ECT*

EUROPEAN CENTRE FOR THEORETICAL STUDIES
IN NUCLEAR PHYSICS AND RELATED AREAS
TRENTO, ITALY
Institutional Member of the ESF Expert Committee NuPECC

Castello di Trento ("Trint"). watercolour, 19.8 x 27.7, painted by A. Dürer on his way back from Venice (1495) British Museum, London.

Spin and Orbital Angular Momentum of Quarks and Gluons in the Nucleon

25-29 August, 2014

Main Topics

Introduction to quark and gluon angular momentum
Spin decompositions and sum rules
Gauge independence of matrix elements
Angular momentum, GPDs and TMDs
Angular momentum in phenomenological models
Angular momentum on the lattice
Angular momentum via TMD and GPD measurements in DIS and SIDIS
Angular momentum via TMD measurements in hadronic collisions

Key Participants

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Organizers

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Study of Strongly Interacting Matter
HadronPhysics3

Director of the ECT*: Professor Wolfram Weise (ECT*)

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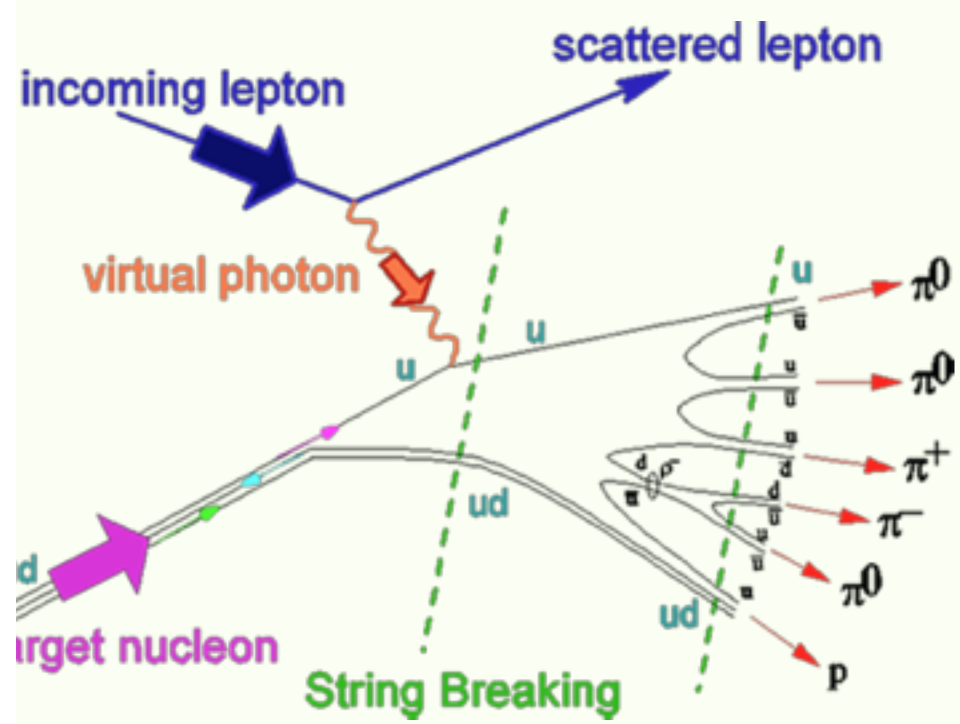
Spin-orbit interactions, TMDs and Bessel weighted asymmetries

Leonard Gamberg
Penn State University



Outline

- Merit of Weighted Asymmetries
- Merit of Bessel Weighting & moments and Fourier Transform of TMDs-- FT &"S/T" pic of SIDIS
- BWA in Parton model connection w/ conventional weighting
- Impact on studying BW and TMD evolution
- **Sketch** ... **Elements TMD Factorization-SIDIS**
- Cancellation Soft & Pert. some other univ. factors BWA-JMY
- Cancellation of Universal & flavor indep. factors in BWAs-Collins
- Use of Bessel Weighting BW of experimental observables $A_{LL}(b_T)$

Comments on Weighting

- Using technique of weighting enables one to disentangle in a model independent way the CS in terms of transverse momentum moments of TMDs

- Convert the convolutions in the cross section into simple products
  Kotzinian, Mulders PLB 97, Boer, Mulders PRD 98

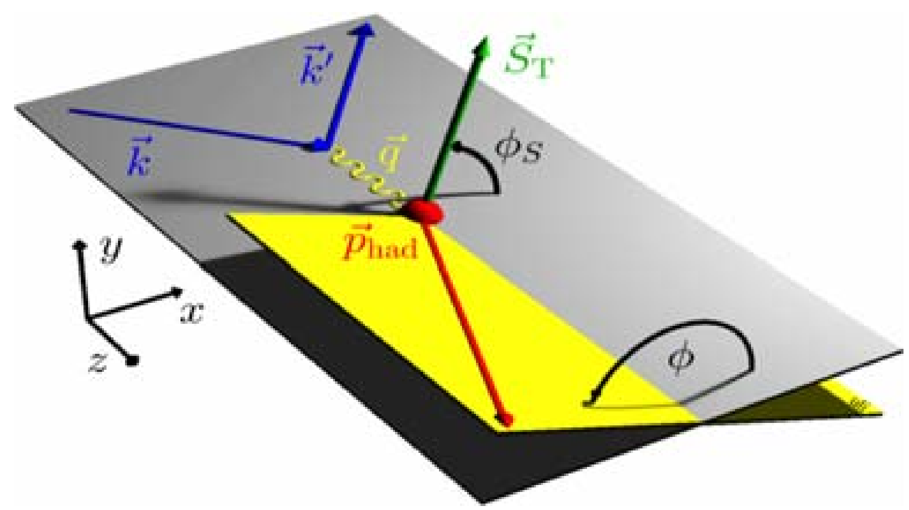
- Bessel Weighting solves problem of infinite contribution from large transverse momentum that arise from using "conventional weighting
  Boer, Gamberg,Musch,Prokudin JHEP 2011

- Explore impact these BWA have on studying the scale dependence of the SIDIS cross section at small to moderate transverse momentum where the TMD framework is expected to give a good description of the cross section **Boer, Gamberg, Musch,Prokudin JHEP 2011**

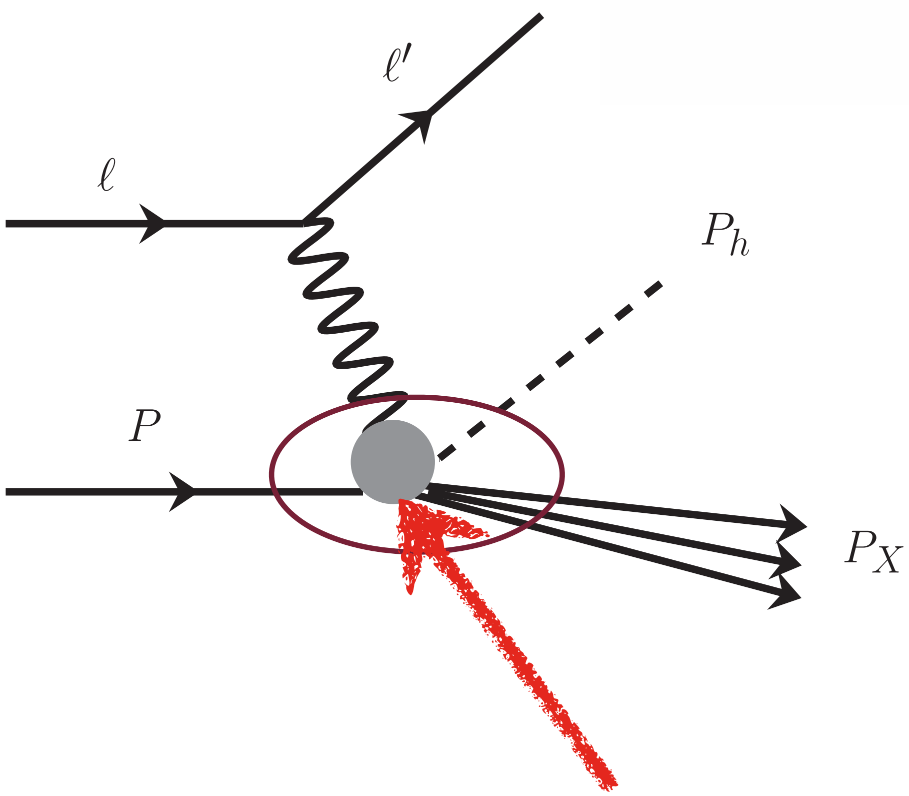
Part 1

- Demonstrate BW results in model indep. deconvolution of TMDs
- Consider in GPM first
- Demonstrate how this rep results in model indep. observables BWAs generalization of conventional WAs Kotzinian & Mulders PLB97

Factorization Parton Model



Kotzinian NPB 95,
Mulders Tangermann NPB 96,
Boer & Mulders PRD 97
Bacchetta et al JHEP 08



Factorize

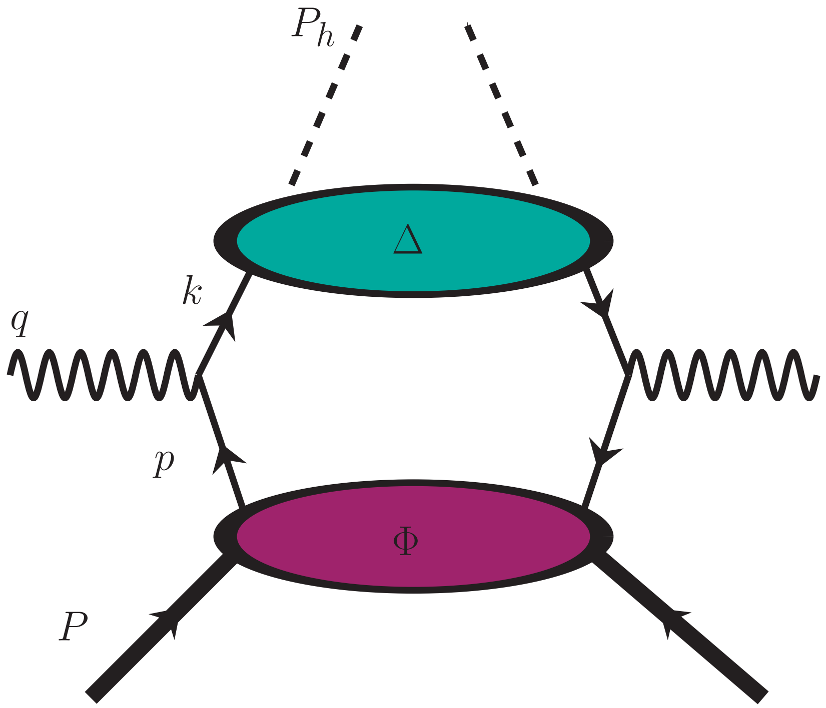
$$\frac{d\sigma}{dx_B\, dy\, d\psi\, dz_h\, d\phi_h\, |\boldsymbol{P}_{h\perp}|\, d|\boldsymbol{P}_{h\perp}|} = \frac{\alpha^2}{x_B\, y\, Q^2}\, \frac{y^2}{(1-\varepsilon)} \left(1+\frac{\gamma^2}{2x_B}\right) L_{\mu\nu}W^{\mu\nu},$$

Factorization P_T of hadron small sensitive to intrinsic transv. momentum of partons

$$W^{\mu\nu}(q,P,S,P_h) = \int \frac{d^2\mathbf{p}_T}{(2\pi)^2}\int \frac{d^2\mathbf{k}_T}{(2\pi)^2}\delta^2\left(\mathbf{p}_T - \frac{\mathbf{P}_{h\perp}}{z_h} - \mathbf{k}_T\right)\mathrm{Tr}\left[\Phi(x,\mathbf{p}_T)\gamma^\mu\Delta(z,\mathbf{k}_T)\gamma^\nu\right]$$

$$\Phi(x,\mathbf{p}_T) = \int dp^-\Phi(p,P,S)|_{p^+=x_BP^+}\,,\qquad \Delta(z,\mathbf{k}_T) = \int dk^-\Delta(k,P_h)|_{k^-=\frac{P^-}{z_h}}$$

Small transverse momentum



<u>Purely Kinematic-integrate over small momentum component</u>

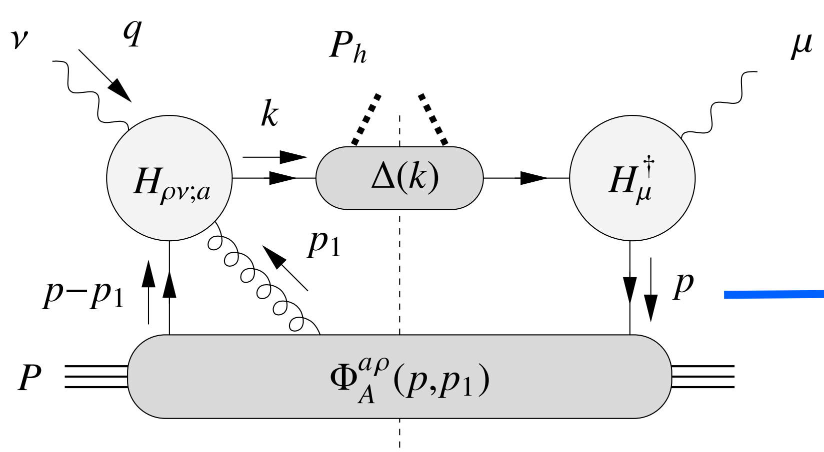
Must also respect gauge invariance

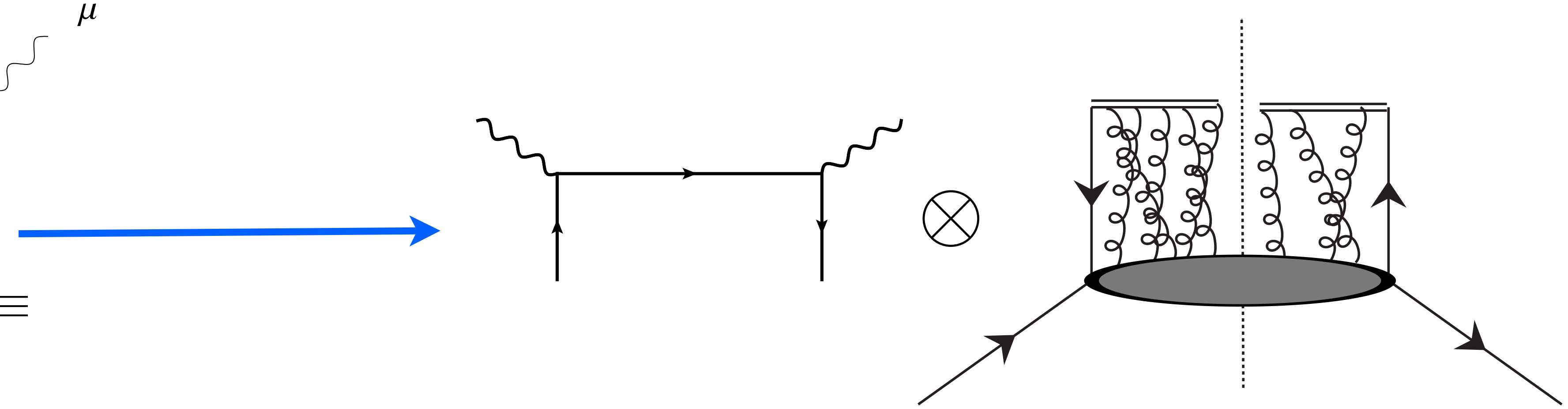
Minimal requirement satisfy **color** gauge invariance

Minimal Requirement for PARTON MDL Factorization

Gauge link determined re-summing leading gluon interactions btwn soft and hard
Efremov,Radyushkin Theo. Math. Phys. 1981, Collins, Soper NPB 1981, 1982,Collins PLB 2002,
Belitsky, Ji, Yuan NPB 2003, Boer, Bomhof, Mulders Pijlman, et al. 2003 - 2008- NPB, PLB, PRD,

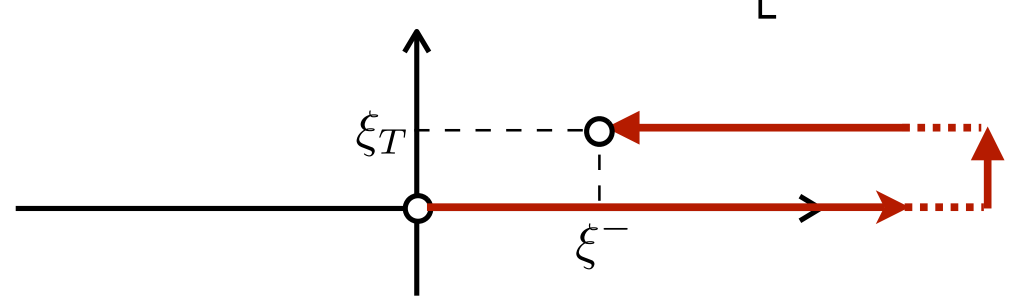
$$\Phi^{[\mathcal{U}[C]]}(x,p_T) = \int \frac{db^- d^2 b_T}{2(2\pi)^3} e^{i\left(p^+ b^- - p_T\cdot b_T\right)} \langle P|\,\overline{\psi}(0)\mathcal{U}^{[\mathcal{C}]}_{[0,b]}\psi(b^-,b_T)|P\rangle\Big|_{b^+=0}$$





