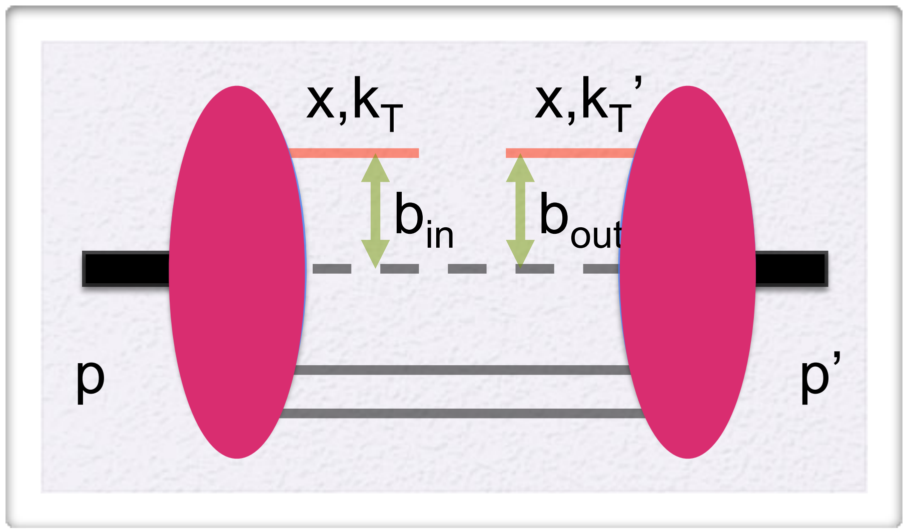
- Transverse Momentum Distributions

$f(x) \rightarrow f(x, k_\perp)$

Partonic meaning

- **Generalized Parton Distributions**

  $f(x) \rightarrow f(x, (P'-P)^2, n.(P'-P))$



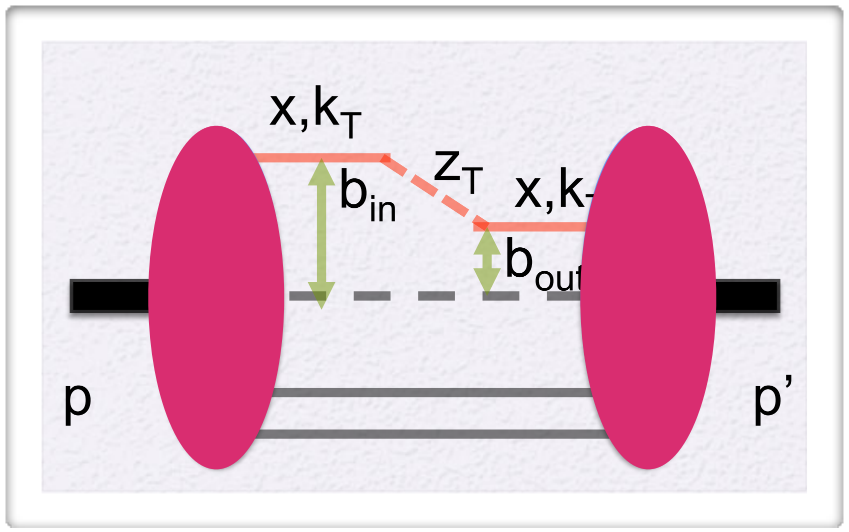
$$\Delta_T = P'_T - P_T = k'_T - k_T$$

$$\int \frac{d^2\Delta_T}{(2\pi)^2}\, e^{i\Delta_T \cdot b_T} \Rightarrow b_T = \frac{b_{T,in} + b_{T,out}}{2}$$

Average

- **Transverse Momentum Distributions**

  $f(x) \rightarrow f(x, k_\perp)$



$$\bar{k}_T = \frac{k_T + k'_T}{2}$$

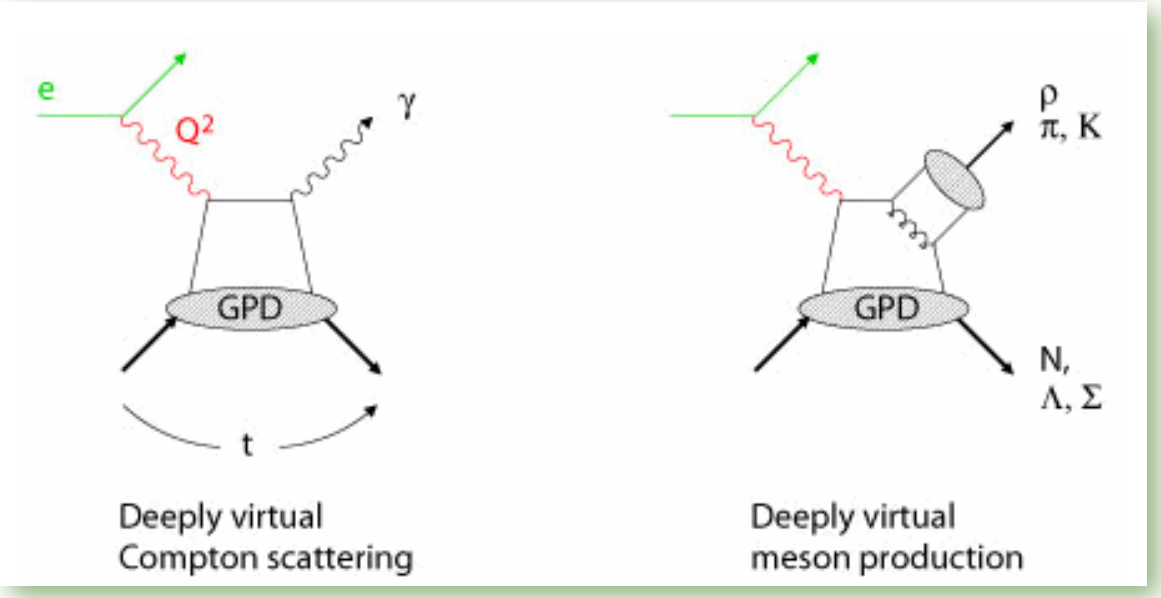
$$\int d^2k_T\, e^{-ik_T \cdot z_T} \Rightarrow z_T = b_{T,in} - b_{T,out}$$

Shift

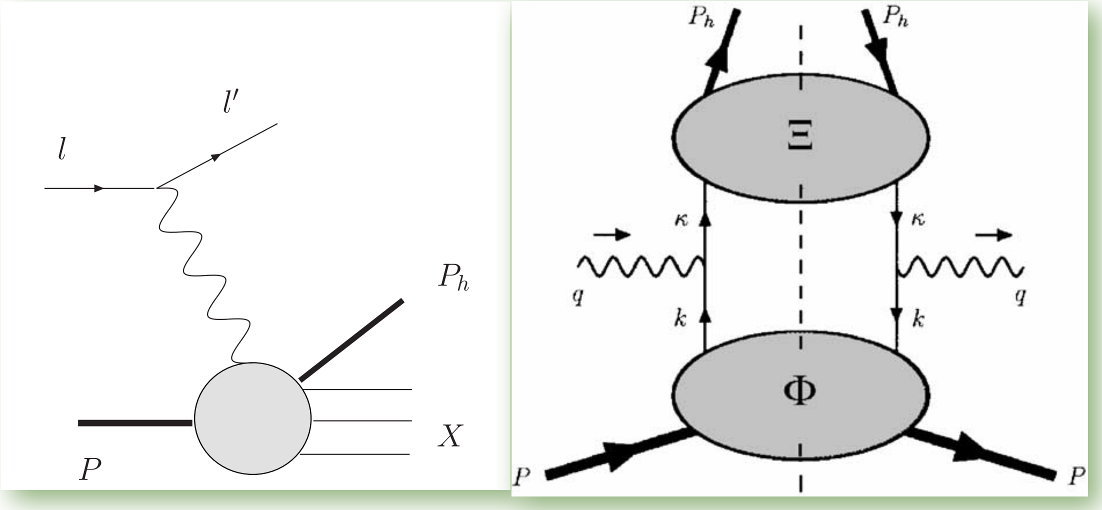
Impact parameter space

Generalized Functions

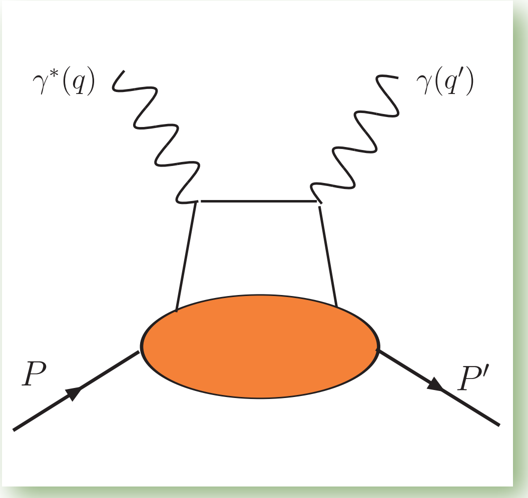
Exclusive processes



Semi-inclusive processes



Momentum transfer b/w initial and final state



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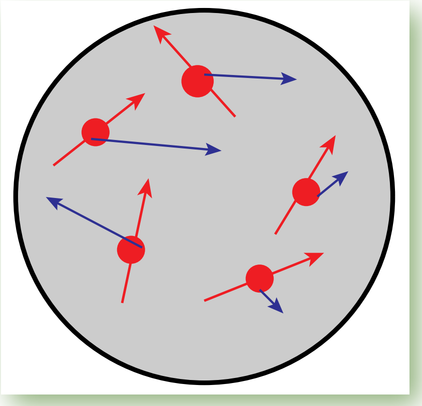
Intrinsic quark transverse motion



- **Transverse Momentum Distributions**

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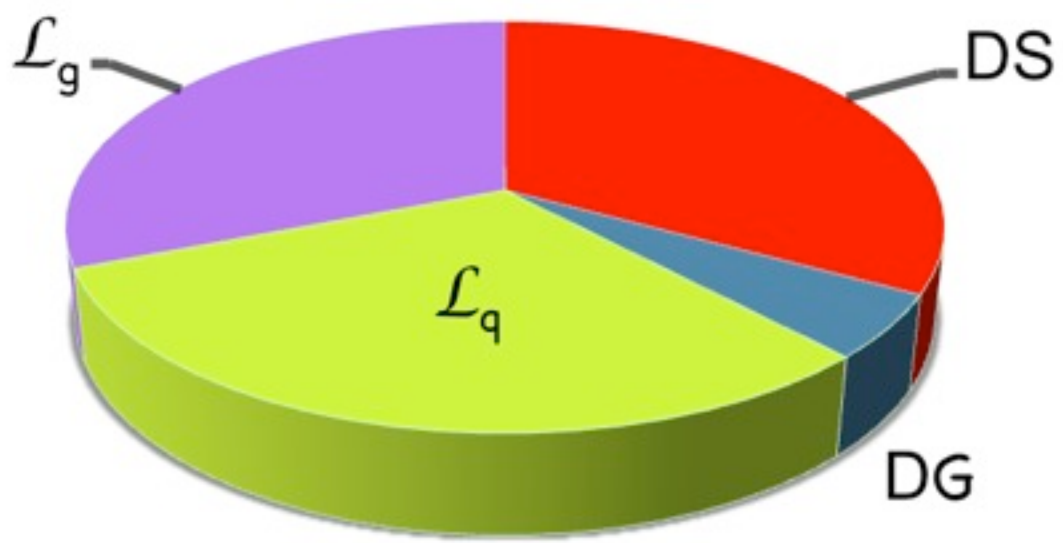
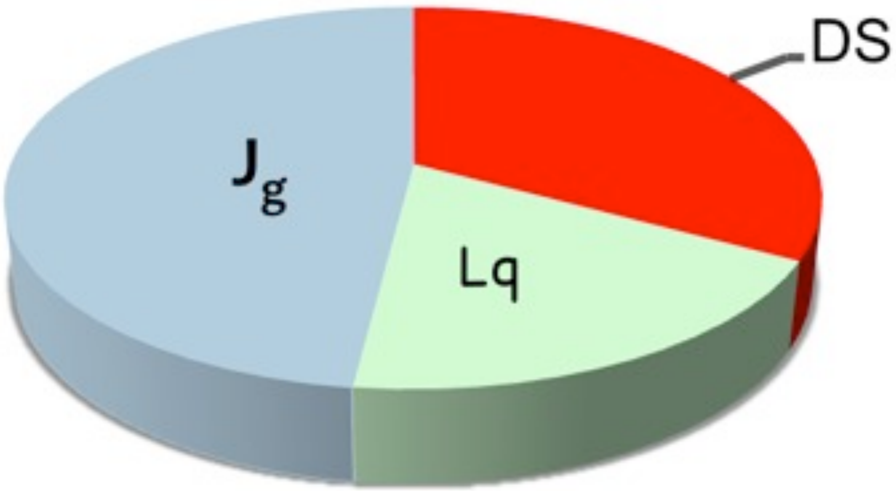
The one and only (motivation): Proton spin decomposition



Jaffe-Manohar



Ji

Find the differences!
Purpose of this workshop.

Beyond the OAM chronicle:
what can we experimentally access and what is its physical content?

That's our pragmatic approach

OAM definitions

See Cédric Lorcé's publications
and
Ji, Xiong & Yuan, PRL109

Wigner functions, natural framework

$$\hat{\mathcal{W}}(\vec{r},k) = \int d^4\xi\, e^{ik\cdot\xi}\,\bar{\Psi}_{GL}(\vec{r}-\xi/2)\gamma^+\Psi_{GL}(\vec{r}+\xi/2)$$

GL=choice of gauge link
r=phase-space position
k=phase-space 4-mmt

quantum average ➔

$$\langle\hat{O}\rangle = \int d\vec{r}dk\, O(\vec{r},k)\,\hat{\mathcal{W}}(\vec{r},k)$$

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$$\langle \hat{O} \rangle = \int d\vec{r} dk \, O(\vec{r}, k) \, \hat{\mathcal{W}}(\vec{r}, k)$$

- gauge-invariant
- canonical

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quantum average →

$$\langle\hat{O}\rangle = \int d\vec{r}dk\, O(\vec{r},k)\,\hat{\mathcal{W}}(\vec{r},k)$$

- **gauge-invariant**

$$L_{FS} = \frac{\langle PS|\int d^3\vec{r}\,\bar{\psi}(\vec{r})\gamma^+(\vec{r}_\perp \times i\vec{D}_\perp)\psi(\vec{r})|PS\rangle}{\langle PS|PS\rangle}$$
$$= \int (\vec{b}_\perp \times \vec{k}_\perp) W_{FS}(x,\vec{b}_\perp,\vec{k}_\perp)\,dx\,d^2\vec{b}_\perp d^2\vec{k}_\perp$$

Ji, Xiong & Yuan, PRL109

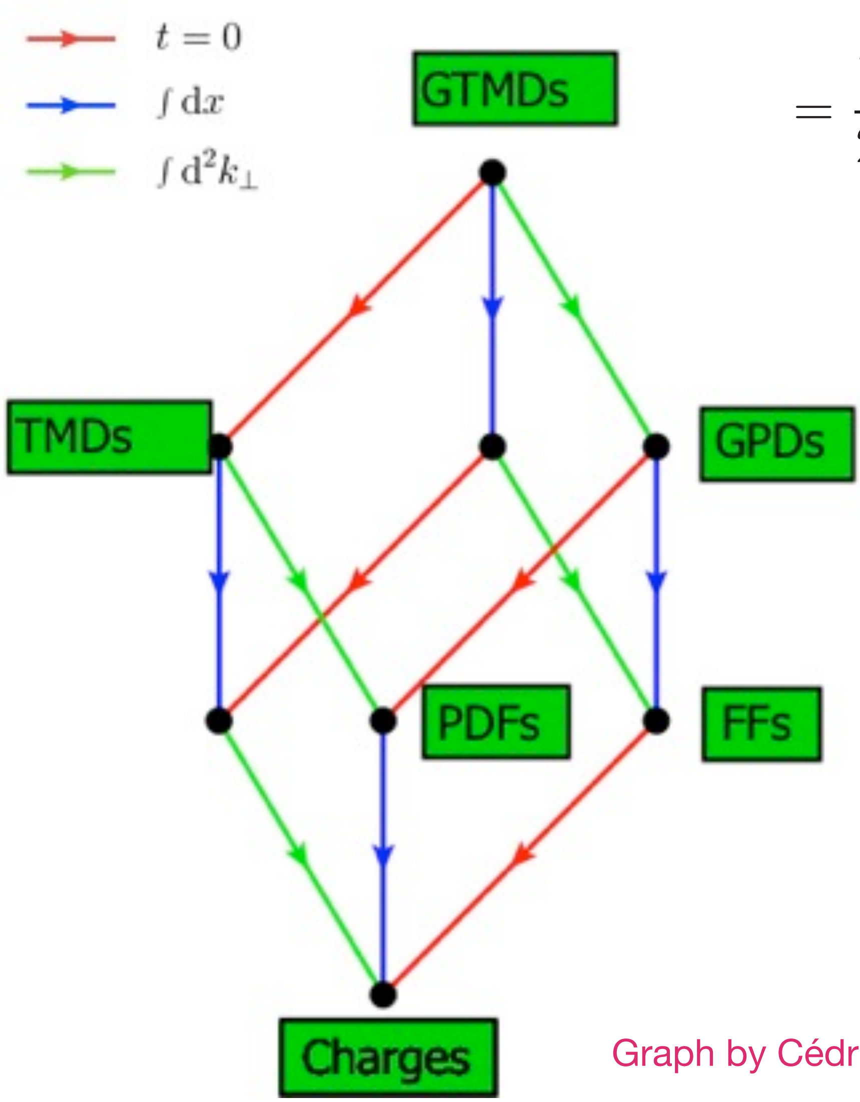
- **canonical**

$$l_q = \frac{\langle PS|\int d^3\vec{r}\,\overline{\psi}(\vec{r})\gamma^+(\vec{r}_\perp \times i\vec{\partial}_\perp)\psi(\vec{r})|PS\rangle}{\langle PS|PS\rangle}$$
$$= \int (\vec{b}_\perp \times \vec{k}_\perp) W_{LC}(x,\vec{b}_\perp,\vec{k}_\perp)\,dx\,d^2\vec{b}_\perp d^2\vec{k}_\perp$$