- **The path $[C]$ is fixed by hard subprocess within hadronic process.**

$$W^{\mu\nu}(q,P,P_h;S) = \int d^4p\, d^4k\, \delta^4(p+q-k)\mathrm{Tr}\left[\Phi^{\mathcal{U}^{[C]}_{[\infty;b]}}(p)\gamma^\mu(p,k)\Delta(k)\gamma^\nu(p,k)\right]$$



Factorization Parton Model-predicts existence of T-odd PDFs and TSSAs--Boer-Mulders PRD 1998

Partonic picture Structure Functions momentum CONVOLUTION

$$\mathcal{C}\big[w\,f\,D\big] = x \sum_q e_a^2 \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, \delta^{(2)}\big(\boldsymbol{p}_T - \boldsymbol{k}_T - \boldsymbol{P}_{h\perp}/z\big)\, w(\boldsymbol{p}_T, \boldsymbol{k}_T)\, f^a(x, p_T^2)\, D^a(z, k_T^2)$$

$$F_{UU,T} = \mathcal{C}\big[f_1 D_1\big],$$

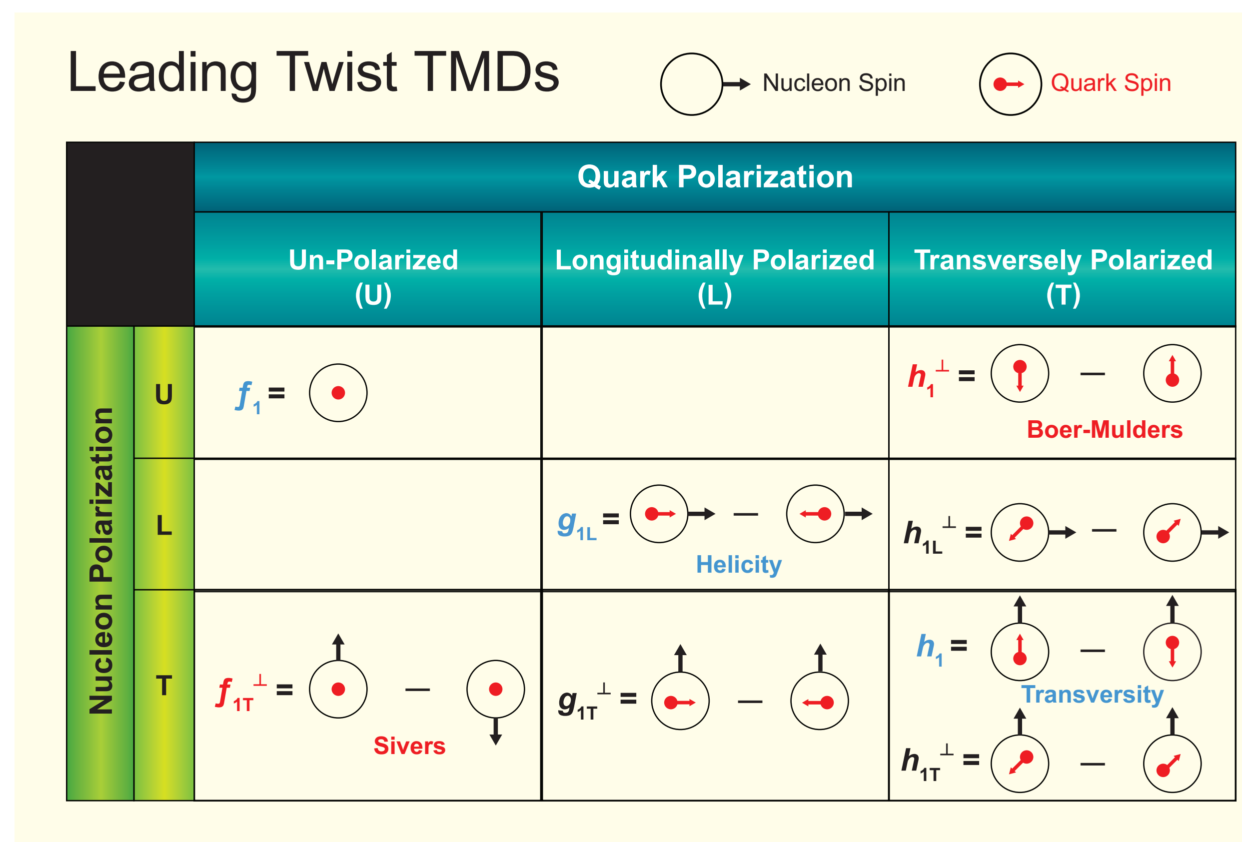
$$F_{LL} = \mathcal{C}\big[g_{1L} D_1\big],$$

$$F_{UT,T}^{\sin(\phi_h-\phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M} f_{1T}^{\perp} D_1\right],$$

$$F_{UT}^{\sin(\phi_h+\phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h} h_1 H_1^{\perp}\right],$$

$$F_{UL}^{\sin 2\phi_h} = \mathcal{C}\left[-\frac{2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T) - \boldsymbol{k}_T\cdot\boldsymbol{p}_T}{M M_h} h_{1L}^{\perp} H_1^{\perp}\right],$$

$$F_{UU}^{\cos 2\phi_h} = \mathcal{C}\left[-\frac{2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T) - \boldsymbol{k}_T\cdot\boldsymbol{p}_T}{M M_h} h_1^{\perp} H_1^{\perp}\right],$$



SIDIS CS & leading and subleading twist structure functions

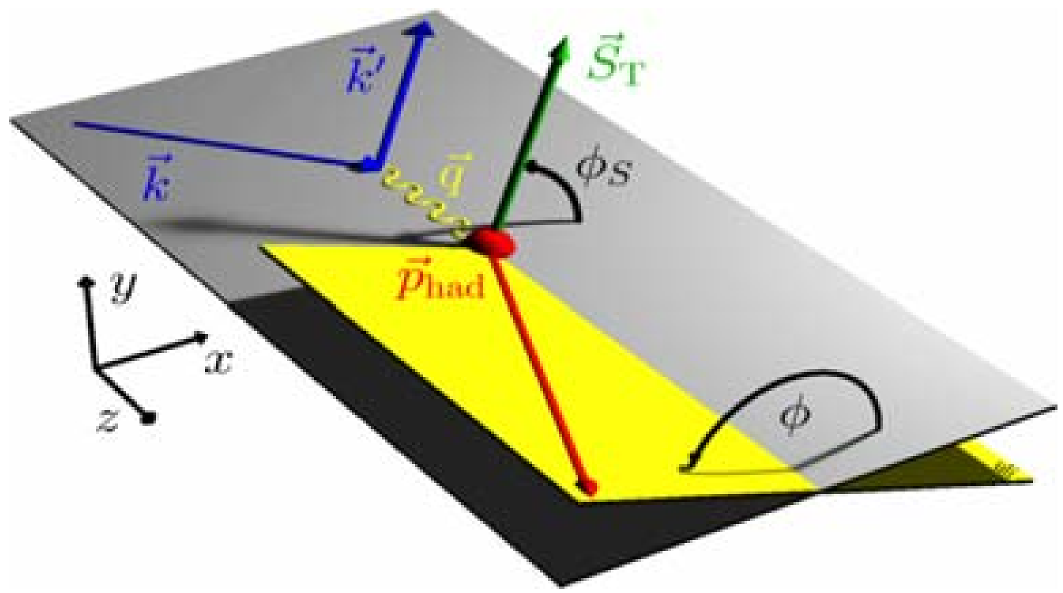
$$
\begin{aligned}
\frac{d\sigma}{dx_B\,dy\,d\psi\,dz_h\,d\phi_h\,dP_{h\perp}^2} &= \frac{\alpha^2}{x_B y Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x_B}\right)\Bigg\{F_{UU,T}+\varepsilon F_{UU,L}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} \\
&+ \quad \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}+\lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h} \\
&+ \quad S_\parallel\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h}+\varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right] \\
&+ \quad S_\parallel\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right] \\
&+ \quad |\boldsymbol{S}_\perp|\Bigg[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)}+\varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) \\
&+ \quad \varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)}+\varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} \\
&+ \quad \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Bigg] \\
&+ \quad |\boldsymbol{S}_\perp|\lambda_e\Bigg[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S} \\
&+ \quad \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Bigg]\Bigg\},
\end{aligned}
$$

Bacchetta et al JHEP 08 *Also See Gunars Talk*

Observables TSSA in SIDIS-CS expressed thru structure functions

$$\frac{d^6\sigma}{dxdydzd\phi_h dP_{h\perp}^2} \sim \left\{F_{UU,T} \cdots + \ldots |S_\perp| \left(\sin(\phi_h - \phi_S) F_{UT,T}^{\sin(\phi_h-\phi_S)} + \sin(\phi_h+\phi_S)\varepsilon F_{UT}^{\sin(\phi_h+\phi_S)} \ldots\right) \ldots\right\}$$

Kotzinian NPB 95,
Mulders Tangermann NPB 96,
Boer & Mulders PRD 97
Bacchetta et al JHEP 08



Spin asymmetry projected from cross section

$$A_{XY}^{\mathcal{F}} \equiv 2\frac{\int d\phi_h\, d\phi_S\, \mathcal{F}(\phi_h,\phi_S)\ \left(d\sigma^{\uparrow} - d\sigma^{\downarrow}\right)}{\int d\phi_h d\phi_S\, (d\sigma^{\uparrow} + d\sigma^{\downarrow})},$$

XY-polarization e.g.

$$\mathcal{F}(\phi_h,\phi_S) = \sin(\phi_h - \phi_S).$$

Weighted asymmetries proposed as *model independent deconvolution* of CS in terms of moments of TMDs

Kotzinian, Mulders PLB 97, Boer, Mulders PRD 98

$$A_{UT,T}^{w_1 \sin(\phi_h-\phi_S)} = 2\,\frac{\int d|\boldsymbol{P}_{h\perp}|\,|\boldsymbol{P}_{h\perp}|d\phi_h\, d\phi_S\, w_1(|\boldsymbol{P}_{h\perp}|)\ \sin(\phi_h-\phi_S)\ \{d\sigma(\phi_h,\phi_S)-d\sigma(\phi_h,\phi_S+\pi)\}}{\int d|\boldsymbol{P}_{h\perp}|\, d\phi_h\, |\boldsymbol{P}_{h\perp}|d\phi_S\, w_0(|\boldsymbol{P}_{h\perp}|)\ \{d\sigma(\phi_h,\phi_S)+d\sigma(\phi_h,\phi_S+\pi)\}},$$

e.g. $$w_1(\boldsymbol{P}_{h\perp}) = \frac{|\boldsymbol{P}_{h\perp}|}{zM}\sin(\phi_h-\phi_S)$$

$$A_{UT}^{\frac{|\boldsymbol{P}_{h\perp}|}{z_h M}\sin(\phi_h-\phi_s)} = -2\,\frac{\sum_a e_a^2\ f_{1T}^{\perp(1)}(x)\ D_1^{a(0)}(z)}{\sum_a e_a^2\ f_1^{a(0)}(x)\ D_1^{a(0)}(z)}$$

Undefined w/o subtractions prescription-need regularization to subtract infinite contribution at large transverse momentum

Problem

$$f_{1T}^{\perp(1)}(x) = \int d^2k_T \frac{k_T^2}{2M} f_{1T}^{\perp}(x, k_T)$$

Problem

$$f_{1T}^{\perp(1)}(x) = \int d^2k_T \frac{k_T^2}{2M} f_{1T}^{\perp}(x, k_T)$$

- Sivers tail $f_{1T}^{\perp}(x, k_T) \sim \dfrac{M^2}{\left(k_T^2 + M^2\right)^2}$

Bacchetta et al. JHEP 08, Aybat Collins Rogers Qiu PRD 2012,

- Moment diverges

- <u>Bessel Weighting solves problem of infinite contribution from large transverse momentum that arise from using "conventional weighting</u>

Solution

- Propose generalize Bessel Weights-”BW”
- BW procedure has advantages
- Provides a regularization of infinite contributions at lg transverse momentum when b_T is non-zero for moments
    - ★ Structure functions are simple product rather $\mathcal{P}[\quad]$ than convolution $\mathcal{C}[\quad]$

e.g. BW Example Sivers Function

"Deconvolution"-Structure function simple product " $\mathcal{P}$ "

$$F_{UT,T}^{\sin(\phi_h-\phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M} f_{1T}^{\perp} D_1\right],$$

<u>"dipole structure"</u>

$$\mathcal{C}\left[w\, f\, D\right] = x \sum_a e_a^2 \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, \delta^{(2)}\left(\boldsymbol{p}_T - \boldsymbol{k}_T - \boldsymbol{P}_{h\perp}/z\right) w(\boldsymbol{p}_T, \boldsymbol{k}_T)\, f^a(x, p_T^2)\, D^a(z, k_T^2)$$

★ $$F_{UT,T}^{\sin(\phi_h-\phi_S)} = -x_B \sum_a e_a^2 \int \frac{d|\boldsymbol{b}_T|}{(2\pi)} |\boldsymbol{b}_T|^2 J_1(|\boldsymbol{b}_T|\,|\boldsymbol{P}_{h\perp}|)\, Mz\ \tilde{f}_{1T}^{\perp a(1)}(x, z^2\boldsymbol{b}_T^2)\, \tilde{D}_1^a(z, \boldsymbol{b}_T^2).$$

$\tilde{f}_1,\ \tilde{f}_{1T}^{\perp(1)}$, and $\tilde{D}_1$ are Fourier Transf. of TMDs/FFs and finite

Boer, LG, Musch, Prokudin JHEP 2011

Transversity and Collins

$$F_{UT}^{\sin(3\phi_h-\phi_S)} = \mathcal{C}\left[\frac{2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)\,(\boldsymbol{p}_T\cdot\boldsymbol{k}_T)+\boldsymbol{p}_T^2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)-4\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)^2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)}{2M^2M_h}\,h_{1T}^{\perp}H_1^{\perp}\right]$$

Write out in "cylindrical polar"- is traceless irreducible tensor no mixture of Bessels just "J_3"

$$F_{UT}^{\sin(3\phi_h-\phi_S)} = x_B \sum_a e_a^2 \int \frac{d|\boldsymbol{b}_T|}{(2\pi)}\,|\boldsymbol{b}_T|^4\,J_3(|\boldsymbol{b}_T|\,|\boldsymbol{P}_{h\perp}|)\,\frac{M^2M_h z^3}{4}\,\tilde{h}_{1T}^{\perp a(2)}(x,z^2\boldsymbol{b}_T^2)\,\tilde{H}_1^{\perp a(1)}(z,\boldsymbol{b}_T^2)\,.$$

Simple product " $\mathcal{P}$ "

Structure Functions deconvolute

$$
\begin{aligned}
\mathcal{F}_{UU,T} &= \mathcal{P}[\tilde{f}_1^{(0)} \ \tilde{D}_1^{(0)}] \,, \\
\mathcal{F}_{UT,T}^{\sin(\phi_h-\phi_S)} &= -\mathcal{P}[\tilde{f}_{1T}^{\perp(1)} \ \tilde{D}_1^{(0)}] \,, \\
\mathcal{F}_{LL} &= \mathcal{P}[\tilde{g}_{1L}^{(0)} \ \tilde{D}_1^{(0)}] \,, \\
\mathcal{F}_{LT}^{\cos(\phi_h-\phi_s)} &= \mathcal{P}[\tilde{g}_{1T}^{(1)} \ \tilde{D}_1^{(0)}] \,, \\
\mathcal{F}_{UT}^{\sin(\phi_h+\phi_S)} &= \mathcal{P}[\tilde{h}_1^{(0)} \ \tilde{H}_1^{\perp(1)}] \,, \\
\mathcal{F}_{UU}^{\cos(2\phi_h)} &= \mathcal{P}[\tilde{h}_1^{\perp(1)} \ \tilde{H}_1^{\perp(1)}] \,, \\
\mathcal{F}_{UL}^{\sin(2\phi_h)} &= \mathcal{P}[\tilde{h}_{1L}^{\perp(1)} \ \tilde{H}_1^{\perp(1)}] \,, \\
\mathcal{F}_{UT}^{\sin(3\phi_h-\phi_S)} &= \frac{1}{4}\mathcal{P}[\tilde{h}_{1T}^{\perp(2)} \ \tilde{H}_1^{\perp(1)}] .
\end{aligned}
$$

$$
\mathcal{P}[\tilde{f}^{(n)}\tilde{D}^{(m)}] \equiv x_B \sum e_a^2 \, (zM|\boldsymbol{b}_T|)^n \, (zM_h|\boldsymbol{b}_T|)^m \, \tilde{f}^{a(n)}(x, z^2\boldsymbol{b}_T^2) \, \tilde{D}^{a(m)}(z, \boldsymbol{b}_T^2)
$$

Also ...

★ CS has simple S/T interpretation as a multipole expansion in terms of $b_T\,[\mathrm{GeV}^{-1}]$ conjugate to $\boldsymbol{P}_{h\perp}$

★ CS has simpler *S/T* interpretation--multipole expansion in terms of $b_T\,[\mathrm{GeV}^{-1}]$ conjugate to $\boldsymbol{P}_{h\perp}$

$$
\begin{aligned}
&\frac{d\sigma}{dx_B\,dy\,d\phi_S\,dz_h\,d\phi_h\,|\boldsymbol{P}_{h\perp}|d|\boldsymbol{P}_{h\perp}|} = \\
&\frac{\alpha^2}{x_B y Q^2}\,\frac{y^2}{(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x_B}\right)\int\frac{d|\boldsymbol{b}_T|}{(2\pi)}|\boldsymbol{b}_T|\Big\{J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{UU,T}+\varepsilon\,J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{UU,L} \\
&+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{UU}^{\cos\phi_h}+\varepsilon\cos(2\phi_h)\,J_2(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{UU}^{\cos(2\phi_h)} \\
&+\lambda_e\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{LU}^{\sin\phi_h} \\
&+S_\parallel\Big[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{UL}^{\sin\phi_h}+\varepsilon\sin(2\phi_h)\,J_2(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{UL}^{\sin 2\phi_h}\Big] \\
&+S_\parallel\lambda_e\Big[\sqrt{1-\varepsilon^2}\,J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{LL}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{LL}^{\cos\phi_h}\Big] \\
&+|\boldsymbol{S}_\perp|\Big[\sin(\phi_h-\phi_S)\,J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\Big(\mathcal{F}_{UT,T}^{\sin(\phi_h-\phi_S)}+\varepsilon\,\mathcal{F}_{UT,L}^{\sin(\phi_h-\phi_S)}\Big) \\
&\quad+\varepsilon\,\sin(\phi_h+\phi_S)\,J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{UT}^{\sin(\phi_h+\phi_S)} \\
&\quad+\varepsilon\,\sin(3\phi_h-\phi_S)\,J_3(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{UT}^{\sin(3\phi_h-\phi_S)} \\
&\quad+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{UT}^{\sin\phi_S} \\
&\quad+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,J_2(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{UT}^{\sin(2\phi_h-\phi_S)}\Big] \\
&+|\boldsymbol{S}_\perp|\lambda_e\Big[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{LT}^{\cos(\phi_h-\phi_S)} \\
&\quad+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{LT}^{\cos\phi_S} \\
&\quad+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,J_2(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{LT}^{\cos(2\phi_h-\phi_S)}\Big]\Big\}
\end{aligned}
$$

Sivers

"Generalized Parton Model"

Bessel weighting-projecting out Sivers
orthogonality of Bessel Fncts.

$$\frac{\mathcal{J}_1^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)}{zM} = \frac{2\,J_1(|\boldsymbol{P}_{hT}|\mathcal{B}_T)}{zM\mathcal{B}_T}$$

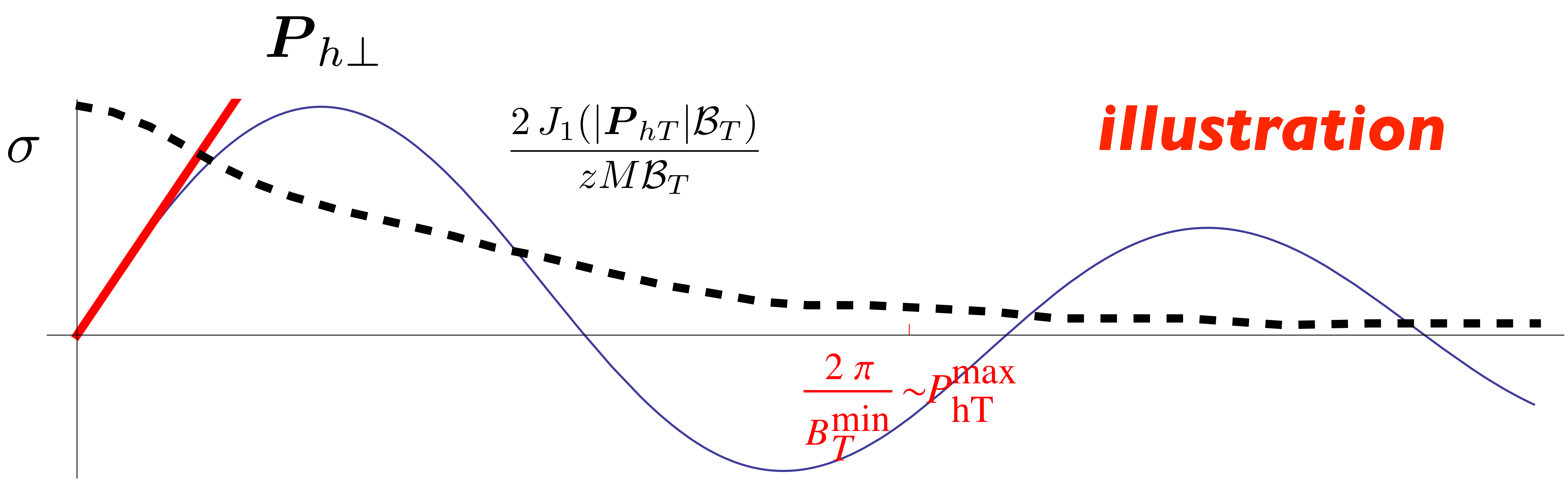
$$A_{UT}^{\frac{\mathcal{J}_1^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)}{zM}\sin(\phi_h-\phi_S)}(\mathcal{B}_T) \quad =$$

$$2\frac{\int d|\boldsymbol{P}_{h\perp}|\,|\boldsymbol{P}_{h\perp}|\,d\phi_h\,d\phi_S\,\frac{\mathcal{J}_1^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)}{zM}\sin(\phi_h-\phi_S)\,(d\sigma^{\uparrow}-d\sigma^{\downarrow})}{\int d|\boldsymbol{P}_{h\perp}|\,|\boldsymbol{P}_{h\perp}|\,d\phi_h\,d\phi_S\;\mathcal{J}_0^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)\;(d\sigma^{\uparrow}+d\sigma^{\downarrow})}$$

$$A_{UT}^{\frac{\mathcal{J}_1^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)}{zM}\sin(\phi_h-\phi_s)}(\mathcal{B}_T) \quad = \quad -2\frac{\sum_a e_a^2\,\tilde{f}_{1T}^{\perp(1)a}(x,z^2\mathcal{B}_T^2)\,\tilde{D}_1^a(z,\mathcal{B}_T^2)}{\sum_a e_a^2\;\tilde{f}_1^a(x,z^2\mathcal{B}_T^2)\,\tilde{D}_1^a(z,\mathcal{B}_T^2)}$$

More sensitive to low $\boldsymbol{P}_{h\perp}$ region

$\mathcal{B}_T$ can serve as a lever arm to enhance the low $\boldsymbol{P}_{h\perp}$ description and possibly dampen lg. momentum tail of cross section. We can use it to scan the cross section



Traditional weighted asymmetry recovered ... UV divergent

$$\lim_{\mathcal{B}_T \to 0} w_1 = 2J_1(|\boldsymbol{P}_{h\perp}|\mathcal{B}_T)/zM\mathcal{B}_T \longrightarrow |\boldsymbol{P}_{h\perp}|/zM$$

$$A_{UT}^{\frac{|\boldsymbol{P}_{h\perp}|}{z_h M}\sin(\phi_h-\phi_s)} = -2\frac{\sum_a e_a^2\, f_{1T}^{\perp(1)}(x)\ D_1^{a(0)}(z)}{\sum_a e_a^2\, f_1^{a(0)}(x)\ D_1^{a(0)}(z)}$$

Bacchetta et al. JHEP 08

undefined w/o regularization

Correlator w/explicit *spin orbit* correlations

$$
\begin{aligned}
\tilde{\Phi}^{[\gamma^+]}(x, \boldsymbol{b}_T) &= \tilde{f}_1(x, \boldsymbol{b}_T^2) - i\,\epsilon_T^{\rho\sigma} b_{T\rho} S_{T\sigma}\, M \tilde{f}_{1T}^{\perp(1)}(x, \boldsymbol{b}_T^2)\,, \\
\tilde{\Phi}^{[\gamma^+\gamma^5]}(x, \boldsymbol{b}_T) &= S_L\, \tilde{g}_{1L}(x, \boldsymbol{b}_T^2) + i\, \boldsymbol{b}_T \cdot \boldsymbol{S}_T M\, \tilde{g}_{1T}^{(1)}(x, \boldsymbol{b}_T^2)\,, \\
\tilde{\Phi}^{[i\sigma^{\alpha+}\gamma^5]}(x, \boldsymbol{b}_T) &= S_T^\alpha\, \tilde{h}_1(x, \boldsymbol{b}_T^2) + i\, S_L\, b_T^\alpha M\, \tilde{h}_{1L}^{\perp(1)}(x, \boldsymbol{b}_T^2) \\
&\quad + \frac{1}{2}\left(b_T^\alpha b_T^\rho + \frac{1}{2}\, \boldsymbol{b}_T^2\, g_T^{\alpha\rho} \right) M^2\, S_{T\rho} \tilde{h}_{1T}^{\perp(2)}(x, \boldsymbol{b}_T^2) \\
&\quad - i\, \epsilon_T^{\alpha\rho} b_{T\rho} M \tilde{h}_1^{\perp(1)}(x, \boldsymbol{b}_T^2)\,,
\end{aligned}
$$

N.B. b_T Transverse sep. of quarks in correlator

We write correlator w/ *b*

$$e^{ik_T\cdot z_T} f_{TMD}(x,k_T) \to e^{ik_T\cdot b_T} f_{TMD}(x,k_T)$$

$$\Phi^{[\mathcal{U}[C]]}(x,p_T) = \int \frac{db^- d^2 b_T}{2(2\pi)^3} e^{i\left(p^+ b^- \, - \, p_T\cdot b_T\right)} \langle P|\,\overline{\psi}(0)\,\mathcal{U}^{[\mathcal{C}]}_{[0,b]}\,\psi(b^-, b_T)|P\rangle\Big|_{b^+=0}$$

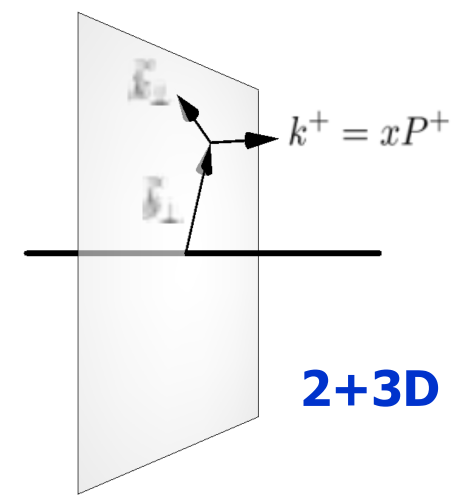
★ Collins Soper (81), Collins, Soper, Sterman (85), Boer (01) (09) (13), Ji,Ma,Yuan (04), Collins-Cambridge University Press (11), Aybat Rogers PRD (11), Abyat, Collins, Qiu, Rogers (11), Aybat, Prokudin, Rogers (11), Bacchetta, Prokudin (13), Sun, Yuan (13), Aidala, Field, Gamberg, Rogers (14)

Let’s be clear about the 

Phase-space (Wigner) distribution

GTMDs ⟶

$x, \vec{k}_\perp, \Delta$

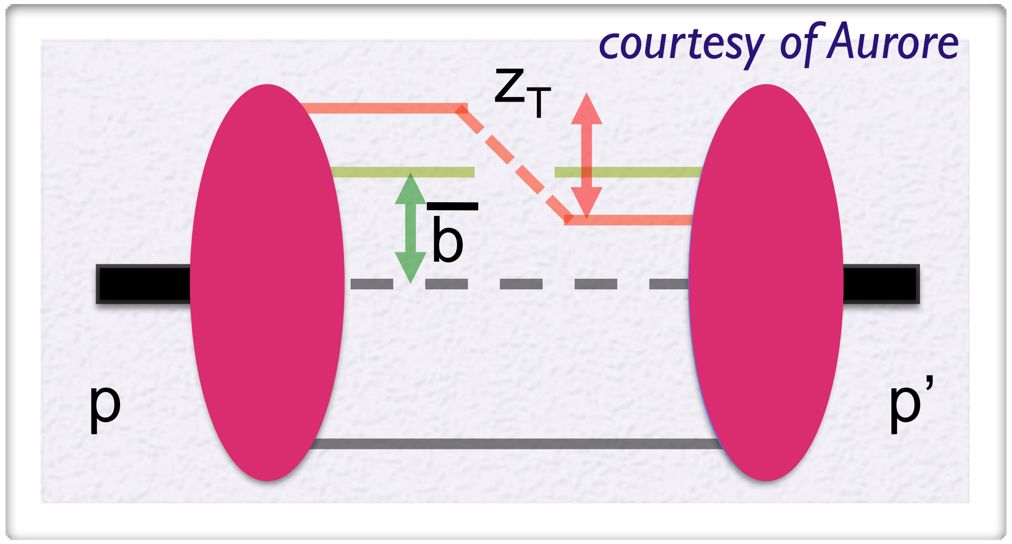


courtesy of Cedric

2+3D

TMD from GTMD

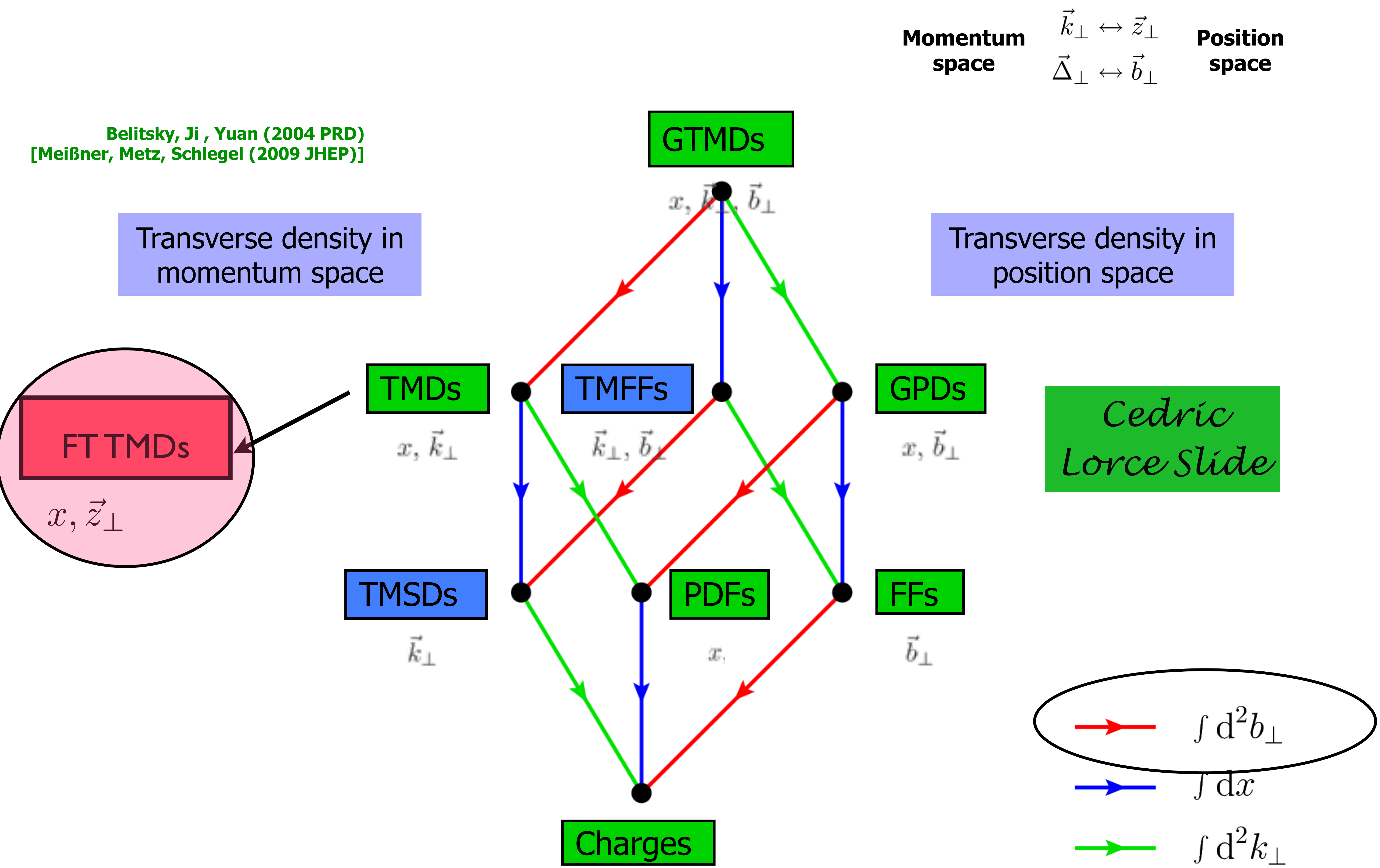
$$\int d^2\bar{b}_T$$



$$\int d^2\bar{b}_T \int \frac{d^2\Delta}{(2\pi)^2} e^{-i(\Delta\cdot\bar{b}-\bar{k}_T\cdot z_T)} F_{GTMD}(x,\Delta,\bar{k}_T) = e^{-ik_T\cdot z_T} F_{GTMD}(x,0,k_T)$$