(Generalization)^2 of distributions

Wigner function quantized at light-cone time ⟶ Generalized TMDs

$$W^{[\Gamma]}_{\lambda\lambda'}(P,x,\vec{k}_T,\Delta,N;\eta) = \int dk^-\, W^{[\Gamma]}_{\lambda\lambda'}(P,k,\Delta,N;\eta)$$
$$= \frac{1}{2}\int \frac{dz^-\, d^2\vec{z}_T}{(2\pi)^3}\, e^{ik\cdot z}\, \langle p',\lambda'|\, \bar{\psi}\Big(-\frac{1}{2}z\Big)\, \Gamma\, \mathcal{W}\Big(-\frac{1}{2}z,\frac{1}{2}z\,|\,n\Big)\, \psi\Big(\frac{1}{2}z\Big)\, |p,\lambda\rangle\Big|_{z^+=0}$$



Graph by Cédric Lorcé

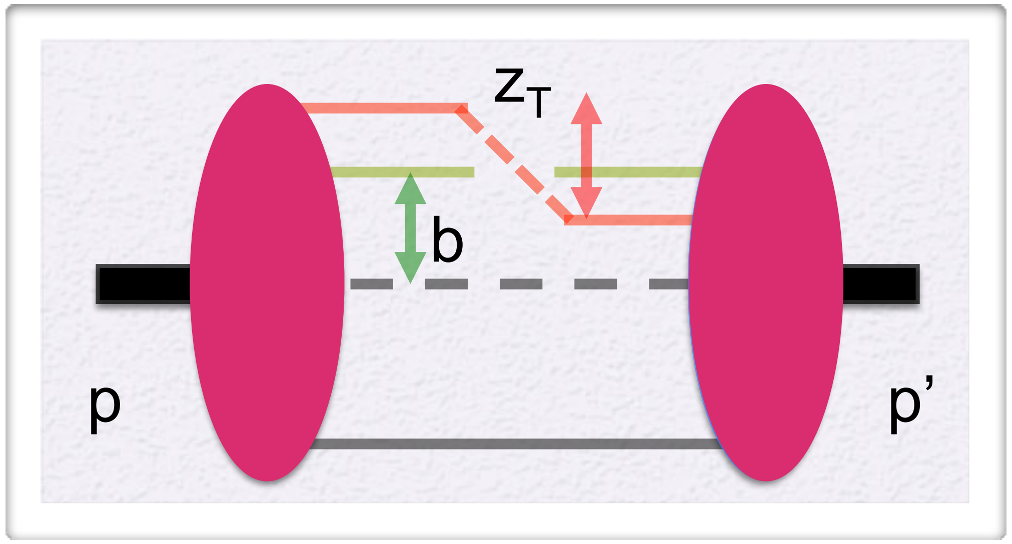
GTMDs account for both $k_\perp$ & Δ

Meissner, Metz, Schlegel & Goeke, JHEP 0808 (2008) 038
Meissner, Metz & Schlegel, JHEP 0908 (2009) 056

Partonic meaning

GTMDs account for both $k_\perp$ & Δ

2 transverse momenta



$$\bar{k}_T = \frac{k_T + k'_T}{2} \Rightarrow z_T = b_{T,in} - b_{T,out}$$

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Average + Shift

Impact parameter space

Interpretation in terms of 2-body scattering??

OAM and GTMDs

- **Still need to define/find process related to GTMD** (à la Goloskokov & Kroll, EPJC 53 ??/ see S. Liuti's talk)
- **Purely theoretical object**
- **Don't know behavior with a possible factorization**
- **No constraint from pQCD so far**

- **Only limits to GPDs & TMDs.**

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- Orbital Angular Momentum
- Wigner functions b/c of quantum average

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Lorcé & Pasquini, PRD84
Lorcé, Pasquini, Xiong & Yuan, PRD85

Main motivations:

- **Orbital Angular Momentum**
- **Wigner functions b/c of quantum average**

$$\ell_z^q \equiv \langle \hat{L}_z^q \rangle^{[\gamma^+]}(\vec{e}_z) = \int dx d^2k_\perp d^2b_\perp (\vec{b}_\perp \times \vec{k}_\perp)_z \rho^{[\gamma^+]q}(\vec{b}_\perp, \vec{k}_\perp, x, \vec{e}_z)$$

$$\ell_z^q = -\int dx d^2k_\perp \frac{\vec{k}_\perp^2}{M^2} F_{1,4}^q(x, 0, \vec{k}_\perp^2, 0, 0)$$

New structure only from GTMD

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New structure only from GTMD

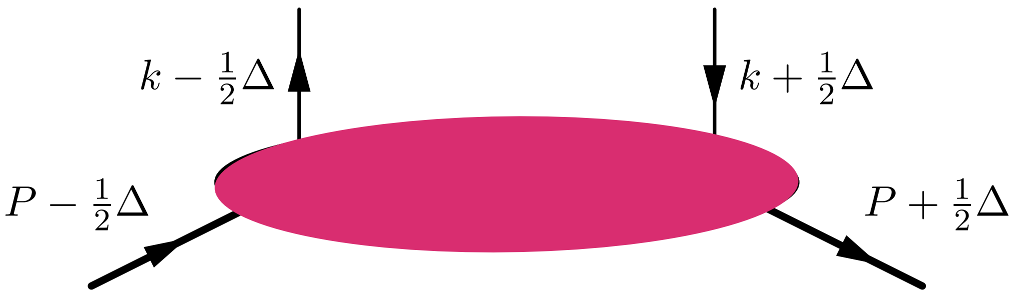
But something doesn't match our intuition...

Classification of GTMDs

Recipe from M. Diehl, EPJC 19

Meissner, Metz & Schlegel [JHEP 0908 (2009) 056]

- Lorentz scalar
- Hermiticity
- Charge-conjugation
- Parity conservation



$$
\begin{aligned}
W^{\gamma^+}_{\Lambda\Lambda'} &= \frac{1}{2P^+}\Big[\overline{U}(p',\Lambda')\gamma^+ U(p,\Lambda)F_{11} + \overline{U}(p',\Lambda')\frac{i\sigma^{i+}\Delta^i_T}{2M}U(p,\Lambda)(2F_{13}-F_{11}) \\
&+ \overline{U}(p',\Lambda')\frac{i\sigma^{i+}\bar{k}^i_T}{2M}U(p,\Lambda)(2F_{12}) + \overline{U}(p',\Lambda')\frac{i\sigma^{ij}\bar{k}^i_T\Delta^j_T}{M^2}U(p,\Lambda)\,F_{14}\Big] \\
&= \delta_{\Lambda,\Lambda'}F_{11} + \delta_{\Lambda,-\Lambda'}\frac{-\Lambda\Delta_1 - i\Delta_2}{2M}(2F_{13}-F_{11}) + \delta_{\Lambda,-\Lambda'}\frac{-\Lambda\bar{k}_1 - i\bar{k}_2}{2M}(2F_{12}) + \delta_{\Lambda,\Lambda'}i\Lambda\frac{\bar{k}_1\Delta_2 - \bar{k}_2\Delta_1}{M^2}F_{14}
\end{aligned}
$$

related to GPDs

related to the Sivers fct

new correlation of $k_\perp$ & Δ

Discrete symmetries

Behavior on the variables constrained by discrete symmetries

Parton correlations defined on the light-cone:

→ customary to check combined P and T invariance to constrain LC variables

→ the observable must be P and T-invariant

Helicity vs. LF helicity

A single particle state is assumed to transform similarly,

$$P|\vec{p}\, s\rangle = \eta_P | -\vec{p}\, s\rangle, \qquad P|\vec{p}\, h\rangle = \eta_P | -\vec{p} - h\rangle,$$

→ from the matrix element, we get that h↛-h

→ overall the LF combination of discrete symmetries is conserved

Parity relations

- Helicity amplitudes of 2-body scattering ➔ $A_{\Lambda',\,\lambda';\,\Lambda,\,\lambda} : q'(k',\lambda') + N(p,\Lambda) \to q(k,\lambda) + N'(p',\Lambda')$
- 16 HA related through parity relations ➔ $A_{-\Lambda',\,-\lambda';\,-\Lambda,\,-\lambda} = (-1)^{\eta}\, A^{*}_{\Lambda',\,\lambda';\,\Lambda,\,\lambda}$
- leaving 8 independent amplitudes.
- so, the combinations ↘

$$-i\,\frac{\bar{k}_1\Delta_2 - \bar{k}_2\Delta_1}{M^2} F_{14} \;=\; \left(A_{++,++} + A_{+-,+-} - A_{-+,-+} - A_{--,--}\right)/4$$

$$i\,\frac{\bar{k}_1\Delta_2 - \bar{k}_2\Delta_1}{M^2} G_{11} \;=\; \left(A_{++,++} - A_{+-,+-} + A_{-+,-+} - A_{--,--}\right)/4$$