$$\equiv e^{-ik_T\cdot z_T} f_{TMD}(x,k_T)$$

Partonic Structure of Nucleon



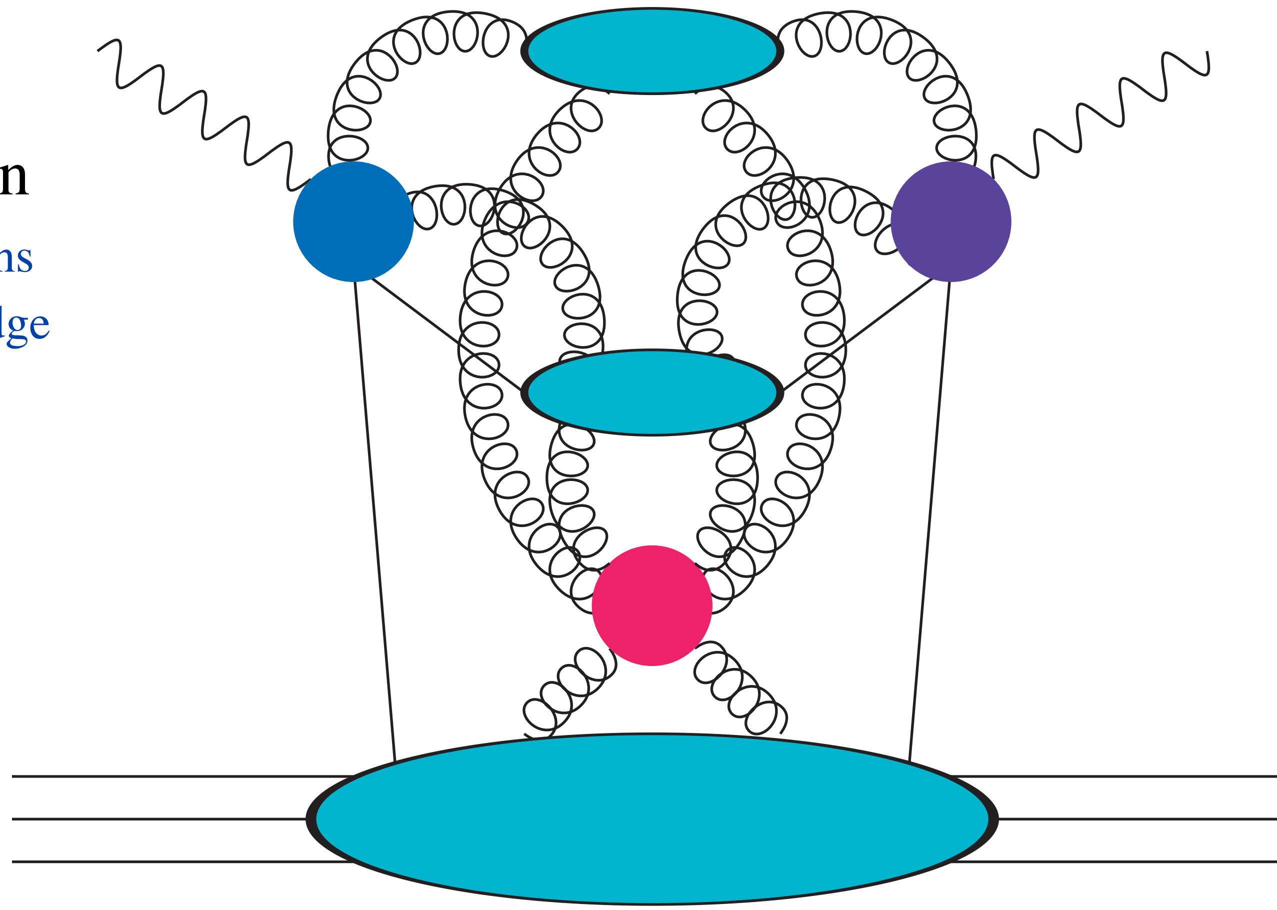
Part 2

- Impact on studying BW and TMD evolution
- Explore impact these BWA have on studying the scale dependence of the SIDIS cross section at small to moderate transverse momentum where the TMD framework is expected to give a good description of the cross section **Boer, Gamberg, Musch, Prokudin JHEP**
- **SKETCH TMD EVOLUTION ....**

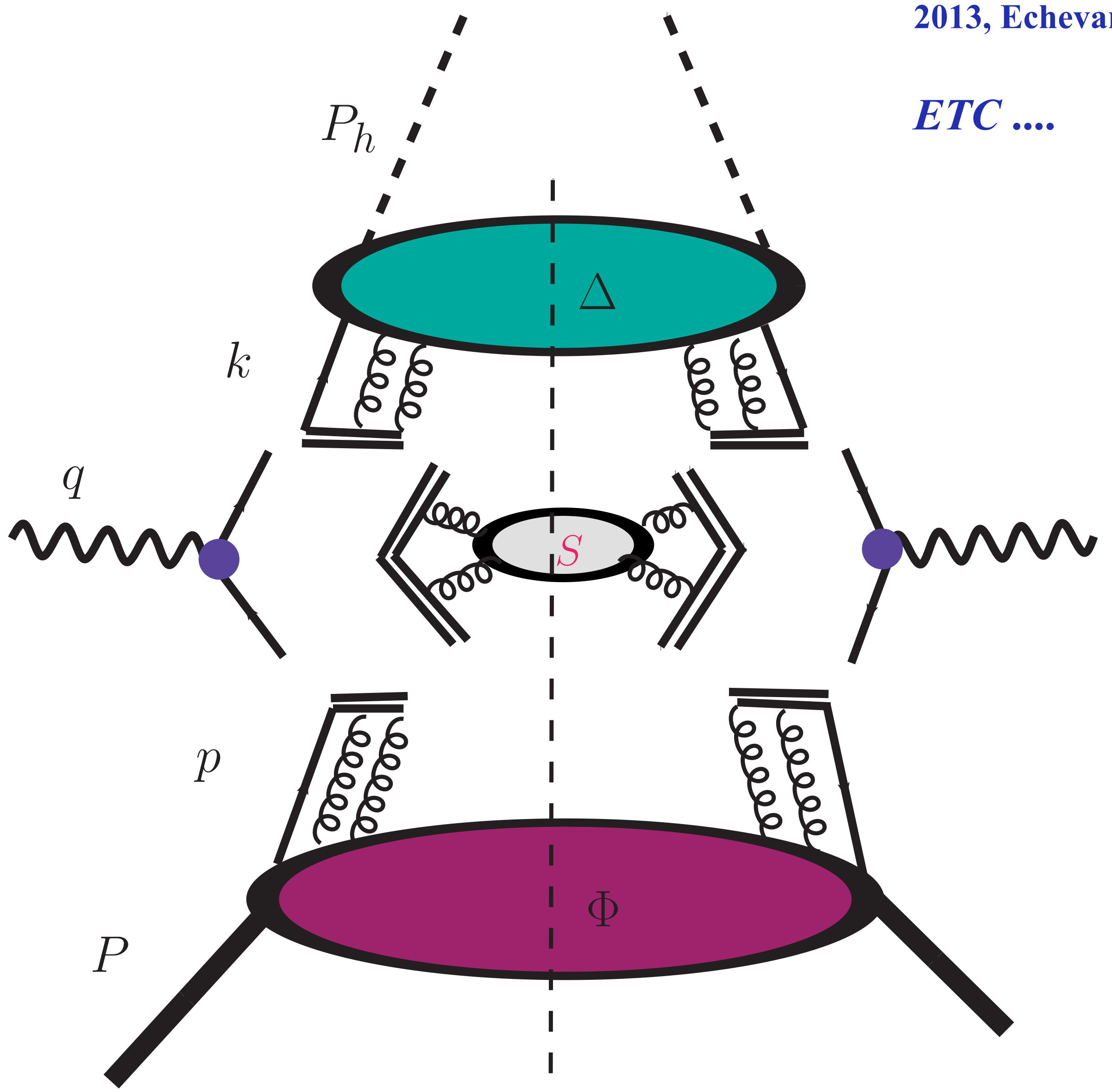
QCD Factorization Procedure Beyond Parton Model includes Glue

- Leading Regions-power counting Libby Sterman PRD 1978 (see Collins PRD 1980 nongauge theories, Collins Soperp NPB& CSS formalism 1982-85... Collins 2011 Cambridge Univ. Press)
- "Reduced Diagrams"

- Apply Ward Identities get factorized form
  - Soft Factor w/ gauge links
  - TMDs w/ gauge links



Further Beyond Parton Model "tree level" factorization



Collins Soper NPB 1981,1982, CSS NPB 1985, Collins, Hautman PLB 00, Collins Metz PRL 2004, Collins Oxford Press 2011, Boer NPB 2001, 2009,2013, Ji, Ma, Yuan PLB 2004, PRD 2005, Ibildi, Ji, Yuan PRD 2004, Cherednikov, Karanikas, Stefanis NPB 2010, Abyat, Rogers PRD 2011, Abyat, Collins, Qiu, Rogers PRD 2012, Collins Rogers 2013, Echevarria, Idilbi, Scimemi JHEP 2012

ETC ....

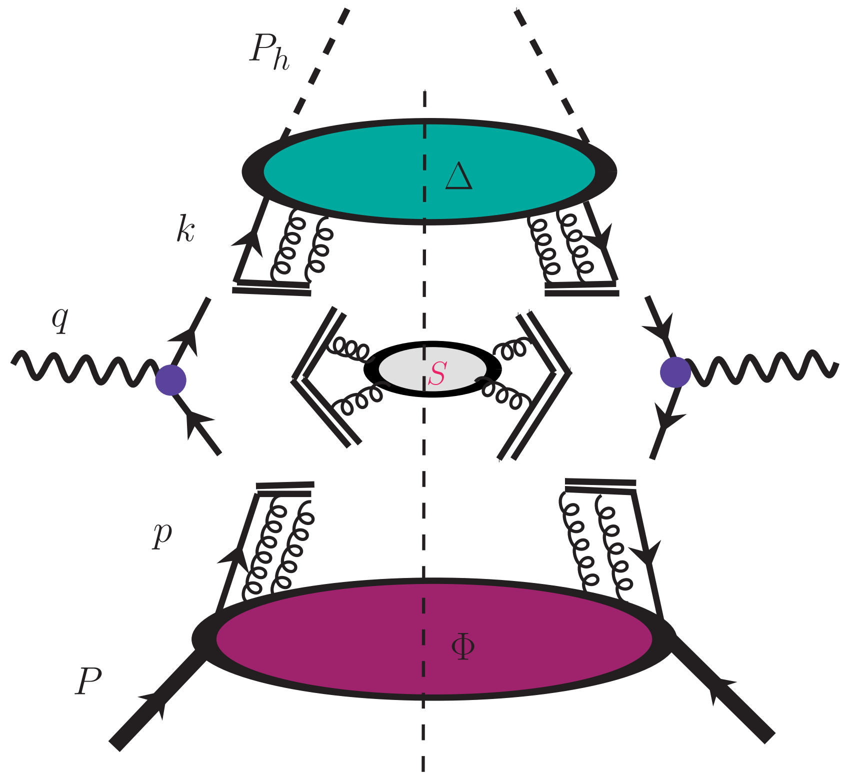
- TMDs w/Gauge links: color invariant
- *In addition Soft factor*

- Extra divergences at one loop and higher
- Extra parameters needed to regulate light-cone divergences
  soft & collinear divergences
- Modifies convolution integral introduction of soft factor
- Some effects of evolution cancel in Bessel weighted asymmetries

Comments on Soft factor

- Collective effect soft gluons associated with distribution or frag function-factorizes into a matrix of Wilson lines in QCD vacuum
- Subtracts rapidity divergences from TMD pdf and FF
- Considered to be universal in hard processes
  (Collins Soper 81, ... , Collins & Metz PRL 04, Ji, Ma, Yuan PRD 05)
- At tree level (zeroth order α_s) unity-parton model and level pheno analyses of experimental data (e.g. Anselmino et al PRD 05 & 07, Efremov et al PRD 07)
- Correct description of energy scale dependence of cross section and asymmetries in TMD picture, soft factor must be included
  (Ji, Ma, Yuan 2004, Collins Camb. Univ. Press 2011, Abyat, Collins, Rogers PRD 2011)
- However, possible to consider observables where its affects cancels e.g. weighted asymmetries Boer, LG, Musch, Prokudin JHEP 2011

Momentum space convolution



CS 81, Idilbi, Ji, Ma, Yuan PRD 05 ....

Hard

$$\mathcal{C}\left[H; w f S D\right] \equiv x_B H(Q^2, \mu^2, \rho) \sum_a e_a^2 \int d^2p_T\, d^2K_T\, d^2\ell_T\, \delta^{(2)}(zp_T + K_T + \ell_T - P_{h\perp})\, w\left(p_T, -\frac{K_T}{z}\right)$$
$$\times f^a(x, p_T^2, \mu^2, x\zeta, \rho)\, \underline{S(\ell_T^2, \mu^2, \rho)}\, D^a(z, K_T^2, \mu^2, \hat{\zeta}/z, \rho)$$

TMD

Soft

FF

Soft factor deconvoluted in Fourier Bessel rep cross sec.

$\mathcal{P}$ versus $\mathcal{C}$

$$\frac{d\sigma}{dx_{_B}\, dy\, d\phi_S\, dz_h\, d\phi_h\, d|\boldsymbol{P}_{h\perp}|^2} \propto \frac{\alpha^2}{x_{_B} Q^2} \int \frac{d|\boldsymbol{b}_T|}{(2\pi)} |\boldsymbol{b}_T|\, \tilde{\mathcal{S}}(\boldsymbol{b}_T^2) \Big\{ \quad \ldots$$

$$+\, J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{P}[\tilde{f}_1\;\; \tilde{D}_1]$$

Soft factor is

- spin blind
- flavor blind
- factors in $\mathcal{P}$
- Universal

Idilbi,Ji,Ma,Yuan PRD 05

$$+\, |\boldsymbol{S}_\perp|\, \sin(\phi_h - \phi_S)\, J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{P}[\tilde{f}_{1T}^{\perp(1)}\;\; \tilde{D}_1]$$

$$+\varepsilon\cos(2\phi_h)\, J_2(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{P}[\tilde{h}_1^{\perp(1)}\;\; \tilde{H}_1^{\perp(1)}]$$

$$+ \quad \ldots 15 \quad \text{more} \quad \text{structure} \quad \text{functions}$$

Products in terms of " $\boldsymbol{b}_T$ moments "

$$\mathcal{F}_{UT,T}^{\sin(\phi_h-\phi_S)} = H_{UT,T}^{\sin(\phi_h-\phi_S)}(Q^2,\mu^2,\rho)\; \tilde{S}^{(+)}(\boldsymbol{b}_T^2,\mu^2,\rho)\; \mathcal{P}[\tilde{f}_{1T}^{(1)}\, \tilde{D}_1^{(0)}] + \tilde{Y}_{UT,T}^{\sin(\phi_h-\phi_S)}(Q^2,\boldsymbol{b}_T^2)\ .$$

Comment

- Y term corrects the structure functions at $P_T \sim Q$, where the factorized structure fnct. does a good job in the $P_T << Q$
- *We will focus the kinematic regime $P_T << Q$ where TMD factorization is appropriate*

Bessel Weighting & cancellation of soft factor

Bessel weighting-projecting out Sivers using orthogonality of Bessel Fncts.

$$\frac{\mathcal{J}_1^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)}{zM} = \frac{2\, J_1(|\boldsymbol{P}_{hT}|\mathcal{B}_T)}{zM\mathcal{B}_T}$$

$$A_{UT}^{\frac{\mathcal{J}_1^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)}{zM}\sin(\phi_h-\phi_S)}(\mathcal{B}_T) \quad =$$

$$2\frac{\int d|\boldsymbol{P}_{h\perp}|\,|\boldsymbol{P}_{h\perp}|\,d\phi_h\,d\phi_S\,\frac{\mathcal{J}_1^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)}{zM}\sin(\phi_h-\phi_S)\left(d\sigma^{\uparrow}-d\sigma^{\downarrow}\right)}{\int d|\boldsymbol{P}_{h\perp}|\,|\boldsymbol{P}_{h\perp}|\,d\phi_h\,d\phi_S\;\mathcal{J}_0^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)\;\left(d\sigma^{\uparrow}+d\sigma^{\downarrow}\right)}$$