... are Not indpt in CoM frame

F_{14} & G_{11}: Non zero b/c imaginary

$\Rightarrow A_{++,++} \neq A_{--,--}$ & $A_{+-,+-} \neq A_{-+,-+}$

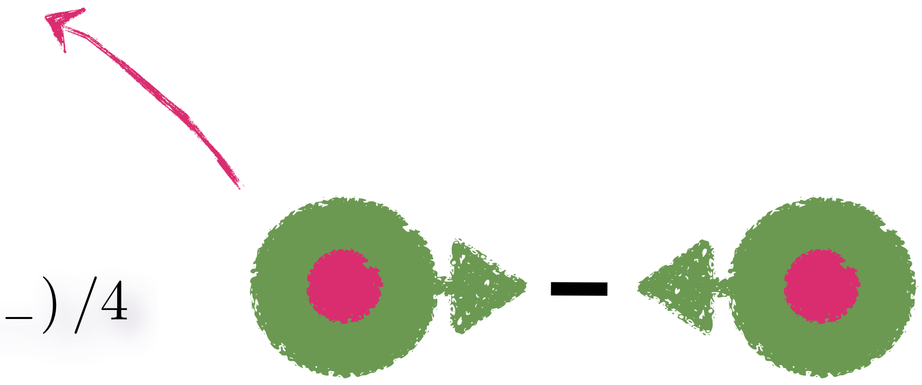
Example: the bag model

$\mathrm{Re}(A_{+-,+-}$ & $A_{++,++}) = \mathrm{Re}(A_{-+,-+}$ & $A_{--,--})$

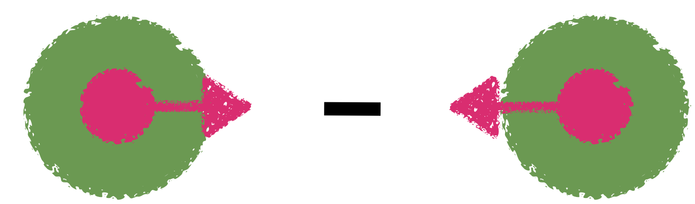
$\mathrm{Im}(A_{+-,+-}$ & $A_{++,++}) = -\mathrm{Im}(A_{-+,-+}$ & $A_{--,--})$

U/L polarized quarks in L/U polarized target

$$-i\,\frac{\bar{k}_1\Delta_2-\bar{k}_2\Delta_1}{M^2}F_{14}=\left(A_{++,++}+A_{+-,+-}-A_{-+,-+}-A_{--,--}\right)/4$$



$$i\,\frac{\bar{k}_1\Delta_2-\bar{k}_2\Delta_1}{M^2}G_{11}=\left(A_{++,++}-A_{+-,+-}+A_{-+,-+}-A_{--,--}\right)/4$$



In terms of Generalized Parton Correlation Functions...

$$\begin{aligned}W^{\gamma^+}_{\Lambda',\Lambda} &= \overline{U}(p',\Lambda')\Bigg[\overbrace{\frac{P^+}{M}(A_1^F+xA_2^F-2\xi A_3^F)}^{type1}+\overbrace{\frac{i\sigma^{+k}}{M}A_5^F+\frac{i\sigma^{+\Delta}}{M}A_6^F}^{type2}+\overbrace{\frac{P^+ i\sigma^{k\,\Delta}}{M^3}(A_8^F+xA_9^F)}^{type3}\\ &\quad+\underbrace{\frac{P^+ i\sigma^{k\,N}}{M^3}(A_{11}^F+xA_{12}^F)+\frac{P^+ i\sigma^{\Delta\,N}}{M^3}(A_{14}^F-2\xi A_{15}^F)}_{type4}\Bigg]U(p,\Lambda)\\ &= A^{[\gamma^+]}_{\Lambda'+;\Lambda+}+A^{[\gamma^+]}_{\Lambda'-;\Lambda-}\end{aligned}$$

F14

has a different behavior under Parity

Gauge-link to be thought of...

Usual distribution functions depend on a vector *n* that comes from the gauge link

Proposed most general correlator:

Goeke et al, Phys.Lett. B567

$$\Phi_{ij}(P,k,S|n) = \int \frac{d^4\xi}{(2\pi)^4}\, e^{ik\cdot\xi}\langle P,S|\bar{\psi}_j(0)\mathcal{W}(0,\xi|n)\psi_i(\xi)|P,S\rangle.$$

Choice of *n* → leading contribution of the correlator in view of factorization theorems

→ turns out that DIS & SIDIS are LC dominated

→ the hard photon selects the "+"-direction as leading contribution

PDFs & TMDs: z-direction cannot be arbitrarily rotated.

GPCFs: no known probes

n unconstrained: chosen to reproduce PDF & TMD limits

Leading-order ⇔ 2-body scattering

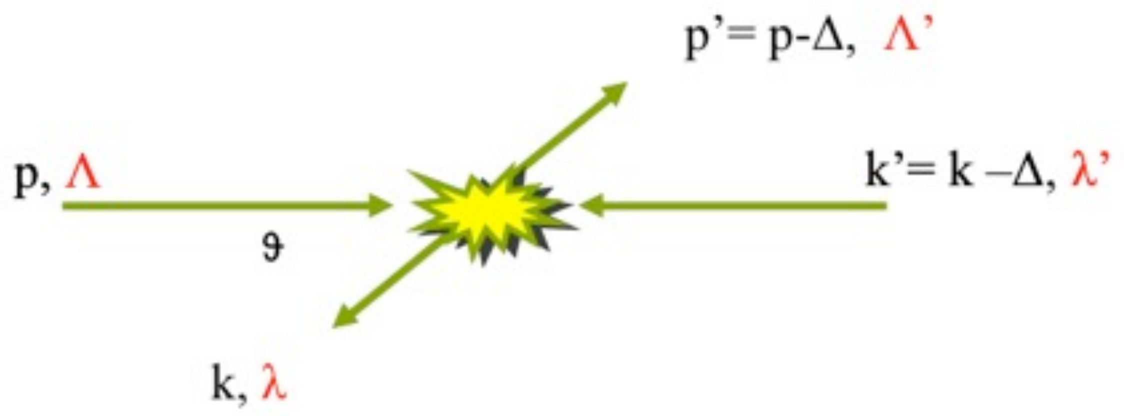
Landshoff, Polkinghorne and Short, NPB28

CoM 2-body scattering must occur on a plane

Helicity amplitudes do not contain info on the LF

Our statement:

if it cannot be explained by 2-body scattering ⇒ the dependence in *n* introduces a 3rd body



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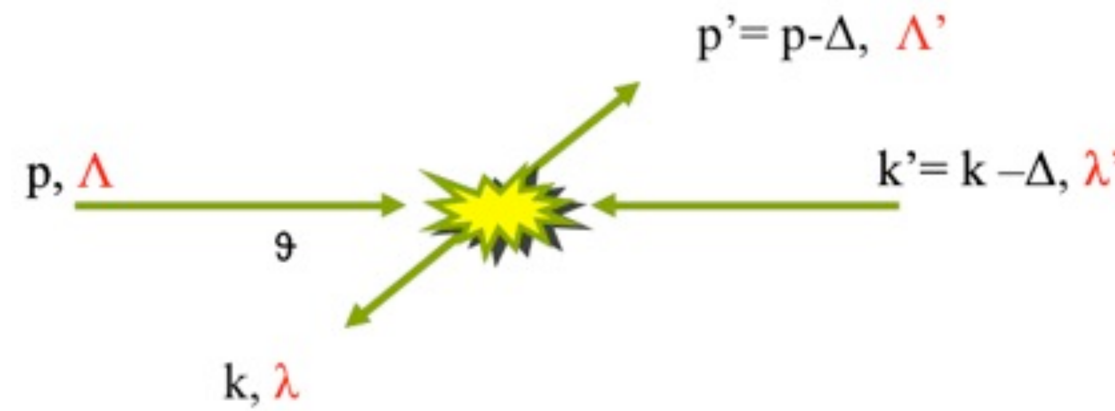
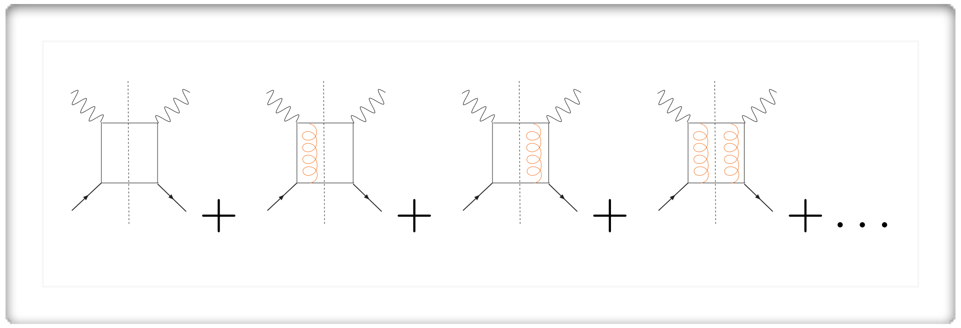


Example:

Sivers function

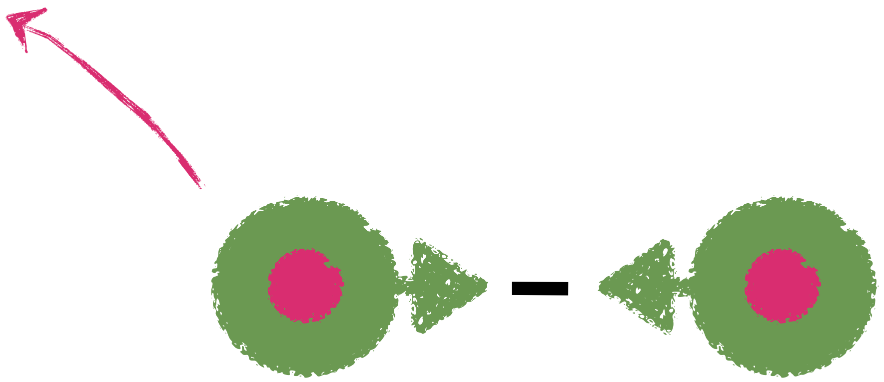
- T-odd, so not allowed as if 2-body scattering
- Needs a third body: different symmetries, more flexibility
- That 3rd body comes from the gauge link→ final state interaction



Here?

What is the concept of *twist* when adding scales ?

Unpolarized quarks in longitudinally polarized target



What happens in models? Is it zero?

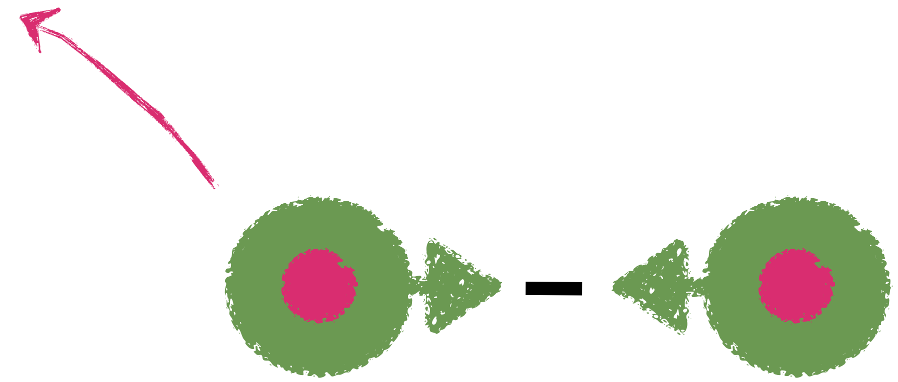
Not in quark models
E.g. in the bag...

$$i\frac{\left(\vec{k}_T \times \vec{\Delta}_T\right)_z}{M^2} F_{14}^u \propto i\frac{2}{3}\frac{\left(\vec{k}_T \times \vec{\Delta}_T\right)_z}{k_3 k'} t_1(k) t_1(k')$$

A. Rajan et al, in preparation

Other calculations are also non-zero, but I cite the one I control.
See Meissner et al, Lorce et al, Kanazawa et al, Mukherjee et al and A. Rajan et al [in preparation]]for more results....

Unpolarized quarks in longitudinally polarized target



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Why is it so?

We think

- the ``low-energy quark" (constituent, preconfined...) implies complex dynamics
- expansion in twist does not match model content

- interpretation still in progress

Observables with EW currents

Warning:

it is a sketched observable, we only care for the helicity amps

See S. Liuti's talk for more info on observables and GTMDs

We consider the HELICITY AMPLITUDES for the EW DIS processes

$$T_{\Lambda_{(W,Z)}\Lambda,\Lambda'_{(W,Z)}\Lambda'} = \mathrm{Amp}\left[\ (W^{\pm},\, Z^0) + \mathrm{Nucleon} \to (W^{\pm},\, Z^0)' + \mathrm{Nucleon}'.\ \right]$$

The hard part comes from the subprocess

$$g^{\pm 1, \pm 1}_{\lambda,\lambda'} = \mathrm{Amp}\left[\ (W^{\pm},\, Z^0) + \mathrm{quark} \to (W^{\pm},\, Z^0) + \mathrm{quark}'.\ \right]$$

for transversely polarized vector bosons

and the quark EW current

$$J^{\mu} = \bar{\psi}\gamma^{\mu}(g_V \mathbb{1} - g_A \gamma^5)\psi$$

X.D. Ji, Nucl.Phys.B402

Observables with EW currents

X.D. Ji, Nucl.Phys.B402

AC et al, drafting

Parity conserving structure

$$G_1 \propto T_{1+,1+} - T_{1-,1-} - T_{-1+,-1+} + T_{-1-,--}$$

$$G_1 \propto g^{1,1}_{++} \otimes (A_{+,+;+,+} - A_{-,+;-,+}) - g^{-1-1}_{--} \otimes (A_{+,-;+,-} - A_{-,-;-,-})$$

with $$g^{\pm1,\pm1}_{\pm\pm} = \frac{q^- \sqrt{2k^+ k'^+}}{\hat{s}} [(g'_V g_V + g'_A g_A) \mp (g'_V g_A + g'_A g_V)]$$

$$\begin{aligned} G_1 \propto\ & (g'_V g_V + g'_A g_A) \otimes (A_{++,++} - A_{-+,-+} + A_{--,--} - A_{+-,+-}) \\ -\ & (g'_V g_A + g'_A g_V) \otimes (A_{++,++} - A_{-+,-+} - A_{--,--} + A_{+-,+-}) \end{aligned}$$

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$$G_1 \propto T_{1+,1+} - T_{1-,1-} - T_{-1+,-1+} + T_{-1-,--}$$

$$G_1 \propto g^{1,1}_{++} \otimes (A_{+,+;+,+} - A_{-,+;-,+}) - g^{-1-1}_{--} \otimes (A_{+,-;+,-} - A_{-,-;-,-})$$

with $g^{\pm 1,\pm 1}_{\pm\pm} = \frac{q^- \sqrt{2k^+ k'^+}}{\hat{s}} [(g'_V g_V + g'_A g_A) \mp (g'_V g_A + g'_A g_V)]$

P-even struct.

$$\begin{aligned} G_1 \propto\ & (g'_V g_V + g'_A g_A) \otimes (A_{++,++} - A_{-+,-+} + A_{--,--} - A_{+-,+-}) \\ -\ & (g'_V g_A + g'_A g_V) \otimes (A_{++,++} - A_{-+,-+} - A_{--,--} + A_{+-,+-}) \end{aligned}$$

G_{11}

F_{14}

P-odd struct.

Observables with EW currents

X.D. Ji, Nucl.Phys.B402

AC et al, drafting

Parity conserving structure

$$G_1 \;\propto\; T_{1+,1+} - T_{1-,1-} - T_{-1+,-1+} + T_{-1-,--}$$

$$G_1 \;\propto\; g^{1,1}_{++} \otimes (A_{+,+;+,+} - A_{-,+;-,+}) - g^{-1-1}_{--} \otimes (A_{+,-;+,-} - A_{-,-;-,-})$$

with $g^{\pm1,\pm1}_{\pm\pm} = \frac{q^- \sqrt{2k^+ k'^+}}{\hat{s}} \left[(g'_V g_V + g'_A g_A) \mp (g'_V g_A + g'_A g_V) \right]$

P-even struct.

$$\begin{aligned} G_1 \;\propto\; & (g'_V g_V + g'_A g_A) \otimes (A_{++,++} - A_{-+,-+} + A_{--,--} - A_{+-,+-}) \\ & - (g'_V g_A + g'_A g_V) \otimes (A_{++,++} - A_{-+,-+} - A_{--,--} + A_{+-,+-}) \end{aligned}$$

G_{11}

F_{14}

P-odd struct.

Parity non-conserving structure

P-even struct.