Sivers asymmetry with full dependences

$$A_{UT}^{\frac{\mathcal{J}_1^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)}{zM}\sin(\phi_h-\phi_s)}(\mathcal{B}_T) \quad =$$

$$-2\frac{\tilde{S}(\mathcal{B}_T^2,\mu^2,\rho^2)H_{UT,T}^{\sin(\phi_h-\phi_S)}(Q^2,\mu^2,\rho)\sum_a e_a^2\,\tilde{f}_{1T}^{\perp(1)a}(x,z^2\mathcal{B}_T^2;\mu^2,\zeta,\rho)\,\tilde{D}_1^a(z,\mathcal{B}_T^2;\mu^2,\hat{\zeta},\rho)}{\tilde{S}(\mathcal{B}_T^2,\mu^2,\rho^2)H_{UU,T}(Q^2,\mu^2,\rho)\sum_a e_a^2\,\tilde{f}_1^a(x,z^2\mathcal{B}_T^2;\mu^2,\zeta,\rho)\,\tilde{D}_1^a(z,\mathcal{B}_T^2;\mu^2,\hat{\zeta},\rho)}$$

Circumvents the problem of ill-defined p_T moments

$$A_{UT}^{\frac{\mathcal{J}_1^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)}{zM}\sin(\phi_h-\phi_s)}(\mathcal{B}_T) \quad =$$

$$-2\frac{\tilde{S}(\mathcal{B}_T^2,\mu^2,\rho^2)H_{UT,T}^{\sin(\phi_h-\phi_S)}(Q^2,\mu^2,\rho)\sum_a e_a^2\,\tilde{f}_{1T}^{\perp(1)a}(x,z^2\mathcal{B}_T^2;\mu^2,\zeta,\rho)\,\tilde{D}_1^a(z,\mathcal{B}_T^2;\mu^2,\hat{\zeta},\rho)}{\tilde{S}(\mathcal{B}_T^2,\mu^2,\rho^2)H_{UU,T}(Q^2,\mu^2,\rho)\sum_a e_a^2\,\tilde{f}_1^a(x,z^2\mathcal{B}_T^2;\mu^2,\zeta,\rho)\,\tilde{D}_1^a(z,\mathcal{B}_T^2;\mu^2,\hat{\zeta},\rho)}$$

Traditional weighted asymmetry recovered but UV divergent

$$\lim_{\mathcal{B}_T\to 0} w_1 = 2J_1(|\boldsymbol{P}_{h\perp}|\mathcal{B}_T)/zM\mathcal{B}_T \longrightarrow |\boldsymbol{P}_{h\perp}|/zM$$

$$A_{UT}^{\frac{|\boldsymbol{P}_{h\perp}|}{z_h M}\sin(\phi_h-\phi_s)} = -2\frac{\sum_a e_a^2\, f_{1T}^{\perp(1)}(x)\;D_1^{a(0)}(z)}{\sum_a e_a^2\, f_1^{a(0)}(x)\;D_1^{a(0)}(z)}$$

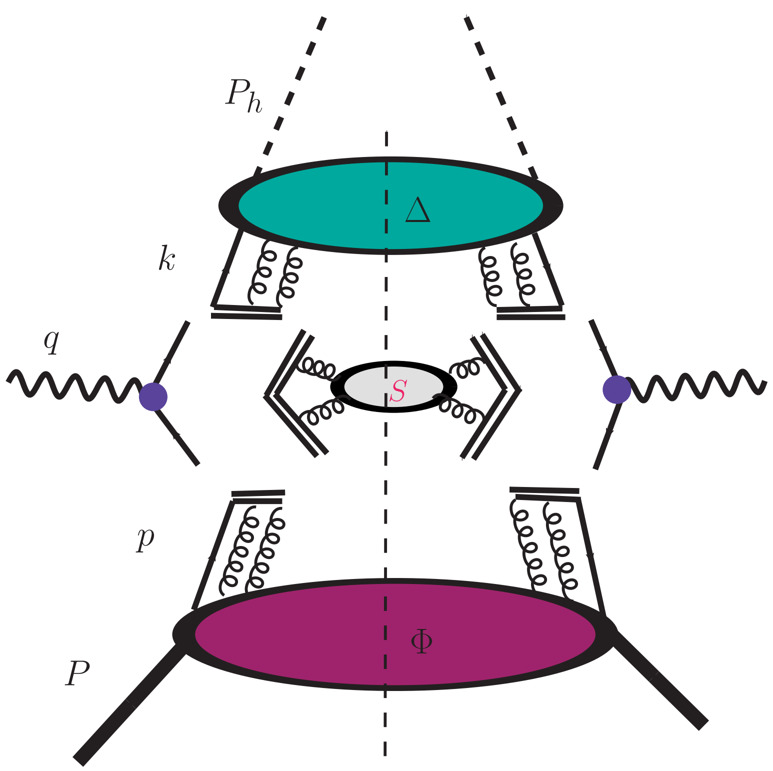
Bacchetta et al. JHEP 08

undefined w/o regularization

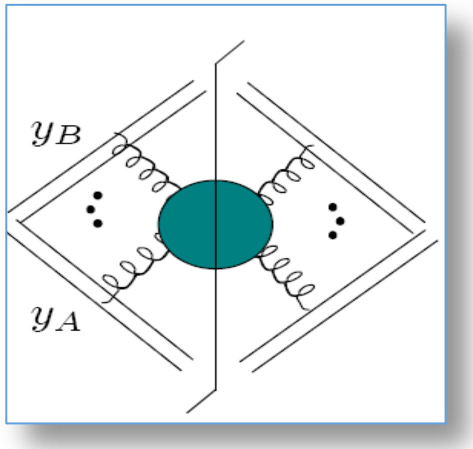
How does this emerge in CSS + JCC 2011 Factorization formulation

- *Here we see the cancellation of spin independent & "Universal" parts of the evolution kernel*

Again …. Emergence of Soft Factor in CS



- Lightlike Wilson lines in TMDs
  - Infinite rapidity QCD radiation in the wrong direction.
  - In soft factor/fragmentation function too.



.... Emergence of Soft Factor in CS

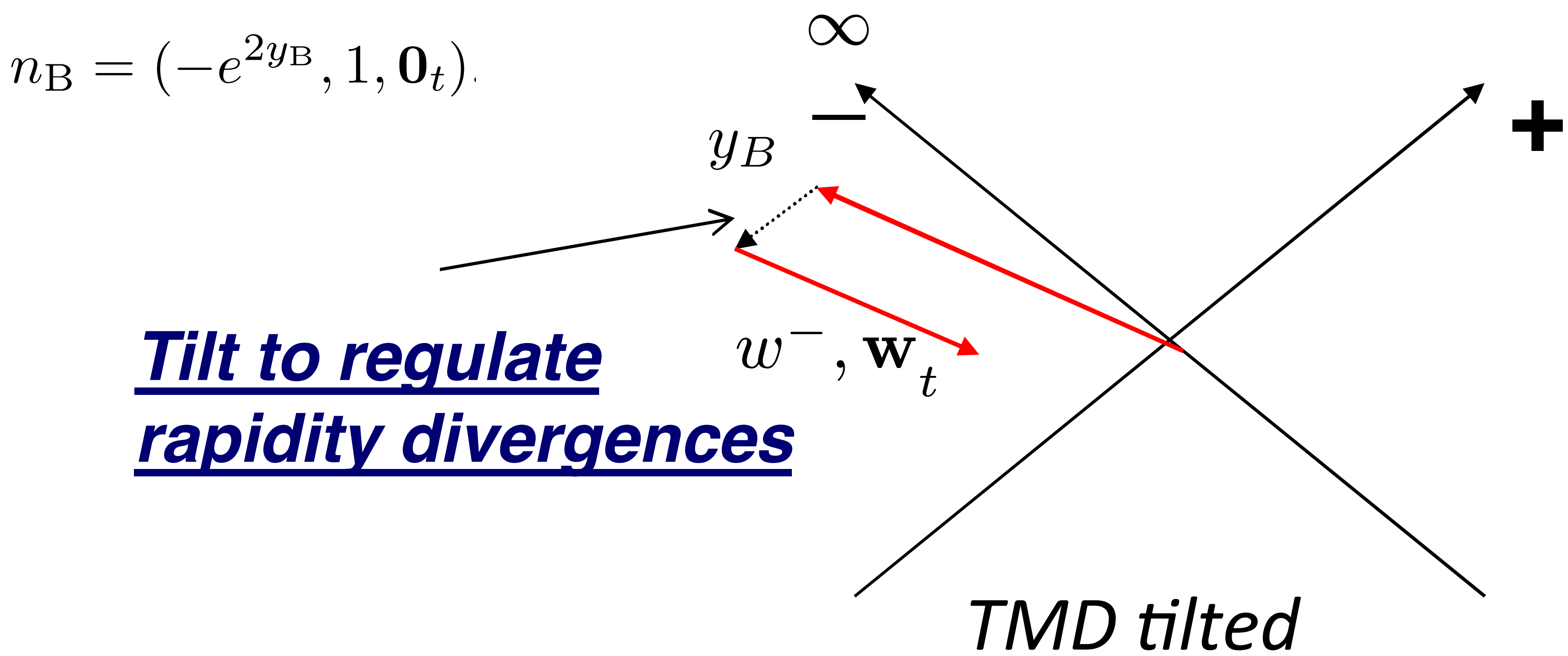
$$d\sigma = |\mathcal{H}|^2 \, \frac{\tilde{F}_1^{\rm unsub}(y_1 - (-\infty)) \times \tilde{F}_2^{\rm unsub}(+\infty - y_2)}{\tilde{S}(+\infty, -\infty)}$$

Collins Act Pol. 2003
Ji Ma Yuan 2004, 2005

TMDs are still "entangled" not yet fully factorized
Use its properties to fully factorize and perform evolution

Collins 2011 Cam. Univ. Press see also Aybat Rogers PRD 2011

Introduce rapidity scale parameter to regulate



$$\zeta_F = 2M_p^2 x^2 e^{2(y_P - y_s)} \Longleftrightarrow y$$

Emergence of Soft Factor in TMDs

Start with only the hard part factorized:

$$d\sigma = |\mathcal{H}|^2 \, \frac{\tilde{F}_1^{\mathrm{unsub}}(y_1 - (-\infty)) \times \tilde{F}_2^{\mathrm{unsub}}(+\infty - y_2)}{\tilde{S}(+\infty, -\infty)}$$

Soft factor repartitioned
This is done to both

1) cancel LC divergences and
2) separate "right & left" movers i.e. factorize

$$d\sigma = |\mathcal{H}|^2 \, \left\{ F_1^{\mathrm{unsub}}(y_1 - (-\infty)) \sqrt{\frac{\tilde{S}(+\infty, y_s)}{\tilde{S}(+\infty, -\infty)\tilde{S}(y_s, -\infty)}} \right\} \times \left\{ \tilde{F}_2^{\mathrm{unsub}}(+\infty - y_2) \sqrt{\frac{\tilde{S}(y_s, -\infty)}{\tilde{S}(+\infty, -\infty)\tilde{S}(+\infty, y_s)}} \right\}$$

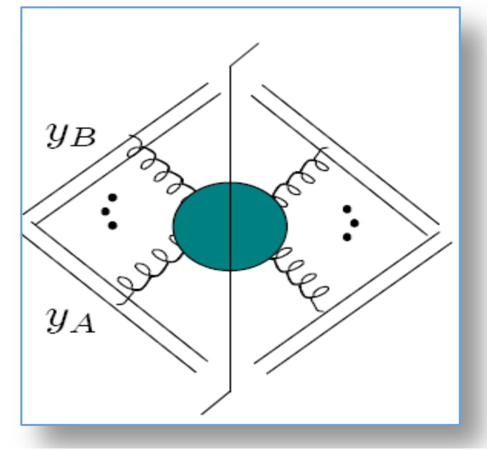
Separately Well-defined

Factorization to TMD Evolution...CSS + JCC 2011

Evolution follows from their operator definition

Collins-Soper Equation:

$$\frac{\partial \ln \tilde{F}(x, b_T, \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_T; \mu)$$



Now effects of Soft factor soft gluon radiation in evolution *kernal*

$$\tilde{K}(b_T; \mu) = \frac{1}{2} \frac{\partial}{\partial y_n} \ln \frac{\tilde{S}(b_T; y_n, -\infty)}{\tilde{S}(b_T; +\infty, y_n)}$$

Along with .... Renormalization group Equations

$$\frac{d\tilde{K}}{d\ln\mu} = -\gamma_K(g(\mu))$$

$$\frac{d\ln\tilde{F}(x, b_T; \mu, \zeta)}{d\ln\mu} = -\gamma_F(g(\mu); \zeta/\mu^2)$$

.... and RGE

Solve Collins Soper & RGE eqs. to obtain TMD Evolution kernal

Solve Collins Soper & RGE eqs. obtain TMD Evolution kernal

however....one more element ....

Partition the perturbative and nonperturbative parts of evolution Kernal

One TMD factorization entire range of P_T or b_T

Collins Soper Sterman NPB 85

- TMD formalism of Collins 2011 interpolates/matches the "TMD" and collinear picture
- Maximizes the perturbative content while providing a TMD formalism that is applicable over the entire range of P_T

$$\mathbf{b}_* = \frac{\mathbf{b}_T}{\sqrt{1 + b_T^2/b_{\max}^2}}, \qquad \mu_b = \frac{C_1}{b_*}.$$

Partition the perturbative and nonperturbative parts of evolution Kernal $\tilde{K}(b_T, \mu)$

Collins Soper Sterman NPB 85

$$\mathbf{b}_* = \frac{\mathbf{b}_T}{\sqrt{1 + b_T^2/b_{\max}^2}}, \qquad \mu_b = \frac{C_1}{b_*}.$$

$$\tilde{K}(b_T;\mu) = \tilde{K}(b_*;\mu_b) - \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \gamma_K(g(\mu')) - g_K(b_T)$$

$b_{\max}$ chosen so that b_* doesnt go too far beyond the pertb. region maximize perturbative content

Nonperturbative part of evolution Kernal

$$\tilde{K}(b_T, \mu)$$

Collins Soper Sterman NPB 85

Totally universal related to derivative of soft factor independent of x & hadron

$$\tilde{K}(b_T;\mu) = \tilde{K}(b_*;\mu_b) - \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \gamma_K(g(\mu')) - g_K(b_T)$$

$$\mathbf{b}_* = \frac{\mathbf{b}_T}{\sqrt{1 + b_T^2/b_{\max}^2}}, \qquad \mu_b = \frac{C_1}{b_*}.$$

$b_{\max}$ chosen so that b_* doesn't go too far beyond the pertb. region maximize perturbative content

Structure Function *beyond* Parton Model

Evolved Structure Function & *TMDs*

$$\mathcal{F}_{UU}(x,z,b,Q^2) = \sum_a \tilde{F}^a_{H1}(x,b_T,\mu,\zeta_F)\tilde{D}^a_{H2}(z_h,b_T,\mu,\zeta_D)H_{UU}(Q^2,\mu^2)$$

Non-perturbative large b_T behavior

Totally universal related to derivative of soft factor independent of x & hadron

$$\tilde{F}_{H_1}(x,b_T;Q,Q^2) = \tilde{F}_{H_1}(x,b_*;\mu_b,\mu_b^2)\exp\left\{-g_1(x,b_T;b_{\max}) - g_K(b_T;b_{\max})\ln\left(\frac{Q}{Q_0}\right) + \ln\left(\frac{Q}{\mu_b}\right)\tilde{K}(b_*;\mu_b) + \int_{\mu_b}^{Q}\frac{d\mu'}{\mu'}\left[\gamma_{\rm PDF}(\alpha_s(\mu');1) - \ln\left(\frac{Q}{\mu'}\right)\gamma_K(\alpha_s(\mu'))\right]\right\}$$

$$\tilde{D}_{H_2}(z,b_T;Q,Q^2) = \tilde{D}_{H_2}(z,b_*;\mu_b,\mu_b^2)\exp\left\{-g_2(z,b_T;b_{\max}) - g_K(b_T;b_{\max})\ln\left(\frac{Q}{Q_0}\right) + \ln\left(\frac{Q}{\mu_b}\right)\tilde{K}(b_*;\mu_b) + \int_{\mu_b}^{Q}\frac{d\mu'}{\mu'}\left[\gamma_{\rm FF}(\alpha_s(\mu');1) - \ln\left(\frac{Q}{\mu'}\right)\gamma_K(\alpha_s(\mu'))\right]\right\}$$

One TMD formalism for entire range of PT

$$\frac{d\sigma}{dP_T^2} \propto \mathcal{H}(\alpha_s(Q)) \int d^2 b_T e^{i b_T \cdot P_T} \; \tilde{F}_{H_1}(x, b_T; Q, Q^2)\, \tilde{D}_{H_2}(z, b_T; Q, Q^2) \;\; + \;\; Y_{\rm SIDIS}$$

$$\frac{d\sigma}{dP_T^2} \propto \; \text{F.T.} \exp\left\{ -g_{\rm PDF}(x, b_T; b_{\rm max}) - g_{\rm FF}(z, b_T; b_{\rm max}) - 2 g_K(b_T; b_{\rm max}) \ln\left(\frac{Q}{Q_0}\right) + \right.$$

$$\left. + \; 2\ln\left(\frac{Q}{\mu_b}\right) \tilde{K}(b_*; \mu_b) + \int_{\mu_b}^{Q} \frac{d\mu'}{\mu'} \left[\gamma_{\rm PDF}(\alpha_s(\mu'); 1) + \gamma_{\rm FF}(\alpha_s(\mu'); 1) - 2\ln\left(\frac{Q}{\mu'}\right) \gamma_K(\alpha_s(\mu')) \right] \right\}$$

$$+ \; Y_{\rm SIDIS}$$

Sivers Structure Function

$$F_{UT}(x,z,b,Q) = (\tilde{C}_{f/i} \otimes f^{(1)}_{1T\,i/P})(x,b_\star;\mu_b)(\tilde{C}_{j/H} \otimes d_{H/j})(z,b_\star;\mu_b)e^{-S^{pert}(b_*,Q)}e^{-S^{NP}_{UT}(b,Q,x,z)}$$

★ Abyat, Collins, Qiu, Rogers PRD (11), $b_* = \dfrac{b}{\sqrt{1+(b/b_{max})^2}}$

$$e^{-S^{NP}_{UT}}(b,Q,x,z) = \exp\left\{-\left[g_1(x,b_T;b_{\max}) + g_2(z,b_T;b_{\max}) + 2g_k(b_T)\ln\left(\frac{Q}{Q_0}\right)\right]\right\}_{UT}$$

Non perturbative factor contribution must be fit

$\left\{g_i\right\} \to 0$ as $\mathrm{b} \to 0$ perturbative

CSS NPB 85

Unpolarized and Sivers evolve in same way !!!