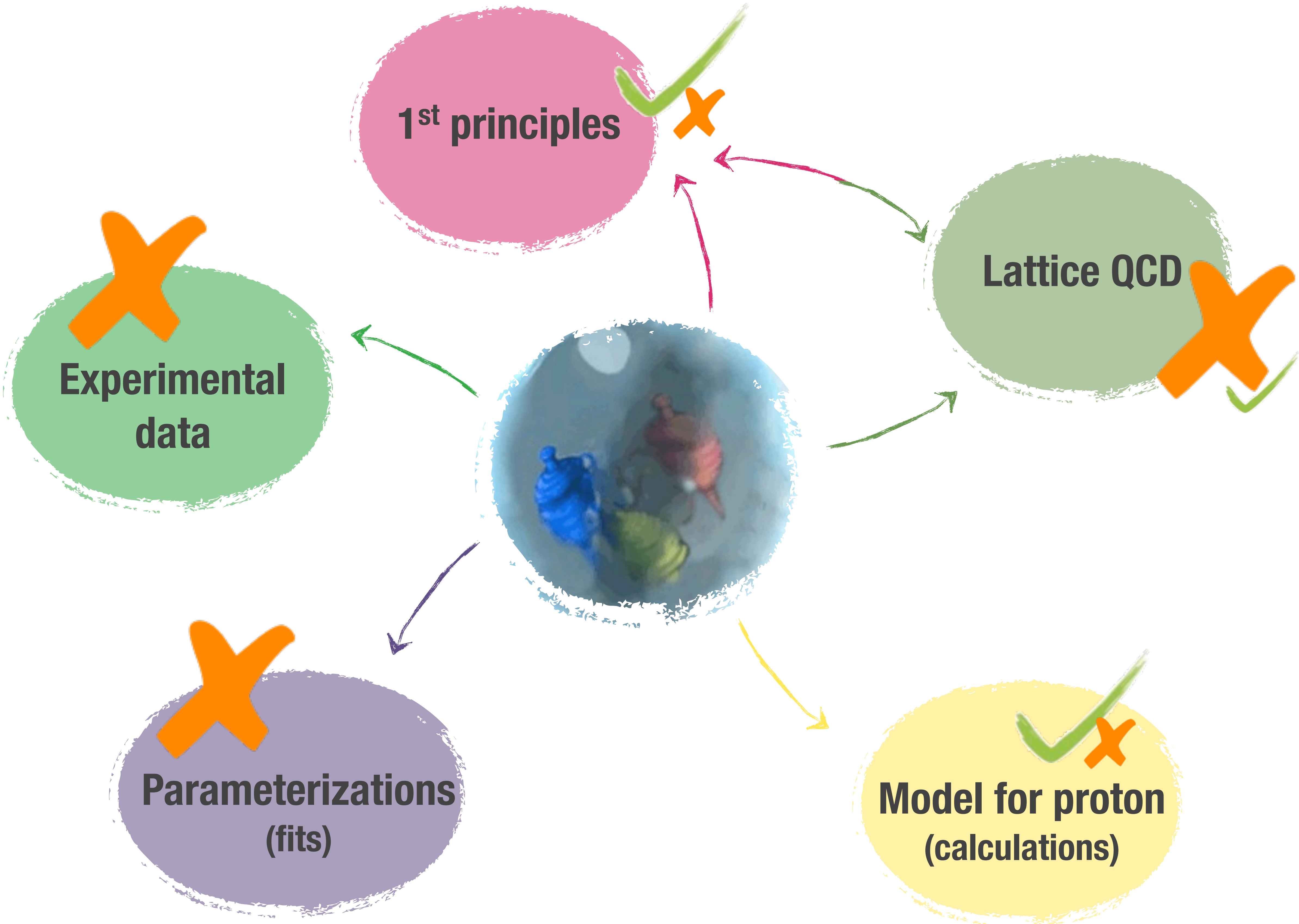
$$\begin{aligned} A_1 \;\propto\; & (g'_V g_V + g'_A g_A) \otimes (A_{++,++} - A_{-+,-+} - A_{--,--} + A_{+-,+-}) \\ & - (g'_V g_A + g'_A g_V) \otimes (A_{++,++} - A_{-+,-+} + A_{--,--} - A_{+-,+-}), \end{aligned}$$

F_{14}

G_{11}

P-odd struct.

How can we access OAM?



Higher-twist contributions

- Helicity amplitude combinations of "F_{14}" do exist
- **Final state interactions** transform differently under parity
  - It comes at twist-3 with the structure $\langle \mathbf{S}_L \times \boldsymbol{\Delta}_T \rangle$
  - Helicity amplitudes here follow $A^{tw3}_{\Lambda'\pm,\Lambda\pm} \to A^{tw2}_{\Lambda'\pm,\Lambda\mp}$
  - so that we can build the ``LU" structure in terms of twist-3 GTMDs

$$-\frac{4}{P^+}\left[\frac{\bar{\mathbf{k}}_T\cdot\boldsymbol{\Delta}_T}{\Delta_T}F_{27}+\Delta_T F_{28}-\left(\frac{\bar{\mathbf{k}}_T\cdot\boldsymbol{\Delta}_T}{\Delta_T}G_{27}+\Delta_T G_{28}\right)\right] = A^{tw3}_{++,++}+A^{tw3}_{+-,+-}-A^{tw3}_{-+,-+}-A^{tw3}_{--,--}$$

Great!

Let's go back to GPDs...

Great news is that those GTMDs do admit a GPD limit!

twist-3 GPDs

$$
\begin{aligned}
2\widetilde{H}_{2T} + E_{2T} &= \int d^2\mathbf{k}_T \left[\left(\frac{\mathbf{k}_T \cdot \boldsymbol{\Delta}_T}{\Delta_T^2} \right) F_{21} + F_{22} \right] \\
\widetilde{E}_{2T} &= -2 \int d^2\mathbf{k}_T \left[\left(\frac{\mathbf{k}_T \cdot \boldsymbol{\Delta}_T}{\Delta_T^2} \right) F_{27} + F_{28} \right] = G_2 = \widetilde{H}^3_- \\
2\widetilde{H}'_{2T} + E'_{2T} &= \int d^2\mathbf{k}_T \left[\left(\frac{\mathbf{k}_T \cdot \boldsymbol{\Delta}_T}{\Delta_T^2} \right) G_{21} + G_{22} \right] \\
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MMS
Kiptily & Polyakov EPJC37
Belitsky *et al* NPB629

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- gauge-invariant
- canonical

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2\widetilde{H}'_{2T} + E'_{2T} &= \int d^2\mathbf{k}_T \left[\left(\frac{\mathbf{k}_T \cdot \mathbf{\Delta}_T}{\Delta_T^2} \right) G_{21} + G_{22} \right] \\
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\end{aligned}$$

MMS
Kiptily & Polyakov EPJC37
Belitsky *et al* NPB629

- **gauge-invariant**

$$L_{FS} = \frac{\langle PS| \int d^3\vec{r}\, \bar{\psi}(\vec{r}) \gamma^+ (\vec{r}_\perp \times i\vec{D}_\perp) \psi(\vec{r}) |PS\rangle}{\langle PS|PS\rangle} = \int (\vec{b}_\perp \times \vec{k}_\perp) W_{FS}(x, \vec{b}_\perp, \vec{k}_\perp) dx d^2\vec{b}_\perp d^2\vec{k}_\perp$$

Ji, Xiong & Yuan, PRL109

- **canonical**

$$l_q = \frac{\langle PS| \int d^3\vec{r}\, \overline{\psi}(\vec{r}) \gamma^+ (\vec{r}_\perp \times i\vec{\partial}_\perp) \psi(\vec{r}) |PS\rangle}{\langle PS|PS\rangle} = \int (\vec{b}_\perp \times \vec{k}_\perp) W_{LC}(x, \vec{b}_\perp, \vec{k}_\perp) dx d^2\vec{b}_\perp d^2\vec{k}_\perp$$

Let's go back to GPDs...

Great news is that those GTMDs do admit a GPD limit!

twist-3 GPDs →

$$
\begin{aligned}
2\widetilde{H}_{2T} + E_{2T} &= \int d^2\mathbf{k}_T \left[\left(\frac{\mathbf{k}_T \cdot \mathbf{\Delta}_T}{\Delta_T^2}\right) F_{21} + F_{22}\right] \\
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MMS
Kiptily & Polyakov EPJC37
Belitsky *et al* NPB629

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$$L_{FS} = \frac{\langle PS| \int d^3\vec{r}\, \bar{\psi}(\vec{r})\gamma^+(\vec{r}_\perp \times i\vec{D}_\perp)\psi(\vec{r})|PS\rangle}{\langle PS|PS\rangle} = \int (\vec{b}_\perp \times \vec{k}_\perp) W_{FS}(x, \vec{b}_\perp, \vec{k}_\perp)\, dx\, d^2\vec{b}_\perp\, d^2\vec{k}_\perp$$

related to twist-2 & twist-3 GPDs

Ji, Xiong & Yuan, PRL109

- **canonical**

$$l_q = \frac{\langle PS| \int d^3\vec{r}\, \overline{\psi}(\vec{r})\gamma^+(\vec{r}_\perp \times i\vec{\partial}_\perp)\psi(\vec{r})|PS\rangle}{\langle PS|PS\rangle} = \int (\vec{b}_\perp \times \vec{k}_\perp) W_{LC}(x, \vec{b}_\perp, \vec{k}_\perp)\, dx\, d^2\vec{b}_\perp\, d^2\vec{k}_\perp$$

related to twist-2 GPDs
&
its gauge-invariant extension is twist-3

Relation to GPDs

- **Ji's Sum Rule PRL97**

$$J_{q(g)} = \frac{1}{2}\int_{-1}^{1} dx\, x(H_{q(g)}(x) + E_{q(g)}(x))$$

Sum Rule

$$\Rightarrow L_q = \frac{1}{2}\int_{-1}^{1} dx\, x\,(H_q(x) + E_q(x)) - \frac{1}{2}\int_{-1}^{1} dx\, \widetilde{H}(x)$$

- **Penttinen *et al* PLB491**

Sum Rule

$$\int dx\, x\, G_2^q(x) = \frac{1}{2}\left[-\int dx x(H^q(x) + E^q(x)) + \int dx \tilde{H}^q(x)\right]$$

$$= -L_q$$

- **Hatta *et al* JHEP10**

WW approx

$$L_q(x) = x\int_x^1 \frac{dy}{y}(H_q(y) + E_q(y)) - x\int_x^1 \frac{dy}{y^2}\widetilde{H}_q(y)$$

We've pointed out an observable in DVCS to access G_2

Conclusions

- The combination $A_{++,++}+A_{+-,+-}-A_{-+,-+}-A_{--,--}$ at twist-2 cannot be explained by 2-body scattering
- The combination $A_{++,++}+A_{+-,+-}-A_{-+,-+}-A_{--,--}$ at twist-3 is related to Ji's OAM
  - from unp. quarks in L pol proton to transverse direction corr. with FSI/3rd body
- The Helicity Amps $A_{++,++}+A_{+-,+-}-A_{-+,-+}-A_{--,--}$ appear with parity-odd structure in EW DIS

- Outlook
  - What is the role of the gauge link?
  - soon model interpretations

Canonical vs. gauge-invariant

- In WW approximation, doesn't matter

$$L_q(x) = L_q^{WW}(x) + \overline{L}_q(x)$$
$$\mathcal{L}_q(x) = L_q^{WW}(x) + \overline{\mathcal{L}}_q(x)$$

genuine twist-3 contribution

- Anyway, we know very little about twist-3 GPDs, so WW is fine for now

except for some model calculations

- genuine twist-3 contributions are expected to be smaller than the WW's

Canonical vs. gauge-invariant

- In WW approximation, doesn't matter

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genuine twist-3 contribution

- Anyway, we know very little about twist-3 GPDs, so WW is fine for now

  except for some model calculations

- genuine twist-3 contributions are expected to be smaller than the WW's

- To evaluate $L_q^{WW}(x)$, we can
  - use a parameterization for twist-2 GPDs (Goldstein, Gonzalez-Hernandez & Liuti, PRD84)
  - apply WW formula

OAM density

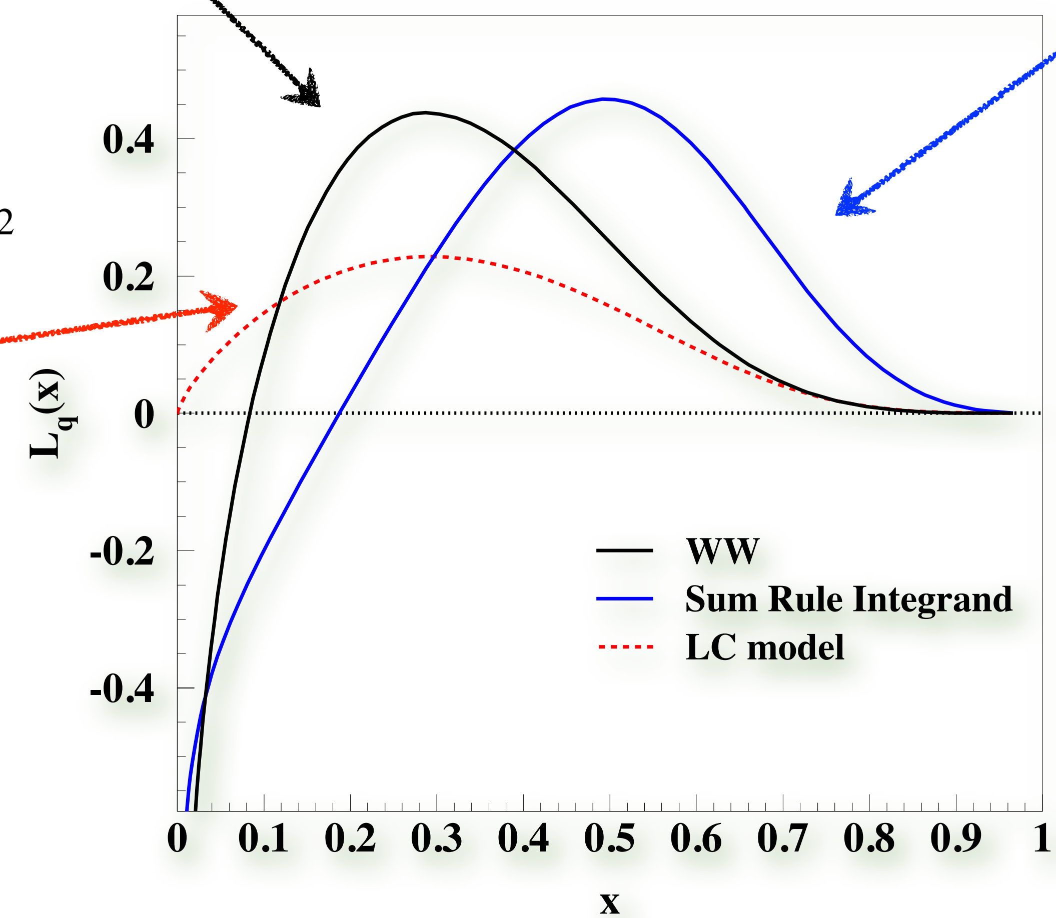
$$L_q(x) = x\int_x^1 \frac{dy}{y}(H_q(y)+E_q(y)) - x\int_x^1 \frac{dy}{y^2}\widetilde{H}_q(y)$$

$$\text{“}L_q(x)\text{”} \to \frac{1}{2}\,x(H_q(x)+E_q(x)) - \frac{1}{2}\,\widetilde{H}(x)$$

$$\mathscr{L}_q^z = \frac{g^2}{16\pi^3}\int_0^1 dx \int d^2\overrightarrow{k_\perp}(1-x)|\Psi^{\uparrow}_{\frac{-1}{2}}|^2$$

=0.11

Burkardt & BC



Black and blue give the same integrated result L_q^{WW}=0.13

OAM density

$$L_q(x) = x\int_x^1 \frac{dy}{y}(H_q(y)+E_q(y)) - x\int_x^1 \frac{dy}{y^2}\widetilde{H}_q(y)$$

$$\text{"}L_q(x)\text{"} \rightarrow \frac{1}{2}\,x(H_q(x)+E_q(x)) - \frac{1}{2}\,\widetilde{H}(x)$$

$$\mathscr{L}_q^z = \frac{g^2}{16\pi^3}\int_0^1 dx \int d^2\vec{k}_\perp (1-x)|\Psi^{\uparrow}_{\frac{-1}{2}}|^2$$

=0.11

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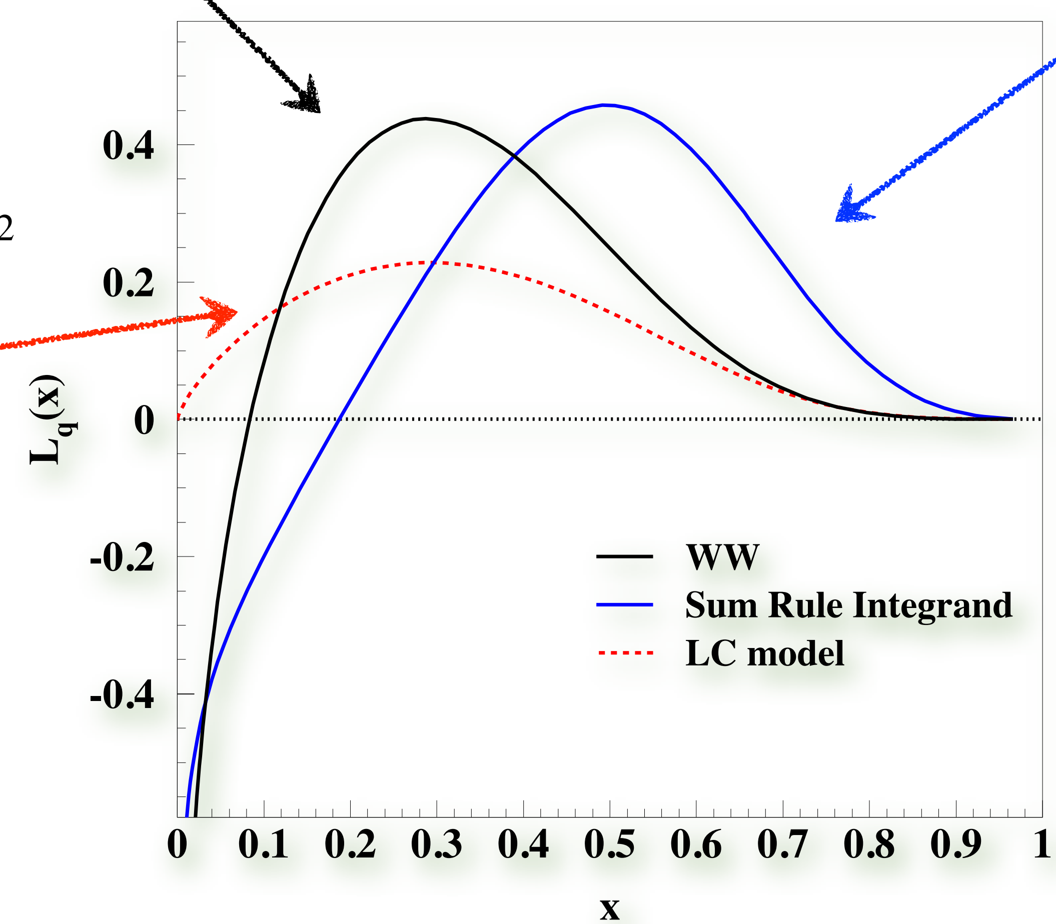


Black and blue give the same integrated result L_q^{WW}=0.13

Ok, so, now, can we access G_2?