Recall correlator in *b*-space From Bessel Transform

$$\tilde{\Phi}^{[\gamma^+]}(x, \boldsymbol{b}_T) \;=\; \tilde{f}_1(x, \boldsymbol{b}_T^2) - i\,\epsilon_T^{\rho\sigma} b_{T\rho} S_{T\sigma}\, M\, \tilde{f}_{1T}^{\perp(1)}(x, \boldsymbol{b}_T^2)$$

$$\frac{\partial\, \tilde{\phi}^i_{f/P}(x, \mathbf{b}_\mathrm{T}; \mu, \zeta_F)\epsilon_{ij} S_T^j}{\partial \ln\sqrt{\zeta_F}} = \tilde{K}(b_T; \mu)\, \tilde{\phi}^i_{f/P}(x, \mathbf{b}_\mathrm{T}; \mu, \zeta_F)\epsilon_{ij} S_T^j.$$

Sivers BWA: Cancellation of Universal *NP* and flavor blind hard contributions

When $\Lambda^2_{QCD} \ll P_h^2 \ll Q^2$

$$
\begin{aligned}
&\mathcal{A}_{UT}(x,z,b,Q^2) \\
&= \frac{\tilde{f}_{1T}^{\perp(1)}(x, z^2\boldsymbol{b}^2, \mu_0^2, Q_0)\tilde{D}_1(z_h, \boldsymbol{b}^2, \mu_0^2, Q_0)\tilde{H}_{UT}(\mu_0^2, Q_0)e^{-S^{\text{pert}}(b_*, Q)}e^{-2g_k(b_T)\ln\left(\frac{Q}{Q_0}\right)}}{\tilde{f}_1(x, z^2\boldsymbol{b}^2, \mu_0^2, Q_0)\tilde{D}_1(z_h, \boldsymbol{b}^2, \mu_0^2, Q_0)\tilde{H}_{UU}(\mu_0^2, Q_0)e^{-S^{\text{pert}}(b_*, Q)}e^{-2g_k(b_T)\ln\left(\frac{Q}{Q_0}\right)}}
\end{aligned}
$$

BWA less sensitivity to Evolution

In prep. Boer, Gamberg, B. Musch, A. Prokudin....

First Attempts at implementation

Prepared for submission to JHEP

Studies of Transverse Momentum Dependent Parton Distributions and Bessel Weighting

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Abstract: In this paper we present a new technique for analysis of transverse momentum dependent parton distribution functions, based on the Bessel weighting formalism. The procedure is applied to studies of the double longitudinal spin asymmetry in semi-inclusive deep inelastic scattering using a new dedicated Monte Carlo generator which includes quark intrinsic transverse momentum within the generalized parton model. Using a fully differential cross section for the process, the effect of four momentum conservation is analyzed using various input models for transverse momentum distributions and fragmentation functions. We observe a few percent systematic offset of the Bessel-weighted asymmetry obtained from Monte Carlo extraction compared to input model calculations, which is due to the limitations imposed by the energy and momentum conservation at the given energy/Q^2. We find that the Bessel weighting technique provides a powerful and reliable tool to study the Fourier transform of TMDs with controlled systematics due to experimental acceptances and resolutions with different TMD model inputs.

Keywords: SIDIS, parton intrinsic transverse momentum, azimuthal moments

preprint: JLAB-THY-14-1945

Use of Bessel Weighting

- Bessel Weighting of Experimental Observables
- MC study by Mher Aghasyan & Harut Avakain using MC generated events for A_{LL} (b_T)

Bessel Weighting of Experimental Observables

$$\tilde{\sigma}^{\pm}(b_T) = 2\pi \int \frac{d\sigma^{\pm}}{d\Phi} J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|) P_{h\perp}\, dP_{h\perp},$$

using the shorthand notation for the differential phase space factor $d\Phi \equiv dx\, dy\, d\psi\, dz\, dP_{h\perp} P_{h\perp}$

$$A_{LL}^{J_0(b_T P_{h\perp})}(b_T) = \frac{\tilde{\sigma}^{+}(b_T) - \tilde{\sigma}^{-}(b_T)}{\tilde{\sigma}^{+}(b_T) + \tilde{\sigma}^{-}(b_T)} \equiv \frac{\tilde{\sigma}_{LL}(b_T)}{\tilde{\sigma}_{UU}(b_T)} = \sqrt{1-\varepsilon^2}\, \frac{\sum_a e_a^2 \tilde{g}_{1L}^a(x, z^2 b_T^2) \tilde{D}_1^a(z, b_T^2)}{\sum_a e_a^2 \tilde{f}_1^a(x, z^2 b_T^2) \tilde{D}_1^a(z, b_T^2)}$$

$$S_{||}\lambda_e = \pm 1$$

The experimental procedure to study the structure functions in b_T -space amounts to discretizing the b-space cross section.

Above results in an expression of sums and differences of Bessel functions for a given set of experimental events.

The details on this formulation are given in *arXiv:0828.xyz* and in extra slides
The resulting expression for the spin asymmetry is

$$A_{LL}^{J_0(b_T P_{h\perp})}(b_T) = \frac{\sum_{j^+}^{N^+} J_0(b_T P_{h\perp\, j^+}) - \sum_{j^-}^{N^-} J_0(b_T P_{h\perp\, j^-})}{\sum_{j^+}^{N^+} J_0(b_T P_{h\perp\, j^+}) + \sum_{j^-}^{N^-} J_0(b_T P_{h\perp\, j^-})}$$

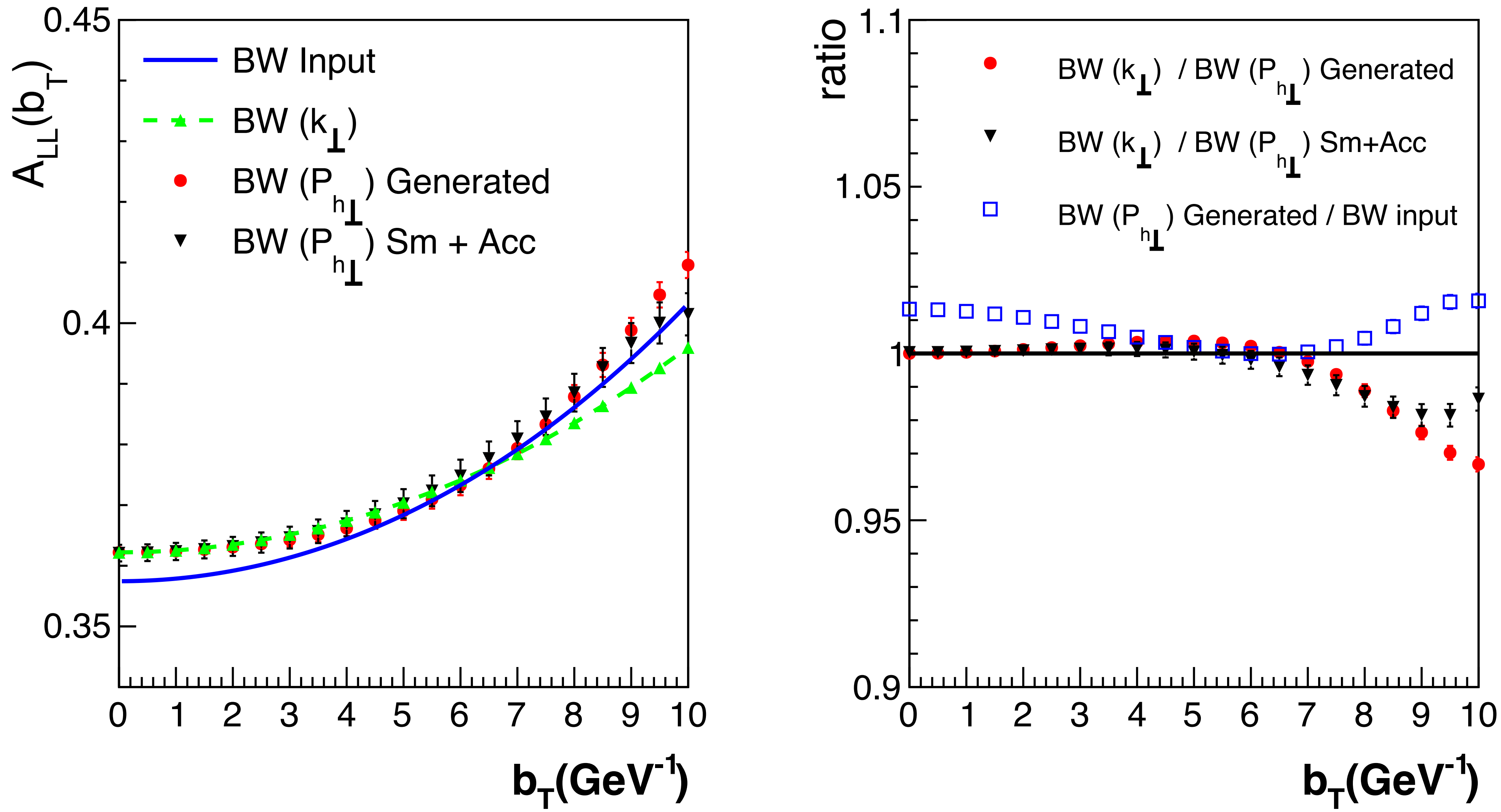
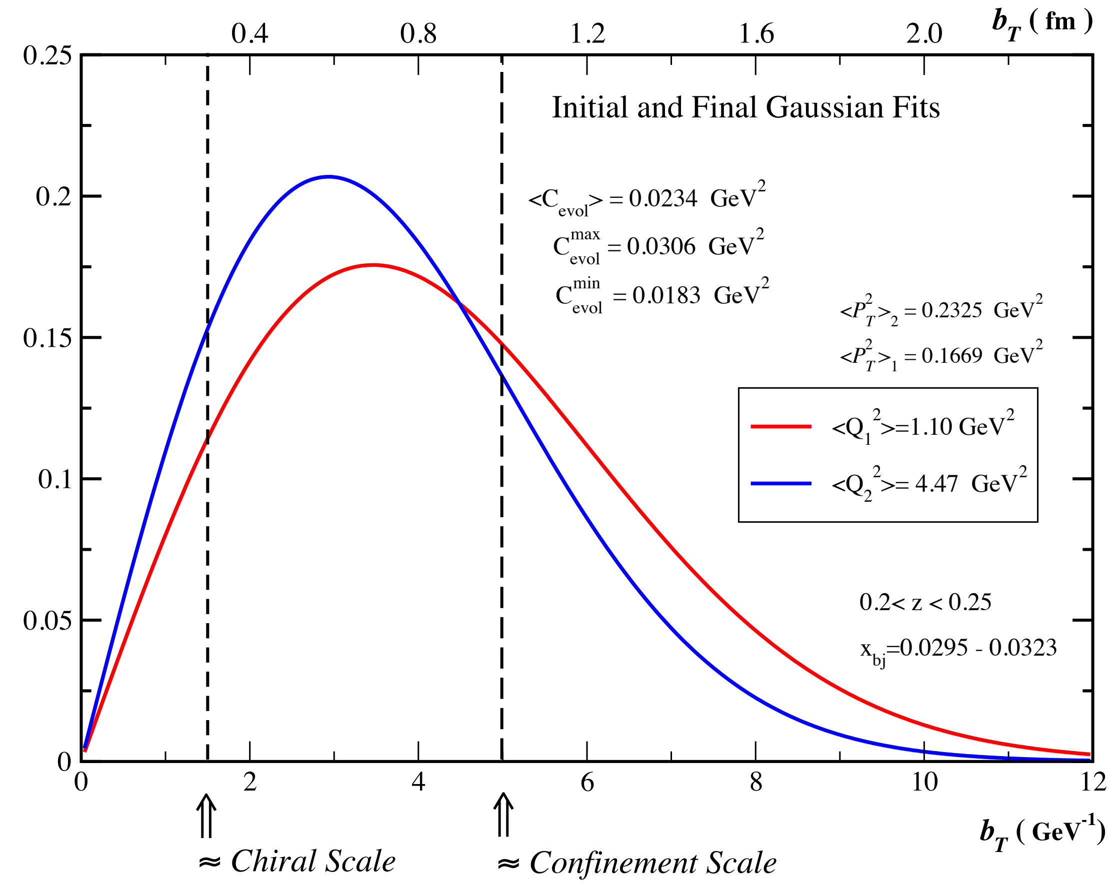
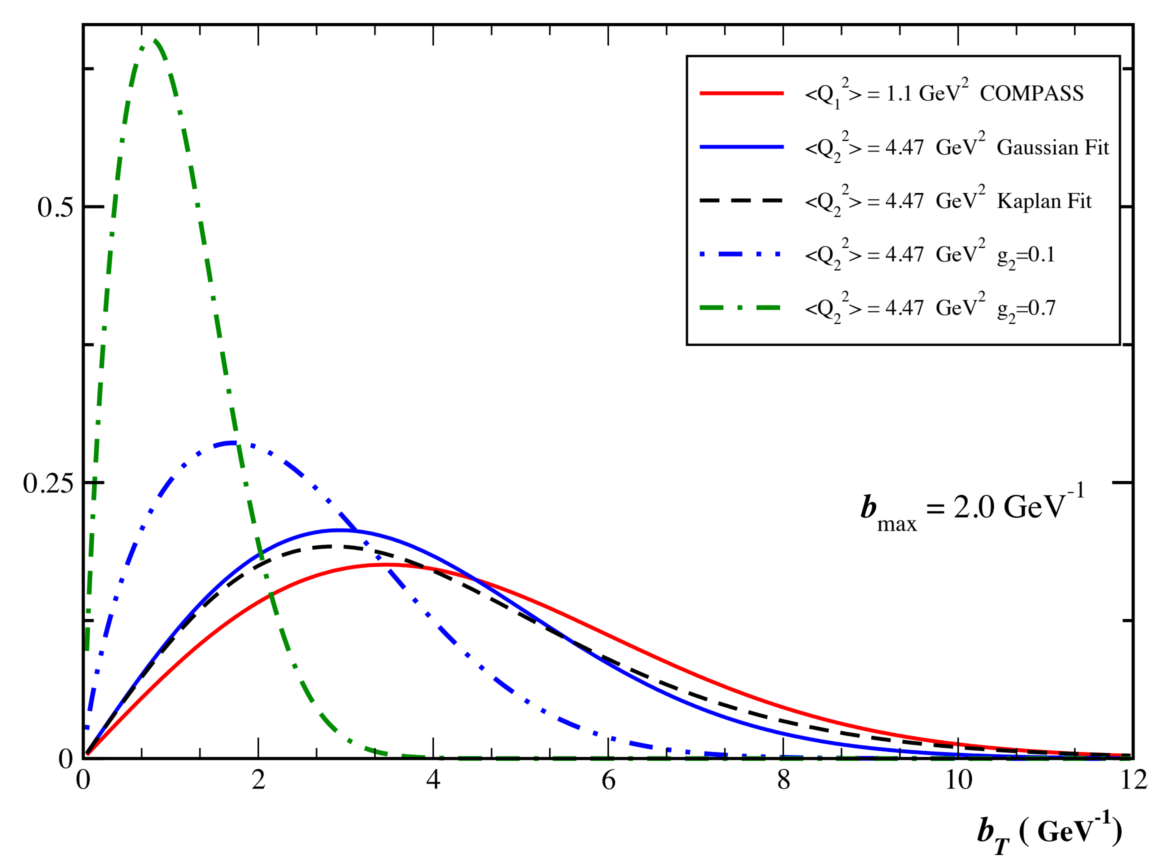


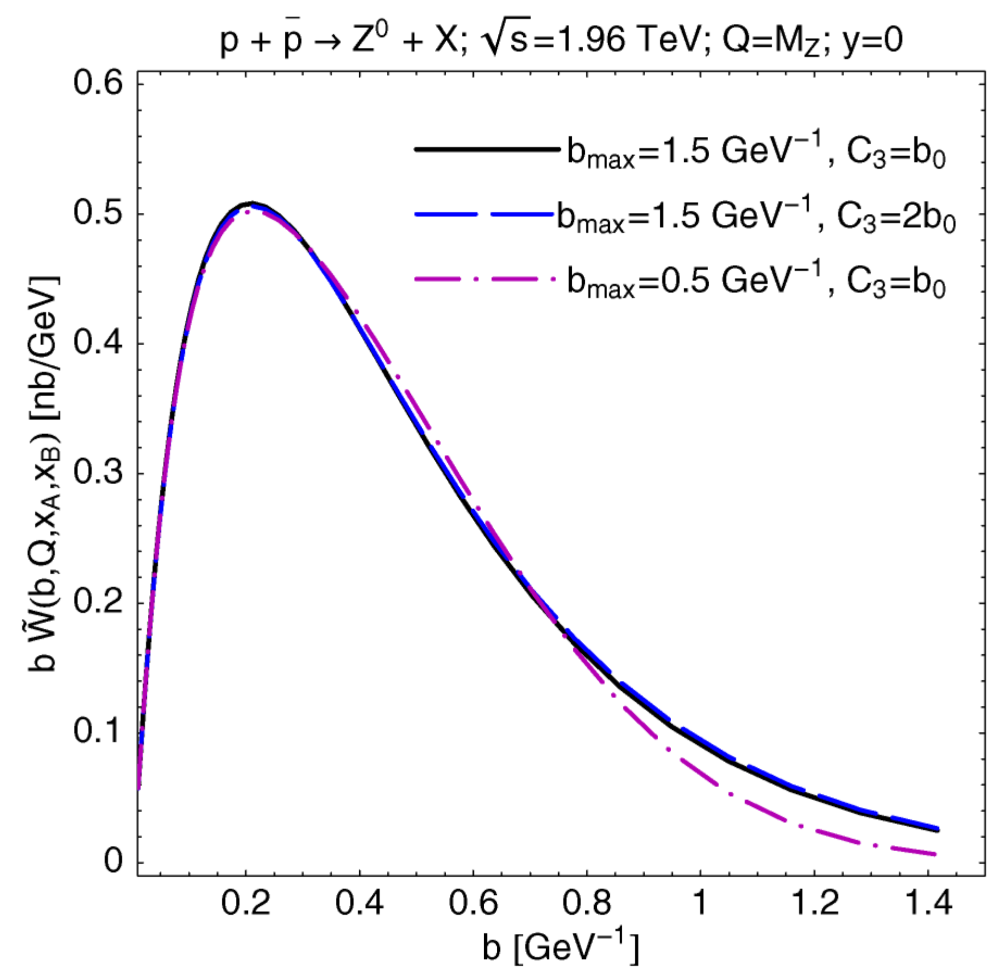
Figure 7. (Color online) Left panel: The ratio of Fourier transforms $\tilde{g}_{1L}/\tilde{f}_1$ and the Bessel weighted asymmetry $A_{LL}^{J_0(b_T P_{h\perp})}$ plotted versus b_T. The solid curve (blue) is the Fourier transform of the input to the Monte Carlo given by Eq. (2.9), red points are generated Monte Carlo events using Eq. 2.10, and triangles down (black) represent results of Monte Carlo events after experimental smearing and acceptance at $\langle x\rangle = 0.22$, and $\langle z\rangle = 0.51$. The triangles up with dashed curve (green) are results of the Monte Carlo without inclusion of fragmentation functions. Right panel: Ratios that represent accuracies of our results.

$$A_{LL}^{J_0(b_T P_{h\perp})}(b_T) = \frac{\sum\limits_{j^+}^{N^+} J_0(b_T P_{h\perp\, j^+}) - \sum\limits_{j^-}^{N^-} J_0(b_T P_{h\perp\, j^-})}{\sum\limits_{j^+}^{N^+} J_0(b_T P_{h\perp\, j^+}) + \sum\limits_{j^-}^{N^-} J_0(b_T P_{h\perp\, j^-})}$$



See, for example, Fig. of Konychev and Nadolsky and compare this with Fig. 3, where contributions from bT < 2.0 GeV^{-1} dominate.





Bo-Qiang Ma and Zhun Lu PRD 87 2013
model calculation

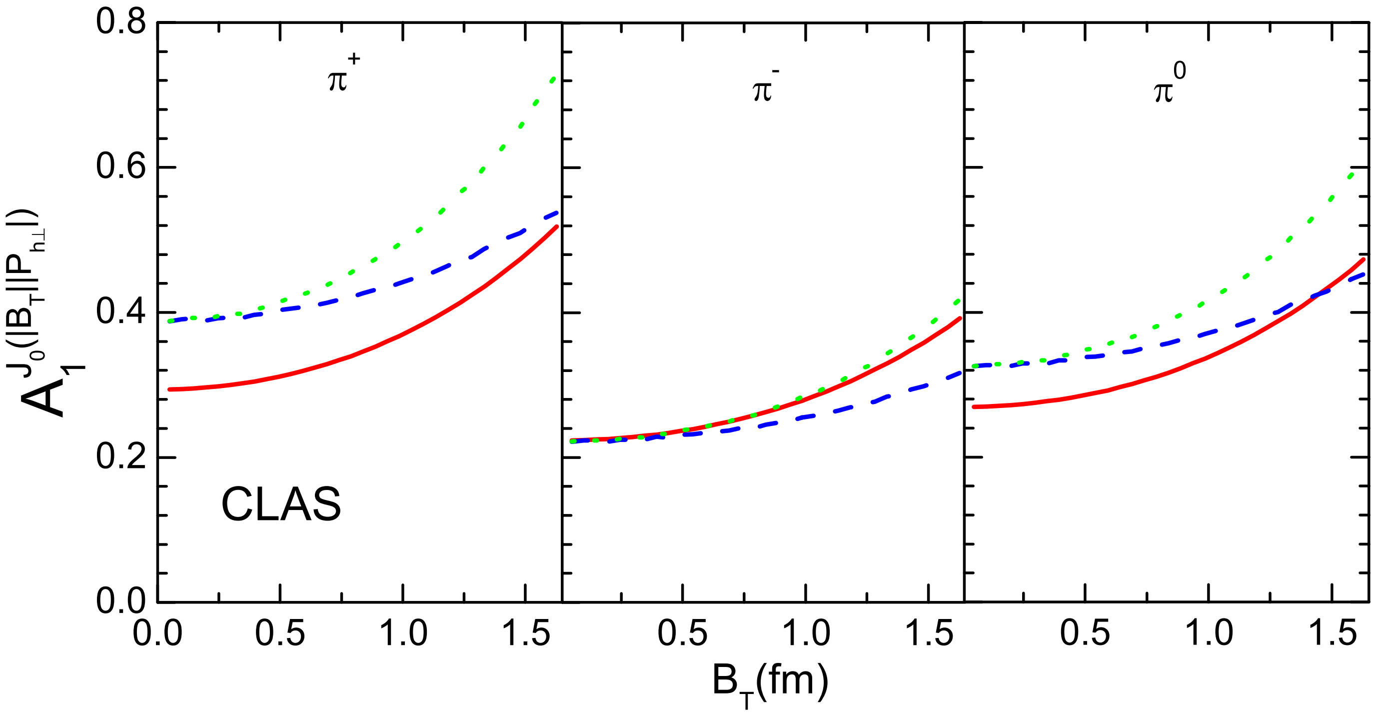


FIG. 5 (color online). The Bessel-weighted DSAs $A_1^{J_0(|\mathcal{B}_T||\boldsymbol{P}_{h\perp}|)}$ for π^+, π^-, and π^0 productions as functions of $\mathcal{B}_T$ at CLAS. The solid lines are from approach 2 of the light-cone diquark model, while the dashed line and the dotted lines are from the Gaussian ansatz for the TMD helicity distributions with $\langle p_T^2 \rangle_g^q = 0.17$ GeV and 0.10 GeV^2, respectively.

Cancellation of Soft Factor on level of the (no time to cover) Matrix elements ***(summarize)***

- So far we get ratios of moments of TMDs and FFs that are free/insensitive to soft gluon radiation
- It was not necessary to specify explicit def. of TMDs and FFs
- We also analyze ratio of moments of TMDs directly on level of matrix elements of TMDs & FFs
- Again we find cancellation of soft factors in ratio
- Impact for Lattice calculation of moments of TMDS, Musch, Ph. Hagler, M. Engelhardt, J.W. Negele, A. Schafer arXiv 2011

Conclusions

- Propose generalized Bessel Weights
- Theoretical weighting procedure-advantages
- Introduces a free parameter $\mathcal{B}_T\,[\mathrm{GeV}^{-1}]$ that is Fourier conjugate to $\boldsymbol{P}_{h\perp}$
- Provides a regularization of infinite contributions at lg. transverse momentum when $\mathcal{B}_T^2$ is non-zero
- Soft, Hard CS, eliminated from weighted asymmetries, Sudakov dpnds coupling of *b* & *Q*
- Possible to compare observables at different scales.... could be useful for an EIC

Extras

Fully Differential Monte Carlo for SIDIS

$$\ell(l) + N(P) \to \ell(l') + h(P_h) + X,$$

$$\frac{d\sigma}{dx\,dy\,dz\,d^2\boldsymbol{p}_\perp d^2\boldsymbol{k}_\perp d\phi_{l'}} = 2\,K(x,y)J(x,Q^2,\boldsymbol{k}_\perp^2)$$
$$\times\, x\sum_a e_a^2\left[f_{1,a}(x,\boldsymbol{k}_\perp^2)D_{1,a}(z,\boldsymbol{p}_\perp^2) + \lambda\sqrt{1-\varepsilon^2}g_{1L,a}(x,\boldsymbol{k}_\perp^2)D_{1,a}(z,\boldsymbol{p}_\perp^2)\right]$$

Anselmino et al. PRD 71 2005

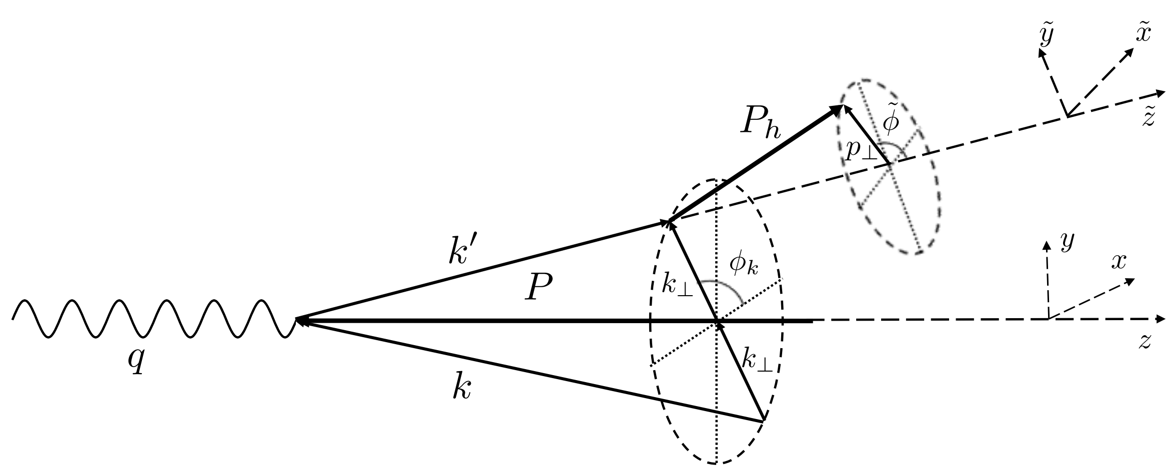


Figure 1. Kinematics of the process. q is the virtual photon, k and k' are the initial and struck quarks, $k_\perp$ is the quark transverse component. P_h is the final hadron with a $p_\perp$ component, transverse with respect to the fragmenting quark k' direction.

Input distributions to MC

In case the dependence is assumed to be a Gaussian, x and z dependent widths are assumed, so that TMDs take the following form,

$$f_1(x, \boldsymbol{k}_\perp^2) = f_1(x) \frac{1}{\langle k_\perp^2(x) \rangle_{f_1}} \exp\left(- \frac{\boldsymbol{k}_\perp^2}{\langle k_\perp^2(x) \rangle_{f_1}} \right) , \tag{3.3}$$

$$g_{1L}(x, \boldsymbol{k}_\perp^2) = g_{1L}(x) \frac{1}{\langle k_\perp^2(x) \rangle_{g_1}} \exp\left(- \frac{\boldsymbol{k}_\perp^2}{\langle k_\perp^2(x) \rangle_{g_1}} \right) , \tag{3.4}$$

$$D_1(z, \boldsymbol{p}_\perp^2) = D_1(z) \frac{1}{\langle p_\perp^2(z) \rangle} \exp\left(- \frac{\boldsymbol{p}_\perp^2}{\langle p_\perp^2(z) \rangle} \right) , \tag{3.5}$$

where $f(x)$ and $D(z)$ are corresponding collinear parton distribution and fragmentation

$$f_1(x, \boldsymbol{k}_\perp^2) = f_1(x) / \left(1 + 20.82\ k_\perp^2 + 126.7\ k_\perp^4 + 1285\ k_\perp^6\right) .$$

Boffi & Pasquini PRD76 2007

We then generate events using the cross section from Eq. (3.2) for both Gaussian and non-Gaussian initial distributions respectively, and we show the resulting transverse momentum distributions in Figs. 2 and 3. Note that the generator we construct is implemented with on mass-shell partons and with four momentum conservation imposed.

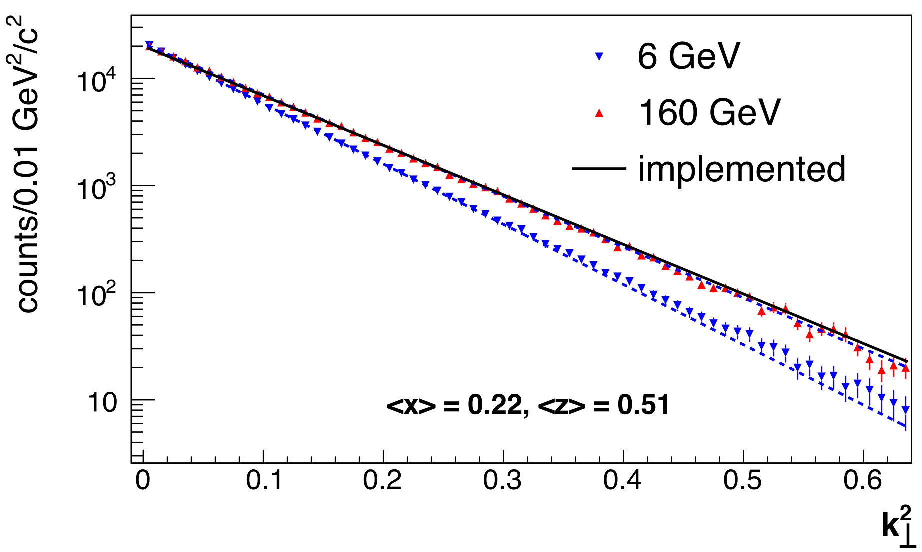


Figure 2. (Color online) The solid line is the Gaussian input distribution implemented using Eq. (3.3), with red triangles coming from the Monte Carlo at 160 GeV initial lepton energy, blue triangles coming from the Monte Carlo at 6 GeV. The dashed line represents the fit to the Monte Carlo distributions which returned values of $C = 0.527$ and $C = 0.444$ at 160 GeV and 6 GeV respectively.