DVCS @ HERMES (JHEP06)

formalism from Belitsky *et al* NPB629

$$G_2 = \widetilde{E}_{2T} = \widetilde{H}^3_-$$

follow the arrows

HERE IS THE OBSERVABLE

$$\mathcal{A}_{\mathrm{UL}}(\phi) \simeq \sum_{n=1}^{3} A_{\mathrm{UL}}^{\sin(n\phi)} \sin(n\phi) + A_{\mathrm{UL}}^{\cos(0\phi)},$$

$$\mathcal{A}_{\mathrm{LL}}(\phi) \simeq \sum_{n=0}^{2} A_{\mathrm{LL}}^{\cos(n\phi)} \cos(n\phi).$$

Asymmetry Amplitude	Contributory Fourier-Coefficients	Power of $\frac{1}{Q}$ Suppression	Dominant CFF Dependence	Twist Level
$A_{\mathrm{UL}}^{\sin(2\phi)}$	$s^{\mathrm{I}}_{2,\mathrm{LP}}$	2	$\mathrm{Im}\,\mathcal{C}^{\mathrm{I}}_{\mathrm{LP}}$	3
	$s^{\mathrm{DVCS}}_{2,\mathrm{LP}}$	2	$\mathrm{Im}\,\mathcal{C}^{\mathrm{DVCS}}_{\mathrm{T,LP}}$	2
$A_{\mathrm{LL}}^{\cos\phi}$	$c^{\mathrm{I}}_{1,\mathrm{LP}}$	1	$\mathrm{Re}\,\mathcal{C}^{\mathrm{I}}_{\mathrm{LP}}$	2
	$c^{\mathrm{DVCS}}_{1,\mathrm{LP}}$	3	$\mathrm{Re}\,\mathcal{C}^{\mathrm{DVCS}}_{\mathrm{LP}}$	3

$$\begin{Bmatrix} c^{\mathcal{I}}_{2,\mathrm{LP}} \\ s^{\mathcal{I}}_{2,\mathrm{LP}} \end{Bmatrix} = \frac{16\Lambda K^2}{2-x_{\mathrm{B}}} \begin{Bmatrix} -\lambda y \\ 2-y \end{Bmatrix} \begin{Bmatrix} \Re\mathrm{e} \\ \Im\mathrm{m} \end{Bmatrix} \mathcal{C}^{\mathcal{I}}_{\mathrm{LP}}(\mathcal{F}^{\mathrm{eff}}),$$

with $\mathcal{F} = \{\mathcal{H}, \mathcal{E}, \widetilde{\mathcal{H}}, \widetilde{\mathcal{E}}\}$ $\quad \mathcal{F}^{\mathrm{eff}} \equiv -2\xi\left(\frac{1}{1+\xi}\mathcal{F} + \mathcal{F}^3_+ - \mathcal{F}^3_-\right)$

$$\mathcal{C}^{\mathcal{I}}_{\mathrm{LP}} = \frac{x_{\mathrm{B}}}{2-x_{\mathrm{B}}}(F_1+F_2)\left(\mathcal{H} + \frac{x_{\mathrm{B}}}{2}\mathcal{E}\right) + F_1\widetilde{\mathcal{H}} - \frac{x_{\mathrm{B}}}{2-x_{\mathrm{B}}}\left(\frac{x_{\mathrm{B}}}{2}F_1 + \frac{\Delta^2}{4M^2}F_2\right)\widetilde{\mathcal{E}},$$

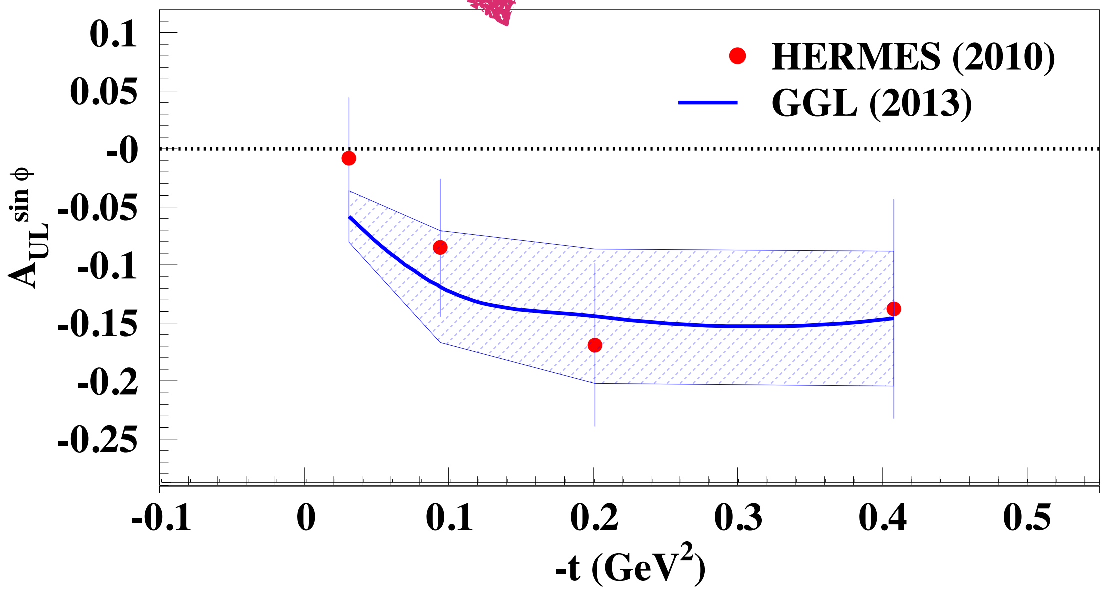
$$\widetilde{\mathcal{H}}^{eff} = -2\xi\left(\frac{1}{1+\xi}\widetilde{\mathcal{H}} + \widetilde{\mathcal{H}}_3^+ - \widetilde{\mathcal{H}}_3^-\right)$$

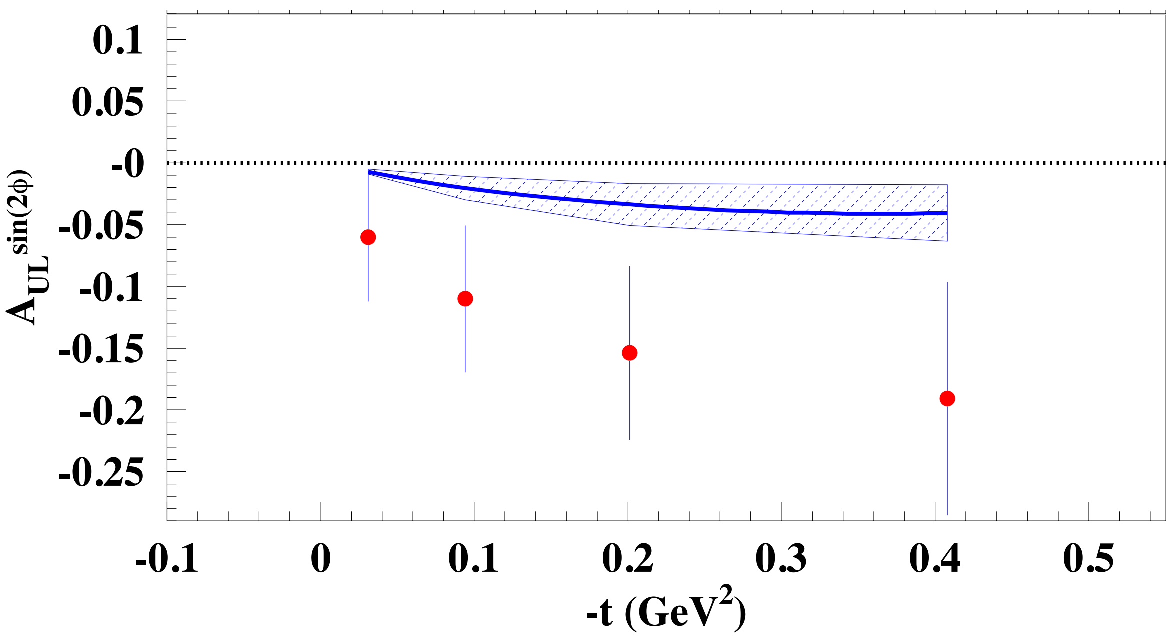
here it is...

OAM from sin(2φ) modulation

- Here we use the WW expression with the GPD fits of GGL

- the sin(φ) is prediction and autoconsistency check. They do great!
- the sin(2φ) is prediction
  - first try to ``access" to OAM !





Nonzero!
and even sizable!

Conclusions

- The combination $A_{++,++}+A_{+-,+-}-A_{-+,-+}-A_{--,--}$ is parity-odd at twist-2
- The combination $A_{++,++}+A_{+-,+-}-A_{-+,-+}-A_{--,--}$ is not parity-odd at twist-3
  - from unp. quarks in L pol proton to transverse direction corr. with FSI/3rd body

- to be translated in terms of Wigner functions (à la Ji, does the gauge link matter?, ...)
- can we go beyond WW approximation?

- Anyhow, we've spotted an observable!
  - TSA for DVCS

$$A_{UL} = \frac{a \sin\phi + b \sin 2\phi}{c_0 + c_1 \cos\phi + c_2 \cos 2\phi}$$

waiting for the promising CLAS data !