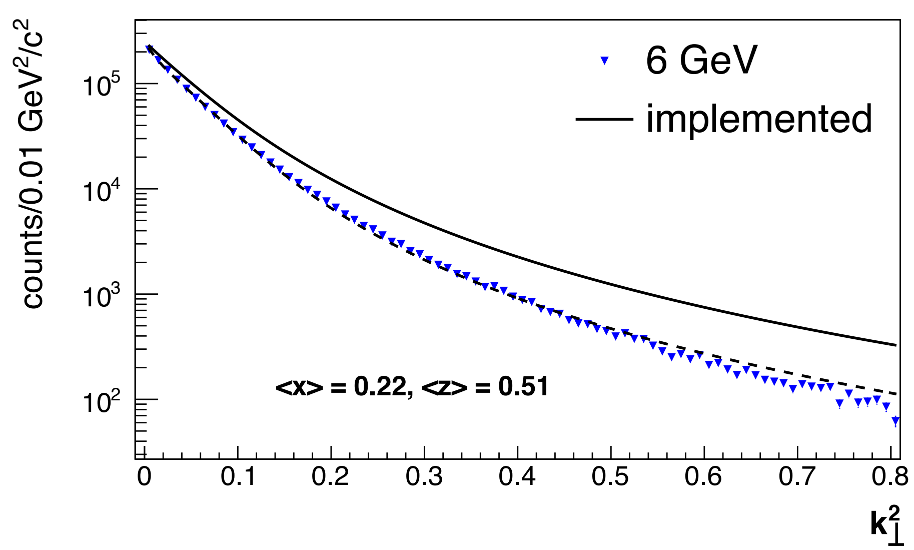


Figure 3. (Color online) The solid line is the implemented non-Gaussian distribution using Eq. (3.6), with $\langle k_\perp^2 \rangle = 0.084\,\mathrm{GeV}^2$, and the dashed curve represents the fit to the Monte Carlo distribution with the value of $\langle k_\perp^2 \rangle = 0.064\,\mathrm{GeV}^2$ at 6 GeV initial lepton beam energy. The available phase space dictated by four momentum conservation results in a deformation of the input distribution.

Project Upol. and Doubly polarized Structure Function

$$\mathcal{F}_{UU,T} = \frac{1}{K(x,y)} \int dP_{h\perp} P_{h\perp} J_0(b_T P_{h\perp}) \left(\frac{d\sigma^+}{d\Phi} + \frac{d\sigma^-}{d\Phi} \right)$$

$$\mathcal{F}_{LL} = \frac{1}{K(x,y)\sqrt{1-\varepsilon^2}} \int dP_{h\perp} P_{h\perp} J_0(b_T P_{h\perp}) \left(\frac{d\sigma^+}{d\Phi} - \frac{d\sigma^-}{d\Phi} \right) ,$$

Now we discretize the momentum integration in Eq. (A.7) and (A.8) for a fixed phase space cell in x, y, z such that the corresponding differential $dx\,dy\,dz$ becomes the bin volume $\Delta x \Delta y \Delta z$. Eqs. (A.7) and (A.8) thus become

$$\begin{aligned}\mathcal{F}_{UU,T} &= x \sum_a e_a^2 \tilde{f}_1(x, z^2 b_T^2) \tilde{D}_1(z, b_T^2) \\ &= \frac{1}{2} \left\{ \frac{1}{\mathcal{N}_0^+} \sum_{i \,\epsilon\, \mathrm{bin}[x,y,z]} \frac{J_0(b_T P_{h\perp\, i}) \Delta n_i^+}{K(x,y)} + \frac{1}{\mathcal{N}_0^-} \sum_{i \,\epsilon\, \mathrm{bin}[x,y,z]} \frac{J_0(b_T P_{h\perp\, i}) \Delta n_i^-}{K(x,y)} \right\} \frac{1}{\Delta x \Delta y \Delta z}\,,\end{aligned} \tag{A.9}$$

and

$$\begin{aligned}\mathcal{F}_{LL} &= x \sum_a e_a^2 \tilde{g}_1(x, b_T^2) \tilde{D}_1(z, b_T^2) \\ &= \frac{1}{2} \left\{ \frac{1}{\mathcal{N}_0^+} \sum_{i \,\epsilon\, \mathrm{bin}[x,y,z]} \frac{J_0(b_T P_{h\perp\, i}) \Delta n_i^+}{K(x,y)\sqrt{1-\varepsilon^2}} - \frac{1}{\mathcal{N}_0^-} \sum_{i \,\epsilon\, \mathrm{bin}[x,y,z]} \frac{J_0(b_T P_{h\perp\, i}) \Delta n_i^-}{K(x,y)\sqrt{1-\varepsilon^2}} \right\} \frac{1}{\Delta x \Delta y \Delta z}.\end{aligned} \tag{A.10}$$

where we sum over the discrete momentum index i, and $\Delta n_i^{\pm}$ are the number of events for polarization $\pm$ as a function of $P_{h\perp i}$.

Substituting Eqs. (A.9) and (A.10) into Eq.(A.4), the experimental procedure to calculate the Bessel weighted asymmetry, $A_{LL}^{J_0(b_T P_{h\perp})}(b_T)$, becomes,

$$\begin{aligned} A_{LL}^{J_0(b_T P_{h\perp})}(b_T) &= \frac{\tilde{\sigma}^+(b_T) - \tilde{\sigma}^-(b_T)}{\tilde{\sigma}^+(b_T) + \tilde{\sigma}^-(b_T)} \\ &= \frac{\sum_{j^+}^{N^+} J_0(b_T P_{h\perp\, j^+}) - \sum_{j^-}^{N^-} J_0(b_T P_{h\perp\, j^-})}{\sum_{j^+}^{N^+} J_0(b_T P_{h\perp\, j^+}) + \sum_{j^-}^{N^-} J_0(b_T P_{h\perp\, j^-})} \\ &\equiv \frac{\tilde{S}^+ - \tilde{S}^-}{\tilde{S}^+ + \tilde{S}^-} \end{aligned} \tag{A.11}$$

where j indicates a sum on events and where $N^{\pm}$ is the number of events with positive/negative products of lepton and nucleon helicities for a given x, y and z, and where $\tilde{S}^{\pm}$ is the sum over events for $\pm$ helicities.

Advantages of Bessel Weighting

"Deconvolution"-CS-struct fncts simple product "$\mathcal{P}$"

$$\frac{d\sigma}{dx_B\, dy\, d\psi\, dz_h\, d\phi_h\, |\boldsymbol{P}_{h\perp}|d|\boldsymbol{P}_{h\perp}|} = \int \frac{d^2\boldsymbol{b}_T}{(2\pi)^2} e^{-i\boldsymbol{b}_T\cdot\boldsymbol{P}_{h\perp}} \left\{ \frac{\alpha^2}{x_B\, y\, Q^2} \frac{y^2}{(1-\varepsilon)} \left(1+\frac{\gamma^2}{2x_B}\right) L_{\mu\nu} \tilde{W}^{\mu\nu} \right\}. \tag{2.11}$$

$$2M\tilde{W}^{\mu\nu} = \sum_a e_a^2 \,\mathrm{Tr}\left(\tilde{\Phi}(x, z\boldsymbol{b}_T)\gamma^\mu \tilde{\Delta}(z, \boldsymbol{b}_T)\gamma^\nu\right) .$$

$$\tilde{\Phi}_{ij}(x, z\boldsymbol{b}_T) \equiv \int d^2\boldsymbol{p}_T\, e^{iz\boldsymbol{b}_T\cdot\boldsymbol{p}_T}\; \Phi_{ij}(x, \boldsymbol{p}_T)$$

$$\tilde{\Delta}_{ij}(z, \boldsymbol{b}_T) \equiv \int d^2\boldsymbol{K}_T\, e^{i\boldsymbol{b}_T\cdot\boldsymbol{K}_T}\; \Delta_{ij}(z, \boldsymbol{K}_T)$$

TMDs in "config" space--Bessel MOMENTS

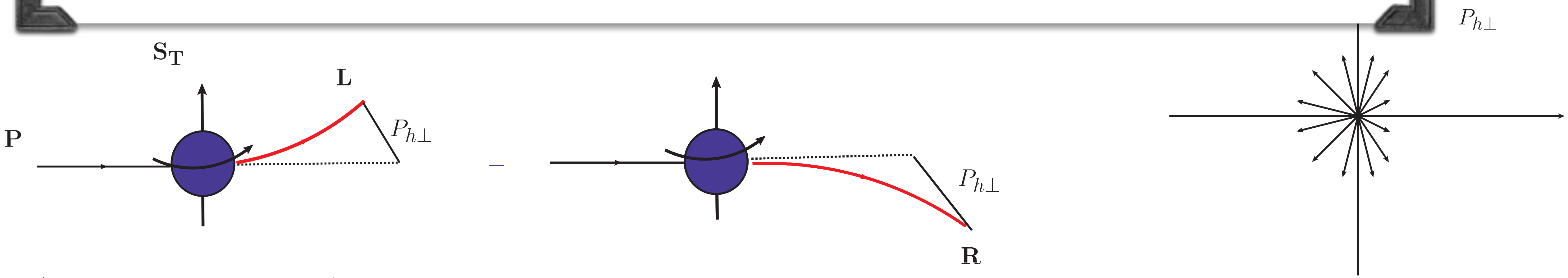
a) F.T. SIDIS cross section w/ following Bessel moments

$$\begin{aligned}
\tilde{f}(x, \boldsymbol{b}_T^2) &\equiv \int d^2\boldsymbol{p}_T \, e^{i\boldsymbol{b}_T\cdot\boldsymbol{p}_T} \; f(x, \boldsymbol{p}_T^2) \\
&= 2\pi \int d|\boldsymbol{p}_T||\boldsymbol{p}_T| \; J_0(|\boldsymbol{b}_T||\boldsymbol{p}_T|) \; f^a(x, \boldsymbol{p}_T^2) \; , \\
\tilde{f}^{(n)}(x, \boldsymbol{b}_T^2) &\equiv n! \left(-\frac{2}{M^2}\partial_{\boldsymbol{b}_T^2}\right)^n \; \tilde{f}(x, \boldsymbol{b}_T^2) \\
&= \frac{2\pi \; n!}{(M^2)^n} \int d|\boldsymbol{p}_T||\boldsymbol{p}_T| \left(\frac{|\boldsymbol{p}_T|}{|\boldsymbol{b}_T|}\right)^n J_n(|\boldsymbol{b}_T||\boldsymbol{p}_T|) \; f(x, \boldsymbol{p}_T^2) \; ,
\end{aligned}$$

b) n.b. connection to $\boldsymbol{p}_T$ moments

$$\tilde{f}^{(n)}(x, 0) = \int d^2\boldsymbol{p}_T \left(\frac{\boldsymbol{p}_T^2}{2M^2}\right)^n f(x, \boldsymbol{p}_T^2) \equiv f^{(n)}(x)$$

What is transverse single spin asymmetry TSSAs



$\sigma^{\downarrow}(x, P_{\perp}) = \sigma^{\uparrow}(x, -P_{\perp})$ Rotational Invariance *“Left-Right” Asymmetry*

$$A_N = \frac{\sigma^{\uparrow}(x, P_{\perp}) - \sigma^{\uparrow}(x, -P_{\perp})}{\sigma^{\uparrow}(x, P_{\perp}) + \sigma^{\uparrow}(x, -P_{\perp})} \equiv \Delta\sigma$$

QCD is Parity Conserving TSSAs Scattering plane transverse to spin

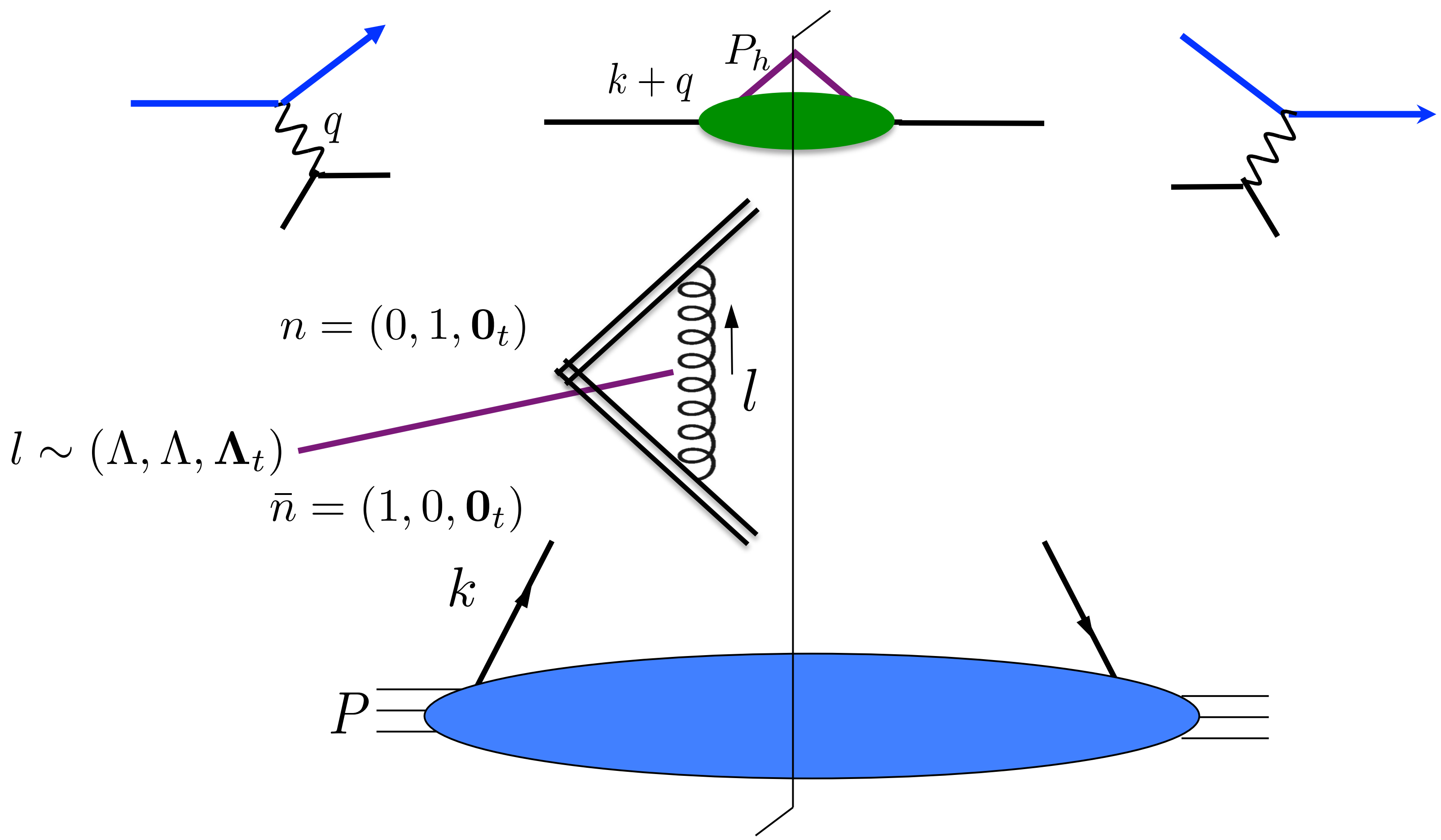
Naively “T-odd”

$$\Delta\sigma \sim i S_T \cdot (\mathbf{P} \times P_{\perp}) \otimes (\text{"}T - odd\text{" QCD} - \text{phases})$$

Spin orbit

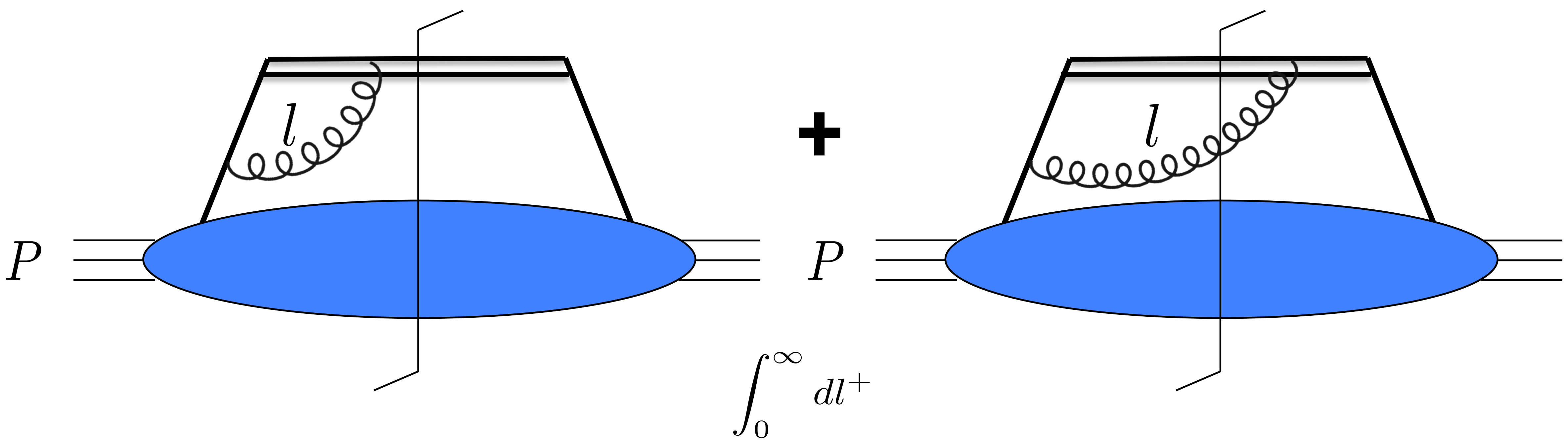
TMD Factorization & treatment of LC/Rapidity divergences
Collins 2011, Aybat & Rogers 2011

Soft Gluons



Power counting "region analysis" soft and collinear factors

Gauge links have light-cone divergences they must cancel or you must regulate



- Divergent contribution at $l^+ = 0$.
- Cancelation in the integral over all l_t.
- What if we don't integrate